THE EFFECT OF PHASE TRANSFORMATION ON FATIGUE CRACK GROWTH OF SHAPE MEMORY ALLOYS

BY

YAN WU

THESIS

Submitted in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering in the Graduate College of the University of Illinois at Urbana-Champaign, 2015

Urbana, Illinois

Adviser:

Professor Huseyin Sehitoglu
Abstract

The fatigue crack growth of shape memory alloys remains a difficult challenge in the scientific community for years. Most of the works on fatigue crack growth are based on materials that don’t undergo phase transformation when an external force is applied. Consequently, the change of crack tip driven force due to phase transformation is not well understood. In this study, the modification of the crack tip driven force was characterized by the stress intensity factor due to tractions on the transformation zone surface. Through modeling, it was found that the modification became more significant at the maximum applied load than at the minimum in a single load cycle. The effective stress intensity factor range was also measured from the displacement fields upon regression. The predicted effective stress intensity factor range through modeling was later compared with the measured one. The results were found to be closed to each other. The closure effect was also measured and determined to be 30% of the maximum load which corresponded to the modeling results as well. All these agreements confirm the validity of the modeling and pointed to the major mechanical factors such as elastic moduli changes, the transformation residual strains, and the transformation domain dimensions that contribute to damage tolerance in shape memory alloys. The results are checked by conducting simulations for two other important shape memory alloys, NiTi and CuZnAl, where the reductions in stress intensity range were found to be lower than NiFeGa explaining the high levels of experimentally determined crack threshold stress intensity range in NiFeGa.
Acknowledgment

I would like to thank Professor Huseyin Sehitoglu for giving me the chance to conduct relevant researches in studying shape memory alloys. His enthusiastic guidance and invaluable suggestions kept the project progressing.

I would also like to thank, particularly, Mr. Luca Patriarca for his guidance through the initial high resolution fatigue crack growth test, Mr. Garrett Pataky for his suggestions on the operation of machines, and Mr. Piyas Bal Chowdhury and Avinesh Ojha for their advice on understanding difficult theoretical concepts. I would also like to thank my other colleagues, Mr. Sertan Alkan, Mr. Pietro Luccarelli, Mr. Silvio Rabbolini, Mr. Emanuele Sgambitterra, and Mr. George Li for accompanying me through this period.

I would like to express my gratitude towards my parents, Jialei Wu and Jie Yan, for their selfless love and on-time encouragement.

Finally, the work on analyzing fatigue crack growth of shape memory alloys was supported by Nyquist Chair funds at University of Illinois and the work on high temperature shape memory alloy was supported by the national science foundation under grant 1-484369-917014-191100.
# Table of Contents

Chapter 1. Introduction ........................................................................................................... 1  
  Section 1.1 Background Motivation .................................................................................. 1  
  Section 1.2 Literature Review of Fatigue Crack Growth of SMAs .................................. 2  
  Section 1.3 Objective and outline .................................................................................... 5  

Chapter 2. Experiments Techniques ..................................................................................... 8  
  Section 2.1. Materials ....................................................................................................... 8  
  2.2.1 Specimen Preparation ............................................................................................. 9  
  2.2.2 Experimental Setup ............................................................................................... 11  
  2.2.3 Digital Image Correlation (DIC) ........................................................................... 13  

Chapter 3 Determination of crack tip driving force ............................................................. 16  
  Section 3.1 Method I - Extraction of Stress Intensity Factor from Displacements using Anisotropic Elasticity via Regression ................................................................................. 16  
  Section 3.2 Method II- Calculation of the Driving Force Changes due to Transformation Shielding in Crack Wake-Equivalent Eigenstrain Determination- Minimum and Maximum Load .................................................. 18  
  3.2.1 Eigenstrain Determination at Minimum and Maximum Load .................................. 18  
  3.2.2 Stress Intensity Calculation due to Internal Traction ............................................... 20  

Chapter 4. Results and Discussion ....................................................................................... 22  
  Section 4.1 Fatigue Crack Growth Results ....................................................................... 22  
  4.1.1 The effective stress intensity factor range of Ni$_2$FeGa ........................................... 22  
  4.1.2 Digital Image Correlation of the Transformation Zones and Strains in Cyclic Loading ....... 25  
  4.1.3 Irreversibility of strain at the crack tip .................................................................... 28  
  4.1.4 Virtual extensometer results .................................................................................. 30  
  Section 4.2 Modeling Results ........................................................................................... 32  
  Section 4.3 Discussions .................................................................................................... 39  

Chapter 5. Conclusions ....................................................................................................... 44  

References ............................................................................................................................ 45  

Appendix A ............................................................................................................................... 49  
Appendix B ............................................................................................................................... 52  
Appendix C ............................................................................................................................... 54  
Appendix D ............................................................................................................................... 57
Chapter 1. Introduction

Section 1.1 Background Motivation

The fatigue crack growth (FCG) still remains as a major issue that arises in the area of material application. Since a various kinds of new materials have been developed to accommodate industrial needs, the study on their FCG behaviors is becoming more important and essential. Shape memory alloys (SMAs) have been manufactured as a very unique class of materials that possess special properties, such as shape memory effect (SME) and pseudoelasticity. SME allows a large amount of strain to be recovered by heating the material above certain temperature. On the other hand, pseudoelasticity enables SMAs to recover from a large amount of strain by removing the applied stress. Mechanistic origin of such effects has been attributed to reversible phase transformation phenomena (austenite to martensite) (Duerig et al., 1990) upon loading/unloading at the microscale. The phase transformation was first reported by Chang and Read (Chang and Read, 1951) in 1932. However, it was not until 1962, when Buehler and co-workers reported the shape memory effect in NiTi (Buehler et al., 1963), that motivated the research and applications of SMAs. Recently, a new class of SMAs (for example, Ni$_2$FeGa), have been of particular interest to scientific and engineering communities due to their unique mechanical properties and potential applications (Cui, 2013). The SME (Chumlyakov et al., 2008; Font et al., 2006; Li et al., 2003), pseudoelasticity (Hamilton et al., 2007a, b; Masdeu et al., 2005; Oikawa et al., 2002), and magnetic property (Morito et al., 2005a; Morito et al., 2005b) of Ni$_2$FeGa have been well studied. There are also literatures reporting the slip (Sehitoglu et al., 2012) and fatigue (Efstathiou et al., 2007) behavior of this material. All these studies have substantially enhanced the understanding of micro-mechanics of deformation in SMAs. However, since the application of SMAs in the area of medical, actuation and nuclear
(Duerig et al., 1990; Otsuka and Wayman, 1998) become more prevalent nowadays, it remains imperative to examine the damage properties as well. Unfortunately, literature on the crack growth behavior of SMA material has been limited, though it is crucial to extend the potential applicability of such materials. The lack of documentation for this particular alloy from this perspective is mainly because that most of the works in this area are based on untransforming materials. As a result, in the current work, we investigated the fatigue crack growth behavior of Ni$_2$FeGa single crystal specifically on its transformation induced closure effect. Such effect is mainly governed by the transformation strain during phase transformation.

**Section 1.2 Literature Review of Fatigue Crack Growth of SMAs**

There has been previous fundamental works on fatigue crack initiation in shape memory alloys (Brown, 1979; Delaey et al., 1978; Hornbogen and Eggeler, 2004; Melton and Mercier, 1979a; Miyazaki et al., 1999; Sade and Hornbogen, 1988; Yang et al., 1977) describing the role of slip, the origin of irreversibilities, and residual martensite, but much less work has been undertaken on fatigue crack growth behavior. Melton and Mercier were the first among all to report on FCG of NiTi at room temperature (Melton and Mercier, 1979b). According to their investigation, crack growth rates and threshold values were found to be similar in stable martensitic and unstable austenitic condition. This fact inspired Dauskardt et al. (Dauskardt et al., 1988) to study specifically on the effect of phase transformation (martensite to austenite) on fatigue crack propagation of NiTi SMA. Their result showed that the threshold values, 2.1 MPa$\sqrt{m}$ to 5.4 MPa$\sqrt{m}$, were higher and FCG rates were lower in stable martensite and austenite than those in pseudoelastic range of NiTi (Dauskardt et al., 1988). According to this result, the phase transformation of SMAs at the crack tip would decrease the crack growth rate. Later, comprehensive study on the effect of temperature, microstructure, and pseudoelasticity on
FCG was conducted by McKelvey and Ritchie (McKelvey and Ritchie, 1999, 2001). They reported similar FCG results to the previous study with a threshold around 2 to 5.4 MPa√m. Other papers confirmed the low fatigue crack growth resistance of shape memory alloys in general (Holtz et al., 1996; Melton and Mercier, 1979a; Vaidyanathan et al., 2000). The reason for degradation of fatigue crack growth resistance in pseudoelastic NiTi is the inhibition of stress induced martensitic transformation due to the negative volume change of NiTi. However, the quantification of phase transformation effect on crack tip toughness cannot be directly determined from these experiment due to the nonlinear elastic behavior. As a result, modeling can be involved to get insight into this question.

