WRC RESEARCH REPORT NO. 67

ANALYSIS OF MULTIPLE-INPUT STOCHASTIC HYDROLOGIC SYSTEMS

Ven Te Chow
Principal Investigator

FINAL REPORT
Project No. B-038-ILL

The work upon which this publication is based was supported by funds provided by the U.S. Department of the Interior as authorized under the Water Resources Research Act of 1964, P.L. 88-379 Agreement No. 14-31-0001-3076

UNIVERSITY OF ILLINOIS
WATER RESOURCES CENTER
Hydrosystems Laboratory
Urbana, Illinois 61801

June, 1973
ABSTRACT

ANALYSIS OF MULTIPLE-INPUT STOCHASTIC HYDROLOGIC SYSTEMS

This report describes the development of a multiple-input stochastic hydrologic system model for the analysis of hydrologic behavior of watersheds. Various components of the hydrologic system are expressed by time series, each containing a trend component, a periodic component, and a stochastic residual component. For modeling the multiple-input system, a Markov-type mathematical formulation is proposed. For illustrative purposes, the model so formulated is applied to the analysis of monthly precipitations and streamflows of the upper Sangamon River basin in Illinois. The results of this study indicate that the proposed model is feasible for the basin and thus can be used for filling missing streamflow data or generating stochastic streamflow sequences.

Chow, Ven Te
ANALYSIS OF MULTIPLE-INPUT STOCHASTIC HYDROLOGIC SYSTEMS
Research Report No. 67 Water Resources Center, University of Illinois at Urbana-Champaign, June 1973,
KEYWORDS--systems analysis/stochastic processes/synthetic hydrology/water resources systems/watershed studies/hydrologic models/hydrology
CONTENTS

Preface .................................................. 1

I. Introduction ........................................... 1

II. Formulation of the Model .............................. 2

III. Analysis of the Data .................................. 4
    A. The Watershed and Hydrologic Data Under Study .... 4
    B. Preliminary Screening of the Data ......................... 4
    C. Analysis Using Cross-Spectra Theory ..................... 8
    D. Analysis Using Principal Components Theory .......... 13

IV. Conclusions and Comments ............................ 20

V. Acknowledgments ....................................... 22

VI. References ........................................... 23

VII. Figures ............................................... 24

Appendix. Computer Programs ............................. 40

The main objective of the Phase-II research is to develop general mathematical models for the simulation of stochastic hydrologic processes. The availability of such models is of paramount importance and valuable use in water resources systems planning and development. For this research, the following major achievements have been made:

1. Completion of the following four major studies:
   (a) Presentation of a lumped stochastic hydrologic system model [reported in paper (a) in item (2) below].
   (b) Development of a system model for residual stochastic hydrologic processes [reported in paper (h) in item (2) below].
   (c) Development of the theory, test, and modeling of stationarity embedded stochastic hydrologic processes [reported in paper (j) in item (2) below and in the Ph.D. thesis by Torelli in item (3) below].
(d) Development of a multiple-input, or spatially distributed stochastic hydrologic system model [reported in this final report].

(2) Publications of the following papers and articles:


(3) Completion of the following theses:


Presentation of seven papers at various technical conferences:

(a) "Analysis of Stochastic Hydrologic Systems," by V. T. Chow and S. J. Kareliotis at the 1970 Annual Meeting of the American Geophysical Union in Washington, D.C.


Many persons participated in this project and contributed to the research. In addition to the Principal Investigator, Ven Te Chow, the following research staff members were involved:

Gonzalo Cortes-Rivera, M.S., Ph.D., Research Assistant in Civil Engineering

Dong Hee Kim, M.S., Research Assistant in Civil Engineering.

Sotirios J. Kareliotis, M.S., Research Assistant in Civil Engineering, then Ph.D., Visiting Assistant Professor of Civil Engineering.

Nobutada Takase, Ph.D., Visiting Scholar in Civil Engineering.

Latino Torelli, M.S., Research Assistant in Civil Engineering.

Taylan A. Ula, B.S., Research Assistant in Civil Engineering.

James S. Windsor, Ph.D., Research Associate in Civil Engineering.

Most of the results obtained from this Phase-II project have been published elsewhere as listed in item (2) above, but the material presented in this report has not been published before.
I. INTRODUCTION

Modeling of stochastic hydrologic systems in the past has been based on the analysis of the record of a single hydrologic process under consideration, such as the modeling by Thomas and Fiering [1962] for streamflow simulation. Moreover, processes like the streamflow are the result of the interaction of many other processes such as precipitation, infiltration, etc., which vary from watershed to watershed and subsequently affecting the modeling of the streamflow record of the various watersheds. To incorporate these interacting processes in the modeling, the watershed can be modeled as a system with the streamflow as the output and the precipitation as the input. Chow and Ramaseshan [1965] have introduced such an approach for the analysis of floods by considering a single-input, single-output system with a time invariant deterministic system structure. Later in the earlier stage of this research program [Chow, 1969; Chow and Kareliotis, 1970] a modeling technique for a single-input, single-output hydrologic system was developed, taking into account the stochastic behavior of the system's structure. The single-input system is a simplification made on the assumption of a uniform spatial distribution of the precipitation. The present study deals with a multiple-input stochastic hydrologic system and the formulation of a mathematical model for such systems. For illustrative purposes, the model so formulated is applied to the analysis of the upper Sangamon River basin in Illinois. The multiple inputs are the hydrologic data obtained from the various recording precipitation stations over the watershed.
II. FORMULATION OF THE MODEL

Let the records of precipitation inputs be represented by a multiple time series \( \{X_{ij}; i = 1,2,\ldots,n; j = 1,2,\ldots,T\} \) where \( i \) is the precipitation station number, \( n \) is the total number of stations, \( j \) is the time periods of the recorded precipitation amount, and \( T \) is the total number of time intervals. Similarly, for the single streamflow output the time series is \( \{Y_j; j = 1,2,\ldots,T\} \).

It is assumed that the random variable \( U_t \) of a time series \( \{U_t; t \in T\} \) representing a stochastic hydrologic process can be expressed mathematically by

\[
U_t = g_t + p_t + U'_t
\]

where \( g_t \) is a trend component, \( p_t \) is a periodic component, and \( U'_t \) is a residual component which remains after the trend and periodic components are removed. The process \( \{U'_t; t \in T\} \) has been defined by Kareliotis and Chow [1972] as "the residual hydrologic stochastic process" of the original stochastic hydrologic process \( \{U_t; t \in T\} \).

Based on the above assumption and notation, each time series of the precipitation inputs can be expressed as

\[
X_{ij} = g_{ij} + p_{ij} + X'_{ij}
\]

and of the streamflow as

\[
Y_j = g_j + p_j + Y'_j
\]

where \( \{X'_{ij}; i = 1,2,\ldots,n; j = 1,2,\ldots,T\} \) and \( \{Y'_{j}; j = 1,2,\ldots,T\} \) are the residual hydrologic stochastic processes of the precipitation inputs and
the streamflow output, respectively. Before any further analysis, the hydrologic data should be first investigated for possible trends and periodic components, using such tools as power spectra. These components will be then removed, if they exist, and the analysis will proceed with the resulting residual hydrologic stochastic processes.

For the analysis of the multiple-input stochastic hydrologic system, a Markov-type mathematical model of the following form is proposed:

\[
y_j' = \sum_{k=1}^{m} a_k y_{j-k} + \sum_{k=0}^{q_1} b_{1k} x_{1,j-k} + \sum_{k=0}^{q_2} b_{2k} x_{2,j-k} + \ldots + \sum_{k=0}^{q_n} b_{nk} x_{n,j-k} + \varepsilon_j
\]

where \(a_1, a_2, \ldots, a_m, b_{10}, b_{11}, \ldots, b_{1q_1}, b_{20}, b_{21}, \ldots, b_{2q_2}, \ldots, b_{n0}, b_{n1}, \ldots, b_{nq_n}\) are coefficients to be determined; \(m\) is an integer depending on the content of significant autocorrelation in the streamflow record; \(q_1\) is also an integer depending on the content of significant crosscorrelation between the streamflow and the precipitation at the \(i\)-th station; and \(\varepsilon_j\) is an independent random residual.

To determine the values of \(m\) and \(q_1\), the theory of multiple cross-spectra may be applied, providing a very fast approach for estimating the existence of any significant autocorrelations and crosscorrelations in the data. Then, the coefficients of all the significant terms in Eq. (4) can be estimated by the least-square principle. A brief description of the multiple cross-spectra theory as it is applied in this study will be presented later.
III. ANALYSIS OF THE DATA

A. The Watershed and Hydrologic Data under Study

To demonstrate the feasibility of the proposed multiple-input stochastic hydrologic system model, the watershed chosen as the hydrologic system to be analysed in this study is the upper Sangamon River basin of 550 sq. mi. in size, above Monticello, Illinois, and located in east central Illinois. Figure 1 shows the map of the Sangamon River basin with the locations of the stream gaging station at Monticello and the precipitation gages where hydrologic data were observed for use in the analysis. The lengths of the available data are shown in Table 1.

B. Preliminary Screening of the Data

The preliminary screening of the data involves the investigation of each hydrologic record for its possible trend and periodic components, and the removal of such components, if they exist. The trend components in all records appear to be nonexistent from a careful observation of the data.

The next step involves the search for the periodic components. This consists of a computation of the power spectrum, or the spectrum, of each hydrologic record. Details of the theory of spectrum and its application for the analysis of time series is presented elsewhere [Chow and Kareliotis, 1970]. It must be reminded, however, that the computation of the spectrum involves the computation of the autocovariances $C_k$ for the time series $\{U_t; t \in T\}$ by the formula:

$$C_k = \frac{1}{T-k} \sum_{t=1}^{T-k} U_t U_{t+k} - \frac{1}{(T-k)^2} \sum_{t=1}^{T-k} U_t \sum_{t=1}^{T-k} U_{t+k}$$

where $k$ is the lag. In the present case, some of the records have intervals
TABLE 1. Available Hydrologic Data for Analysis

<table>
<thead>
<tr>
<th>Stations</th>
<th>Length of Record</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>For Precipitation:</strong></td>
<td></td>
</tr>
<tr>
<td>Bloomington</td>
<td>Jan. 1892 - Dec. 1969</td>
</tr>
<tr>
<td>Clinton</td>
<td>Jan. 1910 - Dec. 1969</td>
</tr>
<tr>
<td>Rantoul</td>
<td>Nov. 1891 - Mar. 1913</td>
</tr>
<tr>
<td></td>
<td>Mar. 1940 - Dec. 1969</td>
</tr>
<tr>
<td>Roberts</td>
<td>Jan. 1911 - Dec. 1969</td>
</tr>
<tr>
<td>Urbana</td>
<td>Sept. 1902 - Dec. 1969</td>
</tr>
<tr>
<td><strong>For Streamflow:</strong></td>
<td></td>
</tr>
<tr>
<td>Monticello</td>
<td>Mar. 1908 - Sept. 1912</td>
</tr>
<tr>
<td></td>
<td>Nov. 1912 - Dec. 1912</td>
</tr>
<tr>
<td></td>
<td>July 1914 - Sept 1969</td>
</tr>
</tbody>
</table>
of missing data. When Eq. (5) is used, the pairs of values of \( U_t \) and \( U_{t+k} \) for which either one or both values are missing should not be taken into account in the summations. A general computer subroutine has been programmed (see Appendix) which can compute the spectrum of any record with or without intervals of the missing data.

