Cooperative Game Theoretic Models for Decision-Making in Contexts of Library Cooperation[1]

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Abstract

This article starts, in Section 1, with a brief summary of Cooperative Economic Game Theory. It covers the following issues: (1) the nature of utility functions, (2) the representation of a decision problem in terms of utility functions, (3) the max-min solution of a decision problem, (4) the extension to multiple participants in the decision, (5) the context of nonzero-sum games, (6) cooperative decision-making, and (7) the role of transferable utilities.

There then is a more detailed summary of the specific measures identified by John F. Nash, Lloyd S. Shapley, and John C. Harsanyi. It includes a brief discussion of their significance in general economic and social decision-making in which negotiation and cooperation have important roles.

There is then a brief review, in Section 2, of contexts in which negotiation and cooperation among libraries is of special economic importance. They include: (1) sharing of resources, (2) cooperative acquisitions, (3) cooperative automation, (4) shared cataloging, (5) shared storage, and (6) preservation and access.

For two of those contexts—cooperative acquisitions and cooperative automation—detailed applications of cooperative game theory are illustrated, including use of specific utility functions to represent the decision problems and show the results of applying the Nash, Shapley, and Harsanyi measures for optimum decision and equitable allocation of resources. Numerical examples are used to make the illustrations as concrete as possible.

The article concludes, in Section 3, with a brief description of the im-

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LIBRARY TRENDS, Vol. 51, No. 3, Winter 2003, pp. 441–461

plementation of the calculations for the two contexts within the LPM—Library Planning Model.

Section 1. Game Theoretic Models for Decision-Making

The crucial reference for game theory is the classic book by John von Neumann and Oscar Morgenstern, *Theory of Games and Economic Behavior* (1944).

Understanding a Decision Problem

The starting point for modeling any decision problem must be an understanding of the problem as it is seen by the decision-maker, a definition of the objectives of the decision-maker, the identification of alternative solutions to the problem, and the formulation of means for representing the objectives in a way that can be used to select among the alternative answers. All of that may sound self-evident and trite, but each of those steps is fraught with difficulty.

Most fundamentally, there are likely to be decision-making problems for the library manager that are not well understood, for which the objectives are by no means evident, and for which the alternative potential answers may not be known. The task in modeling in such cases clearly is complicated and requires an exploration by the library manager with whatever professional assistance, such as systems analysis, can be brought to bear.

Fortunately, though, many of the problems faced by the library manager are, in principle, well understood, as are the potential solutions of them. Even in such cases, though, there still are difficulties in properly representing the objectives. To resolve those difficulties requires definition of an appropriate "utility function."

Utility Functions

A utility function is a means for representing the objectives in a way that can be used to select among the alternative answers. To represent the objectives, two aspects must be recognized. One is the relative importance of the objectives and the second is the scale for assessment individually for each of them. In this respect, it is important to note that an unweighted mix of criteria, such as "the greatest good for the greatest number," is irrational; one cannot in general optimize two objectives simultaneously. To do so, there must be a single criterion, and if there are two or more objectives, that criterion must suitably represent their relative importance. It is that requirement that makes the utility function necessary.[2]

To illustrate, the library manager may have two objectives in mind: (1) to decrease the net cost for providing access to materials and (2) to improve the effectiveness of service in providing that access. On the surface, the two objectives are likely to be in opposition, since decreases in costs are likely to result in decreases in services, but the potential solutions may in fact include some that can to some extent meet both objectives. The utility function is the means for bringing those two objectives into a single criterion for assessing the alternatives.

This example, simple though it is, highlights the difficulties in creating a utility function. First, note that while the first objective is, in principle, quantitative, with net cost measurable in dollars, the second may be essentially qualitative and not adequately assessable in numerical form. Second, note that identifying the relative importance of the two objectives, however they may be assessed, is a near impossibility. Indeed, in any real situation it may shift as the alternative answers represent different combinations of costs and effectiveness.

Despite those difficulties, the process of modeling a decision-making problem requires that there be a utility function, and there are means for resolving the difficulties. First is to translate the problem of comparison among objectives into "quantitative/qualitative" ratios. In the example, that would become a "cost/effectiveness" ratio, a measure of "dollars per service provided."[3] Second is to translate, to the extent possible, the qualitative objectives into quantitative ones. In the example, this might be accomplished by translating "effectiveness" into a combination of measurable characteristics, such as "response time" and "frequency of satisfaction." Third, and most fundamental, is to translate the process of assessment into relative comparisons of alternative options, which might be represented by $U(A) > U(B)$, with $U(X)$ being the utility function, and $U(A)$ and $U(B)$ being the respective "values" for options A and B respectively.

The third means for resolution reflects the fact that the only requirement for the utility function is that it be "order preserving." Specifically, $U(A) > U(B)$ means that option A is preferable to option B (in the order of preferences of the decision-maker). Of course, it may be that two options are of equal preference, and that is represented by $U(A) = U(B)$. The crucial requirement for a utility function is that, for any two options A and B, either $U(A) > U(B)$, $U(A) = U(B)$, or $U(B) > U(A)$. In other words, there must be a means for making the choice and it is not possible for both $U(A) > U(B)$ and $U(B) > U(A)$, so the utility function must preserve the order of preference.

Later, when we discuss the application of game theory to cooperative decision-making among libraries, the specific mixes of quantitative and qualitative objectives appropriate to decisions concerning interlibrary cooperation will be discussed.

