SPACECRAFT ATTITUDE CONTROL USING FEEDBACK PARAMETER OPTIMIZATION

GEORGE CHARLES SCHUMACHER

UNIVERSITY OF ILLINOIS – URBANA, ILLINOIS
This work was supported by the Joint Services Electronics Program (U.S. Army, U.S. Navy, And U.S. Air Force) under Contract DAAB 07-67-C-0199.

Reproduction in whole or in part is permitted for any purpose of the United States Government.

Distribution of this report is unlimited.
ACKNOWLEDGMENT

The author sincerely appreciates the guidance and suggestions of Professor W. R. Perkins. He thanks colleagues at the Coordinated Science Laboratory for their unending assistance, and especially Mr. James E. Heller for the use of several computer subroutines.

Finally, the author acknowledges the support of the National Science Foundation for his graduate studies.
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. THE SPACECRAFT CONTROL PROBLEM.</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Problem Definition</td>
<td>2</td>
</tr>
<tr>
<td>2. THE SYSTEM EQUATIONS.</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Euler's Dynamical Equations and Coordinate Axes.</td>
<td>5</td>
</tr>
<tr>
<td>2.2 The Gyrotorquer Equations.</td>
<td>7</td>
</tr>
<tr>
<td>2.3 Definition of the Spacecraft Attitude [2].</td>
<td>19</td>
</tr>
<tr>
<td>3. SYSTEM SIMULATION AND CONTROLLER STRUCTURE.</td>
<td>26</td>
</tr>
<tr>
<td>3.1 Modeling the Spacecraft with Gyrotorquer Control</td>
<td>26</td>
</tr>
<tr>
<td>3.2 Controller Design and Performance Criteria</td>
<td>29</td>
</tr>
<tr>
<td>4. THE OPTIMIZATION PROCEDURE AND RESULTS.</td>
<td>32</td>
</tr>
<tr>
<td>LIST OF REFERENCES</td>
<td>42</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Block diagram of a manned space station control system.</td>
<td>4</td>
</tr>
<tr>
<td>2.</td>
<td>Configuration of reference coordinates.</td>
<td>8</td>
</tr>
<tr>
<td>3.</td>
<td>Configuration of a single gyro [4].</td>
<td>10</td>
</tr>
<tr>
<td>4.</td>
<td>Transformation between the gyro-gimbal coordinates and the vehicle coordinates</td>
<td>11</td>
</tr>
<tr>
<td>5.</td>
<td>Configuration of the twin gyrorotorquers unit [4].</td>
<td>17</td>
</tr>
<tr>
<td>6.</td>
<td>Single plane Euler angle descriptions.</td>
<td>20</td>
</tr>
<tr>
<td>7.</td>
<td>Multiple plane Euler angle description.</td>
<td>22</td>
</tr>
<tr>
<td>8.</td>
<td>Geometric resolution of Euler angle rates to spacecraft angular rates.</td>
<td>24</td>
</tr>
<tr>
<td>9.</td>
<td>Spacecraft angular rates versus time for different values of the feedback parameters for case one in which ( \Omega_x ) and ( \Omega_z ) have initial non-zero values.</td>
<td>33</td>
</tr>
<tr>
<td>10.</td>
<td>Spacecraft angular rates versus time for different values of the feedback parameters for case two in which a 1,000.0 foot-pound disturbance torque is applied for two seconds about the ( \hat{y} ) axis.</td>
<td>37</td>
</tr>
</tbody>
</table>
Chapter 1

THE SPACECRAFT CONTROL PROBLEM

1.1 Introduction

In present day and future space missions, accurate spacecraft attitude control is a necessity. Before any photographs can be taken, instrument sighting performed, or trajectory corrections made the orientation of the spacecraft must be known and the desired attitude obtained. If solar cells are a source of power the solar panels must be pointed toward the sun. Radio communication is more efficient over interplanetary distances if highly directional antennas are used. Here again, proper vehicle orientation is required. To simulate artificial gravity in manned space stations a controlled angular rate about one axis is desired, while driving the angular rates about the other two axes to zero. The above indicate the importance of control systems aboard a space vehicle.

Generally there are three different phases to the attitude control problem. First, there is the acquisition phase. Large vehicle angular rates obtained during spacecraft-booster separation or after a large disturbance must be reduced, and the craft brought to its proper orientation with respect to the reference frame. Secondly, there is the problem of rotating the vehicle through large angles about its axes to a new desired orientation. Finally, there is the problem of holding the craft in correct position once the desired attitude is obtained.

The type of control system to be designed is highly dependent upon the vehicle configuration and the mission functions. For many missions it is necessary for the vehicle to have its orientation controlled relative to some
celestial body, such as the sun, the earth or some star. In some cases where long lifetime, high reliability and attitude accuracy within a few degrees is desired, a simple passive control method can be used. A passive system is a control system where the controlling forces are generated by an interaction of the physical shape of the vehicle and the fields present, such as the gravity field.

In cases where high pointing accuracy is required an active control system is necessary. This system consists of some attitude sensing device, a torque producing device, and the associated electronics. This paper will be primarily concerned with the design of an active control system for holding the desired vehicle angular rates during small disturbances.

1.2 Problem Definition

In an earth-orbiting manned space station gravity will be simulated by having a controlled constant angular rate about one of the vehicle axes. If the angular rates about the remaining two axes are not kept near zero, the vehicle will undergo a "wobbling" motion causing undesirable acceleration in the station.

Various disturbance torques will always be acting upon a space station while it is in orbit. Turning on and off rotating machinery, crew motion, gravity gradient torques, meteorite collisions, and impacts with docking vehicles all produce torques about the vehicle axes. These torques will produce undesirable angular rates about the axes, thus making a rate control system necessary.

The function of the rate control system to be investigated in the following chapters will be to drive the angular rates about the $\vec{\Omega}_V$ and $\vec{v}_V$
axes to zero, assuming the rate about the $\hat{x}_v$ axis is held constant. The specific example to be used is the Manned Orbital Space Station (MOSS) having the parameters:

\[
I_x = I_z = 9.1 \times 10^6 \text{ slug-feet}^2 \\
I_x = 16.7 \times 10^6 \text{ slug-feet}^2 \\
\omega_{x_0} = \frac{1}{3} \text{ radians/second} \\
2J_r \Omega = 5 \times 10^4 \text{ slug-feet}^2/\text{second}
\]

The spin rate, $\omega_{x_0}$, will produce a $1/4$ G artificial gravity field in the living area of the station. Shuttle space vehicles will be docked symmetrically to the station to avoid high products of inertia. Because of the high frequency of occurrence of the disturbance torques and the requirement for continued control, momentum exchange devices were chosen over a mass expulsion system to produce the correcting torques.

It should be noted that attitude control of a spinning space vehicle cannot be obtained with momentum exchange devices, because the vector sum of the momenta of the control device and the vehicle is a constant when no external torques are applied. Therefore a mass expulsion control system must be used for attitude control. The above mentioned attitude control system will not be investigated, but attitude changes will be recorded for use by such a system.

The general form of the rate control system to be investigated is shown in Figure 1. The problem is to determine the proper feedback parameters to obtain the desired system performance.
Figure 1

Block diagram of a manned space station control system.
Chapter 2
THE SYSTEM EQUATIONS

In this chapter the equations to be used for modeling the system will be derived. Section 2.1 discusses Euler's Dynamical Equations and the various coordinate systems required. In Section 2.2 the equations for the gyrotorquer controllers and for the vehicle are developed. Initially the equations for a single gyro are obtained, and then those for a twin gyro pair acting together. The equations describing the spacecraft attitude are defined in Section 2.3.

2.1 Euler's Dynamical Equations and Coordinate Axes

To define the system equations Euler's Dynamical Equations, or Newton's laws in rotating coordinates, must be used. Considering a set of inertial coordinates, the rotational motion is governed by

\[ \vec{\tau} = \frac{d\vec{L}}{dt}, \]  

(2.1)

where \( \vec{\tau} \) is the vector sum of the torques acting on the vehicle, and \( \vec{L} \) is the total angular momentum of the vehicle about its center of mass. In many cases a more convenient set of axes to use is the vehicle axes which are not necessarily inertial.

It can be shown that for non-inertial coordinates the torque equation in matrix form becomes [2]

\[ \vec{\tau} = \dot{\vec{L}} + \vec{P}, \]  

(2.2)
and for non-varying moments of inertia it simplifies to

\[ \tau = J \dot{\omega} + P J \omega, \]  

(2.3)

where \( \tau^T = [\tau_1 \tau_2 \tau_3] \) is the torque about the non-inertial coordinate axes. \( \omega^T = [\omega_1 \omega_2 \omega_3] \) represents the angular rates about the non-inertial coordinate axes.

\[
P = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}
\]

is a 3x3 matrix.

\[
J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}
\]

is a 3x3 matrix.

