A STUDY OF THE PROJECTIONS ON ELECTRODES AND THEIR EFFECT ON ELECTRICAL BREAKDOWN IN VACUUM

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ERRATA

p. 8  Change first paragraph to read:
"where \( s(y) = v(y) - \frac{1}{2}y \frac{dv(y)}{dy} \). The function \( s(y) \) varies only from 1.0 to 0.833 over the range \( y = 0 \) to \( y = 1 \).
Thus, for measurable values of \( I \) and \( V \), \( s \) is practically a constant and the Fowler-Nordheim plot is essentially a straight line. The slope of this line can be substituted in (2.7) to determine \( B \), if the value of \( \phi \) is known."

p. 15  In line 18, change "face" to "fact."

p. 52  In line 3, change "2MU-2E" to "EMU-2E."
ABSTRACT

Recent studies of electrical breakdown in clean vacuum systems indicate that breakdown is initiated by field emission current and occurs at a value of the electric field which is approximately a constant independent of the electrode geometry and the spacing between the electrodes. It has been postulated that sharp projections on the cathode cause a local enhancement of the electric field and hence almost all of the field emission current is emitted from these projections. Breakdown is believed to occur when a projection is heated to a critical temperature by the field emission currents. Thus, the value of the breakdown field is the field at the tip of such a projection and not the average field at the cathode surface.

The present investigation was undertaken to investigate the characteristics of projections that exist on electrode surfaces and to determine the correlation between the geometry of the projections and the prebreakdown current-voltage characteristics of the electrodes.

Fowler-Nordheim plots of the prebreakdown currents predicted that the projections should be of the order of $10^{-4}$ to $10^{-5}$ cm in diameter and have a length about ten times the diameter. Direct observations of electrode profiles with an electron microscope confirmed that projections of the predicted size and shape did exist on the electrode surface. Electrical breakdown resulted in the disappearance of one or more projections and sometimes caused the formation of new projections.
Under certain conditions large fluctuations in the prebreakdown current were observed. These are believed to be due to changes in the cathode surface caused by ion bombardment. The current fluctuations were accompanied by the formation of fine whisker-like projections on the electrode surfaces. Heating of both electrodes reduced the magnitude of these fluctuations by a considerable amount.

Emission patterns observed on the screen of a modified field emission microscope gave evidence that there were several emitting projections on the cathode rather than one as originally assumed. Calculations of the expected form of a Fowler-Nordheim plot of the combined currents from a group of emitters showed that a linear plot can be obtained and that the slope of the plot is largely determined by the emitters with the larger field enhancement factors.

The modified field emission microscope was also used to investigate a hypothesis which explains the observed reduction in prebreakdown current when gas is admitted to a pressure in the $10^{-4}$ torr region. This explanation is based on the selective sputtering and self-destruction of projections on the cathode by ion bombardment. Observations made with the modified field emission microscope qualitatively confirmed this hypothesis.
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1. INTRODUCTION

The problem of electrical breakdown between electrodes in a vacuum has been studied over a period of at least 50 years. Although vacuum acts as a good insulator between electrodes at low voltages, it is generally found that as the voltage between a given pair of electrodes is increased, there is a more or less reproducible value at which transition to a high-current low-voltage arc takes place. The initiation of the arc is referred to as electrical breakdown, and the value of the voltage at which it occurs is called the breakdown voltage.

In recent years there has been increased interest in this subject because of the increasing requirements for high voltages or high electric fields in evacuated devices such as particle separators, linear accelerators, x-ray tubes, microwave generators, and many others. Despite this large technological interest, or perhaps because of it, there have been many efforts to attain better performance but relatively few efforts to understand the breakdown mechanism. In any case, there have been very few definitive experiments which uniquely identify the processes leading to the initiation of breakdown. A study of the literature (see References 1, 2, and 3 for recent surveys of the literature) reveals that the initiation of breakdown is dependent on a large number of parameters, only some of which can be clearly identified. These include, for example, the electrode material, the distance between the electrodes, the surface roughness, the temperature, the electrode geometry, the gases occluded in the electrodes or adsorbed on the surfaces, etc. In analysing published results, it
often is very difficult to separate out the effects of interrelated parameters. For example, experiments performed by different researchers with the same electrode material often give a greater spread in results than those obtained with a single experimental arrangement using several different electrode materials. Hence, even with such an obvious parameter as the electrode material, it is difficult to correlate differences in breakdown characteristics with variations in a single parameter. Furthermore, it is observed that the characteristics of the arc which is formed are determined not only by the configuration and physical properties of the vacuum-enclosed system but also to a significant degree by the external circuitry and power supply associated with the experiment.

In view of the virtually unlimited variation of the conditions noted above, it is not surprising that the literature of electrical breakdown in vacuum has produced very few theoretical explanations based on first principles. Rather there have been a number of phenomenological descriptions requiring the suitable choice of adjustable parameters to fit the gross features of the experimental observations.

A review of the literature carried out by Alpert and Lee found only one set of experiments in which the initiation of breakdown could be unambiguously associated with a specified physical process. These experiments, by Dyke and his co-workers, were carried out under a very special set of experimental conditions:

(1) The geometry of the electrodes involved a very sharp cathode point with a broad anode at a relatively large distance of separation (the geometry of a field emission microscope).
(2) The material of the cathode, which was the principal electrode, was tungsten, a metal which can be heated in vacuum to achieve atomically clean and hence specifiable surface properties.

(3) Ultrahigh vacuum techniques, which had just been recently developed, were used to maintain known surface and vacuum conditions.

(4) Auxiliary means (the field emission microscope and the electron microscope) were used to examine or verify the nature of the electrode geometry and surface.

While the papers of Dyke et al., written almost a decade ago, were immediately recognized for their definitive quality, the results which they described were so different in a quantitative sense from those which had been observed for broad area electrodes that the results for the different geometries did not seem to be related. For example, Dyke's values for the electric field at breakdown were much higher (one to two orders of magnitude) than those observed for more conventional electrodes whose dimensions were large compared with the gap spacing.

In order to keep the number of experimental parameters to a minimum, Alpert and Lee singled out for special study an experiment reported by Boyle, Kisliuk, and Germer which utilized vacuum and surface conditions commensurate with Dyke, but involved a completely different electrode geometry and gap spacing.

The principal outcome of this study was a phenomenological picture which was applicable to broad area electrodes as well as to point-to-plane geometries, and which provided a single explanation for several heretofore unrelated observations. This picture, which is described in
detail in Section 3, is based on the role of field emission in breakdown. In particular, it involved the assumption put forth by earlier workers that the unexpectedly large field emission typically observed for broad area electrodes originated at submicroscopic points or whiskers which were presumed to form on the electrode surface. Although various features of this picture had been put forth by others, the Alpert and Lee picture tied them together into a single framework to relate in a quantitative manner the observed values of prebreakdown (field emission) currents with observed values of breakdown voltage. As a result, it was possible to predict the breakdown voltage from the measurement of prebreakdown current-voltage relationships.

In a series of experiments which greatly extended the range of observations, Lyman provided strong additional support to the phenomenological picture described above.

The success of this picture suggested and stimulated the present investigation to compare this phenomenological picture with the actual physical situation. As is indicated in Section 4, the over-all aims of the present investigation were (a) to determine whether projections or whiskers of the predicted characteristics do indeed exist on the electrode surfaces; (b) to study their properties and the way in which they affect prebreakdown and breakdown characteristics; and (c) to ascertain, if possible, the processes by which projections are formed.

In order to keep the investigation within reasonable limits and to compare best the results with those experiments mentioned above, it was decided to restrict the study to the use of tungsten electrodes under ultrahigh vacuum conditions.
As is indicated in Sections 4, 5, 6, and 7, the main effort of this work has been to compare the indirect evidence for the properties of electrode projections with a direct observation of the electrodes with an electron microscope. During the course of the experiment certain observations suggested that electrons were emitted from several points on the cathode rather than only one as originally assumed. This stimulated the construction of a modified field emission microscope to investigate the number of emitting points. This tool has proven to be valuable for the study of other related phenomena including the effect of increased gas pressures on the prebreakdown current-voltage characteristics and the formation and destruction of projections on electrodes.
2. THEORETICAL CONSIDERATIONS

2.1 Brief Review of the Theory of Field Emission

The presence of an electric field at a cathode can result in the emission of electrons by the so-called field emission process. In this process, the electrons tunnel through the potential barrier at the surface of the metal rather than going over the barrier as in thermionic or photoelectric emission. A quantum mechanical treatment of this problem was first completed by Fowler and Nordheim who found the functional relationship of the current density, \( J \), to the electric field, \( F \), to be of the form

\[
J = \frac{C^2}{\varphi} \exp\left[-\frac{D\varphi^2}{F}\right]
\]

(2.1)

where \( \varphi \) is the work function of the metal. This was later revised by Nordheim to include the effect of the image potential. Burgess, et al., carried out a more accurate treatment of the image potential and arrived at the following equation:

\[
J = \frac{C^2}{\varphi t(y)} \exp\left[-\frac{D\varphi^2}{F}\right] v(y)/F
\]

(2.2)

where \( C \) and \( D \) are constants and \( v \) and \( t \) are elliptic functions of the variable \( y \). If \( \varphi \) is expressed in electron volts and \( F \) in volts/cm, the values for \( C \), \( D \), and \( y \) are:

\[
C = 1.54 \times 10^{-6}
\]

\[
D = 6.83 \times 10^7
\]

\[
y = 3.79 \times 10^{-4} F^{1/2}/\varphi
\]

(2.3)
Experimentally, the quantities which are measured are the current, $I$, and the voltage, $V$, which are related to the current density and the electric field, respectively, by the equations

\[ I = JA \]  
\[ V = BF \] \hspace{1cm} (2.4) \hspace{1cm} (2.5)

where $A$ is the electrode area and $B$ is a constant determined by the geometry of the electrode. If (2.4) and (2.5) are substituted in (2.2) and, after dividing by $V$, the logarithm of each side is taken, the result is

\[ \log \frac{I}{V} = \log \left[ \frac{AC}{\varphi t (y)^B} \right] - BD\varphi^2 \frac{s(y)}{V} \]  
\hspace{1cm} (2.6)

Assuming for the moment that $t(y)$ and $v(y)$ are constants, it is evident that a plot of $\log \frac{I}{V^2}$ versus $1/V$ should be a straight line. Such a plot is called a Fowler-Nordheim plot. The linearity of a Fowler-Nordheim plot is an indication that the measured current is due to field emission and not some other process.

The elliptic functions $t(y)$ and $v(y)$ are actually not constants. However, the function $t(y)$ varies only from 1.0 to 1.1 as $y$ varies from 0 to 1.* (At $y=1$, the value of the electric field is such that the height of the potential barrier at the surface of the metal is depressed to the Fermi level of the metal.) The function $v(y)$, on the other hand, varies from 1 to 0* for the same range of $y$. However, it has been shown by Houston 16 that the slope of the Fowler-Nordheim plot is given by

\[ \text{slope} = \frac{d \left[ \log \frac{I}{V^2} \right]}{d(1/V)} = -BD\varphi^2 s(y) \] \hspace{1cm} (2.7)

---

* A table of values for $t$, $v$, and $s$ is given in Reference 15.
where \( s(y) = v(y) - \frac{1}{y} \frac{dv(y)}{dy} \). Although \( s \) becomes infinite at \( y = 1 \), in the range \( y = 0 \) to \( y = 0.95 \), \( s \) varies between 0.96 and 1.35. Since it is found experimentally that breakdown usually occurs before \( y \) reaches 0.95, \( s \) is practically a constant for measurable values of \( I \) and \( V \), and the Fowler-Nordheim plot is essentially a straight line. The slope of this line can be substituted in (2.7) to determine \( B \), if the value of \( \varphi \) is known.

2.2 Field Enhancement due to Projections on Electrodes

The presence of a sharp point or whisker on an electrode can cause an enhancement of the electric field at that point. For analytical purposes let us consider a whisker whose shape is a prolate hemispheroid with height \( c \) and base radius \( b \). If the whisker is small with respect to the electrode surface on which it is situated, the potential around the whisker, as derived by Smythe, is

\[
V = F_0 z \left[ 1 - \frac{\coth^{-1} \eta_0^{-1}/\eta}{\coth^{-1} \eta_0^{-1}/\eta_0} \right] \tag{2.8}
\]

where \( \eta_0 = \frac{c}{c_2} = \frac{c}{(c - b) \frac{1}{2}} \) and \( F_0 \) is the field at the surface of the electrode in the absence of the whisker. For brevity in writing, let \( K = \coth^{-1} \eta_0^{-1}/\eta_0 \). The prolate spheroidal coordinates \( \eta \) and \( \xi \) are related to the cylindrical coordinates \( z \) and \( \varphi \) by the expressions

\[
z = c_2 \eta \xi \tag{2.9}
\]

\[
\varphi = c_2 \left[ (1 - \xi^2)(\eta^2 - 1) \right]^{1/2} \tag{2.10}
\]

* A table of values for \( t, v, \) and \( s \) is given in Reference 15.