Representation of the Decision Problem

Given the existence of a utility function, it is then possible to represent the decision problem simply by the assessment of the value of the utility function for each of the alternatives available for solution of the problem. Expressed in that way, the decision problem appears to be almost trivial (even recognizing the possible difficulties in assessing the alternatives).

But, of course, real decision problems are not trivial for the very real reason that there are usually uncertainties that must be recognized. To represent those uncertainties, game theoretic models place the decision problem in the framework of potential contexts over which the decision-maker has no direct control. Thus, while the decision-maker may face and be able to evaluate a set of alternative solutions to a problem, each solution must be assessed for its utility in each context and, more to the point, the likelihood of each context must also be assessed.

The game theoretic model is simply a matrix, the rows of which are the options for alternative solutions, the columns are the contexts, and the elements are the utility function assessments:

Table 1.

	Contexts		
Options	1	2	3
1	U11	U12	U13
2	U21	U22	U23
3	U31	U32	U33
4	U41	U42	U43

For example, the assessments of utility might be as follows:

Table 2.

	Contexts		
Options	1	2	3
1	–3	–4	5
2	–5	2	4
3	1	2	2
4	0	–2	3

The usual frame of reference for a game theoretic model is a competitive game, in which the contexts represent the opponent's strategies for play, and the utilities (if positive) are payments to the decision-maker from the opponent (or, if negative, from the decision-maker to the opponent). Note, that in this case, the player and the opponent each have a utility and that they are negatives of each other: (Uij, Vij), with Vij = –Uij.

With utilities as shown above, the decision-maker might prefer option 1 because its utility is 5 in context 3, but there is the risk of a loss of –4 if the opponent plays context 2. How is the best choice to be made?

Max-Min Solution of the Decision Problem

The classical answer to the choice is "maximize the minimum utility"—the "max-min" solution. That is, for each option across the set of contexts, there is a least utility for the decision-maker and the choice should be that option for which the least utility is the largest. In the numerical example above, the answer is option 3, if the three contexts are equally likely. Note that the set of minimum utilities for the four options is (–4, –5, 1, –2) and the maximum of that set occurs at option 3 in context 1.

If the set of contexts are treated as the potential moves of a competitor, that person is similarly trying to maximize the minimum utility for him (which would be the negatives of the values shown), and the minimum utilities would be (–1, –2, –5), the maximum of which again occurs in context 1, option 3. In either case the result, G, from the game is payment of 1 from the competitor to the decision-maker.

In the example, as a game, the best strategies for the two competitors produce the same solution, option 3 and context 1. Such a game is one with a "saddle-point."

Mixed Strategies. There are games without saddle-points and determining how best to decide for them requires introduction of what are called "mixed strategies" which entail basing the decisions on relative frequencies rather than fixed choices. For example, in the children's game "paper, scissors, rock," the best strategy is to make the choice among the three options as randomly as possible (unless the opponent reveals an evident bias). Using such mixed strategies, the decision process always will have a solution in the form of relative frequencies for each option that will produce at least the minimum expected return (as a counterpart of the max-min solution).

Determination of the best mixed strategy (i.e., best set of relative frequencies for selection of each option by the decision-maker and of the contexts by the opponent) entails solution of a set of linear equalities and inequalities. First, each set of relative frequencies must sum to 1:

$$A_1 + A_2 + \ldots + A_n = 1, \text{ and } B_1 + B_2 \ldots + B_m = 1.$$

Second, each player wants the results, G, from the game to be the best possible for himself:

$$\Sigma\, A_iU_{ij} \geq G, j = 1, 2, \ldots, n \text{ and } \Sigma\, B_iU_{ji} \leq G, j = 1, 2, \ldots, m.$$

The need is to determine the values for the set of frequencies, A_i and B_i, and the value, G, of the game. In general, the solution of a set of linear inequalities (called "linear programming") is an iterative process of searching for values that are potential solutions and then finding the best among them. It is beyond the scope of this article to go into details about that process, and the reader will need to go to a standard text for operations research or linear programming to find them.[4] However, to illustrate the results, consider the following game which does not have a saddle point (i.e., max-min for the options is at option 1, context 2 but min-max for the contexts is at context 1, option 1):

Table 3.

	Contexts		
Options	1	2	3
1	2	–2	3
2	–3	5	–1

The inequalities for the decision-maker are:

$$2A1 - 3A2 \geq G, - 2A1 + 5A2 \geq G, 3A1 - A2 \geq G,$$

Those for the opponent are:

$$2B1 - 2B2 + 3B3 \leq G, -3B1 + 5B2 + B3 \leq G,$$

The solution is:

$$A1 = 2/3, A2 = 1/3, B1 = 7/12, B2 = 5/12, B3 = 0, \text{ and } G = 1/3.$$

The result from each of the inequalities except the third one for the decision-maker is equal to G, but for that one it is greater than G. That means that the opponent does not want to select option 3 under any conditions, which is why B3 should be zero.

Multiple Players

So far, the number of players has been just two—the decision-maker and the opponent. What happens if there are more than two players, say N of them? The crucial point in such games is that players may form coalitions with the objective of gaining advantages by doing so. Of course there is the implication that there will be mutual agreement among the players forming a coalition with respect to the division of utilities among them and that the utilities can be transferred among the participants in a coalition in accordance with that agreement (what are called "transferable utilities").

