# A Sink for Electromagnetic Waves: Harvesting Wave Energy with Resonant Observers

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## Introduction

We use a self-sustaining oscillator as our sender, another two oscillators as the left and right receivers, and a fourth oscillator as an observer. There is an optimal friction constant such that when everything is in resonance, we should find that the minimum power is dissipated in the left and right receivers and maximum power is dissipated in the observer.

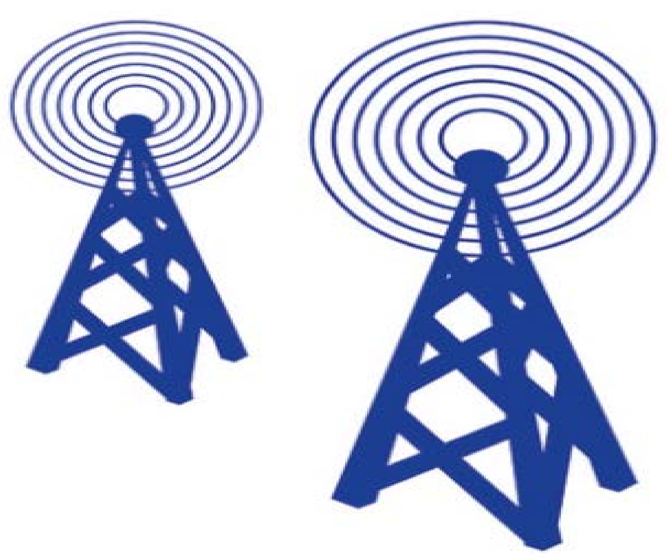

Figure 1. Simple visual of the two oscillators.

## Equations of Motion

These are the equations we use to describe the behavior of the oscillators.

$$\ddot{x} - 0.001\left(1 - \frac{1}{2}\dot{x}^2 - 6x^2\right)\dot{x} + 12\,x = 0.4\,(x_l - x) + 0.4\,(x_r -$$

## No Observer

We first considered the case without an observer. Without the observer present, power flows equally through the left and right receivers.

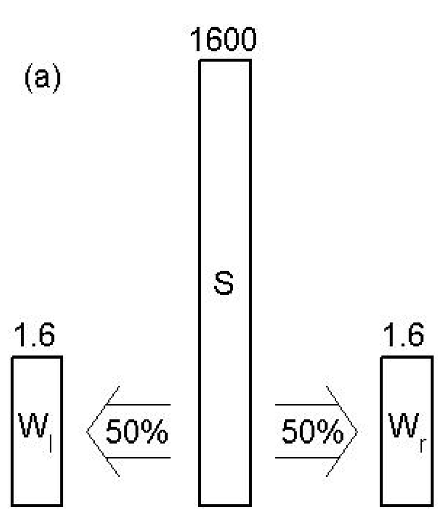


Figure 2. Power flows equally between the left and right receivers.

## With Observer

Now we couple an observer to the right receiver. When we add the observer, we find that 98% of the power flows through the observer and 2% flows through the left receiver. The observer sucks all the power to itself.

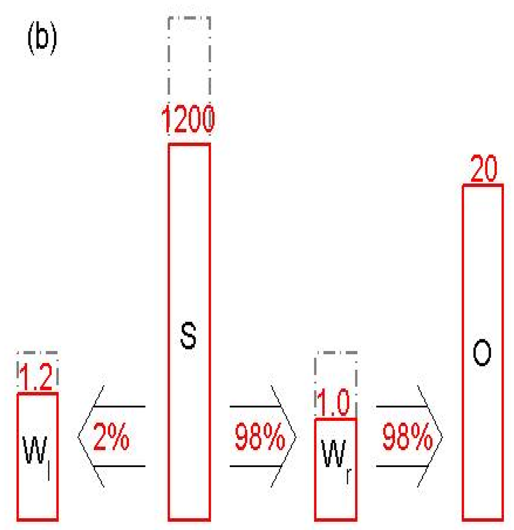


Figure 3. Power flows to the observer

## System with Sender, Receiver, and Observer

Now a system with only one receiver, a sender, and an observer was simulated. The equations used were

**Sender:** $m_1\ddot{x}_1 + \eta_1\left[\frac{m_1\dot{x}_1{}^2}{2} + \frac{c_1 x_1{}^2}{2} - E\right]\dot{x}_1 + c_1 x_1 = k(x_2 - x_1)$

**Receiver:** $m_2\ddot{x}_2 + \eta_2\dot{x}_2 + c_2 x_2 = k(x_1 - x_2)$

**Observer:** $m_3\ddot{x}_3 + \eta_3\dot{x}_3 = k(x_1 - x_3)$

## Simulation and Results

The power through the receiver was simulated first. Keeping $c_1$ fixed, $c_2$ was varied. When the difference between the two values was zero, we see that the power is at a maximum. This is when both are at resonance and thus we have shown that at resonance, the maximum power is produced.

