SCALAR QUEUE CONVERSION: DYNAMIC SINGLE ASSIGNMENT FOR CONCURRENT SCHEDULING

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Scalar Queue Conversion: Dynamic Single Assignment for Concurrent Scheduling

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Abstract

This paper describes scalar queue conversion, a compiler transformation that makes scalar renaming an explicit operation through a process similar to closure conversion. We demonstrate how to use scalar queue conversion to slice a flow graph into two executable parts. When executed, the backward slice creates queues of suspended computations (continuations). At any point in time execution of the backward slice can be suspended and the queued continuations can be invoked to effect the state transformations of the forward slice. In other words, scalar queue conversion finds the concurrency between the backward and forward slices of a given point in the flow graph. We briefly describe our experience using an implementation of scalar queue conversion as the key subroutine in the SUDS automatic parallelization system for the Raw microprocessor. The SUDS compiler implements a generalized form of loop distribution that can distribute loops that contain inner loops with arbitrary (even irreducible) control flow.

1 Introduction

There are two relatively standard approaches for converting sequential imperative programs into equivalent concurrent programs, Tomasulo's algorithm [36, 19], and compiler based program restructuring based on scalar expansion [24]. Both of these techniques are based on the notion that converting a program to a dynamic single assignment form exposes concurrency.

On the other hand, each of these techniques presents the system designer with a set of tradeoffs. In particular, Tomasulo's algorithm guarantees the elimination of scalar storage (anti- and output-) dependences but schedules locally, across a relatively small window of consecutive instructions, and so partially sequentializes control flow. On the other hand, compiler based restructuring techniques can perform global control dependence analysis, and thus find all of the available flow concurrency in a program, but have not, prior to this work, been capable of eliminating scalar storage dependences across arbitrary unstructured control flow. Scalar queue conversion eliminates this tradeoff between Tomasulo's algorithm and compiler based program restructuring techniques.

Informally, renaming turns an imperative program into a functional program. Functional programs have the attribute that every variable is dynamically written at most once. Thus functional programs have no anti- or output- dependences. The cost of renaming is that storage must be allocated for all the dynamically renamed variables that are live simultaneously. The particular problem that any renaming scheme must solve, then, is how to manage the fixed, and finite, storage resources that are available in a real system.

Traditional compiler based renaming techniques, (e.g., scalar expansion), rename only those scalars that are modified in loops with structured control flow and loop bounds that are compile time constants. This enables the compiler to preallocate storage for scalar renaming, but limits the applicability of this technique to structured loops that can be analyzed at compile time. Scalar queue conversion, like modern variants of Tomasulo's algorithm [29, 30], manages scalar renaming resources at runtime with queue data structures. Thus, scalar queue conversion, like Tomasulo's algorithm, is able to rename scalars across arbitrary control
sum = 0
i = 0
do
  partial_sum = 0
  j = 0
  use(i, sum)
do
    use2(sum, partial_sum, i, j)
    partial_sum = partial_sum + 1
    j = next(j)
  c1 = cond1(i, j)
while c1
i = i + 1
sum = sum + partial_sum
  c2 = cond2(i)
while c2
  use(sum)

Figure 1: An example program with a doubly nested loop.

Tomasulo’s algorithm (and trace based scheduling algorithms, in general [15, 18]), however, schedule only locally, and are unable to schedule across the mispredicted branches that exit innermost loops. Thus, Tomasulo’s algorithm is unable to exploit concurrency outside of inner loops. Scalar queue conversion performs global control dependence analysis [14, 12], and is thus able to exploit concurrency in outer loops as well.

Because scalar queue conversion both manages scalar renaming resources at runtime, rather than compile time, and does global control dependence analysis, it is able to exploit concurrency in situations where both Tomasulo’s algorithm and traditional compiler based restructuring algorithms fail. In particular, scalar queue conversion can exploit the concurrency in outer loops of programs with arbitrary unstructured control flow. This is the situation that we have found to be the common case in practice [16]s.

The next section introduces the running example we will use throughout the paper, and defines a few basic terms. Section 3 describes the scalar queue conversion transformation. Section 4 describes unidirectional renaming, the static renaming technique that scalar queue conversion uses to eliminate scalar def-def chains that would otherwise restrict scheduling. Section 5 describes some of the practical issues we encountered in our implementation. Section 6 describes related work. Section 7 concludes.

Figure 2: The control flow graph corresponding to the program in Figure 1.

2 Running Example

The concepts in the rest of this paper are illustrated with respect to an example based on the program shown in Figure 1. We have done our best to choose the example such that it illustrates the relationships between the relevant ideas, but so that it is not so complicated as to overwhelm the reader. The control and data dependence patterns in the example program of Figure 1 are representative of the kind of control and data dependence patterns we found in several sparse matrix creation codes after applying recurrence reassociation. (The variable i corresponds to the row number, j corresponds to the column number of a non-zero entry, and sum and partial_sum represent the reassociated index into the array where non-zero entries are being stored).

We will use a standard control flow graph representation of programs. The flow graph for the code in Figure 1 is shown in Figure 2.

The example problem is as follows. Suppose we want to apply loop distribution to the example program. Roughly speaking, the loop distribution algorithm described in Section 5 starts by identifying the loop carried (cyclic) dependences of the loop (in this case, the variables i and sum), and then creates separate loops for each loop carried dependence. So let us reschedule the loop in Figure 2 into two loops, one that
Figure 3: Partitioning the outer loop into the two sub­sets, 2, 3, 4, 7, 8, 9, 10, 11, 13 and 14, and one corresponding to nodes 1, 5, 6, 12 and 15. In particular, nodes 1 and 12 represent all of the definitions of the variable sum, while nodes 5, 6, 12 and 15 represent all the uses of variable sum.

