THREE ESSAYS IN ECONOMICS AND FINANCE

BY

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DISSERTATION

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Abstract

This dissertation consists of three essays. The first essay is joint work with Dan Bernhardt. We endogenize entry to a security-bid auction, where participation is costly, and bidders must decide given their private valuations whether to participate. We first suppose that the minimum reserve security-bid yields the seller an expected revenue equal to the asset’s stand-alone value to the seller. Demarzo et al. (2005) establish that with a fixed number of bidders, auctions with steeper securities yield the seller more revenues. Counterintuitively, we find that auctions with steeper securities also attract more entry, further enhancing the revenues from such auctions. We then establish that with optimal reserve securities, auctions with steeper securities always yield higher expected revenues.

In the second essay, I consider a situation in which a winning bidder of an equity auction has an investment opportunity after the auction and the seller has private information about the return of the post-auction investment. I show that in such a situation, in contrast to the seminal “linkage principle” by Milgrom and Weber (1982), the seller’s expected revenue may be higher when not disclosing her private information at all than when committing to publicly announce her private information regardless of whether it is positive or negative.

The third essay is joint work with Keiichi Kawai. The securitization of structured finance products entails three types of inefficiency: the issuer’s moral hazard when screening underlying assets (ex-ante inefficiency), the issuer’s incentive to repackage underlying assets into separate securities even when doing so is socially inefficient (interim
inefficiency), and adverse selection in the market (ex-post inefficiency). To analyze the interplay of these inefficiencies and their welfare implications, we consider a situation wherein buying medium-value assets and issuing medium-value securities are first-best. However, we show that the issuer not only buys low-value underlying assets but also repackages underlying assets to issue two types of securities of different values despite paying a socially wasteful cost.
To Mari.
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Chapter 1

Endogenous Entry To Security-Bid Auctions.

1.1 Introduction

DeMarzo, Kremer, and Skrzypacz (2005) (hereafter, DKS) characterize expected seller revenues for general classes of security bid auctions—auctions whose payouts depend on both the security that is bid by the winner of the auction, and the ultimate (stochastic) payoff of the asset won by the bidder. DKS consider a setting where ex-ante symmetric bidders receive i.i.d. signals of the private value of the asset to them if they win the auction. DKS establish that auctions using steeper securities—those whose payments to the seller are more tightly tied to the private valuation of the winning bidder—provide the seller with greater expected revenues. Thus, call options provide greater expected revenues than equity, which, in turn, provides greater expected revenues than debt.

We extend their analysis to a setting where it is costly for a potential bidder to participate in the security-bid auction. Potential bidders know their private valuations when deciding whether to enter. A potential bidder will only participate if the expected payoffs from winning given its signal cover its participation costs; and those expected payoffs
will depend on the class of securities used. A natural conjecture is that since auctions that use steeper securities for payments provide the seller greater expected revenues, it must be that using steeper securities attracts fewer bidders—as more revenues for the seller would seemingly imply less for the winning bidder. Indeed, Gorbenko and Malenko (2011) show that this is what happens when bidders choose which of many ex ante identical auctions to enter before learning about their valuations of the goods being auctioned. Our paper shows that the opposite is true when bidders know their valuations before deciding whether to participate in a single auction: not only do steeper securities extract more revenues from any given set of bidders, but they also attract more bidders, and this increased entry enhances revenue extraction from bidders.

We consider a seller seeking to sell an asset in an open outcry security-bid auction design. Potential bidders receive their signals, and must then weigh whether it is worthwhile to participate in the auction given its security-bid design. Participation demands nontrivial resources—bid preparation costs, time costs, and so on. In addition, we allow for the possibility that the expected net value of the asset with the potential bidder could be less than the stand-alone value to the seller; i.e., “synergies” could be negative.

Because potential bidders with low signals will choose not to participate—and a seller regrets an outcome in which a bidder with negative private valuations wins—the seller must specify a reserve security that a bidder pays with when it is the sole auction participant. This reserve security is the minimum security-bid accepted by the seller. As a benchmark, we first assume that the reserve security is set so that the seller’s expected revenues given a single bidder equal its stand-alone value. A natural interpretation of this reserve security is that the asset is being sold as a result of bankruptcy. The seller, perhaps the firm’s trustee, cannot reject a bid that is expected to have a higher value than the asset’s stand-alone value—and this pins down the reserve security.

---

1 We pose our analysis in an open outcry auction rather than a second-price auction largely to deal with semantics following a single entrant. With a single entrant, the winning bid in an open outcry auction will be the reserve security. Modulo this distinction, our analysis extends to a second-price auction in which bidders know how many others participate in the auction.
Potential bidders enter the auction if and only if their private signals are sufficiently high. The marginal auction participant knows that it will win the auction only if it is the sole entrant. Thus, the expected payoffs that it retains when it pays with the reserve security just compensate for its participation costs.

Our central result is that a security-bid design that features steeper securities attracts more entry. The reasoning behind the seemingly paradoxical result that steeper securities both extract more from winning bidders and attract more entry is as follows. The seller sets the reserve security to break even (relative to its stand-alone value) conditional on the information that only one bidder participates in the auction—that is, conditional on the winner’s type being better than the marginal type who is indifferent to entry. Thus, conditional on a single bidder, the seller expects the same revenues regardless of the security design.

However, precisely because payments of steeper securities are more strongly linked to a private valuation of the winning bidder, extracting more revenues from bidders with higher valuations, when the reserve security breaks even, the seller expects to extract less revenues from bidders with lower valuations. Thus, the steeper is the security design, the less the marginal auction participant expects to pay with the reserve security. Hence, the steeper is the security design, the more willing are bidders with lower valuations to participate.

One might then conjecture that because the marginal entering bidder might have a negative private valuation, this extra entry could harm the seller; i.e., steeper security designs could result in lower expected revenues. This conjecture is also false. Once entry occurs, participation costs are sunk, and a bidder has a weakly dominant strategy to drop out when the expected payoffs that it would retain just equal its stand-alone value (now reduced by the participation costs). Moreover, because each auction participant expects to make enough payoffs to cover participation costs, it must be willing to pay above the reserve security; otherwise, it would be better off not participating. Hence, with multiple
entrants, all losing bidders drop out at security bids above the reserve security. Thus, the greater entry with steeper securities increases the expected revenues that the seller extracts whenever there are multiple bidders. It follows that the greater auction revenues generated by steeper securities are further enhanced when auction participation is endogenized in this way.

This logic is only reinforced when we compare auctions ordered by steepness that employ reserve securities of a fixed given expected value that exceed the seller’s stand-alone value: steeper securities always attract more low-valuation bidders, and this greater participation yields the seller higher expected revenues. Indeed, when the value of reserve securities exceeds the seller’s stand-alone value, rather than being indifferent between auction designs conditional on attracting zero vs. one bidder, in expectation, the seller strictly prefers to attract one bidder rather than none, and steeper securities raise this probability. It follows directly that an auction using steeper securities and associated optimal reserve security always yields the seller higher expected payoffs than an auction using less steep securities and its associated optimal reserve security.

Our paper contributes to the security-bid auction literature (see a review by Skrzypacz (2013)). Hansen (1985) is the first to show that equity auctions yield higher expected revenues than standard cash auctions. DKS extend this result to a general class of security-bid auctions, establishing that the greater is the linkage between a bidder’s private information and the expected payment he would make upon winning, the higher is the expected revenue that a seller receives (Milgrom and Weber (1982)). Other papers that study security-bid auctions include Rhodes-Kropf and Viswanathan (2000), Che and Kim (2010), Kogan and Morgan (2010), Abhishek, Hajek, and Williams (2015) and Liu (2015).

Our paper is most closely related in spirit to Gorbenko and Malenko (2011). They endogenize competition in auction design of simultaneous second-price security-bid auctions between a finite number of sellers. Potential bidders make entry decisions based on
the auction designs, but not knowing their private valuations of the stochastically identical objects being auctioned. The opportunity cost to a bidder of participating in one auction is not participating in another. In this setting, steeper security designs extract more from a given number of bidders, but flatter security designs draw more bidders precisely because steeper securities extract more from bidders. The equilibrium security design typically trades off between these two considerations.

We consider a seemingly similar notion of endogenous entry to a security-bid auction. However, our findings are diametrically opposed. Our setting features a single auction—rather than entry being endogenous due to the opportunity cost of participating in one auction rather than another, entry is endogenous because participation itself is costly. Quite crucially and differently from Gorbenko and Malenko (2011), bidders know their private valuations before deciding whether to participate. We show that with this structure, when the reserve security has a fixed expected value, the steeper is the security design, the greater is entry—a seller does not need to tradeoff between extracting more from a winning bidder and attracting more entry—and the greater are expected revenues. Fishman (1989) builds a sequential entry model in which, in equilibrium, low-valuation bidders use securities and high-valuation bidders use cash. Bidders signal a high value by bidding with cash to preempt competition. Although he too considers entry, his focus is on preemptive bids. In contrast, we focus on the relationship between rent extraction and entry, and revenue comparisons across security-bid auctions.

We next present the model. A brief conclusion follows.

1.2 Model

We modify the framework of DKS by introducing an entry decision by risk-neutral bidders to an open outcry security-bid auction held by a risk-neutral seller. There are $n$ ex-ante identical potential bidders. A bidder incurs cost $\phi > 0$ if it participates in the
auction. Each bidder has a stand-alone value of \( v_B > \phi \), which means that a bidder has enough resources to participate in the auction. The asset being auctioned has a stand-alone value to the seller of \( v \). If bidder \( i \) acquires the asset, it will yield a stochastic payoff of \( Z_i \) at date 2.

At date 0, each potential bidder \( i \) receives a private signal \( \Theta_i \) of the incremental value of the asset to the bidder. Conditional on the asset being acquired by bidder \( i \) of type \( \Theta_i = \theta \), the expected value of \( Z_i \) is normalized to

\[
E (Z_i | \Theta_i = \theta) = v_B + v + \theta - \phi,
\]

where \( Z_i \) is i.i.d. according to a density \( h (\cdot | \theta) \) with full support on \([0, \infty)\). We assume that the family \( \{ h (\cdot | \theta) \} \) has the strict monotone likelihood ratio property (sMLRP): \( h (z | \theta) / h (z | \theta') \) is increasing in \( z \) for \( \theta > \theta' \). That is, higher signals are good news.

The signals \( \Theta_i \) are distributed i.i.d. according to a distribution \( F (\cdot) \) with full support over \([\bar{\theta}, \bar{\theta}]\). We assume that this support satisfies \( \bar{\theta} < \phi < \bar{\theta} \). Thus, the net value of the asset to the bidder may or may not exceed its stand-alone value to the seller. In particular, \( \bar{\theta} > \phi \) means that it is efficient to allocate the asset to a potential bidder with a high private valuation. One can interpret an acquisition by type \( \theta > \phi \) as generating a value-enhancing synergy with an expected value of \( \theta - \phi \). Conversely, it is not efficient for potential bidders with low valuations \( \theta < \phi \) to participate in the auction—it would be better for the seller to retain the asset. Note that our formulation allows for the possibility that \( \bar{\theta} < 0 \). That is, not only may “synergies” fail to cover auction participation costs, but they may be negative in nature.