The diagonal terms of the \( J \) matrix are the moments of inertia, and the off-diagonal terms are the products of inertia.

The following sections are concerned with four three-dimensional, right-handed orthogonal coordinate systems, each dimension being represented by

\( X_i, Y_i, \) and \( Z_i, \)

where

\( i = I, R, V, G \)

for inertial, reference, vehicle, and gyro-gimbal coordinates respectively. It is assumed that the four coordinate systems have the same origin and have only angular rates with respect to each other.

To simplify the problem somewhat we assume that the vehicle is in a circular orbit such that \( \hat{X}_I \) and \( \hat{X}_R \) are coincident, \( \hat{X}_R \) is always tangent to the
orbit direction, and $\hat{\mathbf{r}}$ is pointing toward the earth center (see Figure 2).

For a vehicle traveling at a constant speed the angular rate of the reference frame with respect to the inertial frame is

$$
\Omega_{e} = \begin{bmatrix}
0 \\
0 \\
\omega_0
\end{bmatrix},
$$

(2.4)

where $\omega_0 = 2\pi/T$ radians/second is a constant, and $T$ is the period of the orbit in seconds. Also, for simplification, it is assumed that the gyro and vehicle centers of mass are at the same point. This point is chosen as the origin of the coordinate systems. The vehicle and gyro-gimbal principal axes are used for the coordinate axes, thus eliminating the product of inertia terms from $J$.

2.2 The Gyrotorquer Equations

The momentum exchange devices chosen for the control problem are twin gyrotorquers [3]. In general, a pair of gyros will have its momentum exchange axis along each of the vehicle axes. Thus three pairs of gyros will be required. The basic principle used in employing a momentum exchange device for vehicle control is that the total external torque is equal to the rate of change of the total angular momentum of the vehicle with respect to the inertial axes. When no external torques are present, if the angular momentum of any part of the vehicle is changed, by the law of conservation of angular momentum the remainder of the vehicle will change its momentum so the total angular momentum remains constant. When using gyrotorquers this change in momentum can be obtained by moving the direction of the gyro spin axis.

To obtain the equations required, first consider a single gyro mounted in a vehicle on the $\hat{\mathbf{v}}$ axis. A gyro-gimbal coordinate system is fixed to the
Figure 2
Configuration of reference coordinates.
gimbal and is denoted by \((\hat{X}_G, \hat{Y}_G, \hat{Z}_G)\). The rotor is driven at an angular speed \(\Omega_y\) by an electric motor and is assumed to be perfectly balanced and symmetrical. The chassis supporting the gyro is fixed to the vehicle and thus fixed in the vehicle axes \((\hat{X}_V, \hat{Y}_V, \hat{Z}_V)\). The gimbal is free to rotate about the \(\hat{X}_G\) axis which is fixed and coincident with the \(\hat{X}_V\) axis. The gimbal rotation is represented by the angle \(\beta\), see Figure 3. Angle \(\beta\) is taken to be positive when rotated about the \(\hat{X}_G\) axis in the sense of the right hand rule. The angle \(\beta\) is controlled by the motor shown in Figure 3.

Finally, the relationship between the gyro-gimbal coordinates and the vehicle coordinates must be found. This conversion can be represented by a matrix defined as \(C_{GV}\), where a vector defined in gyro-gimbal coordinates, \(\hat{A}_G\), can be converted to a vector in the vehicle coordinates, \(\hat{A}_V\), by the operation

\[
\begin{bmatrix}
A_{XV} \\
A_{YV} \\
A_{ZV}
\end{bmatrix} = C_{GV}
\begin{bmatrix}
A_{XG} \\
A_{YG} \\
A_{ZG}
\end{bmatrix}
\]  

(2.5)

To determine \(C_{GV}\), the following observations are made from Figure 3. Because the \(\hat{X}_V\) axis and the \(\hat{X}_G\) axis are coincident, it is noted that the \(\hat{Y}_G\), \(\hat{Y}_V\), \(\hat{Z}_G\), and \(\hat{Z}_V\) axes all lie in the same plane which is perpendicular to the \(\hat{X}_G\) and \(\hat{X}_V\) axes. Figure 4 is drawn such that the above determined plane is the plane of the page.
Figure 3
Configuration of a single gyro [4].
Figure 4

Transformation between the gyro-gimbal coordinates and the vehicle coordinates.

From the figure, the following relations can be determined for some vector $\vec{A}$:

$$
\begin{bmatrix}
A_{xv} \\
A_{yv} \\
A_{zv}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \sin\beta & \cos\beta \\
0 & -\cos\beta & \sin\beta
\end{bmatrix}
\begin{bmatrix}
A_{xG} \\
A_{yG} \\
A_{zG}
\end{bmatrix},
$$

(2.6)

where

$$
C_{GV} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \sin\beta & \cos\beta \\
0 & -\cos\beta & \sin\beta
\end{bmatrix}
$$

(2.7)
From Figure 4 it is clear that the relative velocity between the vehicle frame and the gyro-gimbal frame is $\hat{\beta}_G$.

To determine the gyrotorquer equations Euler's Dynamical Equations must be applied to the gimbal.

$$T_G = \begin{bmatrix} I_G + P_L \end{bmatrix}, \quad (2.2)$$

where $T_G^T = [\tau_{XG} \tau_{YG} \tau_{ZG}]$ is a vector containing the component torques applied to the gimbal, with respect to the gimbal coordinate axes. $L_G = [L_{XG} L_{YG} L_{ZG}]$ is the gyro-gimbal assembly angular momentum vector in the gimbal coordinate axes. For this case the matrix

$$P = \begin{bmatrix} 0 & -\omega_{ZG} & \omega_{YG} \\ \omega_{ZG} & 0 & -\omega_{XG} \\ -\omega_{YG} & \omega_{XG} & 0 \end{bmatrix},$$

where the $\omega$'s are the components of the gimbal angular velocity expressed in the gimbal coordinate system.

The gimbal angular momentum vector is composed of two parts: the momentum due to the angular rates of the gimbal itself and the angular momentum of the spinning gyro rotor. The gyro-gimbal coordinates are chosen such that they are the principle axes. Therefore the gimbal angular momentum vector can be represented by

$$L_G^{\text{GIM}} = J_G \omega_G, \quad (2.8)$$

where

$$J_G^{\text{GIM}} = \begin{bmatrix} J_X & 0 & 0 \\ 0 & J_Y & 0 \\ 0 & 0 & J_Z \end{bmatrix}.$$
and

\[ \mathbf{\omega}_G = \begin{bmatrix} \omega_{XG} \\ \omega_{YG} \\ \omega_{ZG} \end{bmatrix} \]

The angular momentum of the spinning rotor is simply

\[ \mathbf{L}_r = \begin{bmatrix} 0 \\ 0 \\ L_{Zr} \end{bmatrix} = \begin{bmatrix} 0 \\ \Omega_r J_{rY} \end{bmatrix} , \]

where \( J_{rY} \) is the moment of inertia of the rotor, and \( \Omega_r \) is the rotor rate of spin. Adding the two vectors, the components of \( \mathbf{L}_G \) are obtained:

\[ L_{XG} = J_X \omega_{XG} \]
\[ L_{YG} = J_Y \omega_{YG} \]
\[ L_{ZG} = J_Z \omega_{ZG} + L_{Zr} \]

Assuming the moments of inertia are time independent, and the spin rate of the gyro rotor is a constant, the time rate of change of the gyro-gimbal assembly momentum vector becomes:

\[ L_{XG} = J_X \dot{\omega}_{XG} \]
\[ L_{YG} = J_Y \dot{\omega}_{YG} \]
\[ L_{ZG} = J_Z \dot{\omega}_{ZG} \]

The simplified form of Euler's Equations can then be applied

\[ \mathbf{L}_G = \frac{\partial \mathbf{\omega}_G}{\partial \mathbf{\omega}_G} + \mathbf{P}_L \]
Expanding (2.11) gives the following results:

\[
\tau_{xG} = J_X \hat{w}_{xG} - (J_Y - J_Z) \omega_{yG} \omega_{zG} + \omega_{yG} L zr
\]

\[
\tau_{yG} = J_Y \hat{w}_{yG} - (J_Z - J_X) \omega_{xG} \omega_{zG} - \omega_{xG} L zr
\]

\[
\tau_{zG} = J_Z \hat{w}_{zG} - (J_X - J_Y) \omega_{xG} \omega_{yG} .
\]

\(\omega_G\) can be expressed in terms of \(\omega_Y\) and \(\dot{\beta}\) by

\[
\omega = C_{GV}^{-1} \omega_Y + 0
\]

Inverting the matrix \(C_{GV}\) of (2.7) and expanding (2.13) to obtain \(\omega_G\) gives

\[
\dot{\omega}_{xG} = \dot{\omega}_{xY} + \dot{\beta}
\]

\[
\dot{\omega}_{yG} = \dot{\omega}_{yY} \sin \beta - \omega_{zY} \cos \beta
\]

\[
\dot{\omega}_{zG} = \dot{\omega}_{zY} \cos \beta + \omega_{zY} \sin \beta .
\]

