ERRATA

p. viii, line 8:  \( C_H \) should read \( C_h \)
p. 3, line 15:  "Bowhis" should read "Bowlus"
p. 4, line 6:  "reaching" should read "reading"
p. 6, last line:  "construction" should read "constriction"
p. 10, line 13:  "construction" should read "constriction"
p. 15, line 11:  "Fig. 4" should read "Fig. 1"
p. 15, line 15:  "was shown" should read "is shown"
p. 19, line 16:  "actual" should read "critical"
p. 19, line 24:  "start" should read "test"
p. 22, line 7:  "constricted" should read "constructed"
p. 26, line 5:  "if \( Q/\sqrt{gD^5} > 0.1 \)" should read "if \( Q/\sqrt{gD^5} < 0.01 \)"
p. 29 and 31, Figs. 10 and 12:  The hydraulic jump position scale along the top of the graphs should be ignored.
p. 33, line 7:  "\( K_c \)" should read "\( K_e \)"
p. 34, Eq. 17:  \[
\frac{h_{LT}}{D} = C_h \frac{Q^2}{gD^5} \left[ \frac{1}{A_2} - \frac{1}{A_1} \right]
\]  should read \[
\frac{h_{LT}}{D} > C_h \frac{Q^2}{2gD^5} \left[ \frac{1}{A_2} - \frac{1}{A_1} \right]
\]
p. 45, last line:  "Fig. 21" should read "Fig. 25"
p. 46, line 6:  "\( S_o < .001 \)" should read "\( S_o = .001 \)"
p. 56, line 3:  Insert "at" after "located"
p. 57, Ref. 9:  "Bowles" should read "Bowlus"
p. 59, last line:  "\( K_c \)" should read "\( K_e \)"
p. 60, Eq. 3-A:  \[
\Delta h = \frac{1}{2} \left( \frac{1}{A_2} - \frac{1}{A_1} \right) \left( \frac{2}{C_D} \right)^2
\]  should read \[
\Delta h = \frac{1}{2} \left( \frac{1}{A_2} - \frac{1}{A_1} \right) \left( \frac{Q}{C_D} \right)^2
\]
ITRC RESEARCH REPORT NO. 74

Development of a Meter for Measurement of Sewer Flow

by

Harry G. Wenzel, Jr.
Associate Professor of Civil Engineering

FINAL REPORT
Project No. B-063-ILL

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UNIVERSITY OF ILLINOIS
WATER RESOURCES CENTER
2535 Hydrosystems Laboratory
Urbana, Illinois 61801

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ABSTRACT

DEVELOPMENT OF A METER FOR MEASUREMENT OF SEWER FLOW

An experimental and analytical study was performed to develop the geometry for a Venturi type flow meter for use in sewer flow measurement. The meter consists of a constriction in the pipe which produces critical flow under open channel flow conditions and acts as a conventional Venturi meter under full flow conditions.

The constriction is constructed using cylindrical segments whose diameter are larger than that of the pipe and which are attached to the sides of the pipe, leaving the invert and crown clear.

Head loss characteristics and experimental rating curves for both open channel and full flow conditions are described. A procedure is presented, based on experimental data, to theoretically construct a rating curve if experimental calibration is impractical. Information is also presented to permit the selection of geometrical parameters for optimum performance for a specific installation.

Wenzel, Harry G.

DEVELOPMENT OF A METER FOR MEASUREMENT OF SEWER FLOW


KEYWORDS--*flow measurement/ *sewer flow/ *flow meter/ discharge measurement/ urban drainage
ACKNOWLEDGEMENTS

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This study has supported two research assistants, Bruce E. Burris and Charles D. Morris. Their help and ideas contributed significantly to the project. In addition three undergraduate students were supported during the duration of the project.

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<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>iii</td>
</tr>
<tr>
<td>List of Figures</td>
<td>vi</td>
</tr>
<tr>
<td>List of Tables</td>
<td>vii</td>
</tr>
<tr>
<td>Notation</td>
<td>viii &amp; ix</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 - Background</td>
<td>1</td>
</tr>
<tr>
<td>1.2 - Objectives</td>
<td>1</td>
</tr>
<tr>
<td>1.3 - Present Methods and Devices</td>
<td>2</td>
</tr>
<tr>
<td>2. THEORY</td>
<td>6</td>
</tr>
<tr>
<td>2.1 - One Dimensional Flow Equations</td>
<td>6</td>
</tr>
<tr>
<td>2.2 - Energy Loss</td>
<td>9</td>
</tr>
<tr>
<td>2.3 - Nondimensional Variables</td>
<td>11</td>
</tr>
<tr>
<td>3. EXPERIMENTAL EQUIPMENT AND PROCEDURE</td>
<td>13</td>
</tr>
<tr>
<td>3.1 - Flow System</td>
<td>13</td>
</tr>
<tr>
<td>3.2 - Flow Meter Geometry</td>
<td>15</td>
</tr>
<tr>
<td>3.3 - Procedure</td>
<td>15</td>
</tr>
<tr>
<td>4. RESULTS</td>
<td>18</td>
</tr>
<tr>
<td>4.1 - Qualitative Description of Hydraulic Characteristics</td>
<td>18</td>
</tr>
<tr>
<td>4.2 - Energy Loss - Open Channel Flow</td>
<td>19</td>
</tr>
<tr>
<td>4.3 - Rating Curves - Open Channel Flow</td>
<td>26</td>
</tr>
<tr>
<td>4.4 - Energy Loss - Full Flow</td>
<td>32</td>
</tr>
<tr>
<td>4.5 - Rating Curves - Full Flow</td>
<td>37</td>
</tr>
<tr>
<td>4.6 - Transition Region</td>
<td>43</td>
</tr>
<tr>
<td>4.7 - Effect of Kinetic Energy Factor</td>
<td>43</td>
</tr>
<tr>
<td>4.8 - Constriction Without Control</td>
<td>45</td>
</tr>
<tr>
<td>4.9 - Effect of Constriction Geometry on Open Channel Flow Capacity</td>
<td>45</td>
</tr>
<tr>
<td>4.10 - Error Analysis</td>
<td>46</td>
</tr>
<tr>
<td>4.11 - Effect of Unsteady Flow</td>
<td>53</td>
</tr>
<tr>
<td>5. CONCLUSIONS AND RECOMMENDATIONS</td>
<td>55</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>57</td>
</tr>
<tr>
<td>APPENDIX A - Theoretical Calibration Procedure</td>
<td>59</td>
</tr>
<tr>
<td>APPENDIX B - Constriction Geometry Relationships</td>
<td>61</td>
</tr>
</tbody>
</table>
**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Generalized Asymmetrical Constriction Geometry</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>Overall View of Pipe System</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>Piezometer Wells</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>Experimental Constrictions</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>Water Surface Profiles Near Constriction</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>Water Surface Profiles Through Constriction</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>Entrance Loss Coefficient vs. Discharge</td>
<td>24 &amp; 25</td>
</tr>
<tr>
<td>8</td>
<td>Open Channel Rating Curve, $r/D = 0.56, Symmetrical</td>
<td>27</td>
</tr>
<tr>
<td>9</td>
<td>Open Channel Rating Curve, $r/D = 0.66, Symmetrical</td>
<td>28</td>
</tr>
<tr>
<td>10</td>
<td>Open Channel Rating Curve, $r/D = 0.8, Symmetrical</td>
<td>29</td>
</tr>
<tr>
<td>11</td>
<td>Open Channel Rating Curve, $r/D = 0.8, Asymmetrical</td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>Open Channel Rating Curve, $r/D = 100, Asymmetrical</td>
<td>31</td>
</tr>
<tr>
<td>13</td>
<td>Head Difference vs. Position for Full Flow, $r/D = 0.56</td>
<td>35</td>
</tr>
<tr>
<td>14</td>
<td>Total Head Loss Coefficient for Full Flow vs. Area Ratio</td>
<td>36</td>
</tr>
<tr>
<td>15</td>
<td>Full Flow Rating Curve, $r/D = 0.56, Symmetrical</td>
<td>38</td>
</tr>
<tr>
<td>16</td>
<td>Full Flow Rating Curve, $r/D = 0.66, Symmetrical</td>
<td>39</td>
</tr>
<tr>
<td>17</td>
<td>Full Flow Rating Curve, $r/D = 0.8, Symmetrical</td>
<td>40</td>
</tr>
<tr>
<td>18</td>
<td>Full Flow Rating Curve, $r/D = 0.8, Asymmetrical</td>
<td>41</td>
</tr>
<tr>
<td>19</td>
<td>Full Flow Rating Curve, $r/D = 100, Asymmetrical</td>
<td>42</td>
</tr>
<tr>
<td>20</td>
<td>Minimum $r/D$ for Control vs. Slope</td>
<td>47</td>
</tr>
<tr>
<td>21</td>
<td>Slope vs. $Q^*/Q_{max}$</td>
<td>48</td>
</tr>
<tr>
<td>22</td>
<td>Slope vs. $Q_{0.05}/Q_{max}$</td>
<td>51</td>
</tr>
<tr>
<td>23</td>
<td>Relative Error in $Q$ vs. $Q/Q^*$</td>
<td>52</td>
</tr>
<tr>
<td>24</td>
<td>Reference Figure - Symmetrical Constriction</td>
<td>64</td>
</tr>
<tr>
<td>25</td>
<td>Constriction Area vs. $r/D$ for Full Flow</td>
<td>65</td>
</tr>
</tbody>
</table>
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Open Channel Flow Tests</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>Water Surface Profiles</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>Lengths Used in Energy Loss Analysis</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>Entrance Loss Coefficients for Full Flow</td>
<td>33</td>
</tr>
</tbody>
</table>
NOTATION