After receiving their private signals, potential bidders simultaneously decide at date 1 whether to participate in a security-bid auction \((S, \xi (S))\) for the asset. \((S, \xi (S))\) specifies a set of feasible bids \( S \) and a reserve security \( \xi (S) \). Bids are made in the form of securities that are contingent on the stochastic payoff \( Z_i \), which is realized at date 2.
The reserve security $$s(S)$$ is the minimum bid accepted under $$S$$; this security is pinned down via a break-even condition that we describe below. We slightly modify DKS’s notion of ordered securities. Let $$S(s,z)$$ denote the payment to the seller when $$Z_i = z$$ is the payoff realized at date 2 for security $$s$$. Bids are restricted to an ordered set of feasible securities $$S = \{S(s,\cdot) : s \in [s(S),s]\}$$ such that (i) for all $$s$$, $$S(s,z)$$ and $$z - S(s,z)$$ are weakly increasing in $$z$$, satisfying $$0 \leq S(s,z) \leq z$$, and (ii) $$\partial ES(s,\theta)/\partial s > 0$$ for all $$\theta$$, and $$ES(s,\theta) \geq v + \theta$$, where $$ES(s,\theta) \equiv E(S(s,Z_i)|\Theta_i = \theta)$$ is the expected value of security $$S(s,\cdot)$$ conditional on $$\Theta_i = \theta$$.

At date 2, the asset payoff $$Z_i = z$$ is realized and payments are made as follows: when bidder $$i$$ is the sole entrant, $$i$$ wins the auction if and only if it submits a feasible bid; i.e., its bid $$s_i$$ weakly exceeds the reserve security $$s(S)$$, paying $$S(s_i,z)$$. When two or more bidders submit feasible bids, the winning bidder $$i$$ pays with the security bid $$s_2$$ of the last bidder to drop out of the auction, paying $$S(s_2,z)$$.

We use the notion of steepness introduced in DKS: an ordered set of securities $$S_A$$ is steeper than $$S_B$$ if for all $$s_A \in S_A$$, $$s_B \in S_B$$, $$s_A \in [s(S_A),\bar{s}_A]$$, and $$s_B \in [s(S_B),\bar{s}_B]$$, $$ES_A(s_A,\theta^*) = ES_B(s_B,\theta^*)$$ implies $$\partial ES_A(s_A,\theta^*)/\partial \theta > \partial ES_B(s_B,\theta^*)/\partial \theta$$. Steeper securities imply that if a bidder with a private valuation $$\theta^*$$ expects to pay the same amount with securities $$s_A$$ and $$s_B$$, then a bidder with a higher private valuation $$\theta > \theta^*$$ expects to pay strictly more with the steeper security $$s_A$$ than with $$s_B$$. Thus, the payment of the steeper security is tied more tightly to the winning bidder’s private valuation.

We first consider bidding decisions conditional on entry, i.e., on paying the participation cost $$\phi$$. The logic in Proposition 1 of DKS yields the following results

- If a bidder $$i$$ with type $$\Theta_i = \theta$$ is the sole entrant, it has a dominant strategy to bid $$s(S)$$ if $$ES(s(S),\theta) \leq v + \theta$$; and not to bid if $$ES(s(S),\theta) > v + \theta$$.

- With multiple bidders, it is a weakly dominant strategy for a bidder $$i$$ of type $$\Theta_i = \theta$$.
to drop out at the bid $s^* (\theta)$ such that $ES (s^* (\theta), \theta) = v + \theta$. Further, $s^* (\cdot)$ increases in $\theta$.

- If the ordered set of securities $S_A$ is steeper than $S_B$, then conditional on the entry of the highest and second-highest types, the expected equilibrium revenue to the seller is greater under $S_A$ than under $S_B$.

Next we consider the entry decisions of bidders à la Samuelson (1985). Due to the participation costs $\phi$, not all potential bidders may enter. Since the equilibrium expected payoff upon entry is increasing in $\theta$ for a given auction $(S, s(S))$, there must be some cutoff $\theta (S)$ such that only bidders with $\theta \geq \theta (S)$ enter the auction. The marginal bidder with type $\theta (S)$ is indifferent between participating or not. Moreover, bidder $\theta (S)$ wins only if all other bidders are of type $\theta < \theta (S)$; that is, bidder $\theta (S)$ wins only if no one else enters. Therefore, the cutoff $\theta (S)$ solves:

$$[E (Z_i | \Theta_i = \theta (S)) - ES (\bar{s} (S), \theta (S))] F^{n-1} (\theta (S)) + (v_B - \phi) \left(1 - F^{n-1} (\theta (S))\right) = v_B.$$  

(1.1)

Lemma 1 $s^* (\theta(S)) > \bar{s} (S)$.

Proof. The left-hand side of (1.1) is decreasing in $\bar{s} (S)$ and would become $v_B - \phi < v_B$ if we replaced $\bar{s} (S)$ with $s^* (\theta (S))$. □

This result implies that on the equilibrium path, if multiple bidders enter the auction, then the winning bidder will pay with a security bid that is at least $s^* (\theta(S))$, which is strictly higher than $\bar{s} (S)$.

The seller accepts any bid that yields higher expected revenue than its stand-alone value $v$. This premise fits well with bankruptcy auctions. When a bankruptcy trustee’s valuation of the asset is $v$, the trustee cannot reject a bid that is thought to yield more than $v$. In equilibrium, when there is a single bidder, that bidder bids $s (S)$, and the seller only
learns that the winning bidder’s type $\theta$ is at least $\theta(S)$. Thus, $\xi(S)$ solves:

$$\int_{\theta(S)}^{\bar{\theta}} ES (\xi(S), \theta) F (d\theta | \theta \geq \theta(S)) = v. \quad (1.2)$$

In an open outcry auction, when there is only one entrant, the seller learns only that the sole entrant’s type is at least $\theta(S)$. These would also be the inferences that a seller would draw in a second-price auction where a bidder knows how many others are participating, because a single bidder knows it will win and pay the reserve regardless of what bid (above the reserve) he submits. In particular, these would be the inferences drawn if a single bidder always bids the reserve.

A bidder’s break-even condition (1.1) can be rewritten as

$$\phi = [v + \theta(S) - ES (\xi(S), \theta(S))] F^{n-1} (\theta(S)). \quad (1.3)$$

That is, the expected payoff from participation must at least compensate for participation costs. The right-hand side is increasing in $\theta(S)$, and from (1.2) it would become $\bar{\theta}$ by substituting $\bar{\theta}$ for $\theta(S)$. Therefore, the assumption that entry costs are not prohibitive, i.e., $\phi < \bar{\theta}$, ensures that $\theta(S) < \bar{\theta}$ holds for all $S$, i.e., a bidder enters with strictly positive probability.$^3$

We now establish that more entry occurs when bids are paid with steeper securities.

**Proposition 2** Suppose the ordered set of securities $S_A$ is steeper than $S_B$. Then auction $(S_A, \xi(S_A))$ attracts more entry than auction $(S_B, \xi(S_B))$: $\theta(S_A) < \theta(S_B)$.

**Proof.** Let $\xi(S_A)$ and $\xi(S_B)$ be the reserve securities under $S_A$ and $S_B$, respectively. By way of contradiction, first suppose $\theta(S_A) = \theta(S_B) = \bar{\theta}$. Then,

$$\int_{\bar{\theta}}^{\bar{\theta}} ES_A (\xi(S_A), \theta) F (d\theta | \theta \geq \bar{\theta}) = \int_{\bar{\theta}}^{\bar{\theta}} ES_B (\xi(S_B), \theta) F (d\theta | \theta \geq \bar{\theta}) \quad (1.4)$$

$^3$Note that $\theta(S)$ may be negative—a bidder with negative synergies may enter when the surplus associated with high-valuation bidders is sufficiently large.
must hold to satisfy (1.2). Also, using (1.1) yields \( ES_A (\bar{s}(S_A), \bar{\theta}) = ES_B (\bar{s}(S_B), \bar{\theta}) \), which, together with the definition of steepness, implies that

\[
\int_{\bar{\theta}}^{\theta} ES_A (\bar{s}(S_A), \theta) F (d\theta | \theta \geq \bar{\theta}) > \int_{\bar{\theta}}^{\theta} ES_B (\bar{s}(S_B), \theta) F (d\theta | \theta \geq \bar{\theta}),
\]

a contradiction to (1.4). Next suppose \( \theta(S_A) > \theta(S_B) \). Then, it follows that

\[
\int_{\theta(S_A)}^{\theta(S_B)} ES_B (\bar{s}(S_B), \theta) F (d\theta | \theta \geq \theta(S_A)) > \int_{\theta(S_B)}^{\theta(S_A)} ES_B (\bar{s}(S_B), \theta) F (d\theta | \theta \geq \theta(S_B))
\]

\[
= \int_{\theta(S_A)}^{\theta(S_A)} ES_A (\bar{s}(S_A), \theta) F (d\theta | \theta \geq \theta(S_A)),
\]

where the equality holds by (1.2). This, together with the definition of steepness, implies \( ES_B (\bar{s}(S_B), \theta(S_A)) > ES_A (\bar{s}(S_A), \theta(S_A)) \); otherwise, the left-hand side of (1.5) would become smaller, a contradiction. Let \( U(\bar{s}(S_j), \theta) \) denote a type \( \theta \)'s expected payoff when the reserve security is \( \bar{s}(S_j) \) for \( j = A, B \). Then, since \( ES_B (\bar{s}(S_B), \theta(S_A)) > ES_A (\bar{s}(S_A), \theta(S_A)) \),

\[
U(\bar{s}(S_A), \theta(S_A)) > U(\bar{s}(S_B), \theta(S_A))
\]

\[
> U(\bar{s}(S_B), \theta(S_B))
\]

\[
= U(\bar{s}(S_A), \theta(S_A)),
\]

a contradiction, where the last inequality holds by \( \theta(S_A) > \theta(S_B) \) and the equality holds by (1.1). Therefore, \( \theta(S_A) < \theta(S_B) \).

To establish existence and uniqueness of equilibrium outcomes, we show that there is a unique \( \bar{s} \) and \( \theta \) that solve the system of equations, (2) and (3). Let \( \bar{s}(\theta) \) be the reserve
security associated with a cutoff type \( \theta \); i.e.,

\[
\int_{\theta} \text{ES} (\bar{s} (\theta), \theta') \, F (d\theta' | \theta' \geq \theta) = v,
\]

for those \( \theta \) large enough that such a security exists. Then, \( \bar{s} (\theta) \) is decreasing and continuous in \( \theta \). Similarly, let \( \hat{\theta} (\bar{s} (\theta)) \) be the cutoff type for reserve security \( \bar{s} (\theta) \) who is indifferent about participation; i.e.,

\[
\phi = v + \hat{\theta} (\bar{s} (\theta)) - \text{ES} (\bar{s} (\theta), \hat{\theta} (\bar{s} (\theta))) F^{n-1} (\hat{\theta} (\bar{s} (\theta))) .
\]

Then, \( \hat{\theta} (\cdot) \) is increasing and continuous in its argument. Therefore, \( \hat{\theta} (\bar{s} (\theta)) \) is decreasing and continuous in \( \theta \). We have already established that \( \hat{\theta} (\bar{s} (\theta)) < \bar{\theta} \), and \( \phi > \theta \) implies that \( \hat{\theta} (\bar{s} (\theta)) > \theta \), so that we have \( \theta < \hat{\theta} (\bar{s} (\theta)) < \bar{\theta} \). Therefore, there exists a unique fixed point that characterizes the equilibrium. \( \square \)

To see the intuition for this result, observe that regardless of the class of securities, the reserve security breaks even for the seller—the seller’s expected revenues when there is a single bidder are the same regardless of the class of securities. However, steeper securities extract a greater share of its revenues from bidders with higher valuations. It follows that steeper securities extract less from bidders with lower valuations. In particular, the steeper is the security design, the less the marginal auction participant expects to pay with the reserve security. Hence, the steeper is the security design, the more willing bidders with lower valuations are to participate—for any realization of bidder signals, auction \((S_A, s(S_A))\) attracts at least as many entrants as auction \((S_B, s(S_B))\).

