OFF-LINE SENSITIVITY TUNING: AN ARC WELDING APPLICATION

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This report presents the sensitivity point tuning theory and its application for off-line optimization of a controller. A consumable electrode gas metal arc welding process is used to implement the controllers and the tuning procedure. A single-input single-output case is treated, first using a PI controller. The nonlinearity of the system, then encourages the realization of a simple nonlinear controller. The tuning procedure is also implemented on a double-input double-output system. The structure of the controller chosen is also a PI with all the crossing terms. Finally the tuned value for this controller is compared to a classical LQ design and the robustness of the method is tested using different operating conditions.
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BY
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ABSTRACT

This report presents the sensitivity point tuning theory and its application for off-line optimization of a controller. A consumable electrode gas metal arc welding process is used to implement the controllers and the tuning procedure. A single-input single-output case is treated, first using a $PI$ controller. The nonlinearity of the system, then encourages the realization of a simple non linear controller. The tuning procedure is also implemented on a double-input double-output system. The structure of the controller chosen is also a $PI$ with all the crossing terms. Finally the tuned value for this controller is compared to a classical $LQ$ design and the robustness of the method is tested using different operating conditions.
I would first like to thank Professor P.V. Kokotović for making my stay in Urbana possible and for all of his help and support. It is my hope that other students from the Ecole Nationnale Supérieure d’Ingénieurs Electriciens de Grenoble (ENSIEG) will also be able to take advantage of the rich research environment I found here.

I would also like to thank Darrel Recker, Jeff Schiano and especially Jim Ross for the cooperation, assistance and encouragement that made these experiments on Arc Welding possible.

All experimental work has been conducted at the United States Army Construction Engineering Research Laboratory (USA-CERL) as part of a joint project with the Illinois Decision and Control Laboratory on consumable-electrode gas metal arc welding. I would like to thank Bob Weber, for the administrative assistance that allowed me to work at CERL.
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Chapter 1

INTRODUCTION

In the classical approach to controller design, the knowledge of the model entirely determines the controller. If the model is poorly known and the disturbance difficult to model, this method cannot give acceptable results. Adaptive control is an alternative, allowing the controller to learn about the process. The sensitivity method presented in this paper tunes the parameters of a controller for optimal response using the real process (and not only the model). This method can also be used to design a simple non-linear controller or to find the optimal parameters of a known non-linear system.

The sensitivity methods, described in Chapter 2, were first developed in the 1960's and have been widely used with the analog computers. Since the development of digital computers, many new methods have been developed to achieve controllers and the use of sensitivity theory has waned. They still represent however, a very attractive way to optimize simple controllers because the real process is used for their adaptation and the adaptation algorithm is simple and independent of the plant order.

Off-line sensitivity optimization has been implemented in the Gas Metal Arc Welding Process described in Chapter 3. The experimental work has been done through a joint project between the United States Army Construction Engineering Research Laboratory (USA-CERL) and the Illinois Decision and Control Laboratory, the ultimate objective of which is to develop an automated welding system that will produce consistently good welds despite variation in materials and experimental conditions.

In Chapter 4, a single-input and single-output system is considered and a $PI$ controller is designed and optimized using the sensitivity function approach. A simple non-linear controller is then implemented to compensate the non-linearity of the process.

In Chapter 5, a two-input and two-output system is described. Sensitivity methods are implemented to optimize a generalized $PI$ controller. Simulations as well as experimental results are presented. The optimal design is then compared with a classical Linear Quadratic approach.

The results of this paper prove the practical advantages of the off-line sensitivity method, even for relatively complex systems.
Chapter 2

OFF LINE SENSITIVITY METHODS

The sensitivity methods for optimization of system parameters have been used for nearly 30 years. Previous work by Kokotović [7] and others [6] presents some simple, practical methods for generation and use of sensitivity function in the design of control systems. These results however, were mainly developed for use by analog computers. With the appearance of the digital computer, new design methods have been developed, and time simulation design is no longer in use. Never the less, sensitivity tuning still represents a practical way to find the optimal set of parameters for a simple controller. It is also an innovative alternative to classical adaptive control.

2.1 Sensitivity Point Tuning Concept

Let us consider a multivariable system function of a parameter vector \( \Theta \)

\[
\Theta = \begin{bmatrix}
\theta_1 \\
\theta_2 \\
\vdots \\
\theta_p
\end{bmatrix}
\]

Let us describe the system by Figure 2.1

\[Z(z) \rightarrow H(z) \rightarrow Y(z) \]

\[w_i(z) \rightarrow \theta_i \rightarrow H(z) \rightarrow Y(z) \]

\[\theta_i \]

**Figure 2.1:** Model of a parametrized system.

For a given input \( Z(z) \) and some small derivation \( \delta \Theta \) of the nominal \( \Theta \) parameters, the output can be expressed as the Taylor expansion

\[
Y(\Theta + \delta \Theta, z) = Y(\Theta, z) + \sum_{i=1}^{p} \frac{\partial}{\partial \theta_i} Y(\Theta, z) \delta \theta_i + \cdots
\]

where \( \frac{\partial}{\partial \theta_i} Y(\Theta, z) \) is define as the sensitivity of the output with respect to the parameter \( \theta_i \).

Note that the Taylor expression can also be written

\[
Y(\Theta + \delta \Theta, z) = Y(\Theta, z) + \nabla_{\Theta} Y(\Theta, z) \delta \Theta + \cdots
\]
If the expression is truncated at the first-order term the difference \( E(Z) \) between the nominal response \( Y(\Theta, z) \) and the derivative response \( Y(\Theta + \delta \Theta, z) \) can be approximated by

\[
E(z) = \nabla_\Theta Y(\Theta, z) \delta \Theta.
\]

Therefore the knowledge of the sensitivity functions \( \frac{\partial}{\partial \Theta} Y(\Theta, z) \) allows the choice of approximate values of \( \delta \Theta \) in order to minimize \( E(z) \). Repetition of this process through several iterations should find the optimal parameter \( \Theta \) that would reduce the magnitude of \( E(z) \) to a minimal value.

