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TRANSPORT PROCESSES OF PARTICLES IN DILUTE  
SUSPENSIONS IN TURBULENT WATER FLOW-PHASE I

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## ABSTRACT

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Modifications to an existing experimental system have been made and have been demonstrated to provide the required resolution and variable parameterization necessary for a detailed study of dilute particle suspensions in a turbulent water flow. These modifications together with the reasons for their necessity are discussed.

Linearization of non-Stokesian drag has been accomplished through the introduction of a diagonal tensor into the Stokes drag force equation. It was found that non-Stokesian effects tend to be of minor importance in the response of water borne particles.

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## INTRODUCTION

A knowledge of the motion of particles suspended in a turbulent fluid field is of great technological interest. Application of this to such topics as atmospheric dispersion of particulate from chimneys, radioactive fallout, fluidized beds and erosion of shorelines demonstrates the importance of understanding this physical phenomenon. Unfortunately, at the present time, there is no satisfactory description which is able to realistically incorporate the many complicated and interrelated phenomena inherent in the process. It is a well known fact that adequate analytical descriptions of single phase fluid turbulence is still lacking. It was with the hope of furthering the understanding of the motion of suspended particles in a turbulent fluid field that an initial study by Jones<sup>1\*</sup> was undertaken. This present study is an extension of his work.

Pertinent to the study of the motion of particles in a turbulent fluid field is the question of whether the presence of the particles affects the fluid turbulence. Such effects could result from a concentrated suspension of particles in which particle-particle interactions occur, from a dilute suspension of relatively large particles (large compared to the smallest characteristic dimension of the turbulent fluid) or from the combination of these affects. Such strongly coupled systems would not be suitable for initial studies of the motion of particles, although it is often in such systems that the technologically attractive

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\* Raised numbers indicate literature listed in the References.

applications are found. A more suitable system, from which fundamental knowledge of the behavior of a particle could be examined, would be one in which the detailed motion of a single (preferably small) particle could be studied. Although the use of single particles would reduce the number of interacting phenomena it would not necessarily guarantee that the particle would not affect the fluid turbulence. However, positive control of the particle's physical properties could conceivably provide suitable parameterization to resolve this interaction phenomenon.

Several analytical studies have been conducted on the motion of small particles in a fluid. The original work of Basset,<sup>2</sup> Boussinesq,<sup>3</sup> and Oseen,<sup>4</sup> on the slow motion of a solid sphere in a fluid at rest, has been extended to include the statistical nature of the particle motion in a stationary turbulent fluid. In its most general form, the Lagrangian equation describing the particle motion becomes a stochastic, nonlinear, integrodifferential equation. There has been little success in solving this equation.

Chao,<sup>5</sup> in studying this stochastic equation in its linearized form, succeeded in obtaining many useful relations for the behavior of the particle. However, Lumley<sup>6</sup> has indicated that linearization, which ignores the "essential" nonlinearity introduced by the appearance of the particle position as an argument of the fluid velocity, may not be meaningful unless the particle is much smaller than the smallest turbulent space-scale of the fluid and its response time is short compared to the shortest turbulent time-scale of the fluid. A conceptually simpler nonlinearity results when the Stokes Drag Law relation is violated. This occurs when high relative motion is present between the particle and its surrounding fluid. Kada and Hanratty,<sup>7</sup> in their study of suspensions

having non-Stokesian drag characteristics, revealed a statistical interaction which developed between the turbulent fluid and the suspended particles. The statistical characteristics of both phases were strongly affected. Recently it has been postulated by Shirazi and Chao<sup>8</sup> that such a statistical interaction may be present even when Stokes drag applies.

Csanady<sup>9</sup> and others in an attempt to describe particle motion in atmospheric situations where the particle's free fall velocity is appreciable have centered their attention on the particle's time autocorrelation coefficient. Their studies indicate that high particle free fall velocity, with the resulting passage of the particle through many different correlated fluid regions - the so-called crossing trajectories effect, causes a drastic reduction in particle correlation and hence reduced particle dispersion. It is thus apparent that considerable variation of opinion exists as to just how particles behave in a turbulent fluid flow.

Several investigators have experimentally examined turbulent fluid flows containing dilute and concentrated particle suspensions. The relative density difference of the fluid and particles has been shown to be of particular significance in all of these studies. It provides the bouyant force through the action of the gravitational field and is closely associated with accelerative forces. The kinematic viscosity of the fluid and the particle dimensions were also shown to be important. In many flow visualization studies small, neutrally bouyant, particles have been successfully employed to map complicated flow patterns in irregular flow geometries. The proceedings of an ASME symposium<sup>20</sup> on the subject includes a variety of applications to turbulent flows. The

general assumption in these applications was that the small particles behaved as small fluid elements and that they followed the flow patterns identically.

Specific studies have been made to determine whether the particles actually do behave as fluid elements. Batchelor, Binnie and Phillips<sup>10</sup> have utilized neutrally bouyant particles in turbulent flow in pipes to examine the nature of Eulerian and Lagrangian velocity averages. They found, by measuring the transit times of particles, that these averages were essentially the same. This conclusion has been interpreted as justifying the use of the equivalence of the Eulerian and Lagrangian averages in turbulent dispersion analyses. Lumley<sup>6</sup> has theoretically proven that their conclusions are justified within the restrictions of their experimental system. Kada and Hanratty<sup>7</sup> have examined the other extreme in which the particle and fluid densities are widely different. They observed that the particles strongly affect the fluid turbulence and that this coupled motion strongly affects the turbulent transport characteristics of the flow field.

Torobin and Gauvin<sup>11</sup> in a series of articles have reviewed and examined the behavior of dilute suspensions in turbulent flows. In a separate article<sup>12</sup> they have reported data on drag coefficients of single spheres moving in steady and accelerated motion in a turbulent fluid. Macroscopic observations in non-Stokesian flow revealed that the drag of the particles was influenced by a disruption of the wake and by a laminar-turbulent transition in the attached boundary layer. No detailed trajectories were measured.

Kennedy<sup>13</sup> and Snyder<sup>14</sup> have examined the dispersion of small particles in a decaying turbulent field. By studying the position and



time that individual particles passed fixed measuring stations, they were able to accumulate sufficient statistical information to evaluate diffusion coefficients, particle velocity autocorrelation functions, integral scales and dissipative scales from the data. However, they were unable to measure the detailed structure of the particle trajectories. Several related references are cited in their theses.

The current project is an extension of work reported in two earlier papers (see references 1 and 8) in which the experimental and analytical aspects of suspended particle motion in a turbulent fluid field were studied. In those studies the particle trajectory monitoring system was severely limited by electronic noise and by the large minimum size of the particles. Since both problems, as well as others, made the findings of these experiments somewhat uncertain, great emphasis in the current project was placed on their successful solution. Analytically, the non-Stokesian drag force expression was linearized through the use of a diagonal tensor whose components, to a first approximation, depend only on the particle's free fall velocity in a quiescent fluid. These experimental and analytical aspects are discussed in detail in the following sections.

## EXPERIMENTAL

In accordance with the objectives of this project, extensive modifications to the existing experimental facilities have been made so as to gain stationary fluid turbulence, successfully fabricate different types of particles, and improve the signal-to-noise level present in the particle tracking system. It is felt that a delineation of these problems together with their solution might prove of benefit to those engaged in similar research work.

Many modifications have been made to the experimental loop that Jones<sup>1</sup> and Shirazi<sup>8</sup> (see Fig. 1) used in their particle monitoring experiments. In the header tank additional wire screening has been placed around the Borda mouth with a view toward increasing flow uniformity into the inlet section. An aluminum test section was used to replace the clear cast acrylic resin tubing test section for fluid turbulence measurements. Five entry stations are equally spaced along its seventeen foot length permitting pitot tube and anemometer measurements to be made. These measurements showed that the fluid turbulence in the test section was not fully developed nor stationary; consequently a stainless steel rake was placed at the Borda mouth so as to "trip" the initial flow causing the more rapid development of stationary turbulence. Further it was found that at the exit from the test section a significant asymmetry existed due to the proximity of an elbow at the exit. A suitable honeycomb resistance was inserted successfully providing symmetry to the exit flow. That stationary, as well as symmetric, turbulence has been achieved throughout the test section can be seen in Fig. 2 where the mean velocity profiles at the test section entrance and exit (at a pipe Reynolds number of 50,000) are compared with Laufer's<sup>15</sup> results.

