SEMI-TOPOLOGICAL ANALYSIS
OF LINEAR NETWORKS

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For the diagnosis of linear networks, topological formulas for
network functions are found convenient. However, we have to generate all
the trees and 2-trees of the network, whose number is generally very big.
In practice, we encounter the situation in which some elements are more
important than others. This paper presents a method of analysis which
enables us to obtain network functions, leaving only several elements
in a symbolical expression, so that we can study the effect of the
variations of those elements.
1. Introduction

By using the well-known Kirchhoff's law, the action of a linear electrical network is fully described as a system of simultaneous linear equations. Network functions are obtained by applying the Cramer's rules in solving simultaneous equations, as seen in Seshu and Reed [1]. If we take a vertex of the network as a reference and the voltage of every other vertices with respect to this reference as unknowns, the coefficient matrix is called the node admittance matrix of the network, which is a \((v-1) \times (v-1)\) matrix, where \(v\) is the number of vertices in the network. Denote this matrix by \(Y_n\), and by \(Y_{jk}\) the minor matrix with respect to the \(jk\)-element of \(Y_n\). To obtain network functions, the computation of \((-1)^{j+k} \det Y_{jk}/\det Y_n\) is essential.

The most expedient method of calculating a determinant of an \(nxn\) matrix may be the triangulation of a matrix. It is known that any square matrix can be brought into a triangular matrix by the successive application of elementary operations. This method roughly requires \(n^3\) times additive operation and \(n^3\) times multiplicative operation.

The triangulation method is not adequate for the symbolical computation. Topological formulas of network functions compensate this weak point, as seen in Seshu and Reed [1], Mayeda [2] and Coates [3]. However, the number of steps involved in the topological computation of network functions is immense; in general, it increases more than exponentially with the number of vertices in \(G\).

In practice, we often want to analyze the effect of the variations of the values of only several elements in the network; in other words, we want to obtain network functions leaving only several factors unknown.
This paper presents a method which meets this purpose. We adopt a topological method only for the subnetwork consisting of those elements which must be left unknown. This method is particularly useful when the number of unknown elements is not very large.
2. Preliminaries

We suppose that a given network $G$ is connected. Let $v$ and $e$ be the number of vertices and edges, respectively, of the network $G$. A tree of the network $G$ is a circuitless subnetwork consisting of $v-1$ edges. A 2-tree $t_{ij,r}$ of $G$ is a circuitless subnetwork of $v-2$ edges in which vertices $i$ and $j$ are connected and both vertices $i$ and $r$ and vertices $j$ and $r$ are separated. If $i = j$, we delete one $i$ from the subscripts. A common tree $t_c$ of $G$ is a tree of $G$ which is a tree of both the current graph $G_i$ and the voltage graph $G_v$ of $G$. (Refer to Mayeda [2] or Coates [3] for the current and voltage graphs.) A common 2-tree $t_{ij,r}$ of $G$ is a 2-tree of $G$ which is a 2-tree separating vertices $j$ and $r$ in the current graph of $G$ and also a 2-tree separating vertices $k$ and $r$ in the voltage graph of $G$.

**Definition 1:** A sub-tree (or sub-2-tree $t_{ij,r}$) is defined to be a set of edges in $G$, including an empty set $\emptyset$, which does not contain any circuit (or which does not contain any circuit and does not connect vertices $i$ and $r$ nor vertices $j$ and $r$). A common sub-tree (or common sub-2-tree $t_{ij,r}$) is a set of edges which is a sub-tree both in the current and voltage graphs of $G$ (or which is a sub-2-tree $t_{ij,r}$ in the current graph and a sub-2-tree $t_{k,r}$ in the voltage graph of $G$). Let $\delta$ be one of them. Then by $|\delta|$ we mean the number of edges contained in $\delta$.

Obviously a set of edges contained in a tree (a 2-tree $t_{ij,r}$, a common tree, or a common 2-tree $t_{ij,r}$ of $G$ is a sub-tree (a sub-2-tree $t_{ij,r}$, a common sub-tree, or a common sub-2-tree $t_{ij,r}$).

The next lemma is easily proved.
Lemma 1: Suppose a given network $G$ is connected. Then each sub-tree is contained in some tree in $G$.

In an electrical network, admittances are assigned to each edge of $G$. Let $D = \{x_1, \ldots, x_p\}$ be the set of unknown admittances, which have to be expressed symbolically in network functions. Other admittances are denoted by $y_1, y_2, \ldots, y_q$, $(p+q = e)$, which are given numerically in a practical case. For convenience, edges will be specified by their assigned admittances. Edges whose assigned admittances are in $D$ are called unknown.

