ANALYSIS OF THE SUSPENDED H-WAVEGUIDE AND OTHER RELATED DIELECTRIC WAVEGUIDE STRUCTURES

T. N. TRINH
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**Report Title:**
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**Controlling Office Name and Address:**
Joint Services Electronics Program

**Report Date:**
January 1983

**Number of Pages:**
153

**Security Classification:**
UNCLASSIFIED

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**DISTRIBUTION STATEMENT (of this Report):**
Approved for public release; distribution unlimited.

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**Abstract:**
In this report, the field and propagation characteristics of many dielectric waveguide structures that are of direct importance to the areas of millimeter-wave integrated circuits are studied. A rigorous method of analysis of the propagation constants and the field distributions of a newly developed suspended H-waveguide based on the mode matching techniques is described. The derived eigenvalue equation for determining the propagation constant of this waveguide structure is very general and can be applied directly to other planar...
dielectric structures of rectangular cross section. However, for applications where only approximate values of the propagation constants are necessary, simple effective parameter approximations are employed. Even though the effective parameter approximations do not provide very accurate results for the propagation constants in some cases, they always provide useful information on the whereabouts and the existences of certain modes which, in turn, can be used as the initial values for iterating in the more rigorous mode matching technique. As frequency increases, the values obtained by the effective parameter technique approach those obtained by the mode matching method.

The loss characteristics of a suspended H-waveguide are also analyzed. The overall low loss figure of this waveguide structure promises wider applications in the millimeter-wave integrated circuits.

Finally, some essential integrable dielectric components, such as the dielectric resonators and the leaky-wave antennas, are studied. A theoretical formulation of the eigenvalue equation based on the spectral domain approach for determining the complex propagation constants of a slot leaky-wave antenna array constructed from a suspended H-waveguide is presented. A technique for improving the radiation characteristics of an image guide leaky-wave antenna is also described.
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by

T. N. Trinh and R. Mittra

Supported by

Joint Services Electronics Program
N0001479C-0424
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1.

1. INTRODUCTION

The renewed interest in the millimeter-wave spectrum is due primarily to the recent advances in the development of solid state devices up to 300 GHz. Attempts have been made to utilize microwave technology in the millimeter-wave frequency range but only with limited successes. The conventional metal waveguides become very lossy and more difficult to fabricate as the wavelengths become shorter. Extremely tight surface tolerance requirements of metal waveguide components at millimeter-wave range contribute to the high cost of conventional systems. Microstrips and other forms of printed circuits have been used successfully at the lower millimeter-wave range [1],[2]. However, in the upper millimeter-wave bands, the extreme miniaturization of the center metal strip or the slot (in a slot-line) makes fabrication very difficult. The dielectric substrate thickness must be very small to avoid higher-order modes. The microstrip lines also suffer high metal loss due to the high concentration of the current density on the metal surfaces and near the edges of the metal strips. This has led to the investigation of alternate forms of low-loss low-cost dielectric-based waveguide designs suitable for integrated circuit applications at the millimeter-wave frequency range.

Several types of dielectric waveguide structures are currently available as the guiding medium for millimeter-wave applications, such as image guide [3], insular guide [4], H-guide [5], and trapped
image guide [6]. In this thesis, another form of dielectric waveguide structure is presented—the suspended H-waveguide.

This suspended H-waveguide is similar in many ways to other waveguide structures. It consists of a guiding dielectric core with dielectric constant $\varepsilon_2$ sandwiched between two low-loss dielectric layers of dielectric constants $\varepsilon_1$ and $\varepsilon_3$ (see Fig. 1). The dielectric arrangement is then shielded to obtain a closed structure. In practice, these two insulating dielectric layers are metal-cladded dielectric sheets which are commercially available. Since $\varepsilon_2 > \varepsilon_1$, $\varepsilon_3$, most of the electromagnetic energy is carried in the guiding dielectric core, and therefore, the conductor loss is smaller than for the microstrip or image guide. Compared with other dielectric-based guiding structures mentioned above, the suspended H-waveguide enjoys a rigid mechanical support due to the metal enclosure. Also, the radiation loss is probably less than for other waveguide structures due to the closed metal walls. For this same reason, cross-talk among circuit elements can be eliminated. Presently, most of the commercial dielectric-base millimeter-wave ICs are shielded, hence, the metal surfaces around the suspended H-waveguide become a natural enclosure. Note that the side walls can be removed if the separation between the dielectric core and these walls is large enough. This allows the adjustment of the relative locations of the dielectric core and other components in the circuit without removing the top dielectric layer. The metal enclosure also allows plenty of ground for dc biasing in solid state applications. Ferrite applications,
Figure 1. Suspended H-Waveguide
such as the phase shifter or field displacement isolator, can readily be integrated since the top and bottom metal plates can be used to support a dc magnetic field. Bonding between dielectric materials may not be needed since the metal enclosure will secure all dielectric layers tightly in place.

Like any other oversized shielded waveguide structure, higher-order modes may arise. This situation may not be desirable for many applications. However, due to the unique structural arrangement, dielectric resonators can be used to eliminate the unwanted modes. Section 5.1 describes the designs of some modified dielectric resonators which can be used in circuits that employ the suspended H-waveguide.

In order to obtain reliable design information for the designs of both active and passive components derived from these waveguides, accurate theoretical predictions of the field and propagation characteristics must be available. A number of theoretical studies on the propagation properties of certain dielectric structures have been presented in the literature based on the effective permittivity and permeability [4],[7]-[9]. These methods are simple and usually provide reasonably accurate propagation information. However, they do not provide information on complete field distributions, and hence, certain loss characteristics. A rigorous mode-matching technique based on the field expansion in each subregion of the guide cross-section and the subsequent matching of the fields at the dielectric interfaces appeared recently in the literature [10]-[14]. Peng et al. [15] proposed a similar approach using the transverse resonant procedures.
In this thesis, a similar mode matching technique is applied to the investigation of the suspended H-waveguide. All relevant characteristic equations for the propagation constant of the waveguide structure are given. It will be shown later that the field and propagation characteristics of other dielectric structures, such as image guide and insular guide, can be obtained directly from the formulation of the eigenvalue problem of the suspended H-waveguide. For generality, a coupled suspended H-waveguide is analyzed, which can then be easily reduced to a single H-waveguide.

The loss characteristics of this waveguide structure are included in Chapter 4. Some components derived from the suspended H-waveguide are described in Chapter 5.
2. METHODS OF ANALYSIS

2.1 Mode Matching

2.1.1. Suspended H-Waveguide

The cross-section of a coupled suspended H-waveguide is shown in Figure 2a. For generality, the thickness of the top and bottom layers and their respective dielectric constants $\varepsilon_1$ and $\varepsilon_3$ are different. The width and the height of the dielectric core are $2w$ and $2h$, respectively. The locations of the top and bottom conductor walls of the guide are $d$ and $l$. Because of the symmetry of the structure about the $x = -a$ plane, even and odd modes can propagate in the coupled structure. The analysis of the coupled lines can then be reduced to that shown in Figure 2b where a perfect conducting (pec) wall implies odd mode propagation, and a perfect magnetic conducting (pmc) wall implies even mode propagation. For $a = b$, the coupled line reduces to that of a single uncoupled structure.

The analysis using the mode matching technique can be applied by separating the cross-section into five subregions (I)-(V). The fields in each region can be appropriately expanded in terms of regional eigenfunctions which satisfy the boundary conditions at the conductor walls. It is well-known that an infinite set of discrete modes is supported by an enclosed metal structure. Of these, only a small number of surface waves exist. The remainder are discrete higher-order modes some of which are propagating and the rest are below cut-off.
Figure 2. (a) Cross section of the coupled suspended H-waveguide.
(b) Structure adopted for analysis.
The total fields are derived from the Hertzian potentials $\Pi_e$ and $\Pi_h$ which have a single x-directed component \cite{16}. The relationships between fields and potentials are:

$$E = \nabla \times \nabla \times \Pi_e - j\omega_0 \nabla \times \Pi_h \tag{1a}$$

$$H = j\omega_0 \varepsilon_i \nabla \times \Pi_e + \nabla \times \nabla \times \Pi_h \tag{1b}$$

where $\varepsilon_i$ is the relative dielectric constant of each region. $\Pi_e$ and $\Pi_h$ are referred to as longitudinal-section electric (LSE) mode and longitudinal-section magnetic (LSM) mode, respectively. Assuming that the propagation is in the z-direction, the field potential eigenfunctions can be expressed as follows ($\exp(-jk_z z)$ suppressed for clarity).

Region I

$$\Pi_e^{(1)} = \sum_{n=0}^{N} A_n \left[ \cos \left( k_{xn} (x+a) \right) \right] \begin{align*}
\sin \left( k_{yn} (y-d) \right) / \sin \left( k_{yn} (h-d) \right) \\
\sin \left( k_{xn} (x-a) \right) 
\end{align*} \tag{2a}$$

$$\Pi_h^{(1)} = \sum_{m=0}^{M} B_m \left[ \sin \left( k_{xm} (x+a) \right) \right] \begin{align*}
\cos \left( k_{ym} (y-d) \right) / \cos \left( k_{ym} (h-d) \right) \\
\cos \left( k_{xm} (x-a) \right) 
\end{align*} \tag{2b}$$

Region II

$$\Pi_e^{(2)} = \sum_{n=1}^{N'} \left[ C_n \cos \left( k_{xn} x \right) + D_n \sin \left( k_{xn} x \right) \right] \sin \left( k_{yn} y \right) / \sin \left( k_{yn} h \right)$$

$$+ \sum_{n=1}^{N'} \left[ C'_n \cos \left( k_{xn} x \right) + D'_n \sin \left( k_{xn} x \right) \right] \cos \left( k_{yn} y \right) / \cos \left( k_{yn} h \right) \tag{3a}$$
\[ \Pi_h^{(2)} = \sum_{m=1}^{M'} \left[ E_m \sin \left( k_{xm} x \right) + F_m \cos \left( k_{xm} x \right) \right] \cos \left( k_{ym} y \right) / \cos \left( k_{ym} h \right) \]
\[ + \sum_{m=1}^{M'} \left[ E_m' \sin \left( k_{xm} x \right) + F_m' \cos \left( k_{xm} x \right) \right] \sin \left( k_{ym} y \right) / \sin \left( k_{ym} h \right) \]

Region III

\[ \Pi_e^{(3)} = \sum_{n=0}^{N} k_n \begin{cases} \cos \left( k_{xn} (x+a) \right) \\ \sin \left( k_{xn} (x+a) \right) \end{cases} \sin \left( k_{yn} (y+1) \right) / \sin \left( k_{yn} (-h+1) \right) \]

\[ \Pi_h^{(3)} = \sum_{m=0}^{M} L_m \begin{cases} \sin \left( k_{xm} (x+a) \right) \\ \cos \left( k_{xm} (x+a) \right) \end{cases} \cos \left( k_{ym} (y+1) \right) / \cos \left( k_{ym} (-h+1) \right) \]

Region IV

\[ \Pi_e^{(4)} = \sum_{n=1}^{N'} \begin{cases} \cos \left( k_{xn} (x+a) \right) \\ \sin \left( k_{xn} (x+a) \right) \end{cases} \begin{cases} \sin \left( k_{yn} \right) \\ \cos \left( k_{yn} \right) \end{cases} + \begin{cases} G_n \sin \left( k_{yn} \right) \\ G_n' \cos \left( k_{yn} \right) \end{cases} \]

\[ \Pi_h^{(4)} = \sum_{m=1}^{M'} \begin{cases} \sin \left( k_{xm} (x+a) \right) \\ \cos \left( k_{xm} (x+a) \right) \end{cases} \begin{cases} \cos \left( k_{ym} \right) \\ \sin \left( k_{ym} \right) \end{cases} + \begin{cases} H_m \cos \left( k_{ym} \right) \\ H_m' \sin \left( k_{ym} \right) \end{cases} \]

Region V

\[ \Pi_e^{(5)} = \sum_{n=1}^{N'} \cos \left( k_{xn} (x-b) \right) \begin{cases} \sin \left( k_{yn} \right) \\ \cos \left( k_{yn} \right) \end{cases} + \begin{cases} I_n \sin \left( k_{yn} \right) \\ I_n' \cos \left( k_{yn} \right) \end{cases} \]

\[ \Pi_h^{(5)} = \sum_{n=1}^{N'} \cos \left( k_{xn} (x-b) \right) \begin{cases} \sin \left( k_{yn} \right) \\ \cos \left( k_{yn} \right) \end{cases} + \begin{cases} I_n \sin \left( k_{yn} \right) \\ I_n' \cos \left( k_{yn} \right) \end{cases} \]
\[
\mu_h^{(5)} = \sum_{m=1}^{M'} \sin \left( \frac{k_{xm}(x-b)}{a+b} \right) \left[ J_m \cos \left( \frac{k_{ym}}{a+b} \right) + J'_m \sin \left( \frac{k_{ym}}{a+b} \right) \right] \tag{6b}
\]

These potentials have been chosen in such a way that the correspondent fields will satisfy all boundary conditions at the metal walls. In (2)-(6), \( k_x^{(i)} \) and \( k_y^{(i)} \) \((i = 1,2,3,4,5)\) are the transverse wavenumbers for the LSM modes, and \( \tilde{k}_x^{(i)} \) and \( \tilde{k}_y^{(i)} \) \((i = 1,2,3,4,5)\) are for the LSE modes. In (2), (3) and (4), the upper line in the brackets implies a pec wall at \( x = -a\), while the lower line implies a pmc wall. The addition of the denominator terms in these potential functions greatly simplifies the final expressions of the eigenvalue equations obtained from the enforcing of the continuity conditions at the \( y = h \) and \( -h \) interfaces. An exact solution for the propagation constant and the field expressions can be obtained if \( N, N', M \) and \( M' \) are infinite.

However, in practice, one must limit these values to finite numbers for numerical computations.

To satisfy the boundary conditions at both side walls, the transverse eigenvalues in regions I and III are given as follows.

\[
\begin{align*}
\text{odd modes (pec at } x = -a) & \quad \text{even mode (pmc at } x = -a) \\
\frac{k_x^{(1)}}{a+b} & \quad \frac{k_x^{(3)}}{a+b} \\
\frac{n\pi}{a+b} & \quad \frac{n+\frac{1}{2}}{a+b} \\
\frac{m\pi}{a+b} & \quad m = 0,1,2,\ldots \\
\end{align*}
\]

\[
\begin{align*}
\text{even mode (pmc at } x = -a) & \quad \text{odd modes (pec at } x = -a) \\
\frac{k_x^{(1)}}{a+b} & \quad \frac{k_x^{(3)}}{a+b} \\
\frac{(m+\frac{1}{2})\pi}{a+b} & \quad \frac{n\pi}{a+b} \\
& \quad n = 0,1,2,\ldots \\
\end{align*}
\]

These odd (antisymmetric) and even (symmetric) modes are decoupled from each other and can exist independently in the coupled structure.
Furthermore, separate eigenvalue equations for $\tilde{k}_x^{(i)}$ and $\tilde{k}_x^{(i)}$ (i = 2, 4, 5) are obtained independently not only for LSE and LSM modes but also for the fields which are even and odd about the $y = 0$ plane.