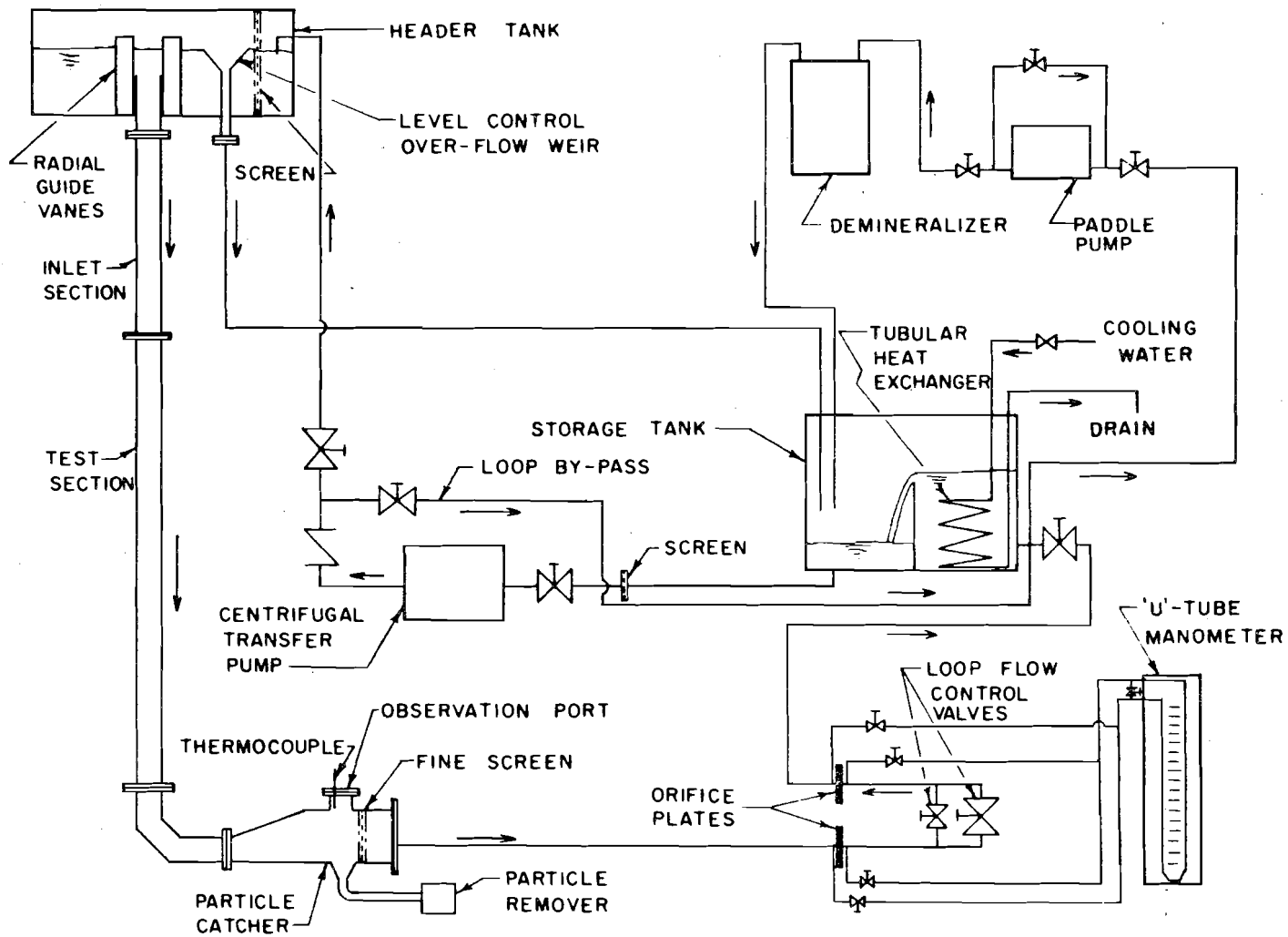


Fig. 1 Detailed Schematic of the Experimental Turbulence Loop

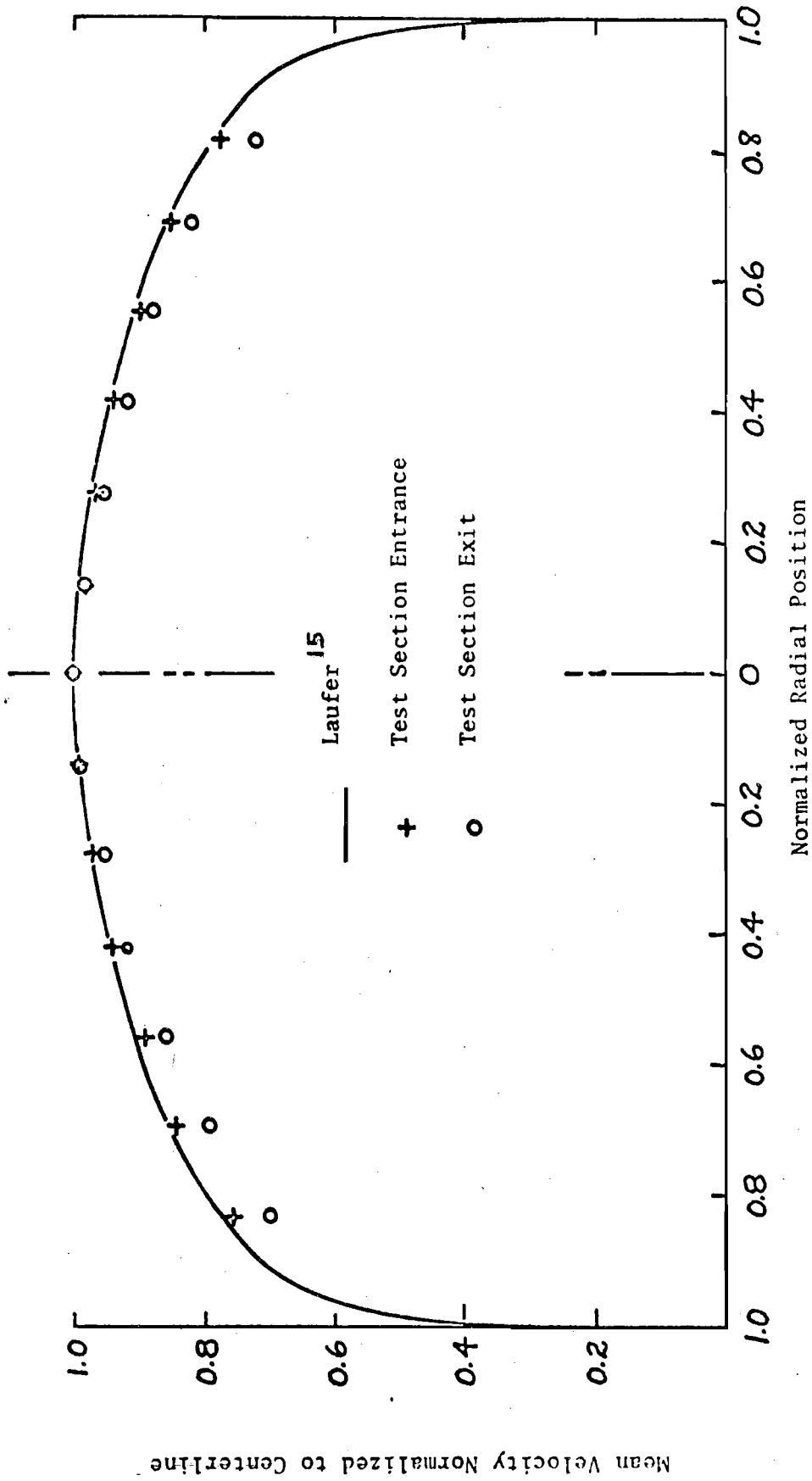


Fig. 2 Comparison of Mean Velocity Profiles at 50,000 Reynolds Number

The replacement of all brass and iron valves, as well as the cast iron pump, with stainless steel components has enabled a de-ionized water condition, which is necessary for successful anemometer operation, to be readily maintained at a  $10^{-6}$  mho-cm level.

At the particle catcher's entrance (see Fig. 1) a funnel screen has been placed to facilitate removal of the rather small, radioactive particles used in the experiment. In addition, fine mesh screening prevents any possible escape of the particles from the catcher should they somehow miss the funnel screen. A particle handling system enabling the removal of them from the particle catcher, the transporting of them up to the header tank, and their injection back into the fluid flow in less than a minute has been designed, built, and tested (see Fig. 3). Such a system allows minimum exposure time to the radioactive particle and thus maximum safety to the experimenters.

Finally, a new particle insertion device which allows the particle to be injected directly into the inlet section just below the Borda mouth through a small tube directed along the centerline was made. This device is mounted to a plate which is positioned immediately above the inlet section near the free fluid surface in the header tank. This new device eliminates the flow disturbances caused by the previous particle insertion device which was placed downstream of the inlet section.

One of the most crucial problems to be met and solved was that of particle choice and fabrication. To have the widest possible parameterization together with adequate signal intensity was the goal of this search for the optimum particle. Previous measurements by Jones and Shirazi were made with particles having radioactivity induced, constant intensity light emission. Since these had relatively weak