The representation of an N-player game is essentially parallel to that for the two-player game, except that there will be N components to the payoff vectors instead of two. That is, instead of simply (U_{ij}, V_{ij}) as a pair of utilities, there will be $(U^1_{ij}, U^2_{ij}, \ldots, U^N_{ij})$ as an N-fold set of utilities with, for the moment, the sum of the utilities being equal to zero. Again, each player has a set of options among which to choose, with a coalition entailing agreed-upon choices among the options for the players forming that coalition.

The question at hand is then, what the value of such a game is as represented by the expected returns for each player, given the possibilities for forming the entire range of coalitions among the players. The answer is a beautiful formula, developed by Lloyd S. Shapley (1953). Consider S as one among the possible coalitions, with s players joining in it, and let v(S) be the sum of the payoffs to the members of the coalition if they cooperate (and do not cooperate with any other player). Then, the payoff that each player can expect from the game is given by:

$$U_i = \Sigma_i\ [v(S)*(s-1)!*(N-s)!/N!] - \Sigma'_i\ [v(S)*s!*(N-s-1)!/N!]$$

where the first sum, Σ_i, is taken over all possible coalitions that include player i and the second sum, Σ'_i, is taken over all coalitions that do not include player i. The sums together include all possible coalitions.

Nonzero-Sum Games

Note that, in the matrix representation of the game theoretic model for the N-person game as shown above, the sum of the utilities equals zero. In particular, for the two-person game, only one utility function has been included and, in the numerical illustration, there are only single numbers in each element of the matrix. Further, in the discussion above, the utility function for the competitor was taken simply as the negative of that for the decision-maker, with the view that the results of the game were simply the transfer from one person to the other.

Clearly, it is possible, even likely, that competitors can have fundamentally different utility functions which cannot be expressed simply as the negatives of each other. If so, the matrix representation must consist of two values in each cell. To illustrate with a two-person game, let U_{ij} be the utility function for the decision-maker and V_{ij} the utility function for the competitor:

Table 4.

	Contexts		
Options	1	2	3
1	U_{11}, V_{11}	U_{12}, V_{12}	U_{13}, V_{13}
2	U_{21}, V_{21}	U_{22}, V_{22}	U_{23}, V_{23}
3	U_{31}, V_{31}	U_{32}, V_{32}	U_{33}, V_{33}
4	U_{41}, V_{41}	U_{42}, V_{42}	U_{43}, V_{43}

The sum of the two utility functions, $U_{ij} + V_{ij}$, would then represent the total value of that combination of options and contexts for both players together. If $V_{ij} = -U_{ij}$, as the prior illustration represented, the game is called a "zero-sum" game. If the two utility functions are not simply the negatives of each other, the determination of strategy by a given player would still be based on maximizing the minimum utility for that player.

As a principle, game theory assumes that the players in a game are "rational," in the sense that they will each make decisions that are best for their individual interests, as expressed by their respective utility functions. That implies, in particular, that the relative frequencies of the options and contexts (as defined above) will be determined by the optimal strategy of the player whose plays they represent. It is further assumed that both players have complete knowledge of the utility functions for each.

There are good reasons to question either of those assumptions in any context more complex than a game. Furthermore, the facts are that while the choice of a play in a game may well be made randomly so that the opponent in making the opposing play is not sure of what it will be, the choice in virtually any real situation is likely not to be based on any element of randomness but instead will be made as directly as possible.

Cooperative Decision-Making

In particular, there are applications of game theory for which the assumption of maximizing individual interests, with max-min as the resulting criterion for choice and with the use of randomization as the means for creating mixed strategies, may be changed. The means for doing so is called "bargaining" and the resulting games are called "cooperative games."

Basically, bargaining is a process of making offers and demands with the objective of achieving total, joint results that are better than can be obtained from simply the competitive game. In such bargaining, of course, the competitive game sits in the background as the fall-back position in the event that bargaining fails and there is no cooperation in arriving at the solution.

Cooperative games are of special importance for libraries for which cooperation in joint solution of operational problems is part of the underlying ethic as well as an economic and operational necessity. These kinds of applications therefore will be considered in the context of national information policy decisions and of library cooperation within them. As the background for that discussion, the following is a brief review of the theory underlying cooperative games.

The basis for the theory of cooperative games was developed by two quite remarkable individuals, each a combination of mathematician and economist—John F. Nash and John C. Harsanyi—who (together with Reinhard Selten) jointly received the Nobel Prize in economics in 1994 for their work. The seminal articles, though, were by Nash, and the following description draws primarily from them, supplemented by material from Harsanyi (Nash, 1950, 1953; Harsanyi, 1977).

Utility Functions in Cooperative Games. As was discussed above, to develop any game-theoretic model, one first needs a measure of utility, a means by which one can express the decision-maker's preferences. While such a utility function normally need only represent and preserve the order of preferences, there are two further requirements for application to cooperative games.

The first added requirement is "transitivity": If A is preferred to B and B is preferred to C, then A is preferred to C. Expressed in terms of the utility function, if $U(A) > U(B)$ and $U(B) > U(C)$, then $U(A) > U(C)$. The second added requirement is "linearity": Given a value p, $0 \leq p \leq 1$, with a possible option represented by $C = p*A + (1 - p)*B$, then the utility of C is the same linear combination of the respective utilities of A and B. Expressed in terms of the utility function, $U(C) = p*U(A) + (1 - p)*U(B)$. Note that the linearity requirement necessitates that the utility function be quantitative.

The Mechanism of a Cooperative Game. The theory developed by Nash treats situations involving individuals whose interests are neither completely opposed nor completely coincident. Decision-making in such situations is expected to require mutual discussion and agreement on a rational plan of joint action.