Matching the tangential field components, which are derived from (1), across the interfaces at $x = tw$, for $-h \leq y \leq h$ yields different eigenvalue equations for $k_x^{(i)}$ and $\tilde{k}_x^{(i)}$:

**pec wall at $x = -a$**

$$\cos (A) \left\{ \chi_1 \cos (B) \sin (W) + \chi_2 \sin (B) \cos (W) \right\}$$

$$+ \chi_5 \sin (A) \left\{ \chi_3 \cos (B) \cos (W) + \chi_4 \sin (B) \sin (W) \right\} = 0 \quad (8)$$

**pmc wall at $x = -a$**

$$\chi_5 \cos (A) \left\{ \chi_3 \cos (B) \cos (W) + \chi_4 \sin (B) \sin (W) \right\}$$

$$- \sin (A) \left\{ \chi_1 \cos (B) \sin (W) + \chi_2 \sin (B) \cos (W) \right\} = 0 \quad (9)$$

where all parameters in Eqs. (8) and (9) are given as follows:

<table>
<thead>
<tr>
<th>LSM modes</th>
<th>LSE modes</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>$k_x^{(4)}(-w+a)$</td>
</tr>
<tr>
<td>B</td>
<td>$k_x^{(5)}(w-b)$</td>
</tr>
<tr>
<td>W</td>
<td>$2k_x^{(2)}w$</td>
</tr>
<tr>
<td>$\chi_1$</td>
<td>$k_x^{(2)}/\varepsilon_k^{(5)}$</td>
</tr>
<tr>
<td>$\chi_2$</td>
<td>-1</td>
</tr>
<tr>
<td>LSM modes</td>
<td>LSE modes</td>
</tr>
<tr>
<td>----------</td>
<td>-----------</td>
</tr>
<tr>
<td>$\chi_3$</td>
<td>1</td>
</tr>
<tr>
<td>$\chi_4$</td>
<td>$\varepsilon_2 k^{(5)/k^{(2)}}_{xmn}$</td>
</tr>
<tr>
<td>$\chi_5$</td>
<td>1</td>
</tr>
</tbody>
</table>

and

$$\varepsilon_1 k^2_0 = k^2_z + k^{(1)^2}_{xyn} + k^{(1)^2}_{ym} = k^2_z + \tilde{k}^{(1)^2}_{xmn} + \tilde{k}^{(1)^2}_{ym} \quad (11a)$$

$$\varepsilon_2 k^2_0 = k^2_z + k^{(2)^2}_{xyn} + k^{(2)^2}_{ym} = k^2_z + \tilde{k}^{(2)^2}_{xmn} + \tilde{k}^{(2)^2}_{ym} \quad (11b)$$

$$\varepsilon_3 k^2_0 = k^2_z + k^{(3)^2}_{xyn} + k^{(3)^2}_{ym} = k^2_z + \tilde{k}^{(3)^2}_{xmn} + \tilde{k}^{(3)^2}_{ym} \quad (11c)$$

$$k^2_0 = k^2_z + k^{(4)^2}_{xyn} + k^{(4)^2}_{ym} = k^2_z + \tilde{k}^{(4)^2}_{xmn} + \tilde{k}^{(4)^2}_{ym} \quad (11d)$$

$$k^2_0 = k^2_z + k^{(5)^2}_{xyn} + k^{(5)^2}_{ym} = k^2_z + \tilde{k}^{(5)^2}_{xmn} + \tilde{k}^{(5)^2}_{ym} \quad (11e)$$

Also, by a simple comparison when applying the continuity conditions, it was found that the following relations hold:

$$k^{(4)}_{yn} = k^{(2)}_{yn} = k^{(5)}_{yn} = k_{yn} \quad (12a)$$

$$\tilde{k}^{(4)}_{ym} = \tilde{k}^{(2)}_{ym} = \tilde{k}^{(5)}_{ym} = \tilde{k}_{ym} \quad (12b)$$

There exist an infinite number of solutions for the eigenvalue equations (8) and (9). If all solutions are taken into account, the
expressions for the potentials in regions II, IV and V form a complete set to represent an exact field distribution in these regions. Each solution constitutes an eigenmode which represents a particular eigenfunction. \( k_x^{(2)} \) and \( k_x^{(2)} \) are always real which implies a transverse standing-wave within the dielectric core, whereas \( k_x^{(1)} \) and \( k_x^{(1)} \) \((i = 4, 5)\) can either be real or imaginary. The imaginary values represent the modes that decay exponentially away from the dielectric core.

For each mode, the relations among the unknown coefficients of the field potentials in regions II, IV and V are obtained. For a reason which will become clear later, these amplitude coefficients are expressed in terms of those in region V. In other words, all unknown coefficients of regions II and IV now are represented by those in region V.

For an LSM mode, the following relations hold:

\[
\begin{align*}
\begin{pmatrix} C_n \\ C'_n \end{pmatrix} &= K_n^{(C-I)} \begin{pmatrix} I_n \\ I'_n \end{pmatrix} \\
\begin{pmatrix} D_n \\ D'_n \end{pmatrix} &= K_n^{(D-I)} \begin{pmatrix} I_n \\ I'_n \end{pmatrix} \\
\begin{pmatrix} G_n \\ G'_n \end{pmatrix} &= K_n^{(G-I)} \begin{pmatrix} I_n \\ I'_n \end{pmatrix}
\end{align*}
\] (13a)

\[
\begin{align*}
\begin{pmatrix} \Delta \text{LSM} \end{pmatrix} \left( \frac{k_x^{(2)}}{k_x^{(4)}} \right) \cos \left( k_x^{(5)}(\omega-b) \right)
\end{align*}
\] (14a)
\[ K_{n}^{(D-I)} = \frac{1}{\Delta_{LSM}} \left[ \begin{array}{c} k_2^{(2)} \\ \epsilon_2 \frac{k_2^{(4)}}{x_n} \end{array} \right] \left[ \begin{array}{c} \cos \left( \frac{k_2^{(4)}}{x_n} (-\omega + \alpha) \right) \\ \sin \left( \frac{k_2^{(4)}}{x_n} (-\omega + \alpha) \right) \end{array} \right] \cos \left( \frac{k_2^{(2)}}{x_n} \omega \right) \\
+ \left[ \begin{array}{c} \sin \left( \frac{k_2^{(4)}}{x_n} (-\omega + \alpha) \right) \\ -\cos \left( \frac{k_2^{(4)}}{x_n} (-\omega + \alpha) \right) \end{array} \right] \sin \left( \frac{k_2^{(2)}}{x_n} \omega \right) \cos \left( \frac{k_5^{(4)}}{x_n} (\omega - b) \right) \right] (14b) \]

\[ K_{n}^{(G-I)} = \frac{1}{\Delta_{LSM}} \left[ \begin{array}{c} \sin \left( \frac{k_2^{(4)}}{x_n} (-\omega + \alpha) \right) \\ \epsilon_2 \frac{k_2^{(4)}}{x_n} \left[ \begin{array}{c} \cos \left( \frac{k_2^{(4)}}{x_n} (-\omega + \alpha) \right) \\ \sin \left( \frac{k_2^{(4)}}{x_n} (-\omega + \alpha) \right) \end{array} \right] \right] \cos \left( \frac{k_2^{(2)}}{x_n} \omega \right) \right] (14c) \]

\[ \Delta_{LSM} = \epsilon_2 \left[ \begin{array}{c} \sin \left( \frac{k_2^{(4)}}{x_n} (-\omega + \alpha) \right) \\ \epsilon_2 \frac{k_2^{(4)}}{x_n} \sin \left( \frac{k_2^{(4)}}{x_n} (-\omega + \alpha) \right) \end{array} \right] \sin \left( \frac{k_2^{(4)}}{x_n} \omega \right) \right] (14d) \]

Likewise, for the LSE modes, the relationships among the unknown complex coefficients in regions II, IV and V are

\[
\begin{bmatrix}
E_m \\
E'_m
\end{bmatrix} = K_{m}^{(E-J)} \begin{bmatrix}
J_m \\
J'_m
\end{bmatrix} \quad (15a)
\]
\[
\begin{align*}
\begin{bmatrix}
F_m' \\
F_m''
\end{bmatrix} &= k_m^{(F-J)}
\begin{bmatrix}
J_m \\
J_m'
\end{bmatrix} \\
\begin{bmatrix}
H_m \\
H_m'
\end{bmatrix} &= k_m^{(H-J)}
\begin{bmatrix}
J_m \\
J_m'
\end{bmatrix}
\end{align*}
\tag{15b}
\]

where

\[
k_m^{(E-J)} = \frac{1}{\Delta_{LSE}} \frac{\tilde{r}^{(2)}_{m}}{k_{xm}^{(4)}} \sin \left[ k_{xm}^{(5)}(w-b) \right]
\tag{16a}
\]

\[
k_m^{(F-J)} = \frac{1}{\Delta_{LSE}} \begin{bmatrix}
\tilde{r}^{(2)}_{m} \\
-k_{xm}^{(4)}
\end{bmatrix} \begin{bmatrix}
\sin \left[ k_{xm}^{(4)}(-w+a) \right] \\
\cos \left[ k_{xm}^{(4)}(-w+a) \right]
\end{bmatrix}
\sin \left[ k_{xm}^{(2)}w \right]
\tag{16b}
\]

\[
k_m^{(H-J)} = \frac{1}{\Delta_{LSE}} \begin{bmatrix}
\cos \left[ k_{xm}^{(4)}(-w+a) \right] \\
-k_{xm}^{(4)}
\end{bmatrix} \begin{bmatrix}
\sin \left[ k_{xm}^{(2)}w \right] \\
\cos \left[ k_{xm}^{(4)}(-w+a) \right]
\end{bmatrix}
\sin \left[ k_{xm}^{(5)}(w-b) \right]
\tag{16c}
\]

and

\[
\Delta_{LSE} = \begin{bmatrix}
\cos \left[ k_{xm}^{(4)}(-w+a) \right] \\
\sin \left[ k_{xm}^{(4)}(-w+a) \right]
\end{bmatrix} \sin \left[ 2k_{xm}^{(2)}w \right] + \begin{bmatrix}
\tilde{r}^{(4)}_{m} \\
k_{xm}^{(4)}
\end{bmatrix} \begin{bmatrix}
\sin \left[ k_{xm}^{(4)}(-w+a) \right] \\
\cos \left[ k_{xm}^{(4)}(-w+a) \right]
\end{bmatrix}
\cos \left[ 2k_{xm}^{(2)}w \right]
\tag{16d}
\]
In (14)-(16), the upper line inside the brackets applies to the pec wall at \( x = -a \) (odd modes), and the lower to the pmc wall (even modes).

The total fields of the composite structure and the longitudinal propagation constant can now be obtained by matching the tangential field components at the plane \( y = \pm h \). In order to satisfy the boundary conditions for all modes, the tangentials \( E_x', H_x', E_z \) and \( H_z \) must be all taken into consideration. Using Eqs. (1)-(6), the resulting eight equations are shown in Appendix A. These equations shall contain the \( x \)-dependence which is different for each region. The \( x \)-variation in each expansion function in regions I and III is chosen to be a sine or cosine type harmonic function to simultaneously satisfy the boundary conditions at both side walls. By appropriately multiplying these functions with either \( \sin \left[ k_x^{(i)}(x+a) \right] \) or \( \cos \left[ k_x^{(i)}(x+a) \right] \), \( i = 1,3 \), where \( k_x^{(i)} \) are given in (7), and integrating over \( x \) from \(-a\) to \( b \) and using the orthogonality properties of these expansion functions, a system of homogeneous linear equations with unknown coefficients \( A_n, B_n, K_n, L_n, I_n, I_n', J_m \) and \( J'_m \) is obtained. To reduce the total number of equations, \( A_n, B_n, K_n \) and \( L_n \) which were derived from Eqs. (A-1), (A-2), (A-5), and (A-6), will be substituted into the remaining four equations. Assuming an equal number of LSE modes in all regions (i.e., \( M = M' \)) and also an equal number of LSM modes (i.e., \( N = N' \)), the reduced system \( 2(N+M) \) homogeneous linear equations with unknown coefficients \( I_n, I_n', J_m \) and \( J'_m \) become:
\[
\left( k_z \sum_{n=1}^{N} \left[ I_n (v) + \frac{k_x (1)}{x v} \left\{ \frac{-1}{1} \right\} P_n (v) \right] \right) (I_n + I'_n)
\]

\[
+ \omega \sum_{m=1}^{M} \left[ \frac{\tilde{k}_y (1) \tan(\tilde{k}_y (1) (h-d))}{\varepsilon_l k_0 - \tilde{k}_x (1) \gamma} \right] Q_m (\mu) - \tilde{k}_y (1) \tan(\tilde{k}_x (1) h) U_m (\mu) \right] J_m
\]

\[
+ \omega \sum_{m=1}^{M} \left[ \frac{\tilde{k}_y (1) \tan(\tilde{k}_y (1) (h-d))}{\varepsilon_l k_0 - \tilde{k}_x (1) \gamma} \right] Q_m (\mu) + \tilde{k}_y (1) \cot(\tilde{k}_x (1) h) U_m (\mu) \right] J'_m = 0
\]

(17)

\[
\omega \sum_{n=1}^{N} \left[ k_y \cot(\tilde{k}_y (1) h) R_n (v) - \frac{\varepsilon_l k_y (1) \cot(\tilde{k}_y (1) (h-d))}{\varepsilon_l k_0 - \tilde{k}_x (1) \gamma} P_n (v) \right] I_n
\]

\[
- \omega \sum_{n=1}^{N} \left[ k_y \tan(\tilde{k}_y (1) h) R_n (v) + \frac{\varepsilon_l k_y (1) \cot(\tilde{k}_y (1) (h-d))}{\varepsilon_l k_0 - \tilde{k}_x (1) \gamma} P_n (v) \right] I'_n
\]

\[
+ \sum_{m=1}^{M} \left[ k_x \left\{ \frac{\tilde{k}_y (1) Q_m (\mu)}{1} \right\} \right] (J_m + J'_m) = 0
\]

(18)
\[
\sum_{n=1}^{N} k \left\{ T_n(\nu) + \frac{\lambda_n}{n} \left\{ \begin{array}{c} \frac{1}{x_n} \\ \frac{1}{y_n} \end{array} \right\} P_n(\nu) \right\} \left(-I_n + I'_n\right)
\]

\[
+ \omega \sum_{m=1}^{M} \left\{ \frac{\tilde{k}_y \tan(\tilde{k}_y h) \ U_m(\mu)}{x_n} + \frac{\tilde{k}_x \tan(\tilde{k}_x h) \ U_m(\mu)}{x_n} \right\} J_m
\]

\[
+ \omega \sum_{m=1}^{M} \left\{ \frac{\tilde{k}_y \cot(\tilde{k}_y h) \ U_m(\mu)}{x_n} - \frac{\tilde{k}_x \cot(\tilde{k}_x h) \ U_m(\mu)}{x_n} \right\} J'_m = 0
\]

\[
\omega \sum_{n=1}^{N} \left\{ k \cot(k_y h) R_n(\nu) + \frac{\epsilon_3 k_y \cot(k_y h) \cot(k_y h) \cot(k_y h) (-h+1)}{} \right\} P_n(\nu) \right\} I_n
\]

\[
+ \omega \sum_{n=1}^{N} \left\{ \frac{\tilde{k}_y \tan(\tilde{k}_y h) \ R_n(\nu) - \epsilon_3 k_y \cot(k_y h) \cot(k_y h) (-h+1)}{x_n} \right\} P_n(\nu) \right\} I'_n
\]

\[
+ \sum_{m=1}^{M} k \left\{ S_m(\mu) + \frac{\frac{1}{x_n} \tilde{k}_x Q_m(\mu)}{x_n} \right\} \right\} \left(J_m - J'_m\right) = 0
\]

\[
(19)
\]

\[
\omega \sum_{n=1}^{N} \left\{ k \cot(k_y h) R_n(\nu) + \frac{\epsilon_3 k_y \cot(k_y h) \cot(k_y h) \cot(k_y h) (-h+1)}{x_n} \right\} P_n(\nu) \right\} I_n
\]

\[
+ \omega \sum_{n=1}^{N} \left\{ \frac{\tilde{k}_y \tan(\tilde{k}_y h) \ R_n(\nu) - \epsilon_3 k_y \cot(k_y h) \cot(k_y h) (-h+1)}{x_n} \right\} P_n(\nu) \right\} I'_n
\]

\[
+ \sum_{m=1}^{M} k \left\{ S_m(\mu) + \frac{\frac{1}{x_n} \tilde{k}_x Q_m(\mu)}{x_n} \right\} \right\} \left(J_m - J'_m\right) = 0
\]

\[
(20)
\]

\[
where
\]

\*

\[
\nu = 0, \ldots, N-1
\]
\[ \mu = \begin{cases} 
1, \ldots, M & \text{for pec wall} \\
0, \ldots, M-1 & \text{for pmc wall} 
\end{cases} \]

The upper value in the brackets applies to the pec wall at \( x = -a \), while the lower line applies to the pmc wall. \( P_n^{(v)}, \ldots, U_m^{(\mu)} \) are the coupling integral terms relating the regions [11], and are shown in Appendix B. The zeroes of the determinant of these \( 2(N+M) \times 2(N+M) \) homogeneous equations are the longitudinal propagation constants \( k_z \)'s of the composite waveguide structure. In (17)-(20), \( k_x^{(i)}, k_y^{(i)}, k_x^{(i)}, k_y^{(i)} \) and \( k_y^{(i)} \) are related to \( k_z \) according to (11). For each \( k_z \), a set of amplitude coefficients \( A_n, \ldots, A_m \) are found within a constant multiplicative factor and, hence, a complete description of the total field in the guide via the potential functions. It should be mentioned that the approach to formulate the eigenvalue problem for determining the propagation constant of the suspended H-waveguide described in this section can also be extended to other more complicated structures whose shapes and boundaries are rectilinear and parallel to either the x- or the y-axis [10].