** A similar treatment has been carried out by T. J. Lewis.
The electric field is found by taking the gradient of the potential which, in generalized coordinates, is expressed by:

\[
\nabla V = \nabla_1 \frac{1}{h_1} \frac{\partial V}{\partial u_1} + \nabla_2 \frac{1}{h_2} \frac{\partial V}{\partial u_2} + \nabla_3 \frac{1}{h_3} \frac{\partial V}{\partial u_3}.
\]

For the case of prolate spheroidal coordinates,

\[
\begin{align*}
  u_1 &= \xi \\
  u_2 &= \eta \\
  u_3 &= \varphi
\end{align*}
\]

\[
\begin{align*}
  h_1 &= c_2 \left( \frac{\eta^2 - \xi^2}{1 - \xi^2} \right)^{1/2} \\
  h_2 &= c_2 \left( \frac{\eta^2 - \xi^2}{\eta^2 - 1} \right)^{1/2} \\
  h_3 &= \rho
\end{align*}
\]

At the surface of the spheroid the field must be normal to the surface, i.e., in the \(\eta\)-direction. Thus

\[
F_{\eta} = \frac{1}{h_1} \frac{\partial V}{\partial \eta} = F_o \xi \left[ (\frac{\eta^2 - 1}{\eta_0^2 - \xi^2})^{1/2} \right. \\
\left. \left( 1 - \coth^{-1} \frac{1}{K} \right) + \frac{1}{K \eta (\eta^2 - 1)^{1/2} (\eta - \xi)^{1/2}} \right].
\]

On the surface of the spheroid, \(\eta = \eta_0\), so the field becomes

\[
F_{\eta} \bigg|_{\eta = \eta_0} = F_o \xi \frac{1}{K \eta_0 [\eta_0^2 - 1)(\eta_0 - \xi)]^{1/2}}.
\]

In the preceding section the enhancement factor was defined as \(\beta = F_A / F_o\). Evaluating (2.12) at the tip of the whisker (\(\xi = 1\)) where the field is a maximum, and taking this as the value of \(F_A\), gives the following expression for \(\beta\):

\[
\beta = \frac{1}{K} \frac{1}{\eta_0 (\eta_0^2 - 1)}.
\]
Since $K$ is a function of $\eta_o$ and $\eta_o = (1 - \frac{b}{c})^\frac{1}{2}$, $\beta$ is just a function of $b/c$, i.e., a function of the ratio of the whisker base radius to its height. A plot of $\beta$ versus $b/c$ is shown in Figure 1.

The relationship between the voltage, $V$, and the average field, $F_o$, is $V = BF_o$, where, as in equation (2.5), $B$ is a geometrical factor which can usually be calculated for a given electrode geometry. The actual field, $F_a$, at the tip of a whisker is just

$$F_a = \beta F_o = \beta \frac{V}{B}. \quad (2.14)$$

If field emission currents from a single whisker on the cathode are measured and plotted in the form of a Fowler-Nordheim plot, the value of $\beta$ can be determined from the slope of the plot. This can be seen by rewriting (2.7), which is the equation of the slope of a Fowler-Nordheim plot, with the factor $B$ replaced by $\frac{B}{\beta}$. The result is

$$\text{slope} = -\frac{B}{\beta} D \frac{3}{2} \Phi_s(y). \quad (2.15)$$

Thus, from the measured value of the slope, the value of $\beta$ can be calculated if the value of $\Phi$ is known. Once $\beta$ is calculated, and knowing the current and voltage at a given point on the Fowler-Nordheim plot, the following additional quantities can be calculated:

1. the value of the actual field, $F_a = \beta \frac{V}{B}$
2. the current density, $J$, corresponding to $F_a$
3. the value of the emitting area, $A = \frac{I}{J}$
4. the value of $b/c$, calculated from (2.13).

The magnitude of the emitting area together with the $b/c$ ratio provide an approximation to the size and shape of the emitting whisker.
Figure 1. The variation of the enhancement factor, \( \beta \), with \( b/c \).
It should be realized that several assumptions have been made in deriving the above information. One assumption is that all the factors, except \( \beta \), in (2.15) are constants. As discussed in the preceding section, \( s(y) \) is not a constant but its variation is only about 50 per cent. This error can be reduced by iterative methods. There can also be variations in the work function, \( \varphi \), due to different crystallographic directions present in the whisker tip. This variation is at most 50 per cent. Experimentally the values found for \( \beta \) are usually greater than 10 and thus the variations in \( \varphi \) and \( s \) are small effects.

Another assumption is that the effective field is constant over the emitting area of the tip. This is certainly not true, since, as shown by equation (2.12), the field is a function of \( \xi \). The measured current is a sum of currents emitted from various parts of the tip, each of which experiences a different field. Thus, the calculated field and current density are average values for the effective emitting area.

The accuracy of the calculated values are also limited by the degree of fit between the actual shape of the whisker and the assumed prolate spheroidal shape. Considering all the possible errors introduced by the assumptions listed above, one would estimate the accuracy of the calculated \( \beta \) value to be only within an order of magnitude. However, even this degree of accuracy provides considerable information about the whisker since the \( \beta \)-factor varies so rapidly with b/c in the region of experimental interest (\( \beta > 10 \)).
3. PHENOMENOLOGICAL PICTURE OF BREAKDOWN INITIATED BY FIELD EMISSION CURRENTS

As mentioned in Section 1, Alpert and Lee, after reviewing the literature, developed a picture of breakdown initiation based on the field emission heating of sharp points on the cathode. The two main experimental works from which this picture was developed are the experiments of Dyke et al., and the experiments of Boyle, Kisliuk, and Germer.

Dyke and his co-workers used a field emission microscope which had essentially a point-to-plane geometry. The point was the emitter (cathode) which was formed by etching the end of a tungsten wire to a radius of about $10^{-5}$ cm. The emitter was examined with an electron microscope to accurately determine the shape of its tip. Knowledge of the shape of the tip made it possible to calculate the relationship between the voltage applied to the field emission microscope and the electric field at the emitter. Knowing this relationship together with the area of the tip, and using an average value (4.5 eV) for the work function of tungsten as found in the literature, Dyke was able to compute values for the field emission current as predicted by the Fowler-Nordheim equation. He was also able to measure field emission currents from the emitter under ultrahigh vacuum conditions after the emitter had been cleaned by heating. At the same time he could observe the emission patterns on the screen of the field emission microscope and thus determine that the tip remained clean during the measurements. The measured values of current obtained in this manner were in excellent agreement with the theoretical values.
Dyke also observed that at a critical current density, electrical breakdown occurred which destroyed the tip of the emitter. From these observations he proposed a theory for the initiation of breakdown based on the heating of the emitter by $I^2R$ losses. According to this theory breakdown occurred when the temperature of the emitter tip reached a critical value (approximately equal to the melting point). For a tungsten emitter, the value of the current density at the breakdown point was between $10^7$ and $10^8$ amp/cm$^2$. It is of interest to note that tungsten wires have been observed to explode at current densities of this magnitude.

In the experiment performed by Boyle, Kisliuk, and Germer (hereafter referred to as B.K.G.), prebreakdown currents and breakdown voltages for small gaps ($10^{-4}$ to $10^{-3}$ cm) between crossed tungsten wires were measured. This experiment was also performed under ultrahigh vacuum conditions and with clean electrodes. B.K.G. found that the prebreakdown currents obeyed the general form of the Fowler-Nordheim equation but that the apparent fields at the cathode were about an order of magnitude too small to give the observed values of current. This effect was attributed to sharp projections or whiskers on the cathode where the actual field $F_A$ was much higher than the average field $F = V/d$. They defined an enhancement factor $\beta$ by the equation

$$F_A = \beta F$$

and were able to determine the value of $\beta$ at various gap spacings. Their explanation of breakdown initiation was based on anode heating due to field-emitted electrons from the cathode.

It might be mentioned in passing that as early as 1936 Ahearn measured prebreakdown currents which obeyed the form of the Fowler-Nordheim equation.
equation and that he also noticed field enhancement effects which he attributed to sharp projections on the cathode. He suggested that breakdown was due to the rupture of one of these projections under the combined effects of \( I^2 R \) heating and the mechanical force exerted by the electric field.

The results of Dyke and of B.K.G. have been analysed by Alpert and Lee. Assuming that the field emission current originated at a whisker on the cathode, as proposed by B.K.G., Alpert and Lee used the data of B.K.G. to calculate the actual breakdown field to be approximately constant at \( 7 \times 10^7 \) v/cm. This value of the field corresponds to a current density of about \( 3 \times 10^7 \) amp/cm² which is in the same range as that observed by Dyke. Thus, in both experiments the actual breakdown field at the cathode emitting surface was approximately a constant.

In the Dyke experiment the anode was at such a relatively large distance (~5 cm) from the cathode that the transit time of an ion (with the possible exception of hydrogen) was longer than the arc formation time (< \( 5 \times 10^{-8} \) sec). This seemed to rule out any effects arising from ions produced at the anode. In fact, when the voltage was applied in pulse lengths which were shorter than the ion transit time, no change was observed in the breakdown characteristics. Furthermore, Dyke performed a mathematical analysis which showed that the experimentally measured current densities were sufficient to raise the temperature of the emitter to a value near the melting point in the observed time.

In view of Dyke's convincing argument in favor of a cathode process for breakdown initiation, combined with the calculations showing that breakdown occurred at about the same value of the electric field in both
the Dyke experiment and the B.K.G. experiment, Alpert and Lee proposed the following phenomenological picture for the initiation of breakdown in the case of broad area electrodes. The local electric field at the cathode is enhanced by the presence of a sharp projection or whisker. The projection is resistively heated by field emission currents drawn from it causing breakdown to occur at a critical current density. Hence, breakdown occurs at a constant field, this field being the actual field at the whisker tip.

Lyman\textsuperscript{11} has recently obtained results in a different range of gap spacings which agree with the model proposed by Alpert and Lee. Working with flat tungsten electrodes about one inch in diameter and with gaps ranging from about $3 \times 10^{-3}$ to $3 \times 10^{-1}$ cm, Lyman has measured prebreakdown currents as well as breakdown voltages under ultrahigh vacuum conditions. By making Fowler-Nordheim plots from his data he was able, by the use of equation (2.7), to obtain the relationship between the applied voltage and the actual field at the emitting area. He thus was able to calculate the actual fields at breakdown which he found to be about $7 \times 10^{7}$ v/cm, in agreement with the Dyke and B.K.G. values. Figure 2 is a plot of breakdown field versus gap spacing showing the data of Dyke, B.K.G., and Lyman. It should also be mentioned that Lyman observed enhancement effects with $\beta$ values ranging from 60 to 220. He calculated the emitting areas to be of the order of $10^{-10}$ cm$^2$. On the basis of the model used in the field enhancement calculations in Section 2.2, these $\beta$ and area values correspond to an electrode projection having a diameter of about $10^{-5}$ cm and a height about 10 times its diameter.
Figure 2. Measured breakdown fields versus gap spacings.
Although there is considerable experimental evidence to support the criterion of a critical breakdown field, there are some details of the breakdown process which are not understood at this time. For example, both Dyke and B.K.G. reported that the formation of an arc was accompanied by an increase in the current of about two orders of magnitude in a time which was less than $5 \times 10^{-8}$ seconds. Both explained this increase as being due to the presence of ions in the vicinity of the cathode. Dyke proposed that the ions, which were thought to have been produced from evaporated cathode material, neutralized the space charge at the cathode. In the B.K.G. case it was proposed that atoms were evaporated from the anode, that these atoms were then ionized, and that the ions in moving to the cathode changed the field at the cathode in such a manner as to enhance the electron emission. In either case it has not been shown in a quantitative manner that ions can produce the observed increase in current.

