Solving Problems in Library and Information Science Using Fuzzy Set Theory

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Abstract

Various mathematical tools and theories have found application in Library and Information Science (LIS). One of these is Fuzzy Set Theory (FST). FST is a generalization of classical Set Theory, designed to better model situations where membership of a set is not discrete but is "fuzzy." The theory dates from 1965, when Lotfi Zadeh published his seminal paper on the topic. As well as mathematical developments and extensions of the theory itself, there have been many applications of FST to such diverse areas as medical diagnoses and washing machines. The theory has also found application in a number of aspects of LIS. Information Retrieval (IR) is one area where FST can prove useful; this paper reviews IR applications of FST. Another major area of Information Science in which FST has found application is Informetrics; these studies are also reviewed. A few examples of the use of this theory in non-LIS domains are also examined.

Background

When an information professional is confronted with a problem, there may be many different ways to tackle it. In the armoury of the profession, there are a number of tools and techniques that can be drawn upon to address the situation. A good problem solver needs to be aware of a wide range of tools that can be used in that particular situation. Tools developed for one specific situation may be applicable to others, though they be quite different. One class of tools that can be applied to library problems are mathematical tools. Mathematical tools are indispensable for solving a

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whole range of different types of problems. These tools include statistics, probability theory, operations research (including, for example, queuing theory), and cluster analysis.

Mathematical tools that are used to solve real-world problems fall into the category of applied mathematics. The essence of applied mathematics is abstraction and modeling. Some aspect of the real world may be modeled by a mathematical theory, which is an approximation of the reality. Development can take place in the mathematical theory, independent of any applications, and the results can then be applied back to the real world. How useful this is depends on how well the mathematical model captures the essence of the reality, and also how well the model has been formulated and developed. A good model will be able to provide useful insights into the real life situation, and will be a good tool for problem solving.

Set Theory is one theory or model that has proved enormously useful in a wide range of situations. Any collection of objects can be regarded as a set. Operations, such as union, intersection, and complementation, can be carried out on these sets. In fact, most of mathematics has Set Theory as its theoretical underpinning. The classical formulation of Set Theory applies to situations where membership of a set is discrete. The "set of red balls" or the "set of white cars" are situations where membership (or not) of the set from a universe of objects is definite. The "set of documents in a filing cabinet" or "the set of books on the library shelf" are also discrete and clear-cut sets. Set Theory has been applied in many situations that can be modeled by discrete membership, and has proven to be a useful tool.

However, there are many situations where classical Set Theory does not provide a good model. If there is some vagueness or fuzziness about the membership of a set, then classical Set Theory may not be useful. The "set of tall people" has the problem of defining exactly what constitutes a tall person: Where do you make the boundary of "tallness" and what happens if someone is marginally shorter than this boundary? The "set of relevant documents" also suffers from this problem. In Library and Information Science, "relevance" is sometimes regarded as dichotomous, but in reality, relevance is a graded concept with documents ranging from highly relevant for a particular purpose, to highly irrelevant, and every degree of relevance in between.[1]

In a seminal paper on what he defined as Fuzzy Sets, Zadeh (1965) attempted to provide a mathematical model that would be better suited to these vague situations. Fuzzy Set Theory (FST) has become a branch of mathematics that generalizes the concept of a set[2] to provide better tools for dealing with the sorts of situations described in the previous paragraph. Though designed to model fuzziness, the theory itself is not fuzzily defined. This essay will give an introduction to what FST is, as well as provide some of the applications for which it has been used. The material for this article has been developed from the first author's Ph.D. thesis (Hood, 1998).

Basic Idea Behind Fuzzy Set Theory

The basic idea behind FST is to generalize the concept of membership of a set. In classical (or crisp) sets, membership of a set can be regarded as a function with only two possible values. That is, an item either belongs to the set or does not. The generalization that is made to produce Fuzzy Sets is to allow the membership function to be multivalued. This allows an item to have a degree of membership in a set. An item can belong to a Fuzzy Set with any degree of membership from none to full. Let us consider the example of a "Fuzzy Set of Relevant Documents" for a particular query. In this case, a highly relevant document may belong to this set with high degree of membership; whereas a marginally relevant document will still belong to the set, but with only a small degree of membership. A totally nonrelevant document will not belong to this set at all.