2.1.2. Related Planar Dielectric Waveguides

The system of homogeneous equations derived for the suspended H-waveguide can directly be applied to other popular rectangular cross-section dielectric waveguide structures as shown in Figure 3. Since the expansion functions of the potentials in the dielectric core region of the suspended H-waveguide are composed of both symmetric
Figure 3. Other dielectric waveguide structures:

(a) Dielectric rectangular guide. (b) Image guide.
(c) Insular guide. (d) Partially-filled dielectric trough waveguide.
and antisymmetric functions with respect to the \( y \)-axis, only a portion of (17)-(20) is needed to calculate the propagation constants of the waveguide structures shown in Figures 3a, 3b, and 3d, whose physical ground planes coincide with the \( y \)-axis. (The rectangular dielectric guide is equivalent to an image guide with twice the aspect ratio of the dielectric guide.) To analyze these structures, the top dielectric layer (\( \varepsilon_1 \)) is replaced by free-space, and the top metal ground plane is moved further away from the dielectric core to minimize its effect on the propagation characteristics. By rearranging Eqs. (17)-(20) into a matrix form, it is easy to isolate the symmetric and nonsymmetric parts, i.e.,

\[
\begin{pmatrix}
M_1 & M_2 \\
M_3 & M_4
\end{pmatrix}
\begin{pmatrix}
I_n \\
J_m
\end{pmatrix}
= 0
\]

(21)

All coefficients of the matrix in (21) are given in (17)-(20). The amplitude coefficients with subscript \( n \) represent the LSM modes, and subscript \( m \) the LSE modes. A quick observation reveals that the equations that correspond to the coefficients of the submatrix in the first or third quadrant of expression (21) are indeed the eigenvalue equations of the structures shown in Figures 3a, b and d, i.e.,

\[
\begin{pmatrix}
M_1 \\
M_3
\end{pmatrix}
\begin{pmatrix}
I_n \\
J_m
\end{pmatrix}
= 0
\]

(22)
The analysis has so far been for a coupled structure. It can be reduced to that of an uncoupled guide simply by moving the left side wall (at \( x = -a \)) to infinity. However, in actual calculation, the left side wall will be moved up against the dielectric core, since in a single guide, the fields can also be even or odd with respect to their centers. A pmc wall at \( x = -w \) will generate a family of even modes in a guide whose total width is now \( 4w \). Likewise, a pec wall at \( x = -w \) corresponds to a family of odd modes with respect to this new guide center. By doing this, the computation can be minimized since the number of LSE and LSM eigenmodes necessary to satisfy the continuity conditions at interfaces of regions increases with increasing ratio \((a+b)/2w\). Further discussion on this subject is given in a later section along with numerical results.

2.2 Effective Permittivity and Permeability Approximations

The mode matching technique described above will yield accurate results for both the field descriptions and the propagation constants if enough terms are taken into account. This procedure is quite involved, and unless enough terms are used, the final results may be in doubt. For many practical engineering applications, where only sufficiently accurate values of \( k_z \) are necessary, the "effective" permittivity or permeability approximation or a combination of the two is quite useful. Even though they do not provide field information, reasonably accurate results for the propagation constants of many complex structures are obtained [4],[12],[15],[17]. The techniques
described here are actually a modification of a one-term approximation of the mode matching technique. They consider two types of modal fields, i.e., LSE (H-wave) and LSM (E-wave) modes, independently. The H-wave leads to the effective dielectric constant (or permittivity) method while the E-wave is associated with the effective permeability method [9]. For abbreviation, when we refer to both the effective permittivity and the effective permeability and their mean values, the effective parameter will be used. The general applications of these approximations can be found in all references cited above. However, it would be appropriate to reintroduce the fundamental concepts again to facilitate their applications. Consider an H-wave (LSE-to-z) obliquely incident upon an infinite interface between two dielectric media I and II with relative dielectric constants and permeabilities \( \varepsilon_1, \varepsilon_2, \mu_1 \) and \( \mu_2 \), respectively (see Fig. 4a). The reflection (and transmission) coefficient of this obliquely incident case can be obtained from that of an equivalent "effective" normally incident one (see Fig. 4b) if an appropriate "effective dielectric constant" can be found, i.e.,

\[
\frac{E_x}{E_1} = \frac{\left( \frac{\varepsilon_1^{*}}{\mu_1} \right)^{\frac{1}{2}} - \left( \frac{\varepsilon_2^{*}}{\mu_2} \right)^{\frac{1}{2}}}{\left( \frac{\varepsilon_1^{*}}{\mu_1} \right)^{\frac{1}{2}} + \left( \frac{\varepsilon_2^{*}}{\mu_2} \right)^{\frac{1}{2}}}
\]  

(23)

For an H-wave, it is apparent that the "effective" concept is valid if the effective dielectric constants in (23) are replaced by:
Figure 4. (a) TE obliquely incident wave.

(b) Equivalent normal incident wave using the effective dielectric constant.
\[ \varepsilon_i^* = \varepsilon_i - \frac{1}{\mu_i} \left( \frac{k_{y_1}}{k_o} \right)^2, \quad i = 1, 2 \] (24)

where \( k_o \) is the free-space wavenumber, and \( k_{y_1} \) is the phase constant in the \( y \)-direction, i.e., \( k_{y_1} = (\varepsilon_i \mu_i)^{1/2} k_o \sin \theta_i \).

For an E-wave (TM) obliquely incident from medium I into II (see Fig. 5a), the equivalent normal incident wave (see Fig. 5b) for the reflection (and transmission) coefficient is obtained if the relative permeability in each medium is replaced by

\[ \mu_i^* = \mu_i - \frac{1}{\varepsilon_i} \left( \frac{k_{y_1}}{k_o} \right)^2, \quad i = 1, 2 \] (25)

In short, the effective dielectric constant (permittivity) must be used for the E-wave and the effective permeability for the H-wave. Unlike the mode-matching technique, the effective parameter approach assumes that there is no coupling between the LSE- and LSM-modes. This, in effect, assumes that the geometrical discontinuities in a dielectric guide occur (alternatively) in one plane only.

This approximate technique will now be applied to the suspended H-waveguide. Consider an LSM (TM-to-y) mode propagating in a coupled structure as shown in Figure 6a. Since this is an LSM-mode, the effective permeability will be utilized. The discontinuities in the \( y \)-direction are first ignored, and the dielectric core region (II) is extended to infinity (see Fig. 6b) so that the characteristic equations for \( k_x \) can be obtained from this two-dimensional dielectric slab. The characteristic equations for \( k_x \) have already been derived.
Figure 5. (a) TM obliquely incident wave.
(b) Equivalent normal incident using the effective permeability.
Figure 6. (a) Cross-sectional structure adopted for the effective parameter approximations.

(b), (c) Models for calculating $k_x$ and $k_y$, respectively.
in Eqs. (8) and (9). Once $k_x$ is obtained, the "effective" permeability is defined according to (25). (Note that the coordinate system in Fig. 6 is different from that as shown in Figs. 4 and 5.) In this case, $\mu_2^*$ is given by

$$\mu_2^*(x) = \mu_0 - \frac{1}{\epsilon_2} \left( \frac{k_x^{(2)}}{k_o^{(2)}} \right)^2$$

(26)

The dielectric core region is now replaced by an infinite slab with an effective permeability $\mu_2^*$ (see Fig. 6c). If tangentials $H_x$ and $E_z$ are matched at the planes $y = \pm h$, the characteristic equation for $k_y$ is obtained:

$$\cos \left( 2k_y^{(2)} h \right) \left\{ \frac{k_y^{(2)}}{\epsilon_1 k_y^{(1)}} \cos \left[ k_y^{(1)} (h-d) \right] \sin \left[ k_y^{(3)} (-h+1) \right] \right\}$$

$$- \frac{k_y^{(2)}}{\epsilon_3 k_y^{(3)}} \sin \left[ k_y^{(1)} (h-d) \right] \cos \left[ k_y^{(3)} (-h+1) \right]$$

$$+ \sin \left( 2k_y^{(2)} h \right) \left\{ \epsilon_2 \sin \left[ k_y^{(1)} (h-d) \right] \sin \left[ k_y^{(3)} (-h+1) \right] \right\}$$

$$+ \frac{\epsilon_1 \epsilon_3}{\epsilon_2} \frac{k_y^{(2)^2}}{k_y^{(1)} k_y^{(3)}} \cos \left[ k_y^{(1)} (h-d) \right] \cos \left[ k_y^{(3)} (-h+1) \right] \right\} = 0$$

(27)

where the transverse propagation constants $k_y^{(i)}$, $i = 1, 2, 3$, are related by

$$k_z^2 = \epsilon_1 k_o^2 + k_y^{(1)^2} = \epsilon_2 \mu_2^*(x) k_o^2 + k_y^{(2)^2} = \epsilon_3 k_o^2 + k_y^{(3)^2}$$

(28)
In Eq. (27), \( k_y^{(1)} \) can be solved numerically. \( k_z \) and, hence, the dispersion characteristics of the original \( H \)-waveguide can then be calculated according to (23).

For LSE\(^X\) (TE-to-x) mode, the process is reversed. Using the effective permittivity approach, the procedure for obtaining the longitudinal propagation constant \( k_z \) is described as follows:

\[
\text{Fig. 6C} \xrightarrow{\text{Eq. (27)}} k_y^{(1)} \xrightarrow{\varepsilon_2^*(y)} \text{Fig. 6b} \xrightarrow{\text{Eqs. (8) & (9)}} k_x^{(1)}
\]

and the following relation:

\[
k_z^2 = \varepsilon_2^*(y) k_o^2 - k_x^{(2)} = k_o^2 - k_x^{(5)}
\]

(29)

The LSE\(^Y\) and LSM\(^X\) modes can be obtained in the same manner by considering

\[
\text{LSE}\(^Y\) : k_x \xrightarrow{\varepsilon_2^*(x)} k_y, k_z
\]

\[
\text{LSM}\(^X\) : k_y \xrightarrow{\mu_2^*(y)} k_x, k_z
\]

(30)

The characteristic equations for \( k_y \) of these two modes are

\[
\cos\left(2k_y^{(2)}h\right)\left\{\begin{array}{l}
k_y^{(2)}k_y^{(1)} \sin\left(k_y^{(1)}(h-d)\right) \cos\left(k_y^{(3)}(-h+1)\right) \\
-k_y^{(3)} \cos\left(k_y^{(1)}(h-d)\right) \sin\left(k_y^{(3)}(-h+1)\right) \\
- \sin\left(2k_y^{(2)}h\right) \cos\left(k_y^{(1)}(h-d)\right) \cos\left(k_y^{(3)}(-h+1)\right) \\
+ \frac{k_y^{(2)^2}}{k_y^{(1)}k_y^{(3)}} \sin\left(k_y^{(1)}(-h+d)\right) \sin\left(k_y^{(3)}(-h+1)\right) \end{array}\right\} = 0
\]

(31)
In summary, the effective parameter approximations are based on three basic assumptions:

1) Only one surface-wave type (either LSE or LSM) can propagate.

2) Each region in the guide can alternatively be treated as a two-dimensional problem, i.e., there is no geometrical discontinuity on one side while investigating the other.

3) $\varepsilon_2 > \varepsilon_1, \varepsilon_3$

The characteristic equations for the propagation constants derived here can be applied to the other guiding structures shown in Figure 3 simply by changing the separation of certain side-wall locations and the relative dielectric constants of certain regions. Usually, the actual value of $k_z$ (using mode-matching technique) lies somewhere between those obtained by the effective permeability and permittivity approximations. An average, e.g., geometrical mean, of these approximate values often produces a more accurate result. Numerical values obtained by the effective parameter approximations and the more rigorous mode matching technique are presented in the next section.
3. NUMERICAL AND EXPERIMENTAL RESULTS

In this chapter, the numerical computations of the propagation constant and field distribution of different waveguide structures are discussed. All calculations were done using the Vax 11/780 computer. Some experimental results are also presented.

3.1 Normalized Propagation Constant of Single and Coupled Suspended H-Waveguides

In an open dielectric waveguide structure, the range of the normalized propagation constant $k_z/k_0$ is between 1 and $\sqrt{\varepsilon_2}$. Any mode with a normalized propagation less than one is a leaky mode. This is the cut-off condition of any normal mode in an open dielectric structure. This phenomenon is not applicable to a partially dielectric-filled shielded structure whose propagation starts at $k_z/k_0$ equal to zero. However, unlike the hollow metal waveguide, the guided modes in a partially dielectric-filled metal structure are neither pure TE nor TM. Therefore, another mode description must be used. In this thesis, the field modes are described as $E_{mn}^y$ if the major transverse electric field is parallel to the y-axis and as $E_{mn}^x$ if the major transverse electric field is parallel to the x-axis. The m and n subscripts are the number of maxima of the dominant electric field in the x- and y-direction, respectively. Due to the high dielectric constant of the core and the large size of the metal shield, the suspended H-waveguide can be considered as an oversized waveguide that can theoretically support few higher-order modes at the upper millimeter-wave frequency range. Unlike the fundamental mode, where a large percentage of the
energy propagating in the guide is concentrated in the dielectric core, and decays exponentially away from the core, the higher-order modes are loosely bounded within the dielectric core causing most of the energy to be concentrated outside the core. Unless over-moded operation is desired, most of these higher-order modes can be eliminated by proper launching into and receiving from the guide. A thin absorbing film can also be placed at the inner side walls to damp out the modes with lateral expansion far beyond the dielectric core that are not permitted for normal propagation [5]. Because of the large separation between the core and the side walls, these absorbing films should not have any effect on the propagation of the fundamental mode. As frequency increases, many of these higher-order modes become tightly bound to the dielectric core as in the case of an open dielectric waveguide structure. For practical means, however, only the fundamental modes of two different polarizations will be investigated in great detail in this thesis. Figure 7 shows the influence of the side-wall location on the propagation constants of the dominant $E_{11}^y$ and $E_{11}^x$ modes in a single suspended H-waveguide. Unless otherwise indicated, in this and many following figures, the top and bottom dielectric layers are the same, i.e., $\varepsilon_1 = \varepsilon_3$, $l = d$, and $a = b$. In Figure 7, $k_z/k_o$ remains virtually unchanged if the ratio $b/w$ is larger than 4.5. This suggests that the side walls have little or no effect on the propagation characteristics of the fundamental modes if the side-wall separation is large enough. Also shown is the effect of the total eigenmodes (equal number of LSM and LSE modes are used) on the convergence
Figure 7. Normalized propagation constants vs. b/w for different numbers of eigenmodes. (a) $E_{11}^\gamma$ mode. (b) $E_{11}^\chi$ mode. $w/h = 1.14$, $h/d = 0.55$, $\varepsilon_2 = 9.8$, $\varepsilon_1 = \varepsilon_3 = 2.3$, $d/\lambda_o = 0.25$. 

---

Stream 1

-3

This is a sample text.

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Stream 2

This is another sample text.

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Stream 3

This is a third sample text.

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Stream 4

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of the propagation constant. For \( N = 6 \) and \( N = 10 \), the results are
almost indistinguishable. A somewhat similar conclusion was also
obtained in investigations of an open dielectric image line by other
authors [11],[13].