The spectra \( f(\omega) \) for the six hydrologic records of precipitation are shown in Figs. 2 through 6. These spectra were computed for a maximum lag \( k = 60 \). Observing these spectra it can be seen that there are high peaks at the 12-month periodicity and smaller peaks but still higher than other peaks at the 6-month periodicity. Similarly for the spectrum of streamflow as computed and shown in Fig. 7, there is only one peak at the 12-month periodicity. Based on these observations, it is concluded that for precipitation the periodic components consist of 12-month and 6-month periodicities, while for streamflow the periodic component is a 12-month periodicity. Thus, for the general case of the hydrologic time series \( \{ U_t; t \in T \} \) Eq. (1) can be written in the form:

\[
U_t = C_1 + C_2 \cos \frac{2\pi t}{12} + C_3 \sin \frac{2\pi t}{12} + C_4 \cos \frac{2\pi t}{6} + C_5 \sin \frac{2\pi t}{6} + U'_t
\]  
(6)

where \( C_1 \) is a coefficient equal to the mean of \( U_t; C_2, C_3, C_4, \) and \( C_5 \) are the coefficients of the periodic components of 12-month and 6-month periodicities; and \( U'_t \) is the residual hydrologic stochastic component with zero mean due to the introduction of the coefficient \( C_1 \).

For each hydrologic record the above coefficients are computed using the least-square method. The values of the coefficient are given in Table 2, which also shows the computed variances of the original hydrologic records and the stochastic residual records. A general computer subroutine was written (see Appendix) for the computations which can also handle hydrologic records with intervals of missing data.
TABLE 2. Coefficients of Time Series Models and Variances of Hydrologic Records

<table>
<thead>
<tr>
<th>Station</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
<th>Variance</th>
<th>Original records</th>
<th>Stochastic residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>For Precipitation:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bloomington</td>
<td>3.0200</td>
<td>-0.9736</td>
<td>-0.0991</td>
<td>-0.0152</td>
<td>-0.3768</td>
<td>4.0118</td>
<td>3.4767</td>
<td></td>
</tr>
<tr>
<td>Clinton</td>
<td>3.2258</td>
<td>-1.0720</td>
<td>-0.2524</td>
<td>-0.0813</td>
<td>-0.4801</td>
<td>4.8724</td>
<td>4.1702</td>
<td></td>
</tr>
<tr>
<td>Rantoul</td>
<td>2.9565</td>
<td>-0.9085</td>
<td>-0.0126</td>
<td>0.1821</td>
<td>-0.3502</td>
<td>3.8022</td>
<td>3.3086</td>
<td></td>
</tr>
<tr>
<td>Roberts</td>
<td>2.7736</td>
<td>-0.9578</td>
<td>-0.1402</td>
<td>-0.0620</td>
<td>-0.3940</td>
<td>3.3672</td>
<td>2.8348</td>
<td></td>
</tr>
<tr>
<td>Urbana</td>
<td>3.0403</td>
<td>-0.8514</td>
<td>-0.0203</td>
<td>-0.0230</td>
<td>-0.3415</td>
<td>3.4456</td>
<td>3.0224</td>
<td></td>
</tr>
<tr>
<td>For Streamflow:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monticello</td>
<td>0.7982</td>
<td>-0.6151</td>
<td>-0.1968</td>
<td>...</td>
<td>...</td>
<td>1.0539</td>
<td>0.8423</td>
<td></td>
</tr>
</tbody>
</table>
C. **Analysis Using Cross-Spectra Theory**

A powerful tool in the analysis of the multiple time series of the residual hydrologic stochastic processes is the cross-spectra theory [Goodman, 1965; Bendat and Piersol, 1966; Kareliotis and Chow, 1972]. Suppose that the multiple time series consists of two time series \( \{x_t; t \in T\} \) and \( \{y_t; t \in T\} \) and their cross-spectrum is

\[
C_{xy}(\omega) = p_{xy}(\omega) + iq_{xy}(\omega)
\]  

(7)

where \( i = \sqrt{-1} \), \( \omega \) is the angular frequency, \( p_{xy}(\omega) \) is the co-spectrum and \( q_{xy}(\omega) \) is the quadrature spectrum. Estimates of co-spectrum and quadrature spectrum are expressed by

\[
L[\hat{p}_{xy}(\omega_j)] = \frac{1}{2\pi} \sum_{k=0}^{m} \frac{1}{2} [C'_{(xy)k} + C'_{(yx)k}] \cos k\omega_j
\]  

(8)

and

\[
L[\hat{q}_{xy}(\omega_j)] = \frac{1}{2\pi} \sum_{k=0}^{m} \frac{1}{2} [C'_{(xy)k} - C'_{(yx)k}] \sin k\omega_j
\]  

(9)

where \( \omega_j = \pi j/m \) with \( j = 0, 1, 2, \ldots, m; \) and

\[
C'_{(xy)0} = C_{(xy)0}
\]

\[
C'_{(xy)m} = C_{(xy)m}
\]

\[
C'_{(xy)k} = 2C_{(xy)k} \quad \text{for } k = 1, 2, \ldots, m-1
\]  

(10)

where \( C_{(xy)k} \) is the cross-covariance between \( \{x_t\} \) and \( \{y_t\} \) of lag \( k \), and \( m \) is taken as less than \( T/10 \). The estimate of \( C_{(xy)k} \) is
The "raw" co-spectrum and quadrature spectrum estimates are "smoothed" by the introduction to Eqs. (8) and (9) of the "Tukey-Hamming" weights given by Blackman and Tukey [1959]:

\[ \lambda_k(\omega_j) = 0.54 + 0.46 \cos(\pi k/m) \]  

Thus, the final smoothed co-spectrum and quadrature spectrum estimates are

\[
\begin{align*}
U_*(\omega_j) &= 0.23 L_*(\omega_{j-1}) + 0.54 L_*(\omega_j) + 0.23 L_*(\omega_{j+1}) \\
U_*(\omega_0) &= 0.54 L_*(\omega_0) + 0.46 L_*(\omega_1) \\
U_*(\omega_m) &= 0.54 L_*(\omega_m) + 0.46 L_*(\omega_{m-1})
\end{align*}
\]  

where \( U_*(\omega) \) and \( L_*(\omega) \) are either the smoothed and raw co-spectrum estimates respectively, or the smoothed and raw quadrature spectrum estimates respectively. For the case of a multiple time series \( \{X_{1t}, X_{2t}, \ldots, X_{nt}; t \in T\} \), Eq. (7) with the above formulas can be used to compute the cross-spectra matrix:

\[
S(\omega) = \begin{bmatrix}
C_{11}(\omega) & C_{12}(\omega) & C_{13}(\omega) & \cdots & C_{1m}(\omega) \\
C_{21}(\omega) & C_{22}(\omega) & C_{23}(\omega) & \cdots & C_{2m}(\omega) \\
C_{31}(\omega) & C_{32}(\omega) & C_{33}(\omega) & \cdots & C_{3m}(\omega) \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
C_{n1}(\omega) & C_{n2}(\omega) & C_{n3}(\omega) & \cdots & C_{nm}(\omega)
\end{bmatrix}
\]
where \( C_{ij}(\omega) \) are the cross-spectrum between the series \( \{X_{it}\} \) and \( \{X_{jt}\} \) for \( i \neq j \) and the spectrum of the series \( \{X_{it}\} \) for \( i = j \).

The matrix \( S(\omega) \) of Eq. (14) can be written also as

\[
S(\omega) = \begin{bmatrix}
S_{11}(\omega) & S_{12}(\omega) \\
S_{21}(\omega) & S_{22}(\omega)
\end{bmatrix}
\]

(15)

where

\[
S_{11}(\omega) = \begin{bmatrix}
C_{11}(\omega) & C_{12}(\omega) \\
C_{21}(\omega) & C_{22}(\omega)
\end{bmatrix}
\]

(16)

\[
S_{12}(\omega) = \begin{bmatrix}
C_{13}(\omega) & \cdots & C_{1m}(\omega) \\
C_{23}(\omega) & \cdots & C_{2m}(\omega)
\end{bmatrix}
\]

(17)

\[
S_{21}(\omega) = \begin{bmatrix}
C_{n1}(\omega) & C_{n2}(\omega) \\
\cdots & \cdots
\end{bmatrix}
\]

(18)

and

\[
S_{22}(\omega) = \begin{bmatrix}
C_{33}(\omega) & \cdots & C_{3m}(\omega) \\
\cdots & \cdots & \cdots
\end{bmatrix}
\]

(19)
Then a new matrix, called "the partial cross-spectra matrix," can be computed as follows:

\[
S_{12k}(\omega) = \begin{vmatrix}
Cr_{11k}(\omega) & Cr_{12k}(\omega) \\
Cr_{21k}(\omega) & Cr_{22k}(\omega)
\end{vmatrix}
\]

\[
= S_{11}(\omega) - S_{12}(\omega) S_{22}^{-1}(\omega) S_{21}(\omega)
\] (20)

The \(S_{12k}(\omega)\) matrix is a 2 \(\times\) 2 matrix and its elements are the partial spectra and partial cross-spectra of the series \(\{X_{1t}\}\) and \(\{X_{2t}\}\) computed by Eq. (20). It must be noted at this point that the computations in Eq. (20) involve the inversion of the square matrix \(S_{22}(\omega)\) but such an inversion exists only when the matrix is non-singular, i.e., its determinant is nonzero.

The significance of computing the partial cross-spectra matrix is that it is thereby possible to compute two parameters: the partial coherence and the partial phase, which provide valuable information about the correlation of the series \(\{X_{1t}\}\) and \(\{X_{2t}\}\) without the influence of the other series of the multiple time series. The partial coherence is given by

\[
Ch_{12k}(\omega) = \frac{|Cr_{12k}(\omega)|^2}{Cr_{11k}(\omega)Cr_{22k}(\omega)}
\] (21)

and the partial phase by

\[
\phi_{12k}(\omega) = \tan^{-1} \frac{\text{imaginary part of } Cr_{12k}(\omega)}{\text{real part of } Cr_{12k}(\omega)}
\] (22)
The partial coherence is a measure of the correlation of the two time series without the influence of the other time series. When plotted against $\omega$, it produces the coherence diagram. Similarly, the partial phase will produce the phase diagram which is a measure of the phase relationship between the two series. [Granger, 1964].

