MEASUREMENT OF ANTENNA RADIATION EFFICIENCY USING IMPROVED WHEELER CAP ALGORITHM

BY

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THESIS

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ABSTRACT

Radiation efficiency measurement can inform the wireless design process by determining the total fraction of accepted power radiated by an antenna. The popular Wheeler cap method can be used to quickly and accurately determine the radiation efficiency of small antennas. However, due to cavity resonant modes and the use of a simplistic equivalent circuit model, the conventional Wheeler cap method yields poor results for electrically larger structures. When efficiency estimates can be obtained, they are often valid only over a narrow frequency band. Higher-order equivalent circuit models may be employed in a modified Wheeler cap algorithm to generate improved radiation efficiency measurements. Guided by advances in broadband modeling of antennas, we demonstrate a new Wheeler cap technique that makes use of parallel admittance subcircuits. We then illustrate the use of this technique to provide improved efficiency estimates at the operating frequencies of electrically larger structures.
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In each segment of the transmit and receive chains of a practical wireless system, signal power is lost. Undesirable yet unavoidable, loss can reduce the performance of a radio system. In broadcast transmitters, loss leads to significant amounts of wasted power and antenna heating. In mobile devices, loss can reduce battery life. Thermal effects associated with loss can further affect system performance. In receivers, loss can even degrade signal-to-noise ratio. Wireless system engineers therefore have significant interest in measuring and reducing losses in their antenna designs.

Antennas dissipate power through two mechanisms: radiation and ohmic (resistive) loss. Conventional wire antennas, including dipoles and monopoles, are typically made with good conductors and often have low ohmic loss at low frequencies, but loss increases with frequency due to the skin effect [1]. Microstrip antennas, which consist of metallization layers on a dielectric substrate, exhibit loss in conductors as well as in the substrate material. Reconfigurable antennas of various types often include control components, such as diodes and varactors, that may contribute to ohmic loss [2]. Some antennas even include resistive components in an attempt to increase bandwidth [3].

Radiation is the other mechanism of power dissipation by an antenna. For a transmitting antenna, on a time-average basis, radially directed real power leaves the vicinity of the antenna. This power outflow can be modeled as dissipation in a resistor. Since radiation is usually the desired characteristic of an antenna, radiation resistance does not have the negative connotations of ohmic resistance. Large, effective radiators often have high radiation resistance values. For example, a thin metal half-wave dipole has an input resistance of $72\,\Omega$, nearly all of which is attributable to radiation resistance [4]. Electrically small antennas—typically poor radiators—have small radiation resistance values ($< 1\,\Omega$).

Radiation efficiency is a parameter that is used to characterize the pro-
portion of power radiated by an antenna to the total power accepted by the antenna. The radiation efficiency value is a property of the antenna alone. It therefore must be independent of the antenna’s impedance match to the rest of the system. The efficiency equation is defined as [4, 5, 6]

\[
\eta = \frac{P_{\text{rad}}}{P_{\text{accepted}}} = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{loss}}}
\]  

(1.1)

Because power can only be dissipated on a time-average basis in real impedances, a radiation efficiency measurement fundamentally involves a comparison of ohmic and radiation resistance values. These resistances change over frequency, and substantial variations can occur over a broad band. Though \(P_{\text{accepted}}\) can be easily determined from impedance measurements, the necessary decomposition into \(P_{\text{rad}}\) and \(P_{\text{loss}}\) quantities is more challenging. Several techniques have been developed to accomplish this and provide experimental estimates of radiation efficiency. Perhaps the technique of greatest research interest is the Wheeler cap method [7].

Every antenna has a quality factor \(Q\) that represents the ratio of stored energy to power dissipated. Antenna design continues to be a challenge because \(Q\), bandwidth, gain, efficiency, and electrical size are all related. Increasing an antenna’s performance in one parameter necessarily requires sacrifices in one or more other areas. To the extent to which other parameters can be held constant, decreasing an antenna’s size will result in reduced efficiency [8]. The effects of the size-efficiency tradeoff are clearly visible in the dipole antenna. The dipole’s simple structure easily lends itself to analytic treatment, and field values can be easily obtained. The ohmic resistance is directly proportional to the physical length of the dipole. The radiation resistance is obtained via the field-derived power radiated into the far field and the known current value. The radiation efficiency for a thin center-fed copper dipole of length 1 meter has been evaluated analytically, using two common radiation resistance models [4], and numerically, via a commercial method of moments solver. In Figure 1.1, a reduction in efficiency is evident as electrical length of the dipole is reduced.

Though an imperfect approximation, the ideal dipole’s fundamental radiation mechanisms are recognizable in many other common antenna families. The size-efficiency tradeoff is well-known to designers of antennas for mobile devices, who struggle with the problem of creating low profile antennas while
minimizing the percentage of power lost to resistive effects.