In Figs. 8 and 9, the phase characteristics are plotted as a
function of \( h/d \), which is the ratio of the dielectric core height to
the total height of the suspended H-guides for different dielectric
constants of the core and the normalized guide height. The values of
\( k_z/k_o \) computed by the mode-matching technique are shown in solid lines
whereas those calculated using the mean value of the effective parameter
approximations are shown in dotted lines. Unless otherwise indicated,
these line designations are used for the rest of this section. For
the fundamental \( E_{11}^x \) mode, both techniques agree well except in the
range where \( h/d \) is small. In most cases, the values of \( k_z/k_o \) of the
\( E_{11}^x \) mode obtained by the effective parameter method are less accurate. For
the latter case, it was found that either the effective permittivity
or the effective permeability approximation alone usually provides a
better result than their mean value. For \( h/d \) less than 0.05, the
field extends far beyond the dielectric core. To obtain accurate
results using the mode-matching approach, the side-wall separation
must be large enough, and many more eigenmodes must be used. The
computation and the convergence, therefore, are very slow. For this
reason, the values of \( k_z/k_o \) for small \( h/d \) are not presented. In this
case, other methods, such as perturbation techniques [20], may be
numerically more efficient.
Figure 8. Normalized propagation constants of $E_{II}^Y$ and $E_{II}^X$ modes of a single suspended H-waveguide vs. $h/d$. $\varepsilon_1 = \varepsilon_3 = 2.3$, $w/\lambda_0 = 0.15$, $l = d$, $a = b$ ($\infty$). Mode matching: solid line; effective parameter: dashed line. (a) $\varepsilon_2 = 9.8$ (b) $\varepsilon_2 = 3.75$. 
Figure 9. Normalized propagation constants of a single suspended H-waveguide vs. h/d. \(\varepsilon_1 = \varepsilon_3 = 2.3, l = d, a = b (\infty)\). Mode-matching: solid line; effective parameter: dashed line. (a) \(2w/\lambda_0 = 0.165\). (b) \(2w/\lambda_0 = 0.2255\).
The effect of the aspect ratio w/h of the dielectric core on the propagation characteristics of an uncoupled suspended H-waveguide is shown in Figs. 10 and 11 for different values of $\varepsilon_2$ and the normalized guide height. In these figures, the ratio of h/d is arbitrarily chosen to be 0.5. For a higher dielectric constant of the core, a crossover occurs for the propagation constants of the $E_{11}^y$ and $E_{11}^x$ modes. As w/h increases, $k_z/k_0$ derived by the effective parameter approximation approaches the mode matching values. This is expected, since for large values of w/h, the waveguide structure closely resembles a dielectric-slab loaded parallel guide whose LSE and LSM modes do not couple. The complete separation of LSE- and LSM-modes is one of the basic assumptions in deriving the effective parameter technique and, therefore, the results obtained by this technique for large w/h should be closer to the actual values. The reverse is true when w/h becomes smaller. When w/h and $\varepsilon_2$ are small, less energy is concentrated in the dielectric core, and the omission of the "corner" regions in deriving the effective parameter technique is no longer valid. For the $E_{11}^y$ mode, the normalized propagation constants derived by both the effective parameter and the mode matching methods agree well. For the $E_{11}^x$ mode, particularly for the lower dielectric constant case and near cut-off, the effective parameter approximation becomes less reliable.

Figures 12 and 13 are the plots of the normalized propagation constant of a single suspended H-waveguide as a function of the ratio $\varepsilon_2/\varepsilon_1$ for different normalized guide heights. The top and bottom
Figure 10. Normalized propagation constants of a single suspended H-waveguide vs. w/h. $\varepsilon_1 = \varepsilon_3 = 2.3$, $l = d$, $a = b$ ($\omega$).

Mode matching: solid line; effective parameter: dashed-line. (a) $\varepsilon_2 = 9.8$; (b) $\varepsilon_2 = 3.75$. 
Figure 11. Normalized propagation constants of a single suspended H-waveguide vs. w/h. $\varepsilon_1 = \varepsilon_3 = 2.3$, $l = d$, $a = b$ (→). Mode-matching: solid line; effective parameter: dashed line. (a) $\varepsilon_2 = 9.8$; (b) $\varepsilon_2 = 3.75$. 
Figure 12. Normalized propagation constants of the fundamental $E_{ll}^y$ and $E_{ll}^x$ modes of a single suspended H-waveguide vs. $\varepsilon_2/\varepsilon_1$, $l = d$, $a = b$ (−∞). Mode-matching: solid line; effective parameter: dashed line.
Figure 13. Normalized propagation constants of the fundamental $E_{yy}$ and $E_{xx}$ modes of a single suspended H-waveguide vs. $\varepsilon_2/\varepsilon_1 \cdot \varepsilon_1 = \varepsilon_3$, $l = d$, $a = b$ ($\rightarrow$). Mode matching: solid line; effective parameter: dashed line.
dielectric layers are identical in this case, i.e., \( \varepsilon_1 = \varepsilon_3 \) and \( l = d \). The frequency in Figure 12 is approximately 1.4 times higher than that in Figure 13. For large \( \varepsilon_2/\varepsilon_1 \), most electromagnetic energy is tightly bound to the center of the dielectric core. The configuration of the core becomes less important since the field strength at the dielectric boundaries is small. For this reason, the propagation constants of the \( E_{11}^y \) and \( E_{11}^x \) modes converge when \( \varepsilon_2 \) becomes large, as shown in Figure 12. In this range, many higher-order modes also exist in the guide. A somewhat similar behavior of the normalized constant is obtained by increasing frequency. Figures 14 and 15 show the dispersion characteristics of the dominant \( E_{11}^y \) and \( E_{11}^x \) modes as a function of the normalized free-space wavenumber \( k_d \). In Figure 15, only the results obtained from the effective permittivity approximation are presented as the dotted lines since the effective permeability method inaccurately predicts a high cut-off of the dominant \( E_{11}^y \) mode. Experimental results are also presented in these figures. The agreement between theoretical and experimental results of the \( E_{11}^y \) mode is very good. A combination of a sliding short along the dielectric core and a reverse coupler at the input of the suspended H-waveguide has been used to record the standing wave pattern in the guide [21]. Energy was launched from a metal waveguide; the dielectric core is made of alumina (\( \varepsilon_r = 9.8 \)) and stycaft (\( \varepsilon_r = 3.75 \)). The insulating top and bottom dielectric layers are copper clad RT/Duroid with an \( \varepsilon_r = 2.3 \). The tapered transition from the suspended H-waveguide to the metal waveguide is shown in Figure 16. All measurements were done in the E-band.
Figure 14. Normalized propagation constants of a single suspended H-waveguide vs. the normalized free-space wavenumber. $\varepsilon_1 = \varepsilon_3$, $l = d$, $a = b$ ($\rightarrow\infty$). Mode matching: solid line; effective parameter: dashed line.
Figure 15. Normalized propagation constants of a single suspended H-waveguide vs. the normalized free-space wavenumber. $\epsilon_1 = \epsilon_3$, $l = d$, $a = b (\rightarrow \infty)$. Mode matching: solid line; effective parameter: dashed line.
Figure 16. Dielectric tapered transition from the suspended H-waveguide to the metal waveguide.
Thus far, only a single guide, which is symmetric both in the x- and y-directions, has been studied. Figure 17 shows the normalized propagation constant of the fundamental $E_{11}^Y$ and $E_{11}^X$ modes of a single suspended H-waveguide whose top and bottom dielectric layers are not identical. In this case, the thickness of the top layer is varied while that of the lower dielectric layer remains constant. $k_x/k_0$ is plotted as a function of $h/d$ for different dielectric constants $\varepsilon_1$ ($\varepsilon_1 = 2.3$ in Fig. 17a and $\varepsilon_1 = 3.75$ in Fig. 17b). For small values of $h/d$ (large separation), the top metal ground plane has virtually no influence on the propagation characteristics of the guide. In these figures, the effective parameter approximation accurately predicts the behavior of the normalized propagation constants and can always be used to check the existence of a particular mode before using the mode-matching method.

The dispersion characteristics of a couple suspended H-waveguide as a function of the normalized free-space wavenumber $k_0 d$ are shown in Figs. 18 and 19 for different dielectric constants $\varepsilon_2$ of the core. As frequency increases, the field strength outside each dielectric core becomes smaller, and the propagation constants of even and odd modes approach the value of an uncoupled structure as the influence of one guide on the other becomes negligible. In these figures, subscripts e and o imply a PMC wall and a PEC wall at $x = -a$, respectively. For example, $E_{11e}^Y$ represents the even $E_{11}^Y$ mode (PMC wall at $x = -a$) while $E_{11o}^Y$ represents the odd $E_{11}^Y$ mode (PEC wall at $x = -a$). The same subscript designation applies to the $E_{11}^X$ mode. In Fig. 18, where the
Figure 17. Normalized propagation constants of the fundamental $E_{11}^y$ and $E_{11}^x$ modes of a single suspended H-waveguide vs. $h/d$. (a) $1/\lambda_o = 2.166$. (b) $1/\lambda_o = 0.2$ Mode matching: solid line; effective parameter: dashed line.
Figure 18. Normalized propagation constants of even and odd $E_{11}^y$ and $E_{11}^x$ modes of a coupled suspended H-waveguide vs. the normalized free-space wavenumber. $l = d$, $b/w = \infty$. 
Figure 19. Normalized propagation constants of even and odd $E^y_{11}$ and $E^x_{11}$ modes of a coupled suspended H-waveguide vs. the normalized free-space wavenumber. $l = d$, $b/w \to \infty$. 
dielectric constant of the core is much higher than that in the surrounding medium, the normalized propagation constants of even and odd modes of both $E_{11}^y$ and $E_{11}^x$ modes converge rapidly as frequency increases. For a lower dielectric constant of the core (see Fig. 19), the fields are weakly bound and, therefore, all modes converge at much higher frequencies. $k_z/k_0$ obtained by the effective parameter approximation always approaches the mode matching's values at higher frequencies. However, in the normalized frequency range investigated here, the difference between the two techniques is substantial. Hence, for clarity, only the dispersion characteristics derived by the mode matching method are presented.

3.2 Normalized Propagation Constant of Other Dielectric Waveguide Structures

It was mentioned previously that the method of analysis for the suspended H-waveguide could also be used for other planar dielectric structures of rectangular cross-section. Information in Figs. 7 and 15 shows that the analysis of a totally shielded structure can be used for an open dielectric structure if the wall separation is large enough. Therefore, by using all or part of Eq. (21), the propagation characteristics of many open dielectric waveguide structures can be obtained. The dispersion curves of some lowest-order modes of an image guide (see Figure 3b) are shown in Figure 20. The range of $k_z/k_0$ is between 1 and $\sqrt{\varepsilon_r}$ since, for an open structure, the propagation constant of any surface wave mode must be greater than the free-space wavenumber and less than the intrinsic phase constant of the dielectric. The
Figure 20. Normalized propagation constants of the fundamental $E_{11}^Y$ and $E_{11}^X$ modes of an image guide vs. the normalized free-space wavenumber. $b/w \to \infty$, $d/h \to \infty$. Mode matching: solid line; effective parameter: dashed line.
theoretical results of both mode matching and of the effective parameter approximation are compared with the experimental results which were published by Solbach et al. [11]. The agreement between theoretical and experimental results is excellent throughout. The propagation constant of the image line derived from the mode-matching method is obtained by recording the zeros of the determinant of (22). The field mode descriptions of an image-guide are the same as those in a suspended H-waveguide. The image-guide has found many applications in millimeter-wave integrated circuits, such as resonators [22], leaky-wave antennas [23], isolators [24], and phase shifter [25].

The partially loaded trough guides (see Fig. 3d) find applications in dielectric leaky-wave antennas [26], and, in many cases, can be used to reduce the radiation loss at a bend in an image-guide transmission line [6],[27]. If the dielectric constant of the dielectric is high enough, backward propagating waves may exist in the structure which can be used for reverse coupling applications. Since this waveguide structure has been extensively studied elsewhere [10], only a typical dispersion curve is presented here (see Fig. 21). The dispersion characteristics of the \( E_{11}^X \) mode in the lower frequency range of this guide are similar to the characteristics of empty parallel metal plates whereas the \( E_{11}^Y \) mode has a high cut-off. The dotted lines represent effective dielectric constant curves and the solid lines are the mode-matching values.

Another important waveguide structure is the rib dielectric waveguide. This waveguide is very similar to an insular guide (see Fig. 3c)
Figure 21. Dispersion curves of the fundamental $E_{11}^y$ and $E_{11}^x$ modes of a partially loaded trough waveguide. $d/h = \infty$. Mode-matching: solid line; effective parameter: dashed line.
except that the dielectric guiding layer and the substrate are made from the same material. Most of the energy concentrates in the dielectric substrate layer. The strip merely acts as a guide to control the lateral expansion of the field. To analyze this structure using the same equations derived for the suspended H-waveguide, the top layer of the suspended H-waveguide is replaced by free space. The top metal ground plane and the side walls must be placed far from the dielectric core so that they have a negligible influence on the propagation of the field modes in the dielectric core region. The effect of the side-wall location on the propagation characteristics has been demonstrated in Figs. 7 and 17. The dispersion characteristics of a single dielectric strip are shown in Fig. 22. Again, only the effective permittivity values are presented since the effective permeability wrongly exhibits a very high cut-off for the fundamental mode. Even the effective permittivity is not accurate in this case as the curves of the \( E_{11}^V \) mode in Fig. 22 show a substantial difference between the effective permittivity and the mode matching techniques. In deriving the effective parameter approximation, we have assumed that most energy is concentrated in the dielectric core. The "corner" regions in the substrate could, therefore, be ignored. This assumption is not valid in a rib dielectric waveguide since most of the energy is actually confined to the substrate region. Consequently, the values of \( k_z/k_0 \) derived by the effective parameter approximation do not represent the true dispersion characteristics of the field modes in a rib dielectric waveguide. If the dielectric constant of the core becomes higher,
Figure 22. Dispersion curves of the fundamental $E_{11}^y$ and $E_{11}^x$ modes of a rib dielectric waveguide. Mode-matching: solid line; effective parameter: dashed line.
as in an insular guide, the effective parameter approximation values will approach the mode matching values as indicated in Figure 17.

3.3 Field Distribution of a Suspended H-Waveguide and Other Dielectric Structures of Rectangular Cross-Section

It is well-known that the effective parameter approximation is adequate for calculations of the propagation constant. It does not, however, provide accurate information on the field distribution of the waveguide structure since the fields in certain regions are totally ignored. The mode matching technique can theoretically provide exact field distributions in the dielectric waveguide if an infinite number of eigenmodes are taken into account. Due to the limitation of computation facilities, an infinite number of these eigenmodes (or terms) must be approximated by a finite series. Hence, the numerical accuracy can be limited by computer storage and computation time. In general, when the waveguide is operated near cut-off, the fields extend far away from the dielectric core. To accurately represent a true open dielectric guide, such as an image-guide or an insular guide, or a partially open structure such as the suspended H-waveguide when using the mode-matching technique, the theoretical metal enclosure must be enlarged to minimize its effect on the propagation and field characteristics of the guide. Consequently, a greater number of eigenmodes must be used to maintain the same degree of accuracy. More terms are necessary since a larger number of slowly varying eigenfunctions in the outer regions are required to satisfy the boundary conditions
across the dielectric interfaces with the rapidly varying fields in the core region. On the other hand, when the dielectric waveguide core has a higher dielectric constant or when the guide is operated far beyond cut-off, the fields decay quickly away from the dielectric core, necessitating smaller wall separation when applying the mode-matching technique. In this case, fewer eigenmodes are necessary.

The effect of the total number of eigenmodes on the field characteristics of a totally open or partially open dielectric waveguide will be discussed in this section. Unless otherwise indicated, equal numbers of the LSM and the LSE eigenmodes are used. Since the field distribution of $H_x$ is very similar to $E_y$ and since $E_x$ is similar to $H_y$, only $H_x$ and $E_x$ are presented as the transverse fields. The plots of $H_x$, $E_x$, $H_z$ and $E_z$ of the $E_{11}^y$ mode versus $y/h$ in a single suspended $H$-waveguide for different $x$-locations in the guide are shown in Figures 23 to 26. For these figures, the modal expansion for the fields consists of ten modes. The top and the bottom dielectric layers are identical, i.e., $l = d$ and $\varepsilon_1 = \varepsilon_3$. All the electric field components in the guide are normalized to the amplitude of the dominant electric field and all the magnetic field components to the amplitude of the dominant magnetic field. By comparing the amplitude of each field component at different $y$-locations, one can visualize the lateral distribution of that particular field. For an $E_{11}^y$ mode, the $x = 0$ plane is equivalent to a pmc wall; therefore, $H_z$ and $E_x$ are equal to zero at this plane. Far away from the dielectric core, e.g., at $x = 2w$, more energy is concentrated in the top and the bottom
Figure 23. Normalized calculated field distribution of $H_x$ of the $E_{11}^y$ mode at different $x$-locations in a single suspended H-waveguide.
Figure 24. Normalized calculated field distribution of $E_x$ of the $E_{11}^y$ mode at different $x$-locations in a single suspended H-waveguide.
Figure 25. Normalized field distribution of $H_z$ of the $E_{11}^y$ mode at different $x$-locations in a single suspended $H$-waveguide.
Figure 26. Normalized field distribution of $E_z$ of the $E_{11}^y$ mode at different $x$-locations in a single suspended H-waveguide.
dielectric insulating layers as indicated by an increasing amplitude
of $H_x$ for $y > h$ as indicated in Figure 23.

Inspection of Figures 23 to 26 indicates that with ten eigenmodes,
the matching of the field components at the $x = t w$ interfaces is
excellent except for the normal electric field component. (For a
complete description of the waveguide parameters, see Figure 2.) Matching
across the $y = th$ interfaces is more difficult since $E_x$ is a continuous
function of $x$ at $y = h^+$, but a discontinuous function of $x$ at $y = h^-$. The matching of the dominant field component $H_x$, however, is excellent
throughout.