Note that the seller would prefer to retain the asset (in expectation) whenever the marginal auction participant is the sole bidder; and, indeed, the marginal participant’s private valuation could even be negative. One might therefore think that because a steeper security design draws more participants with low valuations, it might reduce expected seller revenues. Proposition \(3\) shows that this is not so:
Proposition 3  If the ordered set of securities $S_A$ is steeper than $S_B$, then for any set of signal realizations, $\{\theta_i\}_{i=1}^n$, auction $(S_A, \hat{s}(S_A))$ has at least as much entry as $(S_B, \hat{s}(S_B))$. Moreover,

- If auction $(S_A, \hat{s}(S_A))$ attracts multiple entrants, then it yields the seller higher expected revenue than $(S_B, \hat{s}(S_B))$.

- If auction $(S_A, \hat{s}(S_A))$ attracts zero or one entrant, then it yields the seller the same expected revenue as $(S_B, \hat{s}(S_B))$.

Proof. Let $\theta^2$ be the second-highest type. By Proposition 2 there are three cases: $\theta^2 \leq \theta(S_A)$, $\theta^2 \in (\theta(S_A), \theta(S_B)]$ and $\theta^2 > \theta(S_B)$. When $\theta^2 \leq \theta(S_A)$, at most one bidder enters for both auctions. Then, the seller expects to receive $v$ in both auctions. When $\theta^2 \in (\theta(S_A), \theta(S_B)]$, at least two bidders enter for $(S_A, \hat{s}(S_A))$ yielding a strictly higher revenue than $v$ by Lemma 1, while at most one bidder enters for $(S_B, \hat{s}(S_B))$ yielding a revenue of $v$. When $\theta^2 > \theta(S_B)$, at least two bidders enter both auctions. Then, it follows from DKS that the seller receives higher revenue in the auction with $(S_A, \hat{s}(S_A))$. □

Thus, the greater entry to auctions with steeper securities reinforces their revenue-enhancing advantages.

More generally, the logic underlying Propositions 2 and 3 carries over for any reserve security of a fixed given value that exceeds the seller’s value as a stand-alone entity. Suppose that the reserve security $\hat{s}(S)$ solves:

$$\int_{\theta(S)}^{\hat{\theta}} ES(\hat{s}(S), \theta) F(d\theta|\theta \geq \theta(S)) = \hat{\theta} \in (v, v + \bar{\theta} - \phi).$$

(1.6)

We now establish that more entry occurs when bids are paid with steeper securities.

Proposition 4  If the ordered set of securities $S_A$ is steeper than $S_B$, then auction $(S_A, \hat{s}(S_A))$ attracts more entry than $(S_B, \hat{s}(S_B))$: $\theta(S_A) < \theta(S_B)$. Moreover,

- If auction $(S_A, \hat{s}(S_A))$ either attracts multiple entrants or more entrants than $(S_B, \hat{s}(S_B))$, then it yields the seller higher expected revenue.
• If auctions $(S_A, \hat{s}(S_A))$ and $(S_B, \hat{s}(S_B))$ both attract zero entrants or both attract one entrant, then they yield the seller the same expected revenue.

We omit the proof because it mirrors those for Propositions 2 and 3. Once more, for any reserve security of a fixed value $\hat{v} \in (v, v + \hat{\theta} - \phi)$, the steeper is the security, the greater is the expected portion of that fixed value that comes from high-valuation bidders. That is, the steeper is the security, the smaller is the expected portion of that fixed value that comes from low-valuation bidders. Therefore, steeper securities attract more low-valuation bidders. In turn, this increased participation generates higher expected payoffs for the seller. Indeed, a stronger result obtains, because when the steeper security design attracts a single entrant but no entry occurs with the less steep security design, with the steeper security design, the seller now expects revenues that exceed its stand-alone value since $\hat{v} > v$. We now establish the following proposition.

**Proposition 5** When the seller faces no constraints on the reserve security that it sets for a given class of securities, with the optimal reserve security, the steeper is the security-bid design, the greater are the seller’s expected revenues.

**Proof.** This follows directly from the fact that for a given value of the reserve (including the optimal reserve for the less steep security design), expected revenues are higher with the steeper security design (even though the value of that reserve security need not be the optimal one for the steeper security design.)

### 1.3 Conclusion

We endogenize entry to security-bid auctions, by introducing a cost to participation. We first consider a scenario where, with a single entrant, the minimum security bid accepted is pinned down by a break-even (indifference) condition for the seller. Counterintuitively, we establish that security-bid auctions that use steeper securities for payment, which generate greater expected revenues for the seller for a fixed number of bidders, also make
bidders with worse signals more willing to participate. We show that even when the marginal participant has a negative private valuation, this increased participation reinforces the revenue superiority of such auctions.

We extend this logic to any reserve security of a fixed given value that exceeds the seller’s stand-alone value: steeper securities attract more low-valuation bidders, and increased participation by low-valuation bidders yields the seller higher expected revenues. Furthermore, an auction using steeper securities that sets its optimal reserve security yields higher expected revenues than the one using less steep securities that sets its optimal reserve security.
Chapter 2

Seller’s Information Disclosure Policy in Equity Auctions with Post-Auction Investment Opportunities

2.1 Introduction

According to the seminal “linkage principle” by Milgrom and Weber (1982), a seller can enhance auction revenues by publicly disclosing her private information regarding the value of the good being auctioned. However, this paper shows that, in an auction with certain attributes, the seller may receive higher revenue when not revealing any private information at all than when committing to reveal information publicly regardless of whether it is positive or negative.

The attributes are as follows: (i) instead of making cash bids, bidders make equity offers detailing the share the seller can claim on future cash flows generated by the good being auctioned (i.e., equity or royalty auctions), (ii) the winning bidder has post-auction investment opportunities, and (iii) the seller has private information about the return of the post-auction investment prior to the auction.
Prototypical examples of auctions with the attributes (i) – (iii) are oil and gas lease auctions and corporate takeover contests. In oil and gas lease auctions, a winning bidder makes royalty payment based on production from the lease, and he must choose how much to invest once he acquires the lease. Moreover, the seller has some private information that helps bidders to estimate the value of the tract due to independently exploring the tract prior to the auction. Alternatively, many merger and acquisition deals are paid with stock, and an acquirer must choose how much to invest after the deal. Moreover, it is likely that a target firm has some private information that can help bidding firms estimate to determine the target’s future profitability. In such situations, it is imperative to the seller whether she can benefit by making her private information public.

In equity auctions, bidders are susceptible to a moral hazard problem. If a winning bidder must pay a large fraction of future cashflow generated by the auctioned asset, he has less incentive to make a costly investment in the asset after the auction.

The key intuition behind our counter-example to the linkage principle is that new information affects bidders with different productivity differently who are under the moral hazard problem. In our paper we consider a second-price equity auction in which a winner pays according to the second highest bid. When the second most productive bidder is extremely productive or unproductive, his bid will be so high or low that no new information affects the winning bidder’s investment decision. However, when the second most productive bidder is moderately productive, the new information revealed by the seller will intensify the moral hazard problem faced by the winner. If instead the second most productive bidder is moderately unproductive, the new information will mitigate the moral hazard problem. Therefore, depending on distributions of productivity among bidders, the seller may be better off concealing her private information.

There are a number of papers which show that, under a certain setting, revealing more information does not necessarily enhance a seller’s revenue. Perry and Reny (1999) consider a multi-unit auction setting. Fang and Parreiras (2003) consider a situation when
bidders are financially constrained. Mares and Harstad (2003) consider a situation in which a seller can privately reveal information to a certain set of bidders. Bergemann and Pesendorfer (2007) consider a multiple-signal setting. However, to my knowledge, in all of these research, the reason why revealing more information may lower the seller’s revenue is that information disclosure has an allocation effect; that is, as Board (2009) points out, “the new information can change the order of bidders’ valuations.” This current paper not only offers another situation in which the linkage principle does not hold, but also considers a situation in which new information does not change the order of bidders’ valuations.

Kogan and Morgan (2010) also analyze equity auctions with post-auction investment opportunities, and show that a moral hazard problem arising from equity payment undermines an incentive to invest and thus the resulting revenue may be lower than that of the corresponding debt auction. However, they do not consider a seller’s information disclosure policy.

The remainder of the paper is organized as follows. Section 2.2 describes the model. Section 2.3 derives the equilibrium strategies. Section 2.4 compares the revenues from two different policies: publicly revealing the seller’s private information regardless of whether it is positive or negative, and not revealing at all. Section 2.5 concludes. Most of the proofs are delegated to Section 2.6.

### 2.2 The Model

A seller (female) uses the second-price equity auction to sell a single indivisible asset to at most one of \( N \geq 2 \) bidders (male). If bidder \( i \) acquires the asset, it generates cashflow

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1. Rothkopf and Engelbrecht-Wiggans (1992) consider the same issue of royalty payments in a non-rigorous manner.

2. Our model specification follows the equity auction model in Kogan and Morgan (2010). However, our model has an additional information structure created by a signal \( w \in \{G, B\} \) concerning the profitability of the post-auction investment.
Each bidder bids a fraction \( \alpha_i \in (0, 1) \) and the highest bidder acquires the asset. The winning bidder \( i \) pays \( \alpha_i^2 Z_i \) to the seller while he retains \( (1 - \alpha_i^2) Z_i \), where \( \alpha_i^2 \) is the second-highest bid. Both the seller and bidders are assumed to be risk-neutral.

Prior to the auction, the seller receives a private signal \( w \in \{G, B\} \) concerning the profitability of the asset with a probability \( q \) of \( G \). Also, bidder \( i \) has private information about his productivity \( v_i \in [\underline{v}, \overline{v}] \), where \( v_i \) is distributed according to the distribution function \( F_i \) on \( [\underline{v}, \overline{v}] \). Note that \( v_i \)'s can be dependently and asymmetrically distributed. Upon winning the auction, bidder \( i \) must make an initial investment \( x_0 > 0 \), which is common and known to all the bidders prior to the auction. Once the initial investment is made, bidder \( i \) observes the signal \( w \in \{G, B\} \) that the seller received before the auction.

We can interpret \( w \) as the sort of information that can be obtained only after actually acquiring the asset. After observing the signal \( w \), the winner has an opportunity to make an additional investment \( x \in \{0, 1\} \). Note that, upon making the additional investment decision, the winner knows the signal \( w \) regardless of the seller’s information disclosure policies. If bidder \( i \) with productivity \( v_i \) wins the auction and chooses the additional investment \( x \in \{0, 1\} \) upon receiving the signal \( w \in \{G, B\} \), cashflow \( Z_i \) generated by the asset is

\[
Z_i = \begin{cases} 
  v_i + \beta x & \text{with a probability of } p_w \\
  0 & \text{with a probability of } (1 - p_w),
\end{cases}
\]

where \( p_G > p_B \) and \( p_B \beta > 1 \). Note that, by \( p_B \beta > 1 \), it is socially efficient for any winner to choose \( x = 1 \) regardless of the signal \( w \in \{G, B\} \). Also, we assume that \( p_B \overline{v} > x_0 \).\(^3\)

\(^3\)The assumption of discrete investment is not crucial to the results of this paper. Using continuous investment model with a convex cost function yields qualitatively the same results.
so that ex-post efficiency requires that trade occur even when a bidder with the lowest productivity \( v \) wins the auction and makes no additional investment \( x = 0 \) after receiving the bad signal \( B \).