It is assumed that each participant has a set of possible mixed strategies (i.e., weighted combinations of simple strategies) that represent the actions that can be taken independent of the other participant. Typically the weights for the mixed strategies may be determined by a random process with specified averages.

For each combination of strategies, say (S_1, S_2), there will be resulting utilities $U(S_1, S_2)$ and $V(S_1, S_2)$ for the two players. Each utility is a linear function of S_1 and S_2 (because of the assumed property of linearity for the utility function).

Now, the issue in cooperation is to make a joint decision concerning the choice of S_1 and S_2 that would maximize the joint utility. Nash identifies a process of negotiation by which that joint decision is made and then identifies the properties that any "reasonable" solution must have.

Specifically, (1) there should be a unique solution, (2) any other potential solution cannot be better, (3) order preserving transformations of the utility functions will not change the solution, (4) the solution is symmetrical with respect to the two players, (5) if, for some reason, the set of pairs of strategies should be reduced but still contain the solution, it will continue to be the solution, (6) restricting the strategies for one player cannot increase the value of the solution for that player, (7) there is some way to restrict the strategies for both players without increasing the value of the solution for a given player.

Based on those axioms, Nash proves that there is a solution to the game that will maximize the total utility. The bottom line is that the solution to the game is that pair of strategies that maximizes the product of the possible gains over the fallback positions:

$$\text{Maximize } [U(S_1, S_2) - X_1]*[V(S_1, S_2) - X_2]$$

where X_1 and X_2 are the expected pay-offs for the respective "fallback" positions of the two players (i.e., the results from the strategies which would be used without cooperation). The following table (the example used by Nash in his article) illustrates a set of choices and the two utility values for each.

Table 5.

Choice	Cost to A	Value to B
1	–2	4
2	–2	2
3	–2	1
4	–2	2
5	–4	1
Choice	Value to A	Cost to B
6	10	–1
7	4	–1
8	6	–2
9	2	–2

The crucial point is that by cooperation, the players can do much better, both individually and together, than their respective fallback positions would yield. As Nash identifies, the optimum combination of choices is (1, 2, 3, 4, 6, 7, 8). For that combination, the payoffs are 12 for A and 5 for B, with the criterion product (12 – 0)*(5 – 0) = 60. (The values of zero representing the fallback position of noncooperation.)

One might ask why not include all of the choices except number 5 (in which it is evident that there would be a net loss)? Well, note that the values of the combination (1, 2, 3, 4, 6, 7, 8, 9) are 14 for A and 3 for B. Although the total, at 17, indeed is equal to the total for the optimum choice, it is clear that B is subsidizing A and is not getting all that should come from the collaboration. The criterion product is (14 – 0)*(3 – 0) = 42 and reveals the inequity by being much less than the 60 for the optimum answer.

Risk Factors. In the bargaining process, a crucial element is the relative degree of risk faced by each player at any given point. It is measured by the "risk factors" for each player:

$$R_1 = (U(S_1',S_2') - U(S_1,S_2))/(U(S_1',S_2') - X_1),$$
$$R_2 = (V(S_1',S_2') - V(S_1,S_2))/(V(S_1',S_2') - X_2)$$

If $R_i > R_j$, then player i should prevail over player j in the choice between (S_1', S_2') and (S_1, S_2), since player i has relatively more to gain and player j has relatively more at risk.

Transferable Utilities

However, this does raise the possibility that one might do better. To illustrate the possibilities, in the example given above, let's change the values for choice 9 from (2, –2) to (3, –1). It turns out that there are then two combinations of options that have equal values for the Nash criterion: (1, 2, 3, 4, 6, 7, 8) and (1, 2, 3, 4, 6, 7, 8, 9). The criterion product for the first is still 12*5 = 60, but that for the second is 15*4 = 60. In other words, the Nash criterion for each is 60, but the total utility of the second is 19 versus 17 for the first.

There are two reasons for looking at this new set of values. First, it serves to highlight one of the crucial features of the axioms that underlie the Nash solution. Specifically, the remarkable contribution that Nash made was not only to provide a simple criterion but to prove that it would provide the optimum answer and that it would be unique. How then can we have two options with the same Nash criterion value? The answer is that given two values there are linear combinations of them, lying between them, that are also potential answers.

Thus, let (X_1,Y_1) and (X_2,Y_2) be two options. Then $[a*X_1 + (1-a)*X_2, a*Y_1 + (1-a)*Y_2]$, $a \leq 1$ is also an option. The linearity of the utility function then allows us to calculate the Nash criterion function:

$$N = [a*U(X_1) + (1-a)*U(X_2)]*[a*V(Y_1) + (1-a)*V(Y_2)].$$

To maximize N, set to zero the derivative of it with respect to a:

$$2*a[U(X_1) - U(X_2)]*[V(Y_1) - V(Y_2)] + V(Y_2)*[U(X_1) - U(X_2)] + U(X_2)*[V(Y_1) - V(Y_2)] = 0$$

Then, $a = (1/2)*(V(Y_2)/[V(Y_1) - V(Y_2)] + U(X_2)/[U(X_1) - U(X_2)])$.

In the example given above, $U(X_1) = 12$, $U(X_2) = 15$, $V(Y_1) = 5$, and $V(Y_2) = 4$. In that case,

$$A = (1/2)*[4/(5-4) + 15/(15-12)] = 1/2.$$

The Nash criterion value is then:

$$F = (.5*12 + .5*15)*(.5*5 + .5*4) = 13.5*4.5 = 60.75,$$

and that is the unique maximum value.