We consider only a common tree and a common 2-tree, taking the others to be a special case of them. Let $T$ and $S$ denote the set of common trees and common 2-trees $T_{j, r}^{k, r}$, respectively, of $G$, and $\alpha$ and $\beta$ denote a common sub-tree and sub-2-tree $\tau_{j, r}^{k, r}$, respectively.

We are mainly concerned with a common sub-tree and a common sub-2-tree consisting of edges in $D$ (including an empty one), which will be designated by the modifier "restricted." Let $T(\alpha)$ or $S(\beta)$ be the set of common trees or common 2-trees $T_{j, r}^{k, r}$, respectively, which contain $\alpha$ or $\beta$ both restricted to $D$ but no other edges in $D$.

Lemma 2: $T(\alpha)$ and $S(\beta)$ are equivalence classes of $T$ and $S$, respectively. Distinct restricted common sub-trees and common sub-2-trees $\tau_{j, r}^{k, r}$ define distinct equivalence classes in $T$ and $S$, respectively, unless the classes are empty.

Proof: Every common (2-) tree contains some restricted common sub-(2-) tree. Note here that $\alpha$ and $\beta$ may be empty by definition. Let $t \in T$ and $s \in S$, and $\gamma$ and $\delta$ be the set of all unknown edges contained in $t$ and $s$, respectively. Obviously $\gamma$ (or $\delta$) is a restricted common sub-(2-) tree, and $t \in T(\gamma)$ and $s \in S(\delta)$. 

3. The Network Corresponding to Classes of $T$ and $S$

**Definition 2**: Let $\alpha$ (or $\beta$) be a restricted common sub-(2-) tree of $G$. Let $\delta = D - \delta$ where $\delta = \alpha$ or $\beta$. We define that $G[\delta; \overline{\delta}]$ is the network formed from $G$ by deleting all edges in $\delta$ and coalescing the vertices connected by edges in $\delta$. By the *current or voltage graph of $G[\delta; \overline{\delta}]$* we mean $G_i[\delta; \overline{\delta}]$ or $G_v[\delta; \overline{\delta}]$, respectively, where $G_i$ or $G_v$ is the current or voltage graph of $G$. By a common tree (or a common 2-tree $t_{i,k,r}^j$) of $G[\delta; \overline{\delta}]$ we mean a subnetwork that is a tree both in its current graph and in its voltage graph (or one that is a 2-tree $t_{i,k,r}$ in the current graph and a 2-tree $t_{j,r}$ in the voltage graph of $G[\delta; \overline{\delta}]$).

**Theorem 1**: Let $\alpha$ (or $\beta$) be a restricted common sub-(2-) tree of $G$. Let $t \in T(\alpha)$ and $s \in S(\beta)$. Then $t$ and $s$ can be expressed as $t = t' \cup \alpha$ and $s = s' \cup \beta$, where $t'$ is a common tree of $G[\alpha; \overline{\alpha}]$ and $s'$ is a common 2-tree of $G[\beta; \overline{\beta}]$. Conversely, if $t'$ (or $s'$) is a common (2-) tree of $G[\alpha; \overline{\alpha}]$ (or $G[\beta; \overline{\beta}]$), then $t' \cup \alpha$ (or $s' \cup \beta$) is a common (2-) tree of $G$ which belongs to $T(\alpha)$ (or $S(\beta)$).

**Proof**: By definition $t$ (or $s$) contains $\alpha$ (or $\beta$). Let $t' = t - \alpha$ and $s' = s - \beta$. Then $t'$ (or $s'$) is a common sub-(2-) tree of $G$ which does not contain any edge in $D$ (i.e. any unknown edge). $|t'| = v - 1 - |\alpha|$, while $|s'| = v - 2 - |\beta|$. (See Definition 1 for the definition of $|t'|$, etc.) On the other hand, both $G_i[\alpha; \overline{\alpha}]$ and $G_v[\alpha; \overline{\alpha}]$ (or $G_i[\beta; \overline{\beta}]$ and $G_v[\beta; \overline{\beta}]$) contain $v - |\alpha|$ (or $v - |\beta|$) vertices, where $G_i$ and $G_v$ are the current and voltage graphs of $G$. Therefore $t'$ (or $s'$) is a common (2-) tree of $G[\alpha; \overline{\alpha}]$ (or $G[\beta; \overline{\beta}]$). Since the vertices of $G$ connected by an edge in $\alpha$ (or $\beta$) correspond to one vertex in $G_i[\alpha; \overline{\alpha}]$ or $G_v[\alpha; \overline{\alpha}]$ (in $G_i[\beta; \overline{\beta}]$ or $G_v[\beta; \overline{\beta}]$), the converse is evident.
The next corollary follows immediately.