In the present study the multiple time series consists of the time series of the residual hydrologic stochastic processes of the output streamflow and the input precipitations, i.e., $m = 6$. To study the dependence of the residual hydrologic stochastic process of streamflow with each one of the residual hydrologic stochastic processes of the precipitation, the partial cross-spectra of streamflow and each one of the precipitation records are computed. Thus, in the multiple time series, $\{X_{1t}\} = \{Y_{1}\}$ always represents the record of streamflow and $\{X_{2t}\}$ is the precipitation record which is under investigation for determining its influence on the streamflow. For example, if the precipitation record at Bloomington is investigated, the partial cross-spectra matrix of the streamflow and the Bloomington precipitation record has to be computed. Then $\{X_{1t}\}$ will be the streamflow record, $\{X_{2t}\}$ be the Bloomington precipitation record and $\{X_{3t}\}, \{X_{4t}\}, \{X_{5t}\}$, $\{X_{6t}\}$ be the rest of the precipitation records. A general computer program was prepared (see Appendix) for determining the partial cross-spectra matrix and then the partial coherence and the partial phase according to the formulas mentioned previously, for any number of precipitation stations and any combination of missing data from the records. However, this attempt to compute the partial cross-spectra matrix was unsuccessful because, as it was noticed before, the computation involves the inversion of matrix $S_{22}(\omega)$ of Eq. 19 which happens to be singular; i.e., its determinant is equal or very close to zero. The cause of singularity in matrix $S_{22}(\omega)$ is believed to be attributed to the high correlated precipitation records. It can be
proved very easily that if the time series \( \{X_{3t}\}, \{X_{4t}\}, \{X_{5t}\}, \) and \( \{X_{6t}\} \) are highly correlated then each pair of columns of matrix \( S_{22}(\omega) \) have elements which are proportional to each other and as a result of this the matrix is singular. The high degree of correlation between the precipitation records can be shown to be true by computing the coherence diagrams for a number of pairs of precipitation stations. Four such typical coherence diagrams are plotted in Figs. 8, 9, 10, and 11, which indicate a high degree of correlation among the precipitation stations. As a result of this analysis it is concluded that the precipitation record at one station only is sufficient for the description of the precipitation influence on streamflow.

The analysis here indicates that the record at one station is sufficient to describe the dependence of streamflow on precipitation. Thus, Eq. (4) can be written in the following form:

\[
Y'_j = \sum_{k=1}^{m} a_k Y'_{j-k} + \sum_{k=0}^{q} b_k X'_{i,j-k} + \varepsilon_j \tag{23}
\]

where \( \{X'_{i,j}\} \) can be any of the five residual precipitation records. Looking at Fig. 7 of the spectrum of streamflow, it can be concluded that the content of significant autocorrelation in streamflows is equal to one [Kendall and Stuart, 1966] and therefore, Eq. (23) can be further simplified as follows:

\[
Y'_j = a Y'_{j-1} + \sum_{k=0}^{q} b_k X'_{i,j-k} + \varepsilon_j \tag{24}
\]

To estimate the magnitude of \( q \), the phase diagram is computed between the streamflow and one record of the precipitation stations. Figure 12 shows the phase diagram between the streamflow and the precipitation at Urbana. Since
this diagram does not indicate any trend [Granger, 1964], it can be concluded that \( q = 0 \). Thus, Eq. (24) can be written as

\[
Y'_j = a Y'_{j-1} + b X'_{1j} + \varepsilon_j
\]  

(25)

where \( \{X'_{1j}\} \) is any of the five residual precipitation records. The coefficients \( a \) and \( b \) in Eq. (25) are computed by the least-square method. In the first trial of computations, the record \( \{X'_{1j}\} \) is that at Bloomington and the results indicate that the variance of \( \varepsilon_j \) is equal to 0.4816 while the original variance of \( Y'_j \) was 0.8423 (Table 2); i.e., a reduction in the variance of about 42%. Similarly, for the record at Clinton, the variance is 0.4582 or a 46% reduction; for the record at Roberts, it is 0.5075 or a 40% reduction; and for the record at Urbana, it is 0.4473 or a 47% reduction. The record at Rantoul was not tried because of the too large amount of missing data. As a result of these computations, the precipitation record at Urbana which provides the greatest reduction in variance is recommended for use in Eq. (25). The coefficients for the Urbana record are: \( a = 0.3538 \) and \( b = 0.2897 \).

D. Analysis Using Principal Components Theory

After having concluded that one variable from the five precipitation variables is sufficient for the description of the streamflow dependence on the precipitation, the question on which precipitation station record should be used in a regression equation for the streamflow is then considered. For this question, a principal component transformation of the five precipitation variables is made. That component which indicates the greatest amount of the streamflow variation is then selected.
The principal component transformation is a mathematical technique [Kendall and Stuart, 1966] which is applicable to multiple time series. According to this technique, the original multiple time series is transformed to a new multiple time series where the time series are no longer cross-correlated as in the original multiple time series. Suppose that the multiple time series of the precipitation inputs is \( \{X_{1t}, X_{2t}, \ldots, X_{nt}; t \in T\} \) [noting the difference from the corresponding \( \{X_{2t}, X_{3t}, \ldots, X_{n+1,t}\} \) defined previously for computer programs], or in a matrix form:

\[
X = \begin{bmatrix}
X_{11} & X_{12} & \cdots & X_{1T} \\
X_{21} & X_{22} & \cdots & X_{2T} \\
& \vdots & \ddots & \vdots \\
X_{n1} & X_{n2} & \cdots & X_{nT}
\end{bmatrix}
\]  

(26)

Then, the transformation involves the computation of a new multiple time series:

\[
\zeta = \begin{bmatrix}
\zeta_{11} & \zeta_{12} & \cdots & \zeta_{1T} \\
\zeta_{21} & \zeta_{22} & \cdots & \zeta_{2T} \\
& \vdots & \ddots & \vdots \\
\zeta_{n1} & \zeta_{n2} & \cdots & \zeta_{nT}
\end{bmatrix}
\]  

(27)

where the time series \( \{\zeta_{1t}; t \in T\}, \{\zeta_{2t}; t \in T\}, \ldots, \{\zeta_{nt}; t \in T\} \) are not cross-correlated and they are called "principal components." It must be noted, however, that by cross-correlation in this analysis it is meant cross-correlation of zero lag and the principal component analysis eliminates only the zero lag cross-correlation. But, looking at Fig. 13, which is a typical
phase diagram for the precipitation data, it is obvious [Granger, 1964] that
the precipitation data are only cross-correlated at zero lag. Thus, the
principal component transformation of the precipitation data will provide
a set of time series which are not cross-correlated at any lag.

The computation of matrix $\zeta$ involves the computation of the fol-
lowing matrix:

\[
A = \begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1m} \\
a_{21} & a_{22} & \cdots & a_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \cdots & a_{nm}
\end{bmatrix}
\]  

(28)

which is related to matrix $X$ by the following equation:

\[
\zeta = AX
\]  

(29)

If $C$ is the matrix of covariances and cross-covariances of lag zero of the
data in matrix $X$; i.e.,

\[
C = \begin{bmatrix}
c_{11} & c_{12} & \cdots & c_{1m} \\
c_{21} & c_{22} & \cdots & c_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
c_{n1} & c_{n2} & \cdots & c_{nm}
\end{bmatrix}
\]  

(30)

where $c_{ij}$ is computed according to Eq. (11) for $k = 0$. Then it can be
proved [Kendall and Stuart, 1966] that the rows of matrix $A$ are the latent
vectors of Matrix $C$ and the latent rods of Matrix $C$ are the covariances of
zero lag or the variances of the time series of the multiple time series
\{\xi_{1t}, \xi_{2t}, \ldots, \xi_{nt}, t \in T\}. 
<table>
<thead>
<tr>
<th>Matrix C (dispersion matrix)</th>
<th>Latent roots (eigenvalues)</th>
<th>Latent vectors (eigenvectors)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.47987 2.88683 2.45393</td>
<td>2.41597 10.95903</td>
<td>0.51474 0.56335 0.44055 0.47287</td>
</tr>
<tr>
<td>2.88683 4.18009 2.33947</td>
<td>2.75407 1.15100</td>
<td>0.37937 -0.69895 0.59180 -0.31163</td>
</tr>
<tr>
<td>2.45393 2.33947 2.81457</td>
<td>2.12946 0.80936</td>
<td>-0.48390 -0.32114 0.11031 0.80656</td>
</tr>
<tr>
<td>2.41597 2.75407 2.12946</td>
<td>3.06413 0.61928</td>
<td>-0.59747 0.30163 0.66598 -0.32944</td>
</tr>
</tbody>
</table>
In the present analysis only data from four precipitation stations were used because the data from Rantoul were ignored due to the large amount of missing parts. Table 3 shows the computed matrix C latent rods and vectors. In the literature, it is often to call the latent rods as eigenvalues, the latent vectors as eigenvectors, and Matrix C as dispersion matrix. In this computation the series \( \{X_{1t}\} \) is Bloomington, \( \{X_{2t}\} \) for Clinton, \( \{X_{3t}\} \) for Roberts and \( \{X_{4t}\} \) for Urbana. Then using the computed values for matrix A (Table 3) the new series \( \{\xi_{1t}\}, \{\xi_{2t}\}, \{\xi_{3t}\}, \{\xi_{4t}\} \) were computed.

In order to determine which of the new time series will explain the greatest variation of the streamflow record, the coherence diagrams for each of the time series \( \{\xi_{1t}\} \) and for the residual hydrologic stochastic process of the streamflow were computed. These diagrams are shown in Figs. 14, 15, 16, and 17. It is obvious that the time series \( \{\xi_{1t}\} \) is the best for the purpose due to the high coherence. To determine the lag of the cross-correlation, the phase diagram between the time series \( \{\xi_{1t}\} \) and \( \{Y_t\} \) is plotted in Fig. 18. This graph indicates a zero lag in the crosscorrelation [Granger, 1964].

Going back to Fig. 7 of the spectrum of the streamflow, it can be observed that the streamflow is also serially correlated with lag one due to the shape of the spectrum [Kendall and Stuart, 1966]. Thus, the streamflow residuals will be serially correlated with lag one and cross-correlated to \( \{\xi_{1t}\} \) with lag zero. Based on this conclusion, the following model can be suggested:

\[
Y'_j = a'Y'_{j-1} + b' \xi_{1j} + \varepsilon_j
\]  

(27)

where the coefficients \( a' \) and \( b' \) are computed by the least-square method as \( a' = 0.3488 \) and \( b' = 0.1611 \).
The variance of $e_j$ is found to be 0.4184; i.e., there is a 50% reduction of the original variance of 0.8423. Therefore, the model of Eq. (27) is not any significantly better than the model of Eq. (25) because the reduction in variance is almost the same in both models. Since the modeling procedure for deriving Eq. (25) involves less computations than the one for Eq. (27), the former is recommended for the modeling of the single-input, single-output watershed systems.
IV. CONCLUSIONS AND COMMENTS

As a result of the analysis of the hydrologic data from the upper Sangamon River basin for the proposed modeling of multiple-input stochastic hydrologic systems, the following conclusions and comments may be given:

(a) The hydrologic watershed system can be theoretically simulated as a multiple-input, single-output system and orderly analyzed by the multiple cross-spectra theory.

(b) For the monthly data from the upper Sangamon River basin the single-input, single-output system is the appropriate model due to the highly correlated precipitation inputs.

(c) Principal component transformation of the precipitation data does not provide a component which would be better cross-correlated with the streamflow than any of the original time series of the precipitation inputs.

(d) The monthly precipitation records consist of 12-month and 6-month periodicities while the streamflow record consists only of a 12-month periodicity. The effect of the watershed storage may be the cause for the filtering of the 6-month periodicity.

(e) The residual hydrologic stochastic process of streamflow is serially correlated due to the storage effect of the watershed and it is also cross-correlated to the residual hydrologic stochastic process of precipitations.

(f) The proposed model can be used for filling in missing streamflow data or generating stochastic streamflow sequences.