Is there a legal way to restructure the code to effect this rescheduling? We will demonstrate, in Section 3, that this transformation is legal exactly because the flow of value and control dependences across the partitioning of nodes in the region is unidirectional.

To conserve space, we will assume that the reader is familiar with the standard definitions of dominance, postdominance and dominance frontiers [26], control dependence [14, 12], def-use chains, use-def chains, def-def chains and du-webs. Most of these definitions can be found in a standard undergraduate compiler textbook.

We reserve the terms "def-use, use-def and def-def chains," for dependences between registers (scalars that are provably unaliased). We will also define a particularly conservative set of dependences with respect to memory operations (load and store instructions). We say that any memory operation, \( x \), reaches memory operation, \( y \), if there is a path from \( x \) to \( y \) in the control flow graph. We say there is a memory dependence from \( x \) to \( y \) if at least one of \( x \) and \( y \) is a store instruction. (That is, we don't care about load-load dependences).

We define the conservative program dependence graph as the graph constructed by the following procedure. Take the nodes from the control flow graph. For every pair of nodes, \( x, y \), insert an edge, \( x \rightarrow y \), if there is either a def-use-chain from \( x \) to \( y \), a use-def-chain from \( x \) to \( y \), a def-def-chain from \( x \) to \( y \), a memory dependence from \( x \) to \( y \) or a control dependence from \( x \) to \( y \).

We define the value dependence graph as the graph constructed by the following procedure. Take the nodes from the control flow graph. For every pair of nodes, \( x, y \), insert an edge, \( x \rightarrow y \), if there is either a def-use-chain from \( x \) to \( y \), a def-def-chain from \( x \) to \( y \), a memory dependence from \( x \) to \( y \) or a control dependence from \( x \) to \( y \).

We define the value dependence graph as the graph constructed by the following procedure. Take the nodes from the control flow graph. For every pair of nodes, \( x, y \), insert an edge, \( x \rightarrow y \), if there is either a def-use-chain from \( x \) to \( y \), a def-def-chain from \( x \) to \( y \), a memory dependence from \( x \) to \( y \) or a control dependence from \( x \) to \( y \).

3 Scalar Queue Conversion

In this section we will show that the compiler can restructure a given flow graph code to eliminate the register storage dependences across a class of flow graph partitionings that we call unidirectional cuts. Informally, a unidirectional cut corresponds to a slicing of the
construction scheduling algorithm is less constrained by

ten only once. After this renaming all register storage

er names to "physical" registers, each of which is writ­

algorithm performs a dynamic mapping of "virtual" regis­

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a loop [24]. The loop distribution algorithm in Sec­

how to use scalar queue conversion as the key sub­

ution is best viewed as a schedul­

ing in only one direction across the slice. The ability

to eliminate register storage dependences across uni­

directional cuts means that instruction scheduling algo­

rithms can make instruction ordering decisions irrespec­

of register storage dependences. The increased flexibility

results in schedules that would otherwise be impossible

to construct.

We call this transformation to eliminate register

storage dependences scalar queue conversion, because it

completely generalizes the traditional technique of

scalar expansion [24] to arbitrary unstructured (even

irreducible) control flow, and provably eliminates all

register anti- and output-dependences that would viol­

ate a particular static schedule. In Section 5 we show

how to use scalar queue conversion as the key sub­

routine to enable a generalized form of loop distribu­

tion. Loop distribution is best viewed as a schedul­

ing algorithm that exposes the available parallelism in

a loop [24]. The loop distribution algorithm in Sec­

5 generalizes previous scheduling techniques by

scheduling across code with completely arbitrary con­

rol flow, in particular, code with inner loops. This gen­

eralization is possible only, and exactly, because scalar

queue conversion guarantees the elimination of all reg­

ister anti- and output-dependences.

The intuition behind the transformation is that every

imperative program is semantically equivalent to some func­

tional program [25, 20, 1]. Since a functional program

never overwrites any part of an object (but rather cre­

ates an entirely new object) there are no storage depen­

dences.

Another way to view the transformation is to com­
pare it to the dynamic register renaming performed by

Tomasulo’s algorithm [36, 19]. Tomasulo’s algo­

rithm performs a dynamic mapping of “virtual” regist­

er names to “physical” registers, each of which is writ­

ten only once. After this renaming all register storage

dependences are eliminated, because (conceptually) no

physical register ever changes its value. Thus, the in­

struction scheduling algorithm is less constrained by

register storage dependences.

More concretely, instead of executing a piece of code,

we can defer execution of that piece of code by turning

it a closure. A closure can be thought of as a suspended

computation [25, 31]. It is typically implemented as

a data structure that contains a copy of each part of

the state required to resume the computation, plus a

pointer to the code that will perform the computation.

There are then a set of operations that we can perform

on a closure:

1. We can allocate a closure by requesting a portion of

memory from the dynamic memory allocator that is

sufficient to hold the required state plus code

pointer.

2. We can fill a closure by copying relevant portions

of the machine state into the allocated memory

structure.

3. We can invoke a closure by jumping to (calling) the

closures code pointer and passing a pointer to the

associated data structure that is holding the rele­

vant machine state.