In the following chapter, we will introduce and compare several fundamental techniques for measuring radiation efficiency, including the Wheeler cap method. Chapter 3 illustrates several challenges that can arise during the use of the Wheeler cap method and includes a summary of popular processing steps that can be used to partially overcome these challenges. In Chapter 4, we demonstrate the use of a new circuit model, rooted in characteristic mode theory, for a modified Wheeler cap method. Finally, in Chapter 5 we demonstrate the application of this modified Wheeler cap technique using a variety of representative antennas.

Figure 1.1: Radiation efficiency of a short dipole antenna
A wide range of techniques for radiation efficiency measurement have been developed. Among the most significant are the gain/directivity method, the radiometric technique, and the Wheeler cap method. In this chapter, we explore the advantages and disadvantages associated with each efficiency measurement technique.

2.1 Gain/Directivity Method

One common approach for determining radiation efficiency is derived from the definitions of two far-field antenna parameters, gain and directivity. Gain represents radiation intensity relative to a theoretical radiation intensity that could be attained if all power accepted from a source were radiated isotropically. Directivity indicates radiation intensity in a given direction relative to the average, isotropic radiation intensity \([4]\). For arbitrary \(\theta, \phi\), we observe that

\[
\frac{G(\theta, \phi)}{D(\theta, \phi)} = \frac{U(\theta, \phi)}{P_{\text{rad}}/4\pi} \frac{P_{\text{rad}}}{P_{\text{accepted}}} = \eta
\]

(2.1)

which suggests that dividing a gain measurement by a directivity measurement will produce an efficiency value. Gain can be measured through the gain comparison method. This process involves transmitting a known signal through a secondary antenna and comparing the power received by the antenna under test to that received by a calibrated standard gain antenna. Directivity is measured by obtaining a field strength measurement in \((\theta, \phi)\) and dividing by the integrated total. The necessity of complete, precise three-dimensional pattern measurement makes this calculation method challenging in practice. Repeatability can be poor [6], and an anechoic chamber with a specialized positioning system is required. The gain/directivity method is
widely used for evaluating antennas in industry [9], however, because pattern measurement is routinely performed in the evaluation of a new antenna design. Furthermore, unlike other techniques, the gain/directivity method imposes few restrictions on the electrical size or directivity of the antenna under test.

### 2.2 Radiometric Method

The radiometric method [6] is another well-known radiation efficiency measurement technique. The radiometric approach is based on the idea that an antenna’s input impedance can be accurately modeled by an RLC network. Such a network is referred to as an antenna’s equivalent circuit. The mechanisms by which power is dissipated by the antenna can be modeled as resistors in the equivalent circuit. Power loss due to radiation is modeled as a radiation resistance. Power loss due to losses on the antenna structure itself is represented as a loss resistance. The standard efficiency equation

\[
\eta = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{loss}}} = \frac{P_{\text{radiation resistance}}}{P_{\text{radiation resistance}} + P_{\text{loss resistance}}}
\]

(2.2)

applies. Figure 2.1 contains an equivalent circuit model with radiation and loss resistances in series.

![Figure 2.1: A circuit model for radiometric efficiency measurement](image)

The radiometric method is based on the fundamental observation that noise power is generated in a resistor in direct proportion to the resistor’s effective noise temperature value \( T \) [1]; i.e.,

\[
N = k_B T \Delta f
\]

(2.3)

where \( k_B \) is Boltzmann’s constant, and \( \Delta f \) represents the bandwidth over
which the noise is measured. The total noise power collected by a practical (i.e., lossy) antenna is an additive contribution of the noise powers collected by the radiation resistance and the loss resistance.

\[ N = k_B T_{rad} \Delta f + k_B T_{loss} \Delta f \]  

(2.4)

In a radiometric efficiency measurement, the antenna under test is pointed at two targets with different noise temperatures; the noise power received is compared. Noise power is generated by each resistance in proportion to perceived equivalent noise temperature. The loss resistance is affected by the ambient noise temperature, while the radiation resistance is affected by the noise temperature of the target. For a highly directive antenna, the target may be a small region in space. For an antenna with a wide beamwidth, the target may be very broad. In this case, the effective noise temperature is a result of radiation pattern integration over an extended source region.

The experiment is usually configured such that one target has a known low noise temperature (usually the open sky) and one target has the ambient noise temperature [6], as illustrated in Figure 2.2. In each configuration, the total output noise power is measured over a fixed frequency range \( \Delta f \) using a calibrated instrument.

\[ N_0 = k_B T_{rad|low-noise} \Delta f + k_B T_{loss|ambient} \Delta f \]  

(2.5)

\[ N_1 = k_B T_{rad|ambient} \Delta f + k_B T_{loss|ambient} \Delta f \]  

(2.6)

Because the power generated by the loss resistance does not change, sub-
traction allows the experimenter to calculate the noise power values generated by each resistor. Efficiency is computed as

$$\eta = \frac{N_{\text{rad}}}{N_{\text{rad}} + N_{\text{loss}}} = \frac{\Delta N + k_B T_{\text{rad}|\text{low-noise}} \Delta f}{N_1}$$

(2.7)

where $\Delta N = N_1 - N_0$, and $T_{\text{rad}|\text{low-noise}}$ is usually very small [6].