(g) The proposed approach of the multiple-input, single-output system can be further checked using data of shorter time intervals than one month. For example, weekly data must be more sensitive to the spatial distribution of precipitation than the monthly data used in the present study.
(h) Finally, the method recommended herein can be improved by incorporating additional system components such as evapotranspiration, watershed storage, etc.
V. ACKNOWLEDGMENTS

This report is a result of the research project on "Stochastic Analysis of Hydrologic Systems - Phase II" which began in July 1969 and was completed in June 1973. The work upon which this publication is based was supported in part by funds provided by the United States Department of the Interior as authorized under the Water Resources Act of 1964, Public Law 88-379, Agreement No. 14-31-0001-3076. Under the direction of the Project Investigator, the hydrologic data used in this study were mainly collected by Dr. Gonzalo Cortes-Rivera and the mathematical analysis and computer programming were largely performed by Dr. Sotirios J. Kareliotis.
VI. REFERENCES


VII. FIGURES

Fig. 1. Sangamon River basin above Monticello, Illinois

Fig. 2. Spectrum of Precipitation at Bloomington, Illinois

Fig. 3. Spectrum of Precipitation at Clinton, Illinois

Fig. 4. Spectrum of Precipitation at Urbana, Illinois

Fig. 5. Spectrum of Precipitation at Roberts, Illinois

Fig. 6. Spectrum of Precipitation at Rantoul, Illinois

Fig. 7. Spectrum of Streamflow at Monticello, Illinois

Fig. 8. Partial Coherence of the Precipitation Stochastic Residuals at Bloomington and Clinton, Illinois

Fig. 9. Partial Coherence of the Precipitation Stochastic Residuals at Clinton and Rantoul, Illinois

Fig. 10. Partial Coherence of the Precipitation Stochastic Residuals at Rantoul and Roberts, Illinois

Fig. 11. Partial Coherence of the Precipitation Stochastic Residuals at Roberts and Urbana, Illinois

Fig. 12. Phase Diagram Between the Stochastic Residuals of Streamflow at Monticello, Illinois, and of Precipitation at Urbana, Illinois

Fig. 13. Phase Diagram Between the Stochastic Residuals of Monthly Precipitations at Bloomington and Clinton, Illinois

Fig. 14. Coherence of the Streamflow Stochastic Residuals at Monticello, Illinois, and the Series $\{z_1\}$

Fig. 15. Coherence of the Streamflow Stochastic Residuals at Monticello, Illinois and the Series $\{z_2\}$

Fig. 16. Coherence of the Streamflow Stochastic Residuals and the Series $\{z_3\}$

Fig. 17. Coherence of the Streamflow Stochastic Residuals at Monticello, Illinois and the Series $\{z_4\}$

Fig. 18. Phase Diagram Between the Streamflow Stochastic Residuals and the Series $\{z_1\}$
FIG. I. SANGAMON RIVER BASIN ABOVE MONTICELLO, ILLINOIS
Fig. 2. Spectrum of Precipitation at Bloomington, Illinois

Fig. 3. Spectrum of Precipitation at Clinton, Illinois
Fig. 4. Spectrum of Precipitation at Urbana, Illinois

Fig. 5. Spectrum of Precipitation at Roberts, Illinois
**Fig. 6. Spectrum of Precipitation at Rantoul, Illinois**

**Fig. 7. Spectrum of Streamflow at Monticello, Illinois**
Fig. 8. Partial Coherence of the Precipitation Stochastic Residuals at Bloomington and Clinton, Illinois
Fig. 9. Partial Coherence of the Precipitation Stochastic Residuals at Clinton and Rantoul, Illinois
Fig. 10. Partial Coherence of the Precipitation Stochastic Residuals at Rantoul and Roberts, Illinois
Fig. 11. Partial Coherence of the Precipitation Stochastic Residuals of Roberts and Urbana, Illinois
Fig. 12. Phase Diagram between the Stochastic Residuals of Streamflow at Monticello, Illinois and of Precipitation at Urbana, Illinois
Fig. 13. Phase diagram between the stochastic residuals of monthly precipitations at Bloomington and Clinton, Illinois.
Fig. 14. Coherence of the Streamflow Stochastic Residuals at Monticello, Illinois and the Series \( \{ \xi_1 \} \)
Fig. 15. Coherence of the Streamflow Stochastic Residuals at Monticello, Illinois and the Series \( \{ \xi_2 \} \)
Fig. 16. Coherence of the Streamflow Stochastic Residuals at Monticello, Illinois and the Series \( \{\xi_3\} \)
Fig. 17. Coherence of the Streamflow Stochastic residuals at Monticello, Illinois and the series \( \{ \xi_n \} \)
FIG. 18. PHASE DIAGRAM BETWEEN THE STREAMFLOW STOCHASTIC RESIDUALS AT MONTCALLO, ILLINOIS AND THE SERIES \( \{\xi_j\} \)
APPENDIX: COMPUTER PROGRAMS
Prepared by S. J. Kareliotis
PROGRAM FOR THE COMPUTATION OF SPECTRA

DIMENSION X(12,110),U(1320),COV(61),OMF(61),BN(61),SN(61)
DIMENSION TITLE(20)
DIMENSION M1(10),M2(10)
EQUIVALENCE (X(1,1),U(1))

Z>u> OR Z>U> IS THE HYDROLOGIC VARIABLE

1 READ(5,98,FND=2) (TITLE(I),I=1,20)
98 FORMAT(20A4)
WRITE(6,97) (TITLE(I),I=1,20)
97 FORMAT(*1//4X,20A4)
READ(5,98) (TITLE(I),I=1,20)
WRITE(6,96) (TITLE(I),I=1,20)
96 FORMAT(*4X,20A4)
READ(5,99) N,N12,NN,M
99 FORMAT(4I5)

Z>N> IS THE NUMBER OF YEARS (INPUT)
Z>N12> IS THE NUMBER OF MONTHS, I.E., N12=N*12 (INPUT)
Z>NN> IS THE MAXIMUM LAG FOR THE COVARIANCES INCREASED BY ONE (INPUT)
Z>2M> IS THE NUMBER OF INTERVALS OF MISSING DATA

IF(M,F0,0) GO TO 81
DO 82 I=1,M
82 READ(5,102) M1(I),M2(I)
102 FORMAT(2I5)
81 DO 10 I=1,N
10 READ(5,100) (X(J,I),J=1,12)
FOR X(J,I)=J=THE MONTH, AND I=THE YEAR
100 FORMAT(12F5.2)

COMPUTATION OF AUTOCOVARIANCES

DO 20 K=1,NN
KK=K-1
JK=N12-KK
SX1X2=0.0
SX1=0.0
SX2=0.0
IN=0
DO 21 T=1,JK
IKK=I+KK
IF(M,F0,0) GO TO 83
DO 26 J=1,M
IF((I GE M1(J)) .AND. (I LE M2(J))) .OR. ((IKK GE M1(J)) .AND. (IKK LE M2(J))) GO TO 21
26 CONTINUE
83 SX1X2= SX1X2+U(I)*U(IKK)
SX1=SX1+U(T)
SX2=SX2+U(IKK)
IN=IN+1

DO 20 K=1,NN
KK=K-1
JK=N12-KK
SX1X2=0.0
SX1=0.0
SX2=0.0
IN=0
DO 21 T=1,JK
IKK=I+KK
IF(M,F0,0) GO TO 83
DO 26 J=1,M
IF((I GE M1(J)) .AND. (I LE M2(J))) .OR. ((IKK GE M1(J)) .AND. (IKK LE M2(J))) GO TO 21
26 CONTINUE
83 SX1X2= SX1X2+U(I)*U(IKK)
SX1=SX1+U(T)
SX2=SX2+U(IKK)
IN=IN+1
CONTINUE
C    CJK=IN
20 COV(K)=(SX1*2/CJK)-(SX1+SX2/(CJK**2))
C    COV(K) IS THE AUTOCOVARIANCE OF LAG K=1
WRITE(6,111)
111 FORMAT(/'10X,'15H TABLE OF SPECTRUM,'/5X,'6HPERIOD,5X,'22HRAW SPECTRAL
C    ESTIMATES, 5X, 27H SMOOTHED SPECTRAL ESTIMATES, 4X, 9HFREQUENCY/) 
N1=NN=1
CN1=N1
C
COMPUTATION OF RAW AND SMOOTHED SPECTRA
C    OME(J) IS THE FREQUENCY
C    BNC(J) IS THE RAW SPECTRUM ESTIMATE AT FREQUENCY OME(J)
C    SN(J) IS THE SMOOTHED SPECTRUM ESTIMATE AT FREQUENCY OME(J)
C
DO 11 J=1,NN
C    CJJ=J-1
SS=0.0
AR=CJ*3,14159
DO 12 I=2,N1
C    CJI=I-1
SS=SS+COV(I)*COS(CJ*AR/CN1)
BNC(J)=(COV(1)+2.*SS+CDV(NN)*COS(AR))/(2.*3,14159)
11 OME(J)=AR/CN1
SN(J)=0.46*BNC(2)+0.54*BNC(1)
J=0
WRITE(6,112) J,BNC(1),SN(1),OME(1)
112 FORMAT(/'15X,'15X,'15X,'15X,'15X,'15X,'15X,'15X,'15X,'15X,'15X,'15X,'15X,'15X,'15X,'15X,'15X,'15X,'15X,'15X,'15X,
C    BN(J*13,11X,E12,4,19X,E12,4*,8X,E12,4)
DO 13 I=2,N1
J=I-1
JM=I+1
SN(J)=0.23*BNC(J)+0.54*BNC(I)+0.23*BNC(JM)
13 WRITE(6,112) J,BNC(I),SN(I),OME(I)
SN(NN)=0.46*BNC(NN)+0.54*BNC(NN)
WRITE(6,112) N1,BNC(NN),SN(NN),OME(NN)
GO TO 1
2 STOP
END
PROGRAM FOR THE ESTIMATION OF THE COEFFICIENTS OF THE PRIMARY
DETERMINISTIC COMPONENT AND THE STOCHASTIC RESIDUALS.
THE PRIMARY DETERMINISTIC COMPONENT CONSISTS OF THE 6-MONTH AND
12-MONTH PERIODICITIES.
THE LEAST-SQUARE METHOD IS USED ACCORDING TO THE PROCEDURE OF
R.C. BROWN IN SMOOTHING, FORECASTING AND PREDICTION OF DISCRETE

DIMENSION X(12,110), U(1320), IYR(110), A(5), F(1320,5), FF(5,5), G(5)
DIMENSION TITLE(20), TITL1(20), M1(10), M7(10), FE(1320,5), U(1320)
EQUIVALENCE (X(1,1), U(1))

"X" OR "U" IS THE HYDROLOGIC VARIABLE

THE FUNCTION OF THE PRIMARY DETERMINISTIC COMPONENT IS,
FUNCTION SUM (A(J)*F(I+J)) FOR J=1,...,NM
WHERE I DENOTES THE I-TH TIME INTERVAL
READ(5,95)NM
 1 READ(5,98,FND=2) (TITLE(I), I=1,20)
98 FORMAT(20A4)
  RTAN(5,98) (TITL1(I), I=1,20)
  WRITE(6,97) (TITLE(I), I=1,20)
97 FORMAT("","/",4X,20A4)
  WRITE(6,96) (TITL1(I), I=1,20)
96 FORMAT(4X,20A4)
  READ(5,98) (TITL1(I), I=1,20)
  WRITE(6,96) (TITL1(I), I=1,20)
95 FORMAT(I5)

READ(5,98) (TITL1(I), I=1,20)
  WRITE(6,96) (TITL1(I), I=1,20)
  READ(5,99) N, N12, M
99 FORMAT(215,5X,I5)

"M" IS THE NUMBER OF YEARS (INPUT)
"N12" IS THE NUMBER OF MONTHS, I.E., N12=N*12 (INPUT)
"M" IS THE NUMBER OF INTERVALS OF MISSING DATA

COMPUTATION OF THE MATRIX F(I,J)

DO 60 I=1,N12
  CI=I
  F(I,1)= 1.0
  AR6=3.14159265*CI/3.0
  AR12=AR6/2.0
  F(I,2)=COS(AR12)
  F(I,3)=SIN(AR12)
  F(I,4)=COS(AR6)
  F(I,5)=SIN(AR6)
60 CONTINUE
IF(M,F0,0) GO TO 21
DO 201 I=1,M
201 READ(5,202) M(I),M2(I)
202 FORMAT(2I5)
READ THE DATA

21 DO 10 J=1,N
10 IF(M,F0,0) GO TO 25
L=0
DO 22 I=1,N12
DO 23 J=1,M
IF(C(I,6),M1(J)),AND,(I,LE,M2(J))) GO TO 22
23 CONTINUE
L=L+1
FE(L,J)=1.
FE(L,2)=F(I,1,2)
FF(L,3)=F(I,1,3)
FE(L,4)=F(I,1,4)
FE(L,5)=F(I,1,5)
UE(L)=U(I)
22 CONTINUE
CALL VARM(SMEAN,SVAR,UE,FE,G,FF,L,NM)
GO TO 24
25 CALL VARM(SMEAN,SVAR,UE,F,FF,N12,NM)

COMPUTATION OF THE INVERSE MATRIX OF FF(I,J).