A = cross sectional area, also coefficient

a = abscissa coordinate for center of constriction radius, also coefficient in Colebrook equation

B = width of water surface

b = coefficient in Colebrook equation

C_D = discharge coefficient

C_F = total skin friction coefficient

C_H = total head loss coefficient

c = coefficient in Colebrook equation

D = pipe diameter

f = friction factor

g = acceleration due to gravity

h = head

h_f = friction loss

h_L = total head loss

h_LT = overall head loss for full flow

i = index

K_c = contraction loss coefficient

K_e = expansion or entrance loss coefficient

k_s = equivalent roughness

L = length between reference sections

\ell = length of flat plate

N_R = Reynolds number = 4 \frac{\rho V}{\nu}

p = cartesian abscissa, pressure

P_w = abscissa of water surface

Q = discharge

q = cartesian ordinate
\( q_w \) = ordinate of water surface
\( R_h \) = hydraulic radius
\( R_L \) = plate Reynolds number
\( r \) = constriction radius

\( S \) = horizontal distance between vertical diametrical plane of pipe and point where constriction intersects pipe wall

\( S_o \) = pipe slope
\( S_f \) = friction slope

\( t \) = time

\( V \) = average velocity

\( x \) = distance

\( y \) = depth

\( y_c \) = critical depth

\( y_n \) = normal depth

\( z \) = vertical distance

\( \alpha \) = kinetic energy correction factor

\( \gamma \) = specific weight of water

\( \theta \) = angle

\( \nu \) = kinematic viscosity
1. INTRODUCTION

1.1 — Background

The need for additional urban runoff data is well documented (1,2,3)*. It can be used in the development, calibration and testing of mathematical models, as input to urban water resources management decisions and incorporated in urban drainage control systems.

A suitable meter for continuous in-system flow measurement ideally would be reasonably accurate throughout a range of flow measurement including free surface and pressurized flow conditions, cause minimum reduction in flow capacity, require a minimum of field maintenance, be capable of continuous, real time measurement with remote data transmission and could be built at reasonable cost.

Many devices or methods for sewer flow measurement have been used or proposed. None are ideal in the light of the above requirements. Thus it was proposed to develop a flow meter which was designed to satisfy as many of these requirements as possible at a low cost. The proposed meter is basically a Venturi device or constriction which acts as a Venturi type flume under free surface or part full conditions and as a conventional pipe Venturi meter under pressurized or full flow conditions. The study involves an attempt to optimize the design and to describe its performance characteristics.

1.2 — Objectives

The general objective of this study is to develop a flow meter which is capable of continuous measurement of flow within a sewer pipe under both open channel and full flow conditions. It should require a minimum of attention and be compatible with a remote data recording system.

*Raised numbers in parenthesis refer to reference list.
Although field tests are not included in this study, laboratory tests were performed to determine:

a) The optimum geometry of the meter for specific installation conditions so that a balance between backwater effects and measurement accuracy can be achieved.

b) To evaluate the head loss characteristics of the meter so that rating curves for meter geometrics not tested in the laboratory can be estimated.

c) To describe the characteristics of the rating curves under both open channel, transition, and full flow conditions.

1.3 — Present Methods and Devices

There is a large amount of literature available in the general field of flow meters. A summary of various types is presented by Replogle\(^{(4)}\). The number of methods suitable for sewer flow measurement is a small subset of those summarized by Replogle and some of these, together with their advantages and disadvantages, were discussed by Wenzel\(^{(5)}\). Further discussion in this report will be restricted to methods which can be applied to sewer flow measurement.

Perhaps the simplest methods involve the measurement of only a depth. This may be done at a free overfall assuming critical depth is measured if the pipe slope is mild, or at some point along the pipe where normal depth is assumed to exist. An appropriate equation is then employed to calculate the discharge. These methods of course are not suitable for full flow conditions.

The depth measurement approach can be improved by combining it with a point velocity measurement. By relating the point velocity to the average velocity and the depth measurement to the cross sectional flow area the discharge can be computed.
Dilution methods yield discharge directly and can be used for open channel and full flow measurements. The sudden injection method has been used to measure combined flows in the Minneapolis-St. Paul area\(^6\) in large sewers. The continuous injection method may be more suitable for small sewers where more rapid variation in flow is anticipated. Both methods require use of a concentration measuring device such as a spectrophotometer and thus are not inexpensive.

The weir can be used for open channel measurement, however, it can present problems by trapping debris behind it or causing excessive backwater conditions. The Stevens\(^7\) pipe weir was designed with sloping upstream and downstream faces to facilitate self-cleaning. Another weir produced by N. B. Products is a V-notch type which is wedged into place using a screw type clamp.

Critical flow flumes have been used and studied extensively\(^8\). In 1936 Palmer and Bowhis\(^9\) introduced a flat bottom flume specifically designed for pipes. This type of design has been studied in detail by Wells and Gotaas\(^10\) who have described its performance characteristics.

A different type of critical flow device was introduced by Diskin\(^11\) consisting of a pier-shaped element which is wedged between the crown and the pipe invert. The flow passes on both sides of the constructing pier causing critical flow. The device has the disadvantage of trapping debris at the nose of the pier, requiring frequent inspection.

In 1972 the City of Los Angeles completed tests on the adaptation of a critical depth meter to sewer flow measurement. Critical flow is produced at a reduction in pipe diameter and a change from mild to steep slope\(^12\).

Perhaps the most recent types of meters are based on thermal, acoustic or electromagnetic principles. Two types of thermal meters designed
for measuring both full and open channel flow have been developed. Thermal Instrument Co. (13) has developed a meter which employs a series of sensing elements mounted around the outside of the pipe. Heat conducted from the elements is a function of flow in the boundary layer in the vicinity of the element. A series of depth sensors are also mounted on the outside of the pipe. The integrated reaching of the velocity elements combined with an area determination based on the depth sensors yields the discharge.

Another thermal technique was studied by Hydrospace-Challenger, Inc. (14) which was based on measuring the time-of-flight of thermal pulses generated at various positions around the periphery of the pipe. Prototype tests indicated that significant fluctuations of the output signal occurred resulting in unacceptable accuracy. It was concluded that more investigation of the effects of velocity fluctuations in the boundary layer on the pulse velocity was required.

Acoustic flow meters employ probes which transmit ultrasonic pulses diagonally across the flow in both the upstream and downstream directions. The difference in the pulse velocity in these two directions is proportional to the average fluid velocity across the pulse path. When combined with a depth indicator to permit the flow area to be determined, the discharge can be computed. Commercial units are manufactured by Westinghouse (15) and Badger Meter, Inc. (16).

Badger meters have been installed in 36 and 48 in. diameter storm sewers and the company claims that accuracy is ± 2% of actual reading over an 8:1 range of flows, with no effect of suspended solids up to a concentration of 250 parts per million.

A side contraction flow meter somewhat similar to that described in this report was installed at the Iowa Institute for Hydraulic Research (17).
It was reported to be self-cleaning and capable of measuring part full flow. However no performance details were reported.
2. THEORY

2.1 One-Dimensional Flow Equations

Consider a circular pipe of diameter D on a slope \( S_0 \) with a constricted section as shown in Fig. 1. Reference section 1 is defined immediately upstream from the construction and section 2 is in the construction a distance \( L \) downstream from section 1. In addition the following assumptions are made:

1. Steady flow
2. Hydrostatic pressure distribution at the reference sections
3. Small slope
4. Two and three dimensional effects are negligible or are accounted for as coefficients or energy loss terms in the energy equation

The energy equation can then be written

\[
y_1 + \frac{\alpha_1}{2gA_1^2} \frac{Q^2}{2} + z_1 = y_2 + \frac{\alpha_2}{2gA_2^2} \frac{Q^2}{2} + z_2 + h_L
\]

where \( y = \) depth, \( A = \) cross sectional flow area, \( \alpha = \) kinetic energy correction factor, \( z = \) vertical distance from some datum and \( Q = \) discharge. The term \( h_L \) represents the total energy lost between the reference sections and its evaluation is important and will be discussed in the next section. The equation can be solved for the discharge if all other terms are measured or evaluated.

\[
Q = \left[ \frac{2g(y_1 - y_2 + LS_0 - h_L)}{\frac{\alpha_2}{A_2^2} - \frac{\alpha_1}{A_1^2}} \right]^{1/2}
\]

If open channel flow exists and \( A_2 \) is small enough, critical flow will occur at some point in the construction. If section 2 is defined at
FIG. 1 GENERALIZED ASYMMETRICAL CONSTRICION GEOMETRY
this point a second relationship can be employed

$$\frac{Q^2B_2}{gA_2} = 1$$  \quad (3)$$

where $B_2$ = the width of the free surface at section 2. By substituting Eq. 3 into Eq. 2 along with the known relationships between $A$, $y$ and $B$, the discharge can be implicitly determined by measuring only $y_1$ and evaluating $h_L$, since all other terms are known.