In what follows, we will compare two information disclosure policies: \( D \) and \( ND \).

\( (D) \) Prior to the auction, the seller commits to publicly disclose the signal \( w \) she receives, regardless of whether it is \( G \) or \( B \).

\( (ND) \) Prior to the auction, the seller commits not to disclose the signal \( w \) at all.

The timing of the events is summarized as follows:

1. The seller commits to one of the policies: \( D \) or \( ND \).
2. The seller receives a signal \( w \in \{G, B\} \). She publicly reveals \( w \) if \( D \) is chosen in 1 and does not reveal \( w \) if \( ND \) is chosen in 1. Each bidder receives a productivity signal \( v_i \).
3. The second-price equity auction takes place: the highest bidder \( i \) wins and he learns the second-highest bid \( \alpha^2 \in (0, 1) \).
4. The winning bidder \( i \) makes the initial investment \( x_0 \) and learns the signal \( w \) that is identical to what the seller received in 2.
5. The winning bidder \( i \) chooses whether to make the additional investment \( x \in \{0, 1\} \).
6. Cashflow \( Z_i \) is realized and the winning bidder \( i \) pays \( \alpha^2 Z_i \) to the seller and retains \( (1 - \alpha^2) Z_i \).

### 2.3 Equilibrium Analysis

Let \( \hat{p} \equiv qp_G + (1 - q) p_B \) be the ex-ante expected probability of success, and let \( \Delta p \equiv p_G - p_B \). Let \( \pi(v, w, \alpha^2, x) \) be the expected payoff of a winning bidder with productivity
when he receives a signal $w \in \{G, B\}$, the second-highest bid is $\alpha^2$, and he makes the additional investment $x \in \{0, 1\}$; that is, define

$$\pi(v, w, \alpha^2, x) \equiv \left(1 - \alpha^2\right) p_w (v + \beta x) - x_0 - x.$$ 

Moreover, let

$$\pi(v, w, \alpha^2) \equiv \max \left\{\pi(v, w, \alpha^2, 0), \pi(v, w, \alpha^2, 1)\right\}$$

denote the equilibrium payoff of a winning bidder with productivity $v$ given $w \in \{G, B\}$ and the second-highest bid $\alpha^2$. $\pi(v, w, \alpha^2)$ manifests that there is a moral hazard problem. Although it is socially efficient to additionally invest even when $w = B$, large equity payment $\alpha^2$ undermines the incentive to invest. Since the expected amount of cashflow from the asset is increasing in a bidder’s productivity, a more productive bidder submits a higher bid. However, each bidder must take into account how bidding high will affect the post-auction investment choice. This consideration leads to the following proposition.

**Proposition 6** The equilibrium in weakly undominated strategies in the second-price equity auction with the post-auction investment is for a bidder with productivity $v$ to bid

$$\alpha(v) = \begin{cases} 
1 - \frac{x_0}{p(v)} & \text{if } v > \frac{p_G x_0 \beta}{p} \\
1 - \frac{x_0 + q}{p v + qp_G \beta} & \text{if } v \in \left(\frac{(p_B x_0 - q \Delta p) \beta}{p}, \frac{p_G x_0 \beta}{p}\right) \\
1 - \frac{x_0 + 1}{p (v + \beta)} & \text{if } v < \frac{(p_B x_0 - q \Delta p) \beta}{p}
\end{cases}$$

when the seller chooses ND, and to bid

$$\alpha(v, w) = \begin{cases} 
1 - \frac{x_0}{p_w v} & \text{if } v > x_0 \beta \\
1 - \frac{x_0 + 1}{p_w (v + \beta)} & \text{if } v < x_0 \beta
\end{cases}$$

when the seller chooses D. Furthermore, given the signal $w \in \{G, B\}$ and the second-highest bid
\( \alpha^2 \), the equilibrium additional investment strategy for the winning bidder with productivity \( v \) is

\[
x(v, w, \alpha^2) = \begin{cases} 
1 & \text{if } \alpha^2 \leq 1 - \frac{1}{pw} \\
0 & \text{otherwise.}
\end{cases}
\]

**Proof.** See Section 2.6.

The equilibrium additional investment strategy \( x(v, w, \alpha^2) \) manifests itself in a moral hazard problem. When the winning bidder must pay a large fraction \( \alpha^2 \) of cashflow from the asset, he does not have an incentive to invest. A more productive bidder submits a higher bid, and thereby the incentive to make the post-auction investment is undermined. As a result, he must adjust his conditional expected payoff upon winning accordingly.

More specifically, suppose that the seller does not disclose her private information. A bidder with a large private signal \( v > \frac{p_G x_0 \beta}{p} \) bids so high that he would make no additional investment regardless of the signal \( w \) should he win and pay his own bid. A bidder with an intermediate private signal \( v \in \left( \frac{(p_B x_0 - q \Delta p) \beta}{p}, \frac{p_G x_0 \beta}{p} \right) \) bids so that he would make the additional investment only when \( w = G \) should he win and pay his own bid. A bidder with a small private signal \( v < \frac{(p_B x_0 - q \Delta p) \beta}{p} \) bids so small that he would make the additional investment regardless of the signal \( w \) should he win and pay his own bid.

Similarly, suppose the seller discloses her private information \( w \in \{ G, B \} \). Then, a bidder with a large signal \( v > x_0 \beta \) bids so high that he would make no additional investment should he win and pay his own bid. However, a bidder with a small signal \( v < x_0 \beta \) bids so small that he would make the additional investment should he win and pay his own bid.

Moreover, the following corollary immediately follows from the above proposition.

**Corollary 7** The equilibrium bidding strategies \( \alpha(v) \) and \( \alpha(v, w) \) are strictly increasing and continuous in productivity \( v \).
This corollary implies that the most productive bidder always wins regardless of whether the seller chooses \( D \) or \( ND \); that is, information revealed by the seller does not change the allocation rule, in contrast to the existing papers that offer counter-examples to the linkage principle.

### 2.4 Revenue Comparison

Instead of comparing the ex-ante expected revenues, let us compare the interim expected revenues under the two different disclosure policies \( ND \) and \( D \) by taking the highest private signal \( v_1 \) and the second highest signal \( v_2 \) as given. This allows us to analyze without any assumption on distribution functions \( \{F_i\}_{i \in \{1, \ldots, N\}} \) of signals \( \{v_i\}_{i \in \{1, \ldots, N\}} \). The next proposition provides our key result, which suggests that choosing \( ND \) can yield a higher ex-ante expected revenue than choosing \( D \) for some given distributions \( \{F_i\}_{i \in \{1, \ldots, N\}} \) of signals \( \{v_i\}_{i \in \{1, \ldots, N\}} \) and some given number \( N \) of bidders.

**Proposition 8** Given the realized highest private signal \( v_1 \) and second highest signal \( v_2 \), we can rank the interim expected revenues under the two information disclosure policies \( ND \) and \( D \) as follows:

1. When \( v_2 \) is extremely large; that is, \( v_2 \geq \frac{p_c x_0 \beta}{\hat{p}} \), the second-price equity auction with post-auction investment yields the same interim expected revenue regardless of the seller’s information disclosure policies.

2. When \( v_2 \) is moderately large; that is, \( v_2 \in \left( x_0 \beta, \frac{p_c x_0 \beta}{\hat{p}} \right) \), \( ND \) leads to a higher interim expected revenue than \( D \).

3. When \( v_2 \) is moderately small; that is, \( v_2 \in \left( \frac{(p_B x_0 - q \Delta p) \beta}{\hat{p}}, x_0 \beta \right) \), \( ND \) leads to a higher interim expected revenue than \( D \) if \( v_1 - v_2 > \Delta \hat{d} \); otherwise, \( D \) leads to a higher interim expected revenue.
expected revenue than ND, where

\[ \Delta \hat{\delta} \equiv \frac{(p_B \beta - 1) \left( v_2 + \beta \right) \left( \hat{p} v_2 + q p_G \beta \right)}{\hat{p} v_2 - \left( p_B x_0 - q \Delta p \right) \beta} \, . \]

4. When \( v_2 \) is extremely small; that is, when \( v_2 \leq \frac{(p_B x_0 - q \Delta p) \beta}{\hat{p}} \), the second-price equity auction with post-auction investment yields the same interim expected revenue regardless of the seller’s information disclosure policies.

**Proof.** See Section 2.6.

First, note that, unlike cash auctions, the amount of revenue in equity auctions depends not only on the bid, but also on the realized cashflow generated by the asset being auctioned. When \( v_2 \) is extremely large, the second-highest bidder with productivity \( v_2 \) bids so high regardless of the seller’s information disclosure policies that the winning bidder has no incentive to make the additional investment regardless of \( w \in \{G, B\} \). Similarly, when \( v_2 \) is extremely small, the second-highest bidder with productivity \( v_2 \) bids so low regardless of the seller’s information disclosure policies that the winning bidder makes the additional investment regardless of \( w \in \{G, B\} \). Notice that, when \( v_2 \) is extremely large (small), the second-highest bid is made in anticipation to choose \( x = 0 \) (\( x = 1 \)) regardless of the signal \( w \) should he win and pay his own bid, while the type-\( v_1 \) chooses \( x = 0 \) (\( x = 1 \)) regardless of the signal \( w \). In other word, when \( v_2 \) is extremely large or extremely small, the seller’s information disclosure policies do not affect the equilibrium bid and the investment decision. Consequently, they do not affect the seller’s revenues.

Consider a case in which \( v_2 \) is moderately large: \( v_2 \in (x_0 \beta, p_G x_0 \beta / \hat{p}) \). Then, his bid is also moderately large so that the following is satisfied:

\[ 1 - \frac{1}{p_B \beta} < \alpha (v_2, B) < \alpha (v_2) < 1 - \frac{1}{p_G \beta} < \alpha (v_2, G) . \]
This implies that the additional investment is made only when \( w = G \) under ND while no additional investment is made regardless of \( w \) under \( D \). Consequently, the expected cashflow of the asset is higher under ND. Moreover, type-\( v_2 \) bids more aggressively under ND:

\[
\hat{p}\alpha(v_2) > qp_G\alpha(v_2, G) + (1 - q) p_B\alpha(v_2, B).
\]

Therefore, ND yields a higher interim expected revenue.

Consider a case in which \( v_2 \) is moderately small: \( v_2 \in ((p_B x_0 - q\Delta p) \beta / \hat{p}, x_0\beta) \). Then, his bid is also moderately small so that the following is satisfied:

\[
\alpha(v_2, B) < 1 - \frac{1}{p_B\beta} < \alpha(v_2) < \alpha(v_2, G) < 1 - \frac{1}{p_G\beta}.
\]

This implies that the additional investment is made only when \( w = G \) under ND while the additional investment is made regardless of \( w \) under \( D \). Consequently, the expected cashflow from the asset is higher under \( D \). However, the type-\( v_2 \) bids more aggressively under ND:

\[
\hat{p}\alpha(v_2) > qp_G\alpha(v_2, G) + (1 - q) p_B\alpha(v_2, B).
\]

Thus, ND results in smaller expected cashflow but a more aggressive bid. The latter effect impacts more on the seller’s revenue as \( v_1 \) increases. As a result, for sufficiently large \( v_1 \), the latter effect dominates the former, and therefore ND leads to a higher interim expected revenue.