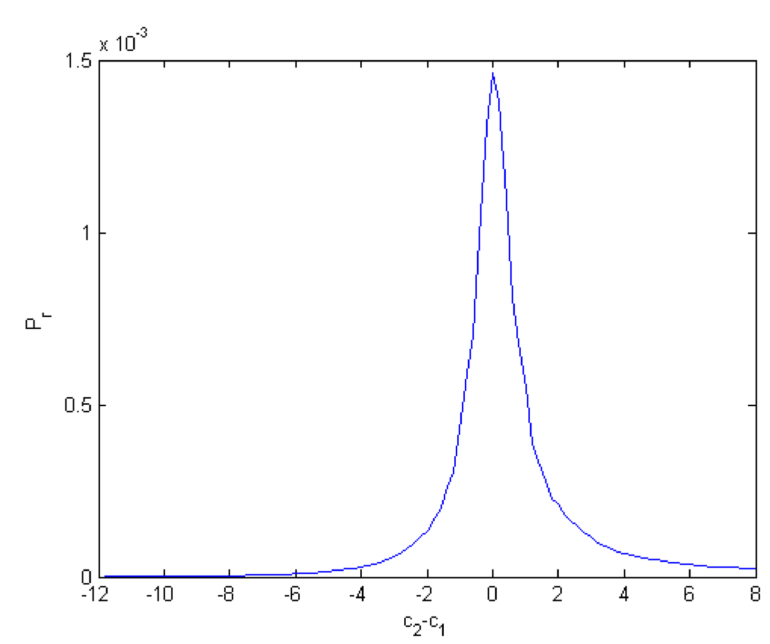


Figure 4. Power in the receiver as a function of the difference in spring-like constants.