Unlike other methods making use of an equivalent circuit model, no assumption is made regarding the structure of the equivalent network. The model may include multiple radiation and loss resistances in series or in parallel, combined with any number of lossless reactive components. As we will observe, this is a key advantage of the radiometric method.

Unfortunately, the radiometric technique is challenging to implement in practice. The technique requires a constant low-noise temperature target. The sky is a sufficient target only for particularly directive antennas, within certain frequency bands [4], and when the clear sky is visible, making it an impractical laboratory technique. Interference from sources in the far field can give the impression that incident power is coming from ambient sources, collected through the loss mechanism. Without correction, this common situation can result in unduly low efficiency values.

### 2.3 Wheeler Cap Method

In 1959, Harold Wheeler described a radiation efficiency technique now known as the Wheeler cap method [7]. For a small radiating element that can be modeled as an ideal dipole, the electric and magnetic fields can be produced, as in [4], by integrating along a uniform current distribution $I$:

$$\mathbf{E} = \frac{I \Delta z}{4 \pi} j \omega \mu \left(1 + \frac{1}{j \beta r} - \frac{1}{\beta^2 r^2} \right) \frac{e^{-j \beta r}}{r} \sin \theta \hat{\theta}$$

(2.8)

$$+ \frac{I \Delta z}{2 \pi} \eta \left(1 + \frac{1}{j \beta r^2} \right) \frac{e^{-j \beta r}}{r} \cos \theta \hat{r}$$

(2.9)

$$\mathbf{H} = \frac{I \Delta z}{4 \pi} j \beta \left(1 + \frac{1}{j \beta r} \right) \frac{e^{-j \beta r}}{r} \sin \theta \hat{\phi}$$

(2.10)

where $r$ represents distance from the antenna, $\beta$ is the phase constant, and $\theta$ is the angle of elevation. For the ideal dipole and other electrically small antennas, electric and magnetic fields associated with stored energy fall off
quickly with distance, leaving $\hat{\theta}$ and $\hat{\phi}$ field components that combine to produce radiated power. Wheeler defined the *radianlength*, $\lambda/2\pi$, to represent the distance at which far field radiation terms begin to dominate field terms associated with near-field effects. The *radiansphere* represents the spherical volume one radianlength from the center of a small antenna. Wheeler argued that a conducting shell placed outside the radiansphere would have a negligible impact on stored energy while eliminating far field radiation [7].

Computing the $\hat{r}$-directed Poynting vector from the fields above, and integrating over a spherical surface of radius $r$ surrounding the antenna,

$$P_r = \frac{(I \Delta z)^2}{12 \pi} \omega \mu \beta \left(1 - \frac{j}{\beta^3 r^3}\right) \quad (2.11)$$

Close to the antenna, the imaginary $1/r^3$ term corresponding to reactive power predominates. In the far field, this term decays rapidly, leaving real-valued radiated power. Plotting the normalized real and reactive components of the power value (Figure 2.3), we observe that the magnitudes are equal for $r = \frac{\lambda}{2\pi}$, or one radianlength. If a conducting shell were placed beyond this boundary, the reactive power intercepted would be a small and rapidly decreasing fraction of the total power.

Figure 2.3: Normalized radiated vs. reactive power for ideal dipole
The Wheeler cap method requires the use of an equivalent circuit, including reactive elements, radiation resistance, and loss resistance, to represent the free space input impedance of the antenna. Placing a conducting cap over the antenna removes the effects of the radiation resistance from its circuit equivalent, either by removing or shorting it. The loss resistance component and reactive elements are not affected. By comparing measured resistances/conductances inside and outside the cap, individual resistance values can be determined. The two required measurements are demonstrated for a monopole antenna in Figure 2.4.

![Diagram of monopole antenna in free space and in Wheeler cap](image)

**Figure 2.4:** Monopole antenna, in free space and in Wheeler cap

Traditionally, a series or parallel RLC circuit model is assumed for the antenna under test. If the reactive elements are in series with the loss mechanism, then $R_{\text{loss}} = \Re\{Z_{\text{cap}}\}$. If the antenna input resistance decreases when the cap is applied, we can assume that the radiation and loss resistance are in series, and

$$R_{\text{rad}} = \Re\{Z_{\text{fs}}\} - \Re\{Z_{\text{cap}}\}$$

(2.12)

$$\eta_{\text{ser}} = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{loss}}} = \frac{\frac{1}{2}|I|^2 R_{\text{rad}} + \frac{1}{2}|I|^2 R_{\text{loss}}}{\frac{1}{2}|I|^2 R_{\text{rad}} + \frac{1}{2}|I|^2 R_{\text{loss}}}$$

(2.13)

$$= \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{loss}}} = 1 - \frac{\Re\{Z_{\text{cap}}\}}{\Re\{Z_{\text{fs}}\}}$$

(2.14)

This scenario is illustrated as Figure 2.5.