Noting that the above argument is made by taking the realized highest signal \( v_1 \) and second highest signal \( v_2 \) as given, let us consider the ex-ante optimality of \( D \) and ND. Proposition 8 implies that, when \( \bar{v} < \frac{(p_B x_0 - q\Delta p) \beta}{\hat{p}} \) or \( \bar{v} > \frac{p_G x_0 \beta}{\hat{p}} \), both N and ND yield the same ex-ante expected revenue. Moreover, when \( \bar{v} \in \left( x_0\beta, \frac{p_G x_0 \beta}{\hat{p}} \right) \), ND yields a higher
ex-ante expected revenue, regardless of the parameters and distribution functions.

However, when \( v < x_0 \beta \), the ex-ante optimality of \( D \) and \( ND \) depends on the parameters \( (N, \beta, x_0, p_G, p_B, q) \) and distributions \( \{F_i\}_{i \in \{1, \ldots, N\}} \) of signals \( \{v_i\}_{i \in \{1, \ldots, N\}} \). To illustrate this, consider a case in which \( N = 2, q = \frac{1}{2}, p_G = \frac{3}{4}, p_B = \frac{1}{4}, x_0 = 3 \) and \( v_i \) is independent and uniformly distributed over \((15, 25)\). Figure 2.1 shows the ex-ante expected revenues under \( D \) and under \( ND \) (solid red line and dashed red line, respectively) and the ex-ante expected probabilities of undertaking investment \( (x = 1) \) under \( D \) and under \( ND \) (solid blue line and dashed blue line, respectively).

Figure 2.1: The ex-ante expected revenues under \( D \) and under \( ND \) (solid red line and dashed red line, respectively) and the ex-ante expected probabilities of undertaking investment \( (x = 1) \) under \( D \) and under \( ND \) (solid blue line and dashed blue line, respectively).

For \( \beta < 7.08718 \), \( ND \) yields a higher ex-ante expected revenue, and for \( \beta \in (7.08718, 25) \), \( D \) yields a higher ex-ante expected revenue. When \( \beta > 25 \), \( \bar{v} < \frac{(p_B x_0 - q \Delta p) \beta}{\beta} \) holds; therefore, both \( N \) and \( ND \) yield the same ex-ante expected revenue.
Several remarks can be made concerning the ex-ante optimality based on Proposition 8. Suppose $ND$ is ex-ante optimal. Then, transferring probability mass away from region 3 in Proposition 8 where $v_1 - v_2 < \Delta \hat{v}$, under an assumption of conditional independence, continues to make $ND$ optimal. Transferring probability mass from region 1 or 4 in Proposition 8 to region 2 or 3 where $v_1 - v_2 > \Delta \hat{v}$ also continues to make $ND$ optimal.

### 2.5 Conclusion

We consider equity auctions with the post-auction investment opportunities when a seller has private information about the return of the post-auction investment. Then we show that in such a situation, in contrast to the seminal linkage principle, the seller’s expected revenue may be higher when not disclosing her private information at all than when committing to publicly announce her private information regardless of whether it is positive or negative.

As noted earlier, unlike the existing literature, no new information changes the allocation rule in our model. The intuition behind our counter-example to the linkage principle is that new information affects bidders with different productivity differently who are under the moral hazard problem. When $v_2$ is extremely large or small, his bid will be so high or low that no new information affects the winning bidder’s investment decision. However, when $v_2$ is moderately large, the new information revealed by the seller will intensify the moral hazard problem faced by the winner. On the other hand, when $v_2$ is moderately small, the new information will mitigate the moral hazard problem. Thus, depending on a combination of the number of bidders and distributions of productivity signals, the seller may be better off concealing her private information.

The policy implication of this paper is that a seller sometimes should not reveal her private information about the profitability of the investment in equity/royalty auctions, or equity/royalty auctions should not be adopted as an auction format in such a situation.
2.6 Proofs of Chapter 2

2.6.1 Proof of Proposition 6

In order to prove Proposition 6, first we need the following two lemmas.

**Lemma 9** Suppose \(E_W[\pi(v, w, \alpha^2)]\) is continuous and non-increasing in \(\alpha^2\) for any \(v\) and \(w\). Also, suppose that for any \(v\) and \(w\) there exists some \(\hat{\alpha}\) such that \(E_W[\pi(v, w, \alpha^2)]\) is strictly decreasing in \(\alpha^2\) for any \(\alpha^2 < \hat{\alpha}\) with \(E_W[\pi(v, w, 0)] > 0\) and \(E_W[\pi(v, w, \hat{\alpha})] < 0\). Then, in the second-price equity auction with the post-auction investment, it is weakly dominant bidding strategy for a bidder with productivity \(v\) to bid \(\alpha(v)\) such that

\[E_W[\pi(v, w, \alpha(v))] = 0.\]

**Proof.** First, by the continuity and strict monotonicity together with the boundary conditions, the equilibrium bidding strategy \(\alpha(v)\) is well defined. Let \(\alpha'\) be the highest competing bid. By bidding \(\alpha(v)\), bidder \(i\) wins and earns a positive payoff \(E_W[\pi(v, w, \alpha^2)]\) if \(\alpha(v) > \alpha'\) and loses if \(\alpha(v) < \alpha'\). Now, suppose bidder \(i\) bids \(\alpha > \alpha(v)\). If \(\alpha^2 > \alpha > \alpha(v)\), then bidder \(i\) still loses. If \(\alpha > \alpha^2 > \alpha(v)\), then bidder \(i\) wins but earns a negative payoff since \(E_W[\pi(v, w, \alpha^2)]\) is strictly decreasing, while had he bid \(\alpha(v)\), he would have earned zero. If \(\alpha > \alpha(v) > \alpha^2\), then he still wins and earns the same payoff \(E_W[\pi(v, w, \alpha^2)]\). Therefore, bidding \(\alpha > \alpha(v)\) is weakly dominated by \(\alpha(v)\). Similarly, one can prove that bidding \(\alpha < \alpha(v)\) is weakly dominated by \(\alpha(v)\). □

**Lemma 10** \(E_W[\pi(v, w, \alpha^2)]\) is continuous and non-increasing in \(\alpha^2\) for any \(v\) and \(w\). Also, for any \(v\) and \(w\) there exists some \(\hat{\alpha}\) such that \(E_W[\pi(v, w, \alpha^2)]\) is strictly decreasing in \(\alpha^2\) for any \(\alpha^2 < \hat{\alpha}\) with \(E_W[\pi(v, w, 0)] > 0\) and \(E_W[\pi(v, w, \hat{\alpha})] < 0\).

**Proof.** First, for any \(v\) and \(w\), \(E_W[\pi(v, w, \alpha^2)]\) is continuous and non-increasing in \(\alpha^2\). Also, \(E_W[\pi(v, w, 0)] > 0\) by \(p_B^V > x_0\). Moreover, for a sufficiently large \(\hat{\alpha}\), we have
\[ E_W [\pi (v, w, \hat{\alpha})] < 0 \] and
\[ E_W [\pi (v, w, \alpha^2)] = (1 - \alpha^2) (q p_G + (1 - q) p_B) v - x_0 \]
is strictly decreasing in \( \alpha^2 \) for any \( \alpha^2 < \hat{\alpha} \). ■

**Proof of Proposition 6**

Now let us prove the latter statement of the proposition. The winning bidder makes the additional investment if and only if
\[
(1 - \alpha^2) p_w (v + \beta) - x_0 - 1 \geq (1 - \alpha^2) p_w v - x_0
\]
or \( \alpha^2 \leq 1 - \frac{1}{p_w \beta} \).

Next, let us prove the equilibrium bidding strategy \( \alpha (v) \) when the seller does not disclose her private information. First suppose \( v > \frac{c x_0 \beta}{p} \). Then
\[
\alpha (v) = 1 - \frac{x_0}{\hat{p} v} > 1 - \frac{1}{p_G \hat{p}}
\]
and hence \( x (v, w, \alpha (v)) = 0 \) for any \( w \in \{G, B\} \). It follows that
\[
q \pi (v, G, \alpha (v)) + (1 - q) \pi (v, B, \alpha (v)) = \hat{p} (1 - \alpha (v)) v - x_0 = 0,
\]
and therefore by Lemma 9 and 10, \( \alpha (v) \) is the equilibrium bidding strategy.

Suppose \( v \in \left( \frac{(p_B x_0 - q \Delta p) \beta}{p}, \frac{p_G x_0 \beta}{p} \right) \). Then
\[
\alpha (v) = 1 - \frac{x_0 + q}{\hat{p} v + q p_G \beta} \in \left( 1 - \frac{1}{p_B \beta}, 1 - \frac{1}{p_G \beta} \right),
\]
and hence \(x(v, G, \alpha(v)) = 1\) and \(x(v, B, \alpha(v)) = 0\). It follows that

\[
q \pi(v, G, \alpha(v)) + (1 - q) \pi(v, B, \alpha(v)) = (1 - \alpha(v)) (\hat{p} v + q \hat{p} G \beta) - x_0 - q = 0,
\]

and therefore by Lemma 9 and 10, \(\alpha(v)\) is the equilibrium bidding strategy.

Suppose \(v < \frac{(p_B x_0 - q \Delta p) \beta}{\hat{p}}\). Then

\[
\alpha(v) = 1 - \frac{x_0 + 1}{\hat{p} (v + \beta)} < 1 - \frac{1}{p_B \beta'}
\]

and hence \(x(v, w, \alpha(v)) = 1\) for any \(w \in \{G, B\}\). It follows that

\[
q \pi(v, G, \alpha(v)) + (1 - q) \pi(v, B, \alpha(v)) = (1 - \alpha(v)) (\hat{p} v + \beta - x_0) - 1 = 0,
\]

and therefore by Lemma 9 and 10, \(\alpha(v)\) is the equilibrium bidding strategy.

We can similarly prove the equilibrium bidding strategy \(\alpha(v, w)\) when the seller discloses her private information \(w \in \{G, B\}\). ■

2.6.2 Proof of Proposition 8

First, since \(p_B < \hat{p} < p_G\), the following parametric relationship holds:

\[
\frac{(p_B x_0 - q \Delta p) \beta}{\hat{p}} < x_0 \beta < \frac{p_G x_0 \beta}{\hat{p}}.
\]

Let us take the realized highest private signal \(v_1\) and second highest signal \(v_2\) as given. Then there are four cases to consider:

(Case 1) \(v_2 > \frac{p_G x_0 \beta}{\hat{p}}\),
(Case 2) \( v_2 \in \left( x_0\tilde{\beta}, \frac{p_G x_0\tilde{\beta}}{\hat{p}} \right) \),

(Case 3) \( v_2 \in \left( \frac{(p_B x_0 - q\Delta p)\tilde{\beta}}{\hat{p}}, x_0\tilde{\beta} \right) \),

(Case 4) \( v_2 < \frac{(p_B x_0 - q\Delta p)\tilde{\beta}}{\hat{p}} \).