The lateral field distributions at different $y$-locations and the
tests of their convergences for different numbers of eigenmodes are
shown in Figures 27 and 28. The general shapes of the distributions
in the insulating layers, e.g., $y = 1.4 h$, and in the dielectric
core region, e.g., $y = 0.5 h$, are very similar. This is a clear
justification for the effective parameter technique in assuming the
dielectric core as an infinite slab in deriving the transverse
characteristic equation of $k_x$. The plots of the fields for $N = 7$ and
$N = 12$ are almost indistinguishable. Except at $y = th$, the matching
of the $H_z$ component at other locations is very good.

Figures 29 to 32 are plots of $H_x$, $H_z$, $E_x$ and $E_z$ at the $x = 0.5w$
plane in a single suspended H-waveguide whose top and bottom dielectric
layers are not identical. The top layer has a dielectric constant
$\varepsilon_1 = 3.75$ while the lower layer has $\varepsilon_2 = 2.3$. The field components
in this structure are no longer symmetric in the $y$-direction. Since
Figure 27. Normalized magnetic field distributions of the $E^{y}_{11}$ mode in the transverse direction at $y = 0.5 \ h$ in a single suspended H-waveguide.
Figure 28. Normalized magnetic field distributions of the $E_{11}^y$ mode in the transverse direction at $y = 0.5 \ h$ in a single suspended H-waveguide.
Figure 29. Normalized field distribution of $H_x$ of the $E_{11}^v$ mode in the vertical direction at $x = 0.5w$ in a single suspended H-waveguide whose top and bottom dielectric layers are dissimilar.
Figure 30. Normalized field distribution of $H_z$ of the $E_{11}^V$ mode in the vertical direction at $x = 0.5w$ in a single suspended H-waveguide whose top and bottom dielectric layers are dissimilar.
Figure 31. Normalized field distribution of $E_x$ of the $E_{11}$ mode in the vertical direction at $x = 0.5w$ in a single suspended H-waveguide whose top and bottom dielectric layers are dissimilar.
Figure 32. Normalized field distribution of $E_z$ of the $E_{11}^y$ mode in the vertical direction at $x = 0.5w$ in a single suspended H-waveguide whose top and bottom dielectric layers are dissimilar.
more energy is concentrated in the regions with higher dielectric constants, the field maxima are shifted slightly toward the top dielectric layer as clearly shown in Figures 29 and 30. As mentioned earlier, $E_y$ and $H_y$ are not presented since their behaviors closely resemble $H_x$ and $E_x$, respectively. Except for the minor field component $E_x$, all field components match very well at the $y = t$ planes for $N = 10$. Typical field distributions in the horizontal direction are shown in Figures 33 and 34. It is interesting to note that, compared to Figure 27, the fields in Figure 33 decay much faster away from the dielectric core since the dielectric constant of the core in the latter case is much higher, and therefore, more energy is concentrated in the core region. The plot of $E_x$ at $y = 0.5h$ clearly shows that more than four eigenmodes are necessary to satisfy the boundary conditions at the $x = \pm w$ interfaces. However, when a few more terms are added, i.e., $N = 10$, the matching of this normal electric field component is very good.

The field distributions of a coupled suspended H-waveguide are similar to those for other coupled dielectric waveguide structures such as the insular guides or the inverted strips. Because of the symmetry of the waveguide structure at the $x = -a$ plane (see Figure 2), the fields are either symmetric or antisymmetric about that plane. For the symmetric mode (or even mode), the vertical plane at $x = -a$ is replaced by an equivalent perfect magnetic conducting (PMC) wall, whereas for the case of the antisymmetric mode (or odd mode), the vertical plane is replaced by an equivalent perfect electric conducting
Figure 33. Normalized magnetic field distribution of the $E_{11}^y$ mode in the horizontal direction at $y = 0.5h$ in a single suspended $H$-waveguide whose dielectric insulating layers are dissimilar.
Figure 34. Normalized field distribution of $E_x$ of the $E_{11}^y$ mode in the horizontal direction at $y = 0.5h$ in a single suspended H-waveguide whose dielectric insulating layers are dissimilar.
(pec) wall. The analysis of the total waveguide structure is, therefore, reduced to that shown in Figure 2b. Since the general behavior of the fields in a coupled structure is well-known, only the symmetric mode \( E_{11e}^y \) is presented here. The dispersion characteristics of a coupled suspended \( H \)-waveguide have been shown in Figures 18 and 19. Figures 35 to 37 are the normalized distributions of four symbolic field components plotted at the \( y = 0.5h \) plane. The field strength of the major magnetic component \( H_x \) (or \( E_y \)) is substantial at the pmc wall as the maxima of the fields are shifted toward the center. As many as ten eigenmodes are necessary to satisfy the boundary condition of the normal electric field component \( E_x \) at the \( x = tw \) interfaces. However, the matching process for \( H_x \) and \( H_z \) converges more quickly.

The transverse field distributions of a single insular guide (see Figure 3c), which is an open dielectric waveguide structure, are plotted for different vertical planes in Figures 38 and 39. In computing the propagation constants and the fields of an insular guide, the metal side walls must be placed far enough from the dielectric core so that their influence on the propagation constant and the field characteristics is negligible. The magnetic field plots in the horizontal direction at \( y = 0.5h \) are shown in Figure 40. The matching of these components is very good even for \( N = 3 \). However, to obtain an accurate field distribution eight or more eigenmodes must be used.

The investigation of the field distribution in many waveguide structures has been concentrated on the \( E_{11}^y \) mode only. The plots of
Figure 35. Normalized magnetic field distributions of the $E_{11e}^y$ mode in a coupled suspended H-waveguide at $y = 0.5h$. 

$\gamma = 0.5h$
$K_0d = 1.25$
$d/h = 2$
$w/h = 1.2$
$a/w = 1.7$

$N = 10 \quad \epsilon_2 = 3.75$
$N = 6 \quad \epsilon_1 = 2.3$
Figure 36. Normalized field distribution of $E_z$ of the $E_{11e}$ mode in a coupled suspended H-waveguide at $y = 0.5h$. 

- $d/h = 2$
- $w/h = 1.2$
- $a/w = 1.7$
- $\kappa_0 d = 1.25$
- $y = 0.5h$
- $\varepsilon_2 = 3.75$
- $\varepsilon_1 = 2.3$

- $N = 10$
- $N = 6$
Figure 37. Normalized field distribution of $E_x$ of the $E_{1le}$ mode in a coupled suspended H-waveguide at $y = 0.5h$. 

- $N=10$
- $N=6$
- $d/h=2$
- $w/h=1.2$
- $a/w=1.7$
- $k_0d=1.25$
- $\gamma=0.5h$
- $\varepsilon_2=3.75$
- $\varepsilon_1=2.3$
Figure 38. Normalized field distribution of $E_x$ of the $E_{11}^y$ mode in the vertical direction for different $x$-locations in a single insular guide.
Figure 39. Normalized field distribution of $H_x$ of the $E_{11}$ mode in the vertical direction for different $x$-locations in a single insular guide.
Figure 40. Magnetic field distributions of the $E_y$ mode in a single insular guide in the horizontal direction at $y = 0.5h$. 
E_x, H_x, E_z, and H_z, respectively, of the E_x^X_11 mode, which has E_x as the major transverse electric field component, in a single suspended H-waveguide are shown in Figures 41 to 44. E_y and H_y are similar to H_x and E_x in this guide and, therefore, are not plotted. Unlike the E_y^Y_11 mode, the longitudinal fields E_z and H_z of the E_x^X_11 mode are very substantial and are comparable to the corresponding major transverse fields (see Figure 45 to 46). At the top and bottom metal ground planes (y = d, -l), the tangential H_z is still very significant. In fact, at these locations, H_z becomes the dominant magnetic field component as shown in Figure 47, which is the plot of the normalized magnetic field distributions at y = d. For this reason, the metal loss for the E_x^X_11 mode is also significant and is primarily due to this magnetic field component. The matching at x = w is very good since, except for the normal electric field component, the fields at x = w^- and x = w^+ always coincide. As in the E_y^Y_11 mode, the matching at the horizontal interfaces y = th is more difficult. Figures 45 and 46 are the plots of the normalized electric and magnetic field components in the horizontal direction at y = 0.5h. The matching at the y = 0.5h plane for N = 8 is excellent for all components.

3.4 Conclusions

The propagation and field characteristics for two fundamental modes that have polarizations orthogonal to each other in many dielectric waveguide structures have been studied. The method of analysis derived for the suspended H-waveguide is general and can be
Figure 41. Normalized field distribution of $E_x$ of the $E_{11}^y$ mode in a single suspended $H$-waveguide. $\varepsilon_1 = \varepsilon_3$. 

- $\varepsilon_2 = 9.8$
- $\varepsilon_1 = 2.3$
- $d/h = 2$
- $w/h = 1.2$
- $l = d$
- $k_0d = 1.152$
- $N = 8$
Figure 42. Normalized field distribution of $H_x$ of the $E_{11}^x$ mode in a single suspended H-waveguide $\varepsilon_1 = \varepsilon_3$. 

$\varepsilon_2 = 9.8$
$\varepsilon_1 = 2.3$
d/h = 2
w/h = 1.2
l = d
$k_0d = 1.152$
N = 8
Figure 43. Normalized field distribution of $E_z$ of the $E_{11}^x$ mode in a single suspended H-waveguide. $\varepsilon_1 = \varepsilon_3$. 

$\varepsilon_2 = 9.8$ 
$\varepsilon_1 = 2.3$ 
d/h = 2 
w/h = 1.2 
l = d 
k_0 d = 1.152 
N = 8
Figure 44. Normalized field distribution of $H_z$ of $E_{11}^x$ mode in a single suspended H-waveguide. $\varepsilon_1 = \varepsilon_3$. 
Figure 45. Normalized electric field components of the $E_{11}^x$ mode in the horizontal direction at $y = 0.5h$. 

$\varepsilon_1 = \varepsilon_3$. 
Figure 46. Normalized magnetic field components of the \( E_{11} \) mode in the horizontal direction at \( y = 0.5h \).

\[ \varepsilon_1 = \varepsilon_3. \]
Figure 47. Normalized magnetic field distributions of the $E_{11}^{x}$ mode at the top metal plane in a single suspended H-waveguide. $\varepsilon_{1} = \varepsilon_{3}$. 

$\varepsilon_{2} = 9.8$
$\varepsilon_{1} = 2.3$
$d/h = 2$
$w/h = 1.2$
$k_{0}d = 1.152$
$l = d$
$N = 8$
$y = d$
applied directly to other single and coupled planar structures of rectangular cross section. Even though the effective parameter approximations do not provide very accurate results for the propagation constants, in some cases they provide very useful information on the whereabouts and the existences of certain modes which, in turn, can be used as the initial values for iterating in the mode matching technique. By doing so, the computation time that could be wasted on a random search by the mode-matching technique can be eliminated. For a high dielectric constant or at high normalized frequency, both techniques agree well. The theoretical and experimental results for many waveguide structures agree well throughout.
4. LOSS CHARACTERISTICS OF A SUSPENDED H-WAVEGUIDE

The major sources of loss in most dielectric-based waveguide structures for millimeter-wave applications operating at frequencies beyond cutoff are the dielectric loss, the metal loss of conducting walls and the radiation loss due to bending. If the radius of curvature is sufficiently large, a general technique for determining the radiation loss has been presented by several authors [28],[29] and, will not be discussed here. The dielectric and metal losses of many dielectric-based waveguide structures, such as the H-guide or the image guide, have been previously investigated [22],[30]-[32]. Therefore, only dielectric and metal loss characteristics of a suspended H-waveguide will be studied in this section.

4.1 Metal Loss

In general, the metal loss is proportional to the square root of the operating frequency and the intensities of the tangential magnetic fields at the metal surfaces. A conventional image guide that supports the $E_{pq}^V$ modes has the dominant magnetic field $H_x$ parallel to the ground plane. Metal losses are therefore dominant as there is no separation between the dielectric core and the metal ground plane. Recently, this metal loss has been reduced by adding a thin dielectric layer of lower dielectric constant between the dielectric core and the metal ground plane [4],[7], since the fields decay rapidly in the
dielectric layer. The proposed suspended H-waveguide utilizes
the same principle to minimize the metal loss. Two low-loss dielectric
layers are inserted between the dielectric core and the metal surfaces.
This principle also applies to the $E_{11}^{ \times }$ mode since the parallel magnetic
field components also decay rapidly in the insulating layers (see
Figures 42 and 44). The increase of dielectric loss due to the addition
of the insulating dielectric layers will be discussed later.

The attenuation in nepers per unit length can be calculated by
the conventional power loss method, i.e.,

$$\alpha_m = \frac{P_L}{2P_T}$$  \hspace{1cm} (32)

where $P_L$ is the metal loss per unit length and $P_T$ is the transmitted
power in the guide. $P_L$ and $P_T$ are given by

$$P_L = \frac{1}{2} R_s \int \left| H_{\text{tan}} \right|^2 \text{dl}$$ \hspace{1cm} (33)

$$P_T = \frac{1}{2} R_e \iint \left( E \times H^* \right) \cdot \hat{z} \text{da}$$ \hspace{1cm} (34)

where $\left| H_{\text{tan}} \right|^2 = \left| H_x \right|^2 + \left| H_y \right|^2 + \left| H_z \right|^2$ at the metal surfaces  \hspace{1cm} (35)

$$R_s = \left( \frac{\omega \mu_0}{2\sigma_m} \right)^{\frac{1}{2}} = \text{surface resistance, and}$$ \hspace{1cm} (36)

$\sigma_m$ is the surface conductivity of metal. Here, it is assumed that the
field configurations in the guide are not affected by the wall conduc-
tivity. The field components in all regions are given in Eqs. (1)-(6).
Figures 48 and 49 are the plots of the attenuation in decibels per meter of the $E_{11}^V$ mode in a single suspended H-waveguide whose top and bottom dielectric layers are identical. In Figure 48, the dielectric constant of the core is 9.8 whereas $\varepsilon_r = 3.75$ in Figure 49. It is evident from these figures that the metal loss is much lower when two dielectric insulating layers are inserted. As explained previously, the tangential magnetic fields decay rapidly toward the metal ground plane in the dielectric layers whose dielectric constants are lower than that of the core. The surface current on the metal plane is therefore small. In the absence of the dielectric layers, i.e., $h/d = 1$, the attenuation increases sharply. This is due to the fact that the dominant tangential magnetic field $H_x$ is constant in the vertical direction throughout the guide.

It is also important that the ratio $h/d$ should not be too small. For small $h/d$, e.g., $h/d = 0.2$, the fields are weakly bound. Consequently, the tangential magnetic fields at the metal planes are significant and higher metal losses occur. Except for $h/d = 1$, the attenuation curves for all other cases in Figure 48 decrease as frequency increases. This feature can be beneficial when the guide is operated at the millimeter-wave frequency range. In Figure 49, where $\varepsilon_r$ is much lower, the attenuation curves first increase then start to turn downward sharply as frequency increases. At lower frequencies, the fields are weakly bound, and the waveguide resembles an empty parallel plate waveguide. The attenuation is roughly proportional to the square root of the frequency. At higher frequencies (or higher dielectric constant of the
Figure 48. Attenuation of the $E_{11}^Y$ mode in decibels per meter due to the metal loss in a single suspended H-waveguide.

\[ \sigma_m = 6.17 \times 10^7. \]
Figure 49. Attenuation of the $E_{11}^y$ mode in decibels per meter due to the metal loss in a single suspended H-waveguide. $\sigma_m = 6.17 \times 10^7$. 

$\varepsilon_2 = 3.75$
$\varepsilon_1 = 2.3$
$w/d = 0.6$
$l = d$
$N = 8$
core), the fields become tightly bound to the dielectric core. The intensities of the tangential magnetic fields at the metal planes are small resulting in smaller metal loss as indicated in Figure 49.

The attenuation curves in decibels per unit length of the $E_{11}^X$ mode in a single suspended H-waveguide are plotted in Figure 50. With the addition of the two dielectric insulating layers, the metal loss is significantly reduced as clearly indicated by the h/d = 0.2, 0.4 and 0.6 curves. Near cutoff, the metal loss of the $E_{11}^X$ mode is very high but decreases sharply as frequency increases. In general, with the exception that the loss in the $E_{11}^Y$ mode is primarily due to the transverse magnetic field component $H_x$ whereas the loss in the $E_{11}^X$ mode is mostly due to the longitudinal field component $H_z$, the metal loss characteristics of the $E_{11}^Y$ mode and the $E_{11}^X$ mode are very similar.