In the case of shape memory alloys, the stress intensity factor and the crack tip displacements that are used to characterize fatigue crack growth change. However, the exact nature of the changes in the driving force has not been derived. Table 1 illustrates the mechanisms that have been forwarded to modify the driving forces in the presence of transformation. The modifications in driving force due to internal tractions (first row) have been derived by Rice-McMeeking-Evans (McMeeking and Evans, 1982a; Rice, 1972a) using weight function theory. The transformation strains drive these tractions. Also as shown in Table 1 (second row), there has been several efforts attempting to calculate the redistribution of stress fields ahead of the crack tip due to the transformation. These analysis (Xiong and Liu, 2007), similar to the work of Irwin on plastic zone size correction (G.R. Irwin, 1960), propose a change in effective crack length, resulting in a change in the stress intensity factor. A number of recent works on the computation (Baxevanis et al., 2014; Baxevanis et al., 2013; Lexcellent and Thiebaud, 2008; Stam and van der Giessen, 1995) and experimental determination of
transformation zones (Creuziger et al., 2008) under monotonic deformation and steady state energy release rates has been undertaken.

In addition to the works listed in Table 1, there are many other works on characterizing the effect of transformation on SMAs. Following closely with the approach of Mcmeeking (McMeeking and Evans, 1982b) and Budiansky et al.(Budiansky et al., 1983), Yi and Gao (Yi and Gao, 2000; Yi et al., 2001), utilizing the constitutive model developed by Sun and Hwang (Sun and Hwang, 1993), were the first to investigate the fracture toughening mechanism on SMAs. In their analysis, the volumetric transformation strain was ignored due to its negligibility compared to the transformation shear strain. Yan et al. (Yan et al., 2002), motivated by the experimental result of Mckelvey and Ritchie (McKelvey and Ritchie, 2001), also included the volumetric transformation strain into the investigation. He concluded that positive volumetric transformation strain will result in a increment in tougheness, while negative a decrement. Subsequently, Stam and Van Der Giessen (Stam and van der Giessen, 1995) and Freed and Banks-Sills (Freed and Banks-Sills, 2007) included the effect of reversible transformation in their analysis, since the irreversible transformation strain left behind the crack tip contributes more to the toughness change. They implemented a cohesive zone model in the finite element analysis and concluded that the reversible transformation would reduce the transformation toughening effect. These results set a good fundation for further investigations. However, they are limited to the assumption of isotropy. The effect of anisotropy is not yet investigated.
Table 1. A summary of the mechanisms at crack tips undergoing transformation under loading. The tractions due to transformation are shown at the austenite to martensite interface. The differences of closure forces at the minimum and maximum stress intensity levels is important in fatigue case (this study).

<table>
<thead>
<tr>
<th>Type of Loading-Mechanism</th>
<th>Schematic</th>
<th>Important Variables</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monotonic Loading-Shielding Associated with Traction</td>
<td>[Diagram of Shielding]</td>
<td>Dimensions of the residual transformation zone in the crack wake, tractions cancel ahead of tip but substantial on crack faces</td>
<td>Rice (Rice, 1972a) McMeeking (McMeeking and Evans, 1982a)</td>
</tr>
<tr>
<td>Monotonic Loading- Driving Force Calculation via FEM</td>
<td>[Diagram of Energy Release Rate]</td>
<td>Energy Release Rate Calculation, $G_u / G_{tip} &gt; 1$ (steady state/tip energy release rate via contour integral)</td>
<td>(Baxevanis et al., 2014; Baxevanis et al., 2013; Stam and van der Giessen, 1995)</td>
</tr>
<tr>
<td>Fatigue Loading- Closure Force Differential at Max. and Min. Loads</td>
<td>[Diagram of Closure Forces]</td>
<td>Elastic moduli (crystallography), transformation zone (verified with DIC), residual transformation strain, reduction of stress intensity range.</td>
<td>This Study</td>
</tr>
</tbody>
</table>

Section 1.3 Objective and outline

In the present study, we investigated fatigue crack growth behavior under cyclic loading on Ni$_{54}$Fe$_{19}$Ga$_{27}$ (at. %) single crystal in [001], [011], and [123] orientation by using two different methodologies. In the first approach (Method I), the displacement fields are measured
in the vicinity of crack tip during fatigue experiments with digital image correlation (DIC). These displacement results can be utilized in turn to determine the 'effective stress intensity' levels (Carroll et al., 2009). In the case of transforming alloys, these measured displacement fields would naturally reflect the crack tip driving force modification in the presence of transformation strains. As an extension of the method using regression, it is worthwhile to measure the contact of crack surfaces during fatigue resulting in crack closure. Such experiments are now possible with the use of virtual extensometers behind the crack tip in conjunction with digital image correlation studies. We explore this possibility as well in the current work accounting for a full range of mechanisms. The results from regression and virtual extensometers agreed in untransforming alloys and a similar agreement is expected in shape memory materials. Alternately, in the second approach (Method II), we compute the modified stress intensity in transforming alloys due to internal tractions. In fatigue loading, one needs to consider tractions at both maximum and minimum loads imposed on the transforming regions by the surrounding untransformed domains. Ideally, both approaches (I and II) should render an ‘effective stress intensity range' that is comparable in magnitude resulting in the true value of the driving force in fatigue.

In summary, utilizing anisotropic elasticity theory, Eshelby’s equivalent inclusion principle (Eshelby, 1957), weight function methods for anisotropic media (Sih et al., 1965), and density functional theory calculations, and extensive digital image correlation results for displacements in crack wake and in transformation zones, we establish the modified stress intensity factor for fatigue crack growth in shape memory alloys. The material properties in terms of the elastic moduli and details of the fatigue crack growth experiment are presented in Section 2. In Section 3, determinations of transformation zone based on strain field obtained via DIC and strain irreversibility are demonstrated. Method I (regression and virtual extensometers) and Method II (modeling) are elaborated on the details in Section 4. The results provide a better
appreciation of the complexity of the crack tip driving force mechanics, and constitute an advancement of the scientific methodology to analyze fatigue crack growth behavior in shape memory alloys.
Chapter 2. Experiments Techniques

Section 2.1. Materials

The materials that are used to investigate the problem is Ni$_2$FeGa single crystal shape memory alloys oriented in [001], [011], and [123]. The dimension of the dog-bone shaped material is included in Figure 1. The single edge-notch tension sample was electrical discharge machined (EDM) with a gage length of 9 mm long, a width of 1.5 mm, and notch length 0.5 mm and width 0.006 mm with the thickness as shown in Figure 1. A summary of Ni$_2$FeGa single crystals with different orientations is included in Table 2.

Figure 1. Dog-bone sample geometry
Table 2. A summary of tested samples of Ni$_2$FeGa single crystals

<table>
<thead>
<tr>
<th>Loading Direction</th>
<th>Normal Direction</th>
<th>Experiment</th>
<th>Magnification</th>
<th>Resolution (μm/pixel)</th>
<th>R-ratio</th>
<th>Stress Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>[001]</td>
<td>[010]</td>
<td>Fatigue Crack Growth Test @ 25C</td>
<td>4.5X</td>
<td>0.983</td>
<td>0.05</td>
<td>47.5Mpa</td>
</tr>
<tr>
<td>[011]</td>
<td>[100]</td>
<td>Fatigue Crack Growth Test @ 25C</td>
<td>10X 20X</td>
<td>0.43 0.21</td>
<td>0.05</td>
<td>47.5Mpa</td>
</tr>
<tr>
<td>[123]</td>
<td>[412]</td>
<td>Fatigue Crack Growth Test @ 25C</td>
<td>10X</td>
<td>0.35</td>
<td>0.05</td>
<td>47.5Mpa</td>
</tr>
</tbody>
</table>

2.2.1 Specimen Preparation

The specimen surface is mechanically polished to a mirror finish by using SiC paper (from P800 to P1500). Sucessively, a fine layer of black paint is airbrushed on the polished surface to create a low resolution speckle pattern using an Iwata micron B airbrush and black paint for DIC analysis. For a high resolution test, silicon powder was used to in this case. An example of the two different speckle patterns is shown in Figure 2.
Figure 2. (a) High resolution speckle pattern applied on [011] single crystal (b) low resolution speckle pattern on [001] single crystal
2.2.2 Experimental Setup

The low resolution fatigue crack growth test was conducted on [001] Ni_{54}Fe_{19}Ga_{27} (at. %) single crystal on an Instron servo-hydraulic load frame shown in Figure 4. The R-ratio and stress range for each test are included in Table 2. Since the material will tend to twin before slip, the maximum stress was picked, based on the stress-strain behavior established by Efstathiou (Efstathiou et al., 2007) in Figure 3, when the strain is approximately 1% in the overall material. An IMI202FT digital camera was used to capture images during the fatigue crack growth test. The resolution of the images is usually 2 micron/pixel. The whole low resolution test is controlled by a customized LabView computer program that allows the images to be associated with their corresponding load levels. The main purpose of low resolution fatigue crack growth test is to capture the relationship between crack growth rate (da/dn) with stress intensity factor range (ΔK).

On the other hand, high resolution test is to focus on the strain contour at both ahead of the crack tip and wake. So, the test is separated into two portions. In the first portion, the specimen is loaded cyclically on Instron servo-hydraulic load frame to initiate the crack. Then, it will be transferred onto a 4.5kN EBSD SEM tester, Figure 5, to grow the crack carefully under surveillance of a microscope. The resolution of the image is at higher magnifications approximately from 0.44 micron/pixel up to 0.22 micron/pixel. The SEM tester is controlled by commercial computer software to manage the loading and unloading process, while the images are taken manually at desired load. For both low resolution and high resolution fatigue crack growth test, the very first picture is usually set to be the reference image for the further DIC analysis.
Figure 3. (a) Stress-strain response of $\text{Ni}_{54}\text{Fe}_{19}\text{Ga}_{27}$ in tension for three orientations considered in this study, (b) stress-strain response to high strains for the [001] case in tension. Note that the maximum transformation strains are rather high as high as 12% in this material.