### 2.2 Sensitivity Functions

Since the system considered is supposed to be linear, a small change \( \delta \theta_i \) in \( \theta_i \) will produce the change \( \delta w_i \) and \( \delta Y \). The system described in Figure 2.1 can then be rewritten as followed.

An equivalent system can be formed considering \( w_i(z) + \delta w_i \) as an additional input.

By superposition, the output will contain only the component due to the change in the parameter when the input \( Z(z) \) is removed.
By taking the limit $\delta \theta_i \to 0$, the output of the new system becomes the desired partial derivative:

![Diagram](image)

This method shows us the way to compute the sensitivity function. It is clear that in order to generate an exact sensitivity function, $w_i$ must be available and the transfer function $H(z)$ must be known.

The complete sensitivity system can be designed as in Figure 2.2.

![Diagram](image)

**Figure 2.2: Complete sensitivity system.**

### 2.2.1 Noise and disturbance

Real systems will have additional inputs due to noise and other disturbances which are not measurable. Such disturbances can be represented as another input $D(z)$ to the system as described in Figure 2.3.

![Diagram](image)

**Figure 2.3: Model of a parametrized system with disturbances.**

Sensitivity functions are obtained setting all the inputs, $Z(z)$ and $D(z)$, to zero. The sensitivity functions therefore remain valid. The disturbances however should not be functions of the parameters $\theta_i$.

The problem of tuning the parameters of a controller, as shown in Figure 2.4, is typically part of the problems described.
2.2.2 Computed sensitivity function

If the model of the system is well known, the sensitivity function can be computed using the model of the process in the sensitivity filter. The computation of these functions can be done on line. It is clear that a specific sensitivity filter should be used for each parameter.

2.2.3 Experimental sensitivity function

If the model of the process is unknown, the sensitivity function cannot be computed, but the real process can be used to obtain the sensitivity function. An initial experiment is conducted to measure all the $w_i$ functions, then a second experiment is completed using $w_i$ as an input to the system.

In this case, however, the disturbance cannot be removed when the sensitivity functions are calculated. Step 2 must be reproduced for each parameter.
2.3 Off-Line Parameter Tuning

Sensitivity functions permit automatic tuning of the parameters of a controller in order to obtain a desired response. Let us define an nominal trajectory $Y_a$ and the error of the closed loop response $E$.

$$E(k) = Y_a(k) - Y(k).$$

The goal of the tuning is to minimized the amplitude of $E(k)$ over an entire period of time ($k \in [0, N]$).

According to the relation

$$E(z) = \nabla_{\Theta} Y(\Theta, z)\delta\Theta$$

we can write

$$\begin{bmatrix} e_1(0) \\ \vdots \\ e_1(N) \\ e_2(0) \\ \vdots \\ e_2(N) \end{bmatrix} = \begin{bmatrix} \frac{\partial y_1(0)}{\partial \theta_1} & \cdots & \frac{\partial y_1(0)}{\partial \theta_p} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_1(N)}{\partial \theta_1} & \cdots & \frac{\partial y_1(N)}{\partial \theta_p} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_2(0)}{\partial \theta_1} & \cdots & \frac{\partial y_2(0)}{\partial \theta_p} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_2(N)}{\partial \theta_1} & \cdots & \frac{\partial y_2(N)}{\partial \theta_p} \end{bmatrix} \begin{bmatrix} \delta \theta_1 \\ \delta \theta_2 \\ \vdots \\ \delta \theta_p \end{bmatrix}$$

or in matrix notation

$$E = S\delta\Theta.$$ 

The solution consists of finding $\delta\Theta$ to minimize the quadratic cost function

$$J = \sum_{k=0}^{N} E'(k)E(k).$$

2.3.1 Least squares estimation

The solution of the least square problem is given by

$$\delta\Theta = (S'S)^{-1}S'E.$$ 

If $S$ is not full rank $(S'S)^{-1}$ does not exist, but this indicates that some columns of $S$ are linearly dependent, and hence some parameter are linearly dependent. It is clear that if this occurs, the number of parameters is too large and there is not a single way of choosing those parameters. The number of parameters should then be reduced. A new set of parameters $\Theta$ can be implemented using

$$\Theta_{new} = \Theta_{old} + \delta\Theta.$$
2.3.2 Gradient algorithm

This parameter update law is given by

$$\Theta_{\text{new}} = \Theta_{\text{old}} + \Gamma \nabla_\Theta J$$

where $J$ is the quadratic criterion defined above and $\Gamma$ is a parameter matrix to adjust the speed of convergence.

$$\nabla_\Theta J = \begin{bmatrix} \sum_{k=0}^{N} E'(k) \frac{\partial E(k)}{\partial \theta_1} \\
\vdots \\
\sum_{k=0}^{N} E'(k) \frac{\partial E(k)}{\partial \theta_p} \end{bmatrix}$$

since

$$\nabla_\Theta J = S'E$$

and therefore, the parameter update law becomes:

$$\Theta_{\text{new}} = \Theta_{\text{old}} + \Gamma S'E.$$

This algorithm is called gradient because $\nabla_\Theta J$ can be interpreted as a direction in which the parameters should be changed. The $\Gamma$ matrix can be interpreted as the step size in the change of parameters. The choice of $\Gamma$ is generally determined through trial-and-error experiments.

2.3.3 Sensitivity tuning

In the rest of this report the sensitivity tuning has been implemented using the following algorithm:

- 1: An initial experiment is conducted using the original set of parameters. During the experiment, the sensitivity functions are calculated using the model of the process.
- 2: At the end of the experiment the parameters are updated using the least square estimation method.
- 3: A second experiment is conducted using the new set of parameters and the new sensitivity functions are computed.
- 4: Steps 2 and 3 are repeated until the parameters converge.