If the pipe is flowing full $A_1$ and $A_2$ are constant. Equation 2 applies if the depth is replaced by the pressure head, $p/\gamma$. By defining the piezometric head, $h$, as

$$h = \frac{p}{\gamma} + z$$  \quad (4)$$

and substituting, Eq. 2 becomes

$$Q = \frac{2g (h_1 - h_2 - h_L)}{0^2 - \frac{a_1}{A_1^2} - \frac{a_2}{A_2^2}}^{1/2}$$  \quad (5)$$

which can be rewritten as

$$Q = C_D \left[ \frac{2gA_2^2 \Delta h}{1 - \left( \frac{A_2}{A_1} \right)^2} \right]^{1/2}$$  \quad (6)$$
where $\Delta h = h_1 - h_2$, $a_2 = a_1$ and $C_D$ is a discharge coefficient which accounts for energy loss and $a$.

2.2 — Energy Loss

A considerable amount has been done regarding energy losses in constriction and measuring flumes. In general the energy loss $h_L$ can be considered to be made up of two components, a friction loss over the length $L$ and a contraction or eddy loss due to the contraction and expansion of the flow as it passes through the entrance transition into the constriction.

A number of relationships have been proposed for open channel expansions and contractions. These include

\[
\begin{align*}
    h_L &= K_c \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \\
    h_L &= K_E \frac{V^2}{2g} \\
    h_L &= K_E \frac{(V_2 - V_1)^2}{2g} \\
    h_L &= K_E \left(1 - \frac{A_1}{A_2}\right) \frac{V_1^2}{2g}
\end{align*}
\]

(7a) \hspace{1cm} (7b) \hspace{1cm} (7c) \hspace{1cm} (7d)

where $K_c$ = a contraction loss coefficient, $K_E$ = an expansion loss coefficient, and $V_1$ and $V_2$ represent velocities upstream and downstream respectively of the expansion or contraction.

The friction loss may be expressed as

\[
h_f = \frac{fLV^2}{8gR_h} = S_f L
\]

(8)
where \( f \) = a friction factor \( R_h \) = the hydraulic radius, and \( S_f \) = the friction slope. Two methods for evaluating \( f \) have been used (8). One method employs the conventional Darcy-Weisbach coefficient which can be expressed in various ways (18). For flows in the transition region and higher the Colebrook equation converted to open channel flow can be used.

\[
\frac{1}{\sqrt{f}} = c \log \left[ \frac{k_s}{a R_h} + \frac{b}{N_R \sqrt{f}} \right]
\]

where \( k_s \) = the roughness, \( N_R \) = the Reynolds number \( 4R_h V/\nu \) where \( \nu \) = the kinematic viscosity, and \( a \), \( b \) and \( c \) are coefficients with typical values of 14.8, 2.51 and 2.1 respectively.

Another method of evaluating \( f \) used by Replogle (19) involves the use of the flat plate boundary layer equations. The assumption is that a boundary begins at the entrance to the transition and grows through the transition into the construction. Initially the boundary layer is laminar, but it becomes turbulent at some critical value of the plate Reynolds number \( R_L = \frac{Vx}{\nu} \) where \( x \) is the distance from leading edge of the plate. For smooth plates with boundary layers which are turbulent from the leading edge onward Schlichting has presented a relationship which agrees well with experimental data (20).

\[
C_f = 0.074 R_L^{-0.2} \quad 5 \times 10^5 < R_L < 10^7
\]

where \( C_f \) = the total skin friction coefficient which is related to \( f \) in Eq. 8 by \( f = 4C_f \). Schlichting also presents a correction term to account for the initial laminar portion of the boundary layer.
where $A$ is a coefficient which vanes with the critical value of $R_L$.

For fully rough flow Schlichting has presented the interpolation formula

$$C_F = 0.074 R_L^{-0.2} - \frac{A}{R_L}$$  \hspace{1cm} (11)

for boundary layers which are turbulent at the leading edge of the plate. In Eq. 12 $L$ is the length of the plate.

It should be emphasized that none of the above equations can
describe exactly the complex energy dissipation process which occurs between
reference sections 1 and 2 in Fig. 1. It is likely that some separation
occurs at the end of the transition section as the flow enters the constric-
tion. If so Eq. 8 would be insufficient and a combination of perhaps one of
Eq. 7 and Eq. 8 could be used. If the boundary layer equations are used to
evaluate $f$ it is difficult to justify the use of a correction term for the
laminar portion since the constriction is not an idealized flat plate. A
boundary layer already exists at the beginning of the transition. Further-
more, since a portion of the constriction perimeter at the invert and crown
consists of the original pipe geometry, the equations should actually be
applied over only the appropriate portions of the constriction. This refine-
ment is also unjustified in view of the other approximations involved.

2.3 — Nondimensional Variables

It is useful to discuss results in dimensionless form so they can
easily be transferred to other pipe sizes. The reference length used to do
this is the pipe diameter $D$. Some dimensionless variables thus formed are
where the primed variables are dimensionless. If the previous flow equations such as Eqs. 1, 2, 5 and 6 were written in dimensionless form they would appear similar to the dimensional form except all variables would be primed and $g$ would not appear since it is absorbed in $V'$ or $Q'$.

Since the results will have greater value in terms of transferability if they are presented in dimensionless form this procedure will be followed.
3. EXPERIMENTAL EQUIPMENT AND PROCEDURE

3.1 — Flow System

The basic system consisted of an 8 in. inside diameter cast acrylic pipe with a straight run of 140 ft. The pipe was suspended from hangers welded to the steel frame of a tilting channel so that the pipe slope could be varied. A 4 ft. long removable section containing the constriction was located 90 ft. from the beginning of the straight run. Figure 2 shows a view of the pipe and hanger system looking upstream from the test section.

Flow was provided from a constant head tank which provided a static head of approximately 50 ft. The maximum discharge which could be generated with a constriction in the pipe was 2.25 cfs. Flow was monitored by means of a set of orifice plates which were calibrated by the standard weighing technique using the weighing tanks available in the laboratory. A check on the discharge reading was provided by a calibrated weir tank at the discharge end of the pipe. The discharges obtained from these methods usually agreed to within 1 percent.

The free surface profile for open channel flow was measured by a series of piezometer taps along the invert of the pipe throughout its length. The taps were spaced at intervals of 1.0 D near the test section and at intervals of 0.5 D within the test section. Each tap was connected to a 6 in. diameter stilling well. Head was read to within 0.001 ft. using a point gage. Figure 3 shows a bank of stilling wells and the piezometer lines can be seen in Fig. 2. For full flow tests the piezometers used were connected to a differential manometer so that difference in piezometric head between any two taps was measured directly.
FIG. 2 OVERALL VIEW OF PIPE SYSTEM

FIG. 3 PIEZOMETER WELLS
3.2 — Flow Meter Geometry

Several types of constriction geometry were considered. It was decided that for self-cleaning purposes the flow lines near the pipe invert should not be disturbed. It was also apparent early in the study that a specific size of constriction was not optimum for all slopes if backwater effects were to be minimized. Therefore a generalized geometry shown in Fig. 1 was adopted. The constriction is generated by a circular section of radius \( r \) (\( r > D/2 \)) whose center is on the horizontal diametrical plane of the pipe and displaced such that the constriction meets the pipe wall at a distance \( S \) from the vertical diametrical plane. Although the asymmetrical case is shown in Fig. 4, the symmetrical case with identical constrictions on either side of the vertical diametrical plane were also studied. A transition entrance and exit section was constructed on a 4:1 slope as shown. A throat length of 4D was shown although a length of 2D was also tested.