Figure 4. a) Low resolution experimental setup b) High resolution experimental setup
Figure 5. The SEM tester that is utilized with high resolution microscope to measure the local crack tip displacements for fatigue experiments.

2.2.3 Digital Image Correlation (DIC)

The images taken during experiments could be used as a source to analyze the fatigue crack growth behavior of this particular class of alloys. During the process, DIC can be implemented to facilitate the analysis. As mentioned beforehand, the speckle pattern on the surface the specimen will create a random array of light intensity level. DIC will compare such light intensity levels between reference and deformed images. However, since same light intensity level could occur among millions of pixel in the pattern, a group of pixels that contains unique light intensity feature need to be selected as a subset. Then, based on the coordination of
the center of the subsets, DIC will give an initial guess on that $(x_0, y_0)$. The new position $(x, y)$ of it can be obtained by Taylor’s expansion expressed as the following.

\[
\begin{align*}
\hat{x} &= x_0 + u_0 + \frac{\partial u}{\partial x} \Delta x + \frac{\partial u}{\partial y} \Delta y + \frac{1}{2} \frac{\partial^2 u}{\partial y^2} \Delta y^2 + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} \Delta x^2 + \frac{1}{2} \frac{\partial^2 u}{\partial x \partial y} \Delta x \Delta y \\
\hat{y} &= y_0 + v_0 + \frac{\partial v}{\partial x} \Delta x + \frac{\partial v}{\partial y} \Delta y + \frac{1}{2} \frac{\partial^2 v}{\partial y^2} \Delta y^2 + \frac{1}{2} \frac{\partial^2 v}{\partial x^2} \Delta x^2 + \frac{1}{2} \frac{\partial^2 v}{\partial x \partial y} \Delta x \Delta y
\end{align*}
\]  

(0.1)

In Equation (0.1), $u_0$ and $v_0$ are the horizontal and vertical displacement respectively, and $\Delta x$, $x-x_0$, and $\Delta y$, $y-y_0$, are the distance between the new position and reference position at the center of the subset. For more details about DIC, one is recommended to read the paper by Sutton (Sutton et al., 2009). All these calculation procedure can be taken care of by a commercial computer software VIC-2D produced by Correlated Solutions. Since the accumulated strain is in great consider when analysis this type of material, the very first image at the minimum load is usually set to be the reference image. The subset size is usually set to be 51 pixels x 51 pixels in order to have a very accurate correlation. The vertical displacement, $v_0$, and horizontal displacement, $u_0$, are extracted from the correlation to be implemented in least square regression, which will be introduced in section 3.1.

In addition, the virtual extensometer method that is available in VIC-2D is another technique for determining the crack closure levels. The technique is illustrated in Figure 5. The relative displacements across the crack faces are measured during loading and unloading. Therefore, DIC measurements allow the determination of the crack opening displacements during loading and unloading. By making such measurements over fine increments, it is possible to precisely determine the applied load level at which the crack opening occurs.
Figure 6. Virtual extensometer setup in VIC-2D
Chapter 3 Determination of crack tip driving force

Section 3.1 Method I - Extraction of Stress Intensity Factor from Displacements using Anisotropic Elasticity via Regression

On study of fatigue crack growth behavior of shape memory alloys, displacement can be extracted by using digital image correlation (DIC) (Sutton et al., 1983) to determine the stress intensity factors. The idea of determining stress intensity factor through DIC was first proposed by McNeil et al. (McNeill et al., 1987). Recently, a least square regression method was developed for calculating stress intensity factors, $K_I$ and $K_{II}$, under isotropic mixed-mode condition (Yoneyama et al., 2006). Later, Carroll et al. (Carroll et al., 2009) included a T-stress term, the second term in the William expansion for stresses (Williams, 1957), into the regression algorithm to determine mode I stress intensity factor, $K_I$ (Carroll et al., 2009). Such method was further refined to characterize anisotropic stress intensity factors (Pataky et al., 2012). Since these methodologies are based on experimental displacement result on the material surface, the crack closure effect is automatically included. In other words, stress intensity factors determined through such techniques are the effective values.

Specifically, these vertical and horizontal results are fitted to the anisotropic displacement fields for cubic crystals. Such solutions are available by Sih et al. (Sih et al., 1965). It is possible to extract the stress intensity by regression fit to the following set of equations given below. We note that the equations include the elastic constants and T stress term. We also note that the orientation of the crack in the [001] specimen is 45° to the loading axis. In this case, both Mode I and Mode II stress intensities can be extracted. In the case of [123] and [011] oriented single crystals, the crack grew nearly normal to the loading axis and the Mode II stress intensity is
small. The crack tip displacements for the [001] and [123] oriented specimens will be shown in
Chapter 4.

The stress intensity factors, $K_1$ and $K_2$, can be extracted from horizontal and vertical
displacements, $u^l$ and $v^l$, through the following equations.

\[
u^l = K_1 \sqrt{2r} \text{Re} \left[ \frac{1}{\mu_1 - \mu_2} \left( \mu_1 p_2 \sqrt{\cos \theta + \mu_2 \sin \theta} - \mu_1 p_1 \sqrt{\cos \theta + \mu_2 \sin \theta} \right) \right] + \\
K_2 \times \sqrt{2r} \text{Re} \left[ \frac{1}{\mu_1 - \mu_2} \left( p_2 \sqrt{\cos \theta + \mu_2 \sin \theta} - p_1 \sqrt{\cos \theta + \mu_2 \sin \theta} \right) \right] + \\
a_{11} Tr \cos \theta + Ar \sin \theta + B_u
\]
(1.1)

\[
v^l = K_1 \sqrt{2r} \text{Re} \left[ \frac{1}{\mu_1 - \mu_2} \left( \mu_2 q_2 \sqrt{\cos \theta + \mu_2 \sin \theta} - \mu_2 q_1 \sqrt{\cos \theta + \mu_2 \sin \theta} \right) \right] + \\
K_2 \times \sqrt{2r} \text{Re} \left[ \frac{1}{\mu_1 - \mu_2} \left( q_2 \sqrt{\cos \theta + \mu_2 \sin \theta} - q_1 \sqrt{\cos \theta + \mu_2 \sin \theta} \right) \right] + \\
a_{12} Tr \cos \theta + Ar \sin \theta + B_v
\]
(1.2)

\[
a_{11} \mu^4 - 2a_{16} \mu^3 + \left( 2a_{12} + a_{66} \right) \mu^2 - 2a_{26} \mu + a_{22} = 0
\]
(1.3)

where Re represents the real part of a complex number, $T$ is the T-stress, $A$ is the rigid body
rotation, $B_u$ and $B_v$ are the rigid body translations in $u^l$ and $v^l$ directions respectively, $a_{11}$, $a_{12}$, $a_{16}$,
$a_{22}$, $a_{26}$ and $a_{66}$ are the compliance components, $r$ and $\theta$ are the polar coordinates with their
origin at the crack tip, and $\mu_1$ and $\mu_2$ are the roots of Equation (1.3). The $p_i$ and $q_j$ in Equation
(1.2) are the anisotropic terms defined in the following ways.
\[ p_i = a_{11} \mu_i^2 + a_{12} - a_{16} \mu_i \]
\[ q_j = a_{12} \mu_j + \frac{a_{22}}{\mu_j} - a_{26} \]

(1.4)

Section 3.2 Method II- Calculation of the Driving Force Changes due to Transformation Shielding in Crack Wake-Equivalent Eigenstrain Determination-Minimum and Maximum Load

As mentioned in Chapter 1, the residual strain ahead of the crack tip or along the crack wake would provide resistance to the crack growth. By implementing eshelby’s methodology, an algorithm is written in matlab to numerically determine the retardation effect resulted from transformation shielding.

3.2.1 Eigenstrain Determination at Minimum and Maximum Load

The strain level measured via digital image correlation (DIC) was shown in Figure 5 for different single crystal orientations. Since the transformed area is surrounded by the matrix material, the DIC result can be interpreted as the total strain, \( e_{\text{nn}}^f \) which is the sum of constrained strain and far field strain. The intrinsic transformation strain, \( e_{kl}^p \) can be calculated by following equation

\[
e_{kl}^p = S_{klnm}^{-1} \left( e_{mn}^f - e_{mn}^o \right)
\]

(3.1)

where \( e_{mn}^o \) is the far-field strain and \( S_{ijkl} \) the Eshelby’s tensor for cubic crystal material. The \( S_{ijkl} \) represents the geometry of the martensite platelets and is treated as a flat ellipsoidal shape. It can be obtained as
$$S_{ijkl} = \frac{1}{8\pi} C_{pqkl} \left( \bar{G}_{qij} + \bar{G}_{piq} \right)$$  \hspace{1cm} (3.2)$$

where the specific terms, $\bar{G}_{qij}$, are given in the book by Mura (Mura, 1987) and also in Appendix A for completeness.