If the input resistance increases when the cap is applied, radiation and loss
resistances are in parallel, and we have

\[ R_{rad} = \frac{\Re\{Z_{fs}\}\Re\{Z_{cap}\}}{\Re\{Z_{cap}\} - \Re\{Z_{fs}\}} \]  \hspace{1cm} (2.15)

\[ \eta_{par} = \frac{P_{rad}}{P_{rad} + P_{loss}} = \frac{|V|^2}{2R_{rad}} = \frac{|V|^2}{2R_{rad} + \frac{|V|^2}{2R_{loss}}} \]  \hspace{1cm} (2.16)

\[ = \frac{R_{loss}}{R_{rad} + R_{loss}} = 1 - \frac{\Re\{Z_{fs}\}}{\Re\{Z_{cap}\}} \]  \hspace{1cm} (2.17)

This scenario is illustrated as Figure 2.6.

Similarly, equations based on conductance can be obtained for circuits in which reactance is parallel to the loss mechanism. If the radiation and loss
mechanisms are modeled in series,

\[
R_{\text{loss}} = \frac{1}{\Re\{Y_{\text{cap}}\}} \quad (2.18)
\]
\[
R_{\text{rad}} = \frac{\Re\{Y_{\text{cap}}\}}{\Re\{Y_{\text{fs}}\}} - \Re\{Z_{\text{fs}}\} \quad (2.19)
\]
\[
\eta_{\text{ser}} = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{loss}}} = 1 - \frac{\Re\{Y_{\text{cap}}\}}{\Re\{Y_{\text{cap}}\} \Re\{Y_{\text{fs}}\}} \quad (2.20)
\]
\[
= \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{loss}}} = 1 - \Re\{Y_{\text{cap}}\} \quad (2.21)
\]

This scenario is illustrated as Figure 2.7.

![Parallel RLC model, with loss mechanism in series](image)

Figure 2.7: Parallel RLC model, with loss mechanism in series

If the radiation and loss mechanisms are modeled in parallel,

\[
R_{\text{loss}} = \frac{1}{\Re\{Y_{\text{cap}}\}} \quad (2.22)
\]
\[
R_{\text{rad}} = \frac{1}{\Re\{Y_{\text{fs}}\} \Re\{Y_{\text{cap}}\}} \quad (2.23)
\]
\[
\eta_{\text{par}} = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{loss}}} = \frac{|V|^2}{2R_{\text{rad}}} + \frac{|V|^2}{2R_{\text{loss}}} \quad (2.24)
\]
\[
= \frac{R_{\text{loss}}}{R_{\text{rad}} + R_{\text{loss}}} = 1 - \Re\{Y_{\text{cap}}\} \quad (2.25)
\]

This scenario is illustrated as Figure 2.8.

The Wheeler cap technique is a relatively simple method to implement in the laboratory. Only a network analyzer (to collect input impedance) and the metal Wheeler caps themselves are required for this technique. Although Wheeler’s original work used spherical and hemispherical shells, cylindrical or rectangular caps are commonly employed and have been shown to pro-
duce minimal distortion [10]. Similarly, the Wheeler cap dimensions can be increased beyond one radian length from the antenna. However, as we will demonstrate in the following chapter, increasing the size of the cap may result in erroneous measurements due to cavity resonance.

Due to the number of variables associated with any antenna under test, it is difficult to specify the accuracy of the Wheeler cap technique. Newman et al. suggested that the traditional method could be expected to be accurate to within 25 percent. With careful configuration, repeatability is approximately 2 to 5 percent [5, 6].

2.4 Wheeler Cap Example

The Wheeler cap technique can be demonstrated in the lab using reference antennas to evaluate its accuracy. Unfortunately, obtaining an accurate ground truth efficiency value for a reference antenna is nearly impossible without specialized equipment. Electromagnetic simulation provides an alternate means of evaluating the accuracy of Wheeler cap algorithms [10]. Ansys HFSS (High Frequency Structure Simulator) is a full-wave EM solver that uses the finite element method. It includes a three-dimensional CAD system for the modeling of antennas and transmission line structures. S-parameter measurements can be generated at defined input ports. HFSS can also generate far field radiation plots and calculate high-precision radiation efficiency values using the gain/directivity method. The Wheeler cap algorithm evaluation procedure consists of the following steps:

1. Simulation of antenna in free space. Collect $S_{11}$ and reference radiation efficiency plots over frequency.

2. Simulation of antenna in Wheeler cap (a PEC or copper surface sur-
rounding the antenna). Collect $S_{11}$ values.


4. Compare processed results to reference efficiency obtained in Step 1.

Figure 2.9: HFSS model of PVC-jacketed monopole over copper ground plane

A short monopole can be used to demonstrate the Wheeler cap technique. A monopole antenna consists of a radiating wire element above a large ground plane. A short monopole is a monopole operating at frequencies well below its resonant frequency; electrically short antennas are often used in mobile devices due to physical size constraints. Because all constituent parts are good conductors, the monopole has low loss resistance. However, the short monopole also has very low radiation resistance and therefore low efficiency.