4.2 Dielectric Loss

The introduction of the dielectric insulating layers in a suspended H-waveguide significantly reduces the metal loss due to finite conductivity of the walls. However, the addition of a large volume of lossy dielectric materials may cause a higher dielectric loss. Therefore, it is necessary to study the dielectric loss characteristics of the guide to obtain a complete overall loss figure for a new waveguide structure.

The power loss technique can also be used to calculate the dielectric loss. However, field computation derived from the mode-matching technique for the power loss approach is very time-consuming.
Figure 50. Attenuation of the $E_{11}^x$ mode in decibels per meter due to the metal loss in a single suspended H-waveguide. $\sigma = 6.17 \times 10^7$. 
By introducing a complex dielectric constant to allow for a small loss in the dielectrics, the attenuation constant $\alpha_d$ associated with lossy dielectrics can be obtained directly by a perturbation technique which is based on the solution of the lossless structure.

The characteristic equation for the propagation constant of a lossless suspended H-waveguide can be symbolically written as

$$D(\varepsilon_1, \varepsilon_2, \varepsilon_3, k_z, \omega) = 0$$  \hspace{1cm} (37)

where $D$ is the determinant of the system of linear and homogeneous equations shown in Eq. (21).

In order to calculate $\alpha_d$, the parameters in (37) are replaced by their complex counterparts. The characteristic equation for the complex propagation constant $\tilde{k}_z = k_z - j\alpha_d$ becomes

$$D(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \tilde{\varepsilon}_3, k_z - j\alpha_d, \omega) = 0$$  \hspace{1cm} (38)

where

$$\tilde{\varepsilon}_1 = \varepsilon_1(1 - j \tan \delta_1)$$

$$\tilde{\varepsilon}_2 = \varepsilon_2(1 - j \tan \delta_2)$$

$$\tilde{\varepsilon}_3 = \varepsilon_3(1 - j \tan \delta_3)$$  \hspace{1cm} (39)

$\tan \delta_i$, $i = 1, 2, 3$, is the dielectric loss tangent in the respective region.

Assuming that $\varepsilon_i \tan \delta_i$ and $\alpha_d$ are small, the left-hand side of (38) can be approximated by a Taylor's series in powers of $j\alpha_i \tan \delta_i$.
and \( j \alpha_d \) about the solution of a lossless structure [33]. Retaining only the first-ordered terms, Eq. (38) becomes

\[
D(\varepsilon_1, \varepsilon_2, \varepsilon_3, k_z, \omega) + j \sum_{i=1}^{3} \varepsilon_i \tan \delta_i \frac{\partial}{\partial \varepsilon_i} D(\varepsilon_1, \varepsilon_2, \varepsilon_3, k_z, \omega) \\
+ j \alpha_d \frac{3}{\partial k_z} D(\varepsilon_1, \varepsilon_2, \varepsilon_3, k_z, \omega) = 0
\]

(40)

The first term of (40) is zero by virtue of (38); consequently,

\[
\alpha_d = - \sum_{i=1}^{3} \frac{\varepsilon_i \tan \delta_i \frac{3D}{\partial \varepsilon_i}}{\frac{3D}{\partial k_z}}
\]

(41)

All the partial derivatives in (41) are evaluated numerically by the finite-difference technique [34] once the longitudinal propagation constant \( k_z \) of the lossless structure has been determined using the mode-matching method. In the calculations of the partial derivatives, it was observed that the solutions converged quickly after a few iterations then began to oscillate. The computations should be terminated before this oscillation occurs.

Figure 51 shows the attenuation constants due to lossy dielectrics of the \( E_{11}^y \) mode in a single, suspended H-waveguide as a function of the normalized free-space wavenumber. It is clear that the dielectric insulating layers cause higher dielectric losses. At lower frequencies, the fields are weakly bound to the low-loss dielectric core. More energy is stored in the substrate regions which have a much higher
Figure 51. Attenuation constant in decibels per meter of the $E_{11}^y$ mode in a single, suspended H-waveguide.
dielectric loss tangent and, therefore, higher dielectric loss. As frequency increases, most of the electromagnetic energy is stored in the core, which results in lower dielectric loss. Since the loss tangents of the substrates are much higher than the loss tangent of the dielectric core in this case, e.g., \( \tan \delta_1 / \tan \delta_2 = 6 \), most of the dielectric loss is due to the substrates. If a lower-loss copper-cladded dielectric sheet is available, the dielectric loss of the suspended H-waveguide should be much lower. However, for given waveguide parameters, the total (dielectric and metal) loss of the suspended H-waveguide (\( h/d = 0.6 \) and 0.8) is still lower than the total loss of a conventional H-guide (\( h/d = 1 \)) as Figures 48 and 51 indicated. For \( h/d = 0.2 \) and 1 in Figure 51, the dielectric loss increases monotonically as a function of frequency.

Figure 52 shows the attenuation constant due to lossy dielectrics in decibels per meter of the \( E_{11}^X \) mode in a single, suspended H-waveguide as a function of the normalized free-space wavenumber. Near cutoff, the dielectric loss is extremely high but decreases sharply as frequency increases. For \( h/d = 0.6 \) and 0.8, the dielectric loss curves of the \( E_{11}^X \) mode in a suspended H-waveguide are very close to the curve of a conventional H-guide (\( h/d = 1 \)). The combined total loss, which consists of the dielectric loss and the metal loss, is, therefore, much less if two dielectric layers are inserted since the metal loss of a suspended H-waveguide is very low as compared to that for a conventional H-waveguide. In Figures 50 and 52, the total loss of the \( E_{11}^X \) mode is lowest when the ratio \( h/d \) is about 0.6.
Figure 52. Attenuation constant in decibels per meter of the $E_{11}^x$ mode in a single, suspended H-waveguide.
In Figures 48 to 52, it appears that the dielectric losses dominate the total loss figure. However, in practice, the surface conductivity at the millimeter wavelength becomes lower [35], and the metal losses become higher. The impact of the suspended H-waveguide should be more profound since the control of the metal losses becomes very important at the millimeter wavelengths.
5. APPLICATIONS

The emergence of the dielectric-based millimeter-wave integrated circuits requires the development of a totally new class of dielectric components. In a conventional microwave circuit, discrete components are connected together by many types of connectors to form a complete system. At the millimeter-wave frequency, a circuit involving many discrete components is usually very expensive and cumbersome. Also, losses due to connectors are usually high.

A dielectric-based millimeter-wave integrated circuit, on the other hand, usually consists of a common ground plane or a common substrate on which the guiding structure and associated components are located. The use of connectors in the circuit is strongly dissuaded. A prime consideration from the structural viewpoint in a millimeter-wave circuit is the adaptability of the guiding structure for other circuit components. By appropriate manipulations of the basic guiding structure, many components can be constructed cheaply and efficiently. For the image guide, insular guide and inverted strip guide, many of those components have been introduced. Applications, such as dielectric resonators [36], directional couplers [37],[38], ferrite isolators [39],[40], phase shifters [41], and leaky-wave antennas [42],[48], have been published in the literature. Other components in which the basic guiding structure has been modified to perform a particular function are described in this section.
5.1 Cylindrical Dielectric Disc Resonators

Because of the unique arrangement of the dielectrics in a suspended H-waveguide, a cylindrical disc resonator can be constructed as described in Figure 53a. A circular dielectric disc of the same height as of the dielectric core is sandwiched between two dielectric insulating layers. Because of the metal enclosure, the dielectric disc is locked securely in place and no bonding material is needed. The dielectric constant of the disc can be different from that of the guiding dielectric core allowing the use of high-permittivity materials for high-Q applications.

The traditional techniques for analyzing the cylindrical dielectric resonators have been based on the open-circuit boundary (magnetic wall) model [44]. Simultaneous applications of the perfect and imperfect magnetic walls resulted in a better prediction of the resonant frequency [45]. Konishi et al. [46] introduced a more complicated method based on the variational procedure and predicted the resonant frequencies within a 1 percent accuracy of the experimental data. Itoh [47] applied the technique originally developed by Marcatili [18] for analyzing the propagation constants of a rectangular dielectric waveguide to accurately predict the resonant frequencies of the TE_015 mode.

The simple analytical technique described by Itoh is used here to analyze the resonant structure shown in Figure 53a. To conform to the waveguide parameters used in previous sections, the structure adopted for analysis with familiar dimensional parameters is depicted as Figure 53b. The dielectric constant of the disc is much higher
Figure 53. (a) Cylindrical dielectric disc resonator. (b) Structure adopted for analysis.
than that of the dielectric insulating layers. When \( \varepsilon_1 \) and \( \varepsilon_3 = 1 \) and the substrate thicknesses \( t \) and \( l \) approach infinity, the structure to be analyzed becomes the resonator in free-space in [44]-[46].

In a high Q resonator, most of the electromagnetic energy is stored in the disc (region 2) and decays exponentially in regions 1, 3 and 4. Much less energy is concentrated in the corner (or the shaded) regions. Consequently, only a very small error is introduced if the matching of the tangential fields is applied to the boundaries of region 2 only. The corner regions will be ignored entirely. Assuming no circumferential variation, the vector potential that has a single component in the y-direction can be written as

\[
\phi = \begin{cases}
A_1 \left[ \begin{array}{c}
\sin \left( k_y^{(1)} (y-d) \right) \\
\cos \left( k_y^{(1)} (y-d) \right)
\end{array} \right] J_0 \left( k_{\rho i} r \right) & \text{region 1} \\
A_2 \sin \left( k_y^{(2)} y \right) + A_3 \cos \left( k_y^{(3)} y \right) \left[ \begin{array}{c}
J_0 \left( k_{\rho i} r \right) \\
K_0 \left( k_{\rho o} r \right)
\end{array} \right] & \text{region 2} \\
A_4 \left[ \begin{array}{c}
\sin \left( k_y^{(3)} (y+1) \right) \\
\cos \left( k_y^{(3)} (y+1) \right)
\end{array} \right] J_0 \left( k_{\rho i} r \right) & \text{region 3}
\end{cases}
\]  

(42a) \hspace{1cm} (42b) \hspace{1cm} (42c) \hspace{1cm} (42d)

where

\[
\varepsilon_1 k_o^2 = k_y^{(1)^2} + k_{\rho i}^2
\]

\[
\varepsilon_2 k_o^2 = k_y^{(2)^2} + k_{\rho i}^2
\]

\[
\varepsilon_3 k_o^2 = k_y^{(3)^2} + k_{\rho i}^2
\]

\[
k_o^2 = k_y^{(2)} - k_{\rho o}^2
\]

(43)
\( \varepsilon_i, i = 1, 2, 3, \) are the relative dielectric constants of the respective regions: \( k_0 = \omega_0 \sqrt{\varepsilon_0 \mu_0} \) = free-space wavenumber; \( A_1, A_2, A_3 \) and \( A_4 \) are the unknown amplitude coefficients; and \( J_0 \) and \( k_0 \) are the Bessel and the modified Hankel function of the second kind of order zero, respectively. The upper line in the brackets in Eqs. (42a) and (42d) is for the TE\(^Y\) mode while the lower line is for the TM\(^Y\) mode. All field components can be derived from (42) [48]. With no circumferential variation, \( E_y = E_\rho = H_\phi = 0 \) for the TE mode, whereas \( H_y = H_\rho = E_\phi = 0 \) for the TM mode. By matching the tangential fields at \( r = a, \) \( -h \leq y \leq h \) and at \( y = \pi h, \) \( 0 \leq r \leq a, \) the following coupled characteristic equations are obtained.

**TE\(^Y\) mode:**

\[
\frac{k_\rho}{k_0} K_0(k_\rho a) J_1(k_\rho a) + k_i^2 J_1(k_\rho a) J_0(k_\rho a) = 0
\]

(44)

and

Equation (31)

**TM\(^Y\) mode:**

\[
\varepsilon_2 \frac{k_\rho}{k_0} K_1(k_\rho a) J_1(k_\rho a) + k_i^2 J_1(k_\rho a) J_0(k_\rho a) = 0
\]

(45)

and

Equation (27)

The resonant frequencies of each mode are obtained by solving the respective set of two characteristic equations simultaneously. The mode designation for the TE and the TM modes are TE\(_{\text{omn}}\) and TM\(_{\text{omn}}\), respectively. The first index is zero since there is no circumferential variation; the second index represents the order of the roots in
the radial direction; and the third index indicates the number of field maxima of the dominant electric field in the y-direction. For example, TE$_{011}$ is the dominant mode instead of TE$_{010}^0$ as described in Reference [46].

The resonant frequencies of the TE$_{011}$ mode plotted as a function of the geometrical dimensions are shown in Figure 54. Figure 55 is a special case when the substrate's thickness approaches infinity and $\varepsilon_1, \varepsilon_3 = 1$. The numerical results predicted by the present method agree well with the experimental results published by Konishi et al. [46]. By comparing the resonant frequency curves in Figures 54 and 55, it is obvious that the removal of the dielectric substrates and the ground planes only causes a small change in the resonant frequencies of the structure. Since the dielectric constant of the dielectric cylindrical disc is much higher than that of the substrate, most of the energy is concentrated in the disc. In the substrate regions, the fields decay extremely fast. Consequently, the removal of the dielectric substrates has a minimal effect on the resonant frequencies of the structure.

Figure 56 shows the curves of the resonant frequencies of the TM$_{011}$ mode in a cylindrical disc resonator as a function of the geometrical dimensions for two different values of the dielectric constants of the disc ($\varepsilon_2 = 35$ and 88).

5.2 Dielectric Ring Resonators

This section describes two novel dielectric ring resonators that can be efficiently integrated into a dielectric-based MMW-IC to provide
Figure 54. Resonant frequencies of the $\text{TE}_{011}$ mode.
Figure 55. Resonant frequencies of the $TE_{011}$ mode.
Figure 56. Resonant frequencies of the TM$_{011}$ mode.
filtering effects. They are particularly useful in a system that employs the suspended H-waveguide as the guiding medium. Figures 57 and 58 show the top views of these resonators and the equivalent signal flow-graph schematics. The shaded regions in Figures 57a and 58a are the waveguide media, such as the dielectric core of a suspended H-waveguide. The metal side walls at ports 3 and 4 are short-circuit terminations. The resonant effect is due to the standing wave created by these terminations. The signal flow graph analysis [49] will be applied to the scattering equations to predict the frequency response of these resonators. The following assumptions are made in our formulations: (a) all guides have identical characteristic impedances, (b) the directional couplers have infinite directivity, and (c) the resonators are perfectly matched.

With these assumptions, the frequency responses of the resonator structure shown in Figure 57 can be approximated by the flow graph analysis as follows:

\[ |S_{21}| = \left| \frac{t(1 - e^{-2\gamma z})}{1 - t^2 e^{-2\gamma z}} \right| \]  \hspace{1cm} (46)

and

\[ |S_{11}'| = \left| \frac{c^2 e^{-\gamma z}}{1 - t e^{-2\gamma z}} \right| \]  \hspace{1cm} (47)

where \( t, c \) = transmission and coupling coefficients of the directional coupler

\[ \gamma = \alpha + jk_z, \] complex propagation constant in the half-ring

\[ z = \text{mean length of the half-ring} \] (see Figure 57a).
Figure 57. (a) Dielectric ring resonator.  
(b) Equivalent signal flow graph schematic.
Figure 58. (a) Dielectric ring resonator.
(b) Equivalent signal flow graph schematic.
\( k_z \) is approximated by the propagation constant of an uncoupled straight guide. \( \alpha \), which is the attenuation constant in the ring, is comprised of the dielectric loss, metal loss and radiation loss. These losses were discussed in Chapter 4. For a symmetric coupler, the transmission and coupling coefficients are given as \([50],[51]\)

\[
t = \cos \phi \ e^{-j\phi/2}
\]

\[
c = j\sin \phi \ e^{-j\phi/2}
\]

where

\[
\phi = \left( \frac{k_{z, \text{even}} - k_{z, \text{odd}}}{2} \right) \lambda_c
\]

and \( \lambda_c \) is the effective coupling length. \( t \) and \( c \) can be calculated either by the mode-matching or the effective parameter techniques. For a nonsymmetric coupler, it is usually impossible to predict \( t \) and \( c \). In this case, an effective coupling factor must be experimentally determined before the frequency response can be calculated. The resonant conditions of this filter occur when \( |S_{21}| = 0 \). From Eq. (46), this corresponds to

\[
n = \frac{\lambda_g}{2}, \quad n = 1, 2, \ldots
\]

where \( \lambda_g \) is the waveguide wavelength at the resonant frequency.

