INITIAL BEHAVIOR
OF THE PSEUDO-SHOCK

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INITIAL BEHAVIOR OF THE PSEUDO-SHOCK

S. M. Yen and B. L. Hicks

Abstract

The behavior of the pseudo-shock at the onset and the early state of relaxation has been studied. The initial relaxation behavior of a gas of elastic spheres has been obtained by solving the Boltzmann equation by Nordsieck's Monte Carlo method for small times (during which less than one molecule in five has collided) with spatially uniform initial velocity distribution function

\[ f(v, \tau=0) = \frac{1}{2} \{ \exp(-\pi|\tilde{v}-\tilde{u}|^2) + \exp(-\pi|\tilde{v}+\tilde{u}|^2) \} \]

where \( f \) and \( \tilde{v} \) are in specialized units and the Mach number

\[ M = \left( \frac{6\pi}{5} \right)^{\frac{1}{2}} \tilde{u} \]

describes the relative separation of the two peaks of this bimodal distribution function. Calculations are reported for \( M = 1, 2, 4 \) and 6. The characteristics of the relaxation of the lateral temperature and the Boltzmann function at the early stage have been compared with those of the large times (during which one molecule has collided up to six times). The initial relaxation of the lateral temperature for \( M = 1, 2 \) and 4 and for the Boltzmann flux for \( M = 1 \) are exponential and the relaxation rate for the lateral temperature seems to be independent of the Mach numbers mentioned. These findings are in agreement with those of large times. However, for \( M = 2 \) and 4, the relaxation of the Boltzmann flux is not exponential near the onset of the relaxation. The relaxation of the distribution
function has been monitored for twelve chosen velocity bins. In most of the velocity bins, the distribution function seems to relax exponentially for the Mach numbers considered. We have also developed analytical expressions for the collision frequency and for two moments of the collision integral at the onset of the relaxation ($\tau = 0$). Comparisons have been made with the corresponding Monte Carlo results to ascertain the accuracy of the latter.
Introduction

The numerical solution of the Boltzmann equation for the pseudo-shock (a simple translational relaxation of a monatomic gas consisting of hard sphere molecules with an initial spatially uniform bi-modal velocity distribution function) has been obtained, and the results on the relaxation for large times (during which one molecule has collided up to six times) of the lateral temperature and the Boltzmann function were given and discussed in detail in an earlier CSL report. Both the lateral temperature $T_L$ and the Boltzmann function $H$ were found to relax exponentially, and the average rate of the relaxation calculated in terms of the number of collisions of a representative molecule of a reference gas appeared to be independent of the Mach number.

The initial out-of-equilibrium distribution chosen for the pseudo-shock is

$$f(v, \tau=0) = \frac{1}{2} \{ \exp(-\pi |\tilde{v}-\tilde{u}|^2) + \exp(-\pi |\tilde{v}+\tilde{u}|^2) \}$$  \hspace{1cm} (1)

where $f$ is the distribution function; $\tilde{v} = \tilde{v}(v_x, v_\perp)$ is the velocity; $\tau$ is the time variable; and the vector $\tilde{u} = u \hat{I}$ lies along the $v_x$ axis. Specialized units are used for $f$ and $\tilde{v}$. The velocity moments of the collision integral for this distribution function can be evaluated analytically; therefore, the relaxation rate at $\tau = 0$ of any property depending on these velocity moments is also known analytically.

In the past year, we have studied the pseudo-shock at the onset ($\tau = 0$) and the early stage of relaxation (at which less than one molecule in five has collided). We have compared the analytical relaxation rates of $(v_\perp^2)$ at $\tau = 0$ with the corresponding Monte Carlo values; and the early
relaxation behavior with that for the relatively larger time mentioned above. In addition, we have monitored the relaxation of the distribution function $f$ for several particular velocity bins.

We use two parameters in our study of the pseudo-shock, $\eta$ and $M$. The parameter $\eta$ is the scaled time variable, related to time $\tau$ by Eq. (4), and represents the number of collisions suffered by a representative molecule of a reference gas in time $\tau$. The parameter $M$, related to $u$ by Eq. (3), is a measure of the initial departure from thermal equilibrium. The present studies of initial behavior have been made for the range of $\eta$ of 0 to 0.2 and for $M = 1, 2, 4, \text{ and } 6$.

**Review of the Problem**

The Boltzmann equation for the pseudo-shock is

$$\frac{df}{d\tau} = a-bf$$

where $(a-bf) =$ collision integral. The results described in this report are evaluated from the numerical solution of the Boltzmann equation for small values of the time variable $\tau$ for a monatomic gas consisting of elastic sphere molecules with the initial out-of-equilibrium condition specified by Eq. (1).