**Corollary 1:** Let \( \alpha \) and \( \delta \) (or \( \beta \) and \( \gamma \)) be restricted common sub-(2-) trees of \( G \) such that \( \alpha \subseteq \delta \) (or \( \beta \subseteq \gamma \)) and that \( \delta \) (or \( \gamma \)) is not a common (2-) tree of \( G \). If class \( T(\delta) \) of the set \( T \) of common trees (or class \( S(\gamma) \) of the set \( S \) of common 2-trees \( t_{k,r}^{j,r} \)) of \( G \) is empty, then class \( T(\alpha) \) (or \( S(\beta) \)) is also empty.

4. The Determinant of the Node Admittance Matrix

Let \( A_i \) and \( A_v \) be the incidence matrices of the current and voltage graphs, respectively, of \( G \) with the same reference vertex whose rows are arranged in the natural order. As seen in Mayeda [2] and Coates [3], the node admittance matrix \( Y_n \) of \( G \) and its minor matrix \( Y_{jk} \) are expressed as

\[
Y_n = A_i Y A_v^t v
\]

\[
Y_{jk} = A_{i-j} Y A_{v-k}^t v
\]

where \( Y \) is the edge admittance matrix of \( G \), which is a diagonal matrix, \( A_{i-j} \) is the current incidence matrix with row \( j \) removed, and \( A_{v-k}^t \) is the transpose of the voltage incidence matrix with row \( k \) removed.

Coalescing two vertices corresponds to adding one row to the other and deleting the former row from the matrix. If one of the two is the reference vertex, simply delete the row corresponding to the other. Removing an edge corresponds to multiplying the corresponding column by zero. Thus we can obtain the incidence matrices \( A_i[\alpha;\overline{\alpha}] \) and \( A_v[\alpha;\overline{\alpha}] \) of \( G_i[\alpha;\overline{\alpha}] \) and \( G_v[\alpha;\overline{\alpha}] \), respectively. Similarly we can obtain \( A_{i-j}[\beta;\overline{\beta}] \) and \( A_{v-k}[\beta;\overline{\beta}] \) of \( G_i[\beta;\overline{\beta}] \) and \( G_v[\beta;\overline{\beta}] \), respectively.
We shall set the rules for coalescing the vertices. Note that a common sub-tree is just a sub-tree in the current graph as well as in the voltage graph.

**Definition 3:** A *free vertex of a sub-tree* $\tau$ of a network is a vertex of degree 1 in $\tau$ which is not the reference vertex; in other words, it is a terminal vertex of the sub-tree $\tau$ different from the reference vertex.

There exists at least one free vertex in each nonempty sub-tree. In coalescing the vertices connected by edges in sub-tree $\tau$, we make it a rule to start with a free vertex of $\tau$ and to coalesce a free vertex into another vertex of $\tau$ which is connected to the former by an edge in $\tau$. Then there still exists at least one free vertex in the resultant sub-tree. Hence we can continue this coalescing process. In coalescing two vertices $l$ and $m$ connected by an edge $x_h$, $1 \leq h \leq p$, one of which, say, vertex $l$, is free, we coalesce vertex $l$ into vertex $m$ by the above rule, and we also make it a rule to put the number $m$ on the new vertex and to assign $l$ or $-l$ to the edge $x_h$, depending on whether $x_h$ is oriented from vertex $l$ to vertex $m$ or from vertex $m$ to vertex $l$, respectively. Thus we can assign a number to each edge belonging to sub-tree $\tau$.