A short monopole ($\Delta z = \lambda/40$ at $f = 100$ MHz) has been simulated in HFSS. The monopole has been surrounded by a thin (3 mm) PVC jacket which provides some dielectric loss. The first model (Figure 2.9) is constructed with an air box and radiation boundary (not shown) to simulate a free space measurement. Figure 2.10 illustrates the second simulated model, in which a rectangular copper Wheeler cap has been placed over the monopole. S-parameters are generated for both simulations.

S-parameter datasets are imported into Matlab in order to implement the Wheeler cap algorithm. After translating both $S_{11}$ datasets to input
impedance values, we observe that the input resistance decreases at every frequency point when the cap is applied. A series circuit model is therefore most appropriate for the short monopole. Applying the conventional Wheeler cap formulation, we obtain the efficiency curve in Figure 2.11. The HFSS-computed radiation efficiency is overlaid for comparison. Excellent agreement is observed between the two plots.

![Figure 2.10: HFSS model of PVC-jacketed monopole in rectangular Wheeler cap](image)

![Figure 2.11: Simulated Wheeler cap efficiency curve for short monopole](image)
CHAPTER 3

WHEELER CAP PROBLEMS

The simplicity of the Wheeler cap formulation leads to a number of well-documented problems with the technique. Naive attempts to use the traditional method have been widely observed to fail for electrically larger antennas [11, 12, 13]. In practice, the placement of a metal shield over the antenna creates a new electromagnetic environment, with distinct coupling and cavity effects. In this chapter, we illustrate the problems associated with the Wheeler cap method. We also demonstrate the application of techniques that have been reported in the literature to partially address these issues.

3.1 Cavity Resonance Modes

The accuracy of the Wheeler cap algorithm can be adversely affected by the presence of cavity resonance modes. The cross-section of the Wheeler cap is that of a hollow waveguide, which supports transverse electric (TE) and transverse magnetic (TM) modes [14]. With proper excitation, electromagnetic waves with frequency above modal cutoff will propagate inside the cap. When the ends are sealed with PEC surfaces, the waveguide becomes a closed cavity. Cavity resonant modes are supported at frequencies based on cap geometry. Since rectangular caps are among the most common, we will consider cavity resonance from this perspective. From the dispersion relation, we have \( k^2 = k_x^2 + k_y^2 + k_z^2 \), where the \( k_i \) are determined by the solutions of the Helmholtz equation satisfying boundary conditions at the PEC cavity walls. The resonant frequencies of a cavity with dimensions \( a \times b \times c \) are then

\[
f_{\text{res}} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}
\]  

(3.1)
where \( m, n, \) and \( p \) represent a series of integers beginning with 0 or 1, depending on the applicable boundary condition. As an example, the spectral occupancy of TE and TM resonant modes for a rectangular cap of dimensions \( 424 \times 424 \times 295 \text{ mm} \) is indicated in Figure 3.1.

![Cavity Mode Resonance Spectrum](image)

Figure 3.1: Cavity resonance mode spectrum (TE and TM modes) for large (424 x 424 x 295 mm) Wheeler cap

Surface currents anywhere in the cap—on intentionally radiating elements, finite ground planes, and cabling—can excite specific modes. The orientation of these currents with respect to the modal field distribution governs the extent to which cavity resonance modes are excited. When the large Wheeler cap described above is used with the previously described short monopole, the antenna acts as a probe feed exciting \( TM^z_{mnp} \) modes, where \( m, n \) are odd.

Cavity resonance can result in spurious \( S_{11} \) measurements. Any attempt to characterize \( S_{11} \) in the Wheeler cap requires the addition of energy at a particular frequency. When energy is not absorbed or radiated but used to excite a resonance, dramatic variation in \( S_{11} \) values can be observed. The spikes in \( S_{11} \) result in invalid radiation efficiency values near frequencies associated with cavity resonance modes. This phenomenon can be observed in the short monopole efficiency estimate in Figure 3.2. Since the dimensions of the Wheeler cap affect the resonant mode frequencies, cap size selection is an important part of experimental design. Because the spectral density of cavity resonance modes increases with frequency, the lowest frequency resonance often acts as an upper limit on the dimensions of the Wheeler cap. Multiple caps may be required for measurements at various frequencies of interest.
3.2 Inadequate Circuit Models

The series and parallel RLC equivalent circuit models traditionally used in the Wheeler cap procedure provide poor impedance approximations for most antennas over a broad band. The real parts of the free space and Wheeler cap input impedances may cross multiple times if the measured bandwidth is sufficiently large. The effects of this phenomenon on efficiency plots, variously described as “dips” or “strange behavior” [11] in the literature, occur when the input resistance peaks have varying widths or are shifted in frequency. The variation has led to confusion regarding the appropriate model for various antennas, including the common microstrip patch antenna [15]. To illustrate the breakdown of the basic RLC models, which often occurs at the intended operating frequencies of the antenna under test, a dual-band PIFA has been designed according to the specifications of Liu et al. [16]. The PIFA is modeled in HFSS (Figure 3.3) and analyzed using the procedure outlined in Chapter 2. The results of the breakdown can be observed in the efficiency plot in Figure 3.4.
3.3 Wheeler Cap Improvements and Modifications

