SURFACE TEXTURES AND NON-NEWTONIAN FLUIDS FOR DECREASED FRICTION IN FULL FILM LUBRICATION

BY

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THESIS

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ABSTRACT

Surface texturing has been studied extensively to decrease friction in lubricated sliding contact. The majority of the previous experimental work has focused on symmetric-depth-surface textures. This work examines both symmetric and asymmetric surface textures in order to determine the effects of asymmetry on friction reduction. Gap controlled experiments with Newtonian fluids were performed on a custom tribo-rheometer in order to measure the normal force production and apparent viscosity reduction due to the surface textures. This work shows that symmetry must be broken in order to produce normal forces with surface textures which are caused by viscous effects for gap based Reynolds Number up to $\text{Re}_h = 1.21$.

The experimental results with Newtonian fluids suggest that there is an optimal surface texture configuration for decreasing friction. In order to determine the optimal texture configuration, numerical methods must be used in order to solve a mathematical model describing the lubricated system. The experimental work shows that the normal forces are produced due to viscous effects, allowing the Reynolds equation to be used as the mathematical model for the lubricated system. A pseudo-spectral method is used to solve the Reynolds equation in cylindrical coordinates. Exponential convergence is shown with this solution method and favorable results to the experimental results are shown, validating the numerical method for use in optimization of the surface textures.

A recent trend in the lubrication industry has been to add polymer additives to base oils in order to increase their effectiveness as lubricants. These polymer additives, however, cause the oil to become a Non-Newtonian viscoelastic lubricant. The final portion of this work presents the first experimental investigation of surface textures with viscoelastic lubricants in order to determine asymmetric and Non-Newtonian effects on friction reduction. Gap controlled
experiments are performed on the same custom tribo-rheometer in order to measure the normal force production and apparent viscosity reduction. This work shows that symmetry must be broken in order to produce normal forces above the viscoelastic response and that an optimal asymmetric texture exists for decreasing friction with viscoelastic lubricants.
ACKNOWLEDGMENTS

I wish to thank God and my family for helping me during my work towards my masters degree. I also wish to thank my advisor, Randy Ewoldt, for allowing me to work on this project and for helping me grow as a researcher. I also wish to thank Paul Fischer, who taught me the fundamentals of computational mechanics, which was the basis for the computational work performed. Finally, I wish to thank Mr. Eric Potter, who taught me the basics of physics and calculus, without which this work would not have been possible.
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CHAPTER 1: Introduction

Friction is a major loss in mechanical systems, which reduces the efficiency of the system. A key focus for many years in fluid power systems has been on determining ways to decrease the frictional losses. One such method that has been extensively studied is adding surface textures to one of the surfaces used in lubricated sliding contact. Studies have also been performed on determining the optimal surface texture profile for decreasing friction in lubricated sliding contact. However, most studies have focused on symmetric textures and no optimization has been performed with asymmetric textures.

Most of the research examining the effects of surface textures used a Newtonian fluid as the lubricant. However, there has been a trend in the lubrication industry to add polymer additives to Newtonian lubricants in order to increase their effectiveness. These polymer additives also have the effect of making the lubricant a viscoelastic Non-Newtonian fluid. No previous work has examined the effects of surface textures with viscoelastic lubricants.

This work presents experimental and numerical analysis of symmetric and asymmetric surface textures with Newtonian fluids for decreasing friction. Chapter 2 presents the experimental work with the surface textures. A custom tribo-rheometer was used to take the experimental measurements of normal force production and apparent viscosity reduction. The experimental results show that symmetry must be broken in order to produce normal forces with surface textures. Chapter 3 presents the numerical work with the surface textures with Newtonian fluids. The Reynolds equation in cylindrical coordinates is solved using a pseudo-spectral
method. The numerical method is shown to have exponential convergence, and a good comparison is seen between the experimental and numerical work.

Finally, this work presents an experimental investigation of surface textures with viscoelastic lubricants in Chapter 4. The same custom tribo-rheometer used to take the experimental measurements in Chapter 2 is also used. This work shows that the normal force production and viscosity reduction are dependent on the asymmetry of the system, suggesting that there is an optimal texture profile for decreasing friction when using a viscoelastic lubricant.
Figure 1: Representation of a symmetric surface texture used in a textured thrust bearing.
Figure 2: Polymer additives in Newtonian base oil. (A) shows the polymers under quiescent conditions. (B) shows the polymers in shear flow, where the polymers have been stretched.
CHAPTER 2: Experiments with Newtonian Fluid

2.1 Introduction

Surface texturing has been used very heavily in recent years in order to reduce the friction in lubricated sliding contact [1,2]. A key focus of applying surface textures has been in lubricating piston rings in internal combustion engines [3] where frictional losses account for up to 50% of the overall mechanical losses of the engine [3,4,5]. Surface textures can decrease the apparent area of contact in boundary lubrication, act as reservoirs that retain lubricant during startup [6,4], increase the film thickness and increase hydrodynamic pressure to further separate the plates [7], and decrease the local shear stress [8], all of which contribute to friction reduction.

Previous experimental studies were performed using primarily symmetric surface textures; i.e. symmetry in shape of the textures as viewed (i) normal to the surface (top view) and (ii) cross section in direction of motion (depth profile). Qiu and Khonsari [9] tested circular and elliptical top profile textures in a thrust bearing like set up and showed that the textures reduced friction from the flat plate reference. Vladescu et. al. [5] tested different top profile textures in reciprocating motion and found that the textures decreased friction by up to 50% in the boundary lubrication regime. Costa and Hutchings [10] also studied textures of varying top profile, and showed that chevron type textures increased the film thickness the best out of the profile types that they tested. Marian, Killan, and Scholz [11] examined square top profile textures and showed that using the textures would increase the load carrying capacity.
Numerical investigations have also been performed with symmetric surface textures. Dobrica et al. [12] studied rectangular type surface textures and showed that surface textures can generate a hydrodynamic lift. Lei et al. [13] studied circular textures and showed that a higher load carrying capacity was generated with bump shaped textures than with dent shaped textures. Shi and Ni [14] examined periodic step profile textures and suggested that there is an optimal texture configuration for supporting normal loads. Shen and Khonsari [15] used a sequential quadratic programming method to determine an optimal top profile shape for producing normal forces due to cavitation under unidirectional and bidirectional sliding.

Asymmetric textures have been somewhat studied with both experiments and computations. Shen and Khonsari [16] examined experimentally and numerically surface textures with a circular top profile and a linearly varying bottom profile under cavitation conditions, and showed that normal forces could be generated with asymmetric textures, but the forces were smaller than with a constant depth circular texture. However, Han et al. [17] and Nanbu et al. [18] also numerically examined linearly varying bottom profile textures, and they showed that normal forces generated with asymmetric textures can be larger than forces generated with symmetric textures.

The majority of the previous work included cavitation in their experiments and numerical modeling. In fact, most rely on cavitation to produce normal forces with the symmetric textures. Stachowiak and Bachelor [19] state that symmetric-depth-profile surface textures must have cavitation in order to produce a load carrying capacity. Using cavitation to produce normal forces might help decrease friction, but the degradation of the surface due to bubble collapse [19,20] could do more harm to the surface than regular frictional wear.
This work experimentally examines the effects of different surface texture depth profiles on decreasing friction through the reduction of apparent viscosity and the production of normal forces. Gap-controlled experiments were performed on a custom tribo-rheometer in order to systematically determine the normal force production and apparent viscosity reduction with varying Reynolds number up to a maximum Reynolds number with respect to gap height \( \text{Re}_n = 1.21 \). Previous experimental work [8,16] only tested with the top plate moving in one direction. In our work here the top plate was allowed to rotate in both directions to verify symmetry in the experimental set up and to determine direction of motion dependence on the normal force and apparent viscosity reduction. Cavitation effects are not apparent in our measurements; therefore the normal force produced is solely from viscous or inertial fluid effects caused by surface textures.

There has been some disagreement in the literature about the dominant effect in producing normal forces with surfaces textures without cavitation. Arghir et. al. [21] suggests that the normal forces are produced due to inertial effects, while Dobrica and Fillon [22] suggest that the normal forces are produced due to viscous effects. Our work here will report normal force production as a function of angular velocity in order to determine which of these effects is apparent.

### 2.2 Materials and Methods

A schematic showing the experimental set up is given in Figure 3. A custom gap controlled tribo-rheometer developed and previously used by our group [8] was used to take the experimental measurement. This set up is similar to an experimental set up used by Kavehpour and McKinley [23] that allowed for gap controlled rheology data be used in tribological
measurements. The custom set up consisted of a gap controlled rotational rheometer (combined motor-transducer, DHR-3, TA Instruments) that could accurately measure the normal force and torque produced due to the shearing of the Newtonian lubricant between two parallel plates. The normal force is measured with a force rebalancing transducer coupled to the top (moving) plate with a manufacturer specified $F_{n,\text{min}} = 5 \text{ mN}, F_{n,\text{max}} = 50 \text{ N}$, and a resolution of 0.5 mN. The top plate was a standard 40 mm stainless steel parallel plate that was allowed to rotate in both directions, while the bottom plate was a custom machined 40 mm 1018 steel plate with varying surface textures that was attached to a temperature controlled Peltier plate using Crystalbond, a thermo-reversible adhesive, and remained fixed. The textures were then aligned to the axis of rotation of the rheometer by manipulating the screws on the Peltier plate such that the misalignment of the system (defined as the difference between the maximum and minimum deflection measured by a dial indicator) was less than 1 µm.

The textures were made by tilting a blank disk to the specified $\beta$ value and using a 1/8” end mill that rotated in a 6 mm diameter circle to machine the texture. A computer aided design (CAD) model of the asymmetric textures is given in Figure 4, where the material was removed in the same way that the textures were manufactured. The CAD model is made using the parameters $2R$, $\beta$, and $\delta$, as shown in Figure 4C. Figure 5 shows the centerline profile of each surface texture tested, and Table 1 gives the geometric parameters for each texture. The centerline profiles were measured using a laser displacement meter manufactured by Keyence, and the geometric parameters were measured using calipers and a KLA Tencor P-11 stylus profilometer. The geometric parameters relate to the $2R$, $\beta$, and $\delta$ parameters of the CAD model through
As can be seen in the table, the dimensional parameters of the textures tested are relatively large; however, Johnston, King, and Ewoldt [8] showed that the non-dimensional geometric ratios \( h/D \) and \( h/W \), where \( h \) is the specified gap) are more important than the dimensional values. Our textures were tested at \( h/D \) of 0.113-0.450, and \( h/W \) of 0.042-0.167, which are in between the minimum and maximum values reported in the literature [7,8,16,17,18,22].