If the mechanism of resistive heating by field emission current is applied to the case of a projection on a broad area electrode (see Appendix A), one finds that for simple geometries the temperature at the tip of the projection depends on the size and shape of the projection as well as on the current density. Hence, one would expect that there should be a variation in the observed current density at breakdown since it would be surprising to find that the projections on the cathode all had the same size and shape. Actually, there is some spread in the observed values of the breakdown field. Lyman's data (see Figure 2), for example, has values ranging from about $5.5 \times 10^7$ to $1.1 \times 10^8$ v/cm. This corresponds to a variation in current density by about three orders of magnitude, which may be enough to account for the variations that occur in the geometry of the projections.
The B.K.G. hypothesis of breakdown states that breakdown occurs when the power density at the anode reaches a critical value. A calculation of the power generated at the anode by the impinging electron beam, using the results of Vibrans to determine the spreading of the beam, indicates that the maximum power density at the anode occurs at a cathode current density of about $10^4$ amp/cm$^2$, which is several orders of magnitude lower than the observed values at breakdown (see Appendix A). Thus, on the basis of these calculations, the anode heating process does not seem to explain the observed results as well as the cathode heating process.

Whatever the actual processes of breakdown are, it is evident from the experimental results that field emission plays a very important role. In fact, on the basis of the phenomenological picture presented by Alpert and Lee, it is possible to predict the breakdown voltage without subjecting the electrodes to an actual breakdown. By making a Fowler-Nordheim plot from the measured prebreakdown (field emission) current, one can determine the relationship between the applied voltage and the actual field at the emitting point. Knowing this relationship, one can calculate the approximate breakdown voltage using the average value of the breakdown field ($7 \times 10^7$ v/cm for tungsten electrodes).
4. EXPERIMENTAL APPROACH

The phenomenological picture for the initiation of breakdown which was presented in Section 3 was based on the hypothesis that sharp projections on electrodes play an important role in the breakdown process. The aims of the present investigation were as follows:

(1) To determine whether or not sharp projections or whiskers actually exist on electrodes.

(2) If they do exist, to determine the size, shape, and other characteristics of these projections and the manner in which they affect the prebreakdown current-voltage characteristics of the electrodes and the breakdown voltage.

(3) To develop methods for investigating the formation and destruction of projections.

Three different methods were used to investigate these questions:

(1) The measurement of the current-voltage characteristics of small gaps between electrodes under clean vacuum conditions.

(2) The direct observation of the profile of an electrode by means of an electron microscope.

(3) The observation of the emission patterns from a cathode as obtained on the screen of a modified field emission microscope.

As indicated in Section 3, the expected size of the electrode projections is of the order of $10^{-5}$ cm which cannot be observed with an optical microscope and thus requires the use of an electron microscope. In order to make direct observations of the cathode profile, the cathode had
to be small enough to fit inside the electron microscope. It was also necessary to limit the cathode area to a size such that the electron microscope observations could be made in a reasonable length of time. It was found that the tip of a .003-inch diameter tungsten wire was a convenient size for the cathode. As it turned out, this small area emphasized an effect which, to the author's knowledge, has not been reported previously. This effect is due to current fluctuations which will be discussed in a later section ("Unstable Current Region").

Tungsten was chosen because it is a refractory metal and thus can be cleaned by heating in vacuum. This choice of material also made it possible to compare results with those of other experimentalists such as Dyke, B.K.G., and Lyman.

The first method used in this investigation of electrode projections was the measurement of prebreakdown currents and breakdown voltages. As described in Section 2.2, a Fowler-Nordheim plot of the prebreakdown current makes it possible to calculate the value of the enhancement factor (for the case of plane parallel electrodes) and the area of the emitting point. It is also possible to calculate the value of the actual electric field at breakdown and compare this with the value of $7 \times 10^7$ V/cm predicted by the phenomenological model.

The electron microscope observations made possible the detection of protuberances on an electrode and the measurement of the size and shape of the protuberances. The measured size and shape of a protuberance could be compared with those predicted on the basis of a Fowler-Nordheim plot of the prebreakdown current.
During the course of the experiment, data was obtained which suggested that the prebreakdown currents might be coming from several points rather than one as it was initially assumed. The modified field emission microscope was then devised as a means of investigating the number of emitting projections. This instrument was later found to be useful also for the study of the creation and destruction of points, and for investigating the effects of increased gas pressure on the emission characteristics of the cathode.
5. CURRENT-VOLTAGE MEASUREMENTS FOR SMALL GAPS

5.1 Vacuum System

The ultrahigh vacuum system used in connection with the current-voltage measurement is shown schematically in Figure 3. Figure 4 is a photograph of the system. The diffusion pump and fore pump were used for the initial evacuation of the system and during the subsequent bakeout at 400°C for about 12 hours with all the valves open. When the temperature dropped to 200°C after bakeout the ovens were removed, the getter-ion pump was turned on, and valves 1 and 3 were immediately closed. This was done to reduce the possibility of contamination of the electrodes by oil vapor. After outgassing the Bayard-Alpert gauge and allowing the system to cool to room temperature, pressures in the low $10^{-10}$ torr region were obtained.

To ensure rapid removal of any gases generated in the electrode envelope, the getter-ion pump was connected to the envelope with one-inch glass tubulation through valve 2 which had an open conductance of about 10 liters/second. Valve 2 was closed whenever the electrode envelope was opened for changes, allowing the getter-ion pump to remain evacuated and thereby eliminating the "hard-starting" problems often found when a getter-ion pump is opened to air. Valves 3, 4, and 5 permitted the introduction of gas into the envelope at a controlled pressure.

The electrode envelope consisted of a glass cylinder three inches in diameter and about six inches long with a metal flange on each end. Demountable flanges which mated with the envelope flanges were used for supporting and positioning the electrodes. Vacuum seals at the flanges were made with gold rings.
Figure 3. Schematic of vacuum system.
Figure 4. Photograph of vacuum system.
5.2 Electrodes

The electrode whose profile was to be studied was mounted on an insert which could be removed from the electrode envelope and placed in a holder for viewing in the electron microscope. A photograph of the electrode and insert is shown in Figure 5. The insert was a slotted hollow molybdenum cylinder with a shoulder at one end. Tungsten pins, .015 inch in diameter, were beaded with 7050 glass and placed in the holes in the shoulder. The insert was then heated in an argon atmosphere until the glass flowed and bonded to the insert. The pins were thus rigidly mounted in the insert but electrically insulated from it. A hairpin filament of .008-inch diameter tungsten wire was spotwelded to the pins. A short length of .003-inch diameter tungsten wire was then spotwelded to the filament such that the free end of this wire was centered on the axis of the cylinder. The tip of this wire formed one of the electrodes (hereafter referred to as the tip electrode) and was normally used as the cathode.

Initially, the flange which was used for supporting the tip electrode was a solid metal flange having two electrical feed-throughs brazed into it. The support arm for the insert was fitted into a collar on the flange and the electrical connections to the tip electrode were made to the feed-throughs. The electrical isolation of the tip electrode from the flange served two purposes. The first purpose was to make possible the heating of the tip electrode by passing current through the filament. By this means the tip electrode could be outgassed and smoothed to some extent. The smoothing effect will be discussed in Section 6. The second purpose for the electrical isolation was to enable the flange to be used
Figure 5. Photograph of tip electrode mounted on the insert.
as a guard ring in order to eliminate leakage currents from the field emission current measurements.

After a period of operation during which the tip electrode was repeatedly heated, it was found that the resistance between the tip electrode and the insert decreased from its original value of several megohms to less than $10^5$ ohms. This value of resistance limited the minimum value of field emission current which could be measured, as will be described later. To eliminate this problem, the original flange was replaced with a flange which contained a three-wire glass press. The support arm was then attached to one of these wires and the electrical connections were made to the other two wires. The flange still served as a guard ring for leakage currents along the glass envelope. The leakage resistance between the tip electrode and the flange with this arrangement was greater than $10^8$ ohms.

The opposing electrode, normally operated as the anode, was the electropolished end of a .100-inch diameter tungsten rod which was brazed into a larger stainless steel rod. This unit was brazed to a collar which was attached to a flange through a monel diaphragm. A driver mechanism used with the diaphragm permitted adjustment of the gap spacing between the electrodes. A dial indicator measured the movement of this electrode to an accuracy of $\pm .00025$ inch. Figure 6 is a photograph showing a close-up view of the electrodes. For gap spacings which were smaller than the tip electrode diameter (.003 inch), the electrode geometry was approximately a parallel plane arrangement.
Figure 6. Close-up view of electrodes.
5.3 Instruments and Circuitry

A schematic of the instruments and circuitry used in measuring the current-voltage characteristics is shown in Figure 7. The output of the high voltage supply (PRL Electronics, model CP-1290-HV) was continuously variable from 0 to 15 kv d.c. and of reversible polarity. Resistors $R_3$ and $R_4$ ($4.7 \times 10^5$ ohms each) limited the current flow during breakdown. Capacitor $C_1$ (.025 μf) was used for additional filtering of the high voltage.

Field emission currents were determined by measuring the voltage drop across resistance $R_L$. This resistance consisted of the leakage resistance between the tip electrode and the flange in parallel with the input resistance of the voltmeter $V_2$ (Millivac, Type MV-17C). The value of $R_L$ was typically about $10^6$ ohms and the lower limit of the voltmeter was about $10^{-4}$ volt so that the smallest measurable emission current was about $10^{-10}$ amp.

The voltage applied to the electrodes was determined by the branch circuit containing resistors $R_1$ and $R_2$, the null detector, and the battery. The resistance of $R_2$ was $10^8$ ohms while $R_1$ was a $10^5$ ohm, 10 turn helipot. The null detector was an experimental d.c. differential voltmeter with a 10 mv full scale deflection on its most sensitive range. With $R_1$ adjusted to produce a null indication, the voltage $V_1$ across the electrodes was given approximately by the expression

$$V_1 = I(R_1 + R_2) \approx \frac{1.5}{R_{1a}} \times R_2,$$

where $R_{1a}$ was the resistance between the helipot tap and ground as indicated by the helipot dial.
Figure 7. Schematic of circuitry for current-voltage measurements.
The accuracy to which the total applied voltage could be measured was limited by the accuracy to which the battery voltage and $R_2$ could be measured. This accuracy was about one per cent. However, changes in the applied voltage could be measured with an accuracy of about 0.1 per cent by means of the helipot. This was useful in determining the linearity of the Fowler-Nordheim plots.

5.4 Experimental Procedure

After completion of a 400°C bakeout and outgassing of the Bayard-Alpert gauge, the system typically reached a pressure in the range $8 \times 10^{-11}$ to $3 \times 10^{-10}$ torr. The tip electrode was then outgassed and the gap setting adjusted for a measurement of the current-voltage characteristics.

To clean the tip electrode, the following heating procedure was used. The tip was brought to a temperature of about 1300°C for a period of 30 minutes for outgassing purposes and then the temperature was raised to about 1600°C for one minute to smooth the tip. Following this, the temperature would be slowly decreased, allowing the pressure to drop into the $10^{-9}$ torr region before cooling the tip completely, thereby keeping the tip relatively free of adsorbed gases. After a cooling period of about 20 minutes the anode was advanced until it touched the tip electrode as indicated by an ohmmeter. This was taken as the zero gap position and the dial indicator was set accordingly. The anode was then backed off and the tip electrode reheated to about 1600°C for one minute to smooth out the spikes caused by touching the electrodes together (see Section 6). The anode was again advanced to touch the tip electrode to check the zero gap position since the tip electrode sometimes shifted position when heated. After several heatings the tip electrode position would generally
stabilize to within ± .00025 inch. After a final heating of the tip electrode the gap was set to the desired position, usually between .001 and .003 inch. Once the gap position had been set, the high voltage was applied to the electrodes, with the tip electrode as the cathode, and the prebreakdown current was measured as a function of the applied voltage.

In some cases it was desirable to observe the projections on the tip electrode in order to compare their characteristics with those predicted on the basis of the Fowler-Nordheim plot. In such cases the run was stopped before a breakdown occurred, and the tip electrode was removed from the electrode envelope in order to examine it in the electron microscope. In other cases the run was continued until a breakdown did occur. The tip electrode was then examined with the electron microscope to determine what changes in the electrode profile has been produced by the breakdown.

When the tip electrode was reassembled in the electrode envelope, it was sometimes advantageous to retain an indication of the gap position. It was then necessary to leave in place the gap positioning assembly, which was not bakeable. To obtain a modified bakeout the electrode envelope was partially wrapped with aluminum foil, leaving one side open to the radiation from a heat lamp. With this method a temperature near 150°C could be obtained. After bakeout the pressure was usually in the low 10^{-8} torr region. The tip electrode was then heated to about 1300°C to outgas it. Following the outgassing of the tip electrode, the gap was set to the desired position, assuming that the tip electrode assembly was remounted in approximately the same position as it was before removal.
5.5 Discussion of Results

In the measurement of the current-voltage characteristics two different types of behavior were observed. These can best be characterized as the "stable current region" and the "unstable current region." In the stable region, which occurred at the lower current ranges, the current behaved as predicted by the Fowler-Nordheim equation. As the voltage was increased toward the breakdown value there was a transition to the second type of behavior in which large current fluctuations occurred but sparks or other visible evidences of breakdown were not observed. Continued increase of the voltage would eventually result in a visible breakdown. It was later found that the current fluctuations could be reduced and sometimes eliminated by heating the anode. In a few runs anomalous Fowler-Nordheim plots were obtained. The anomaly consisted of an abrupt change in the slope of the plot. These characteristics will now be discussed in detail.