Then differentiate (2.14) to obtain \(\ddot{\omega}_G\),

\[
\ddot{\omega}_{xG} = \ddot{\omega}_{xY} + \ddot{\beta}
\]

\[
\ddot{\omega}_{yG} = \ddot{\omega}_{yY} \sin \beta + \dot{\beta} \omega_{yY} \cos \beta - \dot{\omega}_{zY} \cos \beta - \dot{\beta} \omega_{zY} \sin \beta
\]

\[
\ddot{\omega}_{zG} = \ddot{\omega}_{zY} \cos \beta - \dot{\beta} \omega_{zY} \sin \beta + \dot{\omega}_{zY} \sin \beta + \beta \omega_{zY} \cos \beta .
\]

Substituting (2.14) and (2.15) into (2.12), setting \(L_zr = J_{zr} \Omega_Y\), and

simplifying results in the equations:

\[
\tau_{xG} = J_X (\dot{\omega}_{xG} + \ddot{\beta}) + (\omega_{yY} \sin \beta - \omega_{zY} \cos \beta) J_{yY} \Omega_Y + (J_{zG} - J_{yG}) (\omega_{yY}^2 - \omega_{zY}^2) \sin \beta \cos \beta + \omega_{yY} \omega_{zY} (\sin^2 \beta - \cos^2 \beta) .
\]
\[ \tau_{YG} = J_Y (\dot{\omega}_{YYV} \sin \beta + \dot{\omega}_{YYV} \cos \beta - \dot{\omega}_{YZV} \cos \beta + \dot{\omega}_{YZV} \sin \beta) \]
\[ + (J_X - J_Y) (\omega_{XV,YYV} \cos \beta + \omega_{XV,YYV} \sin \beta + \dot{\omega}_{YYV} \cos \beta) \]
\[ + \dot{\omega}_{YZV} \sin \beta - (\omega_{XV} + \dot{\beta}) J_{Yt} \Omega_Y \]

\[ \tau_{ZG} = J_Z (\dot{\omega}_{ZZV} \cos \beta - \dot{\omega}_{ZZV} \sin \beta + \dot{\omega}_{ZZV} \sin \beta + \dot{\omega}_{ZZV} \cos \beta) \]
\[ + (J_Y - J_X) (\omega_{XV,ZZV} \sin \beta - \omega_{XV,ZZV} \cos \beta + \dot{\omega}_{ZZV} \sin \beta - \dot{\omega}_{ZZV} \cos \beta). \]

The above equations describe the torques defined in the gyro-gimbal coordinates due to one gyro mounted in the vehicle on the $\dot{\beta}$ axis. Before the torques from the two gyros can be added, the above torque equations must be transformed to the vehicle frame. Using the transformation

\[ \tau_V = C_{GV} \tau_G \]

and expanding, the torque equations in the vehicle coordinates become:

\[ \tau_{XG} = \tau_{XG} \]
\[ \tau_{YV} = J_Y [\dot{\omega}_{YYV} \sin \beta \omega_{YYV} \cos \beta \sin \beta + 2 \dot{\omega}_{YYV} \sin \beta \cos \beta + \omega_{YZV} (\sin^2 \beta - \cos^2 \beta) + \omega_{YYV} \sin \beta \cos \beta - \omega_{YZV} \cos \beta - \omega_{YZV} \sin \beta] \]
\[ + J_X (\omega_{XV,YYV} + \dot{\omega}_{YYV}) + J_Z [\omega_{XV,YYV} \sin \beta - \omega_{XV,YYV} \sin \beta - \dot{\omega}_{YYV} \sin \beta - \dot{\omega}_{YYV} \cos \beta] \]
\[ - 2 \dot{\omega}_{YYV} \cos \beta \sin \beta - \dot{\omega}_{YYV} \cos \beta - 2 \dot{\omega}_{YYV} \sin \beta \cos \beta + \dot{\omega}_{YYV} \cos \beta \]
\[ + \omega_{YZV} \sin \beta \cos \beta - (\omega_{XV} + \dot{\beta}) J_{Yt} \Omega_Y \sin \beta \] (2.17)
\[ \tau_{ZV} = J_Y [\dot{\omega}_{YYV} \sin \beta \cos \beta - \dot{\omega}_{YYV} (\cos^2 \beta - \sin^2 \beta)] \]
\[ + \dot{\omega}_{YZV} \sin \beta \cos \beta - 2 \dot{\omega}_{YZV} \sin \beta \cos \beta \]
\[ + \omega_{XV,YYV} \sin \beta \omega_{XV,YYV} \cos \beta \sin \beta \]
\[ - J_X (\omega_{XV,YYV} + \dot{\omega}_{YYV}) + J_Z [\omega_{XV,YYV} \cos \beta - \dot{\omega}_{YYV} \sin \beta - \omega_{XV,YYV} \sin \beta \cos \beta] \]
\[ + \omega_{XV,YYV} \sin \beta \cos \beta + \omega_{XV,YYV} \cos \beta \sin \beta (\cos^2 \beta - \sin^2 \beta) \]
The configuration for a twin gyrotorquer unit on the $\hat{v}$ axis is shown in Figure 5. The two gimbals are driven by one motor producing a positive angular displacement for the lower gimbal and a negative angular displacement for the upper gimbal as shown. The gyro rotors are spinning in a manner such that the vector directions of the angular momenta are opposite. Thus the torque equation for the upper gyro can be found using (2.17) where $\beta$, $\dot{\beta}$, $\ddot{\beta}$, and $\Omega_Y$ are replaced by $-\beta$, $-\dot{\beta}$, $-\ddot{\beta}$, and $-\Omega_Y$ respectively.

The total torque, $T = [\tau_{XT} \tau_{YT} \tau_{ZT}]$, which is described in the vehicle coordinates is due to the twin gyro unit on the $\hat{v}$ axis. It can be found by adding the two torque equations in each of the three vector component directions, resulting in:

$$
\tau_{XT} = 2J_X \dot{w}_{XV} + 2(J_z - J_y) w_{YV} \omega_{ZV} (\sin^2 \beta - \cos^2 \beta) + 2J_{YV} \sin \beta J_Y \Omega_Y
$$

$$
\tau_{YT} = 2J_y (\dot{w}_{YV} \sin^2 \beta + 2\dot{\beta} \omega_{YV} \sin \beta \cos \beta - \omega_{XV} \omega_{ZV} \cos^2 \beta)
$$

$$
+ 2J_z (-\omega_{XV} \omega_{ZV} \sin^2 \beta - 2\dot{\beta} \omega_{YV} \cos \beta \sin \beta + \omega_{YV} \cos^2 \beta)
$$

$$
\tau_{ZT} = 2J_z (\dot{w}_{ZV} \cos^2 \beta - 2\dot{\beta} \omega_{ZV} \sin \beta \cos \beta + \omega_{XV} \omega_{YV} \sin^2 \beta)
$$

In general, a twin gyro unit is mounted on each of the vehicle axes. The torques produced about the vehicle axes by the remaining two gyro units can be found by a cyclic permutation of the subscripts and angles in (2.18).
Figure 5
Configuration for the twin gyrotorquer unit [4].
For the gyro unit on the $\hat{Z}_V$ axis the torques can be found by replacing the subscripts X, Y, Z, and the angle $\beta$ by Y, Z, X, and the angle $\gamma$ respectively. Similarly, for the gyro unit on the $\hat{X}_V$ axis the subscripts X, Y, Z, and the angle $\beta$ should be replaced by Z, X, Y, and the angle $\alpha$ respectively. Finally, the torque vector ($\mathbf{T}$) about the vehicle axis due to all three twin gyro units is defined as

$$\mathbf{T} = \mathbf{T}_X + \mathbf{T}_Y + \mathbf{T}_Z.$$  \hspace{1cm} (2.19)

Equations (2.18) and (2.19) will be simplified when the actual control system simulation is discussed in Chapter 3.

In a similar manner the vehicle dynamic equations can be found. Again Euler's Dynamical Equations must be written for the vehicle. Assuming the vehicle moments of inertia are constant, Euler's Equation is

$$\mathbf{I}_S \ddot{\mathbf{\omega}} = \mathbf{P}_N \mathbf{L}_S.$$  \hspace{1cm} (2.20)

$\mathbf{I}_S$ is the total torque vector acting on the vehicle, expressed in the vehicle coordinates. $\mathbf{\omega}_V$ is the vehicle angular velocity vector with respect to the inertial axes, expressed in the vehicle coordinates. The principal axes of the spacecraft are chosen to be the vehicle coordinate axes. Therefore the moment of inertia matrix is

$$\mathbf{I}_S = \begin{bmatrix} I_X & 0 & 0 \\ 0 & I_Y & 0 \\ 0 & 0 & I_Z \end{bmatrix}.$$  \hspace{1cm} (2.21)

The vehicle angular momentum vector is simply

$$\mathbf{L}_S = \mathbf{I}_N \mathbf{\omega}_V.$$  \hspace{1cm} (2.22)
Expanding (2.20) gives

\[ \tau_{SX} = I_X \dot{\omega}_X - (I_Y - I_Z) \omega_Y \omega_Z \]

\[ \tau_{SY} = I_Y \dot{\omega}_Y - (I_Z - I_X) \omega_X \omega_Z \]

\[ \tau_{SZ} = I_Z \dot{\omega}_Z - (I_X - I_Y) \omega_X \omega_Y \]  

(2.23)

which are the final vehicle dynamic equations.