This procedure is called off-line sensitivity tuning. The convergence of this algorithm is explored in [8] including problems due to the use of approximate models to calculate sensitivity functions.

On-line sensitivity tuning has also been successfully implemented in the arc welding process by D. E. Henderson, using the gradient algorithm. For further information see [9].
Chapter 3

DESCRIPTION OF THE ARC WELDING PROCESS

3.1 Gas Metal Arc Welding

Gas metal arc welding is an electric welding process. An arc is created by a direct current power supply in order to melt a consumable electrode and the workpiece to be welded. The region surrounding the weld puddle is purged with a shield gas to prevent oxidation and contamination of the weld joint. The torch is moved along the workpiece in order to complete the welding. For further information about gas metal arc welding, see [4].

![Gas Metal Arc Welding Diagram]

Figure 3.1: Gas Metal Arc Welding.

The goal of the control loop is to achieve a good weld. Many factors can influence the quality of a weld, the most important of which are: welding amperage, arc voltage, travel speed, electrode extension, electrode inclination, electrode size and weld joint position.

Previous research [3] has shown that the shape of the weld puddle plays an important role in the integrity of the weld joint. An image processing system was therefore designed to estimate Puddle Width (PW) and Puddle Area (PA). The control inputs chosen were Arc current (AI) and Travel Rate (TR).
3.2 Experimental Setup

A CCD camera is attached to the torch so as to view the puddle from an angle. An image processing system estimate the Puddle Width (PW) and Puddle Area (PA) using the algorithm presented by Baheti [5]. The puddle boundary can be approximated by an ellipse and the image processing system fits the optimal ellipse to the puddle using a least square estimation.

The Arc Current is not a direct input of the system: the power supply provides a constant voltage and the Arc Current is determined by the wire feed rate. A closed loop was designed to regulate the Arc Current using the wire feed rate. The response time of this loop is approximately 10 times faster than the puddle geometry dynamics. The Arc Current can thereby be considered constant in the puddle geometry loop. A closed loop was also designed to drive the torch velocity.
Chapter 4

THE ARC CURRENT CONTROLLER

4.1 Description of the Problem

In the Gas Arc Welding process, a controller was designed to drive the arc current. This variable is not a direct input of the system, but is determined by the wire feed rate. Figure 4.1 represents the model of the system using the notation: $VM$ for Voltage Motor, $WF$ for Wire Feed and $AI$ for Arc Current.

![Figure 4.1: Model of the Arc Dynamics.](image)

The difficulty of this problem is due to two factors:

- The pinch rollers that drive the wire are eccentric. This produces an important torque disturbance at a frequency around $1.5\, Hz$. This disturbance also affect the arc dynamics since the Wire Feed measured is in fact the Speed of the motor. If the Speed of the motor is constant then the "true" Wire Feed is strictly sinusoid.

- The arc dynamic is highly non-linear and the model is poorly known. This model is also very dependent upon the operating point.

The Structure of the controller chosen was very simple: two loops were closed, one on the Wire Feed and one on the Arc Current. Each loop contains a Proportional and Integral Controller. Figure 4.2 represents the closed loop system.

![Figure 4.2: Model of the Arc current controller.](image)
4.2 Model of the Disturbances

The disturbances can be modelized, as in Figure 4.3. $D_1$ represents a torque disturbance due to the wire spool and the eccentricity of the pinch rollers. $D_2$ represents the sinusoid factor that exists between the speed of the motor (Wire Feed measured) and the “true” Wire Feed. $D_3$ is the noise in the Arc Current measurement.

![Figure 4.3: Model of the disturbances.](image)

4.3 Self Tuning Using Sensitivity Function

4.3.1 The models

Previous work done by B. W. Greene on the Arc Current controller [10], shows that according to the time response of the system, a sampling period of $T = 20ms$ is reasonable, since the response time of the system is approximately 0.2s. The models have been found using identification routines. The nominal model for the arc was found around 360 A.

Model of the Motor: $\frac{9.6549}{z-0.6945}$

Model of the Arc around 360 A: $\frac{0.1153z-0.0501}{z^2-1.5168z+0.5949}$

The identified model of the motor matches almost perfectly with the model found using physical equations of the electric motor. This model of the arc, however, appears to be quite different from those in previous research. Since the identification procedure tries to match a linear model with a non-linear process, the results are not expected to be reliable. The poles and zeros of the models are:

<table>
<thead>
<tr>
<th></th>
<th>Zero</th>
<th>Pole</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor</td>
<td>-</td>
<td>0.6945</td>
</tr>
<tr>
<td>Arc</td>
<td>0.4345</td>
<td>0.7584 ± 0.1405i</td>
</tr>
</tbody>
</table>
The models have stable poles and zeros. The objective of the design is to match the closed loop response with a reference trajectory. The reference model was chosen as the discretization of a continuous-time second order system with an \( \omega_c \) of 4 Hz and a \( \zeta \) of 0.7. This gives us the following transfer function.

\[
\text{Reference Model} : \frac{0.099z+0.0781}{z(z^2-1.3141z+0.4912)}
\]