Closures will be familiar to those who have used lex­i­

cally scoped programming languages. For example, in

C++ and Java closures are called objects. In these lan­
guages closures are allocated by calling operator new,

filled by the constructor for the object’s class, and in­
voked by calling one of the methods associated with the

object’s class.

In the general case we can defer execution of some

subset of the code by creating a closure for each de­

ferred piece of code, and saving that closure on a

queue. Later we can resume execution of the deferred
code by invoking each member of the queue in FIFO

order.

3.1 Unidirectional Cuts

Now we define a cut of the set of nodes in a region, R,
as a partitioning of the set of nodes into two subsets,

A, B such that A ∩ B = Ø and A ∪ B = R. We say that

each cut is unidirectional iff there are no edges x → y

such that x ∈ B and y ∈ A. That is, all the edges either

stay inside A, stay inside B or flow from A to B, and no

edges flow from B to A. For example, given the region

corresponding to the outer loop in Figure 3, the par­
tition {2, 3, 4, 7, 8, 9, 10, 11, 13, 14} and {1, 5, 6, 12, 15} is a

unidirectional cut because there are no def-use chains,

memory or control dependences flowing from the sec­

ond set to the first.

In the following sections we will demonstrate that

by the process of queue conversion we can always trans­

form a unidirectional cut A-B of a single-entry single­

exit region into a pair of single-entry single-exit re­

gions, that produce the same final machine state as the

original code, but have the feature that all of the in­

structions from partition A execute (dynamically) be­

fore all the instructions from partition B.

Any particular value dependence graph might have

many different unidirectional cuts. The criteria for

choosing a specific cut will depend on the reasons for

performing the transformation. In Section 5 we will

discuss one method for efficiently identifying a useful

set of unidirectional cuts for loop distribution.
3.2 Maximally Connected Groups

First we will show that we can create a "reasonable" flow graph that consists only of the nodes from subset A of a unidirectional A-B cut. The property that makes this possible is that every maximally connected group of the nodes from subset B will have only a single exit. Thus we can remove a maximally connected subset of nodes from subset B from the region flow graph and "fix-up" the breaks in the flow graph by connecting the nodes that precede the removed set to the (unique) node that succeeds the removed set.

Given a unidirectional cut A-B of a flow graph then we will call a subset of nodes $\beta \subseteq B$ in the graph a maximally connected group iff every node in $\beta$ is connected in the flow graph only to other nodes of $\beta$ or to nodes of A. That is, given $\beta = B - \beta$ and nodes $b \in \beta$, $a \in A$ there are no edges $b \rightarrow a$ or $a \rightarrow b$. For example, given the unidirectional cut shown in Figure 3 where $A = \{2, 3, 4, 7, 8, 9, 10, 11, 13, 14\}$ and $B = \{1, 5, 6, 12, 15\}$, the maximally connected groups are the subsets $\{1\}$, $\{5, 6\}$, $\{12\}$ and $\{15\}$ of $B$.

But now suppose that we are given a unidirectional cut A-B. This means that there can be no control dependences from B to A. Informally, there are no branches in B that can in any way determine when or if a node in A is executed. Now suppose that we are given a maximally connected group $\beta \subseteq B$. If $\beta$ has an exit edge $b \rightarrow a$ (an edge where $b \in \beta$, $a \in A$), then, because $\beta$ is maximally connected it must be the case that $a \in A$. The node $a$ can not be in B because then $\beta$ would not be maximally connected.

If there are two (or more) such exit edges, $b_0 \rightarrow a_0$ and $b_1 \rightarrow a_1$, where $b_0 \neq b_1$ then it must be the case that there is a branch or set of branches in $\beta$ that causes the flow graph to fork. In particular, $b_0$ and $b_1$ must have different control dependences, and at least one of those control dependences must be on a node inside $\beta$. But $a_1$ and $a_0$ can not be control dependent on any node inside $\beta$, because they are on the wrong side of the A-B cut.

Consider node $a_0$. There is an edge from $b_0$ to $a_0$, thus there is at least one path from $b_0$ to exit that passes through $a_0$. But $a_0$ is not control dependent on $b_0$, so every path from $b_0$ to exit must pass through $a_0$. Thus $a_0$ postdominates $b_0$. Similarly, for every node $b_i \in \beta$ such that there is any path from $b_i$ to $b_0$, it must be the case that $a_0$ postdominates $b_i$.

Consider this set of $b_i \in \beta$ that are on a path to $b_0$. Now, $\beta$ is connected, thus either there must be a path from $b_0$ to $b_1$ or there must be a path from $b_1$ to $b_0$. If there is a path from $b_1$ to $b_0$, then there is a path from $b_1$ to $b_0$ and thus $a_0$ also postdominates $b_1$. Suppose there is no path from $b_1$ to $b_0$, then there must be a path from one of the $b_i$ to $b_1$. But we already know that every path from $b_1$ to exit goes through $a_0$, so every path from $b_1$ to exit must go through $a_0$. Thus $a_0$ postdominates both $b_0$ and $b_1$.

By a similar argument $a_1$ postdominates both $b_1$ and $b_0$. More specifically, $a_1$ immediately postdominates $b_1$, because there is a flow graph edge $b_1 \rightarrow a_1$. Thus $a_0$ must postdominate $a_1$ if it is to also postdominate $b_1$. A similar argument shows that $a_1$ must postdominate $a_0$. Postdominance is a partial order, thus $a_0 = a_1$. So the maximally connected group $\beta$ exits to a unique node in A.