24 CALL INVMAT(NM,FF,0,00001)

COMPUTATION OF THE COEFFICIENTS A(J) OF THE PRIMARY DETERMINISTIC COMPONENT

DO 80 J=1,NM
SS=0.0
DO 81 J=1,NM
81 SS=SS+ G(I)*FF(J,J)
A(I)=SS
80 WRITE(6,400) I,A(I)
400 FORMAT(/10X,"A",I1,="",Fi5.7)

COMPUTATION OF THE STOCHASTIC RESIDUALS

WRITE(6,401)(TITLE(I),I=1,20)
401 FORMAT(/5X*STOCHASTIC RESIDUALS OF",1X,20A4)
-WRITE(6,337)
337 FORMAT (/1X, 4HFVAR, 3X, 3HDEC, 4X, 3HJAN, 4X, 3HFEB, 4X, 13HMAR, 4X, 3HAPR, 4X, 3HJUL, 4X, 3HAUG, 4X, 3HSEP)
   SS1=0,
   SS2=0,
   IF(M1=0) GO TO 26
   DO 84 I=1,L
   DO 85 J=1,NM
   84 U(I)=U(I)+A(J)*FF(I, J)
   SS1=SS1+U(I)
   85 SS2=SS2+U(I)*U(I)
   CL=1-NM
   L=0
   DO 27 I=1,N12
   DO 28 J=1,NM
   IF ((I,GE,M1(J)),AND,(I,LE,M2(J))) GO TO 27
   26 CONTINUE
   L=L+1
   U(I)=U(I)/L
   27 CONTINUE
   GO TO R6
   26 DO 82 I=1,N12
   DO 83 J=1,NM
   82 U(I)=U(I)+A(J)*F(I, J)
   SS1=SS1+U(I)
   83 SS2=SS2+U(I)*U(I)
   CL=1-NM
   86 RMSEAN=SS1/CL
   RVAR=SS2/CL.
   C
   PUNCH 355, (TITLE(I), I=1,20)
   355 FORMAT (*STOCHASTIC RESIDUALS OF*, 1X, 14A4/6A4)
   C
   PUNCH 354, (TITLE1(I), I=1,20)
   354 FORMAT (20A4)
   DO 87 I=1,N
   C
   CARDS TO BE PUNCHED FOR THE STOCHASTIC RESIDUALS
   C
   PUNCH 356, (X(J, I), J=1,12), IYR(I)
   356 FORMAT (12F6.2, 3X, 14)
   87 WRITE(6, 339) IYR(I), (X(J, I), J=1,12)
   339 FORMAT (/X, 14, 12F7.2)
   IF(RMSEAN, LT, 0.001) RMSEAN=0.0
   WRITE(6, 955) SMNEAN, SVAR, RMSEAN, RVAR
   955 FORMAT (////2X, 5HMEAN=E15.7, 3X, 4HVAR=E15.7, ///2X, 13HMEAN OF RES., =E15.7
   1, 7, 3X, 12HVAR OF RES., =E15.7)
   GO TO 1
   2 STOP
   END
SUBROUTINE VARN(SMEAN, SVAR, U, F, G, FF, N12, NM)
DIMENSION U(1320), F(1320, 5), G(5), FF(5, 5)

COMPUTATION OF THE MATRIX FF(I, J) WHICH IS THE PRODUCT OF MATRIX F(L, M) AND ITS TRANSPOSE MATRIX

DO 61 I = 1, NM
DO 62 J = 1, NM
SS = 0.0
DO 63 K = 1, N12
63 SS = SS + F(K, I)*F(K, J)
FF(J, I) = SS
62 FF(I, J) = SS
61 CONTINUE

COMPUTATION OF THE VECTOR G WHICH IS THE PRODUCT OF THE VECTOR U AND THE MATRIX F

DO 70 I = 1, NM
SS = 0.0
DO 71 K = 1, N12
71 SS = SS + U(K)*F(K, I)
70 G(I) = SS

COMPUTATION OF THE SAMPLE MEAN (=SMEAN) AND VARIANCE (=SVAR)

SSI = 0.0
SS2 = 0.0
DO 73 I = 1, N12
SS1 = SSI + U(I)
73 SS2 = SS2 + U(I)*U(I)
CC = N12
SMEAN = SS1/CC
SVAR = (SS2 - (SS1*SS1)/CC)/(CC-1)
RETURN
END
SUBROUTINE INVMA(N2,ST22,ERR)
COMPUTATION OF THE INVERSE MATRIX OF ST22

THE PROGRAM WAS WRITTEN ACCORDING TO THE FLOW CHART GIVEN BY
R.RFC. KEPT, AND J. HURT, IN THE BOOK "NUMERICAL CALCULATIONS AND

N2 IS THE RANK OF MATRIX ST 22

DIMENSION FF(2,4),ST22(2,2)

DO 10 I=1,N2
DO 10 J=1,N2
10 FF(I,J)=ST22(I,J)
N1=N2+1
N11=N2+1
NN=2*N2
DO 101 T=1,N2
DO 102 J=N1,NN
IF(J=N2+1) 104,103,104
103 FF(I,J)=1.0
GO TO 102
104 FF(I,J)=0.0
107 CONTINUE
101 CONTINUE
DO 105 K=1,N11

B=FF(K,K)
KI=K
KD1=K+1
DO 106 I=KD1,N2
IF(ABS(FF(I,K))/=FF(I,K)) 107,106,106
107 B=FF(I,K)
KI=I
106 CONTINUE

CHECK FOR THE CASE OF SINGULAR MATRIX

ERR HAS TO BE SPECIFIED (INPUT)

IF(ABS(R)= ERR ) STOP
108 WRITE(*,110) B
110 FORMAT(///5X,23HSINGULAR MATRIX WITH B=**E15,7)
STOP
109 IF(K1=K1) 111,112,111
111 DD 113 J=K1,NN
B1=FF(K,J)
FF(K,J)=FF(K,J)/B1
113 FF(K1,J)=B1
112 DD 114 J=K1,NN
114 FF(K,J)=FF(K,J)/B
DD 115 T=K1,N2
DD 116 J=K1,NN
116 \text{FF}(I,J) = \text{FF}(I,J) = \text{FF}(I,K) \times \text{FF}(K,J)
115 \text{CONTINUE}
105 \text{CONTINUE}
\text{DO 117 } J = N1, NN
117 \text{FF}(N2,J) = \text{FF}(N2,J) / \text{FF}(N2,N2)
\text{DO 118 } L = 1, N11
K = N2 - N11
K1 = K + 1
\text{DO 119 } J = N1, NN
\text{DO 120 } I = K1, N2
120 \text{FF}(K,J) = \text{FF}(K,J) = \text{FF}(K1,I) \times \text{FF}(I,J)
119 \text{CONTINUE}
118 \text{CONTINUE}
\text{DO 121 } I = 1, N2
\text{DO 122 } J = 1, N2
JJ = N2 + J
122 \text{ST22}(Y,J) = \text{FF}(I,JJ)
121 \text{CONTINUE}
\text{RETURN}
\text{END}
PROGRAM FOR ESTIMATING CROSS-SPECTRA AND PARTIAL CROSS-SPECTRA

COMPLEX ST,STI
DIMENSION X(12,110,6),U(1320,6),M1(10),M2(10),C(6,6,61),UME(61)
DIMENSION C(61,61),SI(61,61),CC(6,6,61),Q(6,6,61),QD(6,6,61)
DIMENSION TITL(20,6,4),ST(6,6,61),STI(6)

EQUIVALENCE(X(1,1,1),U(1,1))

X IS THE RECORD OF PRECIPITATION AND STREAMFLOW RESIDUALS

READ(5,200) NF,NL,NN,NS
200 FORMAT(4(5I1))

NF IS THE FIRST CALENDAR YEAR FOR ALL THE RECORDS
NL IS THE LAST CALENDAR YEAR FOR ALL THE RECORDS
NN IS THE MAXIMUM LAG FOR VARIANCES INCREASED BY ONE
NS IS THE NUMBER OF TIME SERIES OF THE MULTIPLE TIME SERIES
N=NL-NF+1

N IS THE MAXIMUM LENGTH OF RECORD IN YEARS

WRITE(6,400)
400 FORMAT(*T/5X,*THE DATA FOR THE COMPUTATION OF CROSS-SPECTRA ARE IN*/)

READ THE DATA

STREAMFLOW DATA
READ(5,401)(TITLE(I,1,1),I=1,20),(TITLE(I,1,2),I=1,20)
WRITE(6,402)(TITLE(I,1,1),I=1,20),(TITLE(I,1,2),I=1,20)
READ(5,401)(TITLE(I,1,3),I=1,20),(TITLE(I,1,4),I=1,20)
WRITE(6,402)(TITLE(I,1,3),I=1,20),(TITLE(I,1,4),I=1,20)

401 FORMAT(20A4/20A4)
402 FORMAT(*4X,20A4/*4X,20A4)

READ(5,102) NFS,M

NFS IS THE FIRST WATER YEAR FOR STREAMFLOW RECORD
M IS THE NUMBER OF INTERVALS OF MISSING DATA

IF(M,F0.0) GO TO 83
DO 84 J=1,M
84 READ(5,102) M1(J),M2(J)