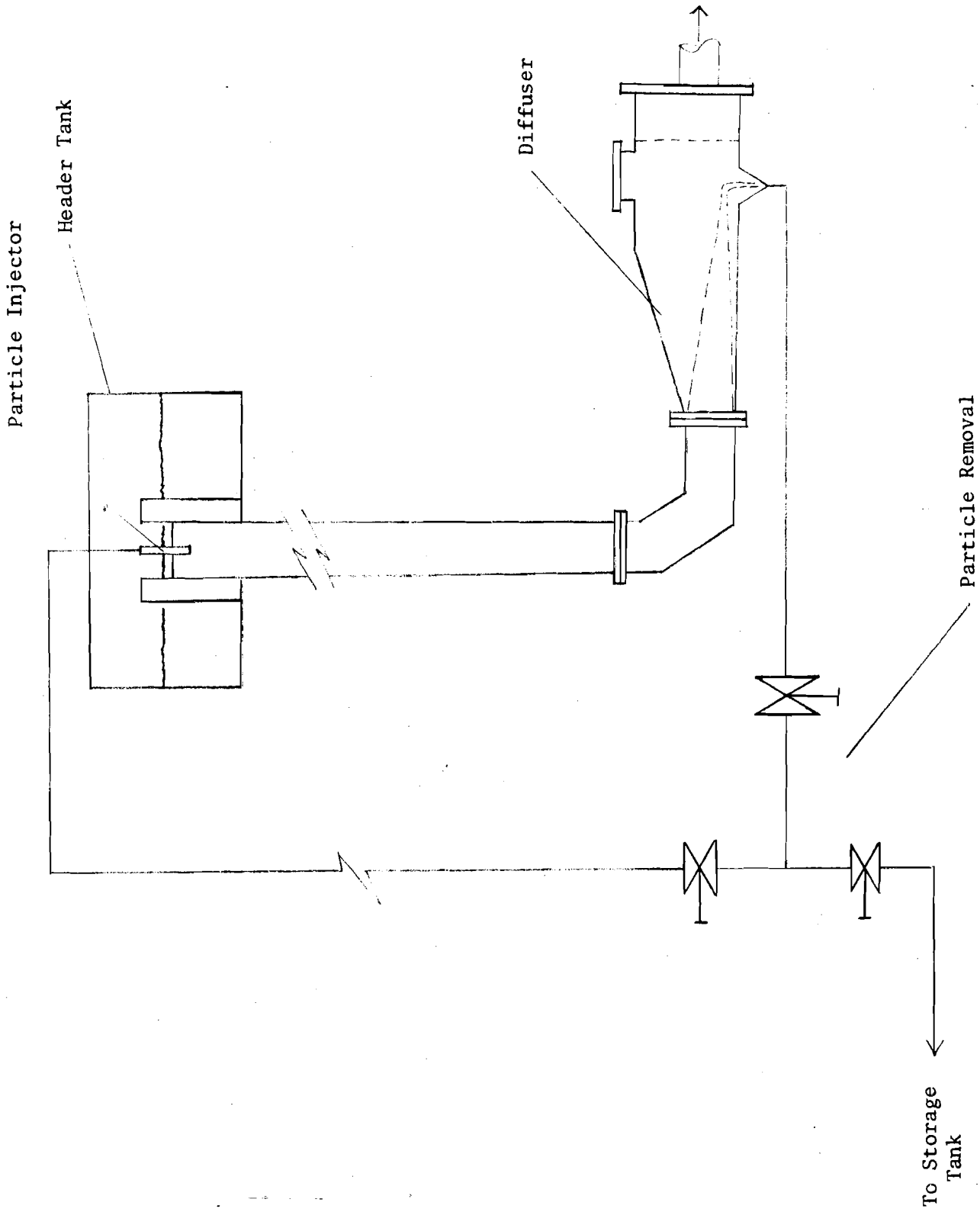


Fig. 3 Particle Injector-Removal System

intensity, this required the use of large particles (about 6 mm in diameter) which raised some doubt as to whether they were sensitive to all the characteristic turbulent fluid motion. Further, since the particles signal was emitted light, the entire monitoring equipment had to be kept in a light tight environment - a bothersome restriction. The weakness of the light intensity also resulted in a near intolerably low signal-to-noise ratio. Several alternatives to this radioactivity induced light signal were considered including the possible use of ultraviolet activation of fluorescently coated particles; however, upon consideration of the advantages of improved signal-to-noise characteristics and the possibility of smaller size it was decided that radioactive particles would be most suitable. Initially cesium 137 sources, having 10 millicuries actively, embedded in a one millimeter plastic microsphere were considered. This isotope provides a suitably long half-life for calibration stability and can be concentrated to give the needed compactness for small particle size. These microspheres were then encapsulated in expandable polystyrene, thus enabling size and density parameterization depending on mold size. Such encapsulation requires immersion of the mold in boiling water for half an hour and unfortunately some 60% of the total cesium 137 activity leaked from the mold into the hot water bath. Apparently the heat together with the radiation damage had broken down the plastic matrix in which the cesium 137 had previously been embedded.

A re-evaluation of possible radioactive particles was then undertaken and it was decided that the difficulty encountered in the use of cesium 137 could be avoided by using nickel plated cobalt 60 particles. These particles are cylindrical in shape with a diameter of .039 inches

and a height of .020 inches, their activity being 10 millicuries and their half-life adequately long. No difficulty was encountered in final particle fabrication using these cobalt particles and as such a general scheme for production of various size and density particles has been evolved. Three mold sizes 2mm, 3mm, and 5 mm in diameter are used. A radioactive cobalt cylinder together with polystyrene and a quantity of thin iron wire (to give the required final particle density) is then added to the desired mold. This mold, as previously noted, is placed in boiling water for approximately thirty minutes. After cooling to room temperature the particle is removed. The resulting particle is sprayed with several coatings of clear varnish. The particle then has a fixed size and density, with no possibility of water absorption during its periods of prolonged immersion. By such methods, particles with sufficient parameterization as well as signal strength may be obtained. Figure 4 shows a schematic of the fabricated particle.

Undoubtedly the most vexing problem of previous experiments has been that of poor signal-to-noise ratios. In Jones<sup>1</sup> work high frequency components, notably that of 60 Hz and 120 Hz, proved extremely troublesome; whereas Shirazi filtered his signals to such an extent that it is questionable whether valid high frequency contributions were retained. Consequently a great deal of effort has been spent attempting to optimize the signal-to-noise with an eye toward retaining true high frequency signals while filtering out the spurious high frequency noise.

Figure 5 shows the particle trajectory data acquisition system used by Jones. The first improvement on this system was the placement of a prefiltering RC network immediately before the differencing amplifiers. Tests of signal gain through the filter bank versus frequency showed



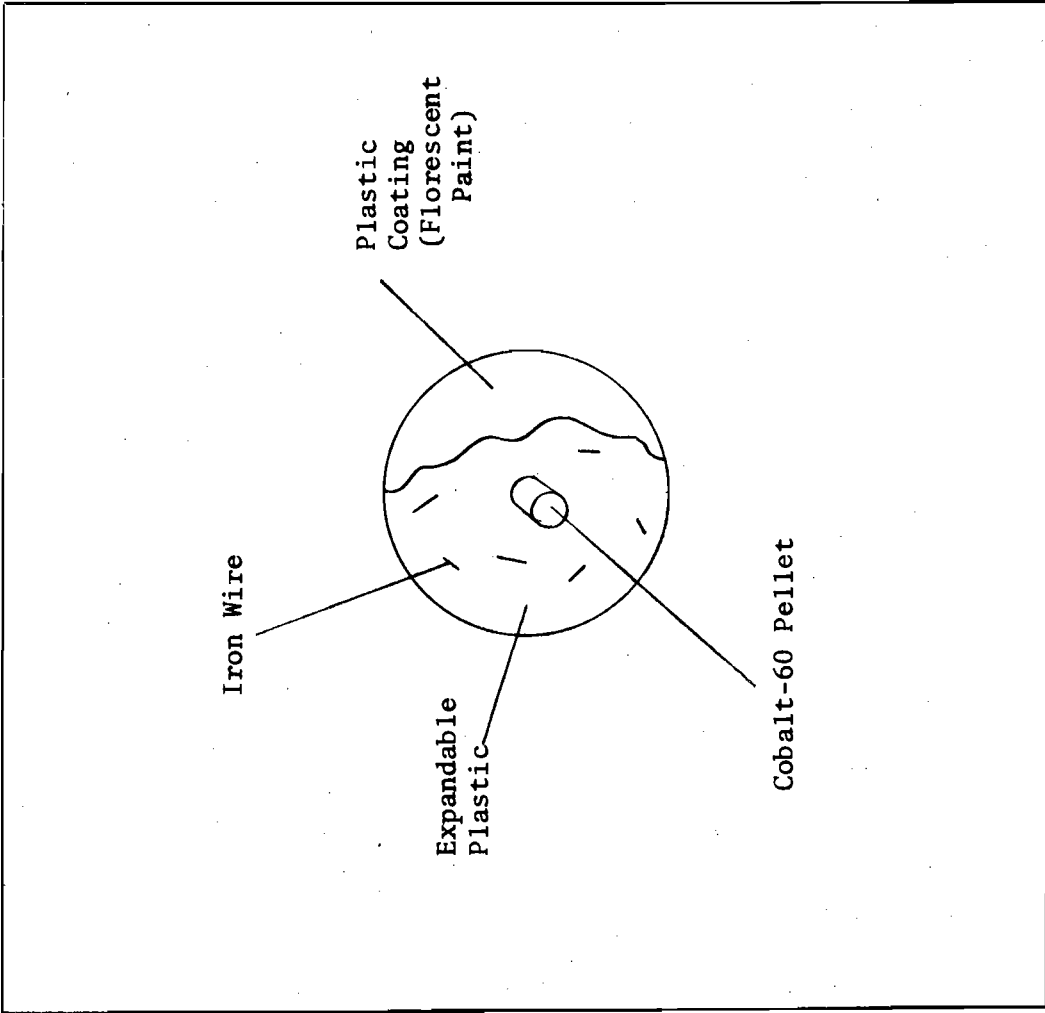


Fig. 4 Schematic of the Fabricated Particle

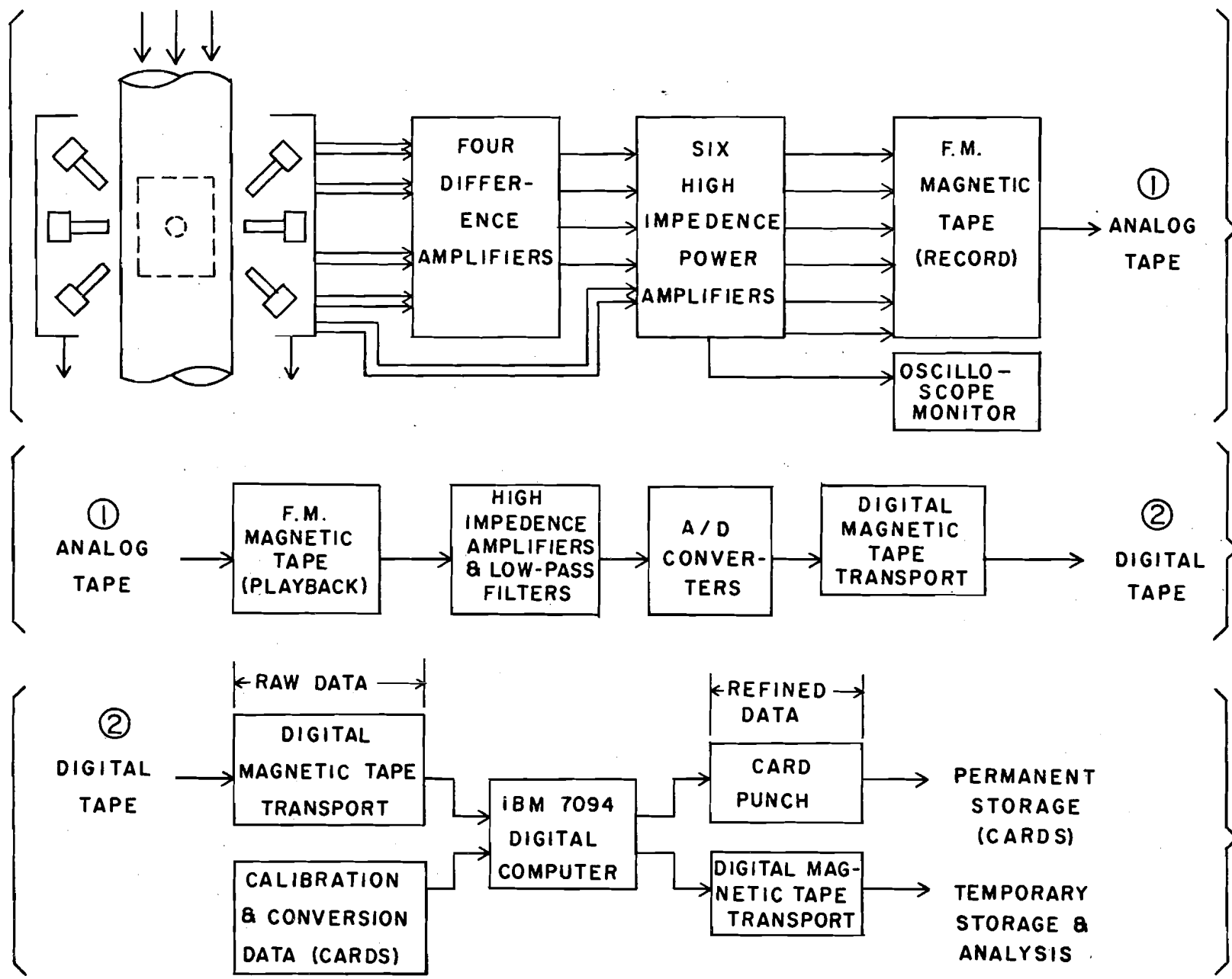


Fig. 5 Particle Trajectory Data Acquisition System

that for a combination of 40 K $\Omega$  resistor and .02  $\mu$ f capacitor no drop in gain occurred until a signal frequency of some 50 Hz was reached. Other filter combinations such as 100 K $\Omega$  and .03  $\mu$ f as well as 300 K $\Omega$  and .05  $\mu$ f showed considerable gain drop beyond a frequency 5 Hz. Since frequencies below ~30 Hz are of particular importance as all the significant fluid turbulent energy is contained in this range, these last two filter combinations are clearly unacceptable; while the 40 K $\Omega$  and 0.02  $\mu$ f combination allows virtually no signal loss and thus was chosen for signal prefiltering.