The units we use are the values, denoted by the subscript 1, of various properties of a reference gas. Thus $n_1$, $T_1$ are the units of number density $n$ and temperature $T$. The unit of length $l_1 = 1/2\pi n_1\sigma^2$. The unit of velocity $C_1 = (2\pi kT_1/m)^{1/2}$. In our units, the parameter $u$ used in Eq. (1) is related to a Mach number $M$ by the equation

$$M = (6\pi/5)^{1/2}u.$$
The reference gas chosen is an equilibrium gas with \( n_1 \) = density of the relaxing gas and \( T_1 \) = temperature corresponding to one of the two distribution functions that constitute the initial symmetrical bimodal distribution function.

As in reference 1, we use the scaled time variable

\[
\eta = \left(\frac{\sqrt{2}}{\pi}\right) t^{\frac{1}{2}}
\]

where \( t = 1 + (5/9)M \), the reduced temperature of the relaxing gas. Thus \( \eta \) is the number of collisions suffered by a representative molecule of an equilibrium gas with \( n = 1 \) and temperature \( t \) in time \( \tau \).

Analytical Calculations for the Onset of the Relaxation

The moment equation corresponding to Eq. (1) is

\[
\frac{d\mathcal{M}(\phi)}{d\tau} = I(\phi)
\]

where \( \phi = \text{function of velocity and/or distribution function} \)

\[\mathcal{M}(\phi) = \text{moment of } f = \int \phi f \, dv\]

\[I(\phi) = \text{moment of } (a-bf) = \int \phi \ (a-bf) \, dv\]

\( I(\phi) \) is thus equal to the relaxation rate of \( \mathcal{M}(\phi) \).

We have obtained the following expressions for \( I(v_\perp^2) \) and \( I(v_\parallel^4) \):

\[
I(v_\perp^2) = \frac{1}{3} \left( \frac{1}{\pi u} \right) \left[ \frac{1}{2} \ \text{erf}\left( \sqrt{\frac{1}{2}} u \right) - \frac{3}{16} \frac{u}{\pi} + \frac{3}{64} \right] + \frac{1}{\sqrt{2}} e^{-2\mu^2} \left[ \pi u^4 + \frac{1}{2} \pi u^2 - \frac{3}{10} \right] \]

(6)
\[ I(v_\perp^4) = \frac{1}{2} u^5 \{ 1 + \text{erf} \sqrt{2\pi u} \} \left[ -\frac{3}{128} \frac{1}{\pi u} + \frac{993}{256} \frac{1}{3 \pi u} - \frac{5}{4} \frac{1}{3 \pi u} - \frac{3}{32} \frac{1}{\pi u} + \frac{11}{16} \frac{1}{4 \pi u} - \frac{3883}{80} \frac{1}{3 \pi u} \right. \\
+ \frac{7}{10^2} + \frac{u}{\sqrt{2}} e^{-2\pi u^2} \left[ \frac{3}{32} \frac{1}{\pi u} + \frac{11}{16} \frac{1}{4 \pi u} - \frac{3883}{80} \frac{1}{3 \pi u} \right. \\
- \left. \left( \frac{1951}{20} \frac{1}{2 \pi u} - \frac{1486}{35} \frac{1}{\pi u^2} - \frac{1508}{63} \right) \right]. \] (7)

We note that \( I(v_\perp^2) = -I(v_\perp^2). \) (Expressions similar to Eqs. (6) and (7) may be obtained for Maxwellian molecules.) The value of \( I(v_\perp^2) \) calculated by using Eq. (6) is tabulated in Table I for \( M = 1, 2, 4 \) and 6.

The collision rate \( v_a = (\int a \, d\bar{v}) \) at the onset of the relaxation may also be evaluated. The expression is

\[ v_a(0) = n_a \left[ \frac{1 + F(M^2/2)}{2} \right] \left( \frac{\sqrt{2}}{\pi} \right), \] (8)

where

\[ F(M) = \frac{1}{2} e^{-\frac{5}{6} M^2} + \left( \frac{\pi}{120} \right)^{\frac{1}{2}} \left[ \frac{5 M^2 + 3}{M} \right] \text{erf} \left[ \left( \frac{5}{6} \right)^{\frac{1}{2}} M \right]. \]

The terminal collision rate

\[ v_a(\infty) = n_a \left( \frac{t^{\frac{1}{2}}}{\pi} \right) \left( \frac{\sqrt{2}}{\pi} \right) \] (9)

Both \( v_a(0) \) and \( v_a(\infty) \) are tabulated in Table II. We note that the ratio \( v_a(\infty)/v_a(0) \) has the range of 1 to \( 4/\sqrt{3\pi} \) for \( M = 0 \) to \( \infty \).