The textures were tested using S600, a Newtonian lubricant manufactured by Cannon Instrument Company with a nominal viscosity \( \eta = 1.4 \text{ Pa s} \). The viscosity of the fluid as a function of temperature is given in the Figure 6, and the highlighted value is the rated viscosity for the temperature used in the experiments.

### 2.3 Risk of Misinterpreting Data

Experimental effects can cause a misinterpretation of the friction reduction of the surface textures. We present the results here, and also show how we can correct for them, in order to give accurate results.

#### 2.3.1 Viscosity Reduction

In order to set the gap for each of the tests performed, a gap zeroing procedure must first be performed. During this procedure, the rheometer head lowers until it measures a specified normal force. When this force is measured, the rheometer sets that gap as the zero plane, and all specified gaps are given with respect to this plane. However, during the gap zeroing procedure, the air trapped between the top and bottom plates is squeezed out, and this squeeze flow can
cause the zero gap normal force to be measured before actual plate-plate contact [24,25,26]. This leads to a gap offset error $\varepsilon$, where the gap between the top and bottom plates is larger than the specified gap.

$$h_i = h_a + \varepsilon$$  \hspace{1cm} (2.2)

Connelly and Greener [25] extensively studied the effects of this gap offset, and showed that a reduction in viscosity for Newtonian lubricants in steady simple shear flow can be seen as the gap is decreased. Using the equation for calculating the viscosity of Newtonian fluids given by Macosko [27]

$$\eta = \frac{2Mh}{\pi\Omega R^4}$$  \hspace{1cm} (2.3)

and substituting Equation (2.2), the gap offset error $\varepsilon$ can be calibrated from

$$\eta_a = \frac{\eta_t}{1 + \frac{\varepsilon}{h_a}}$$  \hspace{1cm} (2.4)

where $\eta_a$ is the measured apparent viscosity, $\eta_t$ is the rated viscosity of the fluid, and $h_a$ is the apparent gap height.

Pipe, Majmudar, and McKinley [26] also examined the effects of decreasing viscosity for decreasing apparent gap height. They gave a reformulation of Equation (2.4) in point slope form, given as

$$\frac{1}{\eta_a} = \frac{1}{\eta_t} + \frac{\varepsilon}{\eta_t} \frac{1}{h_a}$$ \hspace{1cm} (2.5)

Both methods of fitting were used to determine $\varepsilon$ and are shown in Figure 7. Figure 7A shows the method given by Connelly and Greener, Figure 7B shows the method given by Pipe, Majmudar, and McKinley. The gap offset error for our set up was found to be $\varepsilon = 19.0 \pm 0.63 \mu m$. 


This was then used with Equations (2.2) and (2.3) in order to determine the corrected viscosity, which is shown in Figure 7A. As can be seen in the figure, after the correction has been made, the apparent viscosity is a constant as a function of gap height, and matches very closely to the rated viscosity of the fluid. All subsequent viscosity calculations will take this gap-offset error into account so that all reductions in viscosity are solely due to the surface textures.

2.3.2 Normal Force Production

Experimental effects can cause a non-zero normal force to be measured between parallel plates in steady simple shear flow. Andablo-Reyes et. al. [28,29] showed that non-parallelism between the two plates can cause a normal force to be produced, and the normal force is given by

\[
F_N = \Omega \frac{\eta R^4}{h^3} \left( 0.256 \frac{\alpha}{h} \right)
\]  

(2.6)

where \(\alpha\) is the measurement of the non-parallelism (given in \(\mu m\)), \(R\) is the radius of the plate, and \(\Omega\) is the angular velocity. As can be seen from Equation (2.6), this normal force increases as the gap height decreases.

Due to the circular streamlines produced in the rotational rheometer, the fluid inertia can also cause a non-zero normal force to be measured. Macosko [27] gave this force as

\[
F_N = -\frac{3}{40} \pi \rho \Omega^2 R^4
\]  

(2.7)

where \(\rho\) is the fluid inertia.

Finally, if the sample fill is not ideal, surface tension effects can also affect the measurements [30,31]. Usually, this effect is a constant value, and is given as

\[
F_N = C
\]  

(2.8)
Figure 8B shows the magnitude of each of the forces individually, as well as the sum of all the forces. All of these forces must be corrected for in order to report accurate results with the surface textures. The non-parallelism force cannot be eliminated, but can be minimized by minimizing $\alpha$, and has been in this work. The inertia and surface tension effects can be calculated at each angular velocity and can be subtracted from the raw data.

$$F_c - F_{co} = F_{N_{raw}} - \left(-\frac{3}{40}\pi \rho \Omega^2 R^4\right) - C$$  \hspace{1cm} (2.9)

Figure 8A shows the effects of all the above experimental effects on the raw normal force data, and the data after the above corrections have been made. Also shown in Figure 8A is the experimental limit for our system; any normal forces that lie within this band cannot be believed. As can be seen in Figure 8A, after the corrections have been made, the reported normal force between two parallel plates lies within the experimental limit. All subsequent normal force data will have these corrections built in, so that all normal forces reported are solely due to the surface textures.

### 2.4 Results and Discussion

#### 2.4.1 Viscosity Reduction

The apparent viscosity reduction for each of the surface textures is given in Figure 9 and is calculated using Equation (2.3). Also shown in the figure is when the Nahme number, defined as [24]

$$Na = \frac{U^2 \partial \eta}{\kappa \partial T}$$  \hspace{1cm} (2.10)
equals 0.1, where $\kappa$ is the thermal conductivity of the lubricant. This serves to define where viscous heating effects are dominant, and any reduction that occurs beyond this point is neglected.

The apparent viscosity for the flat plate control matches the rated viscosity of the fluid independent of the direction of motion, up until viscous heating effects dominate. The surface textures reduce the apparent viscosity from the flat plate reference independent of the direction of motion, which matches results given by Johnston, King, and Ewoldt [8] and Qiu and Khonsari [9]. The symmetric texture reduces the apparent viscosity more than the asymmetric textures with $\beta=21.7^\circ$, 9.4$^\circ$, and 5.3$^\circ$. This is due to the change in gap height inside the surface texture. The shear stress in cylindrical coordinates for a Newtonian fluid is defined as [27]

$$\tau_{\theta z} = \eta \left( \frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right)$$

(2.11)

where $v_\theta$ is the velocity in the $\theta$ direction and $v_z$ is the velocity in the $z$ direction. If the gap height is much smaller than the radius of the plate, then the shear stress can be approximated by

$$\tau_{\theta z} = \eta \frac{\partial v_\theta}{\partial z}$$

(2.12)

In order to reduce the shear stress, the velocity gradient must be lowered. The velocity spanned by the top plate was the same for all the tests performed. Therefore, $\frac{\partial v_\theta}{\partial z}$ can only be lowered by making the gap height larger. The asymmetric textures increase the gap height linearly from $h$ to $h+D$ such that the maximum spacing between the plates occurs only at one point in the texture. The symmetric texture, on the other hand, increases the gap from $h$ to $h+D$ over the entire length of the texture.
The asymmetric texture with $\beta=14^\circ$ decreased the apparent viscosity the most out of all the textures tested. This is due to the fact that the symmetric texture and asymmetric texture with $\beta=21.7^\circ$ create a large recirculation zone inside the texture. This recirculation zone creates laminar vortices, similar to secondary flow effects. Ewoldt, Johnston, and Caretta [32] have shown that secondary flow effects can cause an increase in apparent viscosity in Newtonian fluids. Therefore, a large recirculation zone inside the textures would cause a higher viscosity to be measured. The asymmetric texture with $\beta=14^\circ$ will have the smallest recirculation zone, while also having the largest change in apparent gap height, causing it to reduce the apparent viscosity the most out of the textures tested. This suggests that there is an optimal texture configuration for decreasing the apparent viscosity with Newtonian lubricants.

2.4.2 Normal Force Production

The normal force produced by all of the surface textures tested in given in Figure 10. Also shown in the figure is the experiment limit for our system; any forces that fall within the experimental limit must be ignored. The normal forces produced by the flat plate lie within the experimental limit for all the gap heights tested, independent of the direction of motion. The forces produced by the symmetric texture lie just outside the experimental limit, independent of the direction of motion. This matches theoretical results by Stachowiak and Bachelor [19], who showed that symmetric textures will not produce a normal force when cavitation is not considered.

The forces produced by all the asymmetric textures are much larger than the experimental limit. There is also a monotonic trend where decreasing the value of $\beta$ leads to an increase in forces produced. It is also seen that when the velocity of the top plate changes directions, the
sign of the forces produced by the asymmetric textures also changes sign, suggesting that cavitation effects are not seen. For the asymmetric textures with $\beta=9.4^\circ$ and $5.3^\circ$, the forces produced are symmetric with respect to $F_N=0$, while for the textures with $\beta=21.7^\circ$ and $14^\circ$, the forces are not symmetric. This is due to the secondary slope from the maximum depth of the texture back up to the non-textured region seen with $\beta=21.7^\circ$ and $14^\circ$ which is not seen with $\beta=9.4^\circ$ and $5.3^\circ$.

The power law scaling of the forces produced with the textures is given in Figure 11. As stated before, there have been some disagreements in the literature about the dominant effect for producing normal forces with surface textures [21,22]. The power law scalings for the dominant effects are given by

\[
\begin{align*}
F_{\text{inertia}} & \sim \Omega^2 \\
F_{\text{viscous}} & \sim \Omega
\end{align*}
\]

(2.13)

The power law scaling seen by our textures suggests that $F_N \sim \Omega$ up to $\text{Re}_h = 1.21$, meaning that the forces are caused by viscous effects. The asymmetric texture with $\beta=21.7^\circ$ shows a different power law scaling when the direction of motion is reversed; this is most likely due to the secondary slope in the texture from the location of maximum depth up to the non-textured region.

### 2.4.3 Non-Dimensional Analysis

The non-dimensional normal force plotted against non-dimensional viscosity is given in Figure 12. The non-dimensional viscosity is found by dividing the apparent viscosity by the true viscosity of the fluid. The non-dimensional force is found by normalizing the normal force produced by a viscous force, since the normal forces due to the textures are produced by viscous
effects. There is a non-monotonic trend relating non-dimensional viscosity to the non-dimensional normal force. The asymmetric texture with $\beta=14^\circ$ shows the lowest non-dimensional viscosity, but produces a smaller normal force than $\beta=9.4^\circ$ and $\beta=5.3^\circ$. This suggests that there is an optimal texture profile for the asymmetric textures for decreasing apparent viscosity and producing normal forces.