Assuming the minimum load to be small, the misfit strain due to modulus mismatch can be neglected. As a result, in the case of minimum load, $e^p_{kl}$, is the equivalent eigenstrain that needs to be calculated. Therefore, the corresponding stress, $\sigma_{ij}$, on the transformation contour can be obtained via

$$\sigma_{ij} = C_{ijkl} e^p_{kl}$$  \hspace{1cm} (3.3)$$

When the maximum load is applied, the eigenstrain effect due to modulus mismatch, $e^*_{mn}$ needs to be taken into account. The equivalent eigenstrain, $e^{**}_{mn}$ which is the sum of $e^*_{mn}$ and $e^p_{kl}$ can be calculated through Eshelby’s equivalent method described below.

$$c_{ijkl} \left( e^0_{kl} + s_{klmn} e^*_{mn} - e^{**}_{kl} \right) = c'_{ijkl} \left( e^0_{kl} + s_{klmn} e^*_{mn} - e^p_{kl} \right)$$

$$e^{**}_{mn} = \left[ \left( c_{ijkl} - c'_{ijkl} \right) s_{klmn} - c'_{ijmn} \right]^{-1} \left[ \left( c_{ijkl} - c'_{ijkl} \right) e^0_{kl} - c_{ijkl} e^p_{kl} \right]$$  \hspace{1cm} (3.4)$$

where $c_{ijkl}$ and $c'_{ijkl}$ are the elastic moduli of cubic austenite and tetragonal martensite for Ni$_2$FeGa, respectively. These tensors are given in the previous section. We note that all tensors are given in the cubic coordinate frame, and the rotations associated with the transformation are accounted for when the moduli are determined.

Upon calculation of the equivalent eigenstrains, the corresponding stress, $\sigma_{ij}$, in the transformation zone can be ascertained as
\[ \sigma_{ij} = C_{ijkl} e_{kl}^{**} \] (3.5)

Using equations above, it is possible to determine the internal tractions along the transformation contour using the Cauchy formula. Further details are given in Appendix A.

3.2.2 Stress Intensity Calculation due to Internal Traction

Knowing the tractions on the surface of the transformation zone, it is possible to numerically calculate the stress intensity change for a specific loading case. By implementing the weight function technique through Equation (3.6) proposed by Bueckner and Rice (Rice, 1972b) the stress intensity factor due to the internal tractions, \( \Delta K_I \), can be written as:

\[ \Delta K_I = \int_{S_p} n_i T_{ij} h_j dS_p \] (3.6)

where \( n_i \) is the outward normal of the transformation zone, \( dS_p \) is the line element on the perimeter of the zone, and \( h_j \) is the anisotropic weight function determined above.

Figure 7. Schematic of the load system on the crack surface. Four zones are considered and the contributions of all four zones are taken into account. Zone 3 and Zone 4 have the most significant influence on the results of stress intensity due to internal traction. \( w \) represents the height of the transformation zone.
According to Rice, the weight function can be readily obtained through Equation (3.7) if the displacement fields, \( u' \) and \( v' \), and stress intensity factor, \( K_1 \) and \( K_2 \), in a reference load system are known.

\[
\begin{align*}
  h_x &= \frac{H}{2K_1} \frac{\partial u'}{\partial l} \\
  h_y &= \frac{H}{2K_1} \frac{\partial v'}{\partial l}
\end{align*}
\]  

(3.7)

The solution for stress intensity factors due to tractions on the crack surface in Figure 6 can be found via Equation 1 and displacement fields are provided by Sih (Sih et al., 1965) as

\[
\begin{align*}
  K_1 &= -\frac{1}{\pi \sqrt{a}} \int_0^a \sigma(x) \left[ \sqrt{\frac{x+a}{a-x}} - \frac{1}{2} \frac{\alpha_o}{\beta_o} \right] dx \\
  K_2 &= -\frac{1}{\pi \sqrt{a}} \int_0^a \sigma(x) \left[ \frac{\alpha_o^2}{2\beta_o} + \frac{1}{2} \frac{a_{12}}{a_{11}} + \left( \alpha_o^2 + \beta_o^2 \right) \right] dx
\end{align*}
\]  

(3.8)

where \( \alpha_o \) and \( \beta_o \) are the real and imaginary component of the roots for Equation (3.9), i.e., for \( \mu_1 = \alpha_o + \beta_o i \) and \( \mu_2 = -\alpha_o + \beta_o i \).

The elastic moduli, \( H \), can be represented in Equation (3.9).

\[
H = -\frac{1}{8} \left( \frac{\mu_1 - \mu_2}{\mu_2} \right) \left\{ \frac{i}{\alpha_o \beta_o} \left[ \frac{a_{12}}{a_{11}} + \left( \alpha_o^2 - \beta_o^2 \right) \right] + 1 \right\}
\]  

(3.9)

Once weight functions are obtained, the corresponding stress intensity factor in that loading system can be determined through Equation (3.6). The summation of stress intensity factors obtained from different parts of the transformation contour (Figure 6) yields the change of stress intensity factor due to transformation effect. Further details are given in Appendix B.
Chapter 4. Results and Discussion

Section 4.1 Fatigue Crack Growth Results

4.1.1 The effective stress intensity factor range of Ni$_2$FeGa

By fitting the experimental data into Equation (1.1) and Equation (1.2), one can extract stress intensity factor range from the experimental displacement result. A comparison of displacement contours between experimental measurement and regression results are shown as the following in Figure 7. The predicted stress intensity factor will tend to be more accurate if the agreement between the two is better.
Figure 8. (a) the vertical displacement (v) for the [011] case, (b) The crack tip displacements normal (v) and horizontal (u) to the crack tip for an inclined crack in a single crystal oriented in [001] direction, (c) the vertical displacement (v) for the [123] case.
Since the low resolution experiment mainly used to determine the relationship between stress intensity factor range, $\Delta K$, and crack growth rate, $da/dN$, was done mostly on [001] single crystal, the corresponding fatigue crack growth result is shown in Figure 8. The data points that rest in the stage II region can be used to determine the coefficients, $C$ and $m$, in a Paris Law shown in Equation (4.1). The way of combining the Mode I and Mode II stress intensity factor range is also shown in Equation (4.1) (Bathias and Pelloux, 1973).

\[
\frac{da}{dN} = C (\Delta K_{\text{eff}})^m \\
= C \left( \Delta K_{I,\text{eff}}^2 + \alpha \Delta K_{II,\text{eff}}^2 \right)^{\frac{m}{2}}
\]  

(4.1)

Since anisotropy needs to be taken into account when analyzing single crystal material, the analysis could be somewhat complicated. As a result, it requires energy release rates, $J_i$, in Equation (4.2) for mode I, $K_{I,\text{eff}}$, and Equation (4.3) for mode II, $K_{II,\text{eff}}$, respectively (Sih et al., 1965). The ratio between these two energy release rates will be utilized to determine $\alpha$, which is included in Equation 4.1.

\[
J_1 = -\frac{\pi K_{I}}{2} a_{22} \text{Im} \left[ \frac{K_I (\mu_1 + \mu_2) + K_{II}}{\mu_1 \mu_2} \right] 
\]  

(4.2)

\[
J_2 = \frac{\pi K_{II}}{2} a_{11} \text{Im} \left[ K_{II} (\mu_1 + \mu_2) + K_I \mu_1 \mu_2 \right] 
\]  

(4.3)

where $a_{11}$ and $a_{22}$ are the compliance coefficients and $\mu_1$ and $\mu_2$ are the roots that need to be solved in Equation (1.3).

A summary is shown in Table 3.

<table>
<thead>
<tr>
<th>Crystallography</th>
<th>R-ratio</th>
<th>C</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td><a href="100">001</a> Single Crystal</td>
<td>0.05</td>
<td>5x10^{-8}</td>
<td>2.4</td>
</tr>
</tbody>
</table>
4.1.2 Digital Image Correlation of the Transformation Zones and Strains in Cyclic Loading

As shown in 4.1.1, displacement was captured, in low resolution, in the vicinity of the crack as the crack advances in order to extract stress intensity factor. The strain fields were monitored by microscope lens in high resolution both ahead and behind the crack tip. These measurements were made at maximum load, minimum load and at intermediate loads. We also employed a higher resolution SEM tester which is a small fatigue machine that can be fitted under an optical lens in an optical microscope or in a scanning electron microscope. The setup for the SEM tester is shown in Figure 9 and was utilized for obtaining images at high magnifications, strains at crack tips and the extent of the residual transformation zones.

Figure 9. Fatigue crack growth result of Ni$_2$FeGa [001] single crystal
Figure 10. The SEM tester that is utilized with high resolution microscope for strain field determination in the vicinity of crack tip

The strain contours for different single crystals, oriented in [001], [011], and [123], are included in Figure 10 and Figure 11. The strain contours at maximum load and minimum load are shown as the following.
Figure 11. The strain fields at the minimum and maximum load of the cycle obtained from fatigue crack growth experiments utilizing digital image correlation, results for fatigue loading in three orientations are displayed. These orientations are [001],[123] and [011].

The crack will tend to propagate at an angle of approximately 45° from the horizontal direction in [001] orientated single crystal, while grow in horizontal direction for the other two single crystals. As shown in Figure 10 and Figure 11, a zone of transformed materials is
generated in the wake of the crack as it advances. The height, shape and strain of the corresponding transformation zone can be readily obtained through these digital image correlation results.