Since the collision rate \( v_a \) is also equal to \( \int b \, f \, d\bar{v} \), we may define the average collision frequency \( \bar{b}_a \) by using the following expression:

\[ \bar{b}_a = v_a / n_a \] (10)

For the pseudo-shock, \( n_a \) is constant; therefore, the ratio \( \bar{b}_a(\infty)/\bar{b}_a(0) \) is equal...
to $v_a(\infty)/v_a(0)$ and also has the range 1 to $4/\sqrt{3}\pi$ for $M = 0$ to $\infty$. We thus observe that the change in average collision frequency between the initial out-of-equilibrium condition ($\eta = 0$) and the equilibrium condition ($\eta \to \infty$) is small for any given Mach number. In the BGK model of the collision integral, the average collision frequency is considered as an unknown parameter.

In the earlier report of the pseudo-shock, $\eta_\perp$, which is the number of collisions of each molecule necessary for relaxation of $t_\perp$ by a factor of $e^{-1}$ and is equal to the reciprocal of the logarithmic slope of the relaxation curve of $t_\perp$, is used to characterize the relaxation of $t_\perp$. For $M \gg 1$, $\eta_\perp$ was found to be $4/\sqrt{3}\pi = 1.303$ which is equal to $b_\perp(\infty)/b_\perp(0)$ for large Mach numbers. The Monte Carlo value of $\eta_\perp$ found for large times in that report was $1.27 \pm 0.044$ collisions. The analytical value of $\eta_\perp$ calculated in this report from the logarithmic slope of the relaxation curve of $t_\perp$ at the onset ($\eta = 0$) was 1.303.

Monte Carlo Results and Discussion

We shall present the results for $M = 1, 2, 4$ and 6 for the early stages of relaxation, that is for $\eta$ less than 0.2. The computer runs and the parameters used are summarized in Table III. We used samples of $N = 2^{17}$ collisions for the set of runs from which we obtained the statistical variation of the various quantities. The interval $\Delta \eta$ used ranged from 0.0168 to 0.0504.

We are interested in four aspects of the relaxation studies:

1. comparing the analytical values of the moment of collision integral $I(v_\perp^2)$ and the collision frequency at the onset ($\eta = 0$) with the corresponding Monte Carlo values;
2. comparing the relaxation of lateral temperature and the Boltzmann function obtained in this study for $\eta < 0.2$ with the corresponding
results obtained for larger times \(^1\) (\(\eta\) ranged from 0.1 to 6); (3) monitoring the initial relaxation of the distribution function at chosen points in the velocity space; and (4) examining the change in collision frequency during the initial period of relaxation. We shall, therefore, divide the presentation of results into four parts: (1) relaxation rates and collision rate at \(\eta = 0\); (2) relaxation of \(t_1\) and \(H\) for small times (including the comparison with results at larger time); (3) relaxation of \(f\) for small times; and (4) collision rates for small times.

We must distinguish several functions that occur in the numerical calculations.\(^1\) To do this we introduce three subscripts \(a\), \(q\), and \(M\) for the number density \(n\), the lateral temperature \(t_1\), and the Boltzmann function \(H\). The subscript \(a\) indicates a wholly analytical calculation. The subscript \(q\) indicates a value derived by numerical integration of an analytical \(f\) over velocity space. The subscript \(M\) indicates a result derived by numerical integration of a Monte Carlo \(f\) over velocity space. We would like to point out here again that the terminal density and temperature are \(n_0\) and \(t_0\) respectively. These are the values that are kept constant during relaxation in our numerical method.

1. Initial Relaxation Rates and Collision Frequencies (\(\eta = 0\))

As indicated by Eq. (5), the moment of collision integral \(I(\theta)\) is equal to the relaxation rate of the velocity moment \(\eta(\theta)\). Since our Monte Carlo results include both \(I(\theta)\) and \(\eta(\theta)\), it is possible to calculate the initial relaxation rates of \(\eta(\theta)\) from \(I(\theta)\) and also directly from the values of \(\eta(\theta)\) at \(\eta = 0\) and \(\eta = \Delta \eta\). (\(\Delta \eta\) is the interval used in our numerical integration and is tabulated in Table III.) We have made such calculations for
\( \phi = v_{\perp}^2 \) and \( \phi = 1nf \). As mentioned earlier, the analytical values of the relaxation rate of \( \phi = v_{\perp}^2 \) were calculated and are given in Table I.