4.4 Experimental Results

4.4.1 Tuning of all parameters

A first set of experiments was conducted, in which the parameters were updated at the end of each experiments. The parameter tuning history is in Table 4.1. Experiment 1, 2, 4 and 5 are plotted in Figure 4.4 and Figure 4.5 shows the plot of the parameter convergence.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>( K_p_a )</th>
<th>( K_i_a )</th>
<th>( K_p_m )</th>
<th>( K_i_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.0000</td>
<td>0.5000</td>
<td>0.0080</td>
<td>0.0020</td>
</tr>
<tr>
<td>2</td>
<td>1.3684</td>
<td>0.5187</td>
<td>0.0181</td>
<td>0.0020</td>
</tr>
<tr>
<td>3</td>
<td>1.7807</td>
<td>0.2659</td>
<td>0.0269</td>
<td>0.0035</td>
</tr>
<tr>
<td>4</td>
<td>1.6155</td>
<td>0.3038</td>
<td>0.0550</td>
<td>0.0033</td>
</tr>
<tr>
<td>5</td>
<td>1.0237</td>
<td>0.3098</td>
<td>0.1375</td>
<td>0.0086</td>
</tr>
<tr>
<td>6</td>
<td>1.7530</td>
<td>0.2845</td>
<td>0.2131</td>
<td>0.0108</td>
</tr>
</tbody>
</table>

The main result of these experiments is that \( K_p_m \) and \( K_i_m \) diverge. In fact they converge to infinity, because in order to have the desired output for the Arc Current, the optimal motor closed loop transfer function must be 1. \( K_p_m \) and \( K_i_m \) therefore converge to infinity. These values are not practically applicable, mainly because the linearity approximation of the system is valid only locally. Experiment 6 could not be conducted because the system was unstable.

4.4.2 Tuning of the arc current parameters

The idea was to freeze \( K_p_m \) and \( K_i_m \) at the optimal practical value found in the first set of experiments and then to find the optimal \( K_p_a \) and \( K_i_a \). The original set of parameters chosen consisted of those used in Experiment 4. A second set of experiment was conducted. The parameter tuning history is in Table 4.2. Figure 4.7 shows the plot of the parameter convergence and Figure 4.6 illustrates 2 of the 7 experiments.
Table 4.2: Second parameter tuning history.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$K_{pa}$</th>
<th>$K_{ia}$</th>
<th>$K_{pm}$</th>
<th>$K_{im}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1.6155</td>
<td>0.3038</td>
<td>0.0550</td>
<td>0.0033</td>
</tr>
<tr>
<td>8</td>
<td>1.9843</td>
<td>0.3906</td>
<td>0.0550</td>
<td>0.0033</td>
</tr>
<tr>
<td>9</td>
<td>2.1194</td>
<td>0.3552</td>
<td>0.0550</td>
<td>0.0033</td>
</tr>
<tr>
<td>10</td>
<td>2.3133</td>
<td>0.3791</td>
<td>0.0550</td>
<td>0.0033</td>
</tr>
<tr>
<td>11</td>
<td>2.4642</td>
<td>0.4018</td>
<td>0.0550</td>
<td>0.0033</td>
</tr>
<tr>
<td>12</td>
<td>2.2347</td>
<td>0.3574</td>
<td>0.0550</td>
<td>0.0033</td>
</tr>
<tr>
<td>13</td>
<td>2.5261</td>
<td>0.4013</td>
<td>0.0550</td>
<td>0.0033</td>
</tr>
</tbody>
</table>

After Experiment 10, we can assume that the parameters have converged to their optimal value, the change in the parameters being due to the noise. Since the optimal parameter for one experiment (using one particular realization of the noise) may not be optimal for another realization, the parameters do not converge but oscillate around the optimal value for any realization of the noise. These oscillations can be reduced through longer experiments.

4.4.3 Gain scheduling

The validity of the final gains has been tested running two experiments at different operating points. Experimental results are given in Figure 4.8. Those experiments also show how the parameters should be changed in order to improve the controller for a wider range of operating values. Table 4.3 shows the parameters at different operating points.

Table 4.3: Parameter values for different operating points.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal current</th>
<th>Low current</th>
<th>High current</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{pa}$</td>
<td>2.5261</td>
<td>3.6730</td>
<td>2.1074</td>
</tr>
<tr>
<td>$K_{ia}$</td>
<td>0.4013</td>
<td>0.3550</td>
<td>0.3954</td>
</tr>
<tr>
<td>$K_{pm}$</td>
<td>0.0550</td>
<td>0.0550</td>
<td>0.0550</td>
</tr>
<tr>
<td>$K_{im}$</td>
<td>0.0033</td>
<td>0.0033</td>
<td>0.0033</td>
</tr>
</tbody>
</table>

These results can help us design a non-linear controller using gain scheduling. Since the change in the operating point affects mainly $K_{pa}$, it is possible to drive $K_{pa}$ by the desired arc current. $K_{pa}$ was chosen as a linear function of the desired arc current.

$$K_{pa} = -0.014 \times AI + 7.54.$$  

The result of this improvement is presented in Figure 4.9. The controller, as expected, performs better after this improvement, especially at a high current. A more precise analysis would optimize this controller, but was not possible due to time constraints. The experiments illustrated in figure 4.8 should be performed allowing only $K_{pa}$ to move and should then be successively repeated in order to find the converged value for the various operating points.
considered. Another consideration is that the linear approximation for the variation of $Kp_a$ is not the optimal choice, rather a quadratic or exponential choice would be more appropriate.

Rather than taking $Kp_a$ as the only parameter to be tuned, $Kp_1$ and $Kp_2$ could be considered as two new parameters to be tuned directly such that

$$Kp_a = Kp_1 * A1 + Kp_2.$$ 

The controller is no longer linear but the sensitivity function relative to $Kp_1$ and $Kp_2$ could still be calculated, and therefore a tuning procedure which optimizes these parameters over the entire operating value of the Arc Current could be realized.

### 4.4.4 Conclusion

The final design implemented was nevertheless effective and was used throughout the rest of the experiments. The Figure 4.10 shows the frequency analysis of the noise in both open and closed loops. The open loop graph shows all the frequency of the different disturbances described in section 4.1. The first set of picks, from 0 to 1 Hz, corresponds to the spool disturbance. The second set of picks, from 1 to 4 Hz, corresponds to the eccentricity of the pinch rollers. The closed loop shows the improvement achieved by rejecting most of the disturbances.