Four Bay Laboratories difference amplifiers having both suitable gain adjustment for calibration and balancing and sharp low pass filtering capability were procured for the system. Comparison of the four amplifiers showed that they were quite similar in their electronic characteristics especially with respect to phase shift. As the 100 Hz filter setting on these difference amplifiers did not produce a decline in signal output until signal frequency exceeded 30 Hz this was used to post filter the differenced signal.

One of the basic sources of noise in earlier work proved to be the photomultiplier tubes which provided the basic signal detection. Indeed it was found that noise was produced due to the physical motion of these detectors. The original tubes also were obtained from surplus and were difficult to keep balanced and calibrated. In addition they were not equipped with scintillation crystals, a necessity for gamma ray sensing. Therefore, new Harshaw Chemical Company photomultipliers were installed. These detectors utilize ruggedized RCA 6199 phototubes on which 1" x 1" cylindrical NaI(Tl) crystals are mounted, the whole assembly being hermetically sealed in aluminum.

Matching of tube pairs was accomplished using a 5 millicurie cesium 137 source. Noise was found to be less than 0.6 mv rms for all the phototubes. The signal-to-noise ratio for this particular source was 22.7 at a detector voltage of 900 volts. After the decision to use cobalt 60 had been taken, signal-to-noise was determined for two cobalt sources: 5 millicuries, which gave a ratio of 20.9 at a detector voltage of 850 volts; and 12 millicuries, whose signal-to-noise ratio at a detector voltage of 800 volts proved to be 39.1. Further, it was found that the power spectral density of the static cobalt 60 noise was nearly identical for both sources; thus the decision to use cobalt sources of 10 millicuries intensity.

Spectrum analysis fo the X-detector difference signal from the static cesium source is shown in Fig. 6 for various combinations of pre- and post-filtering. A typical turbulent water spectrum is observed to lie within the spectrum of the 40 K $\Omega$  and 0.02  $\mu$ f prefiltering and 100 Hz post-filtering, so that this combination of filters retains the important information of the water spectrum but eliminates the higher frequency noise.

As a test of the resolving power of the detector system as a whole, the 5 millicurie cesium particle was spun in a circular trajectory in the x-y plane at frequencies of 4.65, 9.19 and 19.91 Hz and the dynamic power spectra from the detectors observed. One such spectra is shown in Fig. 7. Peaks in the power spectra of the various signal differences occurred at 4.5, 10.0, and 21.5 Hz implying resolutions of 3.23%, 8.82% and 7.99% for these frequencies. As the particle's energy spectrum drops off rather sharply after 5 Hz, resolution in-accuracies of 3% to 6% may be expected.

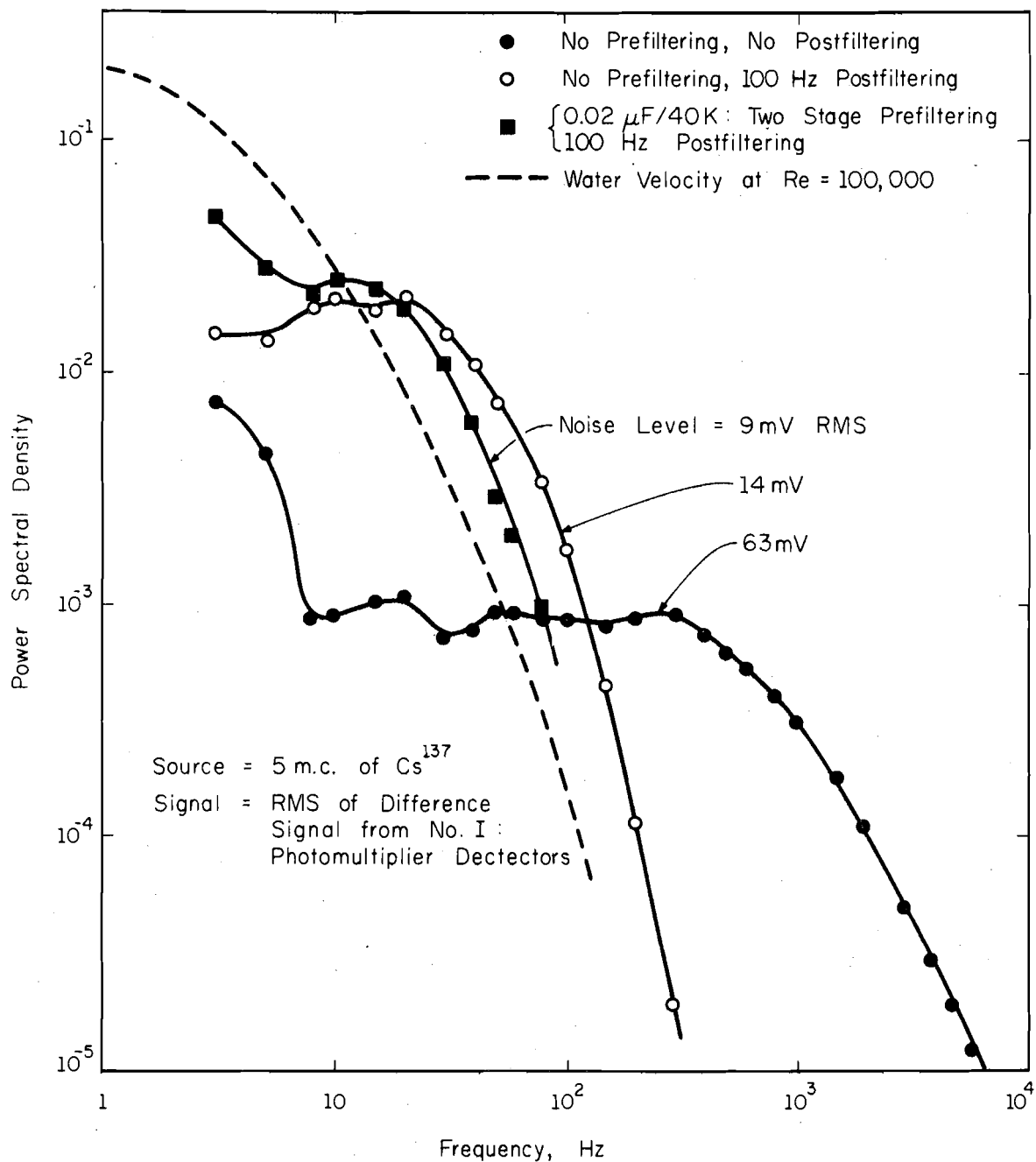


Fig. 6 Power Spectral Density of Static Particle Noise and of Water Velocity from a Turbulent Flow Field