2.3 Definition of the Spacecraft Attitude

To obtain the actual attitude of the spacecraft, its orientation with respect to the reference axes must be defined. The attitude will be related to the angular rates of the spacecraft about the vehicle axes, \( \omega \). The method to be used here is that of defining the Euler angles by a sequence of three ordered rotations of the inertial axes. Beginning with the vehicle axes coincident with the reference axes, the vehicle axes are rotated about the inertial \( \hat{z}_I \) axis by an angle \( \varphi \). The new axes are shown in Figure 6a and are labeled \( \hat{z}', \hat{\eta}', \) and \( \hat{\xi}' (=\hat{z}_I) \). From the figure the following relationships can be found:

\[
\begin{bmatrix}
\hat{z}' \\
\hat{\eta}' \\
\hat{\xi}'
\end{bmatrix} =
\begin{bmatrix}
\cos \varphi & \sin \varphi & 0 \\
-sin \varphi & \cos \varphi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{z}_I \\
\hat{\eta}_I \\
\hat{\xi}_I
\end{bmatrix} =
A_I
\begin{bmatrix}
\hat{z}_I \\
\hat{\eta}_I \\
\hat{\xi}_I
\end{bmatrix}
\]

(2.24)

Then a rotation about the \( \hat{\xi}' \) axis by an angle \( \theta \) gives the new axes shown in Figure 6b. These are labeled \( \hat{\eta}, \hat{\xi}, \) and \( \hat{z}' (=\hat{z}_I) \). The relationships from this figure are as follows:
Figure 6

Single plane Euler angle descriptions.
\[
\begin{bmatrix}
\hat{\zeta}' \\
\hat{\eta}' \\
\hat{\delta}'
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & \cos\theta & \sin\theta \\
0 & -\sin\theta & \cos\theta
\end{bmatrix}
\begin{bmatrix}
\hat{\zeta} \\
\hat{\eta} \\
\hat{\delta}
\end{bmatrix} = A_2 \begin{bmatrix}
\hat{\zeta}' \\
\hat{\eta}' \\
\hat{\delta}'
\end{bmatrix}.
\] (2.25)

Finally a rotation about the \( \hat{\delta} \) axis by an angle \( \phi \) gives the vehicle axes \( \hat{\chi}_v, \hat{\gamma}_v, \) and \( \hat{\zeta}_v \) shown in Figure 6c. The resulting matrix equation is found from this figure:

\[
\begin{bmatrix}
\hat{\chi}_v \\
\hat{\gamma}_v \\
\hat{\zeta}_v
\end{bmatrix} =
\begin{bmatrix}
\cos\phi & \sin\phi & 0 \\
-sin\phi & \cos\phi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{\chi} \\
\hat{\gamma} \\
\hat{\zeta}
\end{bmatrix} = A_3 \begin{bmatrix}
\hat{\chi}' \\
\hat{\gamma}' \\
\hat{\zeta}'
\end{bmatrix}.
\] (2.26)

\( \psi, \theta, \) and \( \phi \) are the Euler angles which define the orientation of the vehicle coordinate axes with respect to the reference coordinate axes. To relate a vector \( \mathbf{x} \) in the reference axes to the vehicle axes the transformation

\[
\mathbf{x}_v = A_3 A_2 A_1 \mathbf{x}_r = C_{RV} \mathbf{x}_r
\] (2.27)

where

\[
C_{RV} = A_3 A_2 A_1 = \\
\begin{bmatrix}
\cos\psi\cos\theta\sin\gamma & \cos\phi\sin\theta & \sin\phi\sin\theta \\
\sin\psi\cos\gamma - \cos\psi\sin\gamma & \cos\phi & \sin\phi \cos\psi \\
\sin\gamma & \sin\phi \sin\psi & \cos\phi \cos\psi
\end{bmatrix}
\] (2.28)

is applied.

It is necessary to relate the Euler angles and the vehicle angular velocities, \( \omega_v \). Referring to Figure 7, it should be noted that \( \hat{\gamma}, \hat{\delta}, \) and \( \hat{\phi} \) are directed along \( \hat{\gamma}_r, \hat{\delta}, \) and \( \hat{\phi}_v \) respectively. Geometric resolution of the components of \( \hat{\gamma}, \hat{\delta}, \) and \( \hat{\phi} \) along the \( \hat{\gamma}_v, \hat{\delta}_v, \) and \( \hat{\phi}_v \) axes will determine \( \omega_x, \omega_y, \) and \( \omega_z \) respectively.
Figure 7
Multiple plane Euler angle description.
Considering first the angular rate $\omega_z$ about the $\hat{z}_v$ axis, note the following (refer to Figure 7 and Figure 8a):

1) $\dot{\theta}$ is perpendicular to $\hat{z}_v$.
2) $\dot{\phi}$ is parallel to $\hat{z}_v$.
3) $\dot{\gamma}$ is in the same plane as $\hat{z}_v$ and $\eta$.

It can be seen from Figure 8a that

$$\omega_z = \dot{\phi} + \dot{\gamma} \cos \theta.$$  \hspace{1cm} (2.29)

Second consider the angular rate $\omega_y$ about the $\hat{y}_v$ axis and note the following (refer to Figure 7 and Figure 8b):

1) $\dot{\phi}$ is perpendicular to $\hat{y}_v$.
2) $\dot{\theta}$ is in the same plane as $\hat{y}_v$ and $\zeta$.
3) $\dot{\gamma}$ is out of the plane of $\hat{y}_v$ and $\zeta$.

From Figure 8b the following result is obtained:

$$\omega_y = \dot{\gamma} \sin \theta \cos \theta - \dot{\theta} \sin \phi.$$  \hspace{1cm} (2.30)

Finally considering the angular rate $\omega_x$ about the $\hat{x}_v$ axis, we can make the following observations (refer to Figure 7 and Figure 8c):

1) $\dot{\phi}$ is perpendicular to $\hat{x}_v$.
2) $\dot{\theta}$ is in the same plane as $\hat{x}_v$ and $\zeta$.
3) $\dot{\gamma}$ is not in the same plane as $\hat{x}_v$ and $\zeta$.

From Figure 8c the following equation is formulated:

$$\omega_x = \dot{\gamma} \sin \theta \sin \phi + \dot{\theta} \cos \phi.$$  \hspace{1cm} (2.31)

Combining (2.29), (2.30) and (2.31) the following matrix equation is formed:
Figure 8

Geometric resolution of Euler angle rates to spacecraft angular rates.
\[
\begin{bmatrix}
\dot{\omega}_x \\
\dot{\omega}_y \\
\dot{\omega}_z
\end{bmatrix}
= \begin{bmatrix}
\sin{\theta}\sin{\varphi} & 0 & \cos{\theta} \\
\sin{\theta}\cos{\varphi} & 0 & -\sin{\theta} \\
\cos{\theta} & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\psi} \\
\dot{\varphi} \\
\dot{\theta}
\end{bmatrix} = Q
\begin{bmatrix}
\dot{\psi} \\
\dot{\varphi} \\
\dot{\theta}
\end{bmatrix}
\] (2.32)

To find the Euler angle rates as functions of the vehicle rates the inverse of Q is used, resulting in

\[
\begin{bmatrix}
\dot{\psi} \\
\dot{\varphi} \\
\dot{\theta}
\end{bmatrix} = \frac{1}{\sin{\theta}}\begin{bmatrix}
\sin{\varphi} & \cos{\varphi} & 0 \\
-\sin{\varphi}\cos{\theta} & -\cos{\varphi}\cos{\theta}\sin{\theta} & \cos{\varphi}\sin{\theta} \\
\cos{\varphi}\sin{\theta} & -\sin{\varphi}\sin{\theta} & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\omega}_x \\
\dot{\omega}_y \\
\dot{\omega}_z
\end{bmatrix}
\] (2.33)

It is important to clarify the difference between \(\omega_V\) and \(\omega\). \(\omega\) is a vector having the components of the spacecraft angular velocities about the vehicle axes (that is the angular velocity of the spacecraft with respect to the reference coordinates). \(\omega_V\) is a vector having the components of the spacecraft angular velocity with respect to the inertial frame. The difference lies in the angular velocity of the reference coordinates with respect to the inertial coordinates. This was found to be \(\omega_{VR}^\Delta\) in (2.4). Using (2.4) and (2.32) the relation between \(\omega_V\) and \(\omega\) is found to be

\[
\omega_V = \omega + C_{RV}
\begin{bmatrix}
0 \\
0 \\
\omega_0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix} + \begin{bmatrix}
\omega_0 \sin{\varphi}\sin{\theta} \\
\omega_0 \cos{\varphi}\sin{\theta} \\
\omega_0 \cos{\theta}
\end{bmatrix}
\] (2.34)
Chapter 3
SYSTEM SIMULATION AND CONTROLLER STRUCTURE

In this chapter the equations for computer simulation of the spacecraft and control system are developed. A performance criteria, which is to be minimized by finding a set of optimum feedback parameters, is also developed.