This example illustrates the practical use of off-line sensitivity tuning. This method is easy to implement and since it takes into consideration all unmodeled disturbances in the criterion, the parameters are optimal for the real process, not for the model of the process. In this case, since the model of the arc is poorly known, off-line sensitivity tuning method provides an advantage over classical design methods. Its parameters are optimal for the rejection of disturbance even though the order of the controller is very low.

Along with a simple gain scheduled controller, designed to optimize the controller for the non-linearity of the problem, and the suggestions given for improvements, the off-line tuning procedure provides an effective method for control design.
Figure 4.4: Arc current experiments (First Set).
Figure 4.5: Parameter convergence (First Set).

Figure 4.6: Arc current experiments (Second Set).
Figure 4.7: Parameter convergence (Second Set).

Figure 4.8: Arc experiments at different operating points.
Figure 4.9: Arc experiments using gain scheduling.

Figure 4.10: Arc experiment: Frequency analysis of the noise.
5.1 Description of the problem

The problem considered that of controlling the puddle geometry on a Gas Metal Arc Welding Process. The two controlled inputs are the Arc Current (AI) and the Travel Rate (TR) of the torch (See Chapter 2). The two measure outputs, which most affect the quality of the welding, are the Puddle Width (PW) and the Puddle Area (PA).

![Figure 5.1: Model of Puddle Geometry.](image)

This process is highly coupled: a step in one input affects almost equally both outputs. The sensitivity method has been applied to this system. The form of the controller was made very simple in order to reduce the number of parameters that could be changed. Figure 5.2 shows the structure of the controller.

![Figure 5.2: Model of the Sensitivity controller.](image)

The controller is a Proportional and Integral controller. Given the errors, $E_{PW}$ and $E_{PA}$, and the integral terms, $V_{PW}$ and $V_{PA}$, the controller can be written:

$$
\begin{bmatrix}
TR \\
AI
\end{bmatrix}
= 
\begin{bmatrix}
K_{p1} & K_{p2} & K_{i1} & K_{i2} \\
K_{p3} & K_{p4} & K_{i3} & K_{i4}
\end{bmatrix}
\begin{bmatrix}
E_{PW} \\
E_{PA} \\
V_{PW} \\
V_{PA}
\end{bmatrix}.
$$
5.2 The models

The models have been found using identifications routines. The goal of these identification was to produce simple models, therefore the structure chosen was a second order transfer function with delays. The image processing system analyzes an image in approximately 0.2s. The sampling period was chosen so that a new measurement would be available (\( T = 0.24s \)). The transfer function founds were:

\[
\text{Travel Rate to Puddle Width : } \frac{10^{-3}(3.0421z-4.5780)}{z^2-1.7455z+0.7820}
\]

\[
\text{Travel Rate to Puddle Area : } \frac{10^{-3}(3.8764z-4.7215)}{z(z^2-1.7455z+0.7820)}
\]

\[
\text{Arc Current to Puddle Width : } \frac{10^{-3}(0.0859z+0.0616)}{z^2-0.4321z-0.3501}
\]

\[
\text{Arc Current to Puddle Area : } \frac{10^{-3}(-0.1031z+0.5402)}{z^2-0.5893z-0.2196}
\]

A plot of the identification experiments can be found in Figure 5.4. The poles and zeros of the model can be found in the following table.

<table>
<thead>
<tr>
<th>Zero</th>
<th>Poles</th>
</tr>
</thead>
<tbody>
<tr>
<td>( PW ) ( TR )</td>
<td>1.5049 ( 0.8728 \pm 0.1423 ) i</td>
</tr>
<tr>
<td>( PA ) ( TR )</td>
<td>1.2180 ( 0.8795 \pm 0.1244 ) i</td>
</tr>
<tr>
<td>( PW ) ( AI )</td>
<td>-0.7167 ( -0.4139 ) and ( 0.8459 )</td>
</tr>
<tr>
<td>( PA ) ( AI )</td>
<td>-0.2589 ( -0.2589 ) and ( 0.8482 )</td>
</tr>
</tbody>
</table>

As can be seen, the models have instable zeros, but the poles are stable. The absolute value of the slower poles is approximately 0.88.

The system also appeared to be highly non-linear. For example, the delay observed on the Puddle Area is dependent on the Travel Rate, therefore the models described are only valid locally. Higher order transfer functions can be found, but the location of the poles becomes very sensitive to the operating point and thereby the model becomes less general.
5.3 Off-line sensitivity tuning

The controller contains 8 parameters and since each parameter influences both outputs, 16 sensitivity functions must be computed. The initial set chosen for the controller was with the crossing terms of the controller null. The controller then becomes 2 independent PI controllers, with each input controlling only one output.

5.3.1 Simulations

The goal of these simulations was to tune the parameters of the controller so that the output of the system follows as closed as possible a reference trajectory. The reference model was chosen according to physical considerations. Uniformity and smoothness of variation are desirable qualities in a weld joint, because they strongly relate to the overall strength of the joint. Based on this information, the reference models were chosen as unitary second order transfer functions with poles of 0.9 for the Puddle Width and 0.95 for the Puddle Area.

\[
\text{Puddle Width : } \frac{0.01}{z^2-1.80z+0.81}
\]

\[
\text{Puddle Area : } \frac{0.0025}{z^5(z^2-1.90z+0.9025)}
\]

The Puddle Area is a factor that has rarely been controlled and therefore its effects on the weld quality are not well defined. The time response for the Puddle Area was then made quite large (approximately 20s). The input of the system was chosen as steps decoupled on each input.