2.5 Conclusions

This work examined the surface texture depth profile effects on apparent viscosity reduction and normal force production. The key results are:

- Surface textures decrease the apparent viscosity from the flat plate reference, matching results previous reported in the literature [8,9].
- When cavitation effects are not considered, the symmetric texture produces forces barely outside the experimental limit, while the asymmetric textures produces forces that are orders of magnitude larger than the experimental limit, showing that symmetry must be broken with surface textures in order to produce observable normal forces.
- The forces produced by surface textures show a power law scaling of 1 with respect to angular velocity, suggesting that the forces are produced by viscous effects.
- The non-dimensional analysis of the viscosity reduction and normal force production suggests that there is an optimal asymmetric texture profile for producing normal forces and decreasing apparent viscosity.

These experimental results serve as a basis for determining the optimal texture surface for decreasing friction in lubricated sliding contact. Asymmetry must be considered when
performing the optimization in order to accurately determine the normal forces produced by the textures.
Figure 3: Experimental set up showing the three different types of surface textures tested. $F_N$ is the measured normal force and $M$ is the measured torque. The drawing is to scale with the gap between the flat and textured surface set to 1mm. In the experiments, the textured plate is stationary while the top plate rotates in both directions.
Figure 4: Computer aided design (CAD) model of the asymmetric surface texture. (A) shows the texture and the surrounding material. (B) shows the texture as a wireframe. (C) shows the centerline cross sectional view of the texture. $2R$ is the machined area used to make the texture, $\beta$ is the angle that the material was tilted to make the texture, and $\delta$ is the depth of the cut. The material is removed in the same way that the experimentally tested plates were made.
Figure 5: Depth profiles of the 6 difference surfaces tested. A flat plate, a symmetric texture, and 4 asymmetric textures with different β values were tested.
Table 1: Measured properties of the surface textures tested.

<table>
<thead>
<tr>
<th></th>
<th>Roughness Measurements</th>
<th>Geometric Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_a$ [$\mu$m]</td>
<td>$R_q$ [$\mu$m]</td>
</tr>
<tr>
<td>Flat Plate</td>
<td>0.064</td>
<td>0.110</td>
</tr>
<tr>
<td>Symmetric</td>
<td>0.276</td>
<td>0.352</td>
</tr>
<tr>
<td>$\beta=21.7^\circ$</td>
<td>0.101</td>
<td>0.137</td>
</tr>
<tr>
<td>$\beta=14^\circ$</td>
<td>0.935</td>
<td>1.23</td>
</tr>
<tr>
<td>$\beta=9.4^\circ$</td>
<td>0.959</td>
<td>1.25</td>
</tr>
<tr>
<td>$\beta=5.3^\circ$</td>
<td>0.883</td>
<td>1.16</td>
</tr>
</tbody>
</table>
Figure 6: Viscosity temperature dependence for the lubricants used in the experiments. S600 is the high viscosity standard with nominal viscosity $\eta=1.4$ Pa s, and S60 is the low viscosity standard with nominal viscosity $\eta=0.14$ Pa s. The highlighted points are the viscosities at the temperature used in the experiments.
Figure 7: Risk of misinterpreting shear stress reduction due to gap offset error. Two different fitting equations were used to determine the gap offset error for our system. (A) used the method given by Connelly and Greener [25]. (B) used the method given by Pipe, Majmudar, and McKinley [26]. Three different gap-zeroing procedures are shown, giving the repeatability of the measurements. The reported gap error is $\varepsilon=19.0 \pm 0.63 \mu m$. 
Figure 8: Risk of misinterpreting normal force production due to experimental effects that produce normal forces between parallel plates. (A) shows the difference between the raw experimental data and the corrected data. (B) shows the experimental effects that can produce normal forces between parallel plates.
Figure 9: Experimental measurements of apparent viscosity with a high viscosity oil standard with nominal shear viscosity $\eta=1.4$ Pa s. (A) is the flat plate control. (B) is the symmetric texture. (C) is asymmetric texture with $\beta=21.7^\circ$. (D) is the asymmetric texture with $\beta=14^\circ$. (E) is the asymmetric texture with $\beta=9.4^\circ$. (F) is the asymmetric texture with $\beta=5.3^\circ$. Viscous heating effects are seen, and are dominant when the Nahme number defined as $Na = \frac{U^2}{\kappa} \frac{\partial \eta}{\partial T} > 0.1$. 
Figure 10: Experimental measurements of normal forces produced with a high viscosity oil standard with nominal shear viscosity $\eta=1.4$ Pa s. (A) is the flat plate control. (B) is the symmetric texture. (C) is asymmetric texture with $\beta=21.7^\circ$. (D) is the asymmetric texture with $\beta=14^\circ$. (E) is the asymmetric texture with $\beta=9.4^\circ$. (F) is the asymmetric texture with $\beta=5.3^\circ$. 
Figure 11: Determining the scaling of the normal forces produced with a high viscosity oil standard with nominal shear viscosity $\eta=1.4$ Pa s. (A) is the flat plate control. (B) is the symmetric texture. (C) is asymmetric texture with $\beta=21.7^\circ$. (D) is the asymmetric texture with $\beta=14^\circ$. (E) is the asymmetric texture with $\beta=9.4^\circ$. (F) is the asymmetric texture with $\beta=5.3^\circ$. An example power law scaling of 1 is shown, which is the scaling of forces due to viscous effects.
Figure 12: Non-dimensional normal force production as a function of non-dimensional viscosity for the symmetric and asymmetric surface textures tested.
CHAPTER 3: Numerical Solution of the Reynolds Equation

3.1 Introduction

Chapter 2 showed that the normal forces produced by the surface textures are due to viscous effects up to $\text{Re}_h = 1.21$, suggesting that the Reynolds equation can be used as the mathematical model for the lubricated system. The Reynolds equation is a single simplified form of the conservation of momentum equations that also satisfies mass conservation. Several numerical studies on the effects of surface textures have been performed using the Reynolds equation [33,34,35]. Journal bearings [36,37], parallel sliders [38], and conforming contacting surfaces [39,18] have been examined, and show that the application of surface textures increases the load carrying capacity of the system. Optimization studies have also been performed in order to determine an optimal texture profile [18,12,15], where the optimal texture profile was selected from a specified set of allowable texture profiles.

A validated numerical method is needed in order to solve the inverse design problem of determining the optimal surface texture for producing normal forces and decreasing apparent viscosity. In our numerical model of the texture system, the Reynolds equation will be used to mathematically model the system, based on the scaling of the normal forces reported in Chapter 2. This equation will need to be solved accurately and at a low computational cost due to the
number of computations that need to be performed in order to fully explore the texture design space.

The most popular method for solving the Reynolds equation with surface textures has been the finite difference method (FDM) [33,34,35,36,37,38,39,40,41]. FDM is the simplest method to implement, and the resulting matrices are sparse because the differences are defined with respect to nearest nodal points. However, the solution using FDM is not guaranteed to be continuous, differentiable, or integrable, since FDM only acts on the function at specified grid points [42]. The solution also converges to the true solution for a given spatial discretization with the leading error term \( \varepsilon \) defined as

\[
\varepsilon \sim \frac{1}{N^a}
\]  

(3.1)

where \( N \) is the number of grid points in an equally spaced mesh and \( a \) is the convergence rate that depends on the type of FDM used; \( a=1 \) for Euler-Forward/Euler-Backward, \( a=2 \) for Central Finite Differences, and \( a>2 \) for higher order differencing schemes. Therefore a large number of grid points is needed in order to decrease the truncation error, resulting in a large computational cost.

The finite volume method (FVM) has also been used [22], and the results can be better than FDM, since FVM is a conservative method. This is because the efflux from one control volume is the influx into the next control volume. The matrices produced using FVM are also sparse, since one control volume only interacts with its nearest neighbors [43]. However, the solution using FVM also cannot be guaranteed to be continuous, differentiable, or integrable, since FVM only acts at discrete points, usually the center of the control volume. It also converges to the true solution for a given spatial discretization with the leading error term defined the same as Equation (3.1) [43].
One method to avoid the problems associated with FDM and FVM is to use the finite element method (FEM) [44]. The resulting solution is guaranteed to be continuous and integrable, since FEM operates on coefficients of basis functions instead of discrete functional points [42]. The resulting matrices are still sparse, since the basis functions are defined to be non-zero locally. The convergence rate in the spatial discretization is the same as Equation (3.1) where $a=4$ for piecewise linear finite elements. However, a large number of grid points are still needed in order to decrease the truncation error, resulting in a large computational cost.

An even better method is the spectral/pseudo-spectral method [45]. In this method, the basis functions are smooth, continuous functions that are defined over the entire domain, guaranteeing the solution to be continuous, integrable, and differentiable. The solution converges to the true solution for a given spatial discretization with the leading error $\varepsilon$ defined as [46,47,48]

$$\varepsilon \sim \frac{1}{e^N}$$  \hspace{1cm} (3.2)

where $N$ is the number of grid points. However, the resulting matrices are no longer sparse, because the basis functions are defined everywhere in the domain. However, since the solution converges exponentially in space, a very small number of grid points are needed in order to guarantee convergence of the system.

This is the first work to present a pseudo-spectral method for solving the steady state Reynolds equation in cylindrical coordinates. The exponential convergence in space allows for a low computational cost, which is ideal for exploring the entire design space for design optimization. Cavitation results are not considered and are outside the scope of this work. The numerical results obtained with the pseudo-spectral method are compared to an analytical solution and experimental data given in Chapter 2 for militextured thrust bearings in order to validate the numerical method.
3.2 Governing Equation

The Reynolds equation in cylindrical coordinates can be derived from the mass conservation for an incompressible fluid and the Navier-Stokes equations in cylindrical coordinates, given as

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( rv_r \right) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0
\]  

(3.3)

\[
\rho \left( \frac{Dv_r}{Dt} - \frac{v_\theta v_\theta}{r} \right) = -\frac{\partial p}{\partial r} + \eta \left( \Delta^2 v_r - \frac{v_r}{r^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} \right)
\]

(3.4)

\[
\rho \left( \frac{Dv_\theta}{Dt} - \frac{v_r v_\theta}{r} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \eta \left( \Delta^2 v_\theta - \frac{v_\theta}{r^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} \right)
\]

\[
\rho \left( \frac{Dv_z}{Dt} \right) = -\frac{\partial p}{\partial z} + \eta \left( \Delta^2 v_z \right)
\]

These equations can be simplified by making the following assumptions:

1) Inertial terms are negligible

2) \( h_0 / R \rightarrow 0 \), where \( h_0 \) is the reference gap height and \( R \) is the reference radius

3) The pressure is invariant in the \( z \) direction

4) The fluid is an isoviscous Newtonian fluid

The simplified Navier-Stokes equations then become

\[
\frac{\partial p}{\partial r} = \eta \frac{\partial^2 v_r}{\partial z^2}
\]

(3.5)

\[
\frac{1}{r} \frac{\partial p}{\partial \theta} = \eta \frac{\partial^2 v_\theta}{\partial z^2}
\]

where \( v_r(r, \theta, z) \) is the velocity in the \( r \) direction, \( v_\theta(r, \theta, z) \) is the velocity in the \( \theta \) direction, and \( v_z(r, \theta, z) \) is the velocity in the \( z \) direction. The boundary conditions are

\[
v_\theta = r \Omega \text{ and } v_r = v_z = 0 \text{ at } z = 0, \quad v_\theta = v_r = v_z = 0 \text{ at } z = h
\]

(3.6)
where $\Omega$ is the prescribed angular velocity of the flat plate and $h(r, \theta)$ is the gap height, which can be a function of both $r$ and $\theta$.