3.1 Modeling the Spacecraft with Gyrorotquer Control

The basic equations needed to model a spacecraft with gyrorotquer control were developed and discussed in Chapter 2. These equations are now applied to the design of the specific Manned Orbital Space Station problem as stated in Chapter 1. It is desirable to find a control system which will drive the vehicle angular rates about the $\dot{\hat{x}}_V$ and $\dot{\hat{y}}_V$ axes to zero. The rate about the $\dot{\hat{x}}_V$ axis is kept at a constant value, $\omega_{X0} = 1/3$ radians/second and $\dot{\omega}_{X0} = 0$.

In applying the theory to the present problem several assumptions are made to simplify the equations that are used. For the purpose of clarity these approximations plus others made in the previous chapter are given below:

1) The principle axes of the spacecraft and the gyros are chosen as coordinate axes.

2) The centers of mass of the vehicle and gyrorotquer units are assumed to be at the same point. This point is chosen as the common origin of all the coordinate axes.

3) The reference coordinates are chosen to be the same as the inertial coordinates. Therefore $\omega_0 = 0$.

4) The gyrorotquer units are all identical, and all gyros are spinning
at the same speed. $\Omega_y$ is a large number compared to the angular rates, $\dot{\omega}_y$.

5) The response speed of the gimbal drive motors is much faster than the spacecraft response. Therefore, the motor dynamics can be neglected.

6) The response time of the sensors can be neglected and therefore is represented by scalars.

7) Because the rates about the $\hat{\chi}_v$ and $\hat{\beta}_v$ axes are to be driven to zero, it is assumed $\dot{\omega}_{yv}$ and $\dot{\omega}_{zv}$ will be small at all times.

8) The vehicle moments of inertia, $I_v$, are much greater than the gyro-gimbal moments of inertia, $J_v$.

Applying the above assumptions, two important equations in Chapter 2 can be simplified in the following manner:

(2.18) becomes

\[
\tau_{XT} = 2\dot{\omega}_{yv}\sin\beta J_{yr} \Omega_y
\]

\[
\tau_{YT} = 2J_{yv}\dot{\omega}_{yv}\sin^2\beta - 2\omega_{xv}J_{yr} \Omega_y \sin\beta + 2J_{zv} \Omega_y \cos^2\beta \tag{3.1}
\]

\[
\tau_{ZT} = 2J_{y} \dot{\omega}_{y} \cos^2\beta + 2J_{zv} \Omega_y \cos\beta
\]

and (2.34) becomes

\[
\dot{\omega}_y = \dot{\omega} \tag{3.2}
\]

Since no attempt is made to control the spin $\omega_x$, no gyrotorquer unit is required on the $\hat{\chi}_v$ axis. Making the proper permutation of subscripts in (3.1), and remembering $\dot{\omega}_{xv} = 0$, the torque equations for the gyrotorquer unit on the $\hat{\beta}_v$ axis can be obtained:
\[
\begin{align*}
\tau_{ZT} &= 2\omega_{X0}\sin\alpha J_{Xr}\Omega_x \\
\tau_{XT} &= 2\omega_{ZV} J_{Xr}\Omega_x\sin\alpha \\
\tau_{YT} &= 2J_r\dot{\omega}_V\cos^2 \alpha + 2\dot{\Omega}_z J_r\Omega_z \cos \alpha.
\end{align*}
\]

Combining (3.1), (3.3), and (2.23); adding the disturbance torque vector, \(\tau_d\); and noting that

\[J_{Xr} = J_{Yr} = J_r\]

\[\Omega_x = \Omega_y = \Omega\]

and

\[I_x \gg J_r\]

the following combined gyrotorquer and vehicle equations are found:

\[
\begin{align*}
J_r \dot{\omega}_r + (I_X - I_Z)\omega_z\omega_y &= \tau_{dY} + 2J_r\Omega(\omega_X \sin \beta - \dot{\alpha} \cos \alpha) \\
I_z \dot{\omega}_y - (I_X - I_Y)\omega_x\omega_z &= \tau_{dZ} - 2J_r\Omega(\omega_X \sin \alpha + \dot{\beta} \cos \beta).
\end{align*}
\]

Inserting the constant values and simplifying, the nonlinear state equations in normal form are

\[
\begin{align*}
\dot{\omega}_Y &= 1.01 \times 10^{-7} [\tau_{dY} - 7.6 \times 10^6 \omega_Z \omega_{X0} + 5 \times 10^4 (\omega_{X0} \sin \beta - \dot{\alpha} \cos \alpha)] \\
\dot{\omega}_Z &= 1.10 \times 10^{-7} [\tau_{dZ} + 7.6 \times 10^6 \omega_X \omega_{Y0} - 5 \times 10^4 (\omega_{X0} \sin \alpha + \dot{\beta} \cos \beta)].
\end{align*}
\]

No attempt will be made to linearize these equations because the expected values of the angles \(\alpha\) and \(\beta\) may be too large so that linear approximations of sine and cosine cannot be made.

Finally the change in the spacecraft attitude is going to be measured.

The initial attitude is such that the reference axes and the vehicle axes
are coincident. This requires that \( \psi_0, \theta_0, \) and \( \phi_0 \) equal \( 0, \frac{\pi}{2}, \) and \( 0 \) radians respectively. The actual attitude can be found using (2.33), which is expanded, simplified, and put in the normal form here:

\[
\begin{align*}
\dot{\psi} &= \frac{\omega_x \sin \phi + \omega_y \cos \phi}{\sin \theta} \\
\dot{\phi} &= -\omega_x \sin \phi \cot \theta - \omega_y \cos \phi \cot \theta + \omega_z \\
\dot{\theta} &= \omega_x \cos \phi - \omega_y \sin \phi .
\end{align*}
\]  

(3.6)

3.2 Controller Design and Performance Criteria

The response of the spacecraft due to the summation of disturbance torques and the torques produced by the gyros is expressed mathematically in (3.5). The torque due to the gyrotorquer units depends on the change in the angles \( \alpha \) and \( \beta \). A control law must be chosen which provides the proper feedback from the vehicle angular rates to the position angles of the gyrotorquers in order to obtain the desired system response.

The control law was chosen to be of the following form:

\[
\begin{align*}
\dot{\alpha} &= -K_1 \omega_y + K_2 \omega_z \\
\dot{\beta} &= -K_3 \omega_z + K_4 \omega_y .
\end{align*}
\]  

(3.7)

The parameters \( K_1, K_2, K_3, \) and \( K_4 \) are the feedback elements to be optimized.

It is assumed that the gimbal drive motor dynamics may be neglected. Therefore angles \( \alpha \) and \( \beta \) are related to the drive motor voltages by

\[
\begin{align*}
V_1 &= K_1 \alpha \\
V_2 &= K_3 \beta .
\end{align*}
\]  

(3.8)
Assuming the maximum available voltage is 500 volts, the parameters $K_1$ and $K_3$ will be limited to 500 volts/radian, and angles $\alpha$ and $\beta$ will be limited to 1 radian. This is reasonable to assume, because for angles of $\alpha$ and $\beta$ greater than $\frac{\pi}{2}(=1.57)$ radians the gyrotorquer units are no longer adequate for control purposes.

It now remains to determine an optimum set of the feedback parameters $K_1$, $K_2$, $K_3$, and $K_4$. To use the modern approach in finding an adequate set of parameter values, a performance criteria must be defined mathematically. This function is either maximized or minimized with respect to the four parameters during the optimization procedure.

There are many criteria by which the system response may be examined. However, since many of the system components have already been chosen, the possible criteria are somewhat reduced. An examination of the problem shows the following three important factors:

1) Once a disturbance has occurred, and the vehicle has obtained non-zero $w_Y$ and $w_Z$, the system should reduce these rates to zero as rapidly as possible.

2) The angles $\alpha$ and $\beta$ must not exceed 1 radian.

3) Because $\alpha$ and $\beta$ are approximately linearly related to the voltage and therefore to the power required, they should be kept as small as possible.