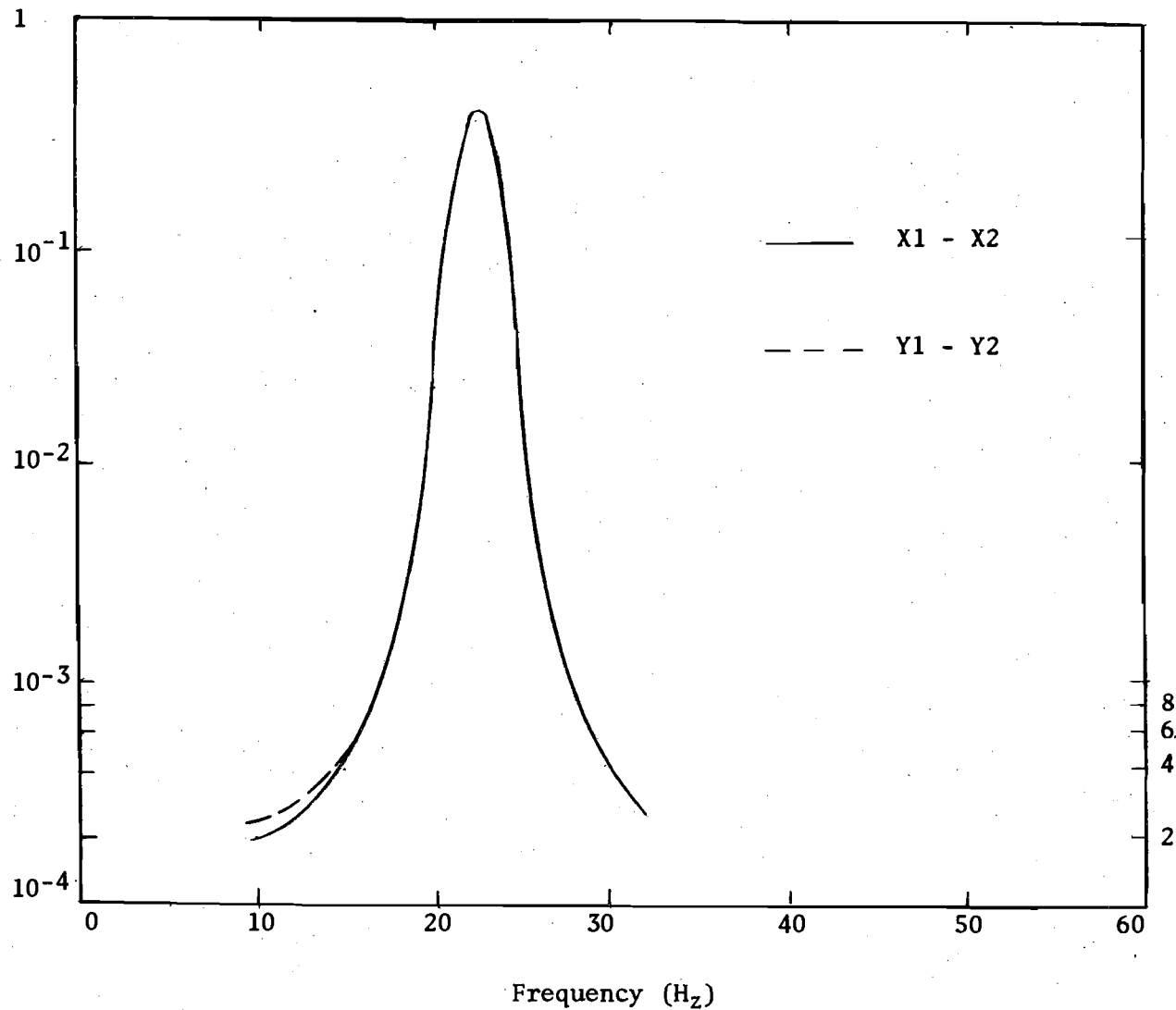


Fig. 7 Resolving Efficiency of the Detecting System

A further improvement on the basic monitoring apparatus has been the addition of a feedback mechanism which uses the differenced signal from one pair of the vertical (i.e. Z-direction) detectors to increase or decrease the monitoring carriage speed enabling it to accurately move along with the mean speed of the particle.

In conclusion, extensive modifications to the experimental system as a whole have been made so as to enable accurate and noise free particle trajectory data to be taken. Changes in the water flow loop have resulted in stationary fluid turbulence and the capacity to monitor the flow field at several positions along the test section length. Successful techniques allow the fabrication of particles with wide parameterization. Pre- and post-filtering of the detector signals has enabled noise to be drastically reduced. Thus, the experimental system has been improved to enable the desired fundamental data to be obtained with the necessary accuracy required to advance the study of particle transport in turbulent fluid flow.

In a companion study designed for detailed measurements of three-dimensional, fully developed, turbulent structure of the velocity field, Burchill<sup>16</sup> measured the Eulerian field of the central core region of pipe flow of water. This information is directly usable for comparison with the turbulent structure of the particle motion and to support the assumption that the flow field employed is essentially homogeneous and isotropic.

## ANALYTICAL

Analytically, emphasis has been directed toward a better understanding of non-Stokesian drag. One of the basic problems in an analytical description of particle motion in a fluid medium is the departure from linearity of the drag force acting on the particle as its Reynolds number is increased beyond unity, the Stokesian limit. This non-linear characteristic can be troublesome when it is included in the particle's dynamic equation of motion and in many cases is only approximately considered or, indeed, neglected entirely. Consequently it was felt that the linearization of this heretofore non-linear force term would be of benefit not only in simplifying the particle's equation of motion, but also in providing a better understanding of the manifestations of non-Stokesian drag. Since this development is to be submitted for publication in the near future (see ref. 17) only a brief outline will be presented herein.

To accomplish linearization of the non-linear drag force equation, a diagonal tensor

$$\delta = \begin{pmatrix} \delta_{xx} & 0 & 0 \\ 0 & \delta_{xx} & 0 \\ 0 & 0 & \delta_{xx} \end{pmatrix} \quad (1)$$

is introduced into the equation, which in tensor notation may be written as:

$$F_i = F_{s,i} \delta_{ij} + G_i \quad (2)$$

where  $F_{s,i}$  is the Stokesian force acting on the spherical particle due to the difference between the fluctuating fluid and fluctuating particle velocities and  $G_i$  is the drag force acting on the particle due to its



free fall. Recalling that the general relationship between the drag coefficient and drag force may be written as:

$$F_i = A K C_d \widehat{(u - u_p)}_i \quad (3)$$

where: A - characteristic area of the particle presented to the fluid flow (i.e. its cross sectional area)

K - characteristic kinetic energy per unit volume

$\widehat{(u - u_p)}_i$  - relative velocity unit vector in the  $i^{\text{th}}$  direction

$$= (u_i - u_{p,i}) / |u_i - u_{p,i}|$$

$u_i$  -  $i^{\text{th}}$  component of the fluid velocity

$u_{p,i}$  -  $i^{\text{th}}$  component of the particle's velocity

From the work of Ingebo<sup>18</sup> it is known that for the realm of particle free fall Reynolds number,  $Re$ , between 6 and 400 the particle's drag coefficient may be expressed as:

$$C_d = \frac{27}{Re^{0.84}} \quad (4)$$

Using this drag coefficient it is then possible to equate (2) and (3) resulting in a determination of the linearization parameters  $\delta_{ij}$ :

$$\delta_{xx} = \delta_{yy} = Re^{0.16} \left[ 1 + \frac{0.16 \bar{u}_{Rz}}{f} \right] \quad (5)$$

$$\delta_{zz} = Re^{0.16} \left[ 1.16 + \frac{0.106 u_{Rz}}{f} \right] \quad (6)$$

where:  $f$  - particle's free fall velocity

$u_{Rz}$  - relative fluctuating velocity between the fluid and the particle

In general, since the particle's free fall velocity will exceed the relative fluctuating velocity difference between the fluid and the particle we may take

$$\delta_{xx} = \delta_{yy} = \text{Re}^{0.16} \quad (7)$$

$$\delta_{zz} = 1.16 \text{Re}^{0.16} \quad (8)$$

Figure 8 shows the predicted values for these parameters. It is apparent that  $\delta_{ii}$  are limited to values below 3.4 and indeed have approximately the same values although, because gravity tends to provide a preferential particle velocity longitudinally in the vertical direction, slight differences between the longitudinal and transverse particle behavior are predicted.

Changes in the particle's response to fluid accelerations due to non-Stokesian drag may now be examined. Following Chao<sup>5</sup> it is possible to determine a particle response function

$$Q(\lambda, \beta, \delta_{ii}) = \frac{(u_i^1)^2 \Omega^{(1)}(\lambda, \delta_{ii})}{(u_{p,i}^1)^2 \Omega^{(2)}(\lambda, \beta, \delta_{ii})} \quad (9)$$

where:  $u_i^1$  - rms of the fluid's fluctuating velocity

$u_{p,i}^1$  - rms of the particle's fluctuating velocity

$$\lambda \equiv \omega/\alpha$$

$\omega$  - circular frequency of particle motion

$$\alpha \equiv \frac{3\nu}{a^2}$$

$a$  - particle radius

$$\beta \equiv \frac{3}{2 \frac{\rho}{\rho_p} + 1}$$

$\rho$  - fluid's density

$\rho_p$  - particle's density

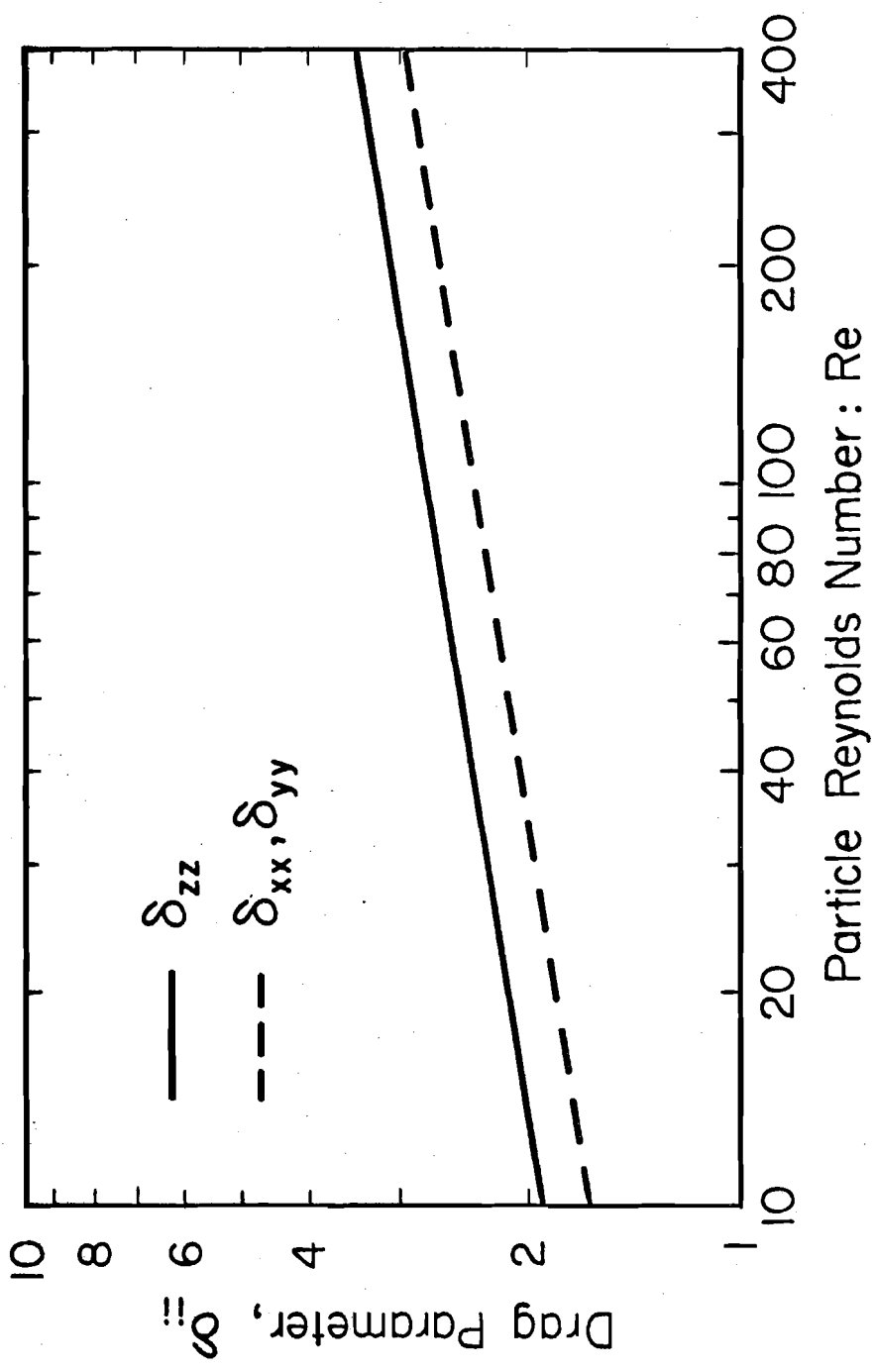


Fig. 8 The Effect of Particle Reynolds Number on the Drag Parameter Based on Ingebo's Drag Law for Longitudinal and Transverse Free Fall Directions

and the spectral energy ratio

$$\frac{\Omega^{(1)}(\lambda, \delta_{ii})}{\Omega^{(2)}(\lambda, \beta, \delta_{ii})} = \frac{\lambda^2 + \sqrt{6} \lambda^{3/2} + 3\lambda + \delta_{ii} \sqrt{6} \lambda^{1/2} + \delta_{ii}^2}{\left(\frac{\lambda}{\beta}\right)^2 + \frac{\sqrt{6}}{\beta} \lambda^{3/2} + 3\lambda + \delta_{ii} \sqrt{6} \lambda^{1/2} + \delta_{ii}^2} \quad (10)$$

Figure 9, 10, and 11 show how non-Stokesian effects (recall  $\delta_{ii} = 1.0$  for Stokesian behavior) affect this spectral energy ratio as particle density increases. From these figures it is evident that non-Stokesian effects tend to increase particle response at decreasing values of  $\lambda$  as particle density increases.