### 4.1.3 Irreversibility of strain at the crack tip

The propagating of the crack during cyclic loading is caused by the dislocation emitted from the crack tip, and not fully recoverable during unloading (Pippan, 1991). The slip irreversibility is the residual left behind after the interaction and, therefore, is related to the crack propagation (Wu et al., 1993).

A strain contour showing the strain accumulation after one cycle is shown in Figure 12. The strains are measured at the beginning and at the end of the same cycle.

![Figure 12. Residual strain accumulation during cycling loading, the DIC images are taken at minimum load at the beginning of the cycle and at the conclusion of the cycle](image-url)
The strain in A and B are measured through DIC and plotted in Figure 13. The strains at minimum loads are numerical determined from digital image correlation software, VIC-2D, at different crack lengths. Since each strain components, normal strain, $\varepsilon_{yy}$, transverse strain, $\varepsilon_{xx}$, and shear strain, $\varepsilon_{xy}$, can be determined through DIC, the equivalent strain can be calculated by using Equation (4.4).

$$\varepsilon_{\text{equivalent}} = \sqrt{\frac{2}{3} \left( \varepsilon_{xx}^2 + \varepsilon_{yy}^2 + 2\varepsilon_{xy}^2 \right)}$$  \hspace{1cm} (4.4)

![Figure 13. The measured accumulation of strains over a wide range of fatigue crack growth.](image)

The accumulated strain per cycle is the difference between the strain levels at points B and A shown above in Figure 13. These data points are mainly captured from the ones at Stage II where material has the most fatigue crack propagation. The change in accumulated strain from
shorter to longer crack length is not very obvious. In other words, the crack growth rate could be relatively small. This also explains the flat slope of fatigue crack growth curve in Figure 8 at Stage II.

4.1.4 Virtual extensometer results

The virtual extensometer method is another technique for determining the crack closure levels which is complimentary to ‘regression’. The technique is illustrated in Figure 5 in Chapter 2. The relative displacements across the crack faces are measured during loading and unloading. Therefore, DIC measurements allow the determination of the crack opening displacements during loading and unloading. By making such measurements over fine increments, it is possible to precisely determine the applied load level at which the crack opening occurs. These results are shown in Figures 14 (a) and Figure 14 (b). According to Figure 14, the profile begins to deviate from previous ones at approximately 35% of the maximum load. As a result, the crack opening load is determined as 35% of the maximum load.
Figure 14. The crack opening displacement profiles, utilizing virtual extensometers, for the sample oriented in [001] direction. The gage location is the distance behind the crack tip. The profiles are given as a fraction of the maximum applied load. The crack opening load is determined as 35% of the maximum applied load.
Section 4.2 Modeling Results

As mentioned in Chapter 3, a model based on Eshelby’s methodology and Rice’s weight function concept has been developed to numerically determine the amount of stress intensity that was reduced from transformation toughening effect. In order to deal with this problem more conveniently, the transformation zone is separated into several different sections as shown in Figure 6.

By applying Equation (3.4), the eigenstrains, $\varepsilon^{**}$, and intrinsic transformation strains, $\varepsilon^p$, for Ni$_2$FeGa, CuZnAl, and NiTi are obtained to be as the following. Since there is no DIC experimental data that is available for CuZnAl and NiTi, the eigenstrains and intrinsic transformation strains are all hypothetical.

Ni$_2$FeGa:

$$
\varepsilon^{**} = \begin{bmatrix}
-0.0002 & 0.0002 & 0 \\
0.0002 & 0.0156 & 0 \\
0 & 0 & -0.005 \\
\end{bmatrix}
$$

$$
\varepsilon^p = \begin{bmatrix}
0.0033 & 0.0027 & 0 \\
0.0027 & 0.0309 & 0 \\
0 & 0 & -0.031 \\
\end{bmatrix}
$$

CuZnAl:

$$
\varepsilon^{**} = \begin{bmatrix}
0.022 & 0.0001 & 0 \\
0.0001 & 0.0024 & 0 \\
0 & 0 & -0.0019 \\
\end{bmatrix}
$$

$$
\varepsilon^p = \begin{bmatrix}
0.0038 & 0.0024 & 0 \\
0.0024 & 0.0428 & 0 \\
0 & 0 & -0.0449 \\
\end{bmatrix}
$$

NiTi:
The net reduction in stress intensity factor as the applied loading is increased is given in Table 4. The contributions from different sectors of the transformation zone are provided. It is noted that the zones that are away from the crack surface, Zone 1, Zone 2, and Zone 3, provide a smaller contribution compared to Zone 4. These results are further elaborated in the next section.

Table 4. Stress intensity factor (\(K(MPa\sqrt{m})\)) values due to tractions on different zone boundaries of Ni\(_2\)FeGa in Figure 6 for the \(a/w=2\) case. One example for demonstrating the calculation procedure is shown in the appendix.

<table>
<thead>
<tr>
<th>Zone #</th>
<th>Zone 1</th>
<th>Zone 2</th>
<th>Zone 3</th>
<th>Zone 4</th>
<th>Total (K_{\text{red}} = \sum_i K_i(MPa\sqrt{m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load(MPa)</td>
<td>(K_1)</td>
<td>(K_2)</td>
<td>(K_3)</td>
<td>(K_4)</td>
<td></td>
</tr>
<tr>
<td>3.34</td>
<td>0.35</td>
<td>0.013</td>
<td>0.35</td>
<td>-3.07</td>
<td>-2.37</td>
</tr>
<tr>
<td>10</td>
<td>0.42</td>
<td>0.018</td>
<td>0.42</td>
<td>-3.69</td>
<td>-2.85</td>
</tr>
<tr>
<td>20</td>
<td>0.63</td>
<td>0.023</td>
<td>0.63</td>
<td>-4.75</td>
<td>-3.49</td>
</tr>
<tr>
<td>30</td>
<td>0.93</td>
<td>0.028</td>
<td>0.93</td>
<td>-7.03</td>
<td>-5.17</td>
</tr>
<tr>
<td>40</td>
<td>1.16</td>
<td>0.033</td>
<td>1.16</td>
<td>-8.75</td>
<td>-6.43</td>
</tr>
<tr>
<td>50</td>
<td>1.46</td>
<td>0.039</td>
<td>1.46</td>
<td>-10.9</td>
<td>-7.98</td>
</tr>
</tbody>
</table>

As we can observe from Table 4, the reduction of stress intensity factor from Zone 2 is the lowest among all the sectors shown in Figure 5. This result matches the one that was obtained from Mcmeeking and Evans (McMeeking and Evans, 1982b).

Three other shape memory alloys were assessed to evaluate the propensity of K reduction. The results are shown in Tables 5 through 6. In the first set of simulations (Table 5) the \(a/w\) ratio was maintained at 2. In Table 6, we consider the reductions in K at both maximum and
minimum loads for different $a/w$ ratios. A noteworthy point is that the reductions in $K$ occurs at both maximum and minimum load, but since the reduction is higher at the maximum load this results in a net decrease in stress intensity range, i.e. $\Delta K_{\text{red}} = K_{\text{red-max}} - K_{\text{red-min}}$.

Table 5. The $\Delta K_{\text{red}} (\text{MPa}$/$\sqrt{m}$) values for alloys noted in the present study.

<table>
<thead>
<tr>
<th>Alloys</th>
<th>$a/w$</th>
<th>$\Delta K_{\text{red}} (\text{MPa}$/$\sqrt{m}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni$_2$FeGa</td>
<td>2</td>
<td>-5.61</td>
</tr>
<tr>
<td>NiTi</td>
<td>2</td>
<td>-2.38</td>
</tr>
<tr>
<td>CuZnAl</td>
<td>2</td>
<td>-1.91</td>
</tr>
</tbody>
</table>

Table 6. Reduction in stress intensity factor ($K_{\text{red}} (\text{MPa}$/$\sqrt{m}$)) values for alloys noted in the present study at minimum and maximum loads.

<table>
<thead>
<tr>
<th>Alloys</th>
<th>$a/w$</th>
<th>$K_{\text{red-min}}$</th>
<th>$K_{\text{red-max}} (\text{MPa}$/$\sqrt{m}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NiTi</td>
<td>0.5</td>
<td>-1.05</td>
<td>-2.59</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-1.47</td>
<td>-3.37</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-1.8</td>
<td>-4.18</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-2.25</td>
<td>-4.54</td>
</tr>
<tr>
<td>Ni$_2$FeGa</td>
<td>0.5</td>
<td>-1.26</td>
<td>-3.06</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-1.89</td>
<td>-6.74</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-2.59</td>
<td>-7.34</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-3.17</td>
<td>-7.77</td>
</tr>
<tr>
<td>CuZnAl</td>
<td>0.5</td>
<td>-1.48</td>
<td>-2.81</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-1.86</td>
<td>-3.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-2.29</td>
<td>-4.2</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-2.62</td>
<td>-4.7</td>
</tr>
</tbody>
</table>

Based on the result show in Table 6, the sensitivity of the model on the change of transformation zone geometry, $a/w$ ratio, and material property, martensite moduli magnitude, was investigated. In Figure 15 (a) and Figure 15 (b), the simulation results were shown based on these two parameters changes for Ni$_2$FeGa case. We note that as the $a/w$ ratio increases, with all other parameters constant, the reduction in both minimum and maximum stress intensity is noted. The overall reduction in stress intensity range saturates with increasing $a/w$ ratio. In
Figure 15 (b) the martensite modulus is pre-multiplied by a factor when $a/w$ ratio is kept at 2. The factor $F = 1$ corresponds to the Ni$_2$FeGa case. As the factor increases the reduction in stress intensity range increases.