The filter shown in Figure 58a is also analyzed by the signal flow graph technique. From Figure 58b, the scattering parameters for this structure can be derived as
\[ |S_{21}| = \left| \frac{c(1+e^{-4\gamma\lambda})}{1 - c^2 e^{-4\gamma\lambda}} \right| \]  
(52)

\[ |S_{1',1}| = \left| \frac{e^{2-2\gamma\lambda}}{1 - c^2 e^{-4\gamma\lambda}} \right| \]  
(53)

where \( \lambda \) is the length of the curved section on each arm of the resonator (see Figure 58a). At resonance, \( |S_{21}| = 0 \), which corresponds to

\[ \lambda \approx \frac{1}{4} \left( n + \frac{1}{2} \right) \lambda_g \quad , \quad n = 1,2,\ldots \]  
(54)

For both resonator configurations, a quick glance at the frequency response expressions reveals that \( S_{21} \) has the characteristics of a band-reject filter, while \( S_{1',1} \) behaves like a band-pass filter. With an additional directional coupler at the input port, these resonators can be used either as a band-pass or as a band-reject filter.

5.3 Suspended H-Waveguide Leaky-Wave Antennas

Another possible application of the suspended H-waveguide is the leaky-wave antenna. The waveguide structure can be converted into a leaky-wave antenna (or a periodic band-reject filter) by edging periodic narrow slots on the top metal surface along the propagation direction. The geometry of the leaky-wave antenna is shown in Figure 59a. This antenna is frequency scannable simply by varying the operating frequency, and can be flush-mounted. Since a complete investigation of this antenna is beyond the scope of this thesis,
Figure 59. (a) Leaky-wave antenna.  
(b) Structure adopted for analysis.
only the formulation of the eigenvalue problem for finding the complex propagation constants in the structure is presented.

The analysis is based on the spectral domain approach in which the Galerkin's procedure is applied in the Fourier transform domain. A dual formulation for an image guide leaky-wave antenna and a band-reject filter has been published recently [52],[53].

As a first step to solve this problem, the dielectric core is transformed into an infinite slab in the x-direction via the effective parameter technique described in detail in Section 2.2. The cross section to be analyzed is shown in Figure 59b. Only the LSM\textsuperscript{Y} mode will be considered here even though the same procedure can be applied to the LSE\textsuperscript{Y} mode.

Because of the periodicity of the structure, the vector potential and all field components must consist of all space harmonic terms. For the LSM\textsuperscript{Y} mode, the Hertzian potential and the field components of interest are

\[ \hat{\mathbf{A}}^e = \sum_{n=-\infty}^{\infty} \phi_n(x,y) \exp(-jk_{zn}z) \]  
\[ E_z \sim \frac{\partial}{\partial z} \left( \frac{\partial}{\partial y} \sum_{n=-\infty}^{\infty} \phi_n(x,y) \exp(-jk_{zn}z) \right) \]  
\[ H_x \sim \varepsilon \sum_{n=-\infty}^{\infty} \phi_n(x,y) \exp(-jk_{zn}z) \]

where \( k_{zn} \) is the propagation constant of the \( n \)-th space harmonic and is related to that of the fundamental mode by

\[ k_{zn} = k_z - ja + \frac{2\pi n}{d}, \quad n = 0, \pm 1, \pm 2 \ldots \]
where $k_z$ is the propagation constant of the fundamental mode, $\alpha$ is the attenuation constant of the structure, and $d$ is the periodic slot spacing. It should be mentioned that the propagation constant of the $n$-th harmonic is different from other harmonics, but the attenuation constant is the same for all harmonics.

In the spectral domain, the potential as well as the field components are Fourier transformed via

$$
\tilde{\phi}_n(k_x, y) = \int_{-\infty}^{\infty} \phi_n(x, y) \exp(ik_x x) \, dx
$$

where $\tilde{\phi}_n$ satisfied

$$
\left( \frac{d^2}{dy^2} - \eta_i^2 \right) \tilde{\phi}_n(k_x, y) = 0, \quad i = 1, 2, 3, 4
$$

From now on, the symbol $\tilde{\phi}$ will be understood as being the Fourier transform of the quantity without it.

The appropriate solutions for the above differential equation are:

$$
\phi_n^{(1)} = A_n \exp(\eta_1(y-h-d)) \quad , \quad y \geq h+d+1
$$

$$
\phi_n^{(2)} = B_n \sinh(\eta_2(y-h-d)) + C_n \cosh(\eta_2(y-h-d)) , \quad h+d \leq y < h+d+1
$$

$$
\phi_n^{(3)} = D_n \sinh(\eta_3(y-h)) + E_n \cosh(\eta_3(y-h)) , \quad h \leq y \leq h+d
$$

$$
\phi_n^{(4)} = F_n \cosh(\eta_4(y)) , \quad 0 \leq y \leq h
$$

where

$$
\eta_1^2 = k_x^2 - k_{zn}^2 - k_o^2
$$

$$
\eta_2^2 = k_x^2 + k_{zn}^2 - \varepsilon_1 k_o^2
$$

$$
\eta_3^2 = k_x^2 + k_{zn}^2 - k_o^2
$$

(62)
\[ \eta^{(3)^2} = k_x^2 + k_{zn}^2 - \mu^* \varepsilon_2 k_o^2 \]

\[ \eta^{(4)^2} = k_x^2 + k_{zn}^2 - \varepsilon_3 k_o^2 \]

In (64), \( \mu^* \) is the effective permeability of the \( \text{TM}^\gamma \) mode in region 2. \( \tilde{A}_n, \ldots, \tilde{F}_n \) are the \( n \)-th harmonics of the transformed potential in each region. Since the boundary conditions with respect to the \( x \)-coordinates are defined for all \( x \)'s in the space domain, they will be defined for all \( k_x \)'s in the transform domain, i.e.,

at \( y = h \) : \( \tilde{H}^{(4)}_x = \tilde{H}^{(3)}_x \) \hspace{1cm} (63a)

\[ \tilde{E}^{(4)}_z = \tilde{E}^{(3)}_z \] \hspace{1cm} (63b)

at \( y = h+d \) : \( \tilde{H}^{(3)}_x = \tilde{H}^{(2)}_x \) \hspace{1cm} (64a)

\[ \tilde{E}^{(3)}_z = \tilde{E}^{(2)}_z \] \hspace{1cm} (64b)

at \( y = h+d+1 \) : \( \tilde{E}^{(2)}_z = \tilde{E}^{(1)}_z = \tilde{E}_z(k_x) \) \hspace{1cm} (65a)

\[ \tilde{H}^{(1)}_x - \tilde{H}^{(2)}_x = \tilde{J}_z(k_x) \] \hspace{1cm} (65b)

where \( \tilde{E}_z(k_x) \) and \( \tilde{J}_z(k_x) \) are the Fourier transforms of the unknown tangential electric field \( E_z(x) \) in the slots and the unknown current \( J_z(x) \) on the metal portion, respectively, i.e.,

\[ E_z(k_x) = \int_{-L/2}^{L/2} E_z(x) \exp(jk_x x) \, dx \] \hspace{1cm} (66)

The \( z \)-dependence is not affected by the Fourier transform and has been excluded to obtain simplified expressions. Since \( \tilde{E}_z(k_x) \) is periodic, we have [54]
\[ \tilde{E}_z(k_x,z) = \sum_{n=-\infty}^{\infty} \tilde{E}_{z,n}(k_x) \exp(-jk_z z) \quad (67) \]

or

\[ \tilde{E}_{z,n}(k_x) = \frac{1}{d} \int_{0}^{W} \tilde{E}_z(k_x,z) \exp(jk_z z) \, dz \quad (68) \]

\( W \) is the width of each slot and \( \tilde{E}_{z,n} \) are the space harmonics of the surface slot field \( \tilde{E}_z \). Using (67) and the boundary conditions (63a)-(65a), the unknown harmonic amplitudes can be expressed in terms of space harmonic \( \tilde{E}_{z,n}(k_x) \). Finally, the last boundary condition (65b) is enforced to obtain the following expression for \( \tilde{J}_z \).

\[ \tilde{J}_z(k_x) = \sum_{n=-\infty}^{\infty} \tilde{C}_{n}(k_x,k_{zn}) \tilde{E}_{z,n}(k_x,k_{zn}) \exp(-jk_z z) \quad (69) \]

where the

\[ \tilde{C}_{n}(k_x,k_{zn}) = \frac{1}{k_{zn}(1)} \left\{ \frac{1 - \frac{\varepsilon_2 \eta(4) \sinh(\eta(2) d)}{\eta(1)} \left[ P_1 \sinh(\eta(2) d) + P_2 \cosh(\eta(2) d) \right]}{\frac{\eta(2) \cosh(\eta(2) d)}{P_1 + \sinh(\eta(2) d) P_2}} \right\} \]

and

\[ P_1 = \varepsilon_2 \eta(4) \cosh(\eta(3) d) \sinh(\eta(3) h) + \varepsilon_3 \eta(4) \sinh(\eta(3) d) \cosh(\eta(3) h) \]
\[ P_2 = \eta(2) \left\{ \frac{2 \eta(4)}{\eta(3)} \sinh(\eta(3) d) \sinh(\eta(3) h) + \varepsilon_2 \varepsilon_3 \cosh(\eta(3) d) \cosh(\eta(3) h) \right\} \]

Before applying the Galerkin's procedure, the unknown \( \tilde{E}_z(k_x) \) is expanded in terms of known basis functions, i.e.,

\[ \tilde{E}_z(k_x,z) = \sum_{m=1}^{M} a_m \tilde{E}_{z,m}(k_x,z) \quad (70) \]

where \( a_m \) is the unknown constant, and the basis functions must be chosen such that in the space domain they are non-zero only in the slots.

For the LSM\( ^{V} \) mode, the basis functions can be chosen as
\[ E_{z,m}(x,z) = \cos\left(\frac{2(m-1)\pi}{L} x\right) \cos\left(\frac{2(2m-1)\pi}{W} z\right), \quad m=1,2,\ldots \]

\[-\frac{L}{2} \leq x \leq \frac{L}{2} \text{ and } -\frac{W}{2} \leq z \leq \frac{W}{2}\]  

\[(71)\]

From (68), (69) and (70), the transform of the unknown current becomes:

\[ J_z(k_x,z) = \sum_{n=-\infty}^{\infty} \sum_{m=1}^{M} a_m \tilde{G}_n(k_x,k_{zn}) \tilde{E}_{z,m,n}(k_x,k_{zn}) \exp(-j k_{zn} z) \]

\[= \sum_{n=-\infty}^{\infty} \tilde{J}_n(k_x,z) \exp(-j k_{zn} z) \]  

\[(72)\]

where \(\tilde{E}_{z,m,n}\) and \(\tilde{J}_n\) are the amplitude coefficients of the \(n\)-th space harmonic of \(\tilde{E}_{z,m}(k_x,z)\) and \(\tilde{J}_z(k_x,z)\), respectively. \(\tilde{E}_{z,m,n}\) is given by

\[\tilde{E}_{z,m,n}(k_x,k_{zn}) = \frac{1}{d} \int_{0}^{W} \tilde{E}_{z,m}(k_x,z) \exp(jk_{zn} z) dz \]

\[= \frac{(2m-1)}{k_{zn}} \frac{\cos(k_x L/2)}{(k_x L)^2 - ((2m-1)\pi)^2} \frac{\sin(k_{zn} W/2)}{(k_{zn} W)^2 - (2(m-1)\pi)^2} \]  

\[(73)\]

Now, over a unit cell, the Galerkin’s procedure is applied by taking the inner product of \(\tilde{J}_z\) and \(\tilde{E}_{z,s}^*\) for different values of \(s\) and sum over all \(n\). The following matrix equation is obtained.

\[\sum_{m=1}^{M} \left\{ \sum_{n=-\infty}^{\infty} \tilde{G}_n(k_x,k_{zn}) \tilde{E}_{z,m,n}(k_x,k_{zn}) \tilde{E}_{z,s,m}(k_x,k_{zn}) dk_x \right\} a_m = 0, \quad a_m = 0 \]

\[s = 1,2,\ldots,M\]  

\[(74)\]

The fact that the left-hand side equals zero comes directly from the Parseval’s theorem, since in the space domain, \(J_z\) and \(E_z\) are zero in the
complementary regions. A nontrivial solution for $a_m$ exists only if the determinant of (74) equals zero which is also the eigenvalue equation that determines the phase constant of the dominant space harmonic. If only one term of the slot field distribution is taken into account, the complex propagation constant is then obtained from the following eigenvalue equation.

$$\sum_{n=-\infty}^{\infty} G_n(k_x, k_{zn}) \left| \tilde{E}_{z,1,n}(k_x, k_{zn}) \right|^2 \, dk_x = 0 \quad (75)$$

The solution for the complex propagation constant of the fundamental mode can be obtained by a numerical procedure which searches the roots of (74) within some numerical tolerance [52]. Since all constituents of (74) and (75) are symmetric with respect to real $k_x$, the integration need be carried out only along half of the $k_x$ axis. Moreover, according to Reference 52, there appears to be little or no contribution to the expression beyond a certain value of $k_x$, and the integration can be truncated accordingly. The range of $|n|$ in these expressions can be as small as 2 [52].

The real part of the propagation constant controls the main beam direction while the attenuation constant $\alpha$ determines the beamwidth and aperture efficiency. It accounts for the radiation loss and is strongly influenced by the size of the slots. Since the amplitude ratio of each element in the slot arrays is determined by $\alpha$, the radiation pattern in the plane that contains the longitudinal axis of the antenna can be computed readily. The radiation pattern in the transverse plane can be approximated by that of a slot antenna. The
angular direction of the leaky beam measured from broadside in the y-z plane is approximated by

\[ \theta_n = \sin^{-1}(\text{Re}(k_{zn}/k_0)) \]  

(76)

This equation can be used as a tool to determine the relationship between the main beam direction and the periodic spacing d of the slot. For antenna applications, the structure’s parameters are usually designed such that only the \( n = -1 \) space harmonic will radiate.

5.4 Horn Dielectric-Filled Leaky-Wave Antennas

The antenna described in this section is designed for integration with the millimeter-wave circuits that employ the image guide or the trapped image guide as the guiding medium. In this structure, the dielectric guide with periodic perturbations on top is embedded in a rectangular trough with metal flare attached along each side as shown in Figure 60. With this arrangement, the antenna behaves like a linear array in the longitudinal (or E-) plane which is the plane containing the longitudinal axis of the antenna. In the transverse (or H-) plane, the radiation pattern resembles that of a horn. Unlike the periodic slots of the antenna described in the previous section, the periodic perturbations in this antenna structure are metallic strips which are painted directly on the dielectric guide. These perturbations cause the guided-wave energy to radiate off the dielectric guiding structure. Each of the strips behaves like an element of a linear array. A complete discussion of this antenna structure can be found in Reference [26]. Some excerpts from that reference are presented here.
Figure 60. Horn dielectric-filled trough leaky-wave antenna.
At each strip, it is desirable that the rate of energy radiated be small. Thus, a portion of the guided-wave energy will propagate through the entire antenna length, which implies a larger aperture. But equally important, the guided energy should be completely radiated by these strips so that there is no energy left at the antenna truncation which would radiate endfire. Experiments show that the rate of energy radiated along the dielectric leaky-wave antenna structure is strongly influenced by the width of the perturbation strips on the top surface of the dielectric guide.

For very narrow strip widths, the radiation from each element is so small that a very long antenna has to be constructed to radiate all energy in an effective manner. On the other hand, if the metal strips are too wide (>0.5 λg), the bulk of radiated energy is produced by the first few strips and, consequently, the effective aperture is very small. Also, for large strip widths, the sidelobes are very high, due probably to the large mismatch at the first strip.

It is well-known that if the attenuation rate of the traveling wave in the z-direction due to the energy leakage at the leaky-wave region of the antenna is a constant (neper/m), the radiation pattern is not symmetric about the main beam direction. Therefore, the energy transfer from the leaky beam to the surface wave, and vice versa, is not optimum [55]. A suitable modification of the geometry in the leaky-wave region of the antenna should be made to obtain a slowly varying leaky rate to increase the conversion efficiency [56]. If the variation of the strip width is correct, all the surface energy
will convert into a symmetric leaky beam (and vice versa). This can be done by tapering the size of the periodic scatterers [55] as shown in Figure 61. Since the mechanisms of perturbation and radiation are not clear, only the widths of these metallic strips were experimentally investigated. The width of the $m$-th strip is found from the following empirical relation:

$$ W_m = \begin{cases} 
(0.15 + 0.015(m-1)) \lambda_g, & m \leq 18 \\
0.4 \lambda_g, & m > 18 
\end{cases} \quad (77) $$

where $\lambda_g$ is the guided wavelength. For slight perturbation, the real part of the propagation constant of the structure is approximated by that of an unperturbed structure.