As an example, consider Figure 4. This figure shows a flow graph containing an irreducible loop. Suppose that we would like to include node 4 (a branch instruction) in set B of a unidirectional A-B cut. We will demonstrate that any maximally connected group $\beta \subseteq B$ that contains node 4 must also contain nodes 8 and 9, and will, therefore, exit through node 10. We can see this by examining Figure 5, which shows control dependence graph corresponding to the flow graph in Figure 4. There is a cycle in the control dependence graph between the two exit branches in nodes 4 and 7. Thus if either of the exit branches for the irreducible

![Figure 4: Any maximally connected subset of nodes from the bottom of a unidirectional cut always exits to a single point. In this case (an irreducible loop) if either node 4 or 7 is in the bottom of a unidirectional cut then so must all the nodes 2, 4, 5, 6, 7, 8 and 9. Thus a maximally connected subset containing node 4 or node 7 will exit to node 10.](image-url)
Given a unidirectional cut A-B of a flow graph we can efficiently find all the maximally connected groups \( \beta \subset B \) as follows. First we scan the edges of the flow graph to find all the edges \( b_j \rightarrow a_i \) where \( b_j \in B \) and \( a_i \in A \). By the argument above the set of nodes \( a_i \) found in this manner represent the set of unique exits of maximal groups \( \beta_i \subset B \). Then for each \( a_i \) we can find the associated maximally connected group \( \beta_i \) by performing a depth first search (backwards in the flow graph by following predecessor edges) starting at \( a_i \), and where we follow only edges that lead to nodes in B.

Figure 5: The control dependence graph for the flow graph in Figure 4 has a cycle between nodes 4 and 7. Thus both nodes must be on the same side of a unidirectional cut of the flow graph.

The loop is included on one side of the unidirectional cut, then the other must as well, because we require that no control dependences in a unidirectional cut flow from B to A.

In addition to creating a flow graph that performs exactly the work corresponding to part A of a unidirectional A-B cut, we can also annotate the flow graph so that it keeps track of exactly the order in which the maximal groups \( \beta_i \subset B \) will be executed. We do this by creating a queue data structure at the entry point of the region flow graph. We call this queue the deferred execution queue.

Every edge \( a_i^v \rightarrow b_j^v, a_i^v \in A, b_j^v \in \beta_i \) in the flow graph represents a point at which control would have entered the maximal group \( \beta_i \). Likewise, every edge \( b_j^v \rightarrow a_i^x, b_j^v \in \beta_i, a_i^x \in A \), represents exactly the points at which control would have returned to region A.

Thus, after creating the sliced flow graph for partition A, by removing the regions \( \beta_i \) from the flow graph (as described in the previous section), we can place an instruction along each edge \( a_i^v \rightarrow a_i^x \) that pushes the corresponding code pointer for the node \( b_j^v \), on to the deferred execution queue. The edges \( a_i^v \rightarrow a_i^x \) execute in exactly the order in which the \( \beta_i \)'s would have executed in the original flow graph. Thus after execution of the sliced flow graph for partition A, the deferred execution queue will contain all of the information we need to execute the code from partition B in exactly the correct order and exactly the correct number of times.

We can accomplish this by converting each \( \beta_i \) into a procedure that contains a flow graph identical to the flow graph that corresponds to the original \( \beta_i \), but returns at each exit point of \( \beta_i \). Then we can recreate the original execution sequence of partition B by popping each code pointer \( b_j^v \), off the front of the deferred execution queue and calling the corresponding procedure.

The queue conversion of our example program is shown in Figure 6. Push instructions for the appropriate maximal group entry points have been inserted along the edges \( \text{begin} \rightarrow 2, 4 \rightarrow 7, 10 \rightarrow 7, 11 \rightarrow 13 \) and \( 14 \rightarrow \text{end} \). The maximal groups \{1\}, \{5, 6\}, \{11, 13\} and \{15\} are each converted into a procedure.

If the underlying infrastructure does not support multiple-entry procedures, then each maximal group \( \beta_i \) can be further partitioned into a set of subprocedures, each corresponding to a maximal basic block of \( \beta_i \). Each subprocedure that does not exit \( \beta_i \) will call \{31\} its successor(s) from \( \beta_i \).
Figure 6: The sliced flow graph for partition A, consisting of nodes 2, 3, 4, 7, 8, 9, 10, 11, 13, 14 and 14. For example, nodes 4 and 10 (the entries to the maximal group consisting of nodes 5 and 6) are connected to node 7, (the single exit node for group 5, 6). Queue conversion annotates the sliced flow graph for A with instructions that record which maximal groups of B would have executed, and in what order. Each maximal group of B is converted into its own procedure.

Figure 7: Cuts in the du-webs for variables i, j, sum and partial_sum given the cut from nodes 2, 3, 4, 7, 8, 9, 10, 11, 13, 14 to nodes 1, 5, 6, 12, 15 (shown in bold). Def-use chains that cross the cut are shown as dotted edges.

Closure Conversion

If it were the case that there were no register storage dependences flowing from B to A then the deferred execution queue would be sufficient. Our definition of a unidirectional A-B cut did not, however, exclude the existence of use-def or def-def chains flowing from region B to region A. Thus, we must solve the problem that partition A might produce a value in register x that is used in region B but then might overwrite the register with a new value before we have a chance to execute the corresponding code from partition B off the deferred execution queue.