Solving Equation (3.5) through direct integration with respect to $z$ and with the applied boundary conditions gives

$$
\begin{align*}
v_r &= \frac{1}{2\eta} \frac{\partial p}{\partial r} \left( z^2 - zh \right) \\
v_\theta &= \frac{1}{2\eta} \frac{1}{r} \frac{\partial p}{\partial \theta} \left( z^2 - zh \right) + \frac{1}{r} \Omega \left( \frac{h - z}{h} \right)
\end{align*}
$$

(3.7)

It can be seen from Equation (3.7) that $v_\theta$ is a linear combination of both simple shear flow and pressure driven flow. Substituting Equation (3.7) into the continuity equation given in Equation (3.3) and applying Liebnitz’s integration rule gives the Reynolds equation in cylindrical coordinates as

$$
\begin{align*}
\frac{1}{r} \frac{\partial}{\partial r} \left( rh^3 \frac{\partial p}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{h^3}{r} \frac{\partial p}{\partial \theta} \right) &= 6\eta \Omega \frac{\partial h}{\partial \theta} + 12 \eta \frac{\partial h}{\partial t}
\end{align*}
$$

(3.8)

If the system is assumed to be in steady state, then the steady form of the Reynolds equation in cylindrical coordinates is given as

$$
\begin{align*}
\frac{1}{r} \frac{\partial}{\partial r} \left( rh^3 \frac{\partial p}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{h^3}{r} \frac{\partial p}{\partial \theta} \right) &= 6\eta \Omega \frac{\partial h}{\partial \theta}
\end{align*}
$$

(3.9)

which is an elliptic non-constant coefficient partial differential equation. If the gap height $h(r, \theta)$ is prescribed, and boundary conditions for the pressure in the $r$ and $\theta$ direction are specified, then Equation (3.9) can be explicitly solved for the pressure, which can then be used to find the velocity field from Equation (3.7). This work will consider two different boundary conditions for the pressure in the $r$ direction.
3.3 Formulation and Solution Procedure

Equation (3.9) is a linear, non-constant coefficient partial differential equation (PDE) for the pressure. The pseudo-spectral method solves the differential equation using a weighted residual technique (WRT) that computes an approximate solution to the differential equation [42]. The WRT is applied in this form [49]:

Let \( X \) be an infinite dimensional space, and let \( v, u \in X \). Then find \( v \) and \( u \) such that

\[
\int \limits_{\Omega} vu \, d\Omega = f(v)
\]  

(3.10)

where \( f \) is some prescribed function of \( v \). The computer cannot actually operate in the infinite dimensional space; therefore introduce \( X_N \subset X \) which is the \( N^{th} \) dimensional subspace of \( X \). Then find \( v_N \) and \( u_N \) such that

\[
\int \limits_{\Omega} v_N u_N \, d\Omega = f(v_N)
\]  

(3.11)

The residual is defined as \( u-u_N \). It can be seen that when calculating the inner product of the residual and \( v_N \) given as

\[
\int \limits_{\Omega} \int \left( u-u_N \right) v_N \, d\Omega = \int \limits_{\Omega} v_N u \, d\Omega - \int \limits_{\Omega} v_N u_N \, d\Omega = f(v_N) - f(u_N) = 0
\]

that the residual is orthogonal to the chosen subspace. Therefore the WRT finds the best function in the approximation space that satisfies the given conditions.

Applying the weighted residual technique to Equation (3.9) yields the weak form of the Reynolds equation:

\[
\int \limits_{-\phi/2}^{\phi/2} \int \limits_{-\phi/2}^{\phi/2} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{h^3}{r} \frac{\partial}{\partial r} \right) r dr d\theta \right) + \int \limits_{-\phi/2}^{\phi/2} \int \limits_{-\phi/2}^{\phi/2} \left( \frac{1}{\rho \omega} \frac{\partial}{\partial \theta} \left( \frac{h^3 \rho \omega}{r} \right) \right) r dr d\theta = 6 \int \frac{\partial h}{\partial \theta} r dr d\theta
\]  

(3.12)
where $\varphi$ is the span in the $\theta$ direction and $R_i$ and $R_o$ are the inner and outer radii respectively. This can be simplified using integration by parts to reduce the degree of differentiability on the left hand side [42] and by applying boundary conditions to obtain

$$
-\int_{-\varphi/2}^{\varphi/2} \int_{R_i}^{R_o} \left( \frac{\partial v}{\partial r} r h^3 \frac{\partial p}{\partial r} + \frac{\partial v}{\partial \theta} h^3 \frac{\partial p}{\partial \theta} \right) dr d\theta = 6\eta \Omega \int_{-\varphi/2}^{\varphi/2} \int_{R_i}^{R_o} v r \frac{\partial h}{\partial \theta} dr d\theta
$$

(3.13)

The integrals in Equation (3.13) can be solved numerically. It is desired to use Gauss-Lobatto-Legendre quadrature to evaluate the integrals. For this quadrature rule, the quadrature weights are optimally chosen and the function evaluations occur at the zeros of the $N^{th}$ order Legendre polynomial, resulting in a quadrature rule that is exact for polynomials of degree $2N-1$ [42]. Because the zeros of the $N^{th}$ order Legendre polynomial exist on the domain [-1,1], a change of variables must be performed in order to evaluate the integrals. The new variables are given as

$$
\xi = \frac{2}{R_o - R_i} \left( r - \frac{R_o + R_i}{2} \right)
$$

$$
\psi = \frac{2}{\varphi} \theta
$$

(3.14)

Substituting Equation (3.14) into Equation (3.13) gives

$$
-\frac{\varphi}{R_o - R_i} \int_{-1}^{1} \int_{-1}^{1} \frac{\partial v}{\partial \xi} r(\xi) h^3 \frac{\partial p}{\partial \xi} d\xi d\psi
$$

$$
-\frac{R_o - R_i}{\varphi} \int_{-1}^{1} \int_{-1}^{1} \frac{\partial v}{\partial \psi} h^3 \frac{\partial p}{\partial \psi} \frac{1}{r(\xi)} d\xi d\psi
$$

(3.15)

$$
= 3\eta \Omega (R_o - R_i) \int_{-1}^{1} \int_{-1}^{1} v r(\xi) \frac{\partial h}{\partial \psi} d\xi d\psi
$$

Finally, the pseudo-spectral method assumes that the functions $v$ and $p$ can be written as linear combinations of basis functions. If the Galerkin method is used, then the basis functions for $v$ and $p$ are the same [42] and $v$ and $p$ can be written as
\[ v = \sum_{i=1}^{N} \sum_{j=1}^{N} v_{ij} \rho_{i}(\xi) \rho_{j}(\psi) \]
\[ p = \sum_{i=1}^{N} \sum_{m=1}^{N} p_{im} \rho_{i}(\xi) \rho_{m}(\psi) \]

where \( \rho \) is the basis function in one dimension, \( v_{ij} \) and \( p_{im} \) are the coefficients evaluated at the grid points, and \( N \) is the total number of basis functions, corresponding to the number of zeros of the \( N \)th order Legendre polynomial. For the pseudo-spectral method, the basis functions are \( N \)th order Lagrange polynomials whose zeros are the zeros of the \( N \)th order Legendre polynomial. The example basis functions in 1-D and 2-D are given in Figure 13. These basis functions are used because \( \rho_{i}(\xi_{j}) = \delta_{ij} \), which will be useful when numerically evaluating the integrals.

Substituting Equation (3.16) into Equation (3.15) gives

\[ v_{ij} \left( \sum_{i,j=1}^{N} \sum_{m=1}^{N} \frac{\varphi}{R_{0} - R_{i}} \int_{-1}^{1} \frac{d \rho_{i}}{d \xi} \rho_{j}(\xi) h^{3} \frac{d \rho_{i}}{d \xi} \rho_{m} d \xi d \psi \right) p_{im} + v_{ij} \left( \sum_{i,j=1}^{N} \sum_{m=1}^{N} \frac{R_{0} - R_{i}}{\varphi} \int_{-1}^{1} \frac{d \rho_{i}}{d \psi} \rho_{j}(\xi) \frac{h^{3}}{r(\xi)} \rho_{i} \frac{d \rho_{m}}{d \psi} d \xi d \psi \right) p_{im} = v_{ij} \left( \sum_{i=1}^{N} 3 \varphi \Omega (R_{0} - R_{i}) \int_{-1}^{1} \rho_{i} \rho_{j}(\xi) \frac{\partial h}{\partial \psi} d \xi d \psi \right) \]

It can be seen that Equation (3.17) can be written in matrix form as

\[ \bar{v}^{T} K \bar{p} = \bar{v}^{T} \bar{f} \]

which is a linear system of equations where \( \bar{v} \) and \( \bar{p} \) are vectors containing the coefficients and

\[ K = \sum_{i,j=1}^{N} \sum_{m=1}^{N} \frac{\varphi}{R_{0} - R_{i}} \int_{-1}^{1} \frac{d \rho_{i}}{d \xi} \rho_{j}(\xi) h^{3} \frac{d \rho_{i}}{d \xi} \rho_{m} d \xi d \psi + \sum_{i,j=1}^{N} \sum_{m=1}^{N} \frac{R_{0} - R_{i}}{\varphi} \int_{-1}^{1} \rho_{i} \frac{d \rho_{j}}{d \psi} \frac{h^{3}}{r(\xi)} \rho_{i} \frac{d \rho_{m}}{d \psi} d \xi d \psi \]

and
\[
\tilde{f} = \sum_{i,j=1}^{N} 3\eta \Omega (R_o - R_i) \int_{-1}^{1} \int_{-1}^{1} \rho_j r(\xi) \frac{\partial h}{\partial \psi} d\xi d\psi
\]  

(3.20)