In Case 1, by Proposition \[6\]

\[
\alpha (v_2) = 1 - \frac{x_0}{\hat{p}v_2} > 1 - \frac{1}{p_G\tilde{\beta}}
\]

and hence \( x (v_1, w, \alpha (v_2)) = 0 \) for any \( w \in \{ G, B \} \) under \( ND \), while

\[
\alpha (v_2, w) = 1 - \frac{x_0}{p_w v_2} > 1 - \frac{1}{p_w\tilde{\beta}}
\]

and hence \( x (v_1, w, \alpha (v_2, w)) = 0 \) for any \( w \in \{ G, B \} \) under \( D \). This and the fact that \( \alpha (v_2) \) and \( (\alpha (v_2, G), \alpha (v_2, B)) \) are the equilibrium strategies imply that

\[
0 = q\pi (v_2, G, \alpha (v_2, G), x = 0) + (1 - q)\pi (v_2, B, \alpha (v_2, B), x = 0)
\]

\[
= q\pi (v_2, G, \alpha (v_2), x = 0) + (1 - q)\pi (v_2, B, \alpha (v_2), x = 0).
\]

By using the definition of \( \pi (v, w, \alpha, x) \), this reduces to the following:

\[
qp_G\alpha (v_2, G) + (1 - q)p_B\alpha (v_2, B) = \hat{p}\alpha (v_2).
\]

Thus, the interim expected revenue under \( D \) can be written as follows:

\[
[qp_G\alpha (v_2, G) + (1 - q)p_B\alpha (v_2, B)] v_1 = \hat{p}\alpha (v_2) v_1,
\]

which is exactly the interim expected revenue under \( ND \).
In Case 2, by Proposition 6,

$$\alpha (v_2) = 1 - \frac{x_0 + q}{p v_2 + q p G} \in \left( 1 - \frac{1}{p B'}, 1 - \frac{1}{p G} \right)$$

and hence $x(v_1, G, \alpha (v_2)) = 1$ and $x(v_1, B, \alpha (v_2)) = 0$ under ND, while

$$\alpha (v_2, w) = 1 - \frac{x_0}{p_w v_2} > 1 - \frac{1}{p_w \hat{\beta}}$$

and hence $x(v_1, w, \alpha (v_2, w)) = 0$ for any $w \in \{G, B\}$ under D. Moreover, we have the following relationship:

$$0 = q \pi (v_2, G, \alpha (v_2), x = 1) + (1 - q) \pi (v_2, B, \alpha (v_2), x = 0)$$
$$= q \pi (v_2, G, \alpha (G, v_2), x = 0) + (1 - q) \pi (v_2, B, \alpha (B, v_2), x = 0)$$
$$> q \pi (v_2, G, \alpha (v_2), x = 0) + (1 - q) \pi (v_2, B, \alpha (v_2), x = 0),$$

where the equalities hold by the break-even conditions of $\alpha (v_2), \alpha (v_2, G),$ and $\alpha (v_2, B),$ and the inequality holds by $x(v_2, G, \alpha (v_2)) \neq 0$ for $v_2 < \frac{p_G x_0 \hat{\beta}}{p}.$ By using the definition of $\pi (v, w, \alpha, x),$ the above inequality implies that

$$qp_G \alpha (v_2, G) + (1 - q) p_B \alpha (v_2, B) < \hat{\rho} \alpha (v_2).$$

This leads to the following:

$$[qp_G \alpha (v_2, G) + (1 - q) p_B \alpha (v_2, B)] v_1 < \hat{\rho} \alpha (v_2) v_1$$
$$< \hat{\rho} \alpha (v_2) \left( v_1 + \frac{q p_G}{\hat{\rho}} \beta \right);$$

that is, the interim expected revenue under $D$ is smaller than that under ND.
In Case 3, by Proposition 6,

\[ \alpha(v_2) = 1 - \frac{x_0 + q}{\hat{p}v_2 + q\hat{p}G} \in \left( 1 - \frac{1}{p_B\hat{p}}, 1 - \frac{1}{p_G\hat{p}} \right) \]

and hence \( x(v_1, G, \alpha(v_2)) = 1 \) and \( x(v_1, B, \alpha(v_2)) = 0 \) under \( ND \), while

\[ \alpha(v_2, w) = 1 - \frac{x_0 + 1}{p_w(v_2 + \beta)} < 1 - \frac{1}{p_w\beta} \]

and hence \( x(v_1, w, \alpha(v_2, w)) = 1 \) for any \( w \in \{G, B\} \) under \( D \). Moreover, we have the following relationship:

\[
0 = q\pi(v_2, G, \alpha(v_2), x = 1) + (1 - q)\pi(v_2, B, \alpha(v_2), x = 0)
\]
\[
= q\pi(v_2, G, \alpha(G, v_2), x = 1) + (1 - q)\pi(v_2, B, \alpha(B, v_2), x = 1)
\]
\[
> q\pi(v_2, G, \alpha(v_2), x = 1) + (1 - q)\pi(v_2, B, \alpha(v_2), x = 1),
\]

where the equalities hold by the break-even conditions of \( \alpha(v_2) \), \( \alpha(v_2, G) \), and \( \alpha(v_2, B) \), and the inequality holds by \( x(v_2, B, \alpha(v_2)) \neq 1 \) for \( v_2 > \frac{(p_Bx_0 + q\Delta p)\hat{p}}{p} \). By using the definition of \( \pi(v, w, \alpha, x) \), the above inequality implies that

\[
qp_G\alpha(v_2, G) + (1 - q)p_B\alpha(v_2, B) < \hat{p}\alpha(v_2).
\]
The interim expected revenue under ND minus the interim expected revenue under D is

\[
\hat{\alpha}(v_2) \left( v_1 + \frac{q p_G}{\hat{p}} \beta \right) - \left[ q p_G \alpha(v_2, G) + (1 - q) p_B \alpha(v_2, B) \right] (v_1 + \beta)
\]

\[= \hat{\rho} \left( 1 - \frac{x_0 + q}{\hat{p} v_2 + q p_G \beta} \right) \left( v_1 + \frac{q p_G}{\hat{p}} \beta \right) - \left[ q p_G \left( 1 - \frac{x_0 + 1}{p_w (v_2 + \beta)} \right) + (1 - q) p_B \left( 1 - \frac{x_0 + 1}{p_w (v_2 + \beta)} \right) \right] (v_1 + \beta)
\]

\[= \hat{\rho} \left( 1 - \frac{x_0 + q}{\hat{p} v_2 + q p_G \beta} \right) \left( v_1 + \frac{q p_G}{\hat{p}} \beta \right) - \left( \hat{\rho} - \frac{x_0 + 1}{v_2 + \beta} \right) (v_1 + \beta)
\]

\[= - (1 - q) (p_B \beta - 1) + (1 - q) (v_1 - v_2) \frac{\hat{p} v_2 - (p_B x_0 - q \Delta p) \beta}{v_2 + \beta} \left( \frac{\hat{p} v_2 + q p_G \beta}{v_2 + \beta} \right),
\]

which is positive if and only if

\[v_1 - v_2 \geq \frac{(p_B \beta - 1) (v_2 + \beta) (\hat{p} v_2 + q p_G \beta)}{\hat{p} v_2 - (p_B x_0 - q \Delta p) \beta}
\]

since \(\hat{p} v_2 > (p_B x_0 - q \Delta p) \beta\).

In Case 4, by Proposition 6

\[\alpha(v_2) = 1 - \frac{x_0 + 1}{\hat{p} (v_2 + \beta)} < 1 - \frac{1}{p_B \beta}
\]

and hence \(x(v_1, w, \alpha(v_2)) = 1\) for any \(w \in \{G, B\}\) under ND, while

\[\alpha(v_2, w) = 1 - \frac{x_0 + 1}{p_w (v_2 + \beta)} < 1 - \frac{1}{p_w \beta}
\]

and hence \(x(v_1, w, \alpha(v_2, w)) = 1\) for any \(w \in \{G, B\}\) under D. This and the fact that \(\alpha(v_2)\)
and \((\alpha(v_2, G), \alpha(v_2, B))\) are the equilibrium strategies imply that

\[0 = q \pi(v_2, G, \alpha(v_2, G), x = 1) + (1 - q) \pi(v_2, B, \alpha(v_2, B), x = 1)
\]

\[= q \pi(v_2, G, \alpha(v_2), x = 1) + (1 - q) \pi(v_2, B, \alpha(v_2), x = 1).
\]
By using the definition of $\pi(v,w,\alpha,x)$, this implies that

$$qp_{G\alpha}(v_2,G) + (1 - q) p_{B\alpha}(v_2,B) = \hat{p}\alpha(v_2).$$

Thus, the interim expected revenue under $D$ is

$$[qp_{G\alpha}(v_2,G) + (1 - q) p_{B\alpha}(v_2,B)](v_1 + \beta) = \hat{p}\alpha(v_2)(v_1 + \beta),$$

which is exactly the interim expected revenue under $ND$. ■
Chapter 3

Moral Hazard and Adverse Selection in the Securitization Process.

3.1 Introduction

Lax lending standards and excessive risk-taking are regarded as the main culprits for the 2007-2008 financial crisis (e.g., see Sewell (2011)). Coval et al. (2009a) found evidence that many structured finance products were overpriced relative to their quality from 2004 to 2007. This may have occurred from errors in evaluating the default probabilities and their systematic risks as explained in Coval et al. (2009b). However, anecdotal evidence suggests that many sellers of structured finance products continued to buy assets when they were aware of their poor quality. They did so because there was a demand for structured finance products that contained those assets.\footnote{For instance, Citigroup’s chief executive at the time commented “When the music stops, in terms of liquidity, things will be complicated. But as long as the music is playing, you’ve got to get up and dance. We’re still dancing” (Nakamoto and Wighton (2007)).} As the Financial Crisis Inquiry Report states:

Lenders made loans that they knew borrowers could not afford and that could cause massive losses to investors in mortgage securities. As early as
September 2004, Countrywide executives recognized that many of the loans they were originating could result in ‘catastrophic consequences.’ Less than a year later, they noted that certain high-risk loans they were making could result not only in foreclosures but also in ‘financial and reputational catastrophe’ for the firm. But they did not stop (United States. Financial Crisis Inquiry Commission (2011), p. xxii).

Among the key features of securitizing structured finance products is that issuers have superior information about the quality of securities they sell and the quality is endogenously chosen by the issuer. Moreover, the repackaging of underlying assets can compound the adverse selection problem by creating more information asymmetry between the issuer and its investors.²

Extensive studies examine different factors that foster lax lending practices and excessive risk-taking during securitization.³ A great deal of papers examine how managerial compensation structures influence risk-taking (e.g., see John and John (1993), Chen et al. (2006), Coles et al. (2006), Bai and Elyasiani (2013), Srivastav et al. (2014), and Brown et al. (2015)). This study provides an alternative and complementary explanation to previous papers.

Our goal is to demonstrate how the endogenous quality choice by an issuer who faces moral hazard affects adverse selection and how additional information asymmetry arising from repackaging affects adverse selection in the securitization process.

Specifically, we consider a situation in which an issuer can buy either low-value assets at zero cost or medium-value assets at cost $c$. After buying the low-value assets, it can issue low-value securities. Similarly, after buying the medium-value assets, it can

²Since most information about the underlying assets in many structured finance products is publicly available, one might think sophisticated investors can value the repackaged securities properly. However, Bernardo and Cornell (1997) find wide variations in valuations by sophisticated investors in auctions of collateralized mortgage obligations, and attribute them to asymmetric information.