102 FORMAT(2I5)
83 K1=NFS-1=NF
IF(K1,F0.0) GO TO 85
DO 86 I=1,K1
86 J=1,12
86 X(J,I)=999.
85 KK1=K1+1
DO 87 J=1,9
87 X(J,KK1)=999.
NN1=N=1
DO 88 I=KK1,NN1
II=I+1
88 READ(5,100) (X(J,I), J=10,12) , (X(J,J), J=1,9)
100 FORMAT(12F6.2)
   DO 89 J=10,12
89 X(J,N)+=999,
   IF(M,J,FQ,0) GO TO 90
   K12=K1*12+9
   DO 91 I=1,M
   M1=M1(I)+K12
   MM2=MM2(I)+K12
   DO 91 J=MM1,MM2
91 U(J,N)+=999,
90 CONTINUE
C PRECIPITATION DATA
   DO 10 IK=2,NS
   READ(5,401) (TITLE(I),I=1,20) , (TITLE(I),I=1,20)
   WRITE(6,402) (TITLE(I),I=1,20) , (TITLE(I),I=1,20)
   READ(5,401) (TITLE(I),I=1,20) , (TITLE(I),I=1,20)
   WRITE(6,402) (TITLE(I),I=1,20) , (TITLE(I),I=1,20)
   READ(5,102) NFS,M
C NFS IS THE FIRST CALENDAR YEAR FOR PRECIPITATION RECORD
   IF(M,FQ,0) GO TO 81
   DO 82 J=1,M
82 READ(5,102) M1(J),M2(J)
81 K1=NF5=NF
   IF(K1,FQ,0) GO TO 70
   DO 71 J=1,K1
   DO 71 J=1,12
71 X(J,I,K)+=999,
70 KK1=K1+1
   DO 72 I=1,KK1,N
72 READ(5,100)X(J,I,K),J=1,12)
   IF(M,FQ,0) GO TO 10
   K12=K1*12
   DO 73 I=1,M
   MM1=M1(I)+K12
   MM2=MM2(I)+K12
   DO 73 J=MM1,MM2
73 U(J,I,K)+=999,
10 CONTINUE
C
C COMPUTATION OF CROSS-COVARIANCES
C
N12=N*12
   DO 13 L=1,N5
   DO 13 M=1,N5
   DO 30 K=1,NN
   KK=K-1
   JK=N12-KK
   SX1X2=0.0
   SX1=0.0
   SX2=0.0
   IN=0
DO 31 I=1,JK
IKK=I+KK
IF((U(I, M), GT, 500.), OR,(U(IKK, I), GT, 500.)) GO TO 31
SX1*P=SX1*X2+U(I, M)*U(IKK, L)
SX1=SX1+U(IKK, L)
SX2=SX2+U(I, M)
IN=IN+1
31 CONTINUE
CJK=IN
30 C(L, M, K)=(SX1*X2-((SX1*SX2)/CJK))/CJK

C(L, M, I) IS THE CROSS-COVARIANCE BETWEEN THE L-TH AND M-TH SERIES FOR LAG I=1

13 CONTINUE
NN1=NN+1
DO 14 J=2,NN1
DO 14 L=1,NS
DO 14 M=1,NS

NOW C(L, M, I) IS MODIFIED AND REPRESENTS A FUNCTION OF C(L, M, I)

14 C(L, M, I)=2.*C(L, M, I)
CNN=NN1
DO 15 J=1,NN
CI=J-1
OM(E)J=3.14159*CI/CNN
OM(E)(J) IS THE FREQUENCY
DO 15 J=1,NN
CJK=J-1

C(CO(I, J)) IS THE COSINE OF OM(E)(J)*(J=1)
C(SI(I, J)) IS THE SINE OF OM(E)(J)*(J=1)

C(CO(I, J)=COS(OM(E)(I)*CJK)
15 SI(I, J)=SIN(OM(E)(I)*CJK)

COMPUTATION OF CROSS-SPECTRA

DO 16 L=1,NS
DO 16 M=1,NS
DO 16 J=1,NN
SS1=0.0
SS2=0.0
DO 17 J=1,NN
SS1=SS1+C(L, M, J)+C(M, L, J)*CO(I, J)
17 SS2=SS2+C(L, M, J)-C(M, L, J)*SI(I, J)
CC(L, M, I)=SS1/2.
16 QQ(L, M, I)=SS2/2.
CC(L,M,I) IS THE RAW ESTIMATE OF CO-SPECTRUM TIMES 2, 3, 14159
BETWEEN THE L-TH AND M-TH SERIES AT FREQUENCY OME(I)

QQ(L,M,I) IS THE RAW ESTIMATE OF QUADRATURE SPECTRUM TIMES
2, 3, 14159 BETWEEN THE L-TH AND M-TH SERIES AT FREQUENCY OME(I)

DO 18 L=1,NS
DO 18 M=L,NS

CC(L,M,I) IS NOW THE SMOOTHED ESTIMATE OF CO-SPECTRUM BETWEEN
THE L-TH AND M-TH SERIES AT FREQUENCY OME(I)

QQ(L,M,I) IS THE SMOOTHED ESTIMATE OF QUADRATURE SPECTRUM BETWEEN
THE L-TH AND M-TH SERIES AT FREQUENCY OME(I)

ST(L,M,I) IS THE ESTIMATED CROSS-SPECTRUM BETWEEN THE L-TH
AND M-TH SERIES AT FREQUENCY OME(I)

CC(L,M,1)=(0.54*CC(L,M,1)+0.46*CC(L,M,2))/(2.*3,14159)
CC(L,M,NN)=0.46*CC(L,M,NN)+0.54*CC(L,M,NN))/(2.*3,14159)
QQ(L,M,1)=(0.54*QQ(L,M,1))/(2.*3,14159)
QQ(L,M,NN)=0.54*QQ(L,M,NN))/(2.*3,14159)
IF(L,NE,M) GO TO 118
ST(L,M,1)=CC(L,L,1)
ST(L,L,NN)=CC(L,L,NN)
GO TO 119

118 ST(L,M,1)=CMPLX(CC(L,M,1)*QQ(L,M,1))
ST(M,L,1)=CONJG(ST(L,M,1))
ST(L,M,NN)=CMPLX(CC(L,M,NN)*QQ(L,M,NN))
ST(M,L,NN)=CONJG(ST(L,M,NN))

119 DO 18 I=2,NN1
CC(L,M,1)=(0.23*CC(L,M,1)+0.54*CC(L,M,1)+0.23*CC(L,M,1+1))/(2.*3,14159)
IF(L,NE,M) GO TO 120
ST(L,L,7)=CC(L,L,1)
GO TO 18

120 QQ(L,M,1)=(0.23*QQ(L,M,1)+0.54*QQ(L,M,1)+0.23*QQ(L,M,1+1))/(2.*3,14159)
ST(L,M,1)=CMPLX(CC(L,M,1)*QQ(L,M,1))
ST(M,L,1)=CONJG(ST(L,M,1))
CONTINUE

COMPUTATION OF PARTIAL CROSS-SPECTRA

NNS=NS=1
DO 60 I=1,NNS
K=L+1
DO 61 M=K,NS
WRITE(6,403)(TITLE(I,L,1),I=1,20)+(TITLE(I,M,1),I=1,20)
403 FORMAT(1X,,5X="CROSS-SPECTRUM BETWEEN"/5X="20A4/1X="AND"

11X,20A4)
61 CALL PART(ST,NNS,NNS,OME)
DO 62 I=1,NNS
DO 62 L=I+1
DO 63 LI=1,NNS
63 STT(LI)=ST(LI+1, I)
   DO 64 LL=2, NS
   LLK=LLL+1
   DO 64 LL=1, NS
64 ST(LLL, LLK, I)=ST(LLL, LLL, I)
   DO 66 LI=1, NS
66 ST(LI, NS, I)=STT(LI)
   DO 67 LI=1, NS
67 STT(LI)=ST(1+LI, I)
   DO 68 LLL=2, NS
   LLK=LL+1
   DO 68 LL=1, NS
68 ST(LLK, LLM, I)=ST(LLL, LLL, I)
   DO 69 LI=1, NS
69 ST(NS*LI, I)=STT(LI)
62 CONTINUE
60 CONTINUE
STOP
END
SUBROUTINE PART(ST, NS, NN, OME)
C THIS SUBROUTINE COMPUTES THE PARTIAL COHERENCE, PHASE ANGLE,
C AND GAIN BETWEEN FIRST AND SECOND TIME SERIES
C COMPLEX ST, ST1, ST2, ST12, ST21, ST22, SN, COH, GAIN
C DIMENSION ST(6, 6, 61), ST1(6, 2, 4), ST21(2, 4, 4), ST22(4, 4), OME(61),
C SN(2, 2, 61)
C DIMENSION A(4, 4), AA(4, 4), B(4, 4)
C
C MAXIMUM LAG FOR THE COVARIANCES INCREASED BY ONE (INPUT)
C ST(I, M, I) IS THE ESTIMATED CROSS-SPECTRUM BETWEEN THE L-TH
C AND M-TH SERIES AT FREQUENCY ONE(I) (INPUT)
C OME(J) IS THE FREQUENCY (INPUT)
C NS = NS = 2
C DO 19 I = 1, NS
C DO 20 L = 1, NS
C LL = L + 2
C DO 21 M = 1, 2
C ST12(M, L) = ST(M, LL, I)
C 21 ST21(L, M) = ST(LL, M, I)
C DO 22 M = 1, NS
C LM = M + 2
C 22 ST22(L, M) = ST(LL, LM, I)
C 20 CONTINUE
C DO 14 I = 1, NS
C DO 14 J = 1, NS
C A(I, J) = ST22(I, J)
C AA(I, J) = ST22(I, J)
C 14 B(I, J) = AIMAG(ST22(I, J))
C CALL INVMAT(NNS, AA, 0, 00001)
C CALL MULT(R, AA, NNS, NNS, NNS)
C CALL MULT(R, AA, NNS, NNS, NNS)
C DO 15 I = 1, NS
C 15 CALL INVMAT(NNS, A, 0, 00001)
C CALL MULT(R, A, NNS, NNS, NNS)
C DO 13 I = 1, NS
C 13 DO 13 J = 1, NS
C AA(I, J) = AA(I, J)
C 13 ST22(I, J) = CMPLX(A(I, J), AA(I, J))
C CALL CMULT(ST12, ST22, 2, NNS, NNS)
C CALL CMULT(ST12, ST21, 2, NNS, 2)
C DO 10 L = 1, 2
C 10 DO 10 M = 1, 2
C SN(L, M, I) = ST(L, M, I) = ST12(L, M)
C SN(L, M, I) IS THE PARTIAL CROSS-SPECTRUM AT FREQUENCY ONE(I)
C BETWEEN THE L-TH SERIES AND M-TH SERIES
C 19 CONTINUE
C DO 11 L = 1, 2
C 11 DO 11 M = 1, 2
C IF(L.EQ.2) AND (M.EQ.1) GO TO 12
C WRITE(6, 112) L, M
C
112 FORMAT(//5x,10HFOR SERIES,1X,I1,1X,3HAN,1X,I1)
   WRITE(6,111)
111 FORMAT(//2x,6HPERIOD,3X,9HFREQUENCY,10X,8HSPECTRUM,17X,9HCORENCE
   1,12X,9HSPACE,15X,9HGAIN/22X,11HCO=0043TREX,2X,10HQUADRATURE,6X,4HR
   2REAL,6X,9HIMAGINARY,20X,4HREAL,2X,9HIMAGINARY/)
   DO 30 I=1,NN
   COH=(SN(L,M,I)*CONJG(SN(L,L,I)))/(SN(L,L,I)*SN(M,M,I))
   COH IS THE CORENCE
   PHA=ATAN(AIMAG(SN(L,M,I))/REAL(SN(L,M,I))
   PHA IS THE PHASE ANGLE
   GAIN=SN(L,M,I)/SN(M,M,I)
   KK=I-1
   A1=REAL(SN(L,M,I))
   A1 IS THE CO=SPECTRUM
   A2=AIMAG(SN(L,M,I))
   A2 IS THE QUADRATURE SPECTRUM
   B1=REAL(COH)
   B1 IS THE CORENCE
   B2=AIMAG(COH)
   B2 IS A CHECK OF THE COMPUTATIONS AND MUST BE EQUAL TO ZERO
   C1=REAL(GAIN)
   C1 IS THE REAL PART OF THE GAIN
   C2=AIMAG(GAIN)
   C2 IS THE IMAGINARY PART OF THE GAIN
   30 WRITE(6,113) KK,ONE(I),A1,A2,B1,B2,PHA,C1,C2
113 FORMAT(4x,12,4x,F7.5,3x,E10.4,3x,2x,E10.4,3x,E10.4,3x,E10.4,3x,E10.4,3x,E10.4,3x,E10.4)
12 CONTINUE
11 CONTINUE
   DO 62 I=1,NN
   DO 63 LI=1,NS
   STT(LI)=ST(LI,2,I)
   DO 64 LLL=1,NS
   LLK=LLL=1
   DO 64 LLM=1,NS
   ST(LLM,LLK,I)=ST(LLM,LLL,I)
   DO 66 LI=1,NS
   ST(LI,NS+I)=STT(LI)
   DO 67 LI=1,NS
   STT(LI)=ST(2,LI,I)
   DO 68 LLL=1,NS
   LLK=LLL=1
   DO 68 LLM=1,NS
   ST(LLM,LLM,I)=ST(LLM,LLM,I)
   DO 69 LI=1,NS
   ST(NS,LI,I)=STT(LI)
   62 CONTINUE
   RETURN
END
SUBROUTINE MULT(A, B, L, M, N)