The problem is that the objects we are pushing and popping on to the deferred execution queue are merely code pointers. Instead, we should be pushing and popping closures. A closure is an object that consists of the code pointer together with an environment (set of name-value pairs) that represents the saved machine state in which we want to run the corresponding code. Thus a closure represents a suspended computation.

Consider the registers (variables) associated with the set of def-use chains that reach into a maximal group $\beta_1 \subset B$. If we save a copy of the values associated with each of these registers along with the code pointer, then we can eliminate all the use-def chains that flow from B to A, and replace them, instead, with use-def chains that flow only within partition A.

To convert each maximal group $\beta_1 \subset B$ into a closure we transform the code as follows.
Figure 8: Closure conversion ensures that each value crossing the cut gets copied into a dynamically allocated structure before the corresponding register gets overwritten.
1. Consider the graph of nodes corresponding to $\beta_i$. For each of the entry nodes $b_{ij}^j$ of this graph find the set of nodes $\beta_{ij} \subset \beta_i$ reachable from $b_{ij}^j$. For each set $\beta_{ij}$ find the set of variables, $\mathcal{V}_{ij} = \{v_{ijkl}\}$ such that there is a def-use chain flowing from partition A into $\beta_{ij}$. (That is, there is a definition of $v_{ijkl}$ somewhere in A and a use of $v_{ijkl}$ somewhere in $\beta_{ij}$). Figure 7 shows that this set can be easily derived from the du-webs corresponding to the flow graph. For example, $\mathcal{V}_{(12)} = \{\text{partial}\_\text{sum}\}$ and $\mathcal{V}_{(15)} = \emptyset$. The maximal group $\beta_{(5,6)}$ has two entry points, (at 5 and 6). In this case it happens that $\mathcal{V}_{(5,6),5} = \mathcal{V}_{(5,6),6} = \{i, j, \text{partial}\_\text{sum}\}$. 

2. Consider each edge $a_{ij}^i \rightarrow a_{ij}^i$ in the sliced flow graph for partition A that corresponds to entry point $b_{ij}^j$ of maximal group $\beta_i$. Along this edge we place an instruction that dynamically allocates a structure with $|\mathcal{V}_{ij}| + 1$ slots, then copies the values $(b_{ij}^j, v_{ij1}, \ldots , v_{ij|\mathcal{V}_{ij}|})$ into the structure, and then pushes this pointer onto the deferred execution queue. Figure 8 demonstrates this process. For example, along the edge 4 $\rightarrow$ 7 we have placed instructions that allocate a structure containing the values of the code pointer, “5”, and the copies of the values contained in variables, $i$, $j$ and $\text{partial}\_\text{sum}$.

3. For each $\beta_i$, we create a procedure that takes a single argument, $c$, which is a pointer to the structure representing the closure. The procedure has the same control flow as the original subgraph for $\beta_i$, except that along each entry we place a sequence of instructions that copies each entry from each slot of the closure into the corresponding variable $v_{ik}$. Figure 8 shows that the two entries to the procedure corresponding to the maximal group $(5, 6)$ have been augmented with instructions that copy the values of variables $i$, $j$ and $\text{partial}\_\text{sum}$ out of the corresponding closure structure.

4. To invoke a closure from the deferred execution queue we pop the pointer to the closure off the front of the queue. The first slot of the corresponding structure is a pointer to the code for the procedure corresponding to $\beta_i$. Thus we call this procedure, passing as an argument the pointer to the closure itself. In Figure 8 this process is shown towards the bottom of the original procedure, where we have inserted a loop that pops closures off the deferred execution queue, and invokes them.

This completes the basic scalar queue conversion transformation. Because a copy of each value reaching a maximal group $\beta_i$ is made just before the point in the program when it would have been used, the correct set of values reaches each maximal group, even when execution of the group is deferred. Additionally, since the copy is created in partition A, rather than partition B, we have eliminated any use-def chains that flowed from partition B to partition A. In the next section we will demonstrate how to generalize the result to eliminate def-def chains flowing from B to A.

4 Unidirectional Renaming

In the previous section we demonstrated that we could transform a unidirectional A-B cut on a single-entry single-exit region into an equivalent piece of code such that all the instructions in partition A run, dynamically, before all the instructions in partition B. Further we demonstrated that we could do this even in the presence of use-def chains flowing from partition B to partition A. In this section we will show that the result can be generalized, in a straightforward way, to A-B cuts where there are additionally def-def chains flowing from partition B to partition A.

The result depends on the fact that given a unidirectional A-B cut, we can insert a new instruction anywhere in the flow graph, and that if we give that instruction a labeling that includes it in partition B, then we will not introduce any new control dependences that flow from partition B to partition A. (The opposite is not true. That is, if we place a new instruction in partition A at a point that is control dependent on an instruction in partition B, then we will introduce a control dependence edge that will violate the unidirectionality of the cut.)

For the remainder of the paper we will assume that each du-web in the program has been given a unique name. This transformation is already done by most optimizing compilers because it is so common for programmers to reuse variable names, even when the variables are completely independent. For example, many programmers reuse the variable name $i$ for the index of most loops. Once the du-webs are calculated we iterate through the set of du-webs for each variable $x$, renaming all the uses and definitions in each node in the $i$th web to $x_i$. Thus we can, without loss of generality, talk about the du-web for a particular variable.