³When we refer to “excessive” risk-taking, we mean that the NPV of the activity is insufficient to compensate fully for risk.
issue medium-value securities. Moreover, after buying the medium-value assets, the issuer can repackage at cost $\epsilon > 0$ to make two types of securities: measure $\gamma \in (0, 1)$ of high-value securities and measure $(1 - \gamma)$ of the low-value securities. Buyers do not observe the issuer’s choices. We restrict our attention to a case wherein always buying the medium-value assets and issuing the medium-value securities without repackaging is first-best, and repackaging the medium-value assets to make two types of securities lowers expected values to the issuer and buyers. We assume all parties are risk neutral. We show that the moral hazard prevents the first-best outcome from being realized in equilibrium. Furthermore, we show that the issuer purchases medium-value assets and repackages them into separate securities with a positive probability, and under some conditions, making the repackaging available as the issuer’s choice increases its equilibrium payoff. Repackaging enhances informational advantage about the quality of the securities being sold.\footnote{Our result re-emphasizes the importance of modeling dynamics in economic situations. It is tempting to discard choices that are inferior in terms of expected values whenever we consider risk-neutral agents; however, such choices can be relevant in decision-making and can be selected with a positive probability in equilibrium.} Thus, the issuer may rather repackage assets and sell them separately than pass through assets even when it must incur a socially wasteful cost.

\textbf{Kawai (2015) and Kawai (2014)} are most closely related to this study. They consider the seller’s endogenous quality choice in dynamic models, and we present a static version of those models. However, we allow sellers to repackage assets into separate securities in the securitization process. In another closely related paper, DeMarzo (2005) considers a situation wherein an issuer has superior information about the value of its assets, and shows that pooling and tranching are preferable to pure-pooling when systematic risk is sufficiently small. However, he does not consider endogenous quality decisions by issuers.

This study proceeds as follows. Section 3.2 presents the model. Section 3.3 derives equilibrium behaviors and discusses their properties. Section 3.4 concludes the study. Most proofs are delegated to the Appendix.
3.2 Model

We model the securitization problem with moral hazard and adverse selection. A risk-neutral issuer (an intermediary) faces a continuum of identical risk-neutral buyers (investors) of measure 1, each of whom is to buy one unit of the security. In our two stage model, in stage 2, without observing the issuer’s stage 1 choices that determine the quality of the security being sold, each buyer makes a take-it-or-leave-it price offer $m$ that the issuer can accept or reject.

In stage 1, the issuer selects one of three alternatives $\{N, S, R\}$ that determine the quality of the security to issue. The issuer can buy either medium-value assets at cost $c$ or low-value assets at zero cost. The issuer can subsequently issue medium-value securities from medium-value assets and low-value securities from low-value assets.

Choice $N$ signifies that the issuer buys low-value assets and issues securities with a low value. If $N$ is chosen, the value of the securities issued is $v_i^L = 0$ to the issuer and $v_b^L > 0$ to the buyers. Choice $S$ signifies that the issuer buys medium-value assets and issues securities with a medium value. If $S$ is chosen, the value of the security is $v_i^M$ to the issuer and $v_b^M$ to the buyers, where $v_b^L < v_b^M$ and $0 < c < v_i^M < v_b^M$.

Alternatively, after buying medium-value assets, the issuer can repackage at cost $\epsilon > 0$ to make two types of securities: measure $\gamma \in (0, 1)$ of high-value securities and measure $(1 - \gamma)$ of low-value securities. This reflects a situation wherein the issuer has and can use superior information regarding the individual qualities of the medium-value assets. Choice $R$ signifies that the issuer buys medium-value assets and repackages them, making high- and low-value securities. The high-value securities yield $v_i^H$ to the issuer and $v_b^H$ to the buyers with $v_i^H < v_b^H$, whereas the low-value securities are the same as before, that is, $v_i^L = 0$ to the issuer and $v_b^L > 0$ to the buyers. Note that buyers always value securities higher than the issuer regardless of the issuer’s choice.

We impose the following parametric assumption:
Assumption 1: (i) $\gamma v^i_H = v^i_{M^r}$, (ii) $\gamma v^i_H + (1 - \gamma) v^i_L \leq v^i_M$, and (iii) $v^i_M - v^i_M - c > v^i_L$.

Condition (i) implies that the expected combined net value of the high- and low-value securities to the issuer from choosing $R$ is smaller than from choosing $S$: $\gamma v^i_H - c - \epsilon < v^i_M - c$. This implies that the issuer prefers to choose $S$ with probability 1 when retaining a security. Condition (ii) states that the expected combined value of the securities to the buyers for $R$ is no greater than that for $S$. Conditions (i) and (ii) together imply that the expected potential gain from trade from $R$ is strictly smaller than that from $S$. In this sense, choice $R$ is inferior to choice $S$ and may seem “irrelevant.” Also, condition (iii) states that the potential gain from trade from choosing $S$ is greater than that from choosing $N$. Thus, conditions (i)–(iii) together guarantee that the efficient outcome is that the issuer always chooses $S$ and a trade always occurs.

Let $q_N$, $q_S$, and $q_R$ be the probabilities that the issuer chooses $N$, $S$, and $R$, respectively. Since each buyer makes a single take-it-or-leave-it offer and possible values to the issuer are $\{0, v^i_M, v^i_H\}$, the buyer never offers $m \notin \{0, v^i_M, v^i_H\}$. So it is sufficient to consider only $m \in \{0, v^i_M, v^i_H\}$. Let $p_H$, $p_M$, and $p_L$ be the fractions of buyers that offer $m = v^i_H$, $m = v^i_M$, and $m = 0$, respectively, where $p_H, p_M, p_L \geq 0$ and $p_H + p_M + p_L = 1$.

Given $(p_H, p_M)$, the issuer’s expected payoffs by choosing $N$, $S$, and $R$ are

$$\pi^i (N; p_H, p_M) = p_H v^i_H + p_M v^i_M, \quad (3.1)$$

$$\pi^i (S; p_H, p_M) = p_H v^i_H + (1 - p_H) v^i_M - c, \quad (3.2)$$

and

$$\pi^i (R; p_H, p_M) = \gamma v^i_H + (1 - \gamma) p_H v^i_H + (1 - \gamma) p_M v^i_M - c - \epsilon$$

$$= p_H v^i_H + (1 - p_H) v^i_M + \frac{(v^i_H - v^i_M) v^i_M}{v^i_H} p_M - c - \epsilon, \quad (3.3)$$

respectively. Also, given $(q_S, q_N, q_R)$, each buyer’s expected payoffs when he offers $m =
$v^i_M, v^i_H$ are

$$
\pi^b (0; q_S, q_N, q_R) = (q_R (1 - \gamma) + q_N) v^b_L
\]

$$
= \left( q_R \frac{v^i_H - v^i_M}{v^i_H} + q_N \right) v^b_L, 
$$

(3.4)

$$
\pi^b (v^i_M; q_S, q_N, q_R) = q_S v^b_M - (1 - q_R \gamma) v^i_M + (q_R (1 - \gamma) + q_N) v^b_L
\]

$$
= q_S v^b_M + q_R \frac{v^i_M^2}{v^i_H} - v^i_M + \left( q_R \frac{v^i_H - v^i_M}{v^i_H} + q_N \right) v^b_L, 
$$

(3.5)

and

$$
\pi^b (v^i_H; q_S, q_N, q_R) = q_R \gamma v^b_H + q_S v^b_M - v^i_H + (q_R (1 - \gamma) + q_N) v^b_L
\]

$$
= q_R \frac{v^i_M v^i_H}{v^i_H} + q_S v^b_M - v^i_H + \left( q_R \frac{v^i_H - v^i_M}{v^i_H} + q_N \right) v^b_L, 
$$

(3.6)

respectively.

We first consider a situation wherein choice $R$ is excessively costly.

**Proposition 11** Suppose $\epsilon \geq \left( v^i_H - v^i_M \right) \left( v^i_M - c \right) / v^i_H$. Then, a unique equilibrium exists. In equilibrium, (i) the issuer chooses $R$ with probability zero, $S$ with probability $q_S = \frac{v^i_M}{v^b_M}$, and $N$ with probability $1 - q_S$, and (ii) fraction $p_M = \left( v^i_M - c \right) / v^i_M$ of buyers offer $v^i_M$, and fraction $1 - p_M$ offer 0. Equilibrium payoffs to the issuer and the buyers are $v^i_M - c$ and $\left( v^b_M - v^i_M \right) v^b_L / v^b_M$, respectively.

**Proof.** Suppose there exists an equilibrium in which $q_R = 0$. In such an equilibrium $q_S \in (0, 1), p_M \in (0, 1), and p_H = 0$.

By (3.4) and (3.5), the buyer is indifferent between offering 0 and $v^i_M$ if and only if $q_S = \frac{v^i_M}{v^b_M}$ when $q_R = 0$. By (3.1) and (3.2), the issuer is indifferent between $N$ and $S$ if and only if $p_M = \left( v^i_M - c \right) / v^i_M$ when $p_H = 0$.

To see that this actually constitutes an equilibrium, we need to check if the issuer wants to choose $q_R = 0$ given $p_M = \left( v^i_M - c \right) / v^i_M$ and $p_H = 0$. Then by (3.3), the ex-ante
equilibrium payoff from choosing $R$ is
\[
v^i_M + \frac{(v^i_H - v^i_M)(v^i_M - c)}{v^i_H} - c - \varepsilon.
\]

Therefore, the issuer chooses $q_R = 0$ if and only if
\[
v^i_M - c \geq v^i_M + \frac{(v^i_H - v^i_M)(v^i_M - c)}{v^i_H} - c - \varepsilon \quad \text{or} \quad \varepsilon \geq \left(\frac{v^i_H - v^i_M}{v^i_M}\right)(v^i_M - c).
\]

Due to asymmetric information, moral hazard prevents the issuer from choosing first-best choice $S$ with probability 1. Note that when $\varepsilon \geq (v^i_H - v^i_M) \times (v^i_M - c) / v^i_H$ holds, $R$ is too costly for the issuer to choose, and the situation is essentially the same when choice $R$ is unavailable to the issuer. Hereafter, we impose the following assumption to investigate the effect of choice $R$:

**Assumption 2:** Choice $R$ is more costly than $N$ but not excessively:
\[
0 < \varepsilon < \frac{v^i_H - v^i_M(v^i_M - c)}{v^i_H}.
\]

### 3.3 Equilibrium

Define the following function:
\[
v \left(v^i_M, v^i_H, v^b_M, v^b_H\right) \equiv \left(v^b_Mv^i_H - v^b_M\right) \left(v^i_H - v^i_M\right) - v^i_M \left(v^b_H - v^i_M\right) \left(v^b_M - v^i_M\right).
\]

For the case in which choice $R$ is more costly than $N$ but not excessively, we obtain the following proposition.

**Proposition 12** 1. Suppose $v \left(v^i_M, v^i_H, v^b_M, v^b_H\right) > 0$. Then, the unique equilibrium strategies
are

\[ (q_N, q_S, q_R) = \left( 0, 1 - q_R, \frac{v_H^i \left( v_M^b - v_M^i \right)}{v_M^b \nu_H^i - \nu_M^2} \right), \]

and \( (p_L, p_M, p_H) = \left( 1 - p_M, \frac{\nu_H^i}{\nu_H^i - \nu_M^i}, 0 \right) \).

The equilibrium expected payoff is \( v_M^i - c \) to the issuer and \( (v_M^b - v_M^i) \left( v_H^i - v_M^i \right) \times \frac{\nu_H^i}{v_M^b v_M^i - v_M^2} \) to the buyers.