(a)

Figure 15 (cont. on next page)
Figure 15. (a) Reduction in maximum and minimum stress intensity levels with increase in crack length, the results are for the [001] Ni$_2$FeGa material and explore the hypothetical variation of residual transformation zone on the results (b) Reduction in maximum and minimum stress intensity levels as a function of martensite modulus factor. The moduli tensor is simply scaled by the factor, F. The F=1 case corresponds to the baseline Ni$_2$FeGa material.

A thorough investigation of stress intensity factor reduction on all three oriented single crystals for Ni$_2$FeGa is shown in Table 7. At each load level, minimum, intermediate, and
maximum, the K reduction was determined and later implemented in the overall stress intensity factor range correction shown in Figure 16 at different crack length.

**Table 7. Reduction in stress intensity factor \( K_{\text{red}}(\text{MPa}\sqrt{m}) \) values for Ni$_2$FeGa for different loading orientations.** The effective stress intensity range \( \Delta K_{\text{eff}} \) values obtained from regression are also given for the \( a/w=2 \) case.

<table>
<thead>
<tr>
<th>Crystal Orientation</th>
<th>[001]</th>
<th></th>
<th>[011]</th>
<th></th>
<th>[123]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Load(MPa)</td>
<td>( K_{\text{red}} )</td>
<td>( \Delta K_{\text{eff}} )</td>
<td>( K_{\text{red}} )</td>
<td>( \Delta K_{\text{eff}} )</td>
<td>( K_{\text{red}} )</td>
<td>( \Delta K_{\text{eff}} )</td>
</tr>
<tr>
<td>3.34</td>
<td>-2.37</td>
<td>0.75</td>
<td>-2.83</td>
<td>0.67</td>
<td>-2.47</td>
<td>0.98</td>
</tr>
<tr>
<td>10</td>
<td>-2.85</td>
<td>1.80</td>
<td>-2.98</td>
<td>2.16</td>
<td>-2.77</td>
<td>5.12</td>
</tr>
<tr>
<td>20</td>
<td>-3.49</td>
<td>3.66</td>
<td>-3.46</td>
<td>3.98</td>
<td>-3.34</td>
<td>8.03</td>
</tr>
<tr>
<td>30</td>
<td>-5.17</td>
<td>5.97</td>
<td>-4.16</td>
<td>5.62</td>
<td>-4.56</td>
<td>12.49</td>
</tr>
<tr>
<td>40</td>
<td>-6.43</td>
<td>10.14</td>
<td>-5.09</td>
<td>6.23</td>
<td>-5.44</td>
<td>14.57</td>
</tr>
<tr>
<td>50</td>
<td>-7.98</td>
<td>13.49</td>
<td>-6.29</td>
<td>7.09</td>
<td>-6.65</td>
<td>17.23</td>
</tr>
</tbody>
</table>

The experimental fatigue crack growth rate results, Figure 8, and predictions of fatigue crack growth rates upon correction of stress intensity range, Table 7, are shown in Figure 16. The effective stress intensity range upon regression of the entire displacement field is also included in Figure 16. The agreement for the theory and regression based stress intensity range is excellent. The reduction in the stress intensity range is approximately 35% of the full range based on regression and also based on theory, i.e. \( \frac{\Delta K_{\text{red}}}{\Delta K} \). We also showed the virtual extensometer results of crack opening displacements in 4.1.4. The virtual extensometer result showed that the crack opening initiates when the current is load is 35% of the maximum load. In this way, the modeling results match with experimental result quite well.
Figure 16. Fatigue Crack Growth Behavior of NiFeGa. The range in effective stress intensity is obtained by regression analysis of crack tip displacements and also via calculation of the shielding effects due to transformation. The effective threshold stress intensity range is $8.3 \text{ MPa} \sqrt{\text{m}}$. The full range of stress intensity is also provided as a reference. The dashed line is added to aid eye.

The theoretical stress intensity factor range is determined through conventional calculation shown in Equation (4.5) and Equation (4.6).

$$\Delta K_I = f\left(\frac{a}{w}\right)\Delta \sigma \sqrt{\pi a}$$  \hspace{1cm} (4.5)

$$\Delta K_{II} = f\left(\frac{a}{w}\right)\Delta \tau \sqrt{\pi a}$$  \hspace{1cm} (4.6)
where $\Delta \sigma$ and $\Delta \tau$ are the normal and parallel force to the crack surface, $a$ is the crack length, and $f\left(\frac{a}{w}\right)$ is the geometric correction factor. For a single-edge notch tension specimen as shown in Figure 1, the geometric correction factor can be written as the following,

$$f\left(\frac{a}{w}\right) = 0.265 \left(1 - \frac{a}{w}\right)^4 + \frac{0.857 + 0.256 \frac{a}{w}}{\left(1 - \frac{a}{w}\right)^{\frac{3}{2}}} \quad (4.7)$$

The points that are picked up for showing the correction of stress intensity factor range in Figure 16 are tabulated in the following table.

**Table 8. Correction of theoretical stress intensity factor range at different crack length**

<table>
<thead>
<tr>
<th>$a$</th>
<th>$da/dN$</th>
<th>$\Delta K_{\text{theory}}$</th>
<th>$\Delta K_{\text{red}}$</th>
<th>$\Delta K_{\text{theory}} - \Delta K_{\text{red}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm</td>
<td>mm/cycle</td>
<td>MPa $\sqrt{m}$</td>
<td>MPa $\sqrt{m}$</td>
<td>MPa $\sqrt{m}$</td>
</tr>
<tr>
<td>0.65</td>
<td>2.74E-06</td>
<td>12.44</td>
<td>-4.5</td>
<td>7.94</td>
</tr>
<tr>
<td>0.68</td>
<td>7.12E-06</td>
<td>12.79</td>
<td>-4.5</td>
<td>8.29</td>
</tr>
<tr>
<td>0.73</td>
<td>9.77E-06</td>
<td>13.40</td>
<td>-4.7</td>
<td>8.70</td>
</tr>
<tr>
<td>0.81</td>
<td>1.17E-05</td>
<td>15.05</td>
<td>-5.5</td>
<td>9.55</td>
</tr>
<tr>
<td>0.87</td>
<td>1.39E-05</td>
<td>16.25</td>
<td>-6.1</td>
<td>10.15</td>
</tr>
<tr>
<td>0.96</td>
<td>1.76E-05</td>
<td>17.35</td>
<td>-6.2</td>
<td>11.15</td>
</tr>
<tr>
<td>1.01</td>
<td>2.12E-05</td>
<td>18.82</td>
<td>-6.3</td>
<td>12.52</td>
</tr>
<tr>
<td>1.10</td>
<td>5.65E-05</td>
<td>24.69</td>
<td>-6.4</td>
<td>18.29</td>
</tr>
<tr>
<td>1.21</td>
<td>8.55E-05</td>
<td>25.56</td>
<td>-6.5</td>
<td>19.06</td>
</tr>
</tbody>
</table>

**Section 4.3 Discussions**

The current project is aimed to understand the behavior of fatigue crack growth on shape memory alloys through both experiment and modeling. A methodology is developed ranging from micro-mechanical modeling to experimental digital image correlation on the material surface. Since the current work on shape memory alloys still focuses on the monotonic tension or compression test to get insight into SME and PE, the fatigue damage tolerance of such
material has been overlooked. The work on fatigue is necessary but more complex compared to unidirectional (monotonic) deformation or a single load-unload cycle. The results of the paper tried to provide greater insight into the processes responsible for the crack growth behavior under fatigue.

In order to establish the problem macroscopically, fatigue crack growth experiments were conducted on Ni$_2$FeGa. The results on the experimental part can be found in previous sections. The single crystals of Ni$_2$FeGa were tested under tension-tension cyclic loading condition with maximum stress 50 MPa and R value 0.05. The tests were conducted at room temperature condition at which the material is at pseudoelastic state. The crack growth rate was measured as a function of effective stress intensity factor range. This characterization was accomplished with the help of Digital Image Correlation by obtaining displacement contour from reference and deformed speckle pattern. Then the effective stress intensity factor was extracted from the displacement data through digital image correlation. The threshold value of this particular class of shape memory alloy (SMA) is measured to be 8.3 MPa$\sqrt{m}$ which higher than NiTi, another well studied SMA. The virtual extensometers were also established along the crack to study the closure effect. To our knowledge this is the first time effective stress intensity range has been determined precisely on shape memory alloys and in particular the Ni$_2$FeGa alloy.

Specific reduction in stress intensity factor was numerically determined with micromechanical analysis. The closure forces arised due to residual strain, measured through DIC, in the crack wake is also determined. Then, they were used to quantify the closure effect through stress intensity factor. The reduction of stress intensity factor is about 35% of the theoretical stress intensity factor. On the other hand, by using virtual extensometer, the crack started to open when the load is about 35% of the maximum load. In other words, the effective stress
intensity factor is about 35% smaller than the theoretical one. This difference is contributed to the shielding effect due to residual displacements in the crack wake. As a result, the difference between theoretical and the calculated reduction stress intensity factor is in a reasonable good agreement with the effective stress intensity factor determined through experiment.