The second reason for looking at this example, though, is that it highlights the potential for bargaining between the players with respect to the distribution of the total maximum utility. For them to bargain, the utilities must be *transferable*, so that player A would be able to give units of utility to player B as an incentive to cooperate in such a way as to increase the total utility.

In the example, player A might agree to give player B one and a half units if they can cooperate on the option that gives 15 units to A and 4 units to B. The result would be that A winds up with 13.5 units and B with 5.5 units. Each is ahead of the option that gave only 12 units to A and 5 units to B.

Later, we will use this example to illustrate the application of cooperative games in the context of decisions concerning library cooperation.

Optimization over Total Utility

So far, the optimization has focused totally on criteria that relate to the individuals separately. But as the discussion just above should demonstrate, there is great potential value if the optimization can consider the total utility, combining those for each of the two players.

Here is where Harsanyi provides another beautiful answer (1977, p. 192). Without going into the details (as given in the reference), the bottom line is to maximize the Harsanyi criterion function:

$$H = [U(S_1, S_2) - X_1]*[V(S_1, S_2) - X_2]*[U(S_1, S_2) + V(S_1, S_2) - (X_1 + X_2)].$$

But beyond the Harsanyi criterion is that of Shapley, as described earlier, which provides the basis for maximum collaboration among all of the participants.

SECTION 2. LIBRARIES WITHIN COOPERATIVE STRUCTURES

We turn now to the potential for use of cooperative game theory in support of cooperation among libraries. Of course, libraries have a long history of cooperation, perhaps best exemplified by the system for interlibrary borrowing and lending. It has been a continuing theme for library management for decades. Today, though, there is an expansion of that tradition into a variety of contexts and purposes and into formalized structures.

Reasons for Library Cooperation

There are several specific reasons for cooperation among libraries:

Sharing of Resources. This is certainly the starting point for library cooperation. It is explicitly represented by the process for interlibrary borrowing and lending that has been formalized for decades. But it has generated a number of supporting tools in the form of union catalogs, union lists of serials, and other cooperative means for determining where desired materials may be available.

Cooperative Acquisitions. This is a means for cooperation that obviously depends upon the sharing of resources, but it goes further by formalizing agreements in which specific institutions take responsibility for identified areas of acquisition. This implies some degree of sharing of funding as well as responsibility, and some formal arrangements include provision for pooling some portions of the acquisitions budgets of the participants.

Automation. The development of automated systems has frequently been a focus of cooperation among libraries. The joint contracting for acquisition of a system, the sharing of costs in implementation and in operation, the sharing of experience and staff expertise—these have been typical ways in which cooperation with respect to automated systems has occurred.

Shared Cataloging. The largest concentrated effort at cooperation among libraries certainly was the development of systems for shared cataloging. That effort is now represented by the international bibliographic databases of OCLC and RLIN. It grew out of the need for cooperation among libraries in the conversion of bibliographic records—catalogs especially—to machine processible forms. The result, of course, is that now virtually every major library has the catalog for its entire collection in an online public access catalog (OPAC).

Shared Storage. The growth of library collections, whether exponential or linear, leads to the problem of allocating materials to alternative places for storage. The costs of storage facilities, though, is great enough that efficiency requires that they be shared by groups of cooperating libraries. Shared storage has therefore been another of the success stories in library cooperation.

Preservation and Access. Perhaps the most dramatic context for library cooperation has been that of preservation and access. The underlying problem is the literal disintegration of the paper in books, especially those produced in the years since the introduction of acidic paper that self-destructs. It has been estimated that as much as 25 percent to 30 percent of the holdings of major research libraries are at risk (Hayes, 1987). To deal with this problem, the Council on Library Resources established the Commission on Preservation and Access as the focus for management of a major cooperative effort. The objectives were identified in testimony at a March 17, 1988 hearing of a Congressional committee: "Commission President Pat Battin proposed a model for a national cooperative microfilming program. A goal of filming 150,000 volumes a year would require 20 institutions to commit to filming 7,500 volumes each. At the 150,000 annual rate, it would take about 20 years to film 3 million volumes—the estimated number of volumes it would be important to save in order to preserve a representative portion of the 10 million or more volumes that will turn to dust by that time" (Commission on Preservation and Access, 1988).

Utility Functions for Library Cooperation

We turn now to the potential for application of cooperative game theories to library cooperation. As was discussed above, to represent a decision-making problem as a game requires that there be a measure of utility for each participant in the game. What are the elements of such a utility model for library cooperation?

Capital Investments and Operating Costs. We start with the most measurable elements, the capital investments and the operating costs associated with alternative options for solution of the decision-making problem. Normally, they will be measured in dollars, or equivalent, and can be readily accumulated.

Sometimes the context for possible cooperation may affect existing capital investments. For example, an effort to cooperate in the development of a joint automated system may need to recognize that a participant already has a system in place and that cooperation might entail changing that system, losing the existing capital investment, and incurring additional capital costs.

Sometimes the context may affect current or future capital investments. An effort to share acquisitions will usually entail a decision by one of the participants to eliminate the capital investment in acquisition and technical processing of materials in a specific subject area, under the assumption that needs for materials in that subject will be met by another participant. This concept underlaid the Farmington Plan, as a national effort in which responsibility for collection development in specific subject areas was to be assigned to specific institutions. The other institutions could then, in principle, count on coverage of the subject fields and concentrate their own budgets on their more specific needs.