5.5a Stable Current Region

The currents in this region were relatively stable and reproducible and obeyed the general form of the Fowler-Nordheim equation. The measured currents were plotted in the form of Fowler-Nordheim plots. In most of the runs the plots came out to be straight lines. Figure 8 shows a typical plot of this kind. By assuming the geometry of the electrodes to be approximately equivalent to parallel plates, values of $\beta$ could be computed from equation (2.15) using a value of 4.5 eV for $\varphi$ and taking $B = d$, the gap spacing. Table I lists the $\beta$ values obtained by this method. With tip electrodes that had been partially smoothed by heating, $\beta$ values ranging from 18 to 52 were calculated. In two cases where the tip electrode had
Figure 8. Fowler-Nordheim plot of currents measured in the stable region.
Table I. β and area values calculated from Fowler-Nordheim plots of the prebreakdown currents.

<table>
<thead>
<tr>
<th>Run</th>
<th>β</th>
<th>Area (cm²)</th>
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<tr>
<td>1</td>
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<tr>
<td>2</td>
<td>106</td>
<td>$7.3 \times 10^{-14}$</td>
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</tr>
<tr>
<td>4</td>
<td>38</td>
<td>$3.3 \times 10^{-13}$</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
<td>$2.0 \times 10^{-10}$</td>
</tr>
<tr>
<td>6</td>
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</tr>
<tr>
<td>7</td>
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<td>$1.4 \times 10^{-10}$</td>
</tr>
<tr>
<td>8</td>
<td>38</td>
<td>$6.3 \times 10^{-12}$</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
<td>$1.5 \times 10^{-11}$</td>
</tr>
<tr>
<td>10</td>
<td>47</td>
<td>$2.1 \times 10^{-11}$</td>
</tr>
<tr>
<td>11</td>
<td>30</td>
<td>$2.3 \times 10^{-13}$</td>
</tr>
<tr>
<td>12</td>
<td>52</td>
<td>$1.0 \times 10^{-13}$</td>
</tr>
<tr>
<td>13</td>
<td>39</td>
<td>$1.1 \times 10^{-14}$</td>
</tr>
<tr>
<td>14</td>
<td>18</td>
<td>$1.5 \times 10^{-12}$</td>
</tr>
</tbody>
</table>
been extremely roughened due to breakdown or other treatment, the values were considerably higher (133 and 106) as would be expected if there were sharper points on the cathode. Area values were also calculated and are listed in Table I. These ranged from $10^{-14}$ to $10^{-10}$ cm$^2$.

5.5b Unstable Current Region

As the voltage was increased toward the breakdown value, a range of values was found in which the field emission current became unstable. This region was characterized by large fluctuations in the d.c. current as indicated by meter $V_2$ (Figure 7), but no visible evidence of breakdown was observed. An oscilloscope connected in parallel with meter $V_2$ showed an a.c. component of the current consisting of flat-topped pulses ranging from $10^{-4}$ to $10^{-5}$ sec in length (see Figure 9). Mixed in with these pulses were longer pulses or steps which appeared as d.c. fluctuations on meter $V_2$. The magnitude of these pulses ranged from 10 to 50 per cent of the d.c. component. A more careful investigation revealed that pulses sometimes occurred at lower values of the voltage in the so-called stable current region, but the magnitude of these pulses was usually only about one per cent of the d.c. level. As far as could be determined, the pulse shapes were not determined by the external circuitry.

In view of the large current fluctuations, attempts to draw a Fowler-Nordheim plot were meaningless in this region of unstable currents. Figure 10 is a Fowler-Nordheim plot illustrating this effect. The linear portion of the plot was obtained in the stable current region. The points in the unstable region are based on average values. Because of the large scatter of points in the unstable region, the slope of the Fowler-Nordheim plot is undefined and it is not possible to calculate values of the actual field.
Figure 9. Tracings of oscilloscope pictures showing typical current pulses observed in the unstable current region.
Figure 10. Fowler-Nordheim plot of currents measured in the stable and unstable regions.
In some cases the transition from the stable to the unstable region was abrupt; in others, more gradual. The actual fields at which the Fowler-Nordheim plot departed noticeably from linearity were calculated and are listed in Table II. The average value of the actual field at the transition point was $4 \times 10^7$ V/cm.

Several observations provide an insight into the cause of the current fluctuations. First of all, the magnitude of the current pulses was roughly proportional to the d.c. level of the current; thus, the fluctuations behaved like a modulation of the field emission current, suggesting a related process rather than a separate phenomenon. Two sets of observations discussed in detail in later sections suggest that this modulation was most likely due to changes in the cathode surface. The first of these observations relates to the flickering of small spots in the emission patterns seen with the modified field emission microscope (see Section 7). It was observed that the flicker was accompanied by current fluctuations. The rate of flicker varied directly with the gas pressure in the system and also with the d.c. level of the current, suggesting that the flicker was induced by ion bombardment. The second set of observations, made with the electron microscope, was that if the electrodes were allowed to operate in the unstable current region for a period of about 15 minutes, a large number of fine whisker-like projections were formed on both the anode and the cathode (see Section 6, Figure 16). This type of whisker formation was observed only after operating in the unstable region, not after stable current operation.

The above observations suggest that the changes in the cathode surface were due to bombardment either by ions produced in the gap from the
Table II. Values of the actual field at the onset of the unstable current region.

<table>
<thead>
<tr>
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<th>$F \left(10^7 \text{ v/cm}\right)$</th>
</tr>
</thead>
<tbody>
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</tr>
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<td>3.2</td>
</tr>
<tr>
<td>23</td>
<td>5.6</td>
</tr>
</tbody>
</table>
residual gas pressure or by charged particles (ions or charged clumps of material) produced at the anode. Since the current fluctuations were greatly diminished by heating the anode (see next section), it was concluded that most of the particles responsible for the bombardment must have come from the anode. These particles could have very well been ions produced by the electron bombardment of adsorbed gases on the anode.\(^{22}\)

This description of the cause of current fluctuations leaves two features to be explained: (1) the whisker formation and (2) the shape of the observed current pulses.

In regard to the first item we note first of all that in general, surfaces which have been subjected to ion bombardment or sputtering have hillocks or etch pits,\(^*\) but whiskers of the type described above have not been reported to the author's knowledge. However, it has been reported that bombardment with ions can cause localized damage to the crystal structure. For example, Ogilvie\(^{26}\) reported that bombardment with 12-4000 eV argon ions produced disoriented crystallites about 100 Å in diameter in thin sheets of silver. Also, Bowden and Brandon\(^{27}\) have reported the formation of dislocation loops about 50 Å to 100 Å in diameter when thin foils of gold, copper, and nickel were bombarded with 100 eV argon ions. Furthermore, it is known that mechanical stress can cause whisker growth (see Franks, ref. 28, for example). Combining the observations of this investigation with the reported observations given above, we obtain the following tentative picture for the whisker formation. Field emission currents from the cathode produce charged particles at the anode which

\(^*\)See, for example, Farnsworth, ref. 23; Wehner, ref. 24; Honig, ref. 25.
cross the gap and bombard the cathode. The resulting damaged areas on
the cathode act as nucleation sites for whiskers which grow due to the
mechanical stress exerted on the electrodes by the electric field. In
a similar manner, particles produced at the cathode produce whiskers at
the anode.

One objection which might be raised to this picture is the produc-
tion of charged particles at the cathode, since this electrode was heated
at the beginning of the experiment and was presumably clean. (In fact,
when both the anode and the cathode were heated, the current fluctuations
were greatly reduced, as described in the next section.) However, bom-
bardment of adsorbed gases by electrons can produce neutral gas atoms as
well as ions. Taking into account the solid angle intercepted by the
cathode and the conductances and pumping speed of the vacuum system, it
would be possible for a monolayer of gas to be deposited on the cathode
in about five minutes with no measurable rise in the pressure, as indicated
on the ionization gauge. Thus, it would be possible, after a relatively
short period of time, to produce ions at the cathode. Another possible
explanation for the production of particles at the cathode is that the
whiskers are pulled off by the electric field and become the bombarding
particles.

Turning now to the question of the shape of the current pulses, we
note that the flat tops on the pulses (see Figure 9) indicate a sudden
increase in the current to a new level which remains constant for a period
of time ($10^{-3}$ to $10^{-1}$ sec), followed by a sudden decrease to the original
level. It was first thought that a current pulse was due to the sudden
formation of a whisker which was immediately heated and destroyed by the
field emission currents drawn from it. However, the thermal time constant for heating a whisker of the typical size observed in this investigation (see Section 6) is of the order of $10^{-9}$ sec which is very much shorter than the observed pulse lengths. A more plausible picture is the following. As indicated earlier, the current pulses appear to be associated with a bombardment of the cathode which causes a localized damage to the crystal structure. It seems reasonable to assume that a crystallite or dislocation loop formed in this manner causes an enhancement of the emission current. (For example, the crystallite could have a work function which is lower than that of the crystalline plane surrounding it. A change of 0.5 eV in the work function can cause an order of magnitude change in the current density.) Of course, in order to produce an observable change in the current, the crystallite would have to be formed on an emitting whisker. It also seems reasonable to assume that not all of these damaged areas act as nucleation sites for whisker growth, particularly if the damaged area is on a whisker, but that some of them may suddenly anneal back into alignment with the surrounding lattice after a period of time commensurate with the observed pulse length, causing the emission current to return to its original value.

The very long pulses or steps which occurred along with the shorter pulses are believed to have been caused by permanent changes in the cathode surface. Such a change could be caused by the formation of a projection, by the removal of a projection due to the mechanical force exerted by the electric field, or by the removal of a small part of a projection due to sputtering effects.
It is of interest to consider why current pulses of this magnitude (relative to the d.c. level) have not been observed previously. Lyman,\textsuperscript{11} for example, has observed pulses with similar characteristics except that the pulse height never exceeded about five per cent of the d.c. level. The explanation may lie in the size of the electrodes. Lyman's cathode area is about $10^5$ times larger than the area of the cathode in the present experiment. If the number of points or whiskers were correspondingly larger on his cathode, a change in the emission characteristics of any one whisker would have a much smaller effect on the total current than in the present case where there are only a few whiskers. Thus the use of small electrodes has introduced a new set of observations. These are strongly suggestive of multiple-whisker phenomena, a topic which is discussed in Section 8.

5.5c Effects due to Heating the Anode

To determine the effect of heating the anode in order to clean it, the original anode was replaced with a hairpin loop of .010-inch diameter tungsten wire spotwelded to two supporting rods which were mounted on a glass press. This press was attached to a flange by means of a stainless steel bellows.

Positioning of this anode was accomplished with three adjusting screws. Since this electrode arrangement was not even approximately equivalent to the plane parallel case, the gap spacing was not measured. The anode was positioned with the adjusting screws while observing the gap with a 10-power optical microscope. The gap was set so that it appeared to be approximately equal to the diameter of the tip electrode, (.003 inch).
This anode was heated in the same manner as the cathode (see Section 5.4). When both electrodes had been heated, it was possible in some of the runs to measure stable currents right up to the point of breakdown. In some of these runs no pulses at all were observed, while in other runs pulses did occur but their magnitude did not exceed five per cent of the d.c. level. The values of the actual field at breakdown were calculated and are listed in Table III. These values have a considerably larger spread than those obtained by other investigators, such as Lyman, for example. Calculations of the resistive heating of a projection (see Appendix A) indicate that $J^2c^2$, instead of $J$, should be a constant, where $c$ is the length of the emitting projection. Although it is not possible to calculate the exact relationship between $c^2$ and the emitting area, it seems reasonable to assume that they are related by a factor which is approximately a constant, since the shape of the projections was approximately a constant (except for runs 1 and 2), judging from the $\beta$ values. Hence, one would expect $J^2A$ to be a constant. However, the values of $J^2A$ obtained in this investigation were not constant, as indicated in Table III.