We compared the analytical values of the moment of the collision integral \( I(v_{\perp}^2) \) calculated by using Eq. (6) with the Monte Carlo values from the computer runs for \( 2^{17} \) collisions. The Monte Carlo results are also given in Table I. We note that for \( M = 1 \) and 2, the analytical value lies within the 50% confidence limits of the Monte Carlo values; however, the corresponding difference is significant for \( M = 4 \) and 6. The Monte Carlo initial relaxation rates of \( \phi = v_{\perp}^2 \) calculated from \( m(v_{\perp}^2) \) were found to be in excellent agreement with the corresponding results from \( I(v_{\perp}^2) \) for the Mach numbers considered.

Since there is no formula for the initial relaxation rate of the Boltzmann function, we are not able to determine the analytical initial relaxation rate as we can for the lateral temperature. We examined the Monte Carlo results on \( I(1+lnf) \) and \( d m(1nf)/d\eta \) at \( \eta = 0 \) and noted that, for \( M = 4 \) and 6, the value of \( I(1+lnf) \) at \( \eta = 0 \) is positive. Since \( I(1+lnf) \) is equal to the relaxation rate of the Boltzmann function, it should have negative values during the relaxation. However, the initial relaxation rates of the Boltzmann function calculated directly from \( m(1nf) \) are negative for all Mach numbers. The initial relaxation rates calculated by using \( I(1+lnf) \) and \( d m(1nf)/d\eta \) are given in Table V.

We have also compared the relaxation rates of \( H \) calculated by using both \( I(1+lnf) \) and \( d m(1nf)/d\eta \) for \( \eta \geq \Delta\eta \). We observe that for \( M = 4 \) and 6: (1) the relaxation rate from the collision integral is positive only for \( \eta = 0 \) and has reasonable values for \( \eta \geq \Delta\eta \); (2) for \( \eta \geq \Delta\eta \), the difference
between the two average relaxation rates calculated by using $I(l+lnf)$ and $\mathcal{M}(lnf)$ is in most cases less than the statistical deviation.

The initial collision frequency ($u_M(0)$) from the Monte Carlo calculation is compared with the analytical value ($u_a(0)$) and tabulated in Table II. The analytical value $u_a(0)$ lies outside the 50% confidence limits for the Monte Carlo value $u_M(0)$, especially for $M = 4$ and 6. In the case of larger Mach numbers, we have noticed that, for a particular run (i.e. with a particular set of random numbers), the disagreement with the analytical value is appreciable whenever the ($a$-$bf$) correction results in large (uncorrected) negative values of $a$ or $bf$ in certain velocity bins.

2. Relaxation of Lateral Temperature and Boltzmann Function

(a) Relaxation of Lateral Temperature

We examined the initial behavior of the relaxation of the lateral temperature by plotting $\log_{10}[t_q(0) - t_q(\eta)]$ vs. $\eta$. Figure 1 shows the plot of $\log_{10}[t_q(0) - t_q(\eta)]$ vs. $\eta$ for $M = 1, 2, \text{and } 4$. The straight lines are drawn through the first point (corresponding to the initial lateral temperature) with average slopes calculated on the basis of the Monte Carlo $I(v_{\perp}^2)$ at $\eta = 0$. The relaxation rate $dt_{\perp}/d\eta$ is related to $I(v_{\perp}^2)$ by the following expression

$$dt_{\perp}/d\eta = (\pi/0.4503 \eta_q(0)t_q(0)^{3/2})I(v_{\perp}^2)$$  \hspace{1cm} (11)

We observe that the lateral temperature appears to relax exponentially for the three Mach numbers investigated (corresponding to curves that are essentially straight lines on semi-logarithmic plots). The finding is in agreement with the results of the earlier study as given in Reference 1. We use the mean deviation (given as $\Delta$ in Fig. 1) of the data with respect to the
straight line for the range of $\eta$ investigated.

The results on the slopes of the logarithmic relaxation curve are given in Table IV. The average Monte Carlo initial slopes for $M = 4$ and 6 are larger than for $M = 1$ and 2. It should be pointed out that for $M = 4$, as shown in Figure 1, if the straight line had been evaluated on the basis of the average Monte Carlo $t_\perp$ in the range of $\eta$ investigated, it would be in much better agreement with the analytical initial relaxation rate calculated according to Eq. (11) and discussed below. We have also compared the initial analytical relaxation rate of $d\ln(t_\perp - t_\infty)/d\eta$ with the average Monte Carlo values. These results are also given in Table IV. The analytical value is calculated according to the following expression:

$$
\frac{d \ln[t_\perp - t_q(0)]}{d\eta} = \frac{\pi^2[I(v_\perp^2)]}{\sqrt{2n_q(0)[t_\perp M(0) - t_q(0)]} t_a^\frac{1}{2}}
$$

(12)

Since our results show that the relaxation of the lateral temperature is exponential, the analytical value may be used to check the accuracy of our Monte Carlo calculation for the relaxation rate of the lateral temperature. We note that the analytical calculation shows that the relaxation rate $d\ln[t_\perp - t_q(0)]/d\eta$ is independent of Mach number. This is in agreement with the finding based on the results for large times.