The above conditions indicate that it is adequate to apply a quadratic performance index of the form

$$I = \int (q_1 w_Y^2 + q_2 w_Z^2 + r_1 \alpha^2 + r_2 \beta^2) \, dt \quad (3.9)$$

The closer the rates $w_Y$ and $w_Z$ and the angles $\alpha$ and $\beta$ are to zero, the
nearer the actual response is to the ideal response of zero radians/second
for \( \omega_Y \) and \( \omega_Z \) given any disturbance. Therefore the optimum set of para-
meters will produce a minimum of the performance index.

Two actual physical situations were investigated. The first case is
one in which a pulse-like disturbance causes the space station to obtain
initial rates of \( \omega_Y = 0.002 \) radians/second and \( \omega_Z = 0.001 \) radians/second.
In the second case involving zero initial angular rates for \( \omega_Y \) and \( \omega_Z \), a
disturbance torque of 1,000 foot-pounds is applied about the \( \hat{y} \) axis for
two seconds. For both cases the purpose of the control system is to reduce
the angular rates to zero as quickly as possible with angles \( \alpha \) and \( \beta \) both
initially zero.

Appropriate penalty weighting of the rates with respect to the angles
was found to be
\[
q_1 = q_2 = 1.0 \times 10^8 \\
\]
\[
\mu_1 = \mu_2 = 1.0 \\
\]
for the first case, and
\[
q_1 = q_2 = 1.0 \times 10^{10} \\
\]
\[
\mu_1 = \mu_2 = 1.0 \\
\]
for case two. Finally the initial starting values for the parameters were
found to be [4]
\[
K_1 = 360.0 \\
K_2 = 1600.0 \\
K_3 = 360.0 \\
K_4 = 0.0 .
\]
Chapter 4

THE OPTIMIZATION PROCEDURE AND RESULTS

There are many methods of calculating optimum parameters [5, 6]. When numerical methods and the high speed digital computer are employed, a strategy for making estimates of the parameter values must be devised. Three of the more familiar methods are the following: gradient, Newton-Raphson, and steepest descent methods. However, a fourth method which eliminates some of the convergence problems of the above three methods and locates an extremal point within a bounded space was developed by H. H. Rosenbrock [7]. A FORTRAN program written by James E. Heller using Rosenbrock's rotating coordinate technique was applied as the search technique.

The results of the optimization for case one, where initially $\omega_Y = 0.002$ radians/second and $\omega_Z = 0.001$ radians/second, are illustrated in Figure 9. All four photographs contain the trajectories of $\omega_Y$ and $\omega_Z$ plotted versus time in seconds for various parameter values. $\omega_Y$ is the "solid" plot, and $\omega_Z$ is the "dotted" plot. The scales of the axes remain the same throughout the series, having 0.001 radians/second for each division on the vertical axis and 1 second per division on the horizontal axis. The total time shown on each plot is thirty seconds. The value of the performance indices given for the following parameters is the value at the end of thirty seconds.

For the initial values of the parameters

\[ K_1 = 360.0 \text{ volts/radian} \]
\[ K_2 = 1600.0 \text{ volts/radian} \]
\[ K_3 = 360.0 \text{ volts/radian} \]
\[ K_4 = 0.0 \text{ volts/radian} \]
Figure 9

Spacecraft angular rates versus time for different values of the feedback parameters for case one in which $\omega_y$ and $\omega_z$ have initial non-zero values.
Figure 9 (Continued)
the system was highly unstable and did not lead to an accurate computer solution. In Figure 9a the parameter values were

\[ K_1 = 247.5 \text{ volts/radian} \]
\[ K_2 = 1605.0 \text{ volts/radian} \]
\[ K_3 = 247.5 \text{ volts/radian} \]
\[ K_4 = 0.0 \text{ volts/radian} \]

The value of the performance index was 1659.511. It should be noted that the system was going unstable after thirty seconds. Figure 9b shows the response for the parameter values

\[ K_1 = 259.6 \text{ volts/radian} \]
\[ K_2 = 1344.0 \text{ volts/radian} \]
\[ K_3 = 259.6 \text{ volts/radian} \]
\[ K_4 = 0.0 \text{ volts/radian} \]

resulting in a value for the performance index of 333.2595. Similarly for Figure 9c where

\[ K_1 = 380.976 \text{ volts/radian} \]
\[ K_2 = 1415.232 \text{ volts/radian} \]
\[ K_3 = 189.950 \text{ volts/radian} \]
\[ K_4 = 3.256 \text{ volts/radian} \]

the performance index value equals 201.569. In Figure 9d the parameters were

\[ K_1 = 500.0 \text{ volts/radian} \]
\[ K_2 = 1394.219 \text{ volts/radian} \]
\[ K_3 = 129.186 \text{ volts/radian} \]
\[ K_4 = 24.21006 \text{ volts/radian} \]

These were found to be an optimum set for the given performance index and parameter boundaries. The final performance index value was 115.6383. After
thirty seconds $\omega_Y = 1.21 \times 10^{-4}$ radians/second and $\omega_Z = -8.7 \times 10^{-5}$ radians/second. The resulting change in the attitude angles were

$\Delta\gamma = 0.0$ radians
$\Delta\varphi = -0.0037$ radians
$\Delta\Theta = 3.72$ radians.

The large change in $\Theta$ is due to the controlled spinning of the vehicle about the $\hat{x}_V$ axis.

It should be noted that the set of parameters found give a suboptimal control system. The upper bound of 500 volts/radian given to $K_1$ was reached. In the optimization procedure, however, $K_1$ was not allowed to take on values greater than this limit. If $K_1$ had been allowed to take on higher values, possibly the performance index would have had a lower value.

The maximum value of both $\omega_Y$ and $\omega_Z$ for the entire time was 0.77 radians, which indicates that a maximum of 386 volts is required to drive the gyro-gimbals. This voltage is well within the allowed range.

From the plots it is noted that during the initial two seconds high accelerations occur about the $\hat{y}_V$ and $\hat{z}_V$ axes. Further investigations should be conducted to see if these accelerations are within acceptable limits or not. One method attempting to reduce these accelerations would be to include a penalty for high angular accelerations in the performance index.

Similar photographs in Figure 10 were taken during the design for case two in which a 1,000 foot-pound disturbance torque is applied for two seconds about the $\hat{x}_V$ axis beginning at one second. As in Figure 9, the photographs show the $\omega_Y$ and $\omega_Z$ trajectories for a total time of thirty seconds. The horizontal scale remains the same at 1 second per division. However the vertical scale for all four plots has been changed to $1.0 \times 10^{-5}$ radians/second.
Figure 10

Spacecraft angular rates versus time for different values of the feedback parameters for case two in which a 1,000.0 foot-pound disturbance torque is applied for two seconds about the $\hat{\psi}$ axis.
Figure 10
(Continued)
for each division. The initial values of both $w_x$ and $w_z$ are zero.

Figure 10a is the response for the initial choice of the parameters,

\[ K_1 = 360.0 \text{ volts/radian} \]
\[ K_2 = 1600.0 \text{ volts/radian} \]
\[ K_3 = 360.0 \text{ volts/radian} \]
\[ K_4 = 0.0 \text{ volts/radian} \]

resulting in a performance index equal to 19.7330. Figure 10b is the response when

\[ K_1 = 383.887 \text{ volts/radian} \]
\[ K_2 = 1638.113 \text{ volts/radian} \]
\[ K_3 = 415.666 \text{ volts/radian} \]
\[ K_4 = 10.734 \text{ volts/radian} \]

and where the performance index equals 12.1553. Similarly for Figure 10c the parameter values were

\[ K_1 = 395.089 \text{ volts/radian} \]
\[ K_2 = 1804.065 \text{ volts/radian} \]
\[ K_3 = 478.452 \text{ volts/radian} \]
\[ K_4 = 10.242 \text{ volts/radian} \]

and the performance index equaled 8.8318. Figure 10d shows the response from a set of parameters for which the performance index had a minimum value. The parameter values were

\[ K_1 = 500.0 \text{ volts/radian} \]
\[ K_2 = 1840.434 \text{ volts/radian} \]
\[ K_3 = 500.0 \text{ volts/radian} \]
\[ K_4 = 7.173 \text{ volts/radian} \]
and the performance index was 7.3367. As in the first case, the final parameters found result in high accelerations during the first three seconds, particularly for $\omega_x$. However, $\omega_y$ reduces much faster toward zero than $\omega_z$, resulting in angular rates of $1.47 \times 10^{-6}$ radians/second and $-1.109 \times 10^{-5}$ radians/second for $\omega_y$ and $\omega_z$ respectively after thirty seconds. As in case one an additional penalty for high angular rates in the performance index would possibly reduce the high initial acceleration and oscillatory response. The largest value of $\alpha$ and $\beta$ over the entire time was only 0.037 radians. Therefore, the boundary value of $K_1$ could be greatly increased and still retain the $\alpha K_1$ and $\beta K_3$ products below 500 volts. The resulting changes in the attitude angles were

\[
\begin{align*}
\Delta Y &= -2.82 \times 10^{-4} \text{ radians} \\
\Delta \phi &= 3.03 \times 10^{-4} \text{ radians} \\
\Delta \theta &= 10.0 \text{ radians}.
\end{align*}
\]

Again the parameters found within the given boundaries result in a suboptimal control system because both $K_1$ and $K_3$ took on the boundary value of 500 volts/radian. When $K_1$ and $K_3$ were allowed to take on larger values, the value of the performance index did decrease.