A possible explanation for the spread in observed values of the breakdown field is based on ion bombardment effects. Since pulses were occasionally observed in these breakdown runs, it seems probable that some ion bombardment occurred and could have affected the breakdown process. For example, the impact of an ion could conceivably sharpen a projection in such a manner as to cause the actual field at its tip to increase beyond the critical value and hence cause a breakdown to occur. Experimentally, this would appear as the termination of the Fowler-Nordheim plot at a value of the field less than the critical value, namely at the
Table III. Values of the actual field at breakdown.

<table>
<thead>
<tr>
<th>Run</th>
<th>$F \times 10^7$ (v/cm)</th>
<th>$J^2 A$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$1.2 \times 10^3$</td>
</tr>
<tr>
<td>9</td>
<td>4.2</td>
<td>$1.3 \times 10^{-2}$</td>
</tr>
<tr>
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<td>6.0</td>
<td>$1.4 \times 10^{-4}$</td>
</tr>
<tr>
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<td>3.4</td>
<td>$3.6 \times 10^{-4}$</td>
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<tr>
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<td>7.3</td>
<td>$3.7 \times 10^2$</td>
</tr>
<tr>
<td>24</td>
<td>7.4</td>
<td>$3.0 \times 10^2$</td>
</tr>
</tbody>
</table>
value of the field which existed at the tip of the projection before the shape of the projection was changed by the ion impact.

The reduction in the current pulses which occurred after the anode had been heated may be related to an effect observed by Pivovar and Gordienko while measuring prebreakdown currents between molybdenum electrodes. They reported that if the electrodes were not outgassed, microdischarges were observed. Although the characteristics of these microdischarges were not precisely described, the microdischarges were attributed to a cascade process such as reported by Arnal in which ions produced at one electrode can cross the gap and produce ions on the opposite electrode. This process seems to occur only under certain conditions, namely when there is a layer of oxide or other contamination on the electrodes such that the impinging ions can produce secondary ions in approximately a one-to-one ratio. After Pivovar and Gordienko heated their anode, the threshold voltage for microdischarges was increased by a factor of about three. They attributed this to the removal of oxides or adsorbed gases thereby reducing the probability of ion production at the anode. The reduction in the current pulses observed in the present experiment is also believed to be due to the removal of oxide or adsorbed gases from the anode.

The pulses that were seen in some of the runs may have been due to ions formed in the gap. Since the ionization cross section falls off with increasing electron energy, the majority of these ions would be produced close to the cathode. Their total energy would thus be considerably less than that of ions produced at the anode and hence the damage produced by them presumably would be less.
It should be pointed out that the microdischarges observed by Pivovar and Gordienko seem to be different from the current pulses seen in the present experiment, since Pivovar and Gordienko also reported that under different conditions they observed a steady current which they attributed to field emission. This steady current was not observed initially. Thus, they observed microdischarges without the presence of field emission current, in contrast to this experiment in which the pulses seemed to be a modulation of the field emission current.

5.5d Anomalous Fowler-Nordheim Plots

In a few runs with one particular tip electrode, the Fowler-Nordheim plots were found to consist of two intersecting straight lines rather than a single straight line (see Figure 11). The mechanism which caused this effect is not definitely understood. The change in slope was in the wrong direction to be explained on the basis of two different emitting points (see Section 8, Figure 21c). The abrupt change in slope was not due to a permanent change in the cathode surface, such as the destruction of a whisker, since the plot was reproducible as the voltage was increased and then decreased. This suggests a reversible process such as the desorption or adsorption of gas. As pointed out in the previous section, it appeared that it would have been possible to form a monolayer of gas on the cathode in about five minutes. Hence, it would be possible, although somewhat unlikely, that the gas desorption and adsorption occurred at just the right rate to cause the results to be reproducible as the voltage was increased and decreased.

Hawley\textsuperscript{1} has reported a similar change in slope, although the bend in his curve is somewhat sharper. (He stated that this effect has also
Figure 11. Anomalous Fowler-Nordheim plot showing an abrupt change of slope.
been reported by Nicholas and by Watts in their Ph.D. theses.) Hawley attributed this effect to adsorbed gases on the electrodes which altered the potential barrier at the surface of the metal and hence modified the field emission process. The electrodes in Hawley's apparatus are described as being "in a fairly dirty state" due to heavy oxide layers and oil vapors. Hence, one might expect the effect of adsorbed gases to be greater in his experiment than in the present investigation.

Another possible explanation is the "hair-raising" effect described by Little and Whitney in which a whisker, originally lying down on the cathode, is pulled upright by the electric field. If this process did occur, it must have been gradual rather than abrupt since no sudden change in the emission current was observed at the point where the slope of the Fowler-Nordheim plot changed.
6. STUDIES OF ELECTRODE PROFILES WITH AN ELECTRON MICROSCOPE

6.1 Method of Viewing the Electrode

The electron microscope used in these studies was an R.C.A. Model 2MU-2E. The resolution of this microscope, as used in this investigation, was about 50 Å.

Initially, it was hoped that both the current-voltage measurements and the observation of the electrode profiles could be made within the electron microscope. However, due to the vacuum conditions in the electron microscope, the electrode surfaces became covered with oil vapor, causing the current measurements to be very erratic. It was then decided to perform the current-voltage measurements in a separate ultrahigh vacuum system.

Along with the determination of whisker size and shape, studies were made of the changes in the electrode surface caused by certain types of treatment such as heating or electrical breakdown. For this purpose it was thought best to view the electrode directly rather than using replica techniques which in themselves might cause changes in the electrode surface. A specimen holder was modified to allow insertion of the electrode into the microscope. Figure 12 is a drawing of the modified specimen holder and the insert on which the electrode was mounted. A photograph of the holder containing the insert and electrode is shown in Figure 13. The electron beam passed along the axis of the holder causing a shadow profile of the tip of the electrode to be produced on the viewing screen. A typical profile, obtained at a low magnification, is shown in Figure 14.
Figure 12. Modified specimen holder for the electron microscope and insert (a).
Figure 13. Tip electrode mounted in the modified specimen holder.
Figure 14. Magnified outline of a typical profile of the tip electrode.
The electrode profile was normally scanned at a magnification of approximately $10^4$ and was photographed in overlapping views. Enlarged prints (2x) were made from the resulting photographic plates. By tracing over these prints it was possible to construct a magnified outline of the tip profile. This outline was used for the measurement of the whisker sizes and also for comparison with other outlines to determine what changes had occurred in the electrode profile.

6.2 Discussion of Observations

In some of the initial observations it was discovered that sharp spikes were formed when the electrodes were touched together as part of the gap adjusting procedure. At first it was thought that this might be due to the current through the electrodes associated with the ohmmeter used to detect the point of contact. However, spikes of the same type were formed when the electrodes were touched together with the ohmmeter disconnected. Figure 15 shows the type of spikes that were produced. Evidently, touching the electrodes together in vacuum caused a cold weld or fusion to occur and sharp spikes were formed by the tearing apart of this weld as the electrodes were separated.

If the tip electrode was heated to about 1600°C for one minute, these sharp projections could be eliminated. Presumably the rate of surface diffusion was increased sufficiently at this temperature to cause the sharp points to become blunted. The projections remaining on the electrode after the heating process were similar in size and shape to those shown in Figure 15a.

Although the heating process eliminated the sharper projections, there were still a large number of projections and bumps remaining on the
(a) A section of the electrode profile before physical contact with the opposite electrode.

(b) Same section after physical contact.

Figure 15. Formation of spikes on the electrode surface due to physical contact with the opposing electrode in vacuum.
cathode. From the outline of a typical profile of the tip electrode, shown in Figure 14, it is apparent that the model used in Section 2.2 for the theoretical calculation of $\beta$, namely an ellipsoidal projection on a plane surface, is a rather crude approximation to the true geometrical situation. Taking into account the crudeness of the model, the observed projections did have the approximate size and shape as predicted by the Fowler-Nordheim plots, but it was not possible to identify any one specific projection as being the actual emitter of the measured current. In fact, there were usually several projections which fit the description predicted by the Fowler-Nordheim plot. In Section 8 a discussion will be given of the concept that the measured prebreakdown current is the sum of the currents from a number of individual points.

If the electrodes were operated for a period of time (~15 minutes) in the unstable current region, a large number of fine whisker-like projections, such as those shown in Figure 16, were produced on the electrodes. Whiskers of this type were formed on the electrode when it was operated as the anode as well as when it was the cathode.

Observation of an electrode profile after an electrical breakdown had occurred revealed that some projections had disappeared, typically the sharper ones, and that usually some new points had been produced, often at a different location on the electrode. Figure 17, taken from a pair of magnified outlines, shows the same section of an electrode profile before and after a breakdown. The disappearance of two sharp projections and a larger bump is apparent.

In the model presented in Section 3, the destruction of a single point is the event which triggers a breakdown. However, once the
Figure 16. Whisker-like projections formed on the electrode after about 15 minutes' operation in the unstable current region.
Figure 17. Removal of electrode projections due to electrical breakdown.
breakdown has been initiated it is perhaps not surprising that additional points or whiskers should be destroyed or created by the discharge. This is discussed in more detail in Section 9.
7. WHISKER STUDIES WITH A MODIFIED FIELD EMISSION MICROSCOPE

7.1 Description of the Apparatus

Various experimental results observed in this present investigation and by other experimenters (see Section 8) have suggested that the measured field emission currents may be due to a number of emitting points or whiskers instead of a single one, as originally proposed. One method of determining the number of emitting points is to coat the anode with a luminescent material and observe the patterns of illuminated spots generated by the impinging electrons. This technique has been used by Little and Whitney with flat electrodes which gave practically no magnification and by Brodie and Weissman using a cylindrical geometry which gave magnification in one dimension. In the present investigation, a spherical geometry was chosen because (1) a field emission microscope was available and could be easily modified, (2) the cathode could be made very similar to the one used for the small gap current-voltage measurements, and (3) magnification in two dimensions could be obtained. The field emission microscope was modified by removing the original emitter and replacing it with a short length of .003-inch diameter tungsten wire spotwelded to a hairpin filament of .010-inch tungsten wire. This cathode was very similar to the one used in the small-gap measurements. Figure 18 is a schematic drawing of this modified field emission-microscope.

This instrument was mounted on the ultrahigh vacuum system described in Section 5.1. The power supply and circuitry used for the current-voltage measurements were the same as those used in the small-gap geometry (see Section 5.2).
Figure 18. Schematic drawing of the modified field emission microscope.
It should be emphasized that the cathode in this instrument was not etched down to a point as is done with a normal emitter. The only treatment given the cathode was to heat it to about 1600°C in vacuum at various times for outgassing purposes.

7.2 Discussion of Observations

7.2a Emission Patterns

Application of a high voltage to the anode permitted the observation of the emission pattern on the screen while the current-voltage characteristics were being measured. The patterns produced on the screen generally consisted of several well-defined spots or regions which were more or less uniformly illuminated. A typical pattern is shown in Figure 19. The number of spots observed in different runs varied from one to about six.

As in the small-gap case, a stable and an unstable current region were found. When operating in the unstable current region, a flickering was detected on the screen. This flickering was caused by the appearance and disappearance of small spots located within one or more of the larger illuminated regions. The rate of flickering increased if either the d.c. level or the gas pressure was increased, suggesting that the flicker was associated with ion bombardment of the emitting points. This was discussed in Section 5.5b. By obtaining the ratio of the total area predicted from a Fowler-Nordheim plot of the prebreakdown current, an approximate value for the magnification factor was obtained. On the basis of this value it was estimated that the size of a flickering spot on the screen corresponded to a region on the cathode of about 20 Å in diameter.
Figure 19. Photograph of a typical emission pattern obtained with the modified field emission microscope.
Occasionally a new illuminated region suddenly appeared on the screen as the voltage was being increased, as if a new whisker had been created. The occurrence of a breakdown usually caused the disappearance of one or more regions and sometimes the appearance of new regions, which agrees with the electron microscope observations of the destruction and formation of projections by a breakdown.

7.2b Current Suppression at Increased Gas Pressures

It has been reported by several workers\textsuperscript{11,33,34,35} that the introduction of a gas to a pressure of about $10^{-4}$ torr causes a substantial decrease in the prebreakdown current and often increases the breakdown voltage. The effect seems to be independent of the nature of the gas, i.e., whether it is a highly adsorbing gas or a noble gas. This factor seems to rule out surface effects such as changes in work function. At these pressures the mean free path is much longer than the typical dimensions of the apparatus so that average volume effects, such as electron scattering or ion production, should be negligible.