This geometry has several advantages. It leaves both the crown and invert clear to permit self-cleaning. It can be built from sections of larger circular pipes or molded on a cylindrical form and the constriction area can be varied over a wide range by varying \( r \) and/or \( S \). For example, for \( r/D > 100 \) the constriction wall is essentially a vertical plane and for \( r/D = 0.5 \) the constriction vanishes.

3.3 — Procedure

The independent variables which were experimentally investigated were discharge, pipe slope, roughness and constriction geometry. For full flow conditions the pipe slope is not significant since piezometric head differences were measured. The various constrictions and slopes used for open channel tests are summarized in Table 1.
(a) \( r/D = 0.8 \), Asymmetrical

(b) \( r/D = 0.56 \), Symmetrical

(c) \( r/D = 0.80 \), Symmetrical

FIG. 4 EXPERIMENTAL CONSTRUCTIONS
Table 1
Open Channel Tests

<table>
<thead>
<tr>
<th>Slope r/D</th>
<th>100</th>
<th>018</th>
<th>0.8</th>
<th>0.66</th>
<th>0.56</th>
</tr>
</thead>
<tbody>
<tr>
<td>.002</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.008</td>
<td>x</td>
<td>x,*</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.015</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

x = smooth wall
* = rough wall
# = short throat (2.25 D)

The throat length for all but the two tests shown was 4D and S/D for all cases was 0.1.

Most of the tests were performed using smooth plexiglass as the surface material. Uniform and full flow tests without a constriction indicating an equivalent roughness value of 0.000167 ft. was appropriate. To simulate a rougher surface, nylon netting was glued to the surface throughout the constriction and upstream for several diameters. Full flow tests without a constriction indicating an equivalent roughness of 0.00333 ft. could be used for this case.

Full flow tests were performed using all constrictions shown in Table 1 plus a roughened symmetrical one with r/D = 0.56.

For each open channel test the discharge was varied such that the depth just upstream from the entrance to the transition, y_1/D, ranged from approximately 0.1 to 0.98. This corresponded to a discharge range of 30:1. Surface profiles were recorded for each flow as well.

For full flow tests a differential manometer was connected between the tap at section 1 and various other taps so that the hydraulic grade line with respect to section 1 could be established. Data were also taken in the transition range between open channel and full flow for both increasing and decreasing discharge increments. Figure 4 shows views of some of the constrictions tested.
4. RESULTS

4.1 - Qualitative Description of Hydraulic Characteristics

Various classifications of flow and types of water surface profiles are possible depending on the slope and roughness of the pipe, the area of the constriction and the downstream control. If the constriction area, $A_z$, is small enough, critical depth will exist in the constriction causing backwater. Table 2 summarizes the possible surface profiles for steady open channel flow assuming no downstream effects from backwater or drawdown at the outlet.

<table>
<thead>
<tr>
<th>Flow Classification</th>
<th>Constriction Control</th>
<th>Surface Profile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Upstream</td>
</tr>
<tr>
<td>Far Upstream</td>
<td></td>
<td>Backwater, $y &gt; y_n$</td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>Backwater</td>
</tr>
<tr>
<td>Subcritical</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td>Hydraulic jump</td>
</tr>
<tr>
<td>Supercritical</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>Uniform</td>
</tr>
<tr>
<td>Supercritical</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The most important characteristic from a practical viewpoint is the upstream backwater caused by the constriction since this will reduce the discharge at which the pipe flows full and thus reduce its capacity if full flow is the criteria on which this is judged. This is discussed quantitatively later in this chapter.
Table 2 describes surface profiles from a one-dimensional viewpoint only. However, the wave patterns set up should be discussed as well. These patterns were most severe with the asymmetrical constriction. When control exists standing waves are established at each abrupt change in geometry which reflect back and forth across the pipe in the downstream direction. However, of even greater magnitude is the swirling or jet action of most of the flow as it passes through the expansion transition. This directs the flow against the pipe wall opposite the transition which rides up the wall forming a wave, and is then reflected back and forth across the pipe as the flow proceeds downstream. Figures 5 and 6 show views of various surface profiles and Fig. 5 (a) shows the wave pattern described above. The pattern for the symmetrical constriction is somewhat similar but not as violent. Since the flow expands on both sides of the pipe a symmetrical pattern is established with reinforcement along the centerline of the flow. It should be pointed out that these wave patterns have no adverse effect on the upstream measuring section since they do not propagate upstream through the actual section.

During most of the tests a free discharge existed at the outlet. If a submerged condition exists the pipe will eventually flow full. However, as long as the depth at the measuring section is less than the pipe diameter, the open channel rating curve applies even though the downstream portion may be flowing full.

4.2 — Energy Loss - Open Channel Flow

The best way to calibrate any meter is to start the full size prototype in place. Since this probably is not feasible for sewer flow measurements an estimate of the rating curve must be made. Of basic importance, therefore, is a means of estimating the energy loss. The effect of
(a) Wave Pattern Downstream from Constriction

(b) Hydraulic Jump Upstream from Constriction

FIG. 5 WATER SURFACE PROFILES NEAR CONSTRUCTION
FIG. 6 WATER SURFACE PROFILES THROUGH CONSTRUCTION
the energy loss term in Eq. 2 can be seen by comparing the solution to Eqs. 2 and 3 with $h_L = 0$ with the experimental data. The results vary somewhat depending on the particular geometry and slope, but the relative error for smooth constrictions is approximately 50%, 20%, and 15% for values of $y_1/D$ of 0.2, 0.5 and 0.9 respectively. Although the error drops sharply as the flow increases, it is still very significant at all discharges and clearly must be accounted for. If the constriction were constricted of material rougher than plexiglass the error would be even larger.

The analysis was made by assuming that the total energy loss was made up of a combination of friction loss over the length $L$ between sections 1 and 2 (see Fig. 1) and an eddy loss caused by contraction and expansion of the flow between the end of the entrance transition and the critical section

$$h_L = \int_0^L S_f \, dx + K_e \left( \frac{V_e^2}{2g} - \frac{V_1^2}{2g} \right)$$

(14)

where $S_f$ is the local value of the friction slope and $K_e$ is an entrance coefficient. The total length $L$ can be broken into three regions and $S_f$ evaluated separately for each. Between the measuring section and the entrance to the transition, $L_1$, $S_f$ can be estimated using Eq. 8 with $f$ evaluated at section 1 using Eq. 9. In the entrance transition, $L_2$, the friction slope is increasing and can be approximated by the average of the values at section 1 and the critical section 2, evaluated using Eqs. 8 and 9. In the constriction, $L_3$, the boundary layer equations can be employed to evaluate $f$ assuming a fully turbulent boundary layer at the end of the entrance transition. Therefore, Eqs. 10 and 12 were used where the length parameter value used was $L_3$ and the velocity was the critical
value. An alternative to this would be to use Eq. 9 to evaluate $f$ at the
critical section. Therefore, the first term of Eq. 14 can be broken down
into three components

$$
\int_0^L S_f \, dx = S_{f_1} L_1 + \frac{L_2}{2} \left( S_{f_1} + S_{f_2} \right) + L_3 S_{f_3}
$$

Evaluation of Eq. 15 requires that the roughness of the pipe and
constriction be known as well as the velocities and depths at sections 1
and 2 and the three lengths. The location of the critical section and
hence $L_3$ varied with discharge and throat length. Generally the critical
section moved upstream with increasing discharge. For purposes of analysis,
constant values were used as shown in Table 3.

<table>
<thead>
<tr>
<th>Table 3 - Lengths Used in Energy Loss Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>r/D</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>100.0*</td>
</tr>
<tr>
<td>0.8*</td>
</tr>
<tr>
<td>0.56</td>
</tr>
<tr>
<td>0.56</td>
</tr>
<tr>
<td>0.66</td>
</tr>
<tr>
<td>0.66</td>
</tr>
<tr>
<td>0.8</td>
</tr>
</tbody>
</table>

*Asymmetrical constriction

With Eq. 15 evaluated $K_e$ in Eq. 14 can be calculated from the
experimental data together with Eq. 2. For successive discharges the only
unknown in Eq. 2 is now $K_e$. The resulting values are shown in Fig. 7.
The data are somewhat scattered because the energy loss term involved is
a small fraction of the total measured head difference between sections 1
and 2 and thus experimental errors have a strong influence. However a trend
FIG. 7 ENTRANCE LOSS COEFFICIENT VS. DISCHARGE
\[ r/D = 0.8 \text{ - SYMMETRICAL} \]

- \( S_e = 0.002 \)
- \( S_e = 0.008 \)

FIG. 7 CONTINUED
is clearly seen with $K_e$ being relatively constant above a dimensionless discharge of 0.1 and increasing as the discharge decreases for the symmetrical case. The results can be described mathematically as

\[
K_e = 0.225 \quad \text{if } Q/\sqrt{gD^5} > 0.1
\]

\[
K_e = -0.375 \log(2.5 Q/\sqrt{gD^5}) \quad \text{if } Q/\sqrt{gD^5} > 0.1
\]

For the asymmetrical case $K_e$ did not rise as strongly for $Q/\sqrt{gD^5} < 0.1$ and a constant value of $K_e = 0.20$ could be used for all discharges. The effect of slope or roughness on these results is not significant in view of the scatter due to experimental error. Furthermore, the variation with $r/D$ is not great either for $0.56 < r/D < 0.8$. The asymmetrical case shows lower loss at the lower discharges perhaps because there is only one possible zone of separation at the constriction entrance.