It should be noted that \( K \) is symmetric and depends only on the 1st derivatives of the basis functions [42]; this is a direct result of using integration by parts in Equation (3.13). Using the Gauss-Lobatto-Legendre quadrature rule yields \( K \) and \( \tilde{f} \) in matrix form as [50]

\[
K = -\frac{\varphi}{R_o - R_i} (I \otimes D) \left( M \otimes M \right) \left( I \otimes R \right) H^3 \left( I \otimes D \right)
\]

\[- \frac{R_o - R_i}{\varphi} (D \otimes I) \left( M \otimes M \right) \left( I \otimes R^{-1} \right) H^3 (D \otimes I)
\]

\[
\tilde{f} = 3 \eta \Omega (R_o - R_i) \left( M \otimes M \right) \left( I \otimes R \right) \left( D \otimes I \right) H
\]

(3.21)

(3.22)

where \( M \) is a diagonal matrix containing the quadrature weights, \( D \) is a full matrix where \( D_{ij} = \frac{d \rho_j}{d \xi_i} \), \( R \) is a diagonal matrix containing the values of \( r \) from \( R_i \) to \( R_o \), \( H \) is a diagonal matrix containing the values of \( h \) at the grid points, \( I \) is the identity matrix, and \( \otimes \) specifies the tensor product allowing the matrices obtained in one dimension to be extended to higher dimensions [50]. It was assumed in this derivation that \( h \) could also be written as

\[
h = \sum_{a=1}^{N} \sum_{b=1}^{N} h_{ab} \rho_a(\xi) \rho_b(\psi)
\]

The above analysis did not take the boundary conditions into account. This can be done using [51,52]

\[
\tilde{v} = B \tilde{v}'
\]

\[
\tilde{p} = B \tilde{p}'
\]

(3.23)

where \( \tilde{v}' \) and \( \tilde{p}' \) are the vectors containing the coefficients at the interior nodes and \( B \) is the matrix \( B_\theta \otimes B_r \) where \( B_\theta \) defines the boundary conditions in the \( \theta \) direction and \( B_r \) defines the boundary conditions in the \( r \) direction. Substituting Equation (3.23) into Equation (3.18) gives
\( \tilde{v}^T \tilde{B}^T K \tilde{p}' = \tilde{v}^T \tilde{B}^T \tilde{f} \)  

(3.24)

which can be rewritten as

\( \tilde{v}^T K' \tilde{p}' = \tilde{v}^T \tilde{f}' \)  

(3.25)

where

\[
K' = B^T KB  \\
\tilde{f}' = B^T \tilde{f}
\]

For non-trivial solutions, Equation (3.25) can then be rewritten as find \( \tilde{p}' \) such that

\[
K' \tilde{p}' = \tilde{f}'
\]

(3.26)

Since \( K' \) and \( \tilde{f}' \) are already known from Equations (3.21), (3.22), and (3.24) for a given \( h(r,\theta) \) and \( \Omega \), Equation (3.26) can be inverted in order to obtain \( \tilde{p}' \) directly, and the full solution \( \tilde{p} \) that satisfies the boundary conditions can be obtained from Equation (3.23). The obtained pressure solution will be used to calculate the normal force on the flat plate through an integration of the pressure over the area. The pressure will also be used to determine the velocity profile through Equation (3.7), which can be used to determine the shear stress on the top plate.

### 3.4 Comparison to Analytic Solution

In order to validate the solution method used in Section 3.3, the numerical solution is compared to an analytic solution of the Reynolds equation in cylindrical coordinates. The analytic solution is obtained by assuming that \( R_o-R_i<<1 \), causing the pressure distribution to not vary in the \( r \) direction, and that \( h = h(\theta) \) only, which will be specified. These assumptions allow Equation (3.9) to be rewritten as

\[
\frac{1}{r} \frac{d}{d\theta} \left( \frac{h^3}{r} \frac{dp}{d\theta} \right) = 6\eta \Omega \frac{dh}{d\theta}
\]

(3.27)
For the analytic solution, the specified gap height is taken to be

\[ h = \frac{h_t - h_0}{\varphi} \theta + \frac{h_t + h_0}{2} \]  

(3.28)

where \( \varphi \) is the span in the \( \theta \) direction, \( h(\theta = \varphi/2) = h_t \), and \( h(\theta = -\varphi/2) = h_0 \). The boundary conditions are

\[ p\left(\theta = -\frac{\varphi}{2}\right) = p\left(\theta = \frac{\varphi}{2}\right) = 0 \]  

(3.29)

Solving Equation (3.27) for the pressure as a function of \( h \) with the boundary conditions gives

\[ p(h) = \frac{6\eta \Omega r^2}{(h_t - h_0) / \varphi} \left(-\frac{1}{h_t} + \frac{h_0 h_t}{(h_0 + h_t) h^2} + \frac{1}{h_0} \left(1 - \frac{h_t}{h_0 + h_t}\right)\right) \]  

(3.30)

which can be solved for \( p(\theta) \) by substituting in Equation (3.28). The \( r \) term in Equation (3.30) can be treated as a constant, because \( \Delta r << 1 \). The normal force acting on the plate is obtained through integrating in both the \( \theta \) and \( r \) direction, \( F_{\text{true}} = \int_{-\varphi/2}^{\varphi/2} \int_{r_i}^{r_f} p r \, dr \, d\theta \) and is the metric used to compare to the numerical solution.

The numerical solution is obtained by solving Equation (3.26) with appropriate boundary conditions. The applied boundary conditions are

\[ p\left(\theta = -\frac{\varphi}{2}\right) = p\left(\theta = \frac{\varphi}{2}\right) = 0, \quad \frac{\partial p}{\partial r} \bigg|_{r = r_i} = \frac{\partial p}{\partial r} \bigg|_{r = r_f} = 0 \]  

(3.31)

which are implemented through \( B_\theta \) and \( B_r \) as

\[
B_\theta = \begin{bmatrix}
0 & 0 & \ldots & 0 \\
1 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & 1 \\
0 & 0 & \ldots & 0
\end{bmatrix} \\
B_r = I
\]  

(3.32)
The boundary conditions in the $r$ direction must be specified, because we are using the original 2-D form of the steady state Reynolds equation given in Equation (3.9) and not the simplified form given in Equation (3.27). The normal force is obtained using Gauss-Lobatto-Legendre quadrature on the computed pressure by 

$$F_{\text{comp}} = \int_{-\frac{\varphi}{2}}^{\frac{\varphi}{2}} p_{\text{comp}} r d\theta,$$

where $p_{\text{comp}}$ is the computed pressure. The error between the computed normal force and the true normal force is given as

$$\varepsilon = \frac{|F_{\text{true}} - F_{\text{comp}}|}{|F_{\text{true}}|} \quad (3.33)$$

and is shown in Figure 14. As can be seen from the figure, the expected convergence rate of $e^{-N}$ is observed [46,47,48], validating the numerical method.

### 3.5 Comparison to Experiments

The numerical results are also compared to experimental results given in Chapter 2 for a millitextured thrust bearing. In the experiments in Chapter 2, the surface textures had a circular top profile and varying depth profiles as shown in Figure 5. Figure 15 shows the geometric quantities used to define the surface textures, and the values used for all the simulated textures are given in Table 1. Examples of the simulated texture surfaces are given in Figure 16.

Periodic boundary conditions in the $\theta$ direction, written as

$$p\left(\theta = -\frac{\varphi}{2}\right) = p\left(\theta = \frac{\varphi}{2}\right)$$

$$\frac{\partial p}{\partial \theta} \big|_{\theta=-\varphi/2} = \frac{\partial p}{\partial \theta} \big|_{\theta=\varphi/2} \quad (3.34)$$

are used to solve Equation (3.26) and are obtained through $B_\theta$ as
It is unclear what boundary conditions should be used in the $r$ direction. In the experiments, the inner radius goes to 0, while in the computations the inner radius is truncated to some small finite value, because of the $1/r$ term in Equation (3.19) which would go to infinity as $R_i \to 0$. For this work, both Dirichlet-Dirichlet and Nuemann-Dirichlet boundary conditions are used in order to determine which one is appropriate. They are given as

$$p(r = R_i) = p(r = R_o) = 0$$  \hspace{1cm} (3.36)

for the Dirichlet-Dirichlet boundary conditions and as

$$\frac{\partial p}{\partial r} |_{r=R_o} = 0, \quad p(r = R_o) = 0$$  \hspace{1cm} (3.37)

for the Nuemann-Dirichlet boundary conditions. Equation (3.36) is implemented numerically through $B_r$ as

$$B_r = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 \\
| & | & \cdots & | \\
0 & 0 & \cdots & 1 \\
0 & 0 & \cdots & 0
\end{bmatrix}$$  \hspace{1cm} (3.38)

for the Dirichlet-Dirichlet boundary conditions and Equation (3.37) is implemented as

$$B_r = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
0 & 1 & 0 & \cdots & 0 \\
| & | & | & \cdots & | \\
0 & 0 & \cdots & 1 & 0
\end{bmatrix}$$  \hspace{1cm} (3.39)
for the Nuemann-Dirichlet boundary conditions.

In order to validate that the solution is calculated correctly, the normal force was calculated using

\[
F_N = N_{\text{tex}} \int_{-\phi/2}^{\phi/2} \int_{R_i}^{R_o} \left( p \left|_{z=0} - \tau_{zz} \left|_{z=0} \right. \right) r \, dr \, d\theta \tag{3.40}
\]

where \( N_{\text{tex}} \) is the total number of surface textures on the thrust bearing, obtained from

\[
N_{\text{tex}} = \frac{2\pi}{\varphi} \tag{3.41}
\]

and \( \tau_{zz} \) is the shear stress on the \( z \) surface in the \( z \) direction, given as

\[
\tau_{zz} = 2\eta \frac{\partial v_z}{\partial z} \tag{3.42}
\]

Using the continuity equation given in Equation (3.3) allows Equation (3.42) to be rewritten as

\[
\tau_{zz} = 2\eta \left( \frac{1}{r} \frac{\partial}{\partial r} \left( rv_z \right) - \frac{1}{r} \frac{\partial v_\theta}{\partial z} \right) \tag{3.43}
\]

Substituting Equation (3.7) into Equation (3.43), performing the differentiation, and evaluating at \( z=0 \) gives that

\[
\tau_{zz} \mid_{z=0} = 0 \tag{3.44}
\]

Substituting Equation (3.44) into Equation (3.44), and noting that the pressure does not change as a function of \( z \) (see assumptions in Section 3.2) gives the final form of the computed normal force as

\[
F_N = N_{\text{tex}} \int_{-\phi/2}^{\phi/2} \int_{R_i}^{R_o} pr \, dr \, d\theta \tag{3.45}
\]

The normal force was plotted as a function of polynomial degree \( N \) in order to determine if the solution method is mesh independent. The results are given in Figure 17 for both the Dirichlet-Dirichlet and Nuemann-Dirichlet boundary conditions. The geometric properties used are given
in Table 1, and the viscosity of the fluid $\eta=0.95$ Pa s. It can be seen that when $N>25$, the solution has converged, which some oscillations in the computed normal force. This is because the simulated textures changed slightly for each change in $N$ due to the method used to generate the simulated textures.