An explanation of this effect, given by Alpert et al.,\textsuperscript{36} is based on the concept of selective ion bombardment of the sharpest points on the cathode. Although the total current is relatively small, it is highly concentrated at the projections or whiskers on the cathode where the current density may approach $10^8$ amp/cm\textsuperscript{2}. This results in the production of an appreciable number of ions in the immediate vicinity of the points. These ions will then bombard the projections and can cause selective sputtering of the projections. Figure 20 is an illustration depicting this effect.
Figure 20. Illustration depicting the selective sputtering of projections by the ions which are produced by field-emitted electrons.
The rate of ion bombardment is given by the following equation:

\[ n_\text{i} = n_\text{e} p K \lambda, \quad (7.1) \]

where \( n_\text{i} \) is the number of ions/cm\(^2\)-sec, \( n_\text{e} \) is the number of electrons/cm\(^2\)-sec, \( p \) is the pressure in torr, \( \lambda \) is the effective path length, and \( K \) is a proportionality constant related to the ionization cross section. To obtain an idea of the magnitude involved, consider the following case:

1. \( p = 5 \times 10^{-5} \) torr.
2. \( n_\text{e} = 10^{24} \) electrons/cm\(^2\)-sec, which corresponds to a current density of \( 2 \times 10^{-5} \) amp/cm\(^2\). This requires a field of about \( 5 \times 10^7 \) v/cm at the whisker.
3. \( \beta = 100 \). Thus, the average field would be \( 5 \times 10^5 \) v/cm.

Taking into account the thermal energy, \( E_\text{t} \), of the ions, the distance, \( \lambda \), from the surface at which at least 10 per cent of the ions formed will hit the projection or whisker, is determined by the equation

\[ \pi b^2 = 0.11 \left[ \frac{E_\text{t}}{e F_\text{o} \lambda} \right]^2, \quad (7.2) \]

where \( b \) is the radius of the whisker base. Taking the thermal energy as .025 eV and \( b \) as \( 10^{-6} \) cm gives

\[ \lambda = 2 \times 10^{-4} \) cm. \]

The electron energy at this distance is

\[ e F_\text{o} \lambda = 100 \) eV. \]

Taking an average value of \( K = 1 \) ion/electron-cm-torr over the path length, and substituting the above factors into equation (7.1), gives
The rate at which neutral atoms strike the surface at this pressure is only about \(2 \times 10^{16}\) atoms/cm\(^2\)-sec. Thus, under these conditions, nearly every gas atom which arrives in the immediate vicinity of the whisker is ionized.

If the whisker bombardment rate is only 0.1 that of the total rate calculated above, the rate at which ions hit the whisker is given by

\[
0.1 n_{\text{e}} \pi b^2 = \pi \times 10^3 \text{ ions/sec}. \tag{7.4}
\]

For a one-to-one sputtering ratio, this rate of ion bombardment would remove nearly a monolayer of whisker material every second.

We thus have a mechanism for the selective sputtering and self-destruction of the whiskers or projections on the cathode. As the projections are eroded away by this mechanism, the enhancement factor is expected to decrease, which in turn causes the observed decrease in current. Eventually an equilibrium should be reached in which the rate of point destruction becomes equal to the rate at which new points are created.

The modified field emission microscope provided a means of checking this description of the current suppression process. Gas (argon) was admitted to a pressure of 1 to 5 \(\times 10^{-5}\) torr with the voltage off. The voltage was then applied and slowly increased. When the current reached a value around \(10^{-6}\) amp it was observed to increase suddenly (with the voltage constant) to about \(10^{-5}\) amp, and then decrease to about 2 or \(5 \times 10^{-7}\) amp. The emission pattern was observed to brighten and darken correspondingly. The time required for this action varied from 1 to 20 minutes. If the voltage was then increased, the over-all process would repeat,
although the magnitudes of the current and the time involved might differ depending on the previous treatment of the cathode. Sometimes increasing the voltage caused a new region of illumination to appear on the screen, but the general behavior was the same.

It seems reasonable to postulate that the initial increase in current is due to a sharpening of the whisker by the first few ion impacts. This is followed by a more gradual dulling process as the whisker is eroded away.

A first attempt was made to obtain some quantitative results relating the length of time needed for the erosion process to the gas pressure. The results were not conclusive. One difficulty is the fact that the emitting area is constantly changing during the erosion process which makes it impossible to keep the current density at a constant value. Also, it may very well be that the erosion process depends on other factors such as the metallurgical character of each whisker or projection, including effects due to imperfections. An attempt to count the number of current pulses in a given length of time as a function of pressure at a fixed current was also inconclusive, probably for the same reasons.

Despite the lack of quantitative results, the qualitative observations were in agreement with the process described above. Further studies seem to be indicated.
8. FIELD EMISSION FROM MULTIPLE POINTS

8.1 Experimental Observations Suggesting Multiple-Point Emission

The treatment of the effect of projections on field emission given in Section 2 was based on the concept that all, or most, of the measured field emission current came from a single emitting point or whisker. This idea originated with Boyle, Kisliuk, and Germer in their interpretation of the fact that their Fowler-Nordheim plots were single straight lines. Their interpretation is discussed in the next section.

The possibility that field emission might be occurring from many points is suggested by several phenomena observed experimentally. In the present investigation, the electron microscope revealed that typically there were several whiskers or projections on the cathode all having the approximate size and shape as predicted from the Fowler-Nordheim plot. Moreover, the patterns seen on the screen of the modified field emission microscope usually consisted of several separate illuminated regions, suggesting emission from several points. (Similar effects were noted by Little and Whitney and by Brodie and Weissman.)

Another phenomenon, observed by Lyman during many of his runs, was the appearance of a number of widely distributed, tiny points of light situated at the anode. The cause of these points has not been established but it seems quite possible that they are connected with the bombardment of the anode by electron beams coming from the cathode. The density of

* E. Silverman reported a blue-green spot due to bremsstrahlung when a copper target was bombarded with a high density, 25 kv electron beam.
these points of light varied from about 10 to 100 per cm$^2$. Similar points of light have been reported by De Geeter.\textsuperscript{38}

Still another observation concerns the radiation emitted from a vacuum capacitor when subjected to high voltage tests. Using x-ray film and a pin-hole camera, Singer\textsuperscript{39} obtained photographs of this radiation which appeared as many well-defined spots on the film. Presumably, electron beams from the cathode were bombarding the anode and creating sources of x-rays which produced the spots on the film. The density of emitting points was estimated to be two or three per cm$^2$.

8.2 The Form of the Fowler-Nordheim Plot for Multiple-Point Emission

To illustrate the argument of B.K.G. relating to the linearity of a Fowler-Nordheim plot, let there be two emitting points on the cathode with $\beta$'s of 100 and 150, respectively. A Fowler-Nordheim plot can be made corresponding to the current emitted by each point and also a plot can be made of the sum of their currents. The resulting plots are shown in Figure 21, where the range of field values is that normally encountered in voltage breakdown measurements. In Figure 21a the emitters were assigned equal areas. In this case the current from the second emitter ($\beta=150$) is so much larger than the other that it completely dominates the combined effect. If the emitter with the smaller $\beta$ is assigned an area much larger than the area of the second emitter, as in Figure 21b, the situation is reversed and the current from the first emitter dominates the combined effect. In the third situation, Figure 21c, the areas have been adjusted to give equal emission currents near the midpoint of the plot. The second emitter now dominates at the lower fields while at the higher fields the first emitter is the dominant one. Note that in this case the
Figure 21a. Fowler-Nordheim plots of currents from two different emitters and their combined current.

$\beta_1 = 100$, $\beta_2 = 150$, $A_1 = A_2 = 1 \text{ cm}^2$
Figure 21b. Fowler-Nordheim plots of currents from two different emitters and their combined current.

\[ \beta_1 = 100, \beta_2 = 150, A_1 = 1.0 \times 10^6 \text{ cm}^2, A_2 = 1 \text{ cm}^2 \]
Figure 21c. Fowler-Nordheim plots of currents from two different emitters and their combined current.

\[ \beta_1 = 100, \beta_2 = 150, A_1 = 2.25 \times 10^3 \text{ cm}^2, A_2 = 1 \text{ cm}^2 \]
Fowler-Nordheim plot of the combined currents is no longer a straight line. From these examples one would conclude that a linear Fowler-Nordheim plot indicates that the measured field emission current comes from a single emitter (or possibly from several identical emitters).

To investigate more thoroughly the plausibility of multiple-point emission, calculations have been carried out to determine the form of the Fowler-Nordheim plot which might be expected from the sum of currents from several emitters. Analytical and numerical (digital computer) solutions were obtained for specific cases.

If it is assumed that there are a number of emitters of varying size on the cathode and that these emitters have a distribution in $\beta$ given by $N(\beta_i)$, then the total current from this set of emitting points will be given by

$$I = K_1 F_0^2 \sum A_i \beta_i^2 N(\beta_i) \exp[-K_0/\beta_i F_0]$$ (8.1)

where $A_i$ and $\beta_i$ are the area and enhancement factors, respectively, of the $i^{th}$ point, and $K_1 F_0^2 \exp[-K_0/\beta_i F_0]$ is a simplified form of the Fowler-Nordheim equation.

If the $\beta$ factors are assumed to have a Gaussian distribution, the number of emitting points, $N(\beta_i)$, which have an enhancement factor equal to $\beta_i$ will be given by

$$N(\beta_i) = N_0 \exp[-(\beta_i - \beta_0)^2/2\sigma^2]$$ (8.2)

where $\beta_0$ is the average value, $\sigma$ is the standard deviation, and $N_0$ is a scale factor. A simplified expression for the approximate value of the total current can be obtained under the following conditions:
(a) the number of emitters is sufficiently large so that the summation in (8.2) can be replaced by an integration,
(b) the standard deviation is not too large,
(c) the individual emitters have equal areas, A.

It can be shown* that the total current under these conditions is of the form

\[ I = K_3 F_0^2 \exp[-K_2/\beta_0 F_0]. \]  

(8.3)

Note that in (8.3) the current has the same functional dependence on \( F_0 \) as in the Fowler-Nordheim equation for a single emitter. Thus, under the conditions given for the derivation of (8.3), it is possible that the combined currents from a large number of emitters will result in a linear Fowler-Nordheim plot.

In many experimental situations the number of emitting points may not be large enough to justify condition (a) above. Also the area factors will probably not be equal. To investigate such cases a digital computer program was written to do the following:

1. Set up a Gaussian distribution in \( \beta \) values.
2. For a given number of emitters (2-100), randomly select from the Gaussian distribution a \( \beta \) value for each emitter.
3. For each emitter randomly select an area factor from the range 1 to 10.
4. For a given value of the apparent field, \( F_0 \), compute the actual effective field \( F_A = \beta F_0 \) and then the field emission current for each emitter.

*See Appendix B.
(5) Sum the currents from the individual emitters to obtain the total current.

(6) Repeat (4) and (5) for various values of $F_0$.

The constants used in the Gaussian distribution (8.2) were

$$
\beta_0 = 44
$$

$$
2\sigma^2 = (44)^2
$$

The selected $\beta$ values were limited to the range $70 \leq \beta \leq 150$ and the value of $N_0$ was adjusted to give $N(150) = 1$.

A similar program was used to calculate the current for the case of a uniform distribution in $\beta$ factors. Initially the $\beta$ factors were randomly selected from a group of values ranging from 100 to 130 and the area factors from a range of 1 to 10. Later the $\beta$ range was changed to 100-150 and the area range to 1-1000.

In each case considered, the result was that the Fowler-Nordheim plot of the calculated currents was a straight line. To check on the possibility that one emitter was dominating the results, as illustrated in Figures 21a and 21b, effective $\beta$ and area values were calculated from each plot. In most of the cases in which only two emitting points were selected, the effective $\beta$ and area values were practically the same as that of the emitter with the larger $\beta$, so that one emitter indeed did dominate the results. This is illustrated in Table IV. However, when the number of selected emitting points was four or greater, it was typically found that the effective $\beta$ was an averaged value with the average weighted toward the higher $\beta$ values in the selected group. The area factors of the individual emitters had some effect on the averaging process. Tables V,
### Table IV. Two emitting points.

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### Table V. Four emitting points.

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VI, and VII list some typical examples. Also listed in Table VI is the fraction of the total current contributed by the individual emitters at two different values of the apparent field (the smallest and the largest field values used in the calculations). Figure 22, which is the Fowler-Nordheim plot of the calculated currents from the group of emitters listed in Table VI, demonstrates the linearity of the plots.