Of further interest is an examination of the difference between non-Stokesian and Stokesian spectral energy ratios

$$\Delta = \frac{\Omega^{(1)}(\lambda, \delta_{ii})}{\Omega^{(2)}(\lambda, \beta, \delta_{ii})} - \frac{\Omega^{(1)}(\lambda, 1)}{\Omega^{(2)}(\lambda, \beta, 1)} \quad (11)$$

Figure 12 shows this difference, for selected values of  $\beta$  and  $\delta$ , as a function of normalized frequency,  $\lambda$ . As can be seen the majority of the non-Stokesian contribution occurs within a definite frequency band. Indeed, as the particles become quite heavy in comparison to the fluid (i.e.  $\beta \rightarrow 0$ ) it may be shown that the maximum contribution to the spectral energy density due to non-Stokesian effects is:

$$\Delta_{\max} = \frac{(1-\beta^2)(\delta_{ii}-1)}{\delta_{ii}+1} \approx \frac{\delta_{ii}-1}{\delta_{ii}+1} \quad (12)$$

which occurs at a normalized frequency of

$$\lambda_{\Delta_{\max}} = \beta \delta_{ii}^{1/2} \quad (13)$$

The behavior of particles of interest to pollution studies has been examined. In Fig. 13, carbon in air, with a typical wind tunnel energy spectrum shown for comparison, is considered. Since the non-Stokesian effects occur in the energy containing region of the turbulence

Spectral Energy Ratio  $\Omega^{(1)}/\Omega^{(2)}$

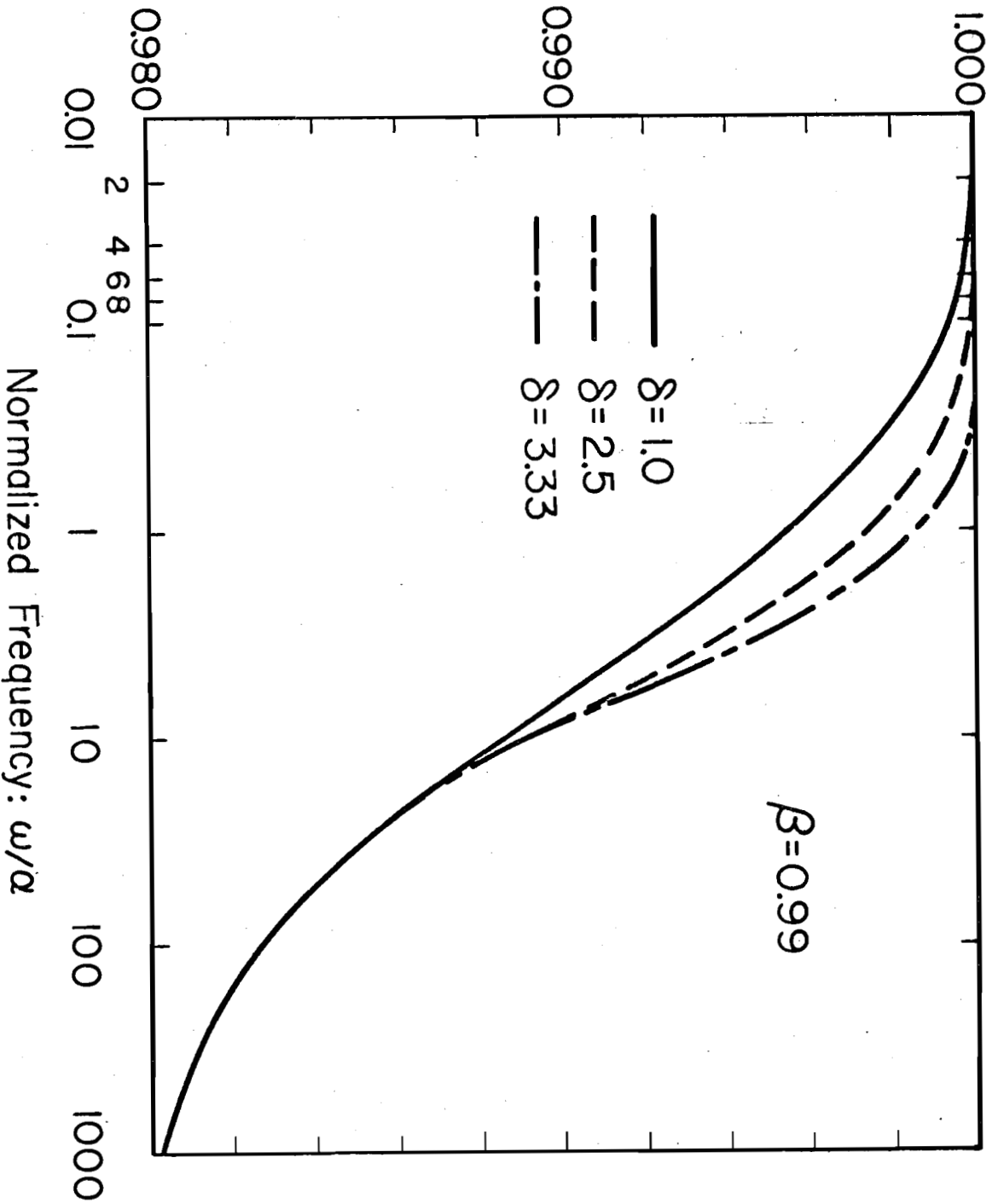


Fig. 9 Particle Response Functions for Near Neutrally Buoyant Particles ( $\beta = 0.99$ )

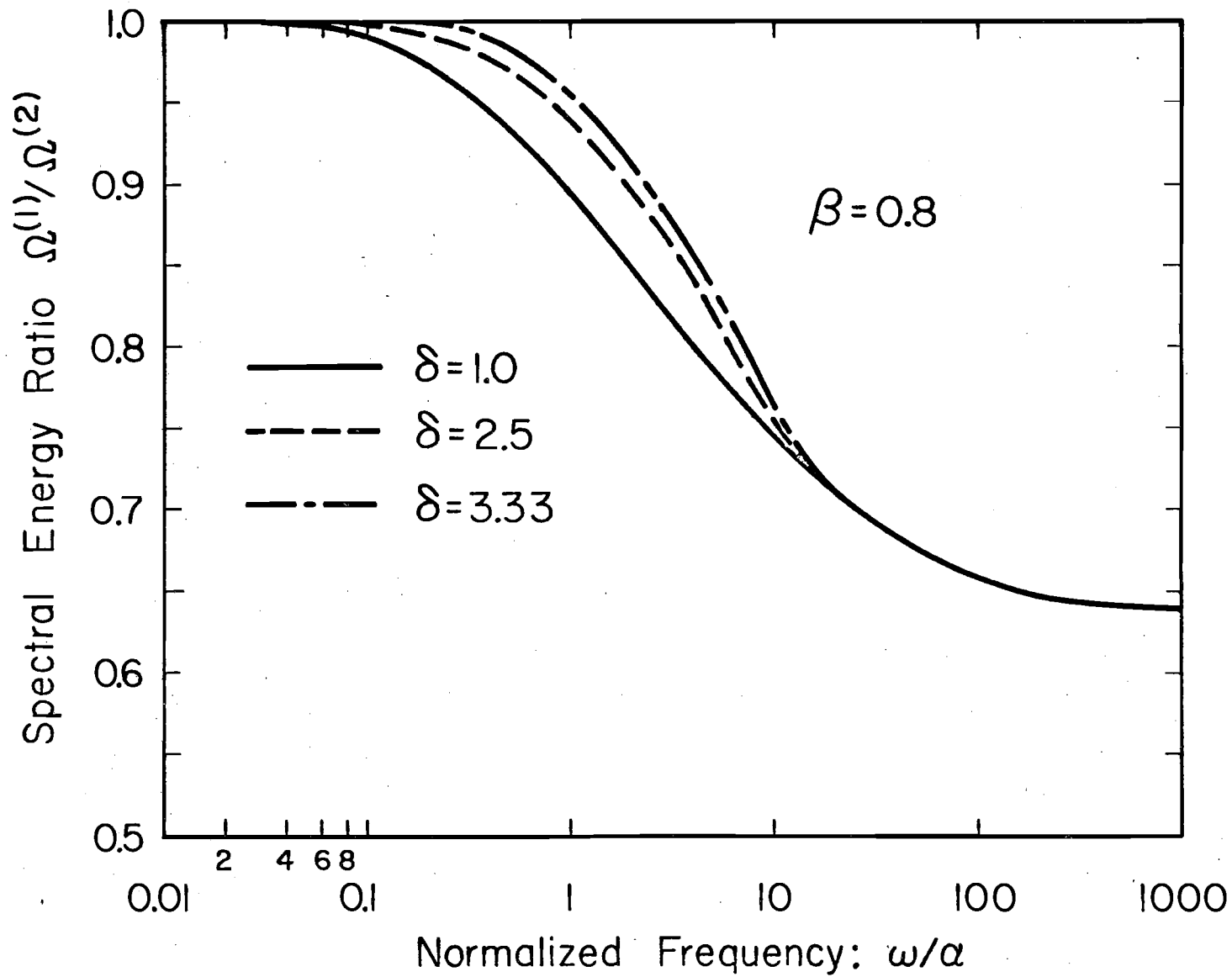


Fig. 10 Particle Response Function for Moderately Heavy Particles ( $\beta = 0.8$ )