**Definition 4:** Suppose that a restricted common sub-tree $\alpha$ consists of edges $x_{i_1}, x_{i_2}, \ldots, x_{i_g}$ in $D$. Let $\sigma_i(i_j)$ and $\sigma_v(i_j)$, $1 \leq j \leq g$, be the absolute values of the numbers assigned to edge $x_{i_j}$ in the current and voltage graphs, respectively. The *sign* $\varepsilon_\alpha$ of a common sub-tree $\alpha$ is defined as

$$
\varepsilon_\alpha = (-1)^{a+b+c+d}
$$
where

\[ a = \text{the total number of minus signs among the numbers assigned to all edges in } \alpha \text{ in both the current and voltage graphs}, \]

\[ b = \sum_{j=1}^{g} (\sigma_i(i_j) + \sigma_v(i_j)) , \]

\[ c = \text{the number of interchanges required of the ordered set } \{\sigma_i(i_1), \sigma_i(i_2), \ldots, \sigma_i(i_g)\} \text{ to be rearranged in the natural order}, \]

\[ d = \text{the number of interchanges required of the ordered set } \{\sigma_v(i_1), \sigma_v(i_2), \ldots, \sigma_v(i_g)\} \text{ to be rearranged in the natural order.} \]

Let \( t \) be a common tree belonging to class \( T(\alpha) \). By \( A_i(t) \) and \( A_v(t) \) we mean the major submatrices of the current and voltage incidence matrices \( A_i \) and \( A_v \), respectively, corresponding to common tree \( t \). Then the sign of common tree \( t \) is, according to Seshu and Reed [1] and Mayeda [2], determined by

\[ \det A_i(t) \det A_v(t) . \]

We can assume without loss of generality that the columns (called unknown columns) corresponding to unknown edges appear first and that the other columns, which correspond to known edges, succeed to them in \( A_i(t) \) and \( A_v(t) \). Hence, the first \( g \) columns of \( A_i(t) \) and \( A_v(t) \) correspond to unknown edges \( x_{i_1}, x_{i_2}, \ldots, x_{i_g} \). We take only the current graph for a while. The row (called a free row) of \( A_i(t) \) corresponding to a free vertex of common sub-tree \( \alpha \) contains exactly one nonzero among the unknown columns. Suppose column \( j \), which corresponds to edge \( x_{i_j} \), of \( A_i(t) \) contains two nonzeros; one in row \( \ell \) and the other in row \( m \), and that row \( \ell \) is free. Adding row \( \ell \)
to row \( m \), which is consistent with the rules fixed in the above, is equivalent to changing the connection of edge \( x_{ij} \) from vertex \( m \) to the reference vertex, since a column which contains only one nonzero in the incidence matrix corresponds to an edge incident with the reference vertex. By repeating this procedure of changing the connection of unknown edges, we obtain the new matrix \( A_1'(t) \) in which each of the unknown columns contains exactly one nonzero. Obviously \( \det A_1'(t) = \det A_1''(t) \). According to our rule of renumbering the shorted edge the nonzero in column \( j \) of \( A_1'(t) \) is located in row \( \sigma_1(i_j) \). The \( c \) times interchanges of unknown columns of \( A_1'(t) \) bring forth the new matrix \( A_1''(t) \) in which the nonzero position in column \( j \) is higher than that in column \( j+1 \), \( j = 1, 2, \ldots, g-1 \). (See Definition 4 for symbol \( c \).) Evidently \( \det A_1''(t) = (-1)^c \det A_1''(t) \). Let \( a_i \) be the number of minus ones among the unknown columns of \( A_1''(t) \). Then \((-1)^{a_i+c} \det A_1''(t) = \det A_1''(t) \), where \( B_1(t) \) is the matrix obtained from \( A_1''(t) \) by changing -1, if any, to +1 in every unknown column. By applying the cofactor expansions with respect to unknown columns successively, we obtain

\[
\det B_1(t) = \prod_{j=1}^{g} (-1)^{\sigma_1(i_j)+j} \det(A_1[\alpha;\bar{\alpha}](t'))
\]

where \( A_1[\alpha;\bar{\alpha}] \) is the incidence matrix of \( G_1[\alpha;\bar{\alpha}] \) and \( t' \) is the sub-tree consisting of all known edges in \( t \). Note that, in obtaining \( G[\alpha;\bar{\alpha}] \), we are following the rules for coalescing the vertices and renumbering the shorted edges. Similarly, we have

\[
\det B_v(t) = \prod_{j=1}^{g} (-1)^{\sigma_v(i_j)+j} \det(A_v[\alpha;\bar{\alpha}](t'))
\]
Thus we obtain the equation
\[
\det A_i(t) \det A_v(t)
= (-1)^f \det(A_i[\alpha;\overline{\alpha}](t')) \det(A_v[\alpha;\overline{\alpha}](t'))
\]
where
\[
f = (a_i + a_v) + \sum_{j=1}^{g} (\sigma_i(i_j) + \sigma_v(i_j) + 2j) + c + d
= a + b + c + d + \sum_{j=1}^{g} 2j.
\]
Since \((-1)^{2j} = +1\) for every \(j\), we have
\[
\det(A_i(t) \cdot A_v^T(t)) = \epsilon_{\alpha} \det(A_i[\alpha;\overline{\alpha}](t') \cdot A_v^T[\alpha;\overline{\alpha}](t'))
\]
where \(\epsilon_{\alpha}\) was defined in Definition 4. Therefore we have proved the following theorem.