TO MULTIPLY MATRICES A AND B AND STORE THE PRODUCT IN A
THE ELEMENTS OF THE MATRICES ARE COMPLEX

L IS THE NUMBER OF ROWS IN A
M IS THE NUMBER OF COLUMNS IN A
N IS THE NUMBER OF COLUMNS IN A
COMPLEX A, B, C
DIMENSION A(4,4), B(4,4), C(4,4)
DO 1 I=1,L
   DO 1 J=1,N
      1 C(I,J)=0,
      DO 10 I=1,L
      DO 10 J=1,N
      DO 10 K=1,M
      10 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      DO 11 I=1,L
      DO 11 J=1,N
      11 A(I,J)=C(I,J)
RETURN
END

SUBROUTINE CMULT(A, B, L, M, N)

DIMENSION A(2,4), B(4,4), C(2,4)
DO 1 I=1,L
   DO 1 J=1,N
      1 C(I,J)=0,
      DO 10 I=1,L
      DO 10 J=1,N
      DO 10 K=1,M
      10 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      DO 11 I=1,L
      DO 11 J=1,N
      11 A(I,J)=C(I,J)
RETURN
END
PROGRAM FOR THE ESTIMATION OF CROSS-SPECTRA

COMPLEX ST, ST
COMPLEX COH, GAIN
DIMENSION X(12,110,6), U(1320,6), M1(10), M2(10), C(6,6,61), DMF(61)
DIMENSION CO(61,61), S(61,61), CC(6,6,61), QQ(6,6,61), D(6,6,61)
DIMENSION TITLE(20,6,4), ST(6,6,61), STT(6)
EQUIVALENCE(X(1,1,1), U(1,1))

X IS THE RECORD OF PRECIPITATION AND STREAMFLOW RESIDUALS

READ(5,200) NF, NL, NN, NS

200 FORMAT(A15)
NF IS THE FIRST CALENDAR YEAR FOR ALL THE RECORDS
NL IS THE LAST CALENDAR YEAR FOR ALL THE RECORDS
NN IS THE MAXIMUM LAG FOR VARIANCES INCREASED BY ONE
NS IS THE NUMBER OF TIME SERIES OF THE MULTIPLE TIME SERIES
N=NL-NF+1
N IS THE MAXIMUM LENGTH OF RECORD IN YEARS
WRITE(6,400)

400 FORMAT('**/*5X,' THE DATA FOR THE COMPUTATION OF CROSS-SPECTRA ARE

**/**

READ THE DATA
STREAMFLOW DATA
READ(5,401) (TITLE(I,1,1), I=1,20), (TITLE(I,1,2), I=1,20)

401 FORMAT(2A4/2A4)
WRITE(6,402) (TITLE(I,1,1), I=1,20), (TITLE(I,1,2), I=1,20)

402 FORMAT('/4X,2A4/'4X,2A4)
READ(5,401) (TITLE(I,1,3), I=1,20), (TITLE(I,1,4), I=1,20)
WRITE(6,402) (TITLE(I,1,3), I=1,20), (TITLE(I,1,4), I=1,20)
READ(5,102) NFS,M

NFS IS THE FIRST WATER YEAR FOR STREAMFLOW RECORD
M IS THE NUMBER OF INTERVALS OF MISSING DATA
IF(M,EQ.,0) GO TO 83

84 READ(5,102) M1(J), M2(J)

102 FORMAT(A15)
83 K1=NFS-1-NF
IF(K1,EQ.,0) GO TO 85

DD 86 I=1,K1
DD 86 J=1,12

86 X(J,I,1)=999.
85 K1=K1+1
DD 87 J=1,9

87 X(J,K1,1)=999.
NN1=M=1
DD 88 I=K1,NN1
II=I+1

88 READ(5,100) (X(J,I,1), J=10,12), (X(JJ,II,1), JJ=1,9)
100 FORMAT(12F6.2)
DO 89 J=10,12
  89 X(J,4,1)=999.
  IF(M.EQ.0) GO TO 90
  K12=1*K12+9
  DO 91 I=1,M
  MM1=41(I)+K12
  MM2=42(I)+K12
  DO 91 J=MM1,MM2

  91 U(J,1)=999.
  90 CONTINUE

  C PRECIPITATION DATA
  IK=2
  READ(5,401)(TITLE(I,IK,1),I=1,20),(TITLE(I,IK,2),I=1,20)
  WRITE(6,402)(TITLE(I,IK,1),I=1,20),(TITLE(I,IK,2),I=1,20)
  READ(5,102) NFS,M
  C NFS IS THE FIRST CALENDAR YEAR FOR PRECIPITATION RECORD
  IF(M.EQ.0) GO TO 81
  DO 82 J=1,M
  82 READ(5,102) M1(J),M2(J)
  81 K1=NFS-NF
  IF(K1.EQ.0) GO TO 70
  DO 71 I=1,K1
  DO 71 J=1,12
  71 X(J,I,IK)=999.
  70 KK1=K1+1
  DO 72 I=KK1,N
  72 READ(5,100)(X(J,I,IK),J=1,12)
  IF(M.EQ.0) GO TO 10
  K12=K1+12
  DO 73 I=1,M
  MM1=41(I)+K12
  MM2=42(I)+K12
  DO 73 J=MM1,MM2
  73 U(J,IK)=999.
  10 CONTINUE

  NN1=NN-1
  CNN=NN1
  DO 15 I=1,NN
  CI=I
  OME(I)=3.14159*CI/CNN
  C OME(J) IS THE FREQUENCY
  DO 15 J=1,NN
  CJ=J

  C OME(I,J) IS THE COSINE OF OME(I)*(J-1)
  C SI(I,J) IS THE SINE OF OME(I)*(J-1)

  COC(I,J)=COS(OME(I)*CJ)
  15 SI(I,J)=SIN(OME(I)*CJ)
COMPUTATION OF CROSS-COVARIANCES

N12=N*12
DO 13 L=1,NS
DO 13 M=1,NS
DO 30 K=1,NN
KK=K-1
JK=N12-KK
SX1X2=0,0
SX1=0,
SX2=0,
IN=0
DO 31 I=1,JK
KK=I+KK
IF((I+M).GT.500.).OR.(U(IKK,L).GT.500.).) GO TO 31
SX1X2=SX1X2+U(I,M)*U(IKK,L)
SX1=SX1+U(IKK,L)
SX2=SX2+U(I,M)
IN=IN+1
31 CONTINUE

CJK=IN
30 C(L,M,I)=(SX1X2-(SX1*SX2)/CJK)/CJK

C(L,M,I) IS THE CROSS-COVARIANCE BETWEEN THE L-TH AND M-TH SERIES FOR LAG I=1

13 CONTINUE
DO 14 I=2,NN1
DO 14 L=1,NS
DO 14 M=1,NS

NOW C(L,M,I) IS MODIFIED AND REPRESENTS A FUNCTION OF C(L,M,I)

14 C(L,M,I)=2.*C(L,M,I)

COMPUTATION OF CROSS-SPECTRA

DO 16 L=1,NS
DO 16 M=L,NS
DO 16 I=1,NN
SS1=0.,0
SS2=0.,0
DO 17 J=1,NN
SS1=SS1+(CC(L,M,J)+CC(M,L,J)+CC(J,L,M))*CO(I,J)
17 SS2=SS2+(CC(L,M,J)-CC(M,L,J)+CC(J,L,M))*SI(I,J)
CC(L,M,I)=SS1/2.
16 DD(L,M,I)=SS2/2.
CC(L,M,I) IS THE RAW ESTIMATE OF CO-SPECTRUM TIMES 2, 3, 14159 BETWEEN THE L-TH AND M-TH SERIES AT FREQUENCY DME(I)
QO(L,M,I) IS THE RAW ESTIMATE OF QUADRATURE SPECTRUM TIMES 2, 3, 14159 BETWEEN THE L-TH AND M-TH SERIES AT FREQUENCY DME(I)

DO 18 M=1,NS
DO 18 L=1,NS

CC(L,M,I) IS THE SMOOTHED ESTIMATE OF CO-SPECTRUM BETWEEN THE L-TH AND M-TH SERIES AT FREQUENCY DME(I)
QO(L,M,I) IS THE SMOOTHED ESTIMATE OF QUADRATURE SPECTRUM BETWEEN THE L-TH AND M-TH SERIES AT FREQUENCY DME(I)
ST(L,M,I) IS THE ESTIMATED CROSS-SPECTRUM BETWEEN THE L-TH AND M-TH SERIES AT FREQUENCY DME(I)

C(L,M,1)=(0.54×CC(L,M,1)+0.06×CC(L,M,2))/(2×3, 14159)
C(L,M,NN)=(0.46×CC(L,M,NN)+0.54×CC(L,M,NN))/(2×3, 14159)
Q(L,M,1)=(0.54×QO(L,M,1))/(2×3, 14159)
Q(L,M,NN)=(0.54×QO(L,M,NN))/(2×3, 14159)

118 ST(L,M,I)=CPLX(C(L,M,1),Q(L,M,1))
ST(L,M,NN)=CPLX(C(L,M,NN),Q(L,M,NN))
DO 18 L=2,NN1
C(L,M,I)=(0.23×CC(L,M,I-1)+0.54×CC(L,M,I)+0.23×CC(L,M,I+1))/(2×3, 14159)
120 Q(L,M,I)=(0.23×QO(L,M,I-1)+0.54×QO(L,M,I)+0.23×QO(L,M,I+1))/(2×3, 14159)
ST(L,M,I)=CPLX(C(L,M,I),Q(L,M,I))