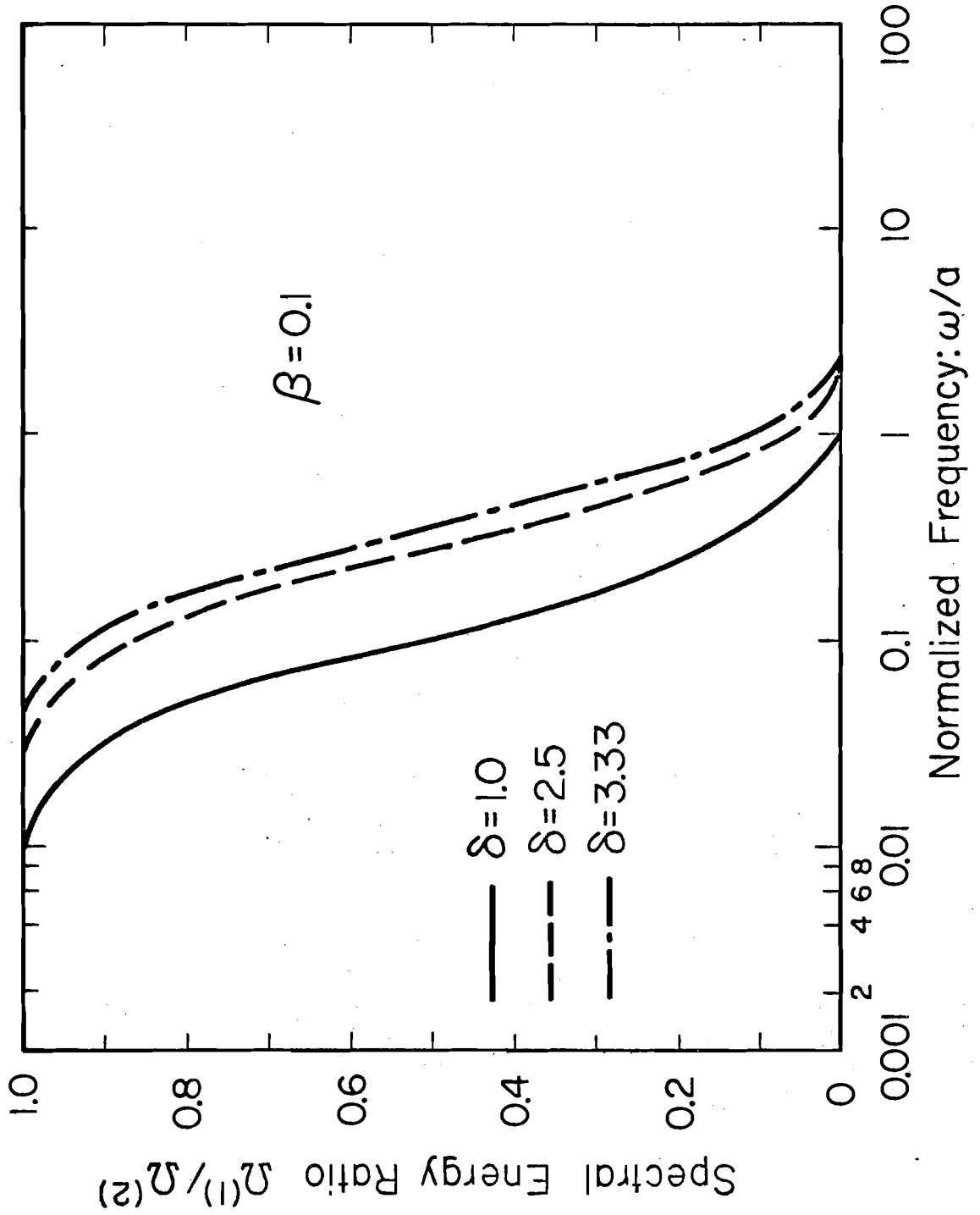


Fig. 11 Particle Response Function for Heavy Particles ( $\beta = 0.1$ )

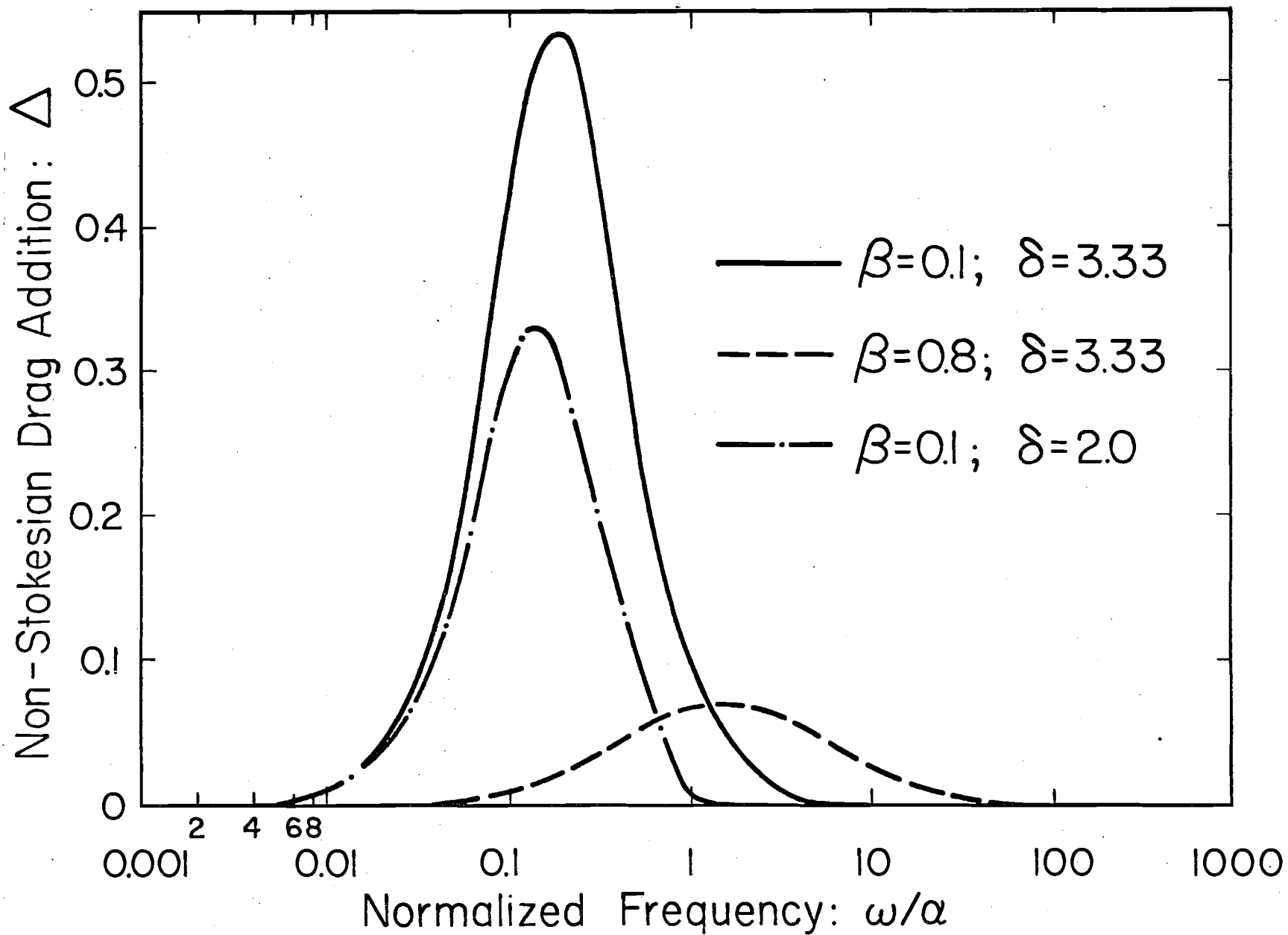


Fig. 12 Non-Stokesian Drag Addition to Stokesian Particle Response Function for Selected  $\beta$  and  $\delta$