The algorithm described in section 2.3.2 was used to tune all the parameters in order to find the optimal controller. Figure 5.5 shows the plot of the first and optimal simulation. Figure 5.6 shows the plot of the convergence of the parameters. A time history of the parameters can be found in Table 5.1.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>( K_{P1} )</th>
<th>( K_{P2} )</th>
<th>( K_{P3} )</th>
<th>( K_{P4} )</th>
<th>( K_{i1} )</th>
<th>( K_{i2} )</th>
<th>( K_{i3} )</th>
<th>( K_{i4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>20.0000</td>
<td>-1.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>20.0000</td>
</tr>
<tr>
<td>1</td>
<td>-4.3446</td>
<td>2.0410</td>
<td>-24.4392</td>
<td>114.0607</td>
<td>-1.3460</td>
<td>0.0250</td>
<td>-17.6966</td>
<td>8.2281</td>
</tr>
<tr>
<td>2</td>
<td>-5.9140</td>
<td>0.1483</td>
<td>-106.8355</td>
<td>33.8946</td>
<td>-1.3147</td>
<td>0.2242</td>
<td>-16.3771</td>
<td>12.6191</td>
</tr>
<tr>
<td>3</td>
<td>-5.9073</td>
<td>0.3919</td>
<td>-100.1119</td>
<td>35.8800</td>
<td>-1.3443</td>
<td>0.2126</td>
<td>-17.1029</td>
<td>12.4256</td>
</tr>
<tr>
<td>4</td>
<td>-5.9121</td>
<td>0.3678</td>
<td>-100.2844</td>
<td>35.8593</td>
<td>-1.3442</td>
<td>0.2122</td>
<td>-17.1074</td>
<td>12.4303</td>
</tr>
<tr>
<td>5</td>
<td>-5.9124</td>
<td>0.3686</td>
<td>-100.2419</td>
<td>35.8579</td>
<td>-1.3442</td>
<td>0.2122</td>
<td>-17.1078</td>
<td>12.4302</td>
</tr>
</tbody>
</table>

According to this table, we can consider that in three steps the parameters have converged to their optimal value. A second set of simulations have been done adding noise to the output.

21
The magnitude of the noise as well as the color of the noise have been chosen to correspond as closely as possible to the real noise. The results are shown in Figure 5.7 and Figure 5.8 and Table 5.2

Table 5.2: Simulated tuning history with noise.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$K_p_1$</th>
<th>$K_p_2$</th>
<th>$K_p_3$</th>
<th>$K_p_4$</th>
<th>$K_i_1$</th>
<th>$K_i_2$</th>
<th>$K_i_3$</th>
<th>$K_i_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>20.0000</td>
<td>-1.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>20.0000</td>
</tr>
<tr>
<td>1</td>
<td>-2.3594</td>
<td>0.7943</td>
<td>9.6960</td>
<td>96.1617</td>
<td>-1.2791</td>
<td>-0.0133</td>
<td>-16.3775</td>
<td>9.7051</td>
</tr>
<tr>
<td>2</td>
<td>-3.7244</td>
<td>-0.0474</td>
<td>-83.3376</td>
<td>4.2532</td>
<td>-1.1621</td>
<td>0.2348</td>
<td>-14.5529</td>
<td>12.1401</td>
</tr>
<tr>
<td>3</td>
<td>-3.2178</td>
<td>0.3886</td>
<td>-30.3967</td>
<td>52.9678</td>
<td>-1.0942</td>
<td>0.2694</td>
<td>-12.9171</td>
<td>13.4405</td>
</tr>
<tr>
<td>4</td>
<td>-3.9160</td>
<td>-2.0195</td>
<td>-85.3143</td>
<td>-70.0732</td>
<td>-1.2250</td>
<td>-0.0133</td>
<td>-16.3775</td>
<td>9.7051</td>
</tr>
<tr>
<td>5</td>
<td>-6.0466</td>
<td>3.0628</td>
<td>-106.1826</td>
<td>50.8661</td>
<td>-1.1359</td>
<td>0.2339</td>
<td>-13.5528</td>
<td>13.1904</td>
</tr>
<tr>
<td>6</td>
<td>-4.5439</td>
<td>-0.5808</td>
<td>-107.2888</td>
<td>5.9683</td>
<td>-1.2237</td>
<td>0.2370</td>
<td>-14.2017</td>
<td>12.3972</td>
</tr>
<tr>
<td>7</td>
<td>-4.6343</td>
<td>0.6234</td>
<td>-84.7324</td>
<td>46.4400</td>
<td>-1.2365</td>
<td>0.1703</td>
<td>-15.9877</td>
<td>11.0924</td>
</tr>
<tr>
<td>8</td>
<td>-1.9908</td>
<td>3.4860</td>
<td>-28.4469</td>
<td>107.7205</td>
<td>-1.2848</td>
<td>0.2169</td>
<td>-17.7017</td>
<td>13.7761</td>
</tr>
<tr>
<td>9</td>
<td>-3.9884</td>
<td>-0.7330</td>
<td>-79.9870</td>
<td>39.0001</td>
<td>-1.3586</td>
<td>0.2004</td>
<td>-18.4830</td>
<td>12.0414</td>
</tr>
</tbody>
</table>

As expected the parameters oscillate around the optimal values. Surprisingly, the parameters have converged to almost the same value as found in the simulation without noise. Since our optimization criterion tries to make the system follow the reference trajectory, the weight of the regulation problem is far less important in the criterion than the weight of the tracking problem. This also means that this controller structure is not very effective in rejecting disturbances due to noise. Since our system has a large amount of delay, the PI structure is not optimal for the problem of regulation.

5.3.2 Experimental results

The Table 5.2 also show us that the proportional parameters are more affected by the noise than are the integral parameters. The variation can be very large, therefore in the experiments, the changes of the proportional parameters have been limited by a factor of 2. In order to reduce those oscillations longer runs would have been effective, but given the length of the plates available, such was not practically possible.