Now consider the du-web for variable $x$ on a unidirectional cut A-B where some of the definitions of $x$ are in A and some of the uses of $x$ are in B. Thus, there is a value dependence flowing from A to B. It may be the case that there are definitions of $x$ in B and uses of $x$ in A, but, because A-B is a unidirectional cut, it cannot be the case that there are any def-use chains reaching from B to A. Thus the du-web has a unidirectional structure, just as the value dependence graph did. (In fact,
Figure 9: An example of statically renaming the variables i, j and partial_sum.

Figure 10: The unidirectionally renamed du-webs for variables i, j and partial_sum.

Another way of seeing this is to observe that each du-web is an induced subgraph of the value dependence graph. For example, in the du-webs shown in Figure 7 one can observe that the def-use chains crossing the cut (shown with dotted edges) all flow in one direction.

The du-web for variable x thus has a structure that is almost renameable, except for those edges in the web that cross the cut. Suppose, however, that we were to place a copy instruction "x' = x" directly after each of the definitions of x from A that reach a use in B. Then we could rename all the definitions and uses of x in B to x'. The program will have exactly the same semantics, but we will have eliminated all of the def-def chains flowing from B to A. We will call such a renaming of a du-web that crosses a unidirectional cut a unidirectional renaming.

An example of a unidirectional renaming is shown in Figure 9. Each time one of the variables i, j and partial_sum is modified it is copied to a corresponding variable i', j' or partial_sum'. The uses of i, j and partial_sum in partition B are then renamed to i', j' and partial_sum'. The du-webs for this unidirectional renaming are shown in Figure 10.

To see how unidirectional renaming eliminates backwards flowing def-def chains, consider Figure 11. We examine the cut from the set of nodes {1, 2, 3, 4, 6, 7} to the set {5, 8}. This is a unidirectional cut because all of the value and control dependences flow from the first set to the second. Figure 12 shows the corresponding du-web for variable x. There is, however, a def-def chain flowing from node 5 to node 7 (against the cut...
Figure 11: The cut separating nodes 1, 2, 3, 4, 6 and 7 from nodes 5 and 8 is unidirectional because all the value and control dependences flow unidirectionally. The def-def chain flowing from node 5 to node 7 does not violate the unidirectionality of the cut.

Figure 12: The du-web for variable x from the flow graph in Figure 11. The cut is unidirectional because all the def-use chains flow in one direction across the cut. Dotted edges show cut edges.

Figure 13: After unidirectionally renaming the variable x the def-def chain between nodes 5 and 7 is eliminated, and replaced instead with a def-def chain from node 5 to node 7'. The new def-def chain does not cross the cut because node 5 and 7' are both in the same partition (indicated by nodes with a bold outline).

Figure 14: After placing the copy instructions for the unidirectional renaming directly after the corresponding definition of each variable produces a correct result, but, in fact, we can do better. We can maintain the program semantics and eliminate the output dependences if we place the copy instructions along any set of edges in the program that have the property that they cover all the paths leading from definitions of x in A that reach uses of x.
5 Implementation Experience

We have implemented scalar queue conversion in the context of SUDS [16], our research system for automatically parallelizing C programs running on the Raw microprocessor [35]. In this section we briefly describe how we used scalar queue conversion as the key subroutine of a generalized form of loop distribution that can reschedule any region of code with arbitrary control flow, including arbitrary looping control flow. In addition, we describe a set of other practical problems that needed to be solved in order to effectively find concurrency. Due to space limitations, the discussion is brief. The intent of this section is to complement our somewhat abstract description of scalar queue conversion with a discussion of its use in a practical setting. For details, refer to Frank's dissertation [16].

5.1 Generalized Loop Distribution

The goal of loop distribution is to transform the chosen region so that any externally visible changes to machine state will occur in minimum time. Roughly speaking, then, we begin by finding externally visible state changes for the region in question, which we call critical definitions. We then find the smallest partition of the value dependence graph that includes the critical node, yet still forms a unidirectional cut with its complement. Finally, we apply scalar queue conversion to create a minimal (and hopefully small) piece of code that performs only the work that cyclically depends on each critical definition.

Consider again the example flow graph from Figure 2 used throughout Section 3. Roughly speaking, this loop has two loop carried dependences, on the variables i and sum. The other variables, e.g., j, partial_sum, c1 and c2, are private to each loop iteration, and thus are not part of the state changes visible external to the loop.

Following this intuitive distinction, we more concretely identify the critical definitions of a region by finding all uses (anywhere in the program) such that at least one definition dR within the region R reaches the use and at least one definition from outside the region dR reaches the use. Then we call the definition dR (the one inside region R) a critical definition. To reiterate, intuitively, the critical definitions represent changes to the part of the state that is visible from outside the region. Critical definitions represent points inside the region at which that visible state is changed. (As opposed to region (loop) invariant and externally invisible (private) state).

For the region corresponding to the outer loop in Figure 2 the critical definitions are the nodes 11 and 12. Nodes 5, 6, 9 and 11, for example, are reached both by node 11 (inside the loop) and node 2 (outside the loop), so node 11 is a critical definition for the loop. Likewise, nodes 5, 6 and 12 are reached both by node 12 (inside the loop) and node 1 (outside the loop), so node 12 is also a critical definition for the loop.

Next, for each critical node we find all nodes in the value dependence graph that have a cyclic dependence with the critical node. That is, given critical node d and node n, if there is a path from d to n in the value dependence graph and a path from n to d in the value dependence graph, then we give n the same priority as d. For example, in the loop in Figure 2 the cyclic path 11 → 13 → 14 → 11 in the value dependence graph indicates that nodes 13 and 14 form a cycle with...
the critical node 11. We assign the nodes in each cyclic critical dependence path to the same partition.