(b) Relaxation of the Boltzmann Function

As indicated earlier, the moments of the collision integral calculated for $\Theta = (1 + \ln f)$ from the Monte Carlo calculations are erroneous at $\eta = 0$ for large Mach number. In studying the relaxation of the Boltzmann function $H$ we therefore calculated the initial rates from $m(\ln f)$. These initial slopes
are given in Table V. The relaxation curves of the Boltzmann flux are plotted in Fig. 2. For \( M = 2 \) and 4, we see that the initial slopes differ appreciably from the mean slopes of the relaxation curves given in Reference 1, and the relaxation of \( H \) does not seem to occur exponentially.

3. Distribution Function

We chose twelve velocity bins in the velocity space in studying the relaxation of the velocity distribution function. We used the average distribution functions in these velocity bins obtained from four runs with sample of \( 2^{17} \) collisions. The values of \( \ln | f(\eta) - f_\infty |_{av} \) were plotted with respect to \( \eta \) for each velocity bin. The distribution function of most of the chosen velocity bins seems to relax exponentially. The decay time in terms of the number of collisions for each of these velocity bins is given in Fig. 3 for \( M = 1, 2, \) and 4 respectively. The critical line on which \( (a - bf) = 0 \) for \( \eta = 0 \) is plotted in these figures. We also plot in Figure 3 the circular arc on which \( \bar{v} = u \). We will discuss the results for each Mach number in detail separately as follows:

(a) \( M = 1 \)

From the plots of \( \ln | f(\eta) - f_\infty |_{av} \) vs \( \eta \), we have observed that for eleven out of twelve velocity bins chosen, the distribution functions appear to relax exponentially (corresponding to curves that are essentially straight lines on the semi-logarithmic plots) and the Monte Carlo mean deviations from the relaxation curve are small. Generally, the decay time, expressed in terms of number of collisions of the reference gas, is of the same order of magnitude for the eleven bins. We chose to show in Figure 4 the relaxation of four velocity bins. The coordinates in the velocity space, \( (v_{xm}, v_{\perp m}) \), for these bins are
(1,1), (9,6), (9,13), and (13,1). In the velocity bin (1,1) (corresponding to the lowest possible speed we are able to study), the relaxation is non-exponential, and the Monte Carlo deviations are large. There are two relaxation phases: one with positive slope during which $f$ increases and one with negative slope during which $f$ decreases. It seems, therefore, that during the initial period more molecules scatter into the velocity bin with very small speed than out of it.

(b) $M = 2$

In six out of ten velocity bins chosen, the velocity distribution function seems to relax exponentially. Three of these six velocity bins have relatively larger speed and two of them have much slower decay time (in terms of number of collisions of reference or equilibrium gas) than others. In two of the ten velocity bins, (9,1) and (9,9), the relaxation curve has some small initial curvature followed by constant slope. In the velocity bin (1,1) the distribution function relaxes slowly initially, as expected, since the molecules in that bin with very low speed have less probability of participating in collisions at first. As $\eta$ increases, the distribution function seems to relax exponentially.

(c) $M = 4$

We selected nine velocity bins. In six of them, the distribution function seems to relax exponentially. The distribution of decay time in the velocity space is quite similar to the case of $M = 2$. For the velocity bin (9,1) with maximum $f$, the decay time for $M = 2$ and 4 is equal. In two other velocity bins, (9,5) and (13,5), which are situated close to the critical line on which $(a - bf) = 0$, the relaxation is non-exponential. Again, for the velocity
bin (1,1) the relaxation start slowly, and the relaxation curve is similar to the corresponding one for $M = 2$.

For $M \gg 1$, we expect the relaxation rate near the onset to be constant along the circular arc $\nu = u$, since, in that case, all pairs of colliding molecules have about the same relative velocity $2u\tilde{t}$ at the onset of relaxation. Two velocity bins chosen for $M = 4$ are on the circular arc. The decay times are 0.688 and 1.35 (in terms of collisions of reference gas); therefore, the relaxation characteristics predicted for very large $M$ were not obtained from the results of $M = 4$. The initial decay time for large $M$ is $4/\sqrt{3\pi} = 1.303$.