Chapter 2 also reported apparent viscosity reduction for the surface textures. This can also be obtained numerically, because the velocity field is known after substituting the obtained pressure into Equation (3.7). The velocity field can then be used to obtain the shear stress; the dominant shear stress $\tau_{\theta z}$ is given as

$$\tau_{\theta z} = \eta \left( \frac{\partial v_{\theta}}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right)$$

(3.46)

Scaling analysis can be performed on Equation (3.46) in order to determine the dominant effect in the shear stress calculation. Using the same non-dimensional values used to derive the Reynolds equation gives

$$\tau_{\theta z} = \frac{\eta \Omega h}{h_0} \frac{\partial \tilde{v}_{\theta}^*}{\partial \tilde{z}^*} + \eta \Omega \frac{h_0}{R} \frac{1}{r} \frac{\partial \tilde{v}_z^*}{\partial \tilde{\theta}}$$

(3.47)

The Reynolds equation was derived based on the assumption that $h_0/R \rightarrow 0$; therefore, the term containing $h_0/R$ in Equation (3.47) must also be neglected. This then gives the shear stress as

$$\tau_{\theta z} = \eta \frac{\partial v_{\theta}}{\partial z}$$

(3.48)

Substituting Equation (3.7) into Equation (3.48) allows the shear stress to be calculated as

$$\tau_{\theta z} = \frac{1}{2r} \frac{\partial p}{\partial \theta} (z^2 - zh) + \eta \Omega r \frac{h - z}{h}$$

(3.49)

In Chapter 2, shear stress on the flat plate was calculated, which is obtained numerically by evaluating Equation (3.49) at $z=0$, giving
\[
\tau_{\theta z} \bigg|_{z=0} = -\frac{1}{2} \frac{\partial p}{\partial \theta} h - \eta \frac{r \Omega}{h} \tag{3.50}
\]

where the negative sign indicates that the shear stress is in the direction opposite to the motion.

A torque balance on the flat plate yields that the total torque measured experimentally is obtained as

\[
M = \int_0^{2\pi} \int_0^{\phi/2} \tau_{\theta z} \bigg|_{z=0} r^2 dr d\theta \tag{3.51}
\]

Macasko [27] showed that this measured torque can then be used to calculate the apparent viscosity as

\[
\eta_a = \frac{2h_0}{\pi R^4} \frac{M}{\Omega} \tag{3.52}
\]

The numerical torque value for the simulations can be obtained through a modified form of Equation (3.51) as

\[
M_{\text{comp}} = N_{\text{rev}} \int_{-\phi/2}^{\phi/2} \int_{0}^{R_i} \tau_{\theta z} \bigg|_{z=0} r^2 dr d\theta \tag{3.53}
\]

which can then be used with Equation (3.52) to calculate the simulated apparent viscosity.

Figure 18 shows a representative pressure profile computed for an asymmetric texture with $\beta=5.3^\circ$ with a fluid with a viscosity $\eta=0.95$ Pa s at an angular velocity of $\Omega=100$ rad/s at a gap height $h_0=250$ µm. The computed pressure profile can then integrated in order to determine the normal force production, which is given as a function of angular velocity for varying gap heights in Figure 19. The normal force shows a power law scaling of 1 with respect to $\Omega$, which matches the trends seen experimentally in Chapter 2. The computed pressure is also used to determine the velocity profile in the surface textures through the use of Equation (3.7). Representative velocity fields are shown in Figure 20 through 2-D slices of the surface texture.
Figure 20A shows $v_\theta$ and $v_z$ as a function of $z$ and $\theta$ at a radius $r=R_c$, and Figure 20B shows $v_r$ and $v_z$ as a function of $r$ and $z$ at $\theta=0$ rad. These velocity profiles are then used to calculate the shear stress on the flat plate, which is used to calculate the apparent viscosity for varying gap heights shown in Figure 21, which matches trends reported in Chapter 2.

The one symmetric and four asymmetric surface textures evaluated experimentally in Chapter 2 are also evaluated numerically with the Reynolds equation. Figure 22 shows the comparison between the experimental normal force and the computed normal force for the two boundary conditions used. It is noted that in Chapter 2 that velocity dependent viscous heating occurred for the Newtonian fluid with a rated viscosity $\eta_0=1.4$ Pa s. This was taken into account numerically by using the apparent viscosity obtained with a flat plate for each tested angular velocity at the different gap heights as the input viscosity for each simulation. Good agreement is shown between the numerical and experimental data for the Dirichlet-Dirichlet boundary conditions with a maximum error of 7.6% for the numerical data. The Nuemann-Dirichlet boundary conditions do not match the experimental data as well, and has a maximum error of 43.8%. This error is due to the pressure being a non-zero value at $r=R_i$, which would result in a much larger pressure field that when integrated would give a much larger normal force than the experimentally measured value.

The difference between the two boundary conditions can also be explained by examining the dependence of the experimentally measured normal force vs. angular velocity. In the experiments, when the angular velocity was less than 5 rad/s (which would be a tangential velocity of 0.1 m/s at the edge of the plate), the measured normal force was below the experimental limit, meaning that the pressure in the system is essentially zero. In the numerical simulations, the tangential velocity at the inner radius is a maximum of 0.1 m/s, which has
already been shown to yield a pressure of 0 Pa. This also explains why setting \( p(r=R_i)=0 \) gives a better result to experimental values than setting \( \frac{\partial p}{\partial r} \big|_{r=R_i} = 0 \).

The computed apparent viscosity is also compared to the experimentally measured apparent viscosity, and the results are given in Figure 23. The viscosity obtained numerically for the flat plate matches exactly to the experimental viscosity value, further validating the numerical method. A good match is obtained with the addition of the surface textures for both boundary conditions with a maximum error of 10.9% observed for the Dirichlet-Dirichlet boundary conditions and a maximum error of 11.2% observed with the Neumann-Dirichlet boundary conditions. The computed numerical apparent viscosity is always lower than the experimentally measured apparent viscosity. This is due to neglecting the \( \eta \frac{1}{r} \frac{\partial v_z}{\partial \theta} \) term in Equation (3.46) due to the scaling arguments used to derive the Reynolds equation. The addition of this term would cause a larger shear stress value to be measured, which would yield a larger apparent viscosity.

Figure 24 shows the validated results for the numerical code when compared to the experiments. This shows that the Dirichlet-Dirichlet boundary condition should be applied in the \( r \) direction in order to obtain good numerical results when compared to experimental data, validating the numerical method and allowing it to be used for determining the optimal texture profile for thrust bearings.

**3.6 Conclusion**

This is the first work to apply the pseudo-spectral method to solve the Reynolds equation in cylindrical coordinates. They key results are:
• The expected exponential convergence was seen when compared to the analytic solution for a thin strip, which shows accurate results can be obtained on course grids.

• Good comparisons seen between the numerical data and experimental data obtained in Chapter 2, with a maximum error of 7.6% for the normal force and 10.9% for the apparent viscosity.

• The Dirichlet-Dirichlet boundary conditions should be used in the $r$ direction in order to obtain accurate data when compared to experiments.

This validated numerical method serves as a basis for future work for the inverse design problem of determining the optimal texture profile for thrust bearings. The validated code can be used with optimization tools to test at any specified $h = h(r, \theta)$ in order to determine the optimal texture profile for producing normal forces and decreasing apparent viscosity in thrust bearings. Accurate results are obtained with the Reynolds equation on a course grid, resulting in low computation times. This will be extremely helpful when performing many numerical simulations to explore the entire design space of the surface texture profile.
Figure 13: Lagrange polynomial basis functions used in solving the Reynolds Equation. (A) is the basis function in 1-D. (B) is the basis function in 2-D.
Figure 14: Error analysis of the computed solution compared the analytic solution of the Reynolds equation across a thin strip defined in Equation (3.28). Exponential decay is observed for the error, matching the expected error convergence for the numerical technique used as seen in Equation (3.2).
Figure 15: Diagram showing the geometric quantities used to define the simulated texture. $R_i$ is the inner radius, $R_o$ is the outer radius, $R_c$ is the radius to the center of the texture, $R_t$ is the radius of the surface texture, and $\phi$ is the periodic spacing of the surface texture. Boundary conditions are also shown at $r=R_i$ and $r=R_o$ and $\theta=-\phi/2$ and $\theta=\phi/2$. 

\[
\begin{align*}
p(r = R_i) &= 0 \\
\frac{\partial p}{\partial r} |_{r=R_i} &= 0 \\
p(\theta = -\frac{\phi}{2}) &= p(\theta = \frac{\phi}{2}) \\
\frac{\partial p}{\partial \theta} |_{\theta=-\frac{\phi}{2}} &= \frac{\partial p}{\partial \theta} |_{\theta=\frac{\phi}{2}} \\
p(r = R_o) &= 0
\end{align*}
\]
Table 2: Geometric parameters used to define all the surface textures for the simulations.