To resolve the issue, a number of investigators have introduced processing techniques that apply corrections to conventionally collected Wheeler cap data.
3.3.1 Rotational techniques

One popular and easily implemented correction method, described by McKinzie in [12], rotates the free-space and Wheeler cap $S_{11}$ responses on the Smith chart. This translation has the same effect as the addition of a transmission line between the antenna and analyzer reference plane. By reorienting the impedance circles along constant resistance or conductance curves on the Smith chart, the rotated $S_{11}$ response better approximates that of a series or parallel RLC circuit. This constant rotation approach produces more accurate results in certain cases. Unfortunately, Matlab simulations demonstrate that constant rotation often merely shifts the breakdown point in frequency. Furthermore, this approach is generally ineffective for multiresonant antennas.

3.3.2 Two-port processing methods

An interesting recent development in antenna modeling for efficiency estimation is the two-port antenna model. In this model, the antenna under test is thought of as a two port system, in which a radio transceiver represents the input and the antenna’s signal environment represents the load attached to the output. The loss mechanism is encapsulated in the two-port definition, while radiation is represented as power delivered to the real part of an attached load impedance. The two-port antenna model is illustrated in Figure 3.5.

![Figure 3.5: Two-port antenna model with external scattering environment (attached load)](image)

The efficiency of the antenna can be expressed as the ratio of power dissipated in the load to the total power accepted by the two-port. This is equivalent to the two-port operating power gain $G$ [1]:

$$G = \frac{\Gamma_{in} \rightarrow \Gamma_{L} \rightarrow [S] \rightarrow Z_L}{\Gamma_{in} \rightarrow \Gamma_{L} \rightarrow [S] \rightarrow Z_L}$$
\[ \eta = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{loss}}} = \frac{P_{\text{rad}}}{P_{\text{accepted}}} = G = \frac{|S_{21}|^2(1 - |\Gamma_L|^2)}{(1 - |\Gamma_{in}|^2)(1 - S_{22}\Gamma_L)^2} \] (3.2)

This efficiency expression can be simplified. First we assume that the load reflection coefficient of the antenna in free space is zero. This is a justifiable assumption because we have enough flexibility in the S-parameter definition to account for any immediate impedance mismatch, while radiation will not be reflected back to the antenna due to the free space assumption. We also assume that the antenna is reciprocal, and therefore \( S_{12} = S_{21} \). The new efficiency equation is then

\[ \eta = G = \frac{|S_{21}|^2}{1 - |\Gamma_{in}|^2} \] (3.3)

Johnston and McRory [17] proposed that the use of multiple Wheeler caps of varying sizes could be used to determine \( S_{21} \). This quantity is not directly measurable, but the two-port input reflection coefficient \( \Gamma_{in} \) is:

\[ \Gamma_{in} = S_{11} + \frac{|S_{21}|^2\Gamma_L}{1 - S_{22}\Gamma_L} \] (3.4)

In a free space measurement, \( \Gamma_L = 0 \), and therefore \( \Gamma_{in,fs} = S_{11} \). This provides the prospect of easily evaluating the quantity

\[ \Delta = |\Gamma_{in} - \Gamma_{in,fs}| = \frac{|S_{21}|^2|\Gamma_L|}{1 - S_{22}\Gamma_L} \] (3.5)

In the \( i \)th Wheeler cap, the output reflection coefficient \( \Gamma_L = e^{i\theta_i} \). Based on the phase of \( \Gamma_L \) relative to \( S_{22} \), there exists a maximum and a minimum \( |1 - S_{22}\Gamma_L| \), and therefore a maximum and a quantity \( \Delta \):

\[ \Delta_{min} = |\Gamma_{in,1} - \Gamma_{in,fs}| = \frac{|S_{21}|^2}{1 + |S_{22}|} \] (3.6)
\[ \Delta_{max} = |\Gamma_{in,2} - \Gamma_{in,fs}| = \frac{|S_{21}|^2}{1 - |S_{22}|} \] (3.7)

By solving the system of equations, \( |S_{21}|^2 \) can be determined. The Johnston efficiency equation is then [17]
Difficulties with the Johnston method include the fact that a cap with a sliding wall must be designed to precise mechanical tolerances. A Johnston cap is necessarily larger than the minimum-sized Wheeler cap due to the need for the sliding wall extension. Testing can be tedious as well, as each antenna must be placed in the cap such that the antenna’s primary reflection path length is affected by the sliding wall.

\[
\eta = \frac{2\Delta_{\text{min}}\Delta_{\text{max}}}{(\Delta_{\text{min}} + \Delta_{\text{max}})(1 - |\Gamma_{\text{in}}|^2)}
\]  

(3.8)

where \( f_{\text{search}} \in f \). In the published Geissler method, the search is conducted
over all frequency points in the Wheeler cap measurement. For multiresonant antennas, the response could be windowed. The efficiency equation is the same as in the Johnston method, Equation 3.8.