All remaining nodes will be assigned to a partition between two critical node partitions. That is, for each node \( n \) find the critical node \( d_{\text{upflow}} \) with the highest priority, such that there is a path from \( n \) to \( d_{\text{upflow}} \) in the value dependence graph. Then assign \( n \) to a partition between \( d_{\text{upflow}} \) and \( d_{\text{downflow}} \)'s parent.

For example, in Figure 2 node 12 depends on node 7. Node 7, in turn, is dependent on nodes 3, 4, 7, 8, 9 and 10. (There exists, for example, the dependence path 4 \( \rightarrow \) 8 \( \rightarrow \) 9 \( \rightarrow \) 10 \( \rightarrow \) 7.) None of these nodes has a path in the value dependence graph leading to any of nodes 11, 13 or 14. Thus we give nodes 3, 4, 7, 8, 9 and 10 a priority between the priority of node 11 and the priority of node 12.

For each partition we have a unidirectional cut from the higher priorities to this partition and those below. Thus we perform scalar queue conversion on each partition (from the bottom up) to complete our code transformation.

### 5.2 Queue Management

The register renaming resources in any real system are of finite size, and thus need to be managed carefully so that they do not overflow. This problem appears in any system where queues are used, and there are several approaches to handling the problem. One approach to managing the deferred execution queues is to alternate execution between operations that increase the queue length (code from the backward slice) until the queue is nearly full, and then run code that decreases the queue length (closures from the forward slice) until the queue is empty. This is similar to the approach used to handle reading and writing from pipes in most UNIX file systems. In fact, in the degenerate case that the total space available for deferred execution queues is only as large as a single closure, this management scheme is equivalent to running the code in the order specified by the original flow graph. The SUDS compiler also strip mines the loop being distributed, which tends to reduce the probability of filling the deferred execution queues.

Generalized loop distribution introduces an additional subtlety, in that multiple queues need to be managed simultaneously. The basic idea of falling back to the sequential schedule given by the original flow graph still works, however, and thus the queues can be managed both correctly and efficiently [7, 6, 16].

### 5.3 Extensions

The transformation described in Section 3 applies only to single exit regions of a flow graph. Scalar queue conversion can be extended to work on multiple exit regions of a flow graph. Interestingly this extension can be effected by applying scalar queue conversion, itself, to separate the multi-exit region from its successors in the flow graph. The generalized loop distribution transformation described above, was also extended with a recurrence reassociation transformation [24].

### 5.4 Memory Dependences

Scalar queue conversion takes a conservative view of memory dependences by inserting edges in the value dependence graph for all load-after-store, store-after-load and store-after-store dependences. These, extra, conservative dependences may restrict the applicability of scalar queue conversion because they might create cycles in the value dependence graph across what would otherwise be unidirectional cuts. In practice we found it necessary to implement four additional transformations to reduce the impact of memory dependences. These include transforming the code by register promotion [10] and apply the transformation that Barua has called "equivalence class unification," [32, 4, 8]. In addition we do a simple form of array privatization, based on the scope information that is typically available in C programs.

Additionally, the SUDS runtime system implements a memory checkpoint repair mechanism, and a concurrency control mechanism based on basic timestamp ordering [5] that is able to detect memory references that violate the expected sequential order. Because of this support, we are able to speculatively eliminate many memory dependence back edges that would otherwise create cycles in the value dependence graph.

### 6 Related Work

The idea of renaming to reduce the number of storage dependences in the dependence graph has long been a goal of parallelizing and vectorizing compilers for Fortran [24]. The dynamic closure creation done by the queue conversion algorithm in Section 3 can be viewed as a generalization of earlier work in scalar expansion [24, 11]. Given a loop with an index variable and a well defined upper limit on trip count, scalar expansion turns each scalar referenced in the loop into an array indexed by the loop index variable. The queue conversion algorithm works in any code, even when there is no well defined index variable, and no way to statically determine an upper bound on the number of times the loops will iterate. Moreover, earlier methods of scalar expansion are heuristic. Queue conversion is the first compiler transformation that guarantees the elimination of all register storage dependences that
create cycles across what would otherwise be a unidirectional cut.

Given a loop containing arbitrary forward control flow, loop distribution [24] can reschedule that graph across a unidirectional cut [21, 17], but since loop distribution does no renaming, the unidirectional cut must be across the conservative program dependence graph (i.e., including the register storage dependences). Queue conversion works across any unidirectional cut of the value dependence graph. Because scalar queue conversion always renames the scalars that would create register storage dependences, those dependences need not be considered during analysis or transformation. It is sometimes possible to perform scalar expansion before loop distribution, but loop distribution must honor any register storage dependences that are remaining.

Moreover, existing loop distribution techniques only handle arbitrary forward control flow inside the loop, and do so by creating arrays of predicates [21, 17]. The typical method is to create an array of three valued predicates for each branch contained in the loop. Then on each iteration of the top half of the loop a predicate is stored for each branch (i.e., "branch went left", "branch went right" or "branch was not reached during this iteration"). Any code distributed across the cut tests the predicate for its closest containing branch. This can introduce enormous numbers of useless tests, at runtime, for predicates that are almost never true.