The concept of a Fuzzy Set can be formally defined mathematically, and interested readers may care to consult Zimmerman (1991), which is one of the current standard texts on FST and its applications. As well as the definition of a Fuzzy Set itself, various classical set operations such as union, intersection, and complementation can be generalized to Fuzzy Sets.

From these basic definitions, many extensions and developments are possible. Zimmerman (1991, p. 6) summarizes the developments that have taken place in FST, along two different lines:

> 1. As a formal theory which, when maturing, became more sophisticated and specified and was enlarged by original ideas and concepts as well as by "embracing" classical mathematical areas such as algebra, graph theory, topology, and so on by generalising (fuzzifying) them.
> 2. As a very powerful modeling language, that can cope with a large fraction of uncertainties of real-life situations. Because of its generality, it can be well adapted to different circumstances and contexts.

Other Measures of Vagueness or Uncertainty

There are other tools and measures of uncertainty and vagueness apart from FST. One such tool that would be quite familiar to most readers is Probability Theory. Probability concerns a set of events which, taken together, are certain, but each separate event only has a degree of certainty. Thus, when tossing a coin, there are only two possible outcomes, as it is certain that the coin will land on either heads or tails. However, Probability Theory tells us that for a nonbiased coin, each of the two events are equally likely, so each has a probability of one half. FST has no such universe of certainty, and therefore is applied in different types of situations. A third tool that can also be used to measure vagueness is Possibility Theory. For a discussion of the distinction between some of the different measures of uncertainty, see Zimmerman (1991, Chap. 8).

Fuzzy Set Theory in Library and Information Science

One of the characteristics of mathematical theories is that they are often applied in a wide range of different situations, beyond the wildest imaginations of the original developers. This is certainly true of FST. The applications of particular interest here are, of course, in the area of Library and Information Science (LIS). Within LIS, FST has been applied to traditional librarianship, as well as to problems in Information Retrieval (IR). Applications have also been made in Bibliometrics and Informetrics, which are closely allied to LIS. Some of these applications will be discussed below.

Library Applications of Fuzzy Set Theory

Following are two examples of FST applications to library decision-making. The first is taken from Egghe & Rousseau (1990) and is based on Turner & O'Brien (1984) and Robinson & Turner (1981).

Many libraries have to make decisions about when and if to bind their periodicals. The decisions may be based on a number of criteria including the number of missing issues, the future expected use of the periodical, etc. Each of these criteria is vague, and can be modeled with a Fuzzy Set. In addition, the decision may be based on the opinions of more than one decision maker. A possible formalization of the methodology is as follows:

> A small committee of experts is formed. For reasons of simplicity, we shall consider a team of two experts. Three criteria will be used:
> –number of citations obtained by the journal, as measured by ISI's (Institute for Scientific Information) citation files;
> –percentage of missing issues;
> –number of circulations (local use) per issue.
> Each committee member must decide on his/her membership function for each of these variables. So, although each of these criteria can be measured in an objective way, the interpretation of the measurements with respect to the ultimate binding decision is subjective and requires an application of concepts borrowed from fuzzy set theory.
> When experts have decided on membership functions, every journal set can be judged on all criteria. This can now be done in a straightforward way and no longer requires a specific intellectual input.
> Finally, each expert must also have decided, beforehand, on the relative importance of each of the three criteria, and the library committee must have decided on the relative importance of each expert (before the data were collected!). This leads to a ranking of journals according to their suitability for binding. (Egghe & Rousseau 1990, pp. 200–201)

A similar approach can be used to making tattletaping decisions of periodicals (Turner, 1981). The methodology outlined above may help the library produce a list of journals that are the most likely candidates for tattletaping.

Information Retrieval Applications of Fuzzy Set Theory

The main application for Fuzzy Sets in LIS to date has been in the area of IR. As mentioned earlier, the concept of "relevance" is essentially a fuzzy concept; for any search, there will be documents that are more relevant or less relevant than others. IR is concerned with retrieving documents that meet some particular user need, or are relevant for some particular situation. The earliest attempts to apply Fuzzy Sets in this area appear to be those of Tahani (1976) and Radecki (1976). An ARIST review of this area is provided by Bookstein (1985) and a survey of the use of FST in IR and databases is given by Kerre, Zenner, & De Caluwe (1986). A theoretical background to the application of Fuzzy Sets to IR is given in Radecki (1983).