4.3 — Rating Curves — Open Channel Flow

Experimental rating curves for the various constrictions studied are shown in Figs. 8-12. The lines shown represent the solution to Eqs. 2 and 3, using Eqs. 14 and 16 to compute $h_L$ for the symmetrical cases and $K_e = 0.20$ for the asymmetrical cases. Of course, since the values of $K_e$ were originally obtained from the data, agreement is expected. The deviations at the lower flows are the result of the approximation involved in using Eq. 16 for all symmetrical cases.

However, the data for the hydraulically steep slopes ($S_o = 0.008, 0.015$), show an interesting effect, particularly at dimensionless discharges less than 0.1. For these cases the hydraulic jump occurs
FIG. 8 OPEN CHANNEL RATING CURVE, $r/D = 0.56$, SYMMETRICAL

<table>
<thead>
<tr>
<th>$S_0$</th>
<th>EXPER</th>
<th>THROAT LENGTH</th>
<th>$L_1/D$</th>
<th>COMPUTED</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.002</td>
<td>+</td>
<td>4 D</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>0.008</td>
<td>o</td>
<td>4 D</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>0.008</td>
<td>•</td>
<td>4 D</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>0.008</td>
<td>△</td>
<td>225 D</td>
<td>1.5</td>
<td>IDENTICAL</td>
</tr>
<tr>
<td>0.008</td>
<td>▲</td>
<td>225 D</td>
<td>0.34</td>
<td></td>
</tr>
</tbody>
</table>

HYDRAULIC JUMP POSITION — DIAMETERS UPSTREAM FROM SEC 1
FIG. 9 OPEN CHANNEL RATING CURVE, $r/D = 0.66$, SYMMETRICAL

HYDRAULIC JUMP POSITION — DIAMETERS UPSTREAM FROM SEC 1

$r/D = 0.66$ SYMMETRICAL

<table>
<thead>
<tr>
<th>$S_0$</th>
<th>EXPER</th>
<th>$L_1/D$</th>
<th>COMPUTED</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.002</td>
<td>+</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>0.008</td>
<td>○</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>0.008</td>
<td>●</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>0.015</td>
<td>△</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>0.015</td>
<td>▲</td>
<td>0.34</td>
<td></td>
</tr>
</tbody>
</table>

HYDRAULIC JUMP POSITION
FOR $S_0 = 0.015$, $L_1/D = 1.33$

FIG. 9 OPEN CHANNEL RATING CURVE, $r/D = 0.66$, SYMMETRICAL
FIG. 10 OPEN CHANNEL RATING CURVE, r/D = 0.8, SYMMETRICAL

HYDRAULIC JUMP POSITION—DIAMETERS UPSTREAM FROM SEC 1

<table>
<thead>
<tr>
<th>S₀</th>
<th>EXPER</th>
<th>Lᵢ/D</th>
<th>COMPUTED</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.002</td>
<td>+</td>
<td>1.2</td>
<td>————</td>
</tr>
<tr>
<td>0.008</td>
<td>○</td>
<td>1.2</td>
<td>————</td>
</tr>
<tr>
<td>0.008</td>
<td>•</td>
<td>0.34</td>
<td>————</td>
</tr>
</tbody>
</table>
FIG. 11 OPEN CHANNEL RATING CURVE, r/D = 0.8, ASYMMETRICAL
FIG. 12  OPEN CHANNEL RATING CURVE, r/D = 100, ASYMMETRICAL
sufficiently close to section 1 that the sequent depth has not yet developed, resulting in an incorrectly low reading for \( y_1 \). This is particularly clear in Fig. 9 at \( S_o = 0.015 \). The hydraulic jump location curve shows that when the beginning of the jump was within approximately 5 D of section 1, the reading of \( y_1 \) was low. This is brought out clearly by observing in Fig. 9 the values of \( y_1 \) when a point approximately 1.0 D downstream is taken as section 1. The calculated rating curve agrees quite well with the later data except for a discharge less than 0.02 whereas disagreement with the experimental data at the original section was as high as 25 percent at the lower discharges. This same effect is seen, but to a lesser extent, for \( S_o = 0.008 \) in Figs. 8, 9 and 10. Figure 8 shows that for a given discharge the jump occurs further upstream on the milder slope, thus reducing its effect. This jump effect indicates that for very steep slopes, section 1 should be located within 0.5 D of the beginning of the transition so this problem can be minimized.

One set of data not shown in Fig. 10 is for a throat length of 2.25 D on a slope of 0.015. That data is almost identical with that shown for the same slope and a throat length of 4 D.

4.4 — Energy Loss — Full Flow

The energy loss under full flow conditions was analyzed similarly to the open channel case. That is, the total head loss in Eq. 5 is considered to be made up of the friction loss between the two measuring sections plus an eddy loss as expressed in Eqs. 14 and 15. However, in this case, \( L_3 \) represents the distance from the end of the entrance transition to the measuring section (section 2) in the constriction. This length was 1.5 D in all cases except for the asymmetrical case for \( r/D = 100 \) where \( L_3 = 1.0 \) D.
The other lengths were as shown in Table 3. The friction factors were estimated using Eqs. 9, 10 and 12 as was done for the open channel flow cases, however the hydraulic radius for the full cross section was used.

By inserting the experimental values for the piezometric head differences between sections 1 and 2 and the measured discharges into Eq. 5 and using Eqs. 9, 10 and 12 to evaluate Eq. 15, it is possible to compute an entrance loss coefficient, $K_c$. The results of this computation assuming $a_1 = a_2 = 1.0$ are shown in Table 4 along with experimental discharge coefficients, $C_D$, from the direct solution of Eq. 6 using experimental data.

<table>
<thead>
<tr>
<th>r/D</th>
<th>Equivalent Roughness (ft)</th>
<th>Experimental $K_e$</th>
<th>Experimental $C_D$</th>
<th>Computed $C_D(K_e=0.20)$</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0.000167</td>
<td>0.160</td>
<td>0.872</td>
<td>0.859</td>
<td>1.5</td>
</tr>
<tr>
<td>0.8*</td>
<td>0.00333</td>
<td>0.160</td>
<td>0.823</td>
<td>0.811</td>
<td>1.5</td>
</tr>
<tr>
<td>0.56</td>
<td>0.000167</td>
<td>0.010</td>
<td>0.945</td>
<td>0.869</td>
<td>8.0</td>
</tr>
<tr>
<td>0.56</td>
<td>0.000167</td>
<td>0.175</td>
<td>0.870</td>
<td>0.862</td>
<td>0.9</td>
</tr>
<tr>
<td>0.66</td>
<td>0.00333</td>
<td>0.290</td>
<td>0.803</td>
<td>0.818</td>
<td>1.9</td>
</tr>
<tr>
<td>0.8</td>
<td>0.000167</td>
<td>0.180</td>
<td>0.882</td>
<td>0.876</td>
<td>0.7</td>
</tr>
<tr>
<td>0.8</td>
<td>0.000167</td>
<td>0.250</td>
<td>0.860</td>
<td>0.879</td>
<td>2.2</td>
</tr>
</tbody>
</table>

*Asymmetrical constriction

The values of $K_e$ shown in Table 4 are averages over the experimental discharge range of approximately $0.4 < Q/\sqrt{gD^5} < 1.25$. The actual values ranged within ±3 percent of those shown, generally increasing with increasing discharge. However, this variation is not significant in comparison to the variation in average $K_e$ values for the various values of r/D. With the exception of r/D = .56 (rough) and 100 the values are comparable to those for $Q/\sqrt{gD^5} > 0.1$ for the open channel flow cases.
Of further interest in the case of full flow is the total overall head loss generated by the constriction. This information is useful in computing the reduction in flow capacity under full flow conditions. In order to determine this head loss, piezometric heads were measured at various positions along the pipe both upstream and downstream from section 1, extending through the constriction and on downstream. This permitted the construction of the hydraulic grade line in the vicinity of the constriction. An example is shown in Fig. 13 where the head at section 1 is used as a reference. By extrapolating the constant slope portion of the hydraulic grade line both upstream and downstream of the constriction so they overlap, the overall head loss for that discharge can be computed. This loss, $h_{LT}$ was correlated to the change in velocity head through the constriction

$$
\frac{h_{LT}}{D} = C_h \frac{Q^2}{gD^5} \left[ \frac{1}{A_2^2} - \frac{1}{A_1^2} \right]
$$