HFSS simulations and antenna measurement show the Geissler technique can provide good efficiency estimates while avoiding the dropouts associated with the series and parallel Wheeler cap models. The estimates are not particularly sensitive to cap size or antenna placement in the cap. Furthermore, the processing requirements—one minimum and maximum search for each free-space frequency point—are relatively low. The Geissler technique is therefore well-suited for producing quick efficiency estimates with reasonable accuracy. However, for electrically larger and multiresonant structures in larger caps, such as those we will consider in Chapter 5, assumptions embedded in the algorithm break down. In these situations, the efficiency results produced by the Geissler method can be corrupted by noise and other artifacts.
The development of advanced equivalent circuit models to represent antenna input impedance is an area of continuing research interest. Higher order circuit models can accurately represent the input impedance of multiresonant antennas over a broad band. Some of these advanced models attempt to accurately represent radiation and loss mechanisms as distinct phenomena.

The Wheeler cap procedure can be adapted for use with higher order circuit models by designating resistive elements in the equivalent circuit network as either radiation or loss resistances. The antenna’s measured free space input impedance corresponds to the input impedance of the complete equivalent circuit. The measurement in the Wheeler cap represents the equivalent circuit with resistors associated with radiation either shorted or removed. Because multiple resistive elements may be responsible for the differences in measured input impedances, response decomposition and numerical fitting techniques are required. All component values must be determined in order to calculate the power dissipated in each resistance. A transfer function approach is used to determine the relative power dissipated in each resistor for each frequency point. The radiation and loss powers are summed and efficiency is computed.

One approach for broadband antenna modeling uses a series cascade of parallel RLC subcircuits [18]. Another method proposed involves the use of multiple series RLC circuits, each coupled to a larger RLC circuit through a transformer [19]. Genetic algorithms and similar techniques are used to select component values that replicate the measured impedance responses. Unfortunately, many of the higher order circuit models suggested have been chosen for mathematical convenience or ease of synthesis, with secondary consideration given to accurate modeling of the electromagnetic phenomena associated with antenna radiation.
4.1 Characteristic Modes in Antenna Theory

In the 1960s and 1970s, Harrington, Garbacz and others investigated a new method of modal analysis, based on the method of moments technique [20]. This approach involved an eigendecomposition of the method of moments impedance matrix

\[ XJ_n = \lambda_n R J_n \]  

(4.1)

The \( J_n \) represent modal currents that can be excited. \( R \) and \( X \) are the real and imaginary parts of the impedance matrix. Characteristic modes can connect far-field pattern characteristics to these orthogonal currents. Each orthogonal eigencurrent contributes an orthogonal pattern to the far field. Characteristic mode analysis is a technique used to identify current modes on a structure that can be potentially excited [21]. Many of these characteristic modes exhibit resonance or radiation characteristics at particular frequencies. Designers can make strategic modifications to their antenna designs that reinforce or eliminate specific currents.

4.2 Improved Parallel Admittance Model

In Chu’s 1948 analysis of radiation from an antenna, a high-pass equivalent circuit model is introduced to represent spherical TM radiating modes [8]. The circuit topology has reappeared in work by Stuart as a model for spherical multielement structures and capped monopole type antennas [22]. Recently, Adams and Bernhard have applied characteristic mode analysis to obtain a similar model for the dipole antenna [23, 24].

The equivalent circuit model proposed by Adams represents an antenna using multiple sets of parallel admittance elements, as shown in Figure 4.1. Each admittance element is intended to reproduce the response of a particular radiating characteristic mode. The number of subcircuits \( N \) is adjusted based on the complexity of the admittance response.

Adopting the Adams HP-2 model for our higher order equivalent circuit, a new Wheeler cap method can be developed. Fitting free space and Wheeler cap responses to the model first requires decomposition of measured admittances, a task accomplished using rational function approximation and vector
4.3 Rational Function Fitting

A rational function approximation technique based on the vector fitting algorithm [25] is applied to break down both admittance responses into sub-admittances $Y_i$ of pole-residue form:

$$Y(s) = \sum_{i=1}^{N} Y_i(s) = \sum_{i=1}^{N} \frac{c_i}{s-a_i} + \frac{c_i^*}{s-a_i^*}$$  (4.2)

The sets of poles $\{a_i\}$ and residues $\{c_i\}$ may include real values and complex conjugate pairs. Antennas are understood to be passive circuits, and as such the real part of each pole is situated in the left half of the complex plane.

The rational fitting procedure is an iterative process that begins with an estimate of the system order and pole locations. The response to be fitted is provided, along with a frequency vector. Residues are then computed that provide a least-squares fit to the desired response. With this set of residues, a new set of poles is computed that provides an improved least-squares fit. A tolerance criterion can be assigned to evaluate the accuracy of each resulting model. If a model that meets the tolerance cannot be obtained within a certain number of iterations, the number of poles is increased and the process is repeated.