The experiments were conducted as in the simulation, except that the change of the proportional parameters were reduced by a factor of 2. Table 5.3 display the parameters tuning history. Figure 5.9 show the plots of the first and last experiments, Figure 5.10 show the plot of the parameters convergence, and Figure 5.11 show the plots of the inputs.

As we can see, the convergence of the parameters is achieved within 5 steps. The proportional parameters seem a much less affected by the presence of noise. The factor of 2 introduced to reduce the oscillation is not only responsible for fewer variations, but also the fact that the real noise has a different shape from the one used in the simulations.
Table 5.3: Experimental tuning history.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$K_{P_1}$</th>
<th>$K_{P_2}$</th>
<th>$K_{P_3}$</th>
<th>$K_{P_4}$</th>
<th>$K_{i_1}$</th>
<th>$K_{i_2}$</th>
<th>$K_{i_3}$</th>
<th>$K_{i_4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>20.0000</td>
<td>-1.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>20.0000</td>
</tr>
<tr>
<td>1</td>
<td>-1.1925</td>
<td>2.3711</td>
<td>35.0486</td>
<td>51.5729</td>
<td>-1.2698</td>
<td>0.2939</td>
<td>-19.0265</td>
<td>9.1279</td>
</tr>
<tr>
<td>2</td>
<td>-2.1026</td>
<td>3.4409</td>
<td>64.0566</td>
<td>92.3653</td>
<td>-1.5284</td>
<td>0.3177</td>
<td>-22.8365</td>
<td>13.1700</td>
</tr>
<tr>
<td>3</td>
<td>-2.0055</td>
<td>5.8256</td>
<td>89.0639</td>
<td>106.0359</td>
<td>-1.6846</td>
<td>0.4255</td>
<td>-24.2017</td>
<td>15.2910</td>
</tr>
<tr>
<td>4</td>
<td>-2.3320</td>
<td>6.9133</td>
<td>119.8094</td>
<td>118.5674</td>
<td>-1.5878</td>
<td>0.2626</td>
<td>-24.1673</td>
<td>15.3913</td>
</tr>
<tr>
<td>5</td>
<td>-2.3059</td>
<td>7.9281</td>
<td>96.0027</td>
<td>139.2306</td>
<td>-1.3231</td>
<td>0.2973</td>
<td>-22.1522</td>
<td>15.6828</td>
</tr>
<tr>
<td>6</td>
<td>-3.3755</td>
<td>6.5336</td>
<td>95.3939</td>
<td>123.1752</td>
<td>-1.4066</td>
<td>0.1418</td>
<td>-22.8554</td>
<td>14.4235</td>
</tr>
<tr>
<td>7</td>
<td>-3.5522</td>
<td>7.4379</td>
<td>92.9888</td>
<td>111.3733</td>
<td>-1.4432</td>
<td>0.2048</td>
<td>-20.4111</td>
<td>13.7923</td>
</tr>
</tbody>
</table>

As expected, the optimal parameters for this set of experiments are different from the ones found in simulation. This follows the goal of finding the optimal parameters for the real process, rather than for the model of the process. As expected also, the result of the tuning is not really optimal for regulation purposes.

As we can see in Figure 5.11 the arc current is operating over the full range of operating values (From 300 A to 420 A). Approximately 60 seconds after the start of the experiment, the current saturates. This point corresponds to a desired output of low Puddle Width and a high Puddle Area. This indicates a long and narrow puddle which is difficult to achieve by the real process.

Figure 5.12 and Figure 5.13 show the plots of the sensitivity functions for the last experiments. The largest functions are for the integral parameters, so a change in one of these parameters most affects the response of the system. It also appears that sensitivity functions related to proportional parameters are more affected by the noise. This also explain why the changes of these parameters are so affected by the realization of the noise.
5.4 Linear Quadratic controller

The purpose of this section is to present a linear quadratic controller and to compare the experimental results with the double PI controller described in the previous chapter. For more information about LQ designs see [1].

5.4.1 Internal model structure

The structure for the command was chosen as in Figure 5.3

\[ Y_{dk} = Y_{ak} - Y_{rk} \]
\[ U_k = MY_{dk} - \bar{U}_k \]
\[ E_k = Y_{pk} - Y_k \]

The command \( \bar{U}_k \) was calculated in order to minimize a quadratic criterion \( J \).

\[ J = \sum_{k=0}^{\infty} (Y_{dk} - Y_k)' Q (Y_{dk} - Y_k) + \bar{U}_k' R \bar{U}_k \]

The optimal command, assuming that \( Z, E \) and \( Yd \) are constant, is given by

\[ \bar{U}_k = -LX_k - PaXa_k - PrXr_k + NY_d + NaZ_k + NrE_k \]

where \( L, Pa, Pr, N, Na \) and \( Nr \) are some constant matrix function of the models and the weight matrix \( Q \) and \( R \). Since \( E \) and \( Yd \) will not be constant, the solution is suboptimal. Finding the real optimal solution would require too many calculations, because \( N \) and \( Nr \) would not be constant matrixes. In order to have no static error, simple calculations can show that \( M \) should be chosen such that \( GM = I \), where \( G \) is the gain matrix of the model of the process. The gain matrix of the Reference Model and the Process Model should also be unitary.
5.4.2 Simulation

The plant model and the reference model were the same as the ones described in the previous section. The regulation model can be interpreted as a low pass filter, sometimes called robustness filter, that filters the difference between the plant and the model of the plant. The poles of this model determine the speed of a disturbance rejection. The models chosen were second order transfer functions with poles at 0.85.

\[
\text{Puddle Width : } \frac{0.0225}{z^2 - 1.70z + 0.7225} \\
\text{Puddle Area : } \frac{0.0225}{z^2 - 1.70z + 0.7225}
\]

The weight matrixes \( Q \) and \( R \) were chosen after iterative simulations so that the output follows the reference trajectory as closely as possible and so that the inputs were neither too large nor oscillating. The final choice was:

\[
Q = \begin{bmatrix}
300000 & 0 \\
0 & 100000
\end{bmatrix} \\
R = \begin{bmatrix}
20 & 0 \\
0 & 1
\end{bmatrix}
\]

Figure 5.14 shows the simulated response of the system and Figure 5.15, the inputs of the system. As expected this controller is better in reducing the amount of noise in the system but this causes a noisy Travel Rate.