Sometimes the context may affect operating costs. As an example, any system for interlibrary borrowing and lending or for document delivery entails substantial costs in both the borrowing and lending institutions. Those operating costs need to be included in any decision concerning shared acquisitions.

A major operating cost in library cooperation is the commitment of the time and energy of the library management and professional staff in negotiation and in governance. Probably the most successful example of library cooperation in the past several decades has been the development of the international bibliographic utilities (as represented by OCLC and RLIN). The impact on both library costs and library effectiveness has been immense. But these efforts have necessitated intense involvement of directors of libraries, catalogers, and reference staff. The expenditures of time by exceptionally valuable persons have been immense. At some time in the process of evaluating options for library cooperation, those costs need to be considered.

Library Effectiveness. Any utility function for assessing options in library cooperation must consider the effect on users and on the overall productivity of the library. Unfortunately, these effects are not easily quantifiable. Of course, some may be, such as "response time" or "frequency of satisfaction." But others, such as "browseability" are not.

Governance. The utility function will need to recognize issues involved in governance. They relate to centralization versus decentralization of decision-making in operation of cooperative enterprises, to the structure for control of policies, and to the relationships of the library to its parent institution. These issues are even less amenable to quantification than those for effectiveness.

Professional Ethics. Underlying all of the contexts of library cooperation is an ethical commitment of librarianship to the very concept itself. It is embedded in the profession and is evidenced in the long-standing commitment to interlibrary borrowing and lending despite the costs and inequities it entails. The major net lenders periodically will complain about the costs they incur and the adverse impact on services to their primary constituencies, but when the decision finally must be made, invariably it is in favor of cooperation.

In a sense, there is an underlying rationale for that professional commitment in the recognition that no library can be all-encompassing and that sharing is the only way to ensure preserving the record of the past and providing access to that record. But there appears to be something more than simply that pragmatic rationale in the view of librarianship that information indeed is a public good.

The Consequent Utility Function. Would that one could readily identify the utility function that will properly weight and combine this combination of quantitative and qualitative factors. In lieu of that, the utility function for application to library cooperation must be an individual assessment of the relative utility of options and, perhaps, a jointly agreed-upon combination of those individual assessments into a mutually acceptable criterion for the group of libraries participating in library cooperation.

Illustrative Applications of Cooperative Games

Two examples will serve to illustrate the potential for use of cooperative games in decision-making concerning library cooperation. One considers cooperative acquisitions and the other considers library automation.

Cooperative Acquisitions. As a start, for simplicity, let's suppose that there are just two institutions considering an agreement to share acquisitions. If one of them will assume responsibility for acquisition in a subject field, the second will save the costs of acquisition and technical processing for that subject. However, each will incur operating costs in meeting the needs of users in the institution served by the second library who need materials in that subject field from the first. The utility measure to be used will be quantitative and based simply on the total costs represented by any given choice.

In Section 1, above, a numerical example was presented to illustrate the choice of optimal mixtures of choices, and the following repeats the table of values but now interprets them as reflecting the net costs or benefits if the options are interpreted as acceptance of subject responsibility.

The interpretation of this table is that there are nine subject areas being considered for cooperative acquisitions. Library A is renowned in the first five fields, and library B, in the last four. If library A were to accept responsibility for one of the first five fields, there would be estimated costs in fulfilling that obligation. Those costs might consist of increased levels of acquisition to meet the joint needs; it definitely would include costs in providing materials to borrowers from library B. On the other hand, library B would save substantial costs in acquisition and technical processing, though

Table 6.

Choice	Cost to A	Value to B
Subject 1	–2	4
Subject 2	–2	2
Subject 3	–2	1
Subject 4	–2	2
Subject 5	–4	1
Choice	Value to A	Cost to B
Subject 6	10	–1
Subject 7	4	–1
Subject 8	6	–2
Subject 9	6	–4

there would be counterbalancing costs in borrowing from library A. The values shown are interpreted as the estimates of those respective costs and benefits.

As was described above, this cooperative game has a solution: Library A accepts responsibility for subject fields 1 through 4 and library B for fields 6 through 8. The remaining fields are left out of the agreement. The net gain both to the individual libraries and in total would be substantially greater than if there were no agreement to cooperate

Now let's complicate the example by including three institutions and ten fields.

Table 7.

Choice Number	A	B	C
1	–2	2	2
2	1	1	–2
3	–2	1	0
4	2	1	–2
5	–4	1	3
6	6	–1	4
7	1	3	–1
8	3	–2	3
9	2	–1	1
10	4	–1	0

This numerical example will be interpreted as follows: There are three libraries (A, B, and C) that are considering a program of cooperative acquisitions. They have identified ten subject fields (choices 1 through 10) as potential candidates. For each choice, if the value for a given library is negative (such as for library A in choice 1), it will be responsible for that subject field. The value is negative for that library because they will now incur, perhaps, additional costs in added acquisitions and, surely, additional costs in providing lending services to users from the other libraries. The values for the other libraries (library B and C in the case of choice 1, for example) are positive because they will now save costs in acquisition in that subject field because they can depend upon the host library (library A for choice 1).

Parenthetically, it should be noted that underlying the choices shown above might be more basic choices reflecting the potential for two-party agreements. For example, choice 1 might be the sum of two more basic choices: (–2,2,2) = (–1,2,0) + (–1,0,2). In this way, if additional libraries were to participate, perhaps without even serving as hosts for subject fields, their impact on costs would be directly represented in a parallel fashion.