For periodic strip spacing, the angular direction $\theta'_n$ of the leaky-wave beam measured from broadside is approximated by Eq. (76), which for clarity, is given again as

$$ \theta'_n = \sin^{-1}(\lambda_0/\lambda_g + n\lambda_0/d) \quad (78) $$

where $d$ is the perturbation spacing between the centers of two adjacent strips, $\lambda_0$ is the free-space wavelength, and $n$ is the index of the space harmonic ($0, \pm 1, \pm 2, \ldots$). For the antenna applications, the structure's parameters are usually designed so that only the $n = -1$ space harmonic will radiate, and all the energy will be radiated through this harmonic. The antenna is frequency scannable if $|\lambda_0/\lambda_g + n\lambda_0/d| < 1$. For a broadside array, $d$ should be made equal to the guided wavelength $\lambda_g$. Figure 62
Figure 61. Tapering the width of the metallic strips and the coordinate system.
Figure 62. Computed and measured main beam directions.
shows the measured and computed main beam directions as a function of the frequency. The entire angular scanning range can be shifted more positively or negatively by changing the perturbation spacing between the adjacent strips.

In the transverse (or H-) plane, the metal flares are attached to both sides of the trough to improve the radiation characteristics in this plane. With $H_x$ as the major magnetic field component in the dielectric guide, the current element on each metal strip is $z$-oriented, and the principal aperture field of the metal flares is $E_z$. Assuming the structure is infinitely long in the $z$-direction allows the metal flares to be modeled as an H-plane sectoral horn [57],[58].

The dielectric guide was designed to fit snugly against the side walls of the metal trough thus eliminating the need for adhesive materials which were used to bond the dielectric guide to the ground plane. The depth of the trough is such that the propagation constant of the structure can be determined using the dielectric-loaded trough guide model. The height of the dielectric guide is then chosen to provide single-mode operation. Under these guidelines and constraints, a dielectric waveguide of dimensions 3.4 x 1.4 mm was constructed. The trough depth was arbitrarily chosen to be 3.4 mm to simplify the construction. It was observed that as long as the trough was not too shallow, the radiation patterns of the structure were almost independent of the trough depth. The length of the flare horn was chosen to be 4 cm. Figure 63 shows the comparison between the measured relative gain of the constructed antenna as a function of the flare angle.
Figure 63. Relative gains of the constructed antenna and an H-plane sectoral horn versus flare angle.
and that of an H-plane sectoral horn. The results of this figure have confirmed the assumption that the H-plane sectoral horn is a reasonable model to predict the flare angle for maximum gain in the H-plane. For the strip width distribution chosen according to (77) and the flare angle selected from Figure 63 for maximum gain, the radiation patterns in both the E- and H-planes of the horn dielectric-filled trough leaky-wave antenna were measured and plotted in Figure 64. The number of the silver strips on top of the dielectric guide was 32. The half-power beamwidths (HPBW) in the E- and H-planes are 4°, 13°, respectively.

The antenna was designed for broadside radiation. The consequent experimental beam angle deviated slightly from the normal direction. This is interesting since the strip width determines the rate of energy radiated off the guide but does not change the angle of radiation significantly. The overall gain of the antenna is the product of the gains in the E- and H-planes and was measured to be 26 dB. The sidelobe levels were at least 25 dB below the main lobe. Since there is no endfire radiation detected, it is assumed that most of the radiated energy is produced by the strip elements.

The main beam level of the antenna is relatively stable throughout the operating frequency range except at around 84 GHz where it falls about 2 dB below the reference level (see Figure 65).
Figure 64. Radiation patterns in the E- and H-planes of the constructed antenna.
Figure 65. Main beam level versus frequency.
6. CONCLUSIONS

In this thesis, the field and propagation characteristics of many dielectric waveguide structures have been analyzed. Both the mode matching and the effective parameter techniques described in this thesis are very general and can be applied directly to many planar dielectric waveguide structures of rectangular cross section. The theoretical and experimental results agree well throughout. A new dielectric waveguide configuration—the suspended H-waveguide—has been introduced along with a detailed loss analysis. The overall low-loss figure and a superior structural layout of this waveguide promise wider applications in the millimeter-wave integrated circuits. Some essential integrable components, such as dielectric resonators and leaky-wave antennas, have been studied. A theoretical formulation of the eigenvalue equation based on the spectral domain approach for determining the propagation constant of a periodic slot waveguide array has also been presented.
MATCHING OF TANGENTIAL FIELD COMPONENTS AT $y = \pm h$

Eight equations resulting from matching the tangential field components $E_x$, $H_x$, $E_z$, and $H_z$ at the planes $y = \pm h$ are

$$
\sum_{n=0}^{N} \left[ \varepsilon_1 k_o^2 - k(xn)^2 \right] \begin{bmatrix}
\cos(k(xn)^{(1)}(x+a)) \\
\sin(k(xn)^{(1)}(x+a))
\end{bmatrix} A_n = \sum_{n=1}^{N'} \left[ \varepsilon_1 k_o^2 - k(xn)^2 \right] \\
\begin{bmatrix}
\cos(k(xn)^{(1)}(x+a)) \\
\sin(k(xn)^{(1)}(x+a))
\end{bmatrix}
$$

\begin{align}
\phi^{(i)}(x) (I_n + I'_n) 
\end{align}

$$
\sum_{m=0}^{M} \left[ \varepsilon_1 k_o^2 - \tilde{k}(xm)^2 \right] \begin{bmatrix}
\sin(\tilde{k}(xm)^{(1)}(x+a)) \\
\cos(\tilde{k}(xm)^{(1)}(x+a))
\end{bmatrix} B_m = \sum_{m=1}^{M'} \left[ \varepsilon_1 k_o^2 - \tilde{k}(xm)^2 \right] \\
\begin{bmatrix}
\sin(\tilde{k}(xm)^{(1)}(x+a)) \\
\cos(\tilde{k}(xm)^{(1)}(x+a))
\end{bmatrix}
$$

\begin{align}
\Phi^{(i)}(x) (J_m + J'_m)
\end{align}

$$
\sum_{n=0}^{N} k z \left[ k(xn)^{(1)} \right] \begin{bmatrix}
-\sin(k(xn)^{(1)}(x+a)) \\
\cos(k(xn)^{(1)}(x+a))
\end{bmatrix} A_n + \sum_{m=0}^{M} \omega_x \omega_y \tilde{k}(1) \\
\begin{bmatrix}
\sin(k(xn)^{(1)}(x+a)) \\
\cos(k(xn)^{(1)}(x+a))
\end{bmatrix}
$$

\begin{align}
\tan(\tilde{k}(ym)^{(1)}(h-d)) B_m = \sum_{n=1}^{N'} \frac{\omega_x}{k z} \Phi^{(i)}(x) (I_n + I'_n) \\
- \sum_{m=1}^{M'} \omega_x \omega_y \tilde{k}(ym) \Phi^{(i)}(x) \left[ -\tan(\tilde{k}(ym)^{(1)} h - \cot(\tilde{k}(ym)^{(1)} h) J_m + \cot(\tilde{k}(ym)^{(1)} J'_m) \right] 
\end{align}
\[
\sum_{n=0}^{N} \omega_1 e_1 y_n \begin{bmatrix}
\cos k_{xn}^{(1)}(x+a) \\
\sin k_{xn}^{(1)}(x+a)
\end{bmatrix} \cot k_{yn}^{(1)}(h-d) A_n + \\
\sum_{m=0}^{M} k_z \bar{k}_{xm}^{(1)} \begin{bmatrix}
\cos \bar{k}_{xm}^{(1)}(x+a) \\
-\sin \bar{k}_{xm}^{(1)}(x+a)
\end{bmatrix} B_m = \sum_{n=1}^{N'} \omega_1 e_1 y_n \phi_{e,n}^{(1)}(x)
\]

\[
\sum_{n=0}^{N} \left( \varepsilon^2 k_0^2 - k_{xn}^2 \right) \begin{bmatrix}
\cos k_{xn}^{(3)}(x+a) \\
\sin k_{xn}^{(3)}(x+a)
\end{bmatrix} K_n
\]

\[
= \sum_{n=1}^{N'} \left( \varepsilon^2 k_0^2 - k_{xn}^{(1)} \right) \phi_{e,n}^{(1)}(x) \left( I_n - I_n^{(1)} \right)
\]

\[
\sum_{m=0}^{M} \left( \varepsilon^2 k_0^2 - \bar{k}_{xm}^{(1)} \right) \begin{bmatrix}
\sin \bar{k}_{xm}^{(3)}(x+a) \\
\cos \bar{k}_{xm}^{(3)}(x+a)
\end{bmatrix} L_m = \sum_{m=1}^{M'} \left( \varepsilon^2 k_0^2 - \bar{k}_{xm}^{(1)} \right) \phi_{h,m}^{(1)}(x) \left( J_m - J_m^{(1)} \right)
\]

(A-4)

(A-5)

(A-6)
\[ \sum_{n=0}^{N} k_{x}^{(3)} \cos[k_{x}^{(3)}(x+a)] \left\{ -\sin[k_{x}^{(3)}(x+a)] \right\} K_{n} - \sum_{m=0}^{M} \omega_{o}^{(3)} k_{y}^{(3)} \cos[k_{x}^{(3)}(x+a)] \sin[k_{x}^{(3)}(x+a)] \]  

\[ \cdot \tan[k_{y}^{(3)}(-h+l)] \]  

\[ L_{m} = \sum_{n=1}^{N} k_{z} \frac{\partial \phi_{e,n}^{(i)}(x)}{\partial x} (-I_{n} + I_{n}') \]  

\[ + \sum_{m=0}^{M} \omega_{o}^{(3)} k_{y}^{(3)} \phi_{h,m}^{(i)}(x)(\tan[k_{y}^{(3)}] J_{m} + \cot[k_{y}^{(3)}] J_{m}') \]  

\[ (A-7) \]

\[ \sum_{n=0}^{N} \omega_{o}^{(3)} k_{y}^{(3)} \sin[k_{x}^{(3)}(x+a)] \left\{ \cos[k_{x}^{(3)}(x+a)] \right\} \]  

\[ \cdot \cot[k_{y}^{(3)}(-h+l)] K_{n} \]  

\[ + \sum_{m=0}^{M} k_{x}^{(3)} \left\{ \cos[k_{x}^{(3)}(x+a)] \right\} \]  

\[ L_{m} = \sum_{n=1}^{N} \omega_{o}^{(3)} k_{y}^{(3)} \phi_{e,n}^{(i)}(x) \]  

\[ (A-8) \]

The upper line in the brackets indicates the puc wall at \( x = -a \) while the lower line applies for a pmc wall. The \( \phi_{e,n}^{(i)}(x) \) and \( \phi_{h,m}^{(i)}(x) \), \( i = 2, 4, 5 \), are the \( x \)-variation components of the Hertzian potentials in respective regions (see Eqs. (3), (4), and (5)). \( \varepsilon_{i}, i = 2, 4, 5 \), is the relative dielectric constant.
APPENDIX B
COUPLING INTEGRAL TERMS

\[ P_n(\nu) = \left( k_0^2 - k_{\text{xn}}(4)^2 \right) k_n^{(G-I)} I_5(\nu) + \left( k_{\text{xn}}^2 - k_{\text{xn}}(2)^2 \right) \]

\[ \cdot \left( k_n^{(G-I)} I_2(\nu) + k_n^{(D-I)} I_3(\nu) + \left( k_0^2 - k_{\text{xn}}(5)^2 \right) I_4(\nu) \right) \]

\[ R_n(\nu) = k_n^{(G-I)} I_1(\nu) + \varepsilon_2 \left( k_n^{(C-I)} I_2(\nu) + k_n^{(D-I)} I_3(\nu) \right) + I_4(\nu) \]

\[ T_n(\nu) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} k_{\text{xn}}^4 k_n^{(G-I)} I_5(\nu) + k_{\text{xn}}(2) k_n^{(G-I)} I_6(\nu) \]

\[ - k_n^{(D-I)} I_7(\nu) + k_{\text{xn}}(5) I_8(\nu) \]

\[ Q_m(\mu) = \left( k_0^2 - k_{\text{xm}}(4)^2 \right) k_m^{(H-J)} I_5(\mu) + \left( k_{\text{xm}}^2 - k_{\text{xm}}(2)^2 \right) \]

\[ \cdot \left( k_m^{(E-J)} I_6(\mu) + k_m^{(F-J)} I_7(\mu) \right) + \left( k_0^2 - k_{\text{xm}}(5)^2 \right) I_8(\nu) \]

\[ S_m(\mu) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \left( k_{\text{xm}}(4) k_m^{(H-J)} I_5(\mu) + k_{\text{xm}}(2) k_m^{(E-J)} I_6(\mu) \right) \]

\[ - k_m^{(F-J)} I_3(\mu) + k_{\text{xm}}(5) I_4(\mu) \]

\[ U_m(\mu) = k_m^{(H-J)} I_5(\mu) + k_m^{(E-J)} I_6(\mu) + k_m^{(F-J)} I_7(\mu) + I_8(\mu) \]

and

**LSM modes**

\[ I_1(\nu) = \int_{-a}^{a} \begin{pmatrix} \cos \left( k_{\text{xn}}(x+a) \right) \\ \sin \left( k_{\text{xn}}(x+a) \right) \end{pmatrix} \psi_\nu(x) \, dx \]
\[ I_2(v) = \int_{-w}^{w} \cos \left( \frac{k(2)x}{x_{vn}} \right) \psi_v^{(1)}(x) \, dx \]

\[ I_3(v) = \int_{-w}^{w} \sin \left( \frac{k(2)x}{x_{vn}} \right) \psi_v^{(1)}(x) \, dx \]

\[ I_4(v) = \int_{-w}^{b} \cos \left( \frac{k(5)(x-b)}{x_{vn}} \right) \psi_v^{(1)}(x) \, dx \]

\[ I_5(v) = \int_{-w}^{-a} \begin{cases} \sin \left( \frac{k(4)(x+a)}{x_{vn}} \right) \\ \cos \left( \frac{k(4)(x+a)}{x_{vn}} \right) \end{cases} \psi_v^{(2)}(x) \, dx \]

\[ I_6(v) = \int_{-w}^{w} \sin \left( \frac{k(2)x}{x_{vn}} \right) \psi_v^{(2)}(x) \, dx \]

\[ I_7(v) = \int_{-w}^{w} \cos \left( \frac{k(2)x}{x_{vn}} \right) \psi_v^{(2)}(x) \, dx \]

\[ I_8(v) = \int_{-w}^{b} \sin \left( \frac{k(5)(x-b)}{x_{vn}} \right) \psi_v^{(2)}(x) \, dx \]

where

\[ \psi_v^{(1)}(x) = \begin{cases} \cos \left( \frac{k(1)(x+a)}{x_{vn}} \right) \\ \sin \left( \frac{k(1)(x+a)}{x_{vn}} \right) \end{cases} \]

\[ \psi_v^{(2)}(x) = \begin{cases} \sin \left( \frac{k(1)(x+a)}{x_{vn}} \right) \\ \cos \left( \frac{k(1)(x+a)}{x_{vn}} \right) \end{cases} \]
LSE modes

\[ \tilde{I}_1(\mu), \ldots, \tilde{I}_8(\mu) \] are obtained by replacing \( k_{xn}^{(1)} \) in \( I_1(\nu), \ldots, I_8(\nu) \) with \( \tilde{k}_x^{(i)}(x) \) by \( \psi_{\nu}^{(i)}(x) \), e.g.,

\[
\tilde{I}_8(\mu) = \int_w^b \sin \left( k_{xm}^{(5)}(x-b) \right) \begin{bmatrix}
\sin \left( k_{x\mu}^{(1)}(x+a) \right) \\
\cos \left( k_{x\mu}^{(1)}(x+a) \right)
\end{bmatrix} \, dx
\]

In these equations, the upper line in \{\} applies to pec wall (odd mode) at \( x = -a \), and the lower line to pmc wall (even mode) at \( x = -a \).

\( k_{xn}^{(i)} \) and \( \tilde{k}_x^{(i)} \) are the solutions of the eigenvalue equations (8) and (9). \( k_{x\nu}^{(1)} \) and \( k_{x\mu}^{(1)} \) are given in (7). \( K_{n}^{(C-L)} \), \ldots, \( K_{m}^{(F-J)} \) are the coupling terms described in Eqs. (14) and (16).
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