Traditionally, the main mathematical tool in IR has been Boolean algebra. Nearly everyone who has done any searching using bibliographic databases (such as those available through the DIALOG information systems), or searched library catalogs or the World Wide Web has used Boolean operators to construct sophisticated searches. In turn, Boolean algebra is based on Set Theory: Each search or index term results in a set of retrieved documents, which can then be combined using the Boolean operators (AND, OR, NOT). An IR system can be regarded as consisting of a set of "documents" and a set of "index terms." Each index term corresponds to a set of documents, which will be a subset of the universe of all documents in the system. This subset will consist of all those documents related to the index term. Traditional Boolean searches correspond to set operations on these index-term subsets.

As has been mentioned earlier, "relevance" is a concept that is not really dichotomous, and can readily be modeled by Fuzzy Set models instead. So Fuzzy IR systems work as follows: When documents are added to the system, index terms are assigned to the document, and each term is assigned a weight, indicating the degree to which that index term is associated with the document. The indexer is then free to indicate that a term applies only partially to a document, without having to make an absolute yes/no decision. Retrieval in a Fuzzy IR system is then based on Fuzzy Set algebra rather than Set algebra. The same Boolean operators are used (AND, OR and NOT), but the operators now rely on fuzzy union, fuzzy intersection, and fuzzy negation, rather than their classic (exact) equivalents.

This approach to IR has a lot of theoretical appeal, as it appears to be a much better model of the underlying process of selection (by users) of "relevant" documents. It is also a (relatively minor) modification of the traditional Boolean retrieval mechanism, so much of the existing infrastructure and mechanisms of IR are still valid. In addition, Fuzzy IR is more flexible in the assignment of index terms, with the use of partially relevant terms as well as fully relevant ones. The output can also be ranked according to relevance. Despite these advantages, there has not been much use made of Fuzzy IR in commercial systems. Reasons for this include: The cost of indexing continues to increase; many of the problems inherent in Boolean retrieval are still problems in Fuzzy IR; the capacity for ranking is not sensitive to all terms in the request; and traditional Boolean systems have done an adequate job in many situations (Bookstein, 1985, p. 124ff.).

Despite a lack of Fuzzy IR usage in most commercial IR systems, research has continued into the development of such systems, and there have been many applications in areas related to IR. Some of these will be listed below.

Expert systems and artificial intelligence. Gaines & Shaw (1985) discuss the history and development of expert systems, and the introduction of concepts from FST into this area. Graham (1991) also describes the use of fuzzy logic in commercial expert systems. FST has also been applied more generally in the area of artificial intelligence (Hofstadter, 1980; Winston, 1984). Nauck & Kruse (1999) use medical data to create fuzzy classification rules.

Knowledge-assisted document retrieval. A number of papers discuss the implementation of a knowledge-assisted document retrieval system (Subramanian, Biswas, & Bezdek, 1986; Biswas et al., 1987a, 1987b).

Relational databases with vague queries. Motro (1988) describes a database system that provides a user interface that permits vague queries based on FST. Some theoretical work done by Hashimoto (1985) can also be applied to Fuzzy databases.

Fuzzy clustering. The use of Fuzzy clustering algorithms in IR is also an area that has received a lot of attention. The excellent monograph by Miyamoto (1990a) provides a good description of the theory behind and the many uses of Fuzzy clustering. Fuzzy clustering can be applied in any situation where normal clustering is useful. Applications of Fuzzy clustering are described in Miyamoto, Miyake, & Nakayama (1983); Nomoto et al. (1987); Miyamoto, Midorikawa & Nakayama (1989); and Nomoto et al. (1990).

Fuzzy thesauri-based retrieval. Particular attention has been paid to the Fuzzy clustering of citations. These clusters can be used to form a thesaurus-like structure. Some work has been done on constructing Fuzzy thesauri to assist in creating queries and searching IR systems. Work in this area is reported in Miyamoto (1989, 1990b).