But, returning to the example as shown, the options available are essentially the several combinations of the choices, of which there are 1,024 (i.e., 2^{10}). The task is to determine the best among those combinations and the resulting distribution of benefits (or costs) among the participants. In general, it would appear that every choice for which the total of values was positive ought to be included, since the group of libraries as a whole would experience a net gain. Whether or not a choice for which there was a total of zero should be included is clearly debatable, but let's see what happens.

For this example, it turns out that the maximum Nash Value occurs if all of the choices are included, including that for which the total of values is negative as well as those for which it is zero. The total individual values are then (11,4,8) with a total for the group of 23 and a Nash Value of 352.

However, the option that excludes choice number 3 has total individual values (13,3,8), with a total for the group of 24 and a Nash Value of 312. It is therefore the option that should be selected if the goal is to maximize the total for the group as a whole.

The Shapley Values are (11.33, 4.33, 8.33), so there would need to be transfers from library A to libraries B and C to provide equity, otherwise, there would be no reason for library B to agree to that option since it would lose in comparison with the Nash Value maximum option (i.e., getting only 3 instead of 4). The Shapley values are calculated as follows:

$$U(A) = (1/3)*11 + (1/6)*15 + (1/6)*19 + (1/3)*24 - (1/3)*4 - (1/3)*8 - (1/6)*12 = 11.33$$

$$U(B) = (1/3)*4 + (1/6)*15 + (1/6)*12 + (1/3)*24 - (1/3)*11 - (1/3)*8 - (1/6)*19 = 4.33$$

$$U(C) = (1/3)*8 + (1/6)*19 + (1/6)*12 + (1/3)*24 - (1/3)*11 - (1/3)*4 - (1/6)*15 = 8.33.$$

Cooperation in Automation. Let's suppose that there are several institutions considering an agreement to cooperate in the installation of a common system for automation in their libraries. If they can agree upon a common system, there should be significant benefits in cooperation. For example, as pointed out earlier, there may be savings from joint contracting in acquisition of the common system, savings in costs in implementation and in operation (such as in shared maintenance and replacement parts), efficiencies in sharing of experience and staff expertise, greater effectiveness in adding later improvements, and easier sharing of common data files.

Of course, balancing such benefits from cooperation may be the fact that each institution has a substantial investment in its current system. Part of that investment may be the residual value in amortization of the initial investment in the system. Another part, one likely to weigh even more heavily, is the fact that the existing system is well-entrenched in the operating procedures of the library and the usage by its patrons.

All in all, the potential is that the benefits from cooperation will be sufficient enough to warrant at least careful evaluation of alternative systems.

There are thus at least four factors to be considered in the utility function for this application of cooperative game theory: (1) existing capital investments at each institution, (2) the costs for installation of each potential candidate for a replacement system, (3) the net benefits (i.e., difference between benefits and operating costs) to be anticipated from each potential candidate for a replacement system, and (4) the benefits to be anticipated from cooperation (which may vary from systems to system) by selection of a common system.

To apply cooperative game theory, it is assumed that those four factors are commensurate, both across factors and across institutions, so that they can be combined by simple arithmetic operations. It is also assumed that each factor is measured by a linear function of the size of the institution so it is expressible in the form V(i,j) = A(i,j) + B(i,j)*Size(k), with the parameters A and B varying by factor (i) and system (j) and the size varying by institution (k). Finally, it is assumed that the parameters for benefits from cooperation are a linear function of the number of institutions selecting a common system so they are expressible in the form A(4,j) = N(j)*A'(j) and B(4,j) = N(j)*B'(j), where N(j) is the number of institutions selecting system Sj and the parameters A'(j) and B'(j) are given for each system Sj.

The following numerical example will illustrate the model for just two institutions (see Table 8).

In this example, the existing investments are, respectively, 6 (for institution J1 in system S1) and 3 (for institution J2 in system S2). The potential third system, S3, does not provide sufficient benefits to overcome the loss of the existing capital investment at J1, but the values in cooperation are sufficient to warrant installation of S1 at J2. However, there needs to be compensation for the loss of investment at J2, and the Shapley values, as shown, provide the basis for such compensation. If the net operating benefits for S3, are increased from 8 to 10, the results are as follows (see Table 9). Note that both institutions lose their existing capital investments,

Table 8.

Institution	Size	Existing Investment	Current System	Best Choices	Net for Best	Shapley Values	Needed Transfers
J1	3	6	1	1	27	24.00	-3.00
J2	2	3	2	1	10	13.00	3.00

	Installation Cost		Net Operating Benefit		Cooperation Benefit	
System	Fixed	Linear	Fixed	Linear	Fixed	Linear
S1	2	2	2	5	1	3
S2	2	1	2	2	1	3
S3	2	4	2	8	1	3

Table 9.

Institution	Size	Existing Investment	Current System	Best Choices	Net for Best	Shapley Values	Needed Transfers
J1	3	6	1	3	22	23.00	1.00
J2	2	3	2	3	16	15.00	-1.00

	Installation Cost		Net Operating Benefit		Cooperation Benefit	
System	Fixed	Linear	Fixed	Linear	Fixed	Linear
S1	2	2	2	5	1	3
S2	2	1	2	2	1	3
S3	2	4	2	10	1	3

but the benefits from both S3 and from cooperation more than compensate. The Shapley values in this case recognize the greater investment loss of institution J1.