4. Collision Frequency

We have examined the collision frequency during the initial period of relaxation. Since the range of the ratio of the asymptotic $\nu_a(\infty)$ to the initial $\nu_a(0)$ is 1 to $4/\sqrt{3\pi}$ for Mach number range of 0 to $\infty$, we expect the variation of $\nu_a$ during the initial period to be small. For the sample size used, $2^{17}$, the Monte Carlo fluctuations of the collision frequency seems to be larger than the difference between $\nu_a(\infty)$ and $\nu_a(0)$; therefore, our Monte Carlo results also seem to indicate that the collision frequency has no significant variation with $\eta$.

Summary

1. We have developed analytical expressions for some moments of the collision integral for the initial out-of-equilibrium condition ($\eta = 0$) of the pseudoshock. These moments of the collision integral may be used to calculate the relaxation rate of the corresponding properties at the onset of the relaxation. Comparisons have been made with the Monte Carlo results to
ascertain the accuracy of the latter.

2. From the Monte Carlo results, we have obtained the moment of collision integral \( I(\phi(\vec{v})) \) of various \( \phi(\vec{v}) \) as well as the moment of the distribution function \( M(\phi(\vec{v})) \). The relaxation rate at any time \( \eta \) may be calculated by either.

3. Monte Carlo results were obtained for \( M = 1, 2, 4 \) and \( 6 \) and for the range of \( \eta \) from 0 to 0.2. We used samples of \( 2^{17} \) collisions and obtained the averages and statistical variations of quantities from sets of four runs.

4. In the earlier study\(^1\) for larger times (\( \eta \) ranged from 0.1 to 6), the relaxation rates were calculated on the basis \( M[\phi(\vec{v})] \). In the present study for smaller times (\( \eta \) ranged from 0 to 0.2), the relaxation rates at the onset (\( \eta = 0 \)) were calculated by using both \( I(\phi(\vec{v})) \) and \( M[\phi(\vec{v})] \) and the values are in agreement with each other except for the Boltzmann function for \( M = 4 \) and 6. For these two Mach numbers, we had to use \( M(1nf) \) to calculate the relaxation rate of the Boltzmann function at the onset since the Monte Carlo calculation of \( I(1nf) \) at these two Mach numbers is inaccurate there.

5. On the basis of average values and confidence limits obtained, the Monte Carlo relaxations of \( t_\perp \) at \( \eta = 0 \) are in good agreement with the analytical values for \( M = 1 \) and 2.

6. The relaxation of \( t_\perp \) in the range of \( \eta \) from 0 to 0.2 appears to support the following findings of the earlier study of \( \eta \) from 0.1 to 6: (a) the relaxation is exponential, and (b) the logarithmic relaxation rate is independent of the parameter \( M \) which is a measure of the initial departure from thermal equilibrium.
7. The relaxation of the Boltzmann function in the range of $\eta$ from 0 to 0.2 appears to be exponential for $M = 1$ only. The relaxation rate at the onset ($\eta = 0$) is more than twice as fast as the relaxation rate of $t_\perp$.

8. The velocity distribution function in most of the twelve velocity bins chosen for each Mach number appears to relax exponentially. The relaxation rate depends on the position of the velocity bin. For molecules with very small speed, the initial relaxation rate is small and the relaxation appears to become exponential as $\eta$ increases.

9. The change in collision frequency is small during the initial relaxation period. The analytical calculation shows that the change in collision frequency during the entire relaxation period is small. The ratio of asymptotic collision frequency (at $\eta = \infty$) to the initial out-of-equilibrium collision frequency (at $\eta = 0$), determined analytically, is $4/\sqrt{3\pi} = 1.303$ for $M = \infty$. 
Footnotes


2. The units of various quantities used in this report are the same as those used in ref. 1 and are given in the next section.

Acknowledgement

We are indebted to Mr. M. Jordan who has obtained most of the Monte Carlo results and made the preliminary analysis of some of these results.
Table I

\( I(v_{1}^2) \) at \( \eta = 0 \)

<table>
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<th>M</th>
<th>Analytical</th>
<th>Monte Carlo Value (From ( I(v_{1}^2) ))</th>
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Table II

Initial Collision Rate, $v(0)$, and Terminal Collision Rate, $v(\infty)$

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<th>$M$</th>
<th>$v_a(0)^*$</th>
<th>$v_a(\infty)^*$</th>
<th>$v_a(\infty)/v_a(0)$</th>
<th>$v_M(0)^{**}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.4502</td>
<td>0.4502</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.5584</td>
<td>0.5615</td>
<td>1.006</td>
<td>0.5691 (.0011)$^\dagger$</td>
</tr>
<tr>
<td>2</td>
<td>0.780</td>
<td>0.8081</td>
<td>1.036</td>
<td>0.8003 (.0068)$^\dagger$</td>
</tr>
<tr>
<td>4</td>
<td>1.260</td>
<td>1.4156</td>
<td>1.124</td>
<td>1.4006 (.0530)$^\dagger$</td>
</tr>
<tr>
<td>6</td>
<td>1.778</td>
<td>2.0629</td>
<td>1.160</td>
<td>2.3060 (.3282)$^\dagger$</td>
</tr>
<tr>
<td>$\infty$</td>
<td>$4/\sqrt{3\pi} = 1.303$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Eqs. (5) and (6) are used for these calculations ($n_a=1$).