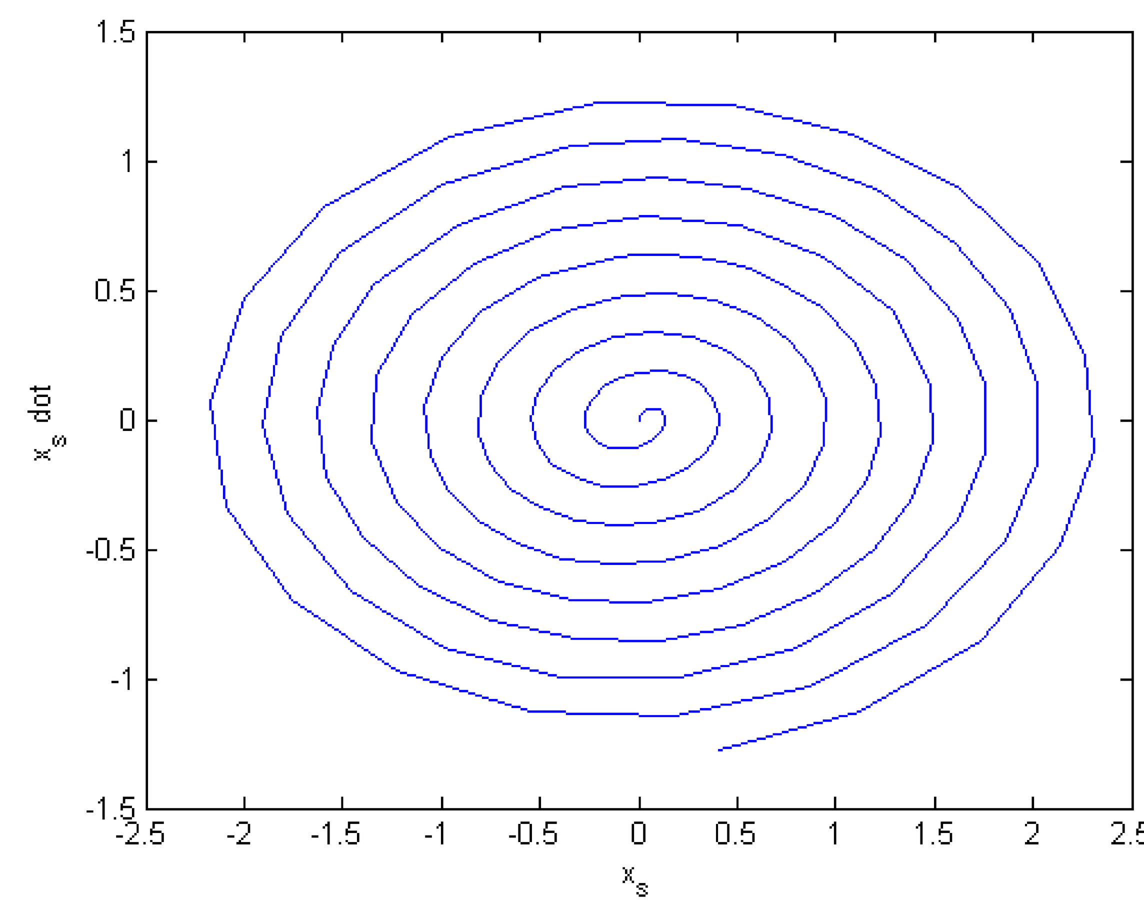


Figure 5. Phase space of the self-sustaining oscillator A.

## Optimal Frictional Constant

Next, the power through the receiver was simulated as a function of varying $\eta_2$ values using the equation $P = \frac{1}{T}\int_0^T \eta_2 \dot{x_B}^2\,dt$. From the simulation below it is clear that there is an optimal $\eta_2$ through which the most power flows through the receiver. As expected, this value is small. This reflects the nature of sensitive dependence in dynamical systems.