Section 3. Implementation in LPM

Processes for solution of cooperative games have been implemented in a program, called LPM—The Library Planning Model. This program is in the form of a Microsoft Excel spreadsheet with extensive Visual Basic macros. It provides a structure within which several models related to library operations and services, management, and planning can be interrelated and easily brought together for application to operations in specific libraries and to several policy contexts.

In particular, LPM provides means for entry of data about the populations served, materials acquired, services provided to the populations served, processes involved in acquiring, cataloging, and preserving materials, and facilities related to both users and materials. From those input data, LPM then derives an estimation of the staff required, both for each category of service or process and in total. Staff estimates are in two categories: direct FTE and indirect FTE and for three levels of personnel (professional, nonprofessional, and hourly). To calibrate the staffing estimates from the LPM, means are included to compare them with actual staffing, distributed both by administrative units and functional areas (using the categories of the model).

LPM also provides means for assessing the needs for facilities to serve users, to store the materials acquired, and for comparing them with the input data for facilities already available. It includes means for applying models for allocation of materials to alternative means for storage and for decisions about the choice between acquisition and access elsewhere.

And LPM includes means for using the cooperative game theoretic models presented in this article.

Implementation of Cooperative Games for Shared Acquisitions

One process is in support of the model for shared acquisitions, representing options (i.e., strategies) that are either independent or are based on combinations of possible choices, such as in the example for shared acquisitions as given above. Note that in the first example there were $2^9 = 512$ possible combinations for two institutions; in the second example, $2^{10} = 1,024$ for three institutions. A given option then is one of those combinations of the nine or ten possible choices. The implementation within LPM will allow up to nineteen choices and up to five institutions. The Nash, Shapley, and Harsanyi criteria have been included in the implementation in LPM.

Implementation of Cooperative Games for Shared Automation

The second process is in support of shared automation or similar contexts. Provision has been made to include up to five institutions and up to six systems. For each institution and each system, the parameters shown in the above illustration need to be entered. That being done, LPM will then determine the optimum selections. In principle, different systems might best be selected by different coalitions, so LPM then determines the Shapley values for that optimum by assessing the optimum choice for all possible coalitions of institutions and combining them as has been discussed in the definition of the Shapley measure.

Conclusion

Game theory has become a powerful tool in decision-making for business and government contexts in which competitive motivations are paramount. Even when "cooperative games" are involved, they are typically seen in the framework of bargaining for best individual advantage.

The value of looking at the potential use of this tool in library contexts is that cooperation is a part of the ethos of the profession. In that respect, it is representative of many kinds of non-profit, non-governmental organizations for which what is good for the group of participants and even for society at large has great weight in decision-making.

The intent of this paper has been to identify some potential applications of cooperative game theory that would illustrate those kinds of values. In doing so, it raises questions about the nature of the utility functions and clearly those questions need to be explored; this is especially the case for those elements of the utility functions that are essentially qualitative. But the framework of game theory provides a context in which such questions can be properly posed.

Beyond that basic intent, the paper has also presented two specific examples of those applications with the objective of showing how some of the questions might be answered in specific situations. The extension to other applications would require developing utility functions appropriate to each. Once that is done, the measures (such as those of Nash, Shapley, and Harsanyi that are discussed in this paper) provide the means for making effective decisions.

Notes

1. This article is based on material from Robert M. Hayes, *Models for Library Management, Decision-Making, & Planning,* San Diego: Academic Press, 2001.
2. It is important to note that game theory fundamentally represents a means to reconcile or combine simultaneous objectives, as represented, for example, by those of the players in a game. The solution of the game is that mix of meeting the simultaneous objectives that is called "Pareto-optimum," meaning "the best that could be achieved without disadvantage for at least one objective." In other words, no objective can be bettered without reducing another objective. There is extensive research on the implications of the criterion of Pareto-optimum and alternatives for it. (See, for example, Schmid, 1987. Reviewed by Boulding, 1979.)
3. It is important to recognize that optimization of a cost/effectiveness measure usually is in the context of boundary conditions (such as "the cost must be less than some maximum" or "the effectiveness must be at least some minimum" or both).
4. Churchman, Ackoff, & Arrow, 1957. (The numerical examples presented in the text above are taken from this textbook.

References

Boulding, K. (1979, September). Book review. *Journal of Economic Issues, 13*(3), 781–785.

Churchman, C. W., Ackoff, R. L., & Arrow, F. L. (1957). *Introduction to operations research.* New York: Wiley.

Commission on Preservation and Access. (1988, June). *Newsletter,* No. 1.

Harsanyi, J. C. (1977). *Rational behavior and bargaining equilibrium in games and social situations.* New York: Cambridge University Press.

Nash, J. (1950, April). The bargaining problem. *Econometrica, 18*(2), 155–162.

Nash, J. (1953, January). Two-person cooperative games. *Econometrica, 21*(1), 128–140.

Hayes, R. (1987). *The Magnitude, costs and benefits of the preservation of brittle books.* Los Angeles: Council on Library Resources.

Hayes, R. M. (2001). *Models for library management, decision-making, and planning.* San Diego, CA: Academic Press.

Schmid, A. A. (1987). *Property, power, and public choice: An inquiry into law and economics.* 2nd ed. New York: Praeger.

Shapley, L. S. (1953). A value for n-person games. In Kuhn, H.W. & Tucker, A. W., *Contributions to the Theory of Games, II. Annals of Mathematics Studies 28* (pp. 307–317). Princeton: Princeton University Press.

Von Neumann, J., & Morgenstern, O. (1944). *Theory of games and economic behavior.* Princeton: Princeton University Press.