18 CONTINUE
DO 95 L=1,NS
DO 95 M=1,NS
WRITE(6,403)(TITLE(I,L,1),I=1,20),(TITLE(I,M,1),I=1,20)
403 FORMAT(*1//"5X","CROSS-SPECTRUM BETWEEN 5X, 20A4/1X,"AND")

11X,20A4)
WRITE(6,111)
111 FORMAT(/"2X,6HPERIOD,3X,9HFREQUENCY,10X,8HSPECTRUM,17X,9HCOHERENCE 1,12X,5HPHASE,15X,4HGAIN/22X,11HCO-SPECTRUM,2X,10HQUADRATURE,6X,4HR 2REAL,6X,9HIMAGINARY,20X,4HREAL,2X,9HIMAGINARY")
DO 95 I=1,NN
KK=I
CDH=ST(L,M,I)+CONJG(ST(L,M,I))/ST(L,L,1)*ST(M,M,1)
CDH IS THE COHERENCE
PHA=ATAN(IMAG(ST(L,M,I)))/REAL(ST(L,M,I))
PHA IS THE PHASE ANGLE
GAIN=ST(L,M,I)/ST(M,M,1)
A1=REAL(ST(L,M,I))
A1 IS THE CO-SPECTRUM
A2 = AIMAG(ST(I,M,T))
A2 is the quadrature spectrum

B1 = REAL(COH)
B1 is the coherence

B2 = AIMAG(COH)
B2 is a check of the computations and must be equal to zero

C1 = REAL(GAIN)
C1 is the real part of the gain

C2 = AIMAG(GAIN)
C2 is the imaginary part of the gain

95 WRITE(6,113) KK,OME(I),A1,A2,B1,B2,PHA,C1,C2
113 FORMAT(4X,12X,F7.5,3X,E10.4,2X,E10.4,3X,E10.4,3X,E10.4,3X,E10.4)
STOP
END
**Program for the computation of the regression coefficient**

```plaintext
DIMENSION X(12,80), Y(12,80), M1(3,1), M2(3), N(12,80), V(960)
DIMENSION C(2,2)
EQUIVALENCE (X(1), U(1)), Y(1), M(1), Z(1), V(1))
READ(5,99) (TITLE(I,1), I=1,20), (TITLE(J,2), J=1,20)
99 FORMAT(20A4/20A4)

WRITE(6,98) (TITLE(I,1), I=1,20), (TITLE(J,2), J=1,20)
98 FORMAT(14/2X, 9X, REGRSSION COEFFICIENT FORM_INPUT=20A4/20A4)
READ(5,97) M
97 FORMAT(5X,15)
DO 10 I=1,M
10 READ(5,96) M1(I), M2(I)
96 FORMAT(215)
DO 11 J=1,59
11 READ(5,100) (X(J,I), I=1,12), IYR(I)
100 FORMAT(12F6.2, 3X, 14)
READ(5,95) (TITLE(I,1), I=1,20), (TITLE(J,2), J=1,20)
95 FORMAT(20A4/20A4)
WRITE(6,93) (TITLE(I,1), I=1,20), (TITLE(J,2), J=1,20)
93 FORMAT(2X, "AND", 20A4/20A4)
DO 12 I=1,59
12 READ(5,100) (Z(J,I), I=1,12), IYR(I)
100 FORMAT(12F6.2, 3X, 14)
DO 13 I=1,M
13 K=1(I)
KK=KK+K
DO 13 J=1,59
13 U(J)=999.
SS1=0,0
SS2=0,0
SS12=0,0
SS21=0,0
DO 20 J=1,705
I=J+2
L=I+1
IF((U(I), GT, 400, ), OR, (U(L), GT, 400, )) GO TO 20
SS1=SS1+U(I)*U(L)
SS12=SS12+U(I)*V(L)
SS2=SS2+V(L)*V(J)
20 CONTINUE
C(1,1)=SS1
C(1,2)=SS12
C(2,1)=SS12
C(2,2)=SS22
```
CALL TNUMMAT(2*C,0.00001)
SS11=0.0
SS21=0.0
DO 21 J=1,705
      L=J+1
      IF((U(J)*GT.400.),U(L)*GT.400.)) GO TO 21
      SS11=SS11+U(J)*U(L)
      SS21=SS21+V(I)*U(L)
21 CONTINUE
A1=C(1,1)*SS11+C(1,2)*SS21
A2=C(2,1)*SS11+C(2,2)*SS21
WRITE(6,101) A1,A2
101 FORMAT(5X,*A1=*E15.7,5X,*A2=*E15.7)
WRITE(*,337)
337 FORMAT(1X,*AYEAR,3X,*MCT,4X,*HNOV,4X,*HDFC,4X,*HJAN,4X,*HFIF,4X,*HMAY,4X,*HJUN,4X,*HJUL,4X,*HAUG,4X,*HSEP)
N=0
SS=0.0
SSS=0.0
DO 61 J=1,705
      L=J+2
      IF((U(J)*GT.400.),OR.(U(L)*GT.400.)) GO TO 22
      W(J)=W(J)+A1*U(L)-A2*V(I)
55 SS=SS+W(J)
      SSS=SSS+W(J)*W(J)
      N=NI+1
      GO TO 61
22 W(J)=9999
61 CONTINUE
WRITE(6,338) IYR(J),W(J),I=4,12
338 FORMAT(1X,*I14,21X,*9F7.2)
PUNCH 357, (W(I),I=4,12), IYR(I)
357 FORMAT(18X,*9F6,2,3X,*I4)
DO 63 I=2,59
      WRITE(6,339) IYR(I),Y(J,I),J=1,12
339 FORMAT(1X,*I4,12F7.2)
63 PUNCH 356, (Y(J,I),J=1,12),IYR(I)
356 FORMAT(12F6,2,3X,*I4)
CN=NI
RMEAN=SS/CN
RVAR=SSS/CN
IF(RMFAAA,LT.0.001) RMEAN=0.0
WRITE(6,355) RMEAN,RVAR
355 FORMAT(///2X,*RMEAN=*E15.7,3X,*RVAR=*E15.7)
STOP
END
PROGRAM FOR THE DETERMINATION OF EIGENVALUES AND EIGENVECTORS
OF THE DISPERSION MATRIX

REAL*4 X(12,80,4),U(960,4),R(16),VA(4),DUM1(10),T(4,4)
DIMENSION TITLE(20,4,2)
EQUIVALENCE(X(1,1,1),U(1,1))

U OR X ARE THE PRECIPITATION DATA

READ(5,99) L,N
99 FORMAT(215)
L IS THE NUMBER OF STATIONS
N IS THE NUMBER OF YEARS OF MONTHLY DATA
WRITE(6,400)
400 FORMAT('THE DATA FOR THE COMPUTATION ARE')
DO 10 K=1,L
READ(5,401)(TITLE(I,K),I=1,20),(TITLE(I,K),I=1,20)
10 CONTINUE
WRITE(6,402)(TITLE(I,K),I=1,20),(TITLE(I,K),I=1,20)
402 FORMAT('DATA ARE:')
22 DO 10 I=1,N
10 READ(5,100) (X(J,I,K),J=1,12)
100 FORMAT(12F6.2)
WRITE(6,101)
101 FORMAT('DISPERSION MATRIX')
N12=N*12
CN=N12
DO 11 K=1,L
11 SUM=0.0
DO 12 I=1,N12
12 SUM=SUM+U(I,K)*U(I,M)
T(K,M)=SUM/CN
13 T(M,K)=T(K,M)
11 WRITE(6,102) (T(K,M),M=1,L)
102 FORMAT(8X,4(F8.5,2X))

CALL FIGENZ(T,R,VA,DUM1,4,4,0)
WRITE(6,103)
103 FORMAT(8X,"LATENT",21X,"VECTORS",8X,"ROOTS")
DO 14 I=1,L
14 WRITE(6,104) VA(I),(R(J),J=K,KK)
104 FORMAT(7X,F8.5,3X,4(2X,F8.5))
STOP
END
PROGRAM FOR THE DETERMINATION OF THE UNCORRELATED TIME SERIES

DIMENSION X(12,80,4), U(960,4), Y(12,80,4), W(960,4), R(4,4), T(4,4),
1 TITLE(20,4,2), EV(4), IYR(40)
EQUIVALENCE(X(1,1,1), U(1,1,1), Y(1,1,1), W(1,1))

U OR X ARE THE PRECIPITATION DATA
Y OR W ARE THE UNCORRELATED OR TRANSFORMED DATA

READ(5,99) L,N
99 FORMAT(2I5)
L IS THE NUMBER OF STATIONS
N IS THE NUMBER OF YEARS OF MONTHLY DATA

READ THE EIGENVECTORS
DO 21 I=1,L
21 READ(5,98) (R(I,J), J=1,L)
98 FORMAT(4F10.5)
WRITE(6,400)
400 FORMAT(1//5X,"THE DATA FOR THE COMPUTATION ARE"//)
DO 10 K=1,L
READ(5,401)(TITLE(I,K,1), I=1,20), (TITLE(I,K,2), I=1,20)
10 FORMAT(20A4/20A4)
WRITE(6,402)(TITLE(I,K,1), I=1,20), (TITLE(I,K,2), I=1,20)
402 FORMAT(//4X,20A4//4X,20A4)
DO 42 J=1,N
42 READ(5,100) (X(J,I,K), J=1,12), IYR(I)
100 FORMAT(12F6.2,3X,I4)
N1=N*12
CN=N12
DO 20 J=1,L
SUMM=0.0
DO 23 I=1,N12
W(I,J)=0.0
DO 22 K=1,L
22 W(I,J)=W(I,J)+R(J,K)*U(I,K)
23 SUMM=SUMM+W(I,J)
EV(J)=SUMM/CN
A=ABS(FV(J))
IF(A.LT.0.00001) EV(J)=0.0
20 CONTINUE
WRITE(6,201)
201 FORMAT(1//28X,"FOR THE TRANSFORMED SERIES"//18X,"DISPERSION MATRIX"
1TRIX="\$2X,"MEAN VALUES")
DO 11 K=1,L
11 FORMAT(12F6.2,3X,I4)
13 M=K,L
SUM=0.0
DO 12 I=1,N12
12 FORMAT(12F6.2,3X,I4)
12 SUM=SUM+W(I,K)*W(I,M)
    T(K,M)=SUM/CN
    A=ABS(T(K,M))
    IF(A,LT,0.0001) T(K,M)=0.0
13 T(M,K)=T(K,M)
11 WRITE(6,102) (T(K,M),M=1,L),XV(K)
102 FORMAT(/8X,4(F8.5,2X),10X,F8.5)

DO 30 K=1,L
    WRITE(6,200) K
200 FORMAT(1X,TRANSFORMED SERIES NO.,X,12/)
    WRITE(6,337)
337 FORMAT(1X,4HYEAR,3X,3HJAN,4X,3HFEB,4X,3HMAR,4X,3HAPR,4X,3HMAY,4X,4X,3HJUN,4X,3HJUL,4X,3HAUG,4X,3HSFP,4X,3HMCT,4X,3HNOV,4X,3HDEC)
    PUNCH 200,K
    PUNCH 354,(TITLE(K,2),I=1,20)
354 FORMAT(20A4)
    DO 30 J=1,N
        WRITE(6,339) IYR(I),(Y(J,I,K),J=1,12)
339 FORMAT(1X,I4,12F7.2)
    30 PUNCH 356, (Y(J,I,K),J=1,12),IYR(I)
356 FORMAT(12F6.2,3X,I4)
STOP
END