** Monte Carlo values obtained from runs for sample $2^{17}$ collisions.

$^{\dagger}$ 50% confidence limit.
### Table III

**Summary of Pseudo-Shock Runs**

<table>
<thead>
<tr>
<th>M</th>
<th>$\Delta \eta$</th>
<th>$\eta/\Delta \eta$</th>
<th>no. of runs</th>
<th>$N^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.0337</td>
<td>1</td>
<td>2</td>
<td>$2^{17}$</td>
</tr>
<tr>
<td></td>
<td>.0168</td>
<td>4</td>
<td>5</td>
<td>$2^{17}$</td>
</tr>
<tr>
<td>2</td>
<td>.0252</td>
<td>8</td>
<td>5</td>
<td>$2^{17}$</td>
</tr>
<tr>
<td></td>
<td>.0504</td>
<td>4</td>
<td>1</td>
<td>$2^{17}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>$2^{19}$</td>
</tr>
<tr>
<td>4</td>
<td>.0283</td>
<td>5</td>
<td>4</td>
<td>$2^{15}$</td>
</tr>
<tr>
<td></td>
<td>.0283</td>
<td>4</td>
<td>4</td>
<td>$2^{17}$</td>
</tr>
<tr>
<td>6</td>
<td>.04126</td>
<td>4</td>
<td>4</td>
<td>$2^{17}$</td>
</tr>
<tr>
<td></td>
<td>.04126</td>
<td>1</td>
<td>1</td>
<td>$2^{19}$</td>
</tr>
</tbody>
</table>

$N^*$ = size of collision sample.
Table IV

Comparison of Slopes of Relaxation Curves

\[ \ln \left[ \frac{t_q(0) - t_q(\eta)}{t_q(\eta)} \right] vs \eta \]

<table>
<thead>
<tr>
<th>M</th>
<th>Analytical*</th>
<th>Monte Carlo Values From Runs of Small Time</th>
<th>Monte Carlo Values From Runs of Large Time**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean Slope</td>
<td>90% Confidence Limit</td>
</tr>
<tr>
<td>1</td>
<td>.7552</td>
<td>.785(^a)</td>
<td>.786(^b)</td>
</tr>
<tr>
<td>2</td>
<td>.7702</td>
<td>.777(^a)</td>
<td>.777(^b)</td>
</tr>
<tr>
<td>4</td>
<td>.7669</td>
<td>.841(^a)</td>
<td>.841(^b)</td>
</tr>
<tr>
<td>6</td>
<td>.7662</td>
<td>.896(^a)</td>
<td>.894(^b)</td>
</tr>
<tr>
<td>av.</td>
<td>.7646</td>
<td>.825</td>
<td>.825</td>
</tr>
</tbody>
</table>

*See Eq. (11)

\(^a\)calculated from \( I_M(v_2) \) (See Table I.)

\(^b\)calculated from \( \frac{d\eta}{d\eta} \) at \( \eta = 0 \)

\(^c\)for both a and b

**See reference 1 for method of calculation.
Table V

Comparison of Slopes of Relaxation Curves

\[ \ln \left[ H_M(\eta) - H'(\omega) \right] \text{ vs } \eta \]

<table>
<thead>
<tr>
<th>Monte Carlo Values from Runs of Small Time</th>
<th>Monte Carlo Values from Runs of Large Time*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Slope</td>
<td>90% Confidence Limit</td>
</tr>
<tr>
<td>M</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.587^a 1.568^b</td>
</tr>
<tr>
<td>2</td>
<td>1.740^a 1.736^b</td>
</tr>
<tr>
<td>4</td>
<td>- 1.746^b</td>
</tr>
<tr>
<td>6</td>
<td>- 1.3699^b</td>
</tr>
</tbody>
</table>

^a calculated from \( I(1+\ln f) \) at \( \eta = 0 \)

^b calculated from \( d M(H)/d\eta \) at \( \eta = 0 \)