(17)

where $C_h$ is a total head loss coefficient. Values of $C_h$ computed from Eq. 17 and values of $h_{LT}$ determined from plots such as Fig. 13 are shown on Fig. 14. Although there was some variation in the results, a correlation of $C_h$ with area ratio was developed as shown.

$$
C_h = 2.0 \log \left( \frac{A_1}{A_2} \right)
$$

(18)

Equations 17 and 18 permit the computation of the total overall head loss generated by the constriction under full flow conditions.
FIG. 13 HEAD DIFFERENCE VS. POSITION FOR FULL FLOW, \( r/D = 0.56 \)

\[ r/D = 0.56, \text{ SYMMETRICAL} \]

\[ Q/\sqrt{gD^5} = 1.078 \]
\[ C_h = -2.0 \log\left(\frac{A_2}{A_1}\right) \]

\[
\frac{h_{LT}}{D} = C_h \frac{Q^2}{2gD^5} \left[ \frac{1}{A_2^2} - \frac{1}{A_1^2} \right]
\]

\[
\text{= RANGE OF DATA}
\]

FIG. 14 TOTAL HEAD LOSS COEFFICIENT FOR FULL FLOW VS. AREA RATIO
4.5 — Rating Curves — Full Flow

The experimental data for the various constrictions are shown in Figs. 15-19. The lines shown were computed using Eq. 5 with an entrance loss coefficient value of 0.20 for all cases since the experimental variation of $K_e$ with constriction shown in Table 4 did not follow any significant trend. Also shown in Table 4 are the predicted values of $C_D$ obtained by computing $\Delta h$ for a specified $Q$ from Eq. 5 and using the results in Eq. 6 to compute $C_D$. It is seen that with the exception of $r/D = 100$, the computed values of $C_D$ are within approximately 2 percent of the experimental values. For this one exception a value of $K_e = 0$ results in a predicted curve which agrees well with the data as shown in Fig. 17.

An equation showing the relationship between the computed value of $C_D$ and the various energy loss terms can be obtained as follows. By eliminating $\Delta h$ from Eqs. 5 and 6 and substituting Eqs. 14 and 15 for $h_2$ assuming $\alpha = 1$ one obtains

$$C_D = \left[ \frac{1}{1 + K_e + \sum_{i=1}^{3} \frac{A_i^2}{A_1-A_2} \frac{\sqrt{fL}}{4R_iA_i^2}} \right]^{1/2} \tag{19}$$

where $i$ represents each of the regions shown in Fig. 1 and the value of $f/4R_iA_i^2$ in region 2 is the average of the values for regions 1 and 3, similar to Eq. 15. Values of $f$ are obtained by assuming a value for $Q$ and using Eqs. 10 or 12. Equation 19 shows that $C_D$ will vary with discharge if any of the values of $f$ vary, as would be the case if the Reynolds number were in the transition region. However, this variation was found to be insignificant over the range of experimental discharges, even for the smooth pipe.
FIG. 15 FULL FLOW RATING CURVE, r/D = 0.56, SYMMETRICAL
$r/D = 0.66$ SYMMETRICAL

- FULL FLOW, SMOOTH
- OPEN CHANNEL & TRANSITION, $S_0 = 0.008$
- COMPUTED FULL FLOW

FIG. 16 FULL FLOW RATING CURVE, $r/D = 0.66$, SYMMETRICAL
FIG. 17 FULL FLOW RATING CURVE, r/D = 0.8, SYMMETRICAL
FIG. 18 FULL FLOW RATING CURVE, r/D = 0.8, ASYMMETRICAL
FIG. 19  FULL FLOW RATING CURVE, $r/D = 100$, ASYMMETRICAL
4.6 — Transition Region

Between the open channel and full flow cases there exists a transition region in which neither rating curve applies. This region is shown in Figs. 15-19. In terms of discharge the region extends from the maximum open channel discharge \( Q/\sqrt{gD^5} \) for \( y_1/D \approx 0.99 \) for an additional dimensionless discharge of approximately 0.1. This region was experimentally established with free flow conditions at the pipe exit. If the pipe exit were submerged the region would be reduced considerably.

It was also observed that for some slopes the transition region for increasing discharge was larger than for the case of decreasing flow. In the latter there was a tendency for the data to remain on the full flow rating curve to discharges lower than the point where the increasing flow data met the rating curve. The data shown in Figs. 15-19 is for the increasing flow case, which gives the larger transition regions.

From an operational viewpoint the transition region can be identified by observing the piezometric heads at sections 1 and 2. When \( y_1/D > 1.0 \), but \( y_2/D < 1.0 \) the flow in the transition region and neither rating curve applies.

4.7 Effect of Kinetic Energy Factor

In all the analysis work it was assumed that the kinetic energy factors at sections 1 and 2, \( \alpha_1 \) and \( \alpha_2 \), were unity. This means that the loss coefficients computed using experimental data include the effect of \( \alpha \). No attempt was made experimentally to measure the velocity profile and hence compute \( \alpha \). Since the throat length in the constriction is relatively short it is reasonable to expect \( \alpha_2 \) to closely approach unity. To estimate the effect on \( Q \) due to a variation of \( \alpha_1 \) from unity one can compute the partial derivative of Eq. 2 with respect to \( \alpha_1 \).
By dividing Eq. 20 by Eq. 2 the relative change in Q for a change in $\alpha_1$ can be obtained

$$\frac{\Delta Q}{Q} = \frac{\Delta \alpha_1}{\alpha_1}$$

(21)

Evaluation of the denominator of Eq. 21 for a wide range of slopes and $r/D$ values indicates a minimum value of approximately 3 for $S_o = .002$, $r/D = 0.54$ to a value of approximately 20 for $r/D = 1.0$. Thus if $\alpha_1 = 1.04$ one could expect no more than approximately a 1 percent error in Q.

It should be pointed out that even though $\alpha$ was taken as unity in the data analysis, its effect has been implicitly taken into account in the computation of the energy loss coefficients. Thus to be more precise, $\Delta \alpha_1$ in Eq. 21 should be the variation of $\alpha$ from the experimental values, i.e. the scale effect, if an estimate of $\Delta Q/Q$ is to be made for the computed rating curves. This would probably reduce $\Delta Q/Q$ even further.

For the full flow case, identical operations with respect to Eq. 5 yield Eq. 21 as well. However, the ratio $A_1/A_2$ is now only a function of constriction geometry and since the pipe is flowing full, the ratio is smaller than for the open channel case. This results in larger estimated errors in Q. For example for $r/D = 0.56$ and $\Delta \alpha_1 = 0.04$, $\Delta Q/Q = 0.028$. 
4.8 - Constriction Without Control

The previous discussion has been concerned only with cases where control or critical depth occurred in the constriction. A test was made using the asymmetrical constriction with \( r/D = 0.8 \) and \( S_o = 0.015 \). Under these conditions critical depth did not occur and the surface profile consisted of a hump in the constriction since the slope was steep. However, since the immediate upstream flow was supercritical, severe wave problems existed at both section 1 and 2, causing serious errors in the measurements of \( y_1 \) and \( y_2 \). These waves caused sufficient problems so that it was concluded that this scheme was inappropriate for steep slopes. For mild slopes the change in depth, \( y_1 - y_2 \), is sufficiently small to generate rather large errors in estimated discharge for a given error in the measurement of \( y_1 \) and \( y_2 \). Furthermore, from a practical standpoint the measurement of an additional depth for open channel flow decreases the reliability of the system. Therefore the use of a meter without control is not recommended.

4.9 - Effect of Constriction Geometry on Open Channel Flow Capacity

In choosing the geometry for a specific installation one must first be assured that the value of \( r/D \) chosen will result in control over the full range of open channel flow. In order to estimate this a computer analysis of the symmetrical case with \( k_s/D = 0.001 \) was performed. The minimum value of \( r/D \) required for control with a symmetrical constriction over the entire possible range of open channel flow at various slopes is shown in Fig. 20. It is seen that the minimum value drops to 0.52 for \( 0.003 < S_o < 0.008 \) and increases on either side of this range. Figure 20 assumes that no backwater effects from pipe exit conditions extend upstream far enough to reach the constriction. For the asymmetrical use, Fig. 21 can be used to obtain an
r/D value which would yield the same full flow area as the symmetrical r/D value obtained from Fig. 20. Since the depth where control is just barely maintained varies and is always less than D, this method of transforming Fig. 20 to the asymmetrical case is an approximation. However, the shape of the depth vs. area curves is similar for the two cases, the approximation is justified. This shows, for example, that for \( S_o < 0.001 \) an \( r/D \geq 0.67 \) is required for a symmetrical constriction and the equivalent \( r/D \) for the asymmetrical case is approximately 100, or a vertical wall. For slopes lower than this the asymmetrical constriction will not produce control unless \( s/D \) is reduced. This is not recommended, however, since it will reduce the free region near the invert needed for self-cleaning.