The results have shown that with the aid of a high speed digital computer proper feedback parameters can be found for a system of nonlinear equations. This is done by finding a minimum or maximum of a mathematical performance index. However, the results also support the fact that in the general case, the values of the optimum set of parameters are highly dependent upon the system initial conditions and the forces acting on the system. Thus all expected combinations of initial conditions and disturbance torques must be examined before the final choice of parameters is made. Finally, the results also demonstrate the difficulty in defining the proper performance
index. In the two examples just given the angular accelerations were not penalized in the performance index. This resulted in a combination of parameters for which high accelerations occurred. Before a suitable set of parameters can be found, further investigation must be done involving several different performance indices as well as various expected initial conditions and system disturbances.
LIST OF REFERENCES


DISTRIBUTION AS OF 22 OCTOBER, 1968

1 Dr A.A. Dougal
Asst Director (Research)
Ofc of Defense Res & Eng
Department of Defense
Washington, D.C. 20301

1 Office of Deputy Director
(Research and Technology)
ODD R&E-OSD
The Pentagon, Room 3-E-144
Washington, D.C. 20301

1 Director
Advanced Research Projects Agency
Department of Defense
Washington, D.C. 20301

1 Director for Information Sciences
Advanced Research Projects Agency
Department of Defense
Washington, D.C. 20301

1 Director for Materials Sciences
Advanced Research Projects Agency
Department of Defense
Washington, D.C. 20301

1 Headquarters
Defense Communications Agency (333)
The Pentagon
Washington, D.C. 20305

20 Defense Documentation Center
Attn: TISIA
Cameron Station, Bldg 5
Alexandria, Virginia 22314

1 Director
National Security Agency
Attn: Librarian C-332
Fort George G. Meade, Maryland 20755

1 National Security Agency
Attn: R4-James Tippett
Office of Research
Fort George G. Meade, Maryland 20755

1 Central Intelligence Agency
Attn: OCR/DD Publications
Washington, D.C. 20505

1 Colonel Kee
AFRSTE
Hq, USAF
Room ID-429, The Pentagon
Washington, D.C. 20330

1 Aerospace Medical Division
AMD (AMRXI)
Brooks Air Force Base, Texas 78235

1 AUL3T-9663
Maxwell AFB, Alabama 36112

1 AFFTC (FTBPP-2)
Technical Library
Edwards AFB, Calif. 93523

1 Hq SAMSO (SMTTA/Lt Nelson)
AF Unit Post Office
Los Angeles, California 90045

1 Lt Col Charles M. Waespy
Hq USAF (AFRDS)D
Pentagon
Washington, D.C. 20330

1 SSD (SSRT/Lt Starbuck)
AFUPO
Los Angeles, California 90045

1 Det #6, OAR (LOOAR)
Air Force Post Office
Los Angeles, California 90045

1 ARL (ARIY)
Wright-Patterson AFB, Ohio 45433
1 Dr H.V. Noble
Air Force Avionics Laboratory
Wright-Patterson AFB, Ohio 45433

1 Mr Peter Murray
Air Force Avionics Laboratory
Wright-Patterson AFB, Ohio 45433

1 AFAL (AVTE/R.D. Larson)
Wright-Patterson AFB, Ohio 45433

2 Commanding General
Attn: STEWS-WS-VT
White Sands Missile Range
New Mexico, 88002

1 RADC (EMLAL01)
Griffiss AFB, New York 13442
Attn: Documents Library

1 Mr H. E. Webb (EMIA)
Rome Air Development Center
Griffiss AFB, New York 13442

1 Academy Library (DFSLB)
U.S. Air Force Academy
Colorado Springs, Colorado 80912

1 Mr Morton M. Pavane, Chief
AFSC Scientific and Liaison Office
26 Federal Plaza
New York, N.Y. 10007

1 Lt Col Bernard S. Morgan
Frank J. Seiler Research Laboratory
U.S. Air Force Academy
Colorado Springs, Colorado 80912

1 Technical Library, AFETR
(ETV, MU-135)
Patrick AFB, Florida 32925

1 AFETR (ETLLG-1)
STINFO Office (For Library)
Patrick AFB, Florida 32925

1 Dr L. M. Hollingsworth
AFCRL (CRN)
L.G. Hanscom Field
Bedford, Massachusetts 01731

1 AFCRL (CRMXLR)
AFCRL Research Library, Stop 29
L.G. Hanscom Field
Bedford, Mass 01731

1 Colonel Robert E. Fontana
Dept of Electrical Engineering
Air Force Institute of Technology
Wright-Patterson AFB, Ohio 45433

1 Colonel A.D. Blue
RTD (RTTL)
Bolling Air Force Base, D.C. 20332

1 Dr I.R. Mirman
AFSC (SCT)
Andrews AFB, Maryland 20331

1 AFSC (SCTR)
Andrews AFB, Maryland 20331

1 Lt Col J.L. Reeves
AFSC (SCBB)
Andrews AFB, Maryland 20331

1 ESD (ESTI)
L.G. Hanscom Field
Bedford, Mass 01731

1 AEDC (ARO, INC)
Attn: Library/Documents
Arnold AFS, Tenn 37389

2 European Office of Aerospace Research
Shell Building
47 Rue Cantersteen
Brussels, Belgium

5 Lt Col Robert B. Kalisch
Chief, Electronics Division
Directorate of Engineering Sciences
Air Force Office of Scientific Research
Arlington, Virginia 22209
1 APCC (PGBS-12)
Eglin AFB, Florida 32542

1 U.S. Army Research Office
Attn: Physical Sciences Division
3045 Columbia Pike
Arlington, Virginia 22204

1 Research Plans Office
U.S. Army Research Office
3045 Columbia Pike
Arlington, Virginia 22204

1 Commanding General
U.S. Army Materiel Command
Attn: AMCRD-TP
Washington, D.C. 20315

1 Commanding General
U.S. Army Strategic Communication Cmd.
Fort Huachuca, Arizona 85613

1 Commanding Officer
U.S. Army Materials Research Agency
Watertown Arsenal
Watertown, Mass. 02172

1 Commanding Officer
U.S. Army Ballistics Research Laboratory
Attn: AMXRD-BAT
Aberdeen Proving Ground
Aberdeen, Maryland 21005

1 Commandant
U.S. Army Air Defense School
Attn: Missile Sciences Division, C & S Dept
P.O. Box 9390
Fort Bliss, Texas 79916

1 Commanding General
U.S. Army Missile Command
Attn: Technical Library
Redstone Arsenal, Alabama 35809

1 U.S. Army Munitions Command
Attn: Technical Information Command
Picatinny Arsenal
Dover, New Jersey 07801

1 Commanding Officer
Harry Diamond Laboratories
Attn: Dr Berthold Altman (AMKDO-TI)
Connecticut Avenue & Van Ness Street N.W.
Washington, D.C. 20438

1 Commanding Officer
U.S. Army Security Agency
Arlington Hall
Arlington, Virginia 22212

1 Commanding Officer
U.S. Army Limited War Laboratory
Attn: Technical Director
Aberdeen Proving Ground
Aberdeen, Maryland 21005

1 Commanding Officer
Human Engineering Laboratories
Aberdeen Proving Ground, Maryland 21005

1 Commandant
U.S. Army Command and General Staff College
Attn: Secretary
Fort Leavenworth, Kansas 66270

1 Dr H. Robl
Deputy Chief Scientist
U.S. Army Research Office (Durham)
Box CM, Duke Station
Durham, North Carolina 27706

1 Commanding Officer
U.S. Army Research Office (Durham)
Attn: CRD-AA-IP (Richard O. Ulsh)
Box CM, Duke Station
Durham, North Carolina 27706