OPACs. Meikle (1995), in a literature review, includes the application of FST to searching on OPACs. Meikle notes that, despite some research effort, there have not been any commercial applications of Fuzzy Sets to OPAC searching to date.

Other. Ahrens (1994) describes tests on a retrieval system (Knowledge Finder) to compare user opinions of the effectiveness of this system versus a traditional Boolean IR system.The results were favourable. Kall & Srinivasan (1990) compare Fuzzy and probabilistic models for user relevance judgements. Klir (1991) also uses Fuzzy Sets in developing a generalized information theory.[3]

Fuzzy Set Theory in Informetrics and Bibliometrics

Bookstein (1997) describes the rationale behind the application of FST to informetrics:

> Informetrics shares with the other social sciences the ubiquity of uncertainty. Key concepts are vague. All mathematical relationships are approximate. Yet we are able to make measurements and learn from them. Clearly, we must have developed adaptations to uncertainty, sometimes explicitly, sometimes intuitively, often inadvertently. (p. 10)

Some examples of the types of uses that FST has found in informetrics are given below in roughly chronological order:

- Zunde & Dexter (1969a, 1969b) apply Fuzzy Set concepts to the measurement of indexing consistency and quality.
- Brusilovsky (1978) characterizes science itself as a "Fuzzy system." As such, forecasting and scientometric studies in general can benefit from the application of FST. Jones (1976) describes a Fuzzy Set characterization of interaction in scientific research that provides a better formalization of the notion of citation than nonfuzzy methods.
- Windsor (1979) uses Fuzzy Sets to create a method to predict the clinical fate of a drug based on an informetric analysis of the patent and nonpatent literature about it.
- Price (1981) uses a Fuzzy Set approach to analyze interactions between various entities such as papers, journals, countries, etc.
- Dobrov & Skofenko (1989) discuss the improvements to the review procedure in making Research and Development (R & D) decisions based on a Fuzzy Set model. They regard FST as a good method of modeling the uncertainty inherent to the process of expert reviews of R & D proposals.
- A Fuzzy Set approach has been used to model a network of research institutions (Korennoi, 1989). Korennoi uses a Fuzzy Cluster Analysis to model the relationship between different research institutions, the similarity measure between institutions being inherently fuzzy.
- The uses Japanese researchers have made of FST in informetric research (and other topics) has been described in two similar articles by Miyamoto, Midorikawa, & Nakayama (1989) and Midorikawa, Miyamoto, & Nakayama (1990).
- Egghe & Rousseau (1990) give the basic concepts behind Fuzzy Sets, and then offer some examples of how Fuzzy Sets may be used in informetric (and bibliometric) analysis.
- The influence of the information scientist Manfred Kochen who, amongst other things, studied Fuzzy Sets, has been examined using citation analysis by Lancaster, Bushur, & Low (1993)—though they did not use FST in their analysis.

- Egghe & Rousseau (2001) provide an exact definition of a bibliography using FST, Lorenz curves, and concentration measures. If a strict delineation is preferred, the fuzzy core can be "defuzzified." They claim that the proposed method does not depend on the subjective notion of "importance" and that the method is completely reproducible.

Other Applications of Fuzzy Set Theory

To provide the reader with a broad idea of where FST is used, this section will offer just a few of the non-LIS examples of Fuzzy Sets applications. Zadeh et al. (1975) cover a number of the applications of FST. Zimmerman (1991), in the second half of his book, outlines a wide range of different applications. He provides a classification of four different types of application (p. 129):

1. Applications to mathematics (i.e., generalizations of traditional mathematics such as topology, graph theory, algebra, logic, etc.). The largest and most important of this type of application is undoubtedly fuzzy logic.
2. Applications to algorithms (e.g., clustering methods, control algorithms, mathematical programming, etc.)
3. Applications to standard models (e.g., "the transportation model," "inventory control models," etc.)
4. Applications to real-world problems of different kinds. The most important of these would include fuzzy expert systems and fuzzy control. Others would include applications to psychology (Kochen, 1975).