From these results one concludes that the linearity of a Fowler-Nordheim plot is not necessarily an indication of emission from a single point. However, if there are several emitting points, the slope of the plot gives an indication of the larger \( \beta \) values in the group. The effective area gives little information about the individual emitters, but it does set an upper limit on the number of emitters since the area of the smallest possible emitter must be at least equal to the area of an atom.

Although the number of emitting points cannot be determined from a Fowler-Nordheim plot, other experimental evidence indicates that in the present investigation (with a cathode area of about \( 10^{-5} \text{ cm}^2 \)) a small number of points were involved. On the basis of the number of illuminated regions observed on the screen of the modified field emission microscope, the number of emitting points ranged from 1 to 10. This is substantiated by the electron microscope observations in the following way. When the area calculated from the Fowler-Nordheim plot was relatively small \((\sim 10^{-13} \text{ cm}^2)\), a few fine whiskers of this size were seen on the cathode. On the other hand, when the area was larger \((\sim 10^{-10} \text{ cm}^2)\), a few protuberances of this size were seen on the cathode but seldom were any fine whiskers observed. Thus, with the cathode used in this study, no cases were observed in which there were a larger number of whiskers or points with a combined area equal to the area calculated from the Fowler-Nordheim plot.
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<td>117</td>
<td>901</td>
<td>$2.7\times10^{-3}$</td>
<td>$6.0\times10^{-2}$</td>
</tr>
<tr>
<td>100</td>
<td>713</td>
<td>$1.4\times10^{-5}$</td>
<td>$9.8\times10^{-3}$</td>
</tr>
<tr>
<td>Effective Values</td>
<td>137</td>
<td>3670</td>
<td>--</td>
</tr>
</tbody>
</table>
### Table VII. Fifty emitting points

**Case A.** $\beta$ values randomly selected from a Gaussian distribution of the form $N(\beta) = N_0 \exp\left[-(\beta-44)^2/(44)^2\right]$; area values randomly selected from a uniform distribution ranging from 1 to 10. Range of selected $\beta$ values, $114 \leq \beta \leq 71$.

<table>
<thead>
<tr>
<th>Five largest $\beta$ values in the selected group</th>
<th>$\beta$</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>114</td>
<td>7.9</td>
</tr>
<tr>
<td></td>
<td>113</td>
<td>7.1</td>
</tr>
<tr>
<td></td>
<td>112</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>111</td>
<td>6.4</td>
</tr>
<tr>
<td></td>
<td>111</td>
<td>9.2</td>
</tr>
</tbody>
</table>

**Effective Values**

|                                                | 110    | 71    |

**Case B.** $\beta$ and area values randomly selected from uniform distributions: $100 \leq \beta \leq 150$, $1 \leq A \leq 1000$.

<table>
<thead>
<tr>
<th>Five largest $\beta$ values in the selected group</th>
<th>$\beta$</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>150</td>
<td>684</td>
</tr>
<tr>
<td></td>
<td>145</td>
<td>498</td>
</tr>
<tr>
<td></td>
<td>144</td>
<td>936</td>
</tr>
<tr>
<td></td>
<td>143</td>
<td>189</td>
</tr>
<tr>
<td></td>
<td>142</td>
<td>887</td>
</tr>
</tbody>
</table>

**Effective Values**

|                                                | 138    | 12,600 |
Figure 22. Fowler-Nordheim plot of the calculated combined currents from the group of emitters listed in Table VI.
This conclusion of emission from multiple points does not make any major change in the proposed model for breakdown initiation. The criterion of a constant breakdown field is still valid since the point which first experiences the critical density will be the one which initiates breakdown. There may be some error introduced in the calculation of the actual breakdown field from the slope of the Fowler-Nordheim plot. When the actual field at the point with the largest $\beta$ factor reaches the critical value, the field calculated from the slope of the Fowler-Nordheim will be somewhat less due to the averaging effect. However, since the Fowler-Nordheim plot is strongly weighted by the larger $\beta$ values in the group, the error introduced will be small.
9. METHODS OF PRODUCING AND ELIMINATING PROJECTIONS ON ELECTRODES

Since projections seem to strongly affect the prebreakdown and breakdown characteristics of electrodes, two interesting questions are:

(a) What is the origin of these projections?
(b) How can projections be eliminated?

It is not difficult to visualize methods for producing electrode projections. To begin with, all machining processes result in some degree of surface roughness. While mechanical and chemical polishing can reduce the degree of roughness, they do not eliminate it completely. Only for specialized geometries, such as the single crystal point electrodes used in field emission or field ion microscopes, can one reduce the roughness to atomic dimensions, and even then there are steps at the edges of lattice planes. In the case of larger area electrodes there are often pits, inclusions, grain boundaries, or other imperfections in the surface even after the best polishing techniques have been used.

Many experiments indicate that the process of testing electrodes by subjecting them to electrical breakdown can in itself change the surface roughness. If there are a few sharp whiskers on the original surface of the cathode, one might expect that these whiskers would initiate breakdown by the field emission mechanism and in the process destroy themselves. However, during the breakdown process it seems possible that metal vapor or droplets can be sprayed on the electrodes producing new projections. Also, if there are loosely bound particles or clumps on an electrode surface, there clumps may be transferred to the opposite electrode where they
can cause sputtering and local evaporation of material which can condense on the electrode surfaces forming more projections.

It is also possible that the mechanical strain exerted on the electrode surfaces by the electric field may produce whiskers. Whiskers have been observed on an electrode surface in the present experiment and also by Little and Whitney after the electrode had been subjected to an electric field. Although the processes involved in whisker formation by this mechanism do not seem to be very well understood, several cases have been reported in which whisker growth has been observed when a metal has been subjected to compressional forces (see Franks, ref. 28, for example).

It is also known from experiments performed with the field emission microscope that the shape of an emitter can change due to the surface migration of atoms resulting from the force exerted by the electric field. Presumably, the same phenomenon can occur with projections on broad area electrodes, and it is of interest to consider what conditions are necessary to cause projections to grow. Calculations pertaining to changes in length by the mechanism of surface diffusion are given in Appendix C. These calculations indicate that the rate of change of length can become appreciable when the temperature exceeds about 1500°K. However, the process in complicated by the non-uniform heating of the projection in the case of resistive heating by field emission currents, and the net result on the length and shape of the projection cannot be accurately predicted.

One method of eliminating points or whiskers that has been demonstrated in this experiment is the admission of gas to pressures of the order of 10^-4 torr while field emission currents are being drawn. The self-destruction of whiskers by this technique was discussed in Section
7.2b. This technique is not without drawbacks, however, since the optimum pressure is very close to the glow discharge region and any transient may trigger off the discharge. If whiskers are removed by operating for a period of time at high pressure and if the gas is then pumped away to prevent a glow discharge, it is quite likely that new whiskers will be formed due to the force exerted by the electric field combined with the bombardment by ions formed from the residual gas in the system.

Although at present it seems very doubtful that whiskers or projections can be completely removed from broad area electrodes, the experimental approach described here provides a tool by which the formation of electrode projections can be studied. Perhaps when more information about the basic mechanisms of whisker or projection formation is obtained, techniques can be developed for controlling or eliminating projections.
10. SUMMARY

This investigation, which was performed with tungsten electrodes under ultrahigh vacuum conditions, has led to or confirmed the following conclusions.

1. There is direct evidence for the existence of sharp projections or whiskers on broad area electrodes which cause a local enhancement of the electric field and hence strongly influence the prebreakdown current and breakdown voltage.

2. The size and shape of the projections as observed with an electron microscope are in approximate agreement with the values predicted from a Fowler-Nordheim plot of the prebreakdown current.

3. Electrical breakdown occurs when the actual electric field at the sharpest point on the cathode reaches a critical value. This is approximately a constant independent of the gap spacing or the geometry of the electrodes. Since the relationship between the applied voltage and the actual field can be determined from the slope of the Fowler-Nordheim plot, the approximate value of the breakdown voltage can be predicted from the current-voltage characteristics.

4. Electrical breakdown causes the destruction of one or more projections and may also create new points.

5. For broad area electrodes, the measured breakdown current is most probably emitted from several points rather than from a
single point. The form of the Fowler-Nordheim plot is
determined to a large extent, but not completely, by the
points with the larger enhancement factors.

6. Under certain conditions, namely when there are adsorbed gases
or oxides on the anode, fluctuations in the field emission
current are observed. These fluctuations are believed to be
the result of surface changes in the cathode due to bombard­
ment by ions (or, perhaps, by charged clumps) which have been
produced at the anode by the impinging electrons. Under these
conditions the formation of whiskers is enhanced. Similar
effects may be produced by the ions formed in the volume between
the electrodes.

7. The introduction of gas at a pressure of about $10^{-4}$ torr causes
an initial increase in the field emission current followed by a
sharp decrease to a value considerably less than the original
value. This is explained in terms of the selective sputtering
and self-destruction of projections on the cathode.

In addition, some insight has been gained into the processes in­
volved in the formation and destruction of whiskers. Ion bombardment and
mechanical stress due to the electric field appear to be major factors in
these processes, but further study is needed to ascertain the physical
mechanisms involved. The modified field emission microscope appears to
be a useful tool for such a study.
11. Lyman, E. M. private communication.
34. Murray, J. Lawrence Radiation Laboratory, University of California, Report UCRL-9506 (unpublished).
35. Sandweiss, J. private communication.


APPENDIX A
ELECTRODE HEATING

A.1 Resistive Heating of a Cathode Projection

Let us consider the resistive heating of a cathode projection by field emission current. If the only heat loss is due to conduction, the basic equation is

\[ KV \frac{\partial^2 T}{\partial t^2} + \frac{1}{V} I^2 R = C \delta \frac{\partial T}{\partial t}, \]

(A-1)

where \( K \) = thermal conductivity
\( T \) = temperature
\( V \) = volume
\( I \) = current
\( R \) = resistance
\( C \) = heat capacity
\( \delta \) = mass density
\( t \) = time.

To simplify the calculations, let us assume that the geometry of the projection being heated is a circular cylinder of radius \( b \) and height \( c \). In this case (A-1) becomes

\[ K \frac{\partial^2 T}{\partial z^2} + \frac{1}{\pi b^4} \frac{I^2 I}{R} = C \delta \frac{\partial T}{\partial t}, \]

(A-2)

where \( \rho \) is the resistivity of the material.

Let us consider only the steady-state solution. The differential equation then becomes
As a first approximation, assume \( \rho \) and \( K \) to be independent of temperature. Then, a solution of (A-3) is

\[
T_s = \frac{-I^2 \rho}{8 \pi^2 b^4 K} \left( z^2 + Bz + D \right) .
\]  (A-4)

To determine the constants \( B \) and \( D \), take the following boundary conditions:

1. At \( z = 0 \), \( T = T_0 \).
   
   Then, \( D = T_0 \).  (A-5)

2. At \( z = c \), \( \frac{dT}{dz} = 0 \).

Then, \( B = \frac{I^2 \rho c}{8 \pi b^4 K} \).  (A-6)

Putting (A-5) and (A-6) into (A-4) and evaluating at \( z = c \) gives the following expression for the temperature \( T_1 \) at the tip of the projection:

\[
T_1 = T_0 + \frac{I^2 \rho c^2}{8 \pi b^4 K} .
\]  (A-7)

If (A-7) is rewritten in terms of the current density, \( J \), the expression becomes

\[
T_1 = T_0 + \frac{J^2 \rho c^2}{2K} .
\]  (A-8)

Thus, the tip temperature depends not only on the current density but also on the length of the projection.

If the shape of the projection is taken to be a prolate hemispheroid, as in Section 2.2, the problem becomes much more difficult because the emitting area cannot be defined as exactly as in the cylindrical case.
The problem can be simplified by assuming that the projection is capped with a hemisphere having a radius $r_1$. Let the prolate hemispheroid be truncated at a point determined by

$$r = z - c$$

(A-9)

where $r$ and $z$ are equivalent to the variables $\rho$ and $z$, respectively, as used in Section 2.2, and are related by equations (2.9) and (2.10). If (A-9) is expressed in terms of $\xi$, two solutions are found:

$$\xi = 1$$

(A-10)

and

$$\xi = \frac{1-b^2/c^2}{1+b^2/c^2}$$

(A-11)

where $b$ is the radius at the base of the projection and $c$ is the length.