**Theorem 2:**
\[
\det Y_n = \det(A_i Y A_v^T)
= \sum_{\alpha} \epsilon_{\alpha} \cdot P(\alpha) \cdot \det(A_i[\alpha;\overline{\alpha}] \cdot Y \cdot A_v^T[\alpha;\overline{\alpha}])
\]
where \(\sum\) signifies the summation over all restricted common sub-trees \(\alpha\) including the empty one \(\phi\), and \(P(\alpha)\) is the product of all admittances assigned to edges in \(\alpha\), which may be called a **common sub-tree product**, and \(P(\phi) = 1\).

5. The Cofactor of the Node Admittance Matrix

As stated in the previous section, the minor matrix \(Y_{jk}\) of the node admittance matrix \(Y_n\) with respect to the \(jk\)-element is given by
We modify the definition of a free vertex for a sub-2-tree \( T_{i,r} \).

**Definition 5:** A free vertex of a sub-2-tree \( T_{i,r} \) of a network is a vertex of degree 1 in \( T_{i,r} \), which is distinct from vertex \( i \) and the reference vertex.

There exists at least one free vertex in each nonempty sub-2-tree. The rules for coalescing the vertices in the previous section still apply. Following the same procedure as in the previous section, we can assign a number to each edge in a sub-2-tree.

**Definition 6:** Suppose that a restricted common sub-2-tree \( \beta \) consists of edges \( x_{i_1}, x_{i_2}, \ldots, x_{i_g} \) in \( D \). Let \( \sigma_i(i_j) \) and \( \sigma_v(i_j) \), \( 1 \leq j \leq g \), be the absolute values of the numbers assigned to edge \( x_{i_j} \) in the current and voltage graphs, respectively. Then the sign \( \varepsilon_{\beta} \) of a common sub-2-tree \( \beta \) is defined as

\[
\varepsilon_{\beta} = (-1)^{a+b+c+d}
\]

where \( a, b, c, \) and \( d \) are those defined in Definition 4.

**Theorem 3:**

\[
\det Y_{jk} = \det (A^t_{i_j} Y A^t_{v_k})
= \sum_{\beta} \varepsilon_{\beta} \cdot Q(\beta) \cdot \det (A_{i_j}^t [\beta; \bar{\beta}] \cdot Y \cdot A^t_{v_k} [\beta; \bar{\beta}])
\]

where \( \Sigma \) indicates the summation over all restricted common sub-2-trees \( \beta \) including the empty one \( \varnothing \), and \( Q(\beta) \) is the product of all admittances associated with edges of \( \beta \), which may be called a common sub-2-tree product, and \( Q(\varnothing) = 1 \).
This theorem is proved in the same way as that for Theorem 2.

6. An Example

As an example, we take a common-emitter transistor circuit, shown in Figure 1, in which the load resistance and the current amplification factor should be taken as variables. The modified graph is shown in Figure 2. The transfer impedance from terminals (1,1') to (2,2') is required. We have

\[
\begin{bmatrix}
 x_1 & x_2 & y_1 & y_2 & y_3 & y_4 \\
 0 & 0 & 1 & 1 & 0 & 0 \\
 1 & -1 & 0 & 0 & 0 & 1 \\
 -1 & 0 & 0 & -1 & 1 & -1 \\
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
 1 & 0 & 1 & 1 & 0 & 0 \\
 0 & -1 & 0 & 0 & 0 & 1 \\
 -1 & 0 & 0 & -1 & 1 & -1 \\
\end{bmatrix}
\]

Restricted common sub-trees are \( \phi, x_1, x_2 \) and \( x_1x_2 \). Obviously \( \epsilon_\phi = \epsilon_{x_2} = +1 \). To obtain \( \epsilon_{x_1} \) and \( \epsilon_{x_1x_2} \), we coalesce vertex 3 into either vertex 2 in the current graph or vertex 1 in the voltage graph, and then vertex 2 into the reference vertex. Hence \( \epsilon_{x_1} = (-1)^{2+3+3} = +1 \) and