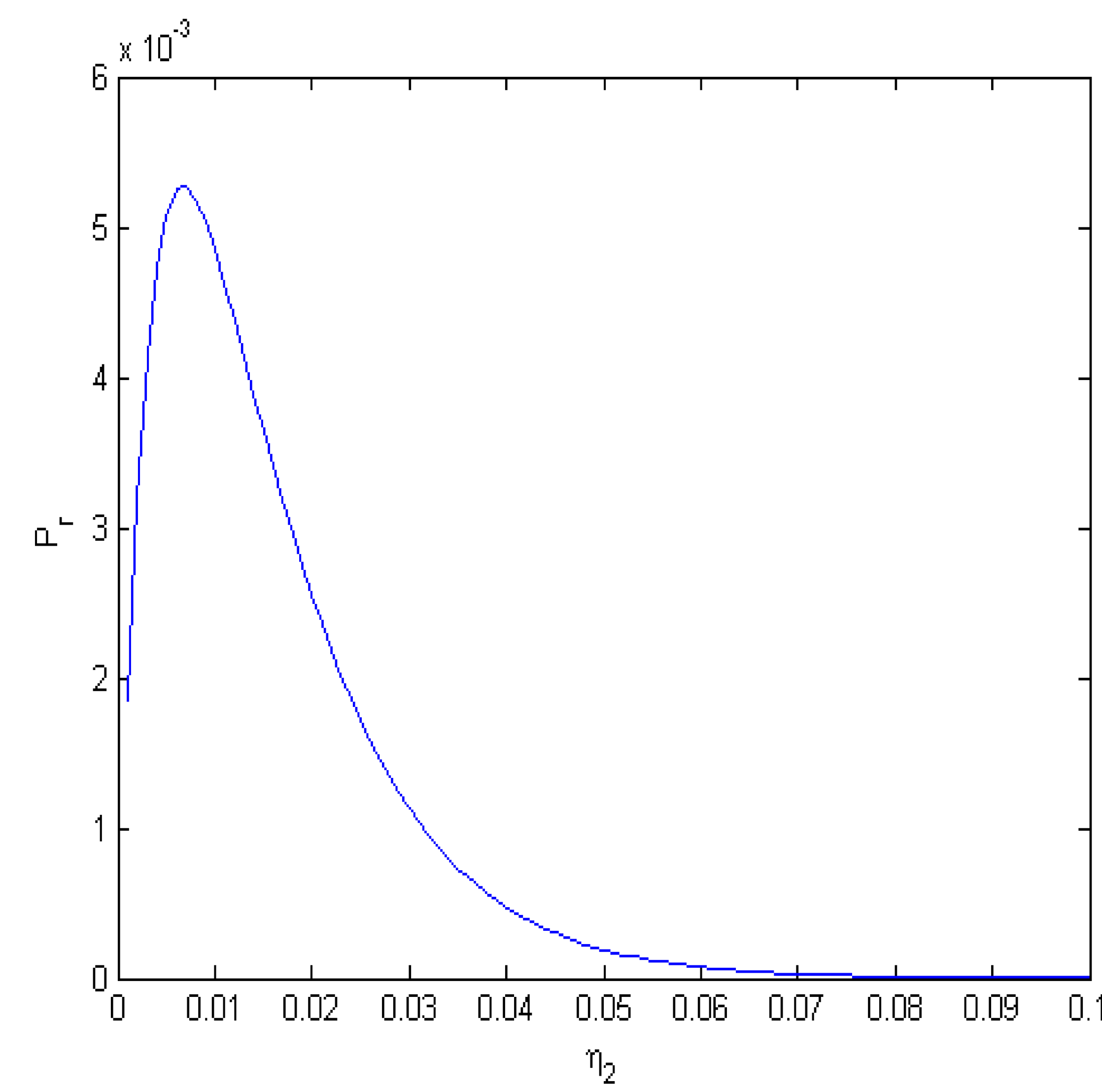


Figure 6. Power flowing through the receiver for different $\eta_2$ values.

## Power at the Observer

Finally, the power at an observer in resonance with the sender and receiver is simulated. As we had hoped, the power at the observer is at a maximum when the power at the receiver is a minimum. The value for $c_1$ is kept fixed while the values for $c_2$ vary. As was shown in Figure 2, when the difference between $c_1$ and $c_2$ is zero, we have maximum power through the observer.

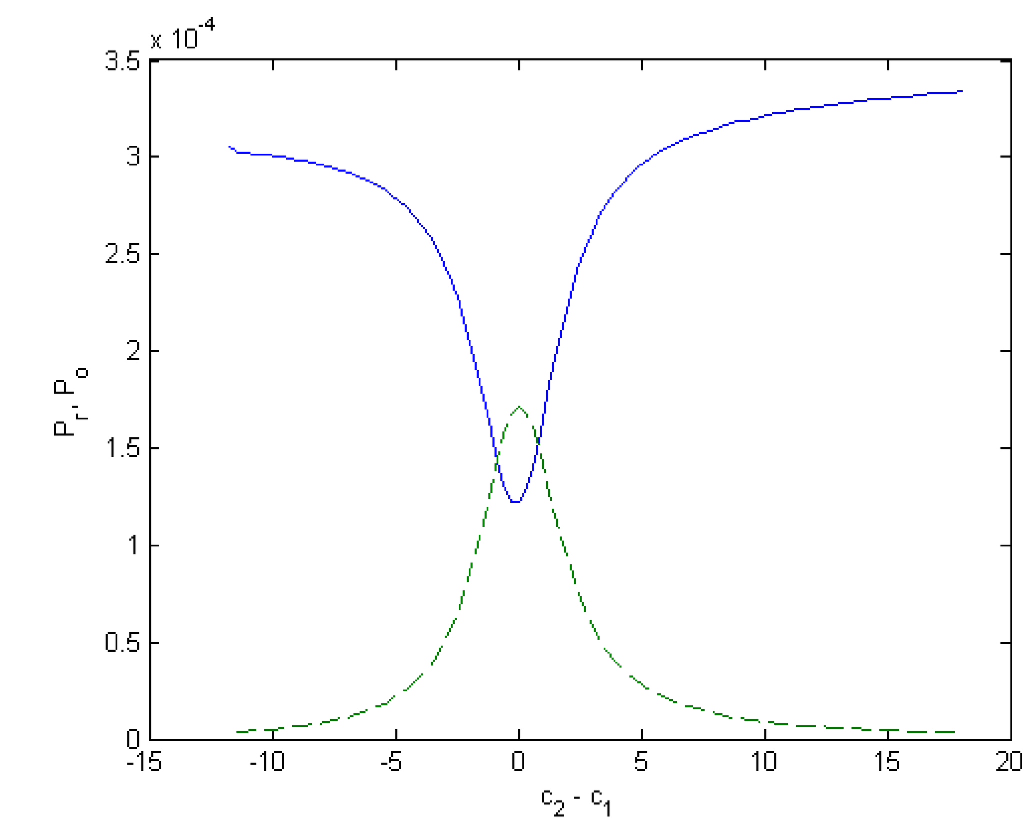


Figure 7. Power at the observer is the dashed line while power at the receiver is the solid line.

## Next Steps

From here, the equations Professor Hubler used in the recently published paper are being verified. Eventually the goal is to see if there is an optimal phase shift in the oscillators. In addition, we want to be able to have many more oscillators coupled to the current receiver and see how the power output behaves at the observer and at each oscillator.

## References

Hubler, A. and Kirsh, T. (2015), Harvesting wave energy with resonant observers. Complexity, 20: 6–7. doi: 10.1002/cplx.21675