2. Suppose \( v \left( v_M^i, v_H^i, v_M^b, v_H^b \right) = 0 \). Then, the unique equilibrium strategies are

\[ (q_N, q_S, q_R) = \left( 0, 1 - q_R, \frac{v_H^i \left( v_M^b - v_M^i \right)}{v_M^b \nu_H^i - \nu_M^2} \right), \]

and \( (p_L, p_M, p_H) = \left( 1 - p_M - p_H^*, \frac{\nu_H^i}{\nu_H^i - \nu_M^i}, p_H^* \right), \)

where \( p_H^* \leq \frac{(v_H^i - v_M^i) \left( v_M^i - c \right) - \nu_H^i}{(v_H^i - \nu_M^i) v_M^i} \).

The equilibrium expected payoff is \( p_H^* \nu_H^i + (1 - p_H^*) \nu_M^i - c \) to the issuer and \( (v_M^b - v_M^i) \left( v_H^i - v_M^i \right) \times \frac{\nu_H^i}{v_M^b v_M^i - v_M^2} \) to the buyers.

3. Suppose \( v \left( v_M^i, v_H^i, v_M^b, v_H^b \right) < 0 \). Then, the unique equilibrium strategies are

\[ (q_N, q_S, q_R) = \left( 1 - q_S - q_R, \frac{v_M^i \left( v_H^b - v_H^i \right)}{v_M^b \left( v_H^b - v_H^i \right)}, \frac{v_H^i \left( v_H^b - v_H^i \right)}{v_M^b \left( v_H^b - v_H^i \right)} \right), \]

and \( (p_L, p_M, p_H) = \left( \frac{c}{v_M^i}, \frac{\nu_H^i}{\nu_H^i - \nu_M^i}, \frac{(v_H^b - v_H^i) \left( v_M^i - c \right) - \nu_H^i}{(v_H^i - \nu_M^i) v_M^i} \right). \)

The equilibrium expected payoff is \( \frac{v_H^i \left( v_M^i - c - \epsilon \right)}{v_M^i} \) to the issuer and \( (v_M^b - v_M^i) \left( v_H^i - v_M^i \right) \times \frac{\nu_H^i}{v_M^b v_M^i - v_M^2} \) to the buyers.

Under Assumption 2, choice R is always selected with a positive probability, although its expected NPV and potential gain from trade are smaller than for choice S. The issuer
can benefit from repackaging since it amplifies the issuer’s informational advantage about the quality of the security being issued. Thus, the issuer may rather repackage than pass through assets, despite having to pay an additional cost.

Let us compare Propositions 11 and 12. Adding $R$ to the issuer’s possible choices decreases the buyers’ expected payoffs from $(v^b_M - v^i_M) v^b_L / v^b_M$ to $(v^b_M - v^i_M) (v^i_H - v^i_M) \times v^b_L / (v^b_M v^i_H - v^i_M^2)$. Moreover, when $v(v^i_M, v^i_H, v^b_M, v^b_H) > 0$, adding $R$ does not change the issuer’s expected payoff. When $v(v^i_M, v^i_H, v^b_M, v^b_H) < 0$, adding $R$ increases the issuer’s expected payoff from $v^i_M - c$ to $v^i_H (v^i_M - c - \epsilon) / v^i_M$. Thus, under Assumption 2, when $v(v^i_M, v^i_H, v^b_M, v^b_H) > 0$, social surplus diminishes by making choice $R$ available. However, when $v(v^i_M, v^i_H, v^b_M, v^b_H) < 0$ and $v^b_L$ is sufficiently small, social surplus increases by making $R$ available since the increase in the issuer’s equilibrium payoff dominates the decrease in the buyers’ equilibrium payoffs.

### 3.4 Conclusion

This examination of securitization has considered issuers’ endogenous quality choice problem in which the quality of securities is unobservable to investors. Our results indicate that issuers succumb to moral hazard and buy low-quality assets even though doing so is not socially optimal. This finding reinforces previous claims that the sellers of structured finance products during the global financial crisis continued to buy assets when they were fully aware of their poor quality.

We also found that issuers repackage assets into separate securities to enjoy additional informational advantage about the quality of securities and, under certain conditions, receive a higher equilibrium payoff. Therefore, the issuer would rather repackage assets and sell them separately than pass through assets so long as the cost incurred in repackaging is not excessive. This advantage created by additional informational asymmetry can
partially explain the popularity of structured finance products.

3.5 Proofs of Chapter 3

To prove Proposition 12, we need the following two lemmas.

Lemma 13 Under Assumption 2, any equilibrium strategy must satisfy the following: (i) $q_R \in (0, 1)$ and $q_S > 0$, (ii) $p_L > 0$ and $p_M > 0$, and (iii) $q_N > 0 \Rightarrow p_H > 0$.

Proof. Proof of (i): We first show that there is no equilibrium in which $q_R = 1$. Suppose $q_R = 1$. Then $p_M = 0$. However, it follows from (3.2) and (3.3) that when $p_M = 0$,

$$\pi^i (R; p_H, p_M = 0) < \pi^i (S; p_H, p_M = 0),$$

which contradicts $q_R = 1$.

We have shown that $q_R = 0$ is not an equilibrium under Assumption 2 in the proof of Proposition 11. Therefore, $q_R \in (0, 1)$ in any equilibrium.

Next we prove $q_S > 0$. Suppose otherwise. Then $p_M = 0$. It then follows that $\pi^i (S; p_H, p_M = 0) > \pi^i (R; p_H, p_M = 0)$, which contradicts $q_R \in (0, 1)$.

Proof of (ii): Suppose $p_L = 0$. Then $p_M = 1 - p_H$ and it follows that

$$\pi^i (N; p_H, p_M = 1 - p_H) > \pi^i (S; p_H, p_M = 1 - p_H),$$

which contradicts $q_S > 0$. Next, if $p_M = 0$, then

$$\pi^i (S; p_H, p_M = 0) > \pi^i (R; p_H, p_M = 0),$$

which contradicts $q_R > 0$. 

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Proof of (iii): Suppose $p_H = 0$. By (i), $\pi^i (S; p_H, p_M) = \pi^i (R; p_H, p_M)$, or

$$p_M = \frac{\epsilon v_H^i}{(v_H^i - v_M^i) v_M^i} < \frac{v_M^i - c}{v_M^i},$$

where the inequality holds by Assumption 2. Therefore, the following inequality holds:

$$\pi^i (N; p_H = 0, p_M) - \pi^i (S; p_H = 0, p_M) = p_M v_M^i - (v_M^i - c) < 0,$$

hence $q_N = 0$. ■

**Lemma 14** Under Assumption 2, any equilibrium strategy must satisfy the following:

$$p_M = \frac{\epsilon v_H^i}{(v_H^i - v_M^i) v_M^i} \quad (3.7)$$

$$p_H \leq \frac{(v_H^i - v_M^i) (v_M^i - c) - \epsilon v_H^i}{(v_H^i - v_M^i) v_M^i} \quad (3.8)$$

where the equality holds if $q_N > 0$.

$$q_S = \frac{v_M^i}{v_M^i} \left(1 - q_R \frac{v_M^i}{v_H^i}\right) \quad (3.9)$$

$$q_R \leq \frac{v_H^i (v_H^i - v_M^i)}{v_M^i (v_H^i - v_M^i)} \quad (3.10)$$

where the equality holds if $p_H > 0$.

**Proof.** Since $q_R > 0$ and $q_S > 0$, we have

$$\pi^i (S; p_H, p_M) = \pi^i (R; p_H, p_M) \quad \text{and} \quad \pi^i (S; p_H, p_M) \geq \pi^i (N; p_H, p_M).$$

Rearranging gives (3.7) and (3.8).
Similarly, it follows by \( p_M > 0 \) and \( p_L > 0 \) that

\[
\pi^b (0; q_S, q_N, q_R) = \pi^b (v^i_M; q_S, q_N, q_R) \quad \text{and} \quad \pi^b (0; q_S, q_N, q_R) \geq \pi^b (v^i_H; q_S, q_N, q_R),
\]

which implies that (3.9) and (3.10). \( \square \)

Now we are ready to prove Proposition 12. Let us first show that the stated set of strategies constitutes an equilibrium. Suppose \( p_M = \) \( \epsilon \) \( v^i_H - v^i_M \). Then, from (3.1)–(3.3) it follows that

\[
\pi^i (S; p_H, p_M) = \pi^i (R; p_H, p_M) \geq \pi^i (N; p_H, p_M),
\]

where the inequality holds if and only if \( p_H < \frac{(v^i_M - v^i_H)(v^i_M - c) - \epsilon v^i_H}{(v^i_H - v^i_M)v^i_M} \) and the equality holds if and only if \( p_H = \frac{(v^i_M - v^i_H)(v^i_M - c) - \epsilon v^i_H}{(v^i_H - v^i_M)v^i_M} \).

Suppose \( (q_N, q_S, q_R) = (0, 1 - q_R, \frac{v^i_H(v^b_M - v^i_M)}{v^b_M(v^i_H - v^i_M)}) \). Then, from (3.4)–(3.6) it follows that

\[
\pi^b (0; q_S, q_N, q_R) = \pi^b (v^i_M; q_S, q_N, q_R) \geq \pi^b (v^i_H; q_S, q_N, q_R),
\]

where the equality holds if and only if \( v (v^i_M, v^i_H, v^b_M, v^b_H) = 0 \) and the inequality holds if and only if \( v (v^i_M, v^i_H, v^b_M, v^b_H) > 0 \). Moreover, suppose

\[
(q_N, q_S, q_R) = \left( 1 - q_S - q_R, \frac{v^i_M(v^b_H - v^i_H)}{v^b_M(v^i_H - v^i_M)}, \frac{v^i_H(v^b_H - v^i_H)}{v^b_M(v^i_H - v^i_M)} \right).
\]

Then, from (3.4)–(3.6) it follows that

\[
\pi^b (0; q_S, q_N, q_R) = \pi^b (v^i_M; q_S, q_N, q_R) = \pi^b (v^i_H; q_S, q_N, q_R).
\]

Thus, the stated set of strategies indeed constitutes an equilibrium.
Next, let us show the uniqueness. By the above series of lemmas, the only possible equilibrium strategies are:

1. $p_M, p_L, q_R, q_S \in (0, 1)$ and $p_H = q_N = 0$.

2. $p_H, p_M, p_L, q_R, q_S \in (0, 1)$ and $q_N = 0$.

3. $p_H, p_M, p_L, q_N, q_R, q_S \in (0, 1)$.

Strategies (1) and (2): Suppose $q_N = 0$. By $1 = q_S + q_R$ and (3.9), it follows

$$q_R = \frac{v^i_H (v^b_M - v^i_M)}{v^b_M v^i_H - v^2_M}.$$ 

Then, (3.10) can be written as

$$\frac{v^i_M (v^i_H - v^i_M)}{v^i_M (v^b_H - v^i_M)} - \frac{v^i_H (v^b_M - v^i_M)}{v^b_M v^i_H - v^2_M} \geq 0$$

or

$$v \left( v^i_M, v^i_H, v^b_M, v^b_H \right) \geq 0,$$

where the equality holds if and only if $p_H > 0$.

Strategy (3): Suppose $p_H > 0$ and $q_N > 0$. Then, the equalities must hold in (3.8) and (3.10). It follows that

$$1 - q_R - q_S$$

$$= \frac{v^b_M v^i_M (v^b_H - v^i_H) - v^b_M v^i_H (v^i_H - v^i_M) - v^2_M (v^b_H - v^i_H)}{v^b_M v^i_M (v^b_H - v^i_M)}$$

$$> 0,$$

which implies $v \left( v^i_M, v^i_H, v^b_M, v^b_H \right) < 0$. Therefore, the stated set of strategies is indeed unique. $\blacksquare$
Bibliography