\( \epsilon_{x_1x_2} = (-1)^{4+(3+3+2+2)+(1+1)} = +1 \). Therefore
Figure 1.
Transistor Circuit

Figure 2.
Modified Graph

\[ y_1 = 0.02 \text{ mho} \]
\[ y_2 = 0.002 \text{ mho} \]
\[ y_3 = 0.04 \text{ mho} \]
\[ y_4 = 2.5 \times 10^{-5} \text{ mho} \]
\[
\det Y_n = \det \begin{bmatrix}
y_1 + y_2 & 0 & -y_2 \\
0 & y_4 & -y_4 \\
y_2 & -y_4 & y_2 + y_3 + y_4
\end{bmatrix}
\]

\[
+x_1 \det \begin{pmatrix}
1 & 1 & 0 & 0 \\
0 & -1 & 1 & 0 \\
0 & 0 & y_3 \\
0 & 0 & y_4
\end{pmatrix}
\begin{pmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4
\end{pmatrix}
= \begin{pmatrix}
1 & 0 \\
0 & 0 \\
1 & 0 \\
-1 & 1
\end{pmatrix}
\]

\[
+x_2 \det \begin{pmatrix}
1 & 1 & 0 & 0 \\
0 & -1 & 1 & -1 \\
0 & 0 & y_3 \\
0 & 0 & y_4
\end{pmatrix}
\begin{pmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4
\end{pmatrix}
= \begin{pmatrix}
1 & 0 \\
1 & -1 \\
0 & 1 \\
0 & -1
\end{pmatrix}
\]

\[
+x_1 x_2 \det \begin{pmatrix}
1 & 1 & 0 & 0 \\
0 & y_2 \\
0 & y_3 \\
0 & y_4
\end{pmatrix}
\begin{pmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4
\end{pmatrix}
= \begin{pmatrix}
1 \\
1 \\
0 \\
-1
\end{pmatrix}
\]

\[
= 2.3 \times 10^{-8} + 9.2055 \times 10^{-4} x_2 + 2 \times 10^{-2} x_1 x_2.
\]

Restricted common sub-2-trees which separate vertices 1 and 1' in the current graph and vertices 2 and 2' in the voltage graph are \( \phi \) and \( x_1 \).

Obviously \( \varepsilon_{\phi} = +1 \). Coalescing vertex 3 into vertex 2 in the current graph or into vertex 1 in the voltage graph, we obtain \( \varepsilon_{x_1} = +1 \). Hence
\[
\det Y_{12} = \det \begin{bmatrix}
0 & -y_4 \\
-y_2 & y_2y_3+y_4
\end{bmatrix}
\]
\[
+ x_1 \det \begin{bmatrix}
0 & -1 & 1 & 0 \\
y_1 & 0 & 0 & 0 \\
y_2 & -1 & 0 & 0 \\
y_3 & 0 & 0 & -1 \\
y_4 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
0 \\
-1
\end{bmatrix}
\]
\[
= -5.0 \times 10^{-8} + 4.0 \times 10^{-2}x_1.
\]
Therefore the transfer impedance from terminals \((1,1')\) to \((2,2')\) is
\[
\frac{5.0 \times 10^{-8} - 4.0 \times 10^{-2}x_1}{2.3 \times 10^{-8} + 9.2055 \times 10^{-4}x_2 + 2 \times 10^{-2}x_1x_2}
\]
since \((-1)^{1+2} = -1.\)