*See reference 1 for method of calculation.
Translational relaxation of the lateral temperature. The abscissa $\eta$ is proportional to the reduced time $\tau$ (see Eq. (4)). It is (cont.)
calculated for a reference gas which is in equilibrium and has the same translational energy as the relaxing gas. The ordinate is \( \log_{10}[t_q(0) - t_{1M}(\eta)] \) and \( [t_q(0) - t_{1M}(\eta)] \) is the excess of the predicted asymptotic value of the lateral temperature \( t_q(0) \) over the value \( t_{1M}(\eta) \) of this temperature calculated by the Monte Carlo solution of the Boltzmann equation. The unit on the axis of the ordinate is \( T_1 \), one of the two constituent streams of gas at \( \eta = 0 \). The straight line shown was drawn through the first point with slope calculated on the basis of \( I(v_1^2) \) at \( \eta = 0 \). (For \( M = 4 \), the dashed line was drawn with a slope equal to the analytical initial slope calculated by using Eq. (12)). \( \Delta \) is the mean deviation of the data points from the linear relaxation curve for the range of \( \eta \) investigated.
Fig. 2 Translational relaxation of the Boltzmann function. The ordinate is $\log_{10}[H_M(\eta) - H'(\infty)]$ and $[H_M(\eta) - H'(\infty)]$ is the excess of the (cont.)
value of the Boltzmann function $H_N(\eta)$ over its predicted asymptotic value $H'(\infty)$. The unit on the axis of the ordinate is the density of one of the two constituent streams of gas at $\eta = 0$. The straight line shown was drawn through the first two points. $\Delta$ is the mean deviation of the data points from that line.
Relaxation of the distribution function in velocity space. The abscissa is $v_x$, the ordinate, $v_A$. The value given in the velocity bin is the decay time for $f$ in that velocity bin expressed in terms of number of collisions of reference (equilibrium) gas. The critical curve on which $(a-bf) = 0$ for $\eta = 0$ is plotted. The circular arc with radius equal to $u$ is also drawn.
Fig. 4(a) Relaxation of the distribution $f$ for Mach number = 1. (cont.)
The ordinate is $\ln|f-f(\infty)|_{av}$ and $|f-f(\infty)|$ is the absolute value of the excess of $f$ over $f(\infty)$. The abscissa $\eta$ is the same as that in Fig. 1 and 2. The position of the velocity bin is identified by $v_{xm}$ and $v_{lm}$. 
Fig. 4(b) Relaxation of the distribution $f$ for Mach number $= 2$. (cont.)
The ordinate is $\ln|f-f(\infty)|_a$ and $|f-f(\infty)|$ is the absolute value of the excess of $f$ over $f(\infty)$. The abscissa $\tau$ is the same as that in Fig. 1 and 2. The position of the velocity bin is identified by $v_{xm}$ and $v_{lm}$. 
Fig. 4(c) Relaxation of the distribution $f$ for Mach number = 4. (cont.)
The ordinate is $\ln |f-f(\infty)|_{av}$ and $|f-f(\infty)|$ is the absolute value of the excess of $f$ over $f(\infty)$. The abscissa $\gamma$ is the same as that in Fig. 1 and 2. The position of the velocity bin is identified by $v_{xm}$ and $v_{ym}$. 
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The behavior of the pseudo-shock at the onset and the early state of relaxation has been studied. The initial relaxation behavior of a gas of elastic spheres has been obtained by solving the Boltzmann equation by Nordieck's Monte Carlo method for small times (during which less than one molecule in five has collided) with spatially uniform initial velocity distribution function

\[ f(v, \tau=0) = \frac{1}{2} \exp(-\pi |v-u|^2 + \exp(-\pi |v+u|)^2) \]

where \( f \) and \( v \) are in specialized units and the Mach number

\[ M = (6\pi/5)^{1/2}u \]

describes the relative separation of the two peaks of this bimodal distribution function. Calculations are reported for \( M = 1, 2, 4 \), and 6. The characteristics of the relaxation of the lateral temperature and the Boltzmann function at the early stage have been compared with those of the large times (during which one molecule has collided up to six times). The initial relaxation of the lateral temperature for \( M = 1, 2 \) and 4 and for the Boltzmann flux for \( M = 1 \) are exponential and the relaxation rate for the lateral temperature seems to be independent of the Mach numbers mentioned. These findings are in agreement with those large times. However, for \( M = 2 \) and 4, the relaxation. The relaxation of the distribution function has been monitored for twelve chosen velocity bins. In most of the velocity bins, the distribution function seems to relax exponentially for the collision frequency and for two moments of the collision integral at
Boltzmann Equation
Monte Carlo
Translational Relaxation
Temperature Relaxation Rate
Entropy Relaxation Rate
Relaxation of Distribution Function
Collision Frequency
Analytical Calculations at the onset of relaxation
the onset of the relaxation \((\tau = 0)\). Comparisons have been made with the corresponding Monte Carlo results to ascertain the accuracy of the latter.