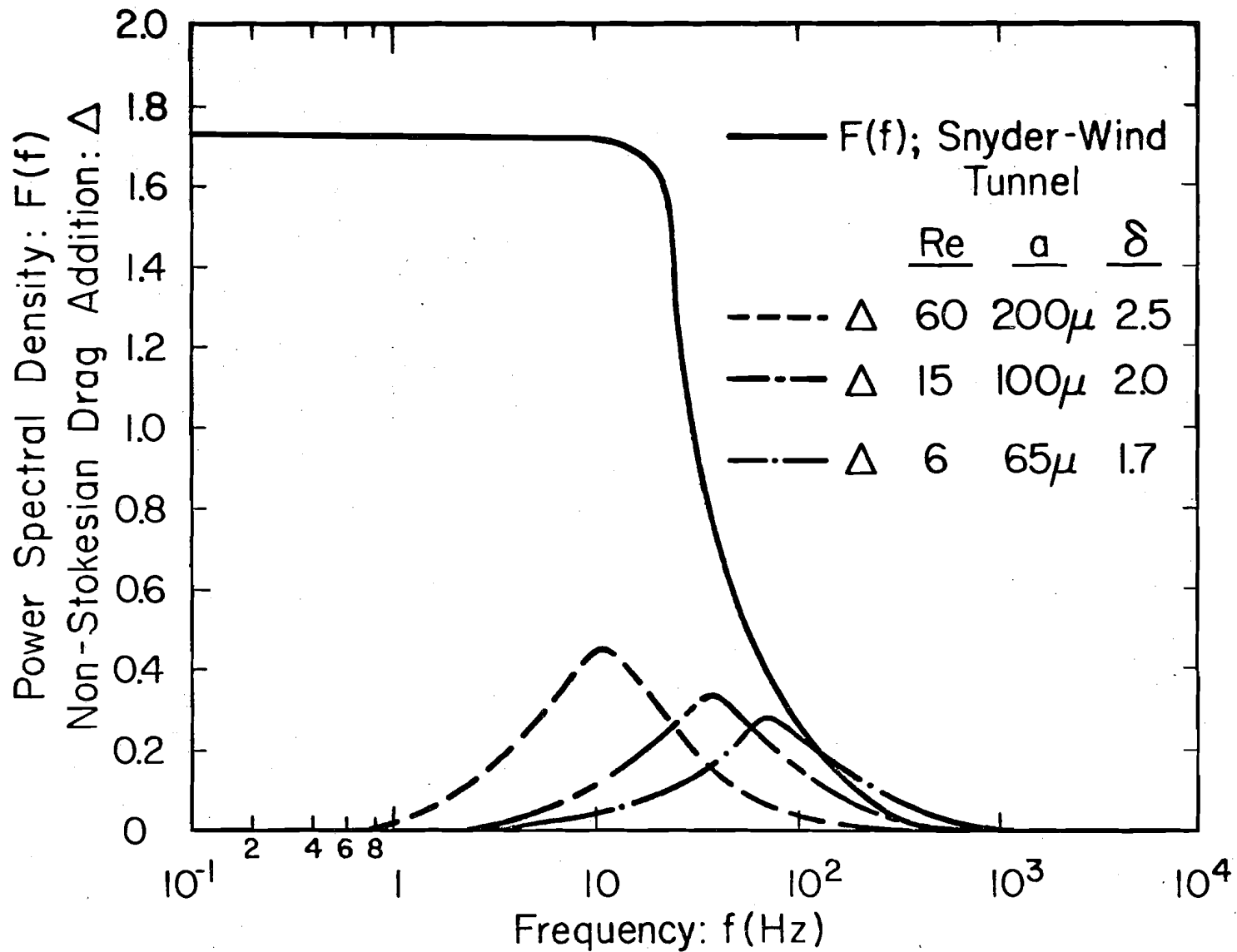


Fig. 13 Non-Stokesian Drag Addition to Stokesian Particle Response Function and Power Spectral Density for Carbon Particles in Air

it is to be expected that they add significantly to particle behavior. However, in Fig. 14 the behavior of sand in water, together with an energy spectrum of turbulence water pipe flow, is shown. It is seen that maximum non-Stokesian contributions come at frequencies much beyond the realm of fluid energy content and, therefore, are probably only of minor significance.

In conclusion, an analytical technique has been developed enabling the linearization of the drag force. This linearization requires that the particle's Reynolds number lie nominally in the range  $6 \leq Re \leq 400$  and that the particle's free fall velocity be larger than the difference between its fluctuating velocity, and that of the fluid field which is always satisfied physically. (See ref. 17 for a discussion of this point.) When applied to the particle's response function it was found that non-Stokesian effects tend to increase the particle's response in a definite frequency band which shifts toward lower frequencies as particle size and weight are increased. It was found that in the case of wind tunnel experiments non-Stokesian effects are of definite importance; whereas they probably are of only minor importance in water experiments.

This application of non-Stokesian drag to the particle's response function is but one of the possible uses of this linearization technique. It is hoped that it may prove useful in other cases where non-Stokesian behavior must be considered.

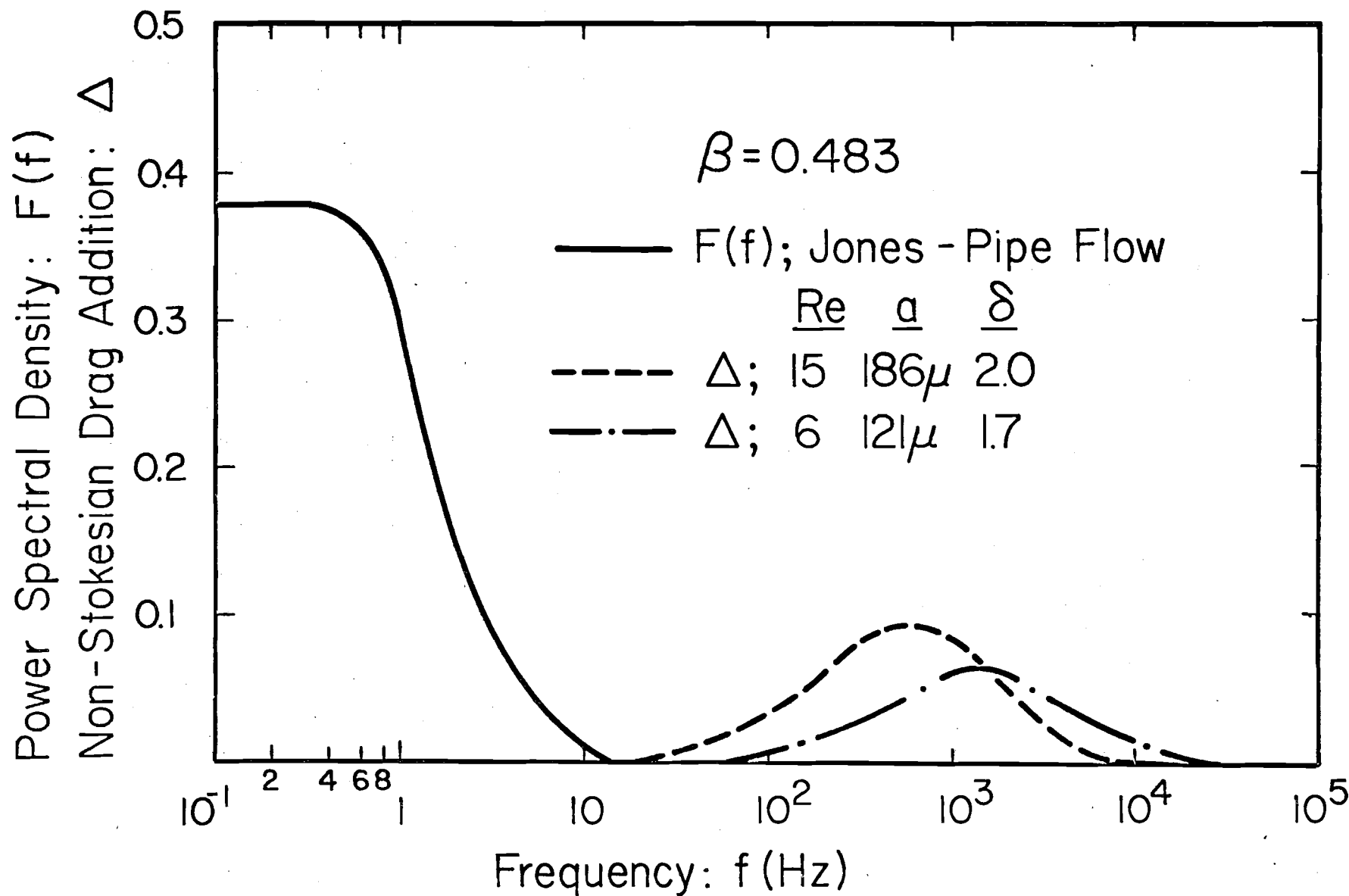


Fig. 14 Non-Stokesian Drag Addition to Stokesian Particle Response Function and Power Spectral Density for Sand Particles in Water

## CONCLUSIONS AND DISCUSSION OF RESEARCH CONTINUATION

As originally conceived this research project had as its primary goals: 1) Experimental system modifications to provide the capability of direct measurement of fluid turbulence in the test section and the required improved resolution of the particle trajectory monitoring system, as well as wide parameterization of the particle characteristics, both of which are essential for these detailed studies of particle behavior in turbulent fluid flows; and 2) Analytical examination of particle motion to enable determination of the basic parameters which characterize particle behavior in such flows. These two rather broad objectives have been accomplished and specific details have been included in the Experimental and Analytical sections of this document. The implications of accomplishing these objectives on continued research in both this specific research study and water resources problems in general are discussed in this section.

In the initial studies<sup>1,8</sup> of suspended particle behavior in turbulent flows, significant difficulties had been encountered, especially in the determination of particle trajectory, due to electronic noise. Improvements to the experimental electronic system have resulted in major reductions in noise-to-signal levels and have enabled particles to be fabricated over a wide variation of physical characteristics. These, characteristics cover the major ranges of suspended particulate of many water resources related problems, i.e. from heavy (sand) to light (organic) particles. The improved system provides relatively noise free and accurate tracking for the particles as they move with the turbulent water.

Since many water pollutants encountered in practical engineering situations (such as sand in water) tend to be relatively heavy and large, they tend to have Reynolds numbers much in excess of the Stokes limit of near unity. Most analytical studies of particle motion, however, are restricted to Stokesian particle behavior because of the difficult non-linear drag aspects of non-Stokesian behavior. Unfortunately results obtained from analyses which ignore non-linear drag for non-Stokesian conditions may not be valid. Consequently, since this project was primarily interested in water borne, non-Stokesian particles, an analytical study was conducted to investigate these effects. The investigation successfully developed a method for linearizing the drag force over an extended particle Reynolds number range. It was found, as far as particle response was concerned, that non-Stokesian effects play a relatively minor role in water born particle transport.

With the accomplishments of this project, future effort will be directed toward utilization of the tools developed herein. The experimental program will provide fundamental data from which the transport phenomena of particles may be studied in detail. A wide variety of experimental parameter variations will be used, including particle size and density, as well as particle shape, so as to better simulate actual solid pollutant materials. Analytically, emphasis will be placed on the detailed formulation of the motion of a particle in a turbulent fluid field in which non-linear drag exists. Two effects seem to be of particular importance. The first is the so-called crossing trajectories effect which describes the loss of correlation and the reduction of dispersion as the particle moves from one region of correlated fluid to another. The second is the inertia effect which describes the reaction

of the particle to fluid accelerations. It is hoped that such a formulation will allow an internally consistent synthesis of both the crossing trajectories effect, as expressed by Csanady,<sup>9</sup> and inertial effects, treated notably by Chao<sup>5</sup>, so as to allow a more complete description of particle motion in a turbulent flow.

One of the advantages obtained by pursuing simultaneously both the analytical and experimental aspects of the continued research is that of having each aspect complement the other. By such combined work the experimentally important parameterization of particle characteristics can be determined from the analytical model and considerable reduction of experimental measurements can result. On the other hand, the experimental observations provide appropriate information from which to construct analytical models.

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