7. Generation of Sub-Trees

We consider the subnetwork \(H\) of \(G\) composed of all the unknown edges. In his recent work [4], the author established a method of generating all the trees of a network by elementary tree transformations along a Hamilton circuit of the associated tree graph, which will be referred to as Procedure T. If the subnetwork \(H\) is connected, we apply this tree generation method directly; if not, we can easily modify the method for the forests of \(H\). (See Seshu and Reed [1] for the definition of a forest.) For simplicity we assume that subnetwork \(H\) is connected.
Pick a tree $\tau^0$ of $H$, and number the edges belonging to $\tau^0$ from 1 to $m$, where $m = |\tau^0|$. Remove the first edge (i.e., $e_1$) from $\tau^0$ and let $\tau_1^0$ denote the resultant sub-tree. Likewise let $\tau_{j_1j_2\ldots j_g}^0$ be the sub-tree formed from $\tau^0$ by removing edges $e_{j_1}, e_{j_2}, \ldots, e_{j_g}$. Continue this process lexicographically with respect to the numbering of the edges; that is, in such an order as $1, 1 \cdot 2, \ldots, 1 \cdot 2 \cdot \ldots \cdot (m-2) \cdot (m-1) \cdot m, 1 \cdot 2 \cdot \ldots \cdot (m-2) \cdot m, 1 \cdot 2 \cdot \ldots \cdot (m-3) \cdot (m-1), \ldots, 1 \cdot m, 2, 2 \cdot 3, \ldots, (m-1) \cdot m, m$. This procedure will be referred to as Procedure L. By the $j_1j_2\ldots j_g$-branch, $1 \leq j_1 < j_2 < \ldots < j_g < m$, we mean a set of sequences between $j_1j_2\ldots j_g (j_g+1)$ and $j_1j_2\ldots j_g$. By Procedure T, obtain another tree $\tau^1$ of $H$ which is adjacent to $\tau^0$. Let $e_{m+1}$ denote the edge which belongs to $\tau^1$ but not to $\tau^0$. Apply the above procedure (Procedure L) to all the edges but $e_{m+1}$ in $\tau^1$, and obtain all the sub-trees containing $e_{m+1}$. We have thus far obtained all the sub-trees which are contained in $\tau^0$ or $\tau^1$. By $\tau^0 \cup \tau^1$, we mean the set of edges belonging to either $\tau^0$ or $\tau^1$. Suppose that all the sub-trees contained in some of trees $\tau^0, \tau^1, \ldots, \tau^{g-1}$ are obtained. Obtain a tree $\tau^g$ adjacent to $\tau^{g-1}$ by Procedure T and let $e^0$ be the edge contained in $\tau^g$ but not in $\tau^{g-1}$. If $e^0 \notin \tau^1 \cup \tau^j$, applying Procedure L to all the edges but $e^0$ in $\tau^g$ we can obtain all the sub-trees in $\tau^g$ containing $e^0$. Note that a sub-tree not containing $e^0$ but contained in $\tau^g$ has been obtained already by the inductive hypothesis. If $e^0 \notin \cup \tau^j$, let $\tau^i, \tau^j, \ldots, \tau^h, 0 \leq j_1 < j_2 < \ldots < j_h < g-1$, be the trees which contain $e^0$. By $e_{j_1}^i, e_{j_2}^i, \ldots, e_{j_h}^i$ we denote the edges which are contained in $\tau^g$ but not in $\tau^j$, $1 \leq i \leq h$. Note $f_0 = 1$. Let $\lambda$ be the collection of all the sets of edges such that a set in $\lambda$ contains at least
one edge in \( \{ e_1^i, e_2^i, \ldots, e_h^i \} \) for every \( i, 0 \leq i \leq h \). A nonempty sub-tree which does not contain any set belonging to collection \( \lambda \) is contained in some tree \( \tau_j^i \), \( 0 \leq j \leq g-1 \), and hence has already been generated according to the hypothesis. We have only to generate all the sub-trees containing a set in \( \lambda \). Applying Procedure \( L \) to the edges belonging to \( \bigcap_{i=1}^{h} \tau_j^i \cap \tau_{g-1} \) and combining a set in \( \lambda \), we can obtain all the sub-trees which are contained in some tree \( \tau_j^i \), \( 0 \leq j \leq g \). Continuing this process we can generate all the nonempty sub-trees of the subnetwork \( H \), since we have Lemma 1.

8. Remarks on Reducing Computation Time

In the previous section, we have obtained a method of generating all the nonempty subtrees of \( H \) without duplication. It can be easily modified to generate the sub-2-trees \( \tau_{ij}^r \), the common sub-trees or the common sub-2-trees \( \tau_{k,r} \) of \( H \).

We consider only common sub-trees and sub-2-trees again. According to Corollary 1, if \( \alpha \) and \( \delta \) (or \( \beta \) and \( \gamma \)) are common sub-(2-) trees and \( \alpha \subseteq \delta \) (or \( \beta \subseteq \gamma \)), then the emptiness of \( T(\delta) \) (or \( S(\gamma) \)) implies the emptiness of \( T(\alpha) \) (or \( S(\beta) \)). Hence, in Procedure \( L \) of the preceding section, we can neglect the \( j_1j_2\ldots j_g \)-branch if \( T(\tau_{ij_1j_2\ldots j_g}^i) \) (or \( S(\tau_{ij_1j_2\ldots j_g}^i) \)) is empty. We are here assuming that \( \tau_{ij_1j_2\ldots j_g}^i \) is a common sub-(2-) tree of \( H \).