The reduction in capacity due to installation of a constriction depends on how "capacity" is defined. In many cases the system is designed for open channel flow. In this case the maximum flow occurs at a normal depth, \( y_n/D = 0.93 \). The discharge at \( y_n/D = 0.93 \) is termed \( Q_{max} \). With a constriction in place the depth at section 1 is always greater than normal and the discharge at which \( y_1/D = 0.99 \) is called \( Q^* \). The ratio \( Q^*/Q_{max} \) is a measure of the reduction in capacity and is shown in Fig. 21 as a function of slope and \( r/D \). It is seen that the variation in \( Q^*/Q_{max} \) is significant, both with \( S_o \) and \( r/D \).

4.10 — Error Analysis

Another variable which affects the choice of geometry is the desired accuracy. An estimate of the relative error in discharge resulting from an error in measurement of \( y_1/D \) for open channel flow can be obtained by considering a simplified form of Eq. 2 in dimensionless form
FIG. 20  MINIMUM \( r/D \) FOR CONTROL VS. SLOPE
FIG. 21 SLOPE VS. $Q^*/Q_{\text{max}}$
In Eq. 22 it is assumed that the slope and energy loss terms in Eq. 2 offset each other and \( a_1 = a_2 = 1.0 \). Also since control is assumed, \( y_2 \) and \( A_2 \) are replaced by critical values \( y_c \) and \( A_c \) respectively. Differentiation of Eq. 22 with respect to \( y_1 \) and dividing the result by Eq. 22 yields

\[
\frac{\Delta Q}{Q} = \Delta y_1 \left[ \frac{1}{2(y_1 - y_c)} + \frac{B_1}{A_1} \left( \frac{1}{A_c^2} - \frac{1}{A_1^2} \right) \right]
\]

Equation 23 shows that, for a specified \( \Delta y_1 \), \( \Delta Q/Q \) will vary with \( y_1 \) and \( y_c \) and hence discharge. In order to describe this variation Figs. 22 and 23 were constructed for the symmetrical case of \( k_s/D = .001 \). Figure 22 shows the ratio of the discharge at which \( \Delta Q/Q = .05 \), denoted \( Q_{.05} \), to \( Q_{\text{max}} \) as a function of slope and \( r/D \) for \( \Delta y_1/D = .01 \). The relative error of .05 was arbitrarily chosen as an acceptable error and \( \Delta y_1/D = .01 \) is a convenient base value. The discharge \( Q_{\text{max}} \) was chosen as a reference value since it can be computed prior to choosing the value of \( r/D \) for the constriction. All discharges below \( Q_{.05} \) are subject to relative errors larger than .05. This is seen in Fig. 23 which shows \( \Delta Q/Q \) as a function of \( Q/Q^* \) for various values of \( r/D \) at a slope of .002 and 0.012. This shows that increasing the slope for a given constriction will reduce the range of discharge which is within specified relative error limit. To put it another more useful way, for a given slope one must increase \( r/D \) to increase the range of discharge below a specified relative error.
It should be emphasized that Figs. 22 and 23 were constructed assuming that the error in measuring $y_1/D$ is 0.01 for all flows. This means that the relative error in $y_1$, $\Delta y_1/y_1$, varies with $y_1$ from 0.01 at $y_1 = D$ to, for example 0.1 at $y_1 = 0.1D$. However, Eq. 23 shows that $\Delta Q/Q$ is directly proportional to $\Delta y_1$ so that information from Figs. 22 and 23 can be linearly adjusted for values of $\Delta y_1/D \neq 0.01$.

Figures 21-23 can be used to decide on an appropriate value for $r/D$ given specified values of $Q^*/Q_{\text{max}}$, $S_o$, and a range of flow which should be measured to an accuracy of 5 percent. For example, assume a maximum allowable reduction in capacity is 20 percent, i.e. $Q^*/Q_{\text{max}} = 0.80$. If $S_o = 0.004$ then from Fig. 21 the largest acceptable $r/D$ value is approximately 0.53. From Fig. 22 the corresponding value of $Q_{0.05}/Q_{\text{max}}$ is approximately 0.32. That is, all flows below $0.32 Q_{\text{max}}$ are subject to a relative error of 5 percent or greater for relative error of 0.01 in measuring $y_1/D$. If it is decided that more accuracy at the lower flows is desired, say $Q_{0.05} = 0.20 Q_{\text{max}}$ is specified, then $r/D = 0.56$ is required which will result in a relative capacity, $Q^*/Q_{\text{max}}$, of 0.65 from Fig. 21 or a capacity reduction of 35 percent.

Under full flow conditions the relative error in discharge can be obtained by differentiating Eq. 6. The result is

$$\frac{\Delta Q}{Q} = \frac{\Delta(h_1 - h_2)}{2(h_1 - h_2)} \quad (24)$$

which states simply that the relative error in discharge is half of the relative error in measuring the head difference between sections 1 and 2.
FIG. 22 SLOPE VS. $Q_{0.05}/Q_{\text{MAX}}$

SYMOMETRICAL CONSTRICTION

$K_s/D = 0.001$

$\Delta Y_I/D = 0.01$

$\gamma / D = 0.52$

SLOPE LIMITS FOR CONTROL
RELATIVE ERROR IN Q $\Delta Q/Q$

SYMMETRICAL CONSTRUCTION

$S_o = 0.002$

$S_o = 0.012$

$K_s/D = 0.0001$

$\Delta Y_1/D = 0.01$

FIG. 23 RELATIVE ERROR IN Q VS. $Q/Q^*$
4.11 — Effect of Unsteady Flow

All previous calculations and experimental work was done assuming steady flow. The difficulties associated with unsteady flow experiments would make the additional information obtained of questionable value because of the errors involved. However, a crude theoretical analysis of the effect of unsteadiness can be made by estimating the magnitude of the unsteady term in the energy equation. This term in dimensional form is \( \frac{1}{g} \frac{\partial V}{\partial t} \) and its effect on the rating curve can be estimated by adding it to the terms in Eq. 2.

\[
Q = \left[ \frac{2g(y_1 - y_2 + LS_0 - h_L - h_a)}{\frac{a_2}{2} - \frac{a_1}{2}} \right]^{(25)}
\]

where \( h_a = \int L \frac{3V}{g} \, dx \). From Eq. 25 it can be seen that if the discharge is increasing the unsteady term is effectively an additional energy loss and the steady flow rating curve overestimates the discharge, whereas if the flow is decreasing the discharge is underestimated.

In order to estimate the magnitude of \( h_a \) consider a dimensionless hydrograph \((Q/Q_p, t/t_p)\) based on the peak discharge \( Q_p \) and its corresponding time \( t_p \). Assume that the maximum slope of the rising limb is given by \( \frac{d(Q/Q_p)}{d(t/t_p)} = 2 \). Assume that \( t_p = 5 \) min and the pipe diameter = 3 ft. Then in dimensionless form \( t'_p = t_p \sqrt{\frac{g}{D}} = 982 \). Let \( Q_p \) assume the maximum dimensionless value for open channel flow in the pipe, which from the rating curves, Figs. 9-13, might be 0.4. Thus

\[
\frac{dQ}{dt}' = 2 \frac{Q_p}{t_p} = 0.00082
\]
where the primes denote dimensionless quantities and A' is taken as 0.5.

This result is a high estimate of the acceleration term and thus serves as a high estimate of the unsteady effect. It is seen that its value is of the same order of magnitude as the slope S_o and the steady flow head loss h_L. Its effect on the discharge can be estimated by differentiating Eq. 2 in dimensionless form with respect to h_a and then dividing by Eq. 2, similar to the process described in the previous section. The result is

$$\frac{\Delta Q}{Q} = -\frac{\Delta h_a}{2(y_1 - y_2 + L S_o - h_L - h_a)}$$

(26)

The following are typical values for the symmetrical constriction r/D = .58, k_s/D = .001 and L/D = 3.0.

- y_1/D = .982
- y_2/D = .695
- h_L/D = .020
- L S_o = .007

Taking Δh_a = .0016 and substituting the above values into Eq. 25 the result is ΔQ/Q = .0029 which shows that the effect of unsteady flow is less than one percent and is not significant.

The above analysis assumed that the unsteady flow was gradually varied. For the case of rapidly varied flow in the form of a surge the constriction will not perform satisfactorily until the surge has passed. This condition should be evident from the values of y_1/D.
5. CONCLUSIONS AND RECOMMENDATIONS

1. A Venturi type flow meter for measurement of sewer flows under both open channel and full flow conditions is feasible.