To get some idea of the broad applicability of Fuzzy Set methods, a list of just a few of the applications in various disciplines or research areas is now provided:

- Linguistics (Jumarie, 1977).
- Fuzzy computer programs (Giles, 1980).
- Fuzzy Set models in inventory control (Kacprzyk & Staniewski, 1982).
- Fuzzy Set models in production control and scheduling (Aliev, 1987).
- Fuzzy Set models in logistics (Klingman, Mote, & Phillips, 1988).
- Fuzzy Sets in psychology (Zetenyi, 1988).
- Support logic programming (Graham, 1989).
- Approximate reasoning (Kienitz, 1990).
- Fuzzy expert systems (Otto, 1990).
- Fuzzy clustering (Trauwaert, Kaufman, & Rousseeuw, 1991).
- Fuzzy dynamic programming (Chung-Ching & Yuan-Yih, 1991).
- Fuzzy logic (Godo, Jacas, & Valverde, 1991).
- Pattern recognition (Pal, 1991).
- Fuzzy control (Moore & Harris, 1992).
- Fuzzy decisions (Lapiga & Polyakov, 1992).
- Fuzzy languages (Gerla, 1992).

- Fuzzy linear programming (Tomsovic, 1992).
- Fuzzy Sets in medical diagnosis (Hassebrock & Prietula, 1992).
- Fuzzy Sets in engineering (Dubois & Prade, 1993).

Literature of Fuzzy Set Theory

Seminal Paper

In 1965, Lotfi A. Zadeh published the seminal paper on FST (Zadeh, 1965). As demonstrated in the previous section, this theoretical work has found applications in a vast array of different disciplines, including medicine, engineering, and information retrieval. As well as applications of this theory, much development to the theory itself has taken place, and this is recorded in the pure mathematical literature.

Main Information Sources: Journals and Books

The key journal for Fuzzy Sets is the journal *Fuzzy Sets and Systems,* which was first published in 1978. Other journals with a significant number of articles concerning Fuzzy Sets include: *Information Sciences; IEEE Transactions on Systems, Man and Cybernetics; Journal of Mathematical Analysis and Applications; International Journal of Human-Computer Studies; European Journal of Operational Research; International Journal of General Systems; Kybernetes; Journal of Fuzzy Mathematics;* and *A I Expert.* A current standard text for FST, now in its second edition, is Zimmerman (1991). Another older, but still quite useful text is Kaufmann (1985).

Historical Information Sources

Quite a few papers give some insight into the history of the development of FST. A number of these were written around 1991, to commemorate the twenty-fifth anniversary of Zadeh's (1965) paper. They include Gupta (1991), Turksen et al. (1991), Hohle (1991), and Hohle & Stout (1991). Other items with some historical content include Krarup (1984); Gaines & Shaw (1985); El-Kafrawy, El-Ramly, & Mahmoud (1986); Gaines (1976); and Shostak (1989). Ostasiewicz (1992) provides a discussion of some of the early work that predates and provides the setting for Zadeh's (1965) seminal paper.

Conclusions

What has been presented here is a mathematical theory, FST, which can be used to model a whole variety of situations in which there is some degree of vagueness or uncertainty. The LIS domain is one area in which this theory has found application. It has been used to assist in decision-making (such as when to bind a periodical), and also in the area of IR. Use has also been made in bibliometrics and informetrics, where some of the quantities being measured have a degree of fuzziness about them.

However, despite the considerable research effort to try to apply FST to solve particular types of LIS problems, and despite the considerable theoretical appeal that this theory has over some of the alternatives, there has been little application in large-scale or commercial systems. More work is needed to take this (and many other) theories from the research papers into practical applications. The benefits that can be gained by using theories such as FST need to be tested and explored, and if found positive, need to be incorporated into the systems in use. Other domains have taken up FST (such as manufacturing and process control) and found it an enormously useful tool. It is time for LIS to do the same.

Notes

1. For recent reviews of the literature of relevance, see Schamber (1994), Saracevic (1996), and Mizzaro (1997).
2. Sets in the original sense of the term are sometimes called "classical" or "crisp" sets to distinguish them from Fuzzy Sets.
3. Some other articles mentioning IR and FST include: Buell (1982); Buell (1985); McCune et al. (1985); Rada (1985); Rousseau (1985); Gauch & Smith (1991); Turtle & Croft (1991); Hassebrock & Prietula (1992); Savoy (1992).

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