<table>
<thead>
<tr>
<th>Geometric Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_i$</td>
<td>1 mm</td>
</tr>
<tr>
<td>$R_o$</td>
<td>20 mm</td>
</tr>
<tr>
<td>$R_c$</td>
<td>14.25 mm</td>
</tr>
<tr>
<td>$R_t$</td>
<td>3 mm</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$2\pi/10$ rad</td>
</tr>
</tbody>
</table>
Figure 16: Simulated texture profiles. (A) is the spatial discretization used to simulate the textures. (B) is an symmetric profile. (C) is an asymmetric profile. One symmetric texture was simulated, and four asymmetric textures were simulated using values given in Tables 1 and 2.
Figure 17: Convergence of normal force as a function of polynomial degree for the two boundary conditions considered. For these simulations, $R_i = 1\text{mm}$, $R_o = 20\text{mm}$, $\eta = 0.95 \text{ Pa s}$, $R_c = 14.25\text{mm}$, $R_t = 3\text{mm}$, and $\phi = \pi/5$. (A) is the convergence with $p(r=R_i)=0$. (B) is the convergence with $\left.\frac{\partial p}{\partial r}\right|_{r=R_i} = 0$. The two boundary conditions have converged when the polynomial degree is greater than 25.
Figure 18: Pressure profile obtained for an asymmetric texture with $\beta=5.3^\circ$ with a fluid with a viscosity $\eta=0.95$ Pa s at an angular velocity of $\Omega=100$ rad/s at a gap height $h_0=250$ µm.
Figure 19: Computed normal force as a function of velocity for an asymmetric texture with $\beta=5.3^\circ$ with a fluid with a viscosity $\eta=0.95 \text{ Pa s}$ at an angular velocity of $\Omega=100 \text{ rad/s}$ at a gap height $h_0=250 \mu\text{m}$. The normal force shows a linear scaling with respect to $\Omega$. Velocity dependent viscous heating effects were seen experimentally and are accounted for numerically by using the velocity dependent apparent viscosity obtained experimentally with a flat plate at each gap height tested.
Figure 20: Velocity profiles obtained for an asymmetric texture with $\beta=5.3^\circ$ with a fluid with a viscosity $\eta=0.95$ Pa s at an angular velocity of $\Omega=100$ rad/s at a gap height $h_0=250$ µm. (A) shows $v_\theta$ and $v_z$ as a function of $z$ and $\theta$ at a radius $r=R_c=14.25$ mm. (B) shows $v_r$ and $v_z$ as a function of $r$ and $z$ at $\theta=0$ rad. (C) shows $v_\theta$ and $v_z$ as a function of $z$ and $\theta$ at a radius $r=R_c=14.25$ mm and is color coded based on the magnitude of the velocity. (D) shows $v_r$ and $v_z$ as a function of $r$ and $z$ at $\theta=0$ rad and is color coded based on the magnitude of the velocity.
Figure 21: Computed apparent viscosity as a function of shear rate for an asymmetric texture with $\beta=5.3^\circ$ with a fluid with a viscosity $\eta=0.95$ Pa s at an angular velocity of $\Omega=100$ rad/s at a gap height $h_0=250$ µm. Velocity dependent viscous heating effects were seen experimentally and are accounted for numerically by using the velocity dependent apparent viscosity obtained experimentally with a flat plate at each gap height tested.
Figure 22: Comparison of forces calculated using the Reynolds equation to forces measured experimentally (Figure 10) for an oil lubricant with nominal viscosity \( \eta = 1.4 \) Pa s. Velocity dependent viscous heating effects were seen experimentally and are accounted for numerically by using the velocity dependent apparent viscosity obtained experimentally with a flat plate at each gap height tested. (A) used \( p(r=R_i) = 0 \). (B) used \( \frac{\partial p}{\partial r} |_{r=R_i} = 0 \). The solid line is where \( F_{\text{Reynolds}} = F_{\text{exp}} \).
Figure 23: Comparison of torque, normalized as apparent viscosity $\eta_a$ from Reynolds equation simulation and experiments (Figure 9) for an oil lubricant with nominal viscosity $\eta=1.4$ Pa s. Velocity dependent viscous heating effects were seen experimentally and are accounted for numerically by using the velocity dependent apparent viscosity obtained experimentally with a flat plate at each gap height tested. (A) used $p(r=R_i)=0$. (B) used $\frac{\partial p}{\partial r} \big|_{r=R_f}=0$. The solid line is where $\eta_{\text{Reynolds}}=\eta_{\text{exp}}$. 
Figure 24: Validated code using the best results from boundary condition $p(r=R_I)=0$ for calculating normal force and apparent viscosity with the Reynolds equation in cylindrical coordinates. (A) is the normal force calculation. (B) is the viscosity calculation. Experimental values are obtained from Chapter 2 Figures 9 and 10. The solid lines are where the computed values equal the experimental values.

Max Error

$\sim 7.6\%$

$\sim 10.9\%$
CHAPTER 4: Experiments with Viscoelastic Fluid

4.1 Introduction

The above studies focused on Newtonian fluids; a recent trend in the lubrication industry is to use polymer additives with Newtonian oils in order to obtain better viscosity-temperature and viscosity-pressure dependence [53, 19, 54] and to increase the load carrying capacity [55]. Numerical studies have been performed using a generalized Newtonian fluid (GNF) as the lubricant in journal bearings [56, 57, 58, 59] and these studies show that the eccentricity of the journal and the load carrying capacity depends on the properties of the lubricant. Experiments have also been performed with the assumption that the lubricant can be modeled as a GNF, and again shows that the bearing performance depends on the properties of the lubricant [60].

GNF models have also been used in the modeling of elastohydrodynamic lubrication (EHL) [19, 61]. Larsson [62] showed that less power is consumed in EHL contact with a fluid that has a smaller pressure-viscosity coefficient. Habchi et. al. [63] also showed that using a GNF model allowed for a more accurate comparison of numerical data to experimental results.

GNF models have also been used in numerical simulations of surface textured thrust bearings. Sharma and Yadav [64] solved a modified Reynolds equation in Cartesian coordinates using a continuous pseudo spectral Galerkin method with a power law fluid as the lubricant. They showed that when the fluid is shear thinning, the load carrying capacity of the textured system is lower than when the surface is perfectly flat (although power law fluid comparisons are
inherently ill-defined, since they do not achieve a low-rate Newtonian viscosity). They also showed that in a textured system, the load carrying capacity is increased above the Newtonian reference if the fluid is shear thickening.

The above models, however, did not take into account the viscoelasticity that occurs in Non-Newtonian lubricants; GNF models only take into account the purely viscous shear thinning/thickening effects of the lubricants. Viscoelasticity, even under steady shear, can lead to first and second normal stress differences, $N_1$ and $N_2$ respectively [65], which can increase the load carrying capacity of the bearing. Tichy [66] derived a modified Reynolds equation in Cartesian coordinates using the upper-convected Maxwell model, which takes into account the viscoelasticity of the lubricant and allows for the generation of first and second normal stress differences. He showed that for a converging flow field, the viscoelastic effects cause an increase in the pressure, leading to a larger load carrying capacity. Zhang and Li [67] performed a perturbation analysis for the flow field, shear stresses, and pressure using the upper-convected Maxwell model, and showed that viscoelasticity can significantly enhance the pressure in thin film flows. Williamson et. al. [68] measured the effects of viscoelastic lubricants in a journal bearing simulator, and showed that viscoelastic effects have a beneficial effect on lubrication characteristics.

The work presented here is the first (experimental) examination of the effects of surface texture depth profiles with a viscoelastic lubricant. Gap controlled experiments were performed on a custom tribo-rheometer in order to systematically examine the friction reduction with varying Reynolds number and Weissenberg number $Wi \equiv \dot{\gamma} \lambda$ where $\dot{\gamma} = \Omega R_o / h$ is the nominal shear rate. The top plate was allowed to rotate in both directions to determine direction of motion dependence of the normal force production and apparent viscosity reduction. Cavitation effects
are not present, so that the normal force is produced solely by the lubricant and the surface textures.

### 4.2 Materials and Methods

The same custom tribo-rheometer experimental set up used in Chapter 2 is used to perform the experiments in this section. The same surface textures used in Chapter 2 are also used to measure the friction reduction with a viscoelastic lubricant.

### 4.3 Risk of Misinterpreting Data

The risks given in Chapter 2 are still present in this experimental work. Therefore, the same corrections must be made as in Chapter 2 to ensure that the reported experimental results are accurate. All subsequent data have been corrected.

### 4.4 Lubricant Non-Newtonian Characterization

The lubricant used in this work is made by dissolving polyisobutylene (PIB) obtained from Sigma Aldrich (a long chain polymer with a molecular weight (MW) ~1million) in a low viscosity mineral oil (supplied by Cannon Instrument Company with nominal viscosity $\eta_0 = 9.6$ mPa s at $T=20^\circ$C) to create a 0.5wt% PIB in mineral oil solution. The concentration of polymer in solution compared to the critical concentration $c/c^* = 0.222$, meaning the lubricant is a dilute polymer solution. Figure 2 shows a depiction of the molecules dissolved in the base oil. Figure 2A shows the molecules under quiescent conditions and Figure 2B shows the molecules under shear flow. In the shear flow, the molecules are stretched, which generates Non-Newtonian effects including shear thinning and normal stresses.
The lubricant was characterized using a 40 mm cone and plate rheometer; this type of geometry was used so that the shear rate is constant throughout the gap \cite{65,27}. The viscosity of the fluid can be calculated from the measured torque on the top geometry using \cite{65,27}

\[
\eta = \frac{3\phi}{2\pi R^3} \frac{M}{\Omega} \tag{4.1}
\]

where \(\phi\) is the cone angle and \(R\) is the radius of the cone. The viscosity measurements of the viscoelastic lubricant and the base oil are given in Figure 25A. The base oil has a constant Newtonian viscosity, independent of the shear rate. The addition of the polymers causes the zero shear viscosity of the fluid to increase by a factor of \(\approx 3\). Shear thinning is also observed in the viscoelastic lubricant, and a Carreau-Yasuda model, given as

\[
\eta(\dot{\gamma}) = \eta_0 + \frac{\eta_\infty - \eta_0}{1 + \left(\frac{\dot{\gamma}}{\lambda}\right)^n} \tag{4.2}
\]

is fit to the viscoelastic lubricant data where \(\eta_0=0.03\) Pa s, \(\eta_\infty=0.00962\) Pa s (which is the viscosity of the base oil), \(\lambda=7.5\) ms, and \(n=0.812\). The viscoelastic timescale \(\lambda=7.5\) ms is important for determining viscoelastic effects in flow, in particular, with the Weissenberger number \(W_i = \lambda \dot{\gamma}\), where here \(W_i\) ranges from \(0.074 \leq W_i \leq 27.88\).

The cone and plate rheometer also allowed the first normal stress difference \(N_1\) to be calculated directly from the measured normal force. Corrections to the raw normal force must be made in the exact same way as those given in Section 2.3.2 in order to eliminate experimental effects. The first normal stress difference can then be calculated as

\[
N_1 = \frac{2(F_c - F_{co})}{\pi R^2} \tag{4.3}
\]

The measurements of \(N_1\) for the viscoelastic lubricant and the base oil are given in Figure 25B. \(N_1\) for the base oil lies below the experimental measurement limit. The addition of the polymers
causes \( N_1 \) to increase beyond the experimental limit and scale approximately as \( N_1 \sim \dot{\gamma}^2 \), which is expected for polymeric liquids at low shear rates [65,27].

4.5 Results and Discussion

4.5.1 Viscosity Reduction

The viscosity reduction results are given in Figure 26 and are calculated using [27]

\[
\eta = \frac{2h_t}{\pi R^2} \left( \frac{3}{4} + \frac{1}{4} \frac{d \ln(M)}{d \ln(\dot{\gamma}_R)} \right) M \Omega
\]

where \( h_t \) is the true gap given by Equation (2.2) and \( \dot{\gamma}_R \) is the edge shear rate given as

\[
\dot{\gamma}_R = \frac{\Omega R}{h_t}
\]

The viscosity from the flat plate reference matches the Carreau-Yadusa model obtained from the characterization in Section 4.4 for all the shear rates tested, and the viscosity is independent of the direction of motion. The addition of the surface textures causes the zero shear viscosity with the textures to be lower than the zero shear viscosity of the Carreau-Yasuda model (horizontal plateaus in Figure 26).