2. Its advantages are low cost and simple operation. Two pressure sensors are required.

3. Its disadvantages are that the capacity of the pipe is reduced due to the added energy loss caused by the constriction and that the chances of clogging are increased.

4. The information in this report can be used to construct theoretical rating curves if experimental calibration is impractical or too costly. The procedure is described in Appendix A. The error in discharge without experimental calibration should be within 5 percent for the upper 75 percent of the open channel flow range if the error in pressure head measurement is within .01 D.

5. The difference in performance between the symmetrical and asymmetrical constrictions is not great. There is slightly lower energy loss for the asymmetrical case, however the wave pattern thus generated is more severe. The area-depth curves for the two cases are quite similar in shape and hence the rating curves are as well. Since a particular design requires a particular area-depth curve (r/D value) for optimum performance, the value of r/D required for the asymmetrical case will simply be larger than the symmetrical case.

6. The throat length should be at least 2.25 D. There is no advantage in a throat length larger than 4.0 D. The entrance and exit transitions should be on a 1:4 slope. Steeper slopes might increase the entrance loss coefficient and decrease the discharge coefficient. The upstream
measuring section should be located approximately $D/3$ upstream from the beginning of the entrance transition. The other measuring section should be located approximately the center of the throat.
REFERENCES


Appendix A

THEORETICAL CALIBRATION PROCEDURE

1. Determine the pipe diameter, slope and roughness for the installation.

2. From Figs. 20-23 determine the best value of r/D for the specified conditions as described in Sec. 4.10.

3. The solution to the energy equation for open channel flow is implicit and requires an iterative procedure. The best general procedure is to assume the discharge and then compute the corresponding $y_1$ value. The energy equation in appropriate dimensionless form is

$$ y_1 = y_2 + \frac{Q^2}{2} \left( \frac{1}{A_2^2} - \frac{1}{A_1^2} \right) - 1S_o + h_L $$

(1-A)

Since $y_2$ is critical it can be determined from Eq. 3 in dimensionless form

$$ \frac{Q^2B_2}{A_2} = 1 $$

(2-A)

A logical procedure is as follows:

a) Assume a normal depth, $y_n$.

b) Compute the discharge using either Manning's or the Darcy-Weisbach equation.

c) Solve Eq. 2-A for $A_2$. This is an implicit solution requiring the geometrical relationship between $y_2$, $B_2$ and $A_2$ as given in Appendix B.

d) Assume $y_1 = 1.2 y_n$. This is an initial guess and is used to compute $A_1$ and part of the head loss.

e) Using Eq. 16 compute $K_c$. 
f) Using Eqs. 15, 8, 9, 10 and 12 together with the appropriate
\( L_i \) values the \( h_L \) term can be evaluated.

g) With the computation of \( LS_0 \), all terms on the right hand side
of Eq. (1-A) are known and the initial value of \( y_1 \) can be
computed. This value should be compared with that assumed in
step 3(c). If they are not within the desired percentage
difference the new value of \( y_1 \) should be used and steps e-g
repeated.

h) A new value of \( y_n \) is assumed and steps 3(b)-(g) repeated.
This should be repeated until the rating curve is defined to
the level of detail desired.

4. The full flow calibration curve is somewhat simpler to compute since
both \( A_1 \) and \( A_2 \) are known. In dimensionless form the energy equation
can be written

\[
\Delta h = \frac{1}{2} \left( \frac{1}{A_2^2} - \frac{1}{A_1^2} \right) \left( \frac{2}{C_D} \right)^2
\]  (3-A)

a) Assume a value of \( Q \) beginning above the highest value on the
open channel rating curve.

b) Using Eq. 19 with \( K_c = 0.2 \) and the appropriate \( L_i \) values
compute \( C_D \). Note that if the friction factors are independent
of Reynolds number, \( C_D \) is constant.

c) Compute the corresponding \( \Delta h \) from Eq. 3-A. If \( C_D \) is constant
the rating curve is determined for all \( Q \). If not assume
increasing values of \( Q \) and repeat 3(b) and 3(c).
Appendix B
CONSTRUCTION GEOMETRY RELATIONSHIPS

In order to compute the theoretical rating curve it is necessary to describe mathematically the various geometrical properties of the constriction. Referring to Fig. 24, the throat cross section for the symmetrical case is constructed from two segments of circles of radius \( r \) with centers displaced a distance from the pipe centerline along its horizontal diametrical plane. A \( p, q \) coordinate system is established with its origin at the center of the pipe. This notation is used to avoid confusion with the depth of flow \( y \) which is measured from the pipe invert. The constriction intersects the pipe at \( p = \pm S, q = \pm q_s \). The angle subtended by the intersection of the water surface extended to intersect the pipe wall is denoted by \( \theta \).

The following relations can be written

\[
\frac{y}{D} = 0.5\left[1 - \cos \left(\frac{\theta}{2}\right)\right] \quad (1-B)
\]

\[
\frac{q_w}{D} = -0.5 \cos \left(\frac{\theta}{2}\right) \quad (2-B)
\]

\[
\theta = 2 \cos^{-1} \left(-\frac{2q_w}{D}\right) \quad (3-B)
\]

where \( q_w \) is the ordinate of any point on the water surface.

The expression for \( a \), the displacement of the constriction radius, in terms of the specified quantities \( r, S \) and \( D \) can be obtained from the simultaneous solution of the equations for the pipe and the constriction. The result is

\[
a = -S + \left(S^2 - 0.25D^2 + r^2\right)^{1/2} \quad (4-B)
\]
The cross section can be divided into three regions as shown in
Fig. 24 and expressions for the area and water surface width written for
each region. Region 1 is bounded below by the pipe invert and above by
$q_{w} = -q_{s}$ where

$$q_{s} = 0.5D \cos \left( \frac{\theta}{2} \right) \tag{5-B}$$

$$\theta_{s} = 2 \sin^{-1} \left( \frac{2S}{D} \right) \tag{6-B}$$

Within region 1 ($q_{w} \leq -q_{s}$)

$$A = \frac{D^{2}}{8} (\theta - \sin \theta) \tag{7-B}$$

$$\frac{B}{D} = \sin \frac{\theta}{2} \tag{8-B}$$

Within region 2 ($-q_{s} \leq q_{w} \leq q_{s}$)

$$A = \frac{D^{2}}{8} (\theta_{s} - \sin \theta_{s}) + q_{w} \left( r^{2} - q_{w}^{2} \right)^{1/2} + q_{s} \left( r^{2} - q_{s}^{2} \right)^{1/2}$$

$$+ r^{2} \left[ \sin^{-1} \left( \frac{q_{w}}{r} \right) + \sin^{-1} \left( \frac{q_{s}}{r} \right) \right] - 2a \left( q_{w} + q_{s} \right) \tag{9-B}$$

$$\frac{B}{D} = 2 \left[ (r^{2} - q_{w}^{2})^{1/2} - a \right] \tag{10-B}$$

Within region 3 ($q_{w} \geq q_{s}$)

$$A = A_{1} + \frac{D^{2}}{8} (\theta - \sin \theta) - \frac{D^{2}}{8} (2\pi - \theta_{s} - \sin \theta_{s}) \tag{11-B}$$
where \( A_1 = \text{Eq. 9-B evaluated at } q_w = q_s \)

\[
\frac{B}{D} = \sin \frac{\theta}{2} \tag{12-B}
\]

The corresponding area relationships for the asymmetrical case for regions 2 and 3 are given by

\[
A_{\text{asym}} = \frac{1}{2} A_{\text{sym}} + \frac{D^2}{16} (\theta - \sin \theta) \tag{13-B}
\]

\[
B_{\text{asym}} = \frac{1}{2} B_{\text{sym}} + \frac{D}{2} \sin \frac{\theta}{2} \tag{14-B}
\]

For full flow the area relationship for the symmetrical case becomes

\[
A = \frac{D^2}{4} (\theta_s - \sin \theta_s) + 2r^2 \sin^{-1} \left( \frac{q_s}{r} \right) + 4Sq_s \tag{15-B}
\]

The asymmetrical case can be easily computed from Eq. 15-B,

\[
A_{\text{asym}} = \frac{A_{\text{sym}}}{2} + \frac{\pi D^2}{8} \tag{16-B}
\]

Figure 25 shows a graphical relationship between area and \( r/D \) for full flow for \( S/D = 0.1 \). This shows that an asymmetrical constriction with \( r/D = 100 \) is equivalent to a symmetrical constriction with \( r/D = 0.66 \), assuming \( S/D = 0.1 \). This means that if a symmetrical constriction with \( r/D > 0.66 \) is required for control or accuracy, an equivalent asymmetrical constriction is not possible if \( S/D = 0.1 \).
FIG. 24 REFERENCE FIGURE - SYMMETRICAL CONSTRUCTION
FIG. 25 CONSTRICITION AREA VS. r/D FOR FULL FLOW