5.4.3 Experimental results

An experiment was conducted to test the LQ design. Figure 5.16 shows the plot of the outputs and Figure 5.17 the plot of the inputs. As expected from the simulated results, this controller is slightly more effective in reducing the noise.
5.5 Thin plate experiments

All the experiments described in the previous sections were done on a one inch plate. The robustness of the two designs has been tested by running the same experiments on a one and an half inch plate. The dynamic of the thin plate is very different from that of the one inch plate, mainly due to the difference in the heat dissipation. Since the process is different, the models used no longer represented the real process.

5.5.1 PI sensitivity tuning

The experimental parameter convergence is in Table 5.4 and its plot in Figure 5.18.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$K_{p1}$</th>
<th>$K_{p2}$</th>
<th>$K_{p3}$</th>
<th>$K_{p4}$</th>
<th>$K_{i1}$</th>
<th>$K_{i2}$</th>
<th>$K_{i3}$</th>
<th>$K_{i4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>20.0000</td>
<td>-1.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>20.0000</td>
</tr>
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<td>-3.2610</td>
<td>80.7152</td>
<td>-1.2058</td>
<td>0.5206</td>
<td>-19.3694</td>
<td>10.4393</td>
</tr>
<tr>
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<td>-1.7291</td>
<td>0.5347</td>
<td>-29.6997</td>
<td>14.1443</td>
</tr>
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<td>-2.0665</td>
<td>0.5305</td>
<td>-34.0132</td>
<td>17.5476</td>
</tr>
<tr>
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<td>4.1545</td>
<td>144.4698</td>
<td>170.1379</td>
<td>-2.1095</td>
<td>0.4233</td>
<td>-36.5818</td>
<td>18.2445</td>
</tr>
<tr>
<td>5</td>
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<td>2.8485</td>
<td>185.8786</td>
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<td>-1.6796</td>
<td>0.7655</td>
<td>-45.2612</td>
<td>25.9920</td>
</tr>
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<td>278.4188</td>
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<td>-1.9574</td>
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<td>-46.7542</td>
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<td>-2.8435</td>
<td>5.7031</td>
<td>337.2880</td>
<td>291.4432</td>
<td>-1.9561</td>
<td>0.6804</td>
<td>-41.9510</td>
<td>29.7277</td>
</tr>
</tbody>
</table>

As can be seen, $K_{p3}$ and $K_{p4}$ grew continuously. This produced large oscillations in the Puddle Width and Puddle Area as shown in Figure 5.20. The other parameters however seem to converge. To analyze the outcome, let us look at the sensitivity function with respect to $K_{p3}$ and $K_{p4}$ for the last experiment (Figure 5.21). The oscillations observed in Figure 5.20 are not present in the sensitivity function, therefore the new parameters were not optimal for reducing these oscillations. The models used were too different from the process. In fact, according to the model used such oscillations were not allowed.

This short example shows us that the model of the plant should not vary greatly from the actual plant. However, it can be noted that only two of the eight parameters had diverged and that freezing the diverging parameters does not yield a reasonable controller.

5.5.2 Linear Quadratic design

The controller described in section 5.4.1 was applied using a thin plate. Figure 5.19 shows the plots of the experimental results. The response appeared to be comparable to the response obtained using the one inch plate. This result seemed to have been due to the structure of the LQ design. More classical designs, particularly those using an observer, have been unsuccessful in using thin plates when designed for thick plates.
Figure 5.4: Identification Experiments.
Figure 5.5: Simulated Responses without noise.
Figure 5.6: Parameters tuning history without noise.

Figure 5.7: Parameters tuning history with noise.
Figure 5.8: Simulated Responses with noise.
Figure 5.9: Experimental Responses.
Figure 5.10: Experimental Parameter tuning history.

Figure 5.11: Experimental Inputs.
Figure 5.12: Experimental Sensitivity functions for the last experiment.
Figure 5.13: Experimental Sensitivity functions for the last experiment.
Figure 5.14: Simulated Response for the LQ design.

Figure 5.15: Simulated Inputs for the LQ design.
Figure 5.16: Experimental Response for the LQ design.

Figure 5.17: Experimental Inputs for the LQ design.
Figure 5.18: Thin Plate: Experimental Parameter tuning history.

Figure 5.19: Thin Plate: Experimental Response for the LQ design.
Figure 5.20: Thin Plate : Experimental Response for the PI design.

Figure 5.21: Sensitivity Function with respect to $K_{p3}$ and $K_{p4}$. 
Off-line sensitivity tuning has been developed to optimize the controller in two different systems. The successful experimental results demonstrate the robustness and practical use of such methods. This tuning procedure provides the advantage of finding a controller which is optimal for the actual process and not only for its model. This method can be easily implemented to find the optimal values of simple controllers.

The Puddle geometry controller presented using a PI, once tuned, was not as effective as the LQ design. Its structure was however very simple and good performance could not be achieved using such a simple controller, especially since the system included delays. For the structure considered, this controller was never the less optimal.

An interesting possibility for the arc welding system would be to extend the results presented here to other more complicated structures of controllers (non linear, dynamic feedback ...). Non-linear models could also be used to represent the system over a larger range of operating points, thereby making the tuning more accurate.
REFERENCES