A combined experimental-theoretical methodology is outlined for a better understanding of the driving force for fatigue crack growth in shape memory alloys. The choice of single crystals allows precise knowledge of the elastic moduli in the austenitic and martensitic phases. In turn, the moduli tensors were used in a micro-mechanical analysis to determine the equivalent eigenstrains in the transformed regions. The equivalent eigenstrain was used to determine the internal tractions at maximum load of the cycle. This led to the calculation of the reduction in stress intensity. Hence, a modified range of stress intensity was determined. The calculations presented represent an improvement over those which do not account for elastic moduli difference and the experimental determination of strain fields at minimum and maximum loads is also novel.

To put perspective on results, the simulations were repeated on two well-known shape memory materials, the NiTi and CuZnAl. The reductions in stress intensity range were lower in NiTi compared to Ni$_{54}$Fe$_{19}$Ga$_{27}$, while the reduction in CuZnAl was substantially lower. These results cannot be directly with experiments in the literature. There is no reported CuZnAl fatigue crack growth data to our knowledge. The literature on NiTi shows threshold levels that are rather low compared to Ni$_{54}$Fe$_{19}$Ga$_{27}$, however, no crack closure measurements were reported to our knowledge for NiTi. So, the effective stress intensities are not available for NiTi.

Fatigue crack growth behavior in shape memory alloys remains a complex topic. The elastic moduli evolves continuously, it is strongly orientation dependent in both austenite and
martensite, and strongly influences the stress intensity levels. The moduli tensors decide the equivalent eigenstrains hence the closure forces. The closure forces vary as a function of cycles accompanying the transformation strains at peak loads and residual transformation strains. In this study we established the modification in stress intensity and established a rigorous estimate of the stress intensity range. In future studies, the crack growth rate needs to be predicted based the magnitude of the irreversibility in displacements at crack tips (Chowdhury et al., 2014a, b). This would require knowledge of the slip and transformation energy barriers in the material such as those presented in a paper by Sehitoglu (Sehitoglu et al., 2012). This approach would need to be taken with care because both transformation and plasticity can occur simultaneously at the crack tips. In the present calculations no explicit consideration of plastic slip was included (Sehitoglu et al., 2012). Plasticity can occur at high stress levels and it needs to be considered in future work.

It is instructive to put the experimental findings of threshold stress intensity in perspective with other metals and alloys. The Ni₅₄Fe₁₉Ga₂₇ (at.%) exhibits effective stress intensity range of 8.3 MPa√m which is higher than other untransforming intermetallic alloys. For example, if we compare with other B2 structures their fatigue thresholds are less than 3 MPa√m in most cases (Ritchie, 1999).

Finally, we comment on the role of martensite to austenite modulus change. Evidence of higher martensite modulus relative to austenite is well documented (Sehitoglu et al., 2002). On the other hand, the martensite modulus is taken as less than the austenite modulus in most constitutive models. The answer is to do with the martensite moduli being measured in the self accomodating state in experiments while the state that is relevant to fatigue and fracture studies is the oriented martensite (Wang and Sehitoglu, 2014). There is a second issue with the FEM
simulations. Inevitably, the constitutive models utilized have been simple for ease of implementation in a FEM code. This creates some difficulty when residual strain buildup due to residual martensite or plastic deformation needs to be considered. There is no provision for these mechanisms in most constitutive models. Finally, there is the matter of orientation dependence. A highly anisotropic material cannot be represented accurately as isotropic with two constants. Constitutive models need to address these issues.
Chapter 5. Conclusions

The work supports the following conclusions:

(1) The new shape memory alloy Ni$_{54}$Fe$_{19}$Ga$_{27}$ displays unusually high fatigue thresholds and excellent fatigue crack growth resistance. The reduction of the stress intensity range associated with the transformation is considerable as shown with an anisotropic micromechanics calculation.

(2) Excellent quantitative correlation is achieved between theory and the experimental measurements of stress intensity range reduction. Utilizing crack tip displacement fields with digital image correlation methods allowed evaluation of the effective stress intensity range in agreement with the virtual extensometers along the crack flanks. These results show that the reduction in stress intensity is 30% of the theoretical value.

(3) Comparisons were made between three shape memory alloys to assess their propensity for shielding associated with phase transformations. It was found that the Ni$_2$FeGa produced higher levels of stress intensity reduction compared to NiTi and CuZnAl alloys. The work underscored the role of elastic moduli in the martensitic and austenitic phases on the calculations of the reduction in stress intensity range.
References


Appendix A

The treatment follows that given by Mura (Mura, 1987). The Eshelby’s tensor calculation is introduced in Equation (3).

\[
S_{ijkl} = \frac{1}{8\pi} C_{pqkl} \left( \overline{G}_{ipjq} + \overline{G}_{jpiq} \right)
\]

For the case of cubic material, the definition of \( \overline{G}_{ipjq} \) is presented in Equation (A1).
\[ \begin{align*}
\bar{G}_{111} &= \bar{G}_{222} \\
&= \frac{2\pi}{a} \int_0^1 \left(1-x^2\right) \left(\frac{1-x^2 + \rho^2 x^2}{pq}\right) \left[\mu^2(1-x^2 + \rho^2 x^2) + \beta \rho^2 x^2\right] dx \\
&\quad + \frac{\pi}{a} \int_0^1 \left(1-x^2\right)^2 \left[\beta(1-x^2 + \rho^2 x^2) + \gamma \rho^2 x^2\right] dx \\
\bar{G}_{333} &= \frac{4\pi}{a} \int_0^1 \frac{\rho^2 x^2}{pq} \left(1-x^2 + \rho^2 x^2\right) \left[\mu^2(1-x^2 + \rho^2 x^2) + \beta(1-x^2)\right] dx \\
&\quad + \frac{\pi}{a} \int_0^1 \frac{\rho^2 x^2}{p(p+q)} (1-x^2)^2 dx \\
\bar{G}_{122} &= \bar{G}_{221} \\
&= \frac{2\pi}{a} \int_0^1 \left(1-x^2\right) \left(1-x^2 + \rho^2 x^2\right)^2 \left[\mu^2(1-x^2 + \rho^2 x^2) + \beta \rho^2 x^2\right] dx \\
&\quad + \left(1-x^2\right) \left[\beta(1-x^2 + \rho^2 x^2) + \gamma \rho^2 x^2\right] dx \\
&\quad - \frac{\pi}{a} \int_0^1 \left(1-x^2\right)^2 \left[\beta(1-x^2 + \rho^2 x^2) + \gamma \rho^2 x^2\right] dx \\
\bar{G}_{133} &= \bar{G}_{233} \\
&= \frac{2\pi}{a} \int_0^1 \rho^2 x^2 \left(1-x^2 + \rho^2 x^2\right) \left[\mu^2(1-x^2 + \rho^2 x^2) + \beta \rho^2 x^2\right] dx \\
&\quad + \left(1-x^2\right) \left[\beta(1-x^2 + \rho^2 x^2) + \gamma \rho^2 x^2\right] dx \\
\bar{G}_{123} &= -\frac{\pi}{a} \int_0^1 \left(1-x^2\right)^2 \left[\mu \left(1-x^2 + \rho^2 x^2\right) + \mu' \rho^2 x^2\right] dx
\end{align*} \]
\[
\bar{G}_{1313} = \bar{G}_{2323} \\
= -\frac{2\pi\mu(\lambda + \mu)}{a} \int_0^1 \frac{\rho^2 x^2 (1-x^2)(1-x^2 + \rho^2 x^2)}{pq} \, dx \\
- \frac{\pi\mu'(\lambda + \mu)}{a} \int_0^1 \frac{\rho^2 x^2 (1-x^2)^2}{p(p+q)} \, dx
\]

\[
\bar{G}_{3311} = \bar{G}_{3322} \\
= \frac{2\pi}{a} \int_0^1 \frac{(1-x^2)}{pq}(1-x^2 + \rho^2 x^2) \left[ \mu^2 (1-x^2 + \rho^2 x^2) + \beta (1-x^2) \right] \, dx \\
+ \frac{\pi\mu'}{2a} \int_0^1 \frac{(1-x^2)^3}{p(p+q)} \, dx
\]

Specific terms in Equation (A1) can be represented as the following,

\[
\begin{align*}
\rho &= a_1/a_3 \\
a &= \mu^2 (\lambda + 2\mu + \mu') \\
b &= a_1^3 \mu \mu' (2\lambda + 2\mu + \mu') \\
c &= a_1^3 \mu^2 (3\lambda + 3\mu + \mu') \\
\beta &= \mu (\lambda + \mu + \mu') \\
\gamma &= \mu' (2\lambda + 2\mu + \mu') \\
p &= \left\{ (1-x^2 + \rho^2 x^2)^3 + b \rho^2 x^2 (1-x^2)(1-x^2 + \rho^2 x^2) \right. \\
&\quad \left.+ \frac{1}{4} (1-x^2)^2 \left[ b (1-x^2 + \rho^2 x^2) + c \rho^2 x^2 \right] \right\}^{1/2}, 0 < x < 1 \\
q &= \left\{ (1-x^2 + \rho^2 x^2)^3 + b \rho^2 x^2 (1-x^2)(1-x^2 + \rho^2 x^2) \right. \\
&\quad \left.+ \frac{1}{4} (1-x^2)^2 \left[ b (1-x^2 + \rho^2 x^2) + c \rho^2 x^2 \right] \right\}^{1/2}, 0 < x < 1
\end{align*}
\] (A2)

where \( \lambda \) is \( C_{12} \), \( \mu \) is \( C_{44} \), \( \mu' \) is \( C_{11}-C_{12}-2C_{44} \), \( a_1, a_2, \) and \( a_3 \) are the semi axis align with the coordinate \( x, y \) and \( z \). For the case of flat ellipsoid \( a_1 > a_2 > a_3 \) \( \rho \) is assumed to be infinity.
Appendix B