The value of interest is the one given by (A-11). The corresponding value of $r_1$ is

$$r_1 = c \frac{2b^2/c^2}{1+b^2/c^2}$$

(A-12)

Let the current density be defined by

$$J = \frac{1}{n r_1}$$

(A-13)

If the temperature is assumed to be constant throughout the hemispherical cap, the steady state temperature of the tip can be found in a manner similar to that used in the cylindrical case. The result is

$$T_1 = T_0 + \frac{6J^2 c^2}{K} \frac{4b^2}{c^2} \frac{(1-b^2/c^2)}{(1+b^2/c^2)^3}$$

(A-14)

Note that again the tip temperature depends on the length of the projection.
In the above calculations, the Nottingham effect and radiation losses were neglected. The Nottingham effect is discussed by Martin et al., who show that this effect is small when the tip temperature exceeds about 1500°K. The heat loss due to radiation is given by

\[ P_r = \sigma A (T_1^4 - T_0^4) , \]  

(A-15)

where \( \sigma = 5.67 \times 10^{-12} \) watts/cm²°K and A is the area. In the cylindrical case, the total power generated in the projection by resistive heating is

\[ I^2 R = \frac{I^2 P_C}{\pi b^2} . \]  

(A-16)

From (A-7) this is found to be equivalent to

\[ I^2 R = \frac{(T-T_0)^2 \pi b^2 K}{c} . \]  

(A-17)

The ratio of the radiation loss to the power generated is

\[ \frac{P_r}{I^2 R} = \frac{\sigma A (T_1^4 - T_0^4) c}{(T_1 - T_0)^2 \pi b^2 K} . \]  

(A-18)

Electrical breakdown is believed to occur when the tip temperature approaches the melting point which is about 3670°K for tungsten. The value of K is about 0.65 watts/cm°K. Assume the following values for the size of the projection:

\[ c = 10^{-4} \text{ cm}, \]
\[ b = 10^{-5} \text{ cm}. \]

Then

\[ \frac{P_r}{I R} \approx 7.7 \times 10^{-4} \text{ A.} \]  

(A-19)

Even if the temperature of the whole projection were at the melting point so that \( A = 2\pi \times 10^{-9} \text{ cm}^2 \), the ratio would only be
\[ \frac{P_r}{T^3 R} \approx 4.8 \times 10^{-4}. \]  

(A-20)

Hence, almost all the heat loss must be due to conduction and the error introduced by the neglect of radiation losses is negligible.

Since electrical breakdown is believed to occur when the tip temperature approaches the melting point, let us determine the required current density as predicted by (A-13). For this numerical example, let \( b/c = 0.1 \) and \( c = 10^{-4} \text{ cm} \). As suggested by Martin et al., the assumption that \( \rho \) and \( K \) were independent of temperature in the above derivation introduced an error which can be partially reduced by evaluating \( \rho \) and \( K \) at \( 0.8T_1 \) and using these values in (A-14). If \( T_1 \) is taken to be \( 3700^\circ\text{K} \) (the melting point of tungsten), then the appropriate values of \( \rho \) and \( K \) are

\[ \rho = 1.2 \times 10^{-4} \text{ ohm-cm}, \]
\[ K = 0.65 \text{ watts/cm}^0\text{K}. \]

If these values are substituted in (A-14) and \( T_0 \) is taken to be \( 320^\circ\text{K} \), the current density, \( J \), required to produce this tip temperature is found to be

\[ J = 2.2 \times 10^8 \text{ amp/cm}^2. \]  

(A-21)

A value of \( c = 10^{-5} \text{ cm} \) increases the predicted value of \( J \) to \( 2.2 \times 10^9 \text{ amp/cm}^2 \). These values are approximately in the range of values observed experimentally.

A.2 Anode Heating by an Electron Beam

The field-emitted electrons from the cathode dissipate most of their energy as heat when they strike the anode. The power density, \( P \), generated
by the impinging electrons is given by

\[ P = J_a V, \]  \hspace{1cm} (A-22)

where \( J_a \) is the current density of the electron beam when it reaches the anode and \( V \) is the voltage applied to the electrodes. The current density at the anode is less than the current density at the cathode due to the spreading of the electron beam caused by geometrical and space charge effects. Calculations of the beam spreading have been carried out by Vibrans for the case of parallel electrodes. Figure 23 is a plot of the anode current density as a function of the cathode current density for several gap spacings. It can be shown that the point of maximum power density occurs at the point of maximum anode current density. Thus, if breakdown were initiated at a critical power density, the cathode current density at breakdown would vary with gap spacing and would be considerably less than the observed values of \( 10^6 \) to \( 10^9 \) amp/cm\(^2\).
Figure 23. Plot of anode current density, $J_a$, versus cathode current density, $J_c$ (after Vibrans\textsuperscript{21}).

(Solid lines, $F_o = 5 \times 10^5$ v/cm; dashed lines, $F_o = 2.5 \times 10^6$ v/cm)
APPENDIX B
MULTIPLE-POINT EMISSION CALCULATIONS

Assume there are many points on the cathode and that the β factors of these points have a Gaussian distribution. Then

\[ N(\beta_i) = N_0 e^{-\frac{(\beta_i - \beta_0)^2}{2\sigma^2}}. \]  \hspace{1cm} (B-1)

The total field emission current is then found by summing the individual currents; thus

\[ I = \sum_i N_0 e^{-\frac{(\beta_i - \beta_0)^2}{2\sigma^2}} A_1 K_1 \beta_i^2 F_0 \frac{e^{-K_3 / \beta_i F_0}}{\beta_i F_0}, \]  \hspace{1cm} (B-2)

where \( K_1 \beta_i^2 F_0 e^{-K_3 / \beta_i F_0} \) is a simplified form of the Fowler-Nordheim equation.

Assume the following:

(1) The individual areas are equal; \( A_1 = A \).

(2) There are a large number of points so that the summation can be replaced by an integration.

Then,

\[ I \approx K_3 \int_{-\infty}^{\infty} e^{-\frac{(\beta - \beta_0)^2}{2\sigma^2}} e^{-K_4 / \beta} \beta^2 d\beta, \]  \hspace{1cm} (B-3)

where \( K_3 = N_0 A K_1^2 F_0 \)

and \( K_4 = K_3 / F_0 \).

To obtain an approximate value for (B-3), change the lower limit to zero and expand the term \( e^{-K_4 / \beta} \) about \( \beta_0 \), i.e.,
$$e^{-K_3/\beta} = e^{-K_3/\beta_0}[1 + \frac{K_3}{\beta_0} (\beta-\beta_0) + \frac{1}{2} \left( \frac{2K_3}{\beta_0} + \frac{K_3^2}{\beta_0^2} \right) (\beta-\beta_0)^2 + \ldots] .$$

These three terms can be substituted in (B-3) and the integration can then be carried out. The result can be simplified by replacing \(2\beta^2\) by \(\beta_0^2\) and by replacing \(K_3\) by \(K_2/\beta_0\). The final result is

$$I \sim K_3 e^{-K_2/\beta_0 F_0} [(0.443\beta^3 + \beta^3 + 0.887\beta)$$

$$+ \frac{K_2}{\beta_0 F_0} (-0.666f^5 - 0.5f^4 + 0.443f^3 + 0.5f^2)$$

$$+ \frac{K_2}{\beta_0 F_0}^2 (0.333f^5 + 0.5f^4 + 0.222f^3)] . \quad (B-4)$$

It is apparent that the terms containing \(K_3/\beta_0 F_0\) inside the brackets must be dropped if \(I\) is to have the same functional dependence on the field as in the Fowler-Nordheim equation. In the measurable range of current \((10^7 < F < 10^8 \text{ v/cm})\), the maximum value of \(K_2/\beta_0 F_0\) is about 50. Thus, the requirement on \(f\) is

$$f < \beta_0 F_0/K_2 \leq 2 \times 10^{-2} .$$

If this condition holds, (B-4) reduces to

$$I \sim K_3 \beta_3 f e^{-K_2/\beta_0 F_0}$$

$$= N_0 A K_1 \beta_0 f (\beta_0 F_0)^2 e^{-K_2/\beta_0 F_0} . \quad (B-5)$$
APPENDIX C
CHANGES IN THE LENGTH OF AN ELECTRODE PROJECTION DUE TO SURFACE DIFFUSION

The forces exerted on surface atoms are related to the chemical potential of the surface as discussed by Herring. If the surface tension is assumed to be constant, and if the surface is assumed to have a single radius of curvature, the general expression for the chemical potential, \( \mu \), as given by Herring, can be simplified to the following form:

\[
\mu = \mu_0 + \left[ \gamma/R - p \right] \Omega
\]  

where \( \mu_0 \) is the chemical potential of a flat surface, \( \gamma \) is the surface tension, \( R \) is the radius of curvature, \( p \) is the applied normal surface stress, and \( \Omega \) is the atomic volume.

If this expression is applied to the case of an electrode projection subjected to an applied stress due to the electric field, \( F \), then

\[
p = \frac{1}{8\pi} F^2 .
\]  

Let us take the shape of the projection to be a prolate hemispheroid with base radius \( b \) and height \( c \), as in Section 2.2. Then, the radius of curvature is given by

\[
R = \frac{c_2 \left( \frac{a^2}{c^2} \right)^{1/2}}{\xi \left( \frac{a^2}{c^2} - 1 \right)^{1/2}} .
\]  

The rate at which atoms diffuse along the surface is given by

\[
\frac{dn}{dt} = \frac{D}{kT_o} \nabla \mu \text{ atoms/cm-sec} ,
\]
where \( D \) is the surface diffusion coefficient, \( A_0 \) is the area per atom, \( k \) is Boltzmann's constant, and \( T \) is the temperature. Since diffusion is along the surface, the gradient of \( \mu \) along the surface is given by

\[
(\nabla \mu)_{\xi} = \frac{\partial \mu}{\partial \xi} = \frac{\Omega}{c_a} \left[ \gamma (\eta_0^2 - 1)^{\frac{1}{2}} \left( \frac{\eta_0^2 + 2b^2}{c_a (\eta_0^2 - b^2)^{\frac{3}{2}}} \right) - \frac{F_0^2 \xi}{4\pi K (\eta_0^2 - 1)} \right], \tag{C-5}
\]

where, as in Section 2.2, \( K = \coth^{-1} \eta_0 - \frac{1}{\eta_0} \).

To simplify the calculations, assume that the projection is capped by a hemisphere of radius \( r_1 \), as is Appendix A, where

\[
r_1 = c \left( \frac{2b^2}{c^2} \right) \left( 1 + \frac{b^2}{c^2} \right). \tag{C-6}\]

The rate of change of the length of the projection, as determined by the surface diffusion of atoms to the hemispherical cap, is given approximately by

\[
\frac{dz}{dt} \approx - \frac{\Omega \, d\eta}{r \, dt}. \tag{C-7}
\]

Equation (C-7) can be rewritten using (C-4) and (C-5) and can then be evaluated at the value of \( \xi \) given in (A-11). The result can be further simplified by noting that typically \( b^2 \ll c^2 \) and hence can be neglected in several of the terms. The final result is

\[
\frac{dz}{dt} \approx - \frac{\Omega^2 D}{56A_0 kTb^2} \left( \frac{\xi}{b} \right)^4 \left[ \frac{1.34\gamma}{c} - \frac{F_0^2 c^2}{4\pi K b^2} \right]. \tag{C-8}
\]

If the projection is to increase in length, the second term in the brackets must be greater than the first. As a numerical example, take \( b/c = 0.1 \) and \( c = 10^{-4} \) cm. The value of \( \gamma \) given by Barbour, et al., is 2900 dynes/cm. With these values, the minimum value of \( F_0 \) which will
produce an increase in length is

\[ F_0 \approx 1.3 \times 10^6 \text{ v/cm}. \quad (C-9) \]

Note that this is the average field at the electrode surface; the actual field at the tip of the projection is about \(6.5 \times 10^7\) v/cm, since a b/c value of 0.1 gives an enhancement factor of about 50. Thus, the field at the tip of the projection is very close to the breakdown value.

For tungsten, Barbour, *et al.*, give the following values for the physical constants appearing in (C-8):

- \( A_0 = 10^{-15} \text{ cm}^2/\text{atom} \)
- \( \Omega = 1.57 \times 10^{-23} \text{ cm}^3/\text{atom} \)
- \( D = D_0 e^{-Q/kT} \)
- \( D_0 = 4 \text{ cm}^2 \)
- \( Q = 5.03 \times 10^{-12} \text{ erg/atom} \)

If these values are substituted in (C-8), the growth rate as a function of temperature can be calculated. As a numerical example, take \( F_0 = 1.4 \times 10^6 \text{ v/cm}, \ b/c = 0.1, \text{ and } c = 10^{-4} \text{ cm} \). The value of \( \frac{dz}{dt} \) is then found to approach \(10^{-7} \text{ cm/sec} \) when the temperature reaches \(1500^\circ K\). This corresponds to a 10 per cent change in length in 100 seconds which is an appreciable growth rate.
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