The computation of the signs of a common sub-tree and a common sub-2-tree can be simplified. Let \( \delta = \{ X_1, X_2, \ldots, X_{g-1}, X_{g+1}, \ldots, X_h \} \) and \( \gamma = \{ X_1, X_2, \ldots, X_{g-1}, X_{g+1}, \ldots, X_h \} \) be common sub-trees or common sub-2-trees such that \( \delta \subset \gamma \) and \( \gamma - \delta = \{ X_g \} \). Assume that \( \sigma_i(j) \) and \( \sigma_{\gamma}(j) \) are those computed for edge \( X_j \), \( 1 \leq j \leq h \), in \( \gamma \), using the notations defined in
Definition 4. By \(\mu_p(j)\) or \(\mu_v(j)\) (or \(\mu_s(j)\) or \(\mu_v(j)\)) we denote the number of \(\sigma_i\)'s or \(\sigma_v\)'s greater (smaller) than \(\sigma_i(j)\) or \(\sigma_v(j)\), respectively, which are located preceding to (succeeding to) the jth position in the ordered sets \(\{\sigma_i(1), \sigma_i(2), \ldots, \sigma_i(j), \ldots, \sigma_i(h)\}\) or \(\{\sigma_v(1), \sigma_v(2), \ldots, \sigma_v(j), \ldots, \sigma_v(h)\}\), both associated with \(\gamma\). As seen in Hohn [5],

\[
\prod_{j=1}^{h} (-1)^{\mu_p(j)} = \prod_{j=1}^{h} (-1)^{\mu_s(j)} \\
\prod_{j=1}^{h} (-1)^{\mu_v(j)} = \prod_{j=1}^{h} (-1)^{\mu_v(j)}
\]

where c and d were defined in Definition 4. Consider the ordered sets \(\{\sigma_i(1), \sigma_i(2), \ldots, \sigma_i(g-1), \sigma_i(g+1), \ldots, \sigma_i(h)\}\) and \(\{\sigma_v(1), \sigma_v(2), \ldots, \sigma_v(g-1), \sigma_v(g+1), \ldots, \sigma_v(h)\}\), which are associated with \(\delta\). Let \(c'\) and \(d'\) be defined for \(\delta\) similarly to \(c\) and \(d\) for \(\gamma\). Then

\[
(-1)^{c' + d'} = (-1)^{c + d}
\]

\[
\prod_{j=1}^{h} (-1)^{\mu_p(j) + \mu_s(j)} = \prod_{j=1}^{h} (-1)^{\mu_v(j) + \mu_v(j)}
\]

since \(\mu_p(g)\) larger \(\sigma_i\)'s or \(\mu_v(g)\) larger \(\sigma_v\)'s precede to \(\sigma_i(g)\) or \(\sigma_v(g)\), respectively, and \(\sigma_i(g)\) or \(\sigma_v(g)\) precedes to \(\mu_s(g)\) smaller \(\sigma_i\)'s or \(\mu_v(g)\) smaller \(\sigma_v\)'s in the ordered set \(\{\sigma_i(1), \ldots, \sigma_i(h)\}\) or \(\{\sigma_v(1), \ldots, \sigma_v(h)\}\) associated with \(\gamma\). If we coalesce the vertices in \(\delta\) in the same manner as in \(\gamma\), the same numbers are assigned to all edges but \(X_g\) in \(\delta\) and \(\gamma\). Therefore the sign of \(\delta\) is given

\[
\xi_\delta = (-1)^{f} \xi_\gamma
\]
where
\[
f = \sigma_i(g) + \sigma_v(g) + \mu_{p_i}(g) + \mu_{s_i}(g)
+ \mu_{p_v}(g) + \mu_{s_v}(g) + \text{no. of minus signs among } \sigma_i(g) \text{ and } \sigma_v(g).
\]

Thus we can reduce the time for computing the signs.

9. Conclusion

We have obtained one method of network analysis. The graph-theoretical manipulation is applied only to the set D of unknown edges.

In the worst case, the number of restricted sub-trees is equal to \(2^D\), and hence the procedure involved in this method increases rather rapidly with the number of unknown edges. However, it is still better than the symbolical computation of the determinant of the node admittance matrix and its cofactor, even applying the Laplace expansion theorem.

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