The symmetric texture decreases the zero shear viscosity the most out of all the textures tested. This is due to the change in gap height inside the surface texture, which increases the effective gap clearance (similar to the Newtonian results in Chapter 2 Figure 9). The shear stress in cylindrical coordinates for a Newtonian fluid is defined as [27]

\[
\tau_{\theta z} = \eta \left( \frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right)
\]

where \( v_\theta \) is the velocity in the \( \theta \) direction and \( v_z \) is the velocity in the \( z \) direction. If the gap height is much smaller than the radius of the plate, then the shear stress can be approximated by
\[ \tau_{\theta z} = \eta \frac{\partial v_\theta}{\partial z} \]  

(4.7)

In order to reduce the shear stress, the velocity gradient must be lowered. The velocity spanned by the top plate was the same for all the tests performed. Therefore, \( \frac{\partial v_\theta}{\partial z} \) can only be lowered by making the gap height larger. The asymmetric textures increase the gap height linearly from \( h \) to \( h+D \) such that the maximum spacing between the plates occurs only at one point in the texture. The symmetric texture, on the other hand, increases the gap from \( h \) to \( h+D \) over the entire length of the texture, and hence decreases the apparent viscosity \( \eta_a \) more than the linearly sloped textures with equal maximum depth \( D \).

4.5.2 Normal Force Production

The results for the normal force production are given in Figure 27. The flat plate produces normal forces above the experimental limit, and the forces are independent of the direction of motion. This direction independence validates the symmetry of the experimental apparatus. The normal forces produced with the flat plate are due to the viscoelasticity of the lubricant, and Macosko [27] showed that the normal forces produced by the flat plate are due to \( N_1 - N_2 \), which are material properties of the lubricant. The forces produced with the symmetric texture are slightly greater than the values produced by the flat plate. This matches results given in Chapter 2, which showed that even with Newtonian lubricants, the normal forces produced by the symmetric textures are slightly above the experimental limit.

The asymmetric textures produce normal forces that are above the flat plate reference, and the magnitude depends on the direction of motion. The sign of the force is always positive, which means that the normal force will always act to separate the two plates. Chapter 2 showed
that with Newtonian lubricants, the forces change sign with asymmetric surface textures and different directions of motion, where a negative normal force would cause the two plates to come together.

The normal forces produced with the asymmetric textures also show that the magnitude of the forces produced depends on the angle \( \beta \) of the asymmetric textures. When \( \beta = 5.3^\circ \), the normal forces are larger than the flat plate limit, and as \( \beta \) increases to \( 9.8^\circ \), the magnitude of the normal force also increases. However, when \( \beta \) is increased above \( 9.8^\circ \) to \( 14^\circ \), the magnitude of the normal force drops. This trend continues when \( \beta \) increases to \( 21.7^\circ \). This non-monotonic dependence on \( \beta \) suggests that an optimal \( \beta \) value exists for producing normal forces with asymmetric surface textures.

**4.5.3 Effective Friction Coefficient**

Typically, friction reduction is reported through a decrease in friction coefficient [5]. For the system studied here, an effective friction coefficient \( \mu^* \) can be defined as

\[
\mu^* = \frac{M}{RF_N}
\]

where \( M \) is the measured torque, which when divided by \( R \) gives an effective tangential force, and \( F_N \) is the measured normal force. The comparison of the effective friction coefficient for the surface textures tested is given in Figure 28. The effective friction coefficient \( \mu^* \) is highest for the flat plate and decreases through the addition of the asymmetric surface textures. A minimum in the effective friction coefficient is seen when \( \beta = 9.8^\circ \), again suggesting that an optimal \( \beta \) exists for decreasing friction with viscoelastic lubricants.
4.6 Conclusion

This work is the first to examine the friction reduction of surface textures with a viscoelastic lubricant. The key results of this work are:

- Surface textures are able to decrease the apparent viscosity beyond the reduction obtained through the shear thinning of the lubricant.
- The viscoelastic lubricant produces forces that are above the experimental limit even when surface textures are not present. This normal force production serves as a bias, resulting in the normal forces remaining positive even when the direction of motion is changed.
- Asymmetric surface textures produce normal forces above the flat plate reference, while the forces produced with symmetric textures are barely above the flat plate reference. This suggests that symmetry must be broken in order to produce normal forces beyond those of simple shear flow.
- The normal force production, viscosity reduction, and effective friction coefficient are dependent on the texture angle $\beta$, suggesting that there is an optimal $\beta$ value for decreasing friction with viscoelastic lubricants.

This work will serve as a basis for determining the dependence on viscoelastic timescales for normal force production and apparent viscosity reduction with surface textures. This work examined one viscoelastic time scale $\lambda$ over a range of Weissenberg number $0.074 \leq Wi \leq 27.88$. There remains a wide parameter space to explore the co-design of viscoelastic fluids [69] and surface textures, which will serve as future work.
Figure 25: Characterization of the viscoelastic lubricant using a cone and plate rheometer. (A) is the viscosity characterization; Non-Newtonian shear thinning is observed when PIB (MW~1 million, 0.5 wt%) is added to the Newtonian base mineral oil. The addition of the polymer increases the zero shear viscosity by a factor of ≈ 3. A Carreau-Yasuda model is fit to the viscosity data with parameters $\eta_0=0.03$ Pa s, $\eta_\infty=0.00962$ Pa s, $\lambda=7.5$ ms, and $n=0.812$. (B) is the first normal stress difference characterization; the addition of the polymers also generates a measurable first normal stress difference $N_1$, whereas $N_1$ for the base oil lies below the experimental detection limits.
Figure 26: Experimental viscosity results with the surface textures and the viscoelastic lubricant. (A) is the flat plate reference. (B) is the asymmetric texture with $\beta=5.3^\circ$. (C) is $\beta=9.8^\circ$. (D) is $\beta=14^\circ$. (E) is $\beta=21.7^\circ$. (F) is the symmetric texture. Filled symbols are present, but are covered by open symbols due to close correspondence.
Figure 27: Experimental normal force results with the surface textures and the viscoelastic lubricant. (A) is the flat plate reference. (B) is the asymmetric texture with $\beta=5.3^\circ$. (C) is $\beta=9.8^\circ$. (D) is $\beta=14^\circ$. (E) is $\beta=21.7^\circ$. (F) is the symmetric texture.
Figure 28: Effective friction coefficient \( \mu^* = \frac{M}{RF_N} \) for the different textures tested with a viscoelastic lubricant at a gap height of 250 μm and an angular velocity \( \Omega = 50 \) rad/s. An optimal low \( \mu^* \) exists at intermediate texture angle between \( 5.3^\circ \leq \beta \leq 14^\circ \).
CHAPTER 5: Conclusions and Future Work

This work experimentally examined the effects of asymmetric and symmetric surface textures on normal force production and viscosity reduction with a Newtonian lubricant (Chapter 2). It showed that symmetry must be broken to produce normal forces with surface textures, and that an optimal texture geometry exists for decreasing friction in lubricated sliding systems. These experimental results were then used to validate a numerical method used to simulate the surface textures (Chapter 3).

The future work with this validated numerical method using the Reynolds equation in cylindrical coordinates is to determine the optimal texture configuration with Newtonian fluids. One of the key challenges with this optimization is that there are two objectives that are used to determine the performance of the texture: normal force production and apparent viscosity reduction. This results in a multi-objective optimization (MOO) function that must be performed in order to determine the optimal texture geometry, which results in multiple optimal design options. Preliminary results for the multiple optimal designs have been studied in collaboration with Professor James Allison and his group at UIUC [70] using CFD results obtained in FLUENT and surrogate modeling technique. Continued collaboration with Professor Allison in the Industrial and Enterprise Engineering Department will help address these MOO function objectives.
This work also was the first work to present experimental analysis of symmetric and asymmetric surface textures with a viscoelastic lubricant (Chapter 4). It showed that the normal stress differences of the lubricant bias the normal forces produced to always be positive, even when the direction of motion changes. It also showed that asymmetric surface textures are able to produce forces that are larger than those produced solely by viscoelastic effects. Finally, it also showed that there is an optimal texture configuration for reducing friction with viscoelastic lubricants (Figure 28).

The future work with the viscoelastic lubricants is to first explore the fluid design space by varying the viscoelastic relaxation time scale. This thesis only examined one time-scale with a single polymer additive at a single concentration. The viscoelastic time-scale of the lubricant is relevant in comparison to the two operational time scales with the textures; the time that the polymer spends inside the texture defined as \( W / (\Omega R_c) \), and the shearing time scale defined as \( 1 / \gamma \). These two time-scales lead to a two dimensional design space for the viscoelastic lubricant [71,69], and changing the time-scale and the surface texture would allow the whole design space to be explored. Future work will be performed by experimentally varying the time-scale and measuring the normal force production and apparent viscosity reduction with the new viscoelastic lubricants.

Future work will also be done on determine the optimal texture configuration with a viscoelastic lubricant. In order to do this, a numerical method must be developed in order to perform the optimization. One of the key challenges will be selecting the correct mathematical constitutive model for describing the viscoelastic lubricant. Different viscoelastic properties are observed with different mathematical models [54,69]. This also affects performing numerical simulations with the viscoelastic lubricants; the mathematical descriptions for viscoelastic fluids
are much more complex than for a Newtonian fluid. This means that the general governing equations are much more complex than the Navier-Stokes equations. However, if reduced order equations, similar to the Reynolds equation, can be derived for the viscoelastic lubricant, the computational complexities will decrease, allowing the optimization to be performed. If this cannot be obtained, then techniques similar to the surrogate modeling used by Rao et. al. [70] could be used with experimental data in order to perform the design optimization.

Finally, future work will be performed in the context of reducing leakage. Feldman et. al. [72,73] studied the effect of surface textures on decreasing leakage in a hydrostatic gas seal, and showed that the surface textures can increase the load carrying capacity of the seal and have a lower leakage than step seals. However, their analysis was limited to symmetric surface textures. Chapter 2 showed that asymmetric textures increase the load carrying capacity more than symmetric textures. The asymmetric textures also have less volume removed inside the texture, which would help decrease leakage within the seal.

Thatte and Salant [74] also studied leakage by examining the viscoelastic behavior of the seal and showed that the viscoelastic behavior of the seal can help prevent leakage. However, this work was performed with a Newtonian lubricant. Viscoelastic lubricants can have laminar secondary flows induced due to the polymers [32], and these secondary flows can also help decrease leakage. This decrease in leakage metric can also be used in the design optimization of the surface textures and viscoelastic lubricant, which will increase the complexity of the MOO function. However, continued work with Professor Allison and his group will help address these design objectives.
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