1 Librarian
U.S. Army Military Academy
West Point, New York 10996

1 The Walter Reed Institute of Research
Walter Reed Medical Center
Washington, D.C. 20012

1 Commanding Officer
U.S. Army Electronics R & D Activity
White Sands Missile Range, New Mexico 88002
<table>
<thead>
<tr>
<th>1</th>
<th>Commanding Officer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Office of Naval Research Branch Office</td>
</tr>
<tr>
<td></td>
<td>219 South Dearborn Street</td>
</tr>
<tr>
<td></td>
<td>Chicago, Illinois 60604</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2</th>
<th>Chief of Naval Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OP-07</td>
</tr>
<tr>
<td></td>
<td>Washington, D.C. 20350</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Commanding Officer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Office of Naval Reserve Branch Office</td>
</tr>
<tr>
<td></td>
<td>207 West 24th Street</td>
</tr>
<tr>
<td></td>
<td>New York, New York 10011</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2</th>
<th>Director, U.S. Naval Security Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Attn: G43</td>
</tr>
<tr>
<td></td>
<td>3801 Nebraska Avenue</td>
</tr>
<tr>
<td></td>
<td>Washington, D.C. 20390</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Commanding Officer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Office of Naval Research Branch Office</td>
</tr>
<tr>
<td></td>
<td>1030 East Green Street</td>
</tr>
<tr>
<td></td>
<td>Pasadena, California 91101</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Commanding Officer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Office of Naval Research Branch Office</td>
</tr>
<tr>
<td></td>
<td>495 Summer Street</td>
</tr>
<tr>
<td></td>
<td>Boston, Massachusetts 02210</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Commanding Officer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Office of Naval Research Branch Office</td>
</tr>
<tr>
<td></td>
<td>1030 East Green Street</td>
</tr>
<tr>
<td></td>
<td>Pasadena, California 91101</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8</th>
<th>Director, Naval Research Laboratory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Technical Information Officer</td>
</tr>
<tr>
<td></td>
<td>Washington, D.C. 20360</td>
</tr>
<tr>
<td></td>
<td>Attn: Code 2000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Commanding Officer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Naval Air Development and Material Center</td>
</tr>
<tr>
<td></td>
<td>Johnsville, Pennsylvania 18974</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Librarian</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S. Naval Electronics Laboratory</td>
</tr>
<tr>
<td></td>
<td>San Diego, California 95152</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2</th>
<th>Librarian</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S. Naval Research Laboratory</td>
</tr>
<tr>
<td></td>
<td>Technical Information Officer</td>
</tr>
<tr>
<td></td>
<td>Washington, D.C. 20360</td>
</tr>
<tr>
<td></td>
<td>Attn: Code 2000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Commander</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Naval Avionics Facility</td>
</tr>
<tr>
<td></td>
<td>Indianapolis, Indiana 46241</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Commander</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Naval Air Development and Material Center</td>
</tr>
<tr>
<td></td>
<td>Johnsville, Pennsylvania 18974</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Librarian</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S. Naval Post Graduate School</td>
</tr>
<tr>
<td></td>
<td>Monterey, California 93940</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Librarian</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S. Naval Underwater Sound Laboratory</td>
</tr>
<tr>
<td></td>
<td>New London, Connecticut 06840</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Librarian</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S. Naval Underwater Sound Laboratory</td>
</tr>
<tr>
<td></td>
<td>Fort Trumbull</td>
</tr>
<tr>
<td></td>
<td>New London, Connecticut 06840</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Librarian</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S. Naval Underwater Sound Laboratory</td>
</tr>
<tr>
<td></td>
<td>Fort Trumbull</td>
</tr>
<tr>
<td></td>
<td>New London, Connecticut 06840</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Commanding Officer &amp; Director</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S. Naval Underwater Sound Laboratory</td>
</tr>
<tr>
<td></td>
<td>Fort Trumbull</td>
</tr>
<tr>
<td></td>
<td>New London, Connecticut 06840</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Head, Technical Division</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S. Naval Counter Intelligence Support Center</td>
</tr>
<tr>
<td></td>
<td>Fairmont Building</td>
</tr>
<tr>
<td></td>
<td>4420 North Fairfax Drive</td>
</tr>
<tr>
<td></td>
<td>Arlington, Virginia</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>Mr Charles Yost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Special Asst to the Director of Research</td>
</tr>
<tr>
<td></td>
<td>National Aeronautics &amp; Space Admin.</td>
</tr>
<tr>
<td></td>
<td>Washington, D.C. 20546</td>
</tr>
<tr>
<td>Institution</td>
<td>Address</td>
</tr>
<tr>
<td>-------------------------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>The John Hopkins University</td>
<td>Applied Physics Laboratory</td>
</tr>
<tr>
<td>8621 Georgia Avenue</td>
<td>Silver Spring, Maryland 20910</td>
</tr>
<tr>
<td>Dr Leo Young</td>
<td>Stanford Research Institute</td>
</tr>
<tr>
<td>Menlo Park, California 94025</td>
<td></td>
</tr>
<tr>
<td>Mr Henry L. Bachmann</td>
<td>Assistant Chief Engineer</td>
</tr>
<tr>
<td>Wheeler Laboratories</td>
<td>122 Cutterhill Road</td>
</tr>
<tr>
<td>Great Neck, New York 11021</td>
<td></td>
</tr>
<tr>
<td>School of Engineering Sciences</td>
<td>Arizona State University</td>
</tr>
<tr>
<td>Tempe, Arizona 85281</td>
<td></td>
</tr>
<tr>
<td>Engineering and Math. Sciences Library</td>
<td>University of California at Los Angeles</td>
</tr>
<tr>
<td>405 Hilgred Avenue</td>
<td>Los Angeles, California 90024</td>
</tr>
<tr>
<td>California Institute of Technology</td>
<td>Pasadena, California 91109</td>
</tr>
<tr>
<td>Attn: Documents Library</td>
<td></td>
</tr>
<tr>
<td>University of California</td>
<td>Santa Barbara, California 93106</td>
</tr>
<tr>
<td>Attn: Library</td>
<td></td>
</tr>
<tr>
<td>Carnegie Institute of Technology</td>
<td>Electrical Engineering Dept</td>
</tr>
<tr>
<td>Pittsburgh, Pa 15213</td>
<td></td>
</tr>
<tr>
<td>University of Michigan</td>
<td>Electrical Engineering Dept</td>
</tr>
<tr>
<td>Ann Arbor, Michigan 48104</td>
<td></td>
</tr>
<tr>
<td>New York University</td>
<td>College of Engineering</td>
</tr>
<tr>
<td>New York, N.Y. 10019</td>
<td>Emil Schafer, Head</td>
</tr>
<tr>
<td></td>
<td>Electronics Properties Infor. Cen</td>
</tr>
<tr>
<td></td>
<td>Hughes Aircraft Co</td>
</tr>
<tr>
<td></td>
<td>Culver, California 90230</td>
</tr>
<tr>
<td>Syracuse University</td>
<td>Dept of Electrical Engineering</td>
</tr>
<tr>
<td>Dept of Electrical Engineering</td>
<td>Syracuse, New York 13210</td>
</tr>
<tr>
<td>Yale University</td>
<td>Engineering Dept</td>
</tr>
<tr>
<td>Engineering Dept</td>
<td>New Haven, Connecticut 06520</td>
</tr>
<tr>
<td>Airborne Instruments Laboratory</td>
<td>Deerpark, New York 11729</td>
</tr>
<tr>
<td>Bendix Pacific Division</td>
<td>11600 Sherman Way</td>
</tr>
<tr>
<td>General Electric Co</td>
<td>Research Laboratories</td>
</tr>
<tr>
<td>Defense, New York 12301</td>
<td></td>
</tr>
<tr>
<td>Lockheed Aircraft Corp</td>
<td>P.O. Box 504</td>
</tr>
<tr>
<td>Raytheon Co</td>
<td>Bedford, Mass 01730</td>
</tr>
<tr>
<td>Dr G. J. Murphy</td>
<td>The Technological Institute</td>
</tr>
<tr>
<td>NorthWestern University</td>
<td>Evanston, Illinois 60201</td>
</tr>
<tr>
<td>Director</td>
<td>Electronics Systems Research Lab</td>
</tr>
<tr>
<td>Purdue University</td>
<td>Lafayette, Indiana 47907</td>
</tr>
<tr>
<td>Director</td>
<td>Microwave Laboratory</td>
</tr>
<tr>
<td>Stanford University</td>
<td>Stanford, California 94305</td>
</tr>
<tr>
<td>Emil Schafer, Head</td>
<td>Hughes Aircraft Co</td>
</tr>
<tr>
<td>Culver, California 90230</td>
<td></td>
</tr>
</tbody>
</table>
SPACERCT ATTITUDE CONTROL USING FEEDBACK PARAMETER OPTIMIZATION

In present day and future space missions, accurate spacecraft attitude control is a necessity. Before any photographs can be taken, instrument sighting performed, or trajectory corrections made the orientation of the spacecraft must be known and the desired attitude obtained. If solar cells are a source of power the solar panels must be pointed toward the sun. Radio communication is more efficient over interplanetary distances if highly directional antennas are used. Here again, proper vehicle orientation is required. To simulate artificial gravity in manned space stations a controlled angular rate about one axis is desired, while driving the angular rates about the other two axes to zero. The above indicate the importance of control systems aboard a space vehicle.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Systems</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parameter Optimization</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attitude Control</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spacecraft</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manned Orbital Space Station</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>