Earlier, the calculation of weight function was introduced as the following.

\[
h_x = \frac{H}{2K_1} \frac{\partial u^l}{\partial l}
\]

\[
h_y = \frac{H}{2K_1} \frac{\partial v^l}{\partial l}
\]

Horizontal displacement \( u^l \) and vertical displacement \( v^l \) can be found earlier. The stress intensity factor \( K_I \) is presented earlier as well as the elastic modulus \( H \). The partial differential terms in equation above can be further expanded in Equation (B1).

\[
\frac{\partial u^l}{\partial l} = \frac{\partial u^l}{\partial \theta} \frac{\partial \theta}{\partial l} + \frac{\partial u^l}{\partial r} \frac{\partial r}{\partial l}
\]

\[
\frac{\partial v^l}{\partial l} = \frac{\partial v^l}{\partial \theta} \frac{\partial \theta}{\partial l} + \frac{\partial v^l}{\partial r} \frac{\partial r}{\partial l}
\]  

(B1)

A schematic showing a point load with a distance \( r \) and oriented at an angle of \( \theta \) from the crack tip is presented in Figure b1.

![Figure B1. A schematic showing arbitrary point loading at the crack tip](image-url)
According to Figure b1, the term $r$ and $\theta$ can be represented in Equation B2.

$$r = \sqrt{(x-l)^2 + y^2}$$  \hspace{1cm} (B2)

$$\theta = \tan^{-1} \frac{y}{x-l}$$

Through Equation (B2), $\frac{\partial \theta}{\partial l}$ and $\frac{\partial r}{\partial l}$ in Equation (B1) can be further determined in Equation (B3).

$$\frac{\partial \theta}{\partial l} = \frac{d}{dl} \left( \frac{y}{x-l} \right) = \frac{y}{1 + \left( \frac{y}{x-l} \right)^2} = \frac{y}{r} = \sin \theta$$

$$\frac{\partial r}{\partial l} = -\frac{1}{2} \sqrt{(x-l)^2 + y^2} \cdot (2x-2) = \frac{-(x-l)}{\sqrt{(x-l)^2 + y^2}} - \cos \theta$$  \hspace{1cm} (B3)

Plugging Equation (B3) back in Equation (B1), Equation (B4) can be obtained.

$$\frac{\partial u^i}{\partial l} = \frac{\partial u^i}{\partial \theta} \cdot \frac{\sin \theta}{r} - \frac{\partial u^i}{\partial r} \cdot \cos \theta$$

$$\frac{\partial v^i}{\partial l} = \frac{\partial v^i}{\partial \theta} \cdot \frac{\sin \theta}{r} - \frac{\partial v^i}{\partial r} \cdot \cos \theta$$  \hspace{1cm} (B4)

By using Equation (B4), the weight function can be calculated by using Equation (B5) in computer programs, like Mathematica, Matlab and etc.

$$h_x = \frac{H}{2K_1} \left( \frac{\partial u^i}{\partial \theta} \cdot \frac{\sin \theta}{r} - \frac{\partial u^i}{\partial r} \cdot \cos \theta \right)$$

$$h_y = \frac{H}{2K_1} \left( \frac{\partial v^i}{\partial \theta} \cdot \frac{\sin \theta}{r} - \frac{\partial v^i}{\partial r} \cdot \cos \theta \right)$$  \hspace{1cm} (B5)
Appendix C

Procedure of the K reduction calculation

1) When there is no applied load on the specimen, the stress intensity factor change will only be affected by the intrinsic inelastic strain, $e^p$.

The strain obtained through DIC at the crack tip is interpreted as the constraint strain in this methodology. For example, at the minimum load, the strain at the beginning of the load cycle is obtained as the following.

$$
\begin{pmatrix}
0.001 & 0.0052 & 0 \\
0.0052 & 0.0092 & 0 \\
0 & 0 & 0.041
\end{pmatrix}
$$

The intrinsic inelastic strain can be determined by Equation (3.1) without any far field strain, $e^{o}_{mn}$ and included as the following.

$$
\begin{pmatrix}
0.001 & 0.0052 & 0 \\
0.0052 & 0.0092 & 0 \\
0 & 0 & 0.041
\end{pmatrix}
$$

By following Equation (3.8) and Equation (3.6), the corresponding stress intensity factor for each zone can be calculated.
Table C1. Stress intensity factor due to the tractions on different zones at minimum load

<table>
<thead>
<tr>
<th>Anisotropic Calculation</th>
<th>Figure</th>
<th>$\Delta K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td><img src="image1.png" alt="Image" /></td>
<td>0.43 MPa(\sqrt{m})</td>
</tr>
<tr>
<td>Zone 2</td>
<td><img src="image2.png" alt="Image" /></td>
<td>0.012 MPa(\sqrt{m})</td>
</tr>
<tr>
<td>Zone 3</td>
<td><img src="image3.png" alt="Image" /></td>
<td>0.43 MPa(\sqrt{m})</td>
</tr>
<tr>
<td>Zone 4</td>
<td><img src="image4.png" alt="Image" /></td>
<td>-3.07 MPa(\sqrt{m})</td>
</tr>
<tr>
<td>Total</td>
<td><img src="image5.png" alt="Image" /></td>
<td>-2.37 MPa(\sqrt{m})</td>
</tr>
</tbody>
</table>

2) When there is an external load applied on the specimen, the stress intensity factor change will only be affected by both intrinsic inelastic strain, $e^p$, and the eigenstrain due to modulus mismatch, $e^e$.

At the maximum load step, the total strain obtained through DIC is demonstrated as the following.

$$
\left[ e_{kl}^D \right] = 
\begin{pmatrix}
0.015 & 0.0235 & 0 \\
0.0235 & 0.045 & 0 \\
0 & 0 & -0.037
\end{pmatrix}
$$
The intrinsic inelastic strain can be determined by Equation (3.1) with a far field strain and included as the following.

\[
\begin{bmatrix}
\epsilon_{el}^p \\
\end{bmatrix} = \begin{bmatrix}
0.0033 & 0.0027 & 0 \\
0.0027 & 0.0309 & 0 \\
0 & 0 & -0.031
\end{bmatrix}
\]

\(\epsilon_{en}^p\) is then substituted into Equation (3.4) to calculate the equivalent transformation strain \(\epsilon_{en}^{**}\).

Then, by plugging the equivalent transformation strain into Equation (3.5), the tractions on each zones can be obtained. Later, the corresponding stress intensity factor for each zone can be calculated through Equation (3.6) and Equation (3.8).

**Table C2. Stress intensity factor due to the tractions on different zones at maximum load**

<table>
<thead>
<tr>
<th>Anisotropic Calculation</th>
<th>Figure</th>
<th>(\Delta K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td><img src="image" alt="Figure Zone 1" /></td>
<td>1.46 MPa(\sqrt{m})</td>
</tr>
<tr>
<td>Zone 2</td>
<td><img src="image" alt="Figure Zone 2" /></td>
<td>0.039 MPa(\sqrt{m})</td>
</tr>
<tr>
<td>Zone 3</td>
<td><img src="image" alt="Figure Zone 3" /></td>
<td>1.46 MPa(\sqrt{m})</td>
</tr>
<tr>
<td>Zone 4</td>
<td><img src="image" alt="Figure Zone 4" /></td>
<td>-10.9 MPa(\sqrt{m})</td>
</tr>
<tr>
<td>Total</td>
<td><img src="image" alt="Figure Total" /></td>
<td>-7.98 MPa(\sqrt{m})</td>
</tr>
</tbody>
</table>
Appendix D

More DIC results

Figure D.1 Axial strain contours of Ni$_2$FeGa [001] single crystal in the case of crack bifurcation at maximum and minimum load. $K_I$=12.7 MPa$\sqrt{m}$, $K_{II}$=12.9 MPa$\sqrt{m}$, $\Delta\sigma$=42.5MPa.
Figure D.2 High resolution (10x) shear strain contours of Ni$_2$FeGa [123] single crystal at maximum and minimum load. $K_I=31.63$ MPa$\sqrt{m}$, $K_{II}=0.17$ MPa$\sqrt{m}$, $\Delta\sigma=42.5$MPa.
Figure D.3 High resolution (10x) axial strain contour of Ni$_2$FeGa [011] single crystal at maximum load with higher magnification (20x) showing the contour at the crack wake. $K_I=9.63$ MPa$\sqrt{\text{m}}$, $K_{II}=0.17$ MPa$\sqrt{\text{m}}$, $\Delta\sigma=42.5$ MPa.
Figure D.4 High resolution (10x) axial strain contour of Ni$_2$FeGa [011] single crystal at minimum load with higher magnification (20x) showing the contour at the crack wake