Queue conversion, on the other hand, creates and queues closures if and only if the dependent code is guaranteed to run. Thus, the resulting queues are (dynamically) often much smaller than the corresponding set of predicate arrays would be. More importantly, queue conversion works across inner loops. Further, because queue conversion allocates closures dynamically, rather than creating static arrays, it can handle arbitrary looping control flow, in either the outer or inner loops, even when there is no way to statically determine an upper bound on the number of times the loops will iterate.

Feautrier has generalized the notion of scalar expansion to the notion of array expansion [13]. As with scalar expansion, Feautrier's array expansion works only on structured loops with compile time constant bounds, and then only when the array indices are affine (linear) functions of the loop index variables. Feautrier's technique has been extended to the non-affine case [22], but only when the transformed array is not read within the loop (only written). The equivalence class unification and register promotion techniques mentioned in Section 5.4 extend scalar queue conversion to work with structured aggregates (e.g., C structs), but not with arrays. Instead, our implementation of scalar queue conversion relies on the memory dependence speculation system mentioned in Section 5.4 to parallelize across array references (and even arbitrary pointer references).

The notion of a unidirectional cut defined in Section 3.1 is similar to the notion, from software engineering, of a static program slice. A static program slice is typically defined to be the set of textual statements in a program upon which a particular statement in the program text depends [38]. Program slices are often constructed by performing a backward depth first search in the value dependence graph from the nodes corresponding to the statements of interest[27]. This produces a unidirectional cut.

In Section 3.2 we proved that we could produce an executable control flow graph that includes exactly the nodes from the top of a unidirectional cut of the value dependence graph. Yang has proved the similar property, in the context of structured code, that an executable slice can be produced by eliding all the statements from the program text that are not in the slice [39]. Apparently it is unknown, given a program text with unstructured control flow, how to produce a control flow graph from the text, elide some nodes from the graph and then accurately back propagate the elisions to the program text [3]. Generalizations of Yang's result to unstructured control flow work only by inserting additional dependences into the value dependence graph [3, 9], making the resulting slices larger and less accurate. The proof in Section 3.2 demonstrates that when working directly with control flow graphs (rather than program texts) this extra work is unnecessary, even when the control flow is irreducible.

Further, executable program slicing only produces the portion of the program corresponding to partition A of a unidirectional cut A-B (that is, it only produces the backward executable slice). In Sections 3.3 and 4 we demonstrated how to queue and then resume a set of closures that reproduce the execution of partition B as well (the set of state transitions corresponding to the forward executable slice).

The reason queue conversion generalizes both loop distribution and executable program slicing is that queue conversion makes continuations [34, 31, 2] explicit. That is, any time we want to defer the execution of a piece of code, we simply create, and save, a closure that represents that code, plus the suspended state in which to run that code. It is standard to compile functional languages by making closures and continuations explicit [31, 2], but this set of techniques is relatively uncommon in compilers for imperative languages.

In fact, scalar queue conversion was anticipated by work from formal programming language semantics that demonstrates that continuation passing style rep-
resentations and SSA form flow graphs of imperative programs are semantically equivalent [20]. Based on this work, Appel has suggested that a useful way of viewing the \( \phi \) nodes at the join points in SSA flow graphs is as the point in the program at which the actual parameters should be copied into the formal parameters of the closure representing the code dominated by the \( \phi \) node [1]. This roughly describes what scalar queue conversion does.

That is, given a maximal group \( \beta \) containing a use of variable \( x \) for which we are going to create a closure, we rename \( x \) to \( x' \) (which can be viewed as the formal parameter). Then we introduce a new closure, containing the instruction \( x' = x \), at the \( \phi \) point which shares an environment containing \( x' \) with \( \beta \). It is useful to view the new closure as simply copying the actual parameter, \( x \), to the formal parameter \( x' \).

A transformation similar to loop distribution, called critical-path reduction has been applied in the context of thread-level speculative systems [37, 33, 40]. Rather than distribute a loop into multiple loops, critical-path reduction attempts to reschedule the body of the loop so as to minimize the amount of code executed during an update to a critical node. While the transformation is somewhat different than that performed by loop distribution, loop distribution and critical-path reduction share the goal of trying to minimize the time observed to update state visible outside the loop body.

Schlansker and Kathail [28] have a critical-path reduction algorithm that optimizes critical paths in the context of superblock scheduling [18], a form of trace scheduling [15]. Vijaykumar implemented a critical-path reduction algorithm for the multiscalar processor that moves updates in the control flow graph [37]. Stefan et al have implemented a critical-path reduction algorithm based on Lazy Code Motion [23] that moves update instructions to their optimal point [33, 40]. As with previous loop distribution algorithms, none of these critical-path reduction algorithms can reschedule loops that contain inner loops.

7 Conclusion

This paper has given an informal description of the scalar queue conversion transformation. We have argued that scalar queue conversion can restructure any unidirectional cut of the true scalar dependences in any flow graph, and reschedule the code so that all of the instructions in the top half of the cut run (dynamically) before all of the instructions in the bottom half. Scalar queue conversion completely eliminates scalar anti- and output-dependences that might otherwise make this rescheduling impossible.

We have described the use of scalar queue conversion in one practical setting, as a subroutine for a generalized form of loop distribution that can reschedule loops with arbitrary control flow, including irreducible control flow and inner loops with trip counts that can not be determined until the loop exits. We believe that scalar queue conversion has many other applications as well. Our ongoing work involves applying scalar queue conversion to automate the process of hiding program slices for the purposes of software security (as in [41]), and applying scalar queue conversion as the basis of a code generator to convert imperative programs to executable data-flow graphs.

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