The generalized vector fitting process is designed to provide rational function approximations for multiport devices. These devices are often represented using $S$, $Z$, or $Y$-parameter matrices. The frequency responses observed at each port often have common poles. For a single-port antenna,
the fitting process is simplified. The Matlab function `rationalfit`, included in the RF Toolbox, uses the mechanics of vector fitting to produce a set of poles and residues for a provided frequency response. Our modified Wheeler cap method applies `rationalfit` to develop rational function approximations for the admittance of the antenna, as collected both in free space and in the Wheeler cap. The rational function fitting process provides a modal decomposition of the input admittance.

Processing both the free space and Wheeler cap collected data, we observe that poles can be found in substantially the same locations. Each pole in the Wheeler cap response can be associated with a corresponding pole at the same frequency in the free space response. The Wheeler cap response may include extra poles representing cavity resonance. The frequencies at which these extra poles appear can be predicted by cap geometry. The cavity resonance poles may be neglected in subsequent processing.

4.4 Model Synthesis

In the next stage of the modified Wheeler cap method, equivalent circuits are constructed for the antenna under test, both in free space and in the Wheeler cap. These equivalent circuit models are combined to produce a free space circuit model for the antenna under test with radiation and loss resistances specifically identified. From the synthesized circuit model, efficiency can be determined.

Each complex conjugate pole-residue pair produced in the rational function approximation step is synthesized as an HP-2 subcircuit. We use a genetic algorithm to determine component values, including $R$, $L$, and $C$, that provide a minimum least-squares fit. The genetic algorithm begins with a population of component values uniformly spaced on some interval believed $a priori$ to contain the true value. In an evaluation step, the admittance response is computed for each member of the population. The least-squares distances of the computed and target admittance responses, taken over frequency, represent the cost function of the genetic algorithm. Population members with component values that generate the lowest cost-function values—the most accurate responses, in a least squares sense—are preserved for consideration in subsequent rounds. Members with the highest cost function are removed.
and replaced with new members. Some new members inherit component values from two “parents” of higher fitness. Other new members are generated by adding a random variation to a constituent component value of an already-successful “parent.” As the genetic algorithm simulation progresses through multiple iterations, the cost function monotonically decreases. The simulation is configured to stop after a certain number of iterations, or when a component value combination is identified that meets a mean square error tolerance.

Figure 4.2: Converting measured admittances to RLC values
The flow chart in Figure 4.2 illustrates the model synthesis process in detail. Because the genetic algorithm is computationally intensive, the synthesis routine is written in the lower-level C programming language. It can be run within the Matlab environment as a precompiled executable using the mex utility.

The free space and Wheeler cap admittance subcircuits are paired based on resonant frequency. As in the traditional Wheeler cap method, a series or parallel loss model is identified based on comparison of the bulk resistance values. The individual resistances $R_{\text{rad}}$ and $R_{\text{loss}}$ are then defined for each pair. In the vector fitting decomposition of the Wheeler cap response, spurious sub-admittance responses due to cavity resonance modes are easily identifiable. By ignoring these undesirable artifacts in the pairing process, they are excluded from further consideration.

Finally, we compute overall efficiency using a transfer function approach, which accounts for the fact that at different frequencies certain subcircuits dominate the total response. This is accomplished by expressing the voltage across each radiation and loss resistance in terms of the system terminal voltages. Efficiency is computed as

$$\eta = \frac{\sum_{i=1}^{N} P_{i,\text{rad}}}{\sum_{i=1}^{N} P_{i,\text{rad}} + \sum_{i=1}^{N} P_{i,\text{loss}}} \quad (4.3)$$

4.5 Illustrated Example

A 2.4 GHz planar inverted-F antenna (PIFA) has been simulated using HFSS and subjected to the simulated Wheeler cap procedure previously described. The substrate foundation, shorting plane, and coaxial feed structure are clearly visible in the free-space HFSS model, shown in Figure 4.3.
$S_{11}$ values are collected in free space and within the Wheeler cap. The Wheeler cap simulated values include a number of spikes due to cavity resonance, as can be seen in Figure 4.4. The spikes manifest as spurious resonances, visible in Figure 4.5, when the S-parameters are converted to admittance responses.

(a) free space  
(b) Wheeler cap

Figure 4.4: HFSS-simulated $S_{11}$ values
The vector fitting process is then applied, decomposing the admittance responses. Overlaying the admittance curves, as in Figure 4.6, we can clearly observe the correspondence between the responses.

Figure 4.6: Decomposed admittance response, with cavity resonance artifacts

The resonances in the Wheeler cap measurement that do not have a corresponding set of poles in the free space measurement are identified and
excluded from subsequent processing steps. Figure 4.7 illustrates the effects of this removal.

![Mode Comparison—Cavity Resonance Removed](image)

Figure 4.7: Decomposed admittance response, cavity resonance artifacts removed

The genetic algorithm is applied to convert each remaining resonance into an appropriate HP-2 subcircuit. The subcircuits are arranged in parallel to provide equivalent circuits for the antenna inside and outside the Wheeler cap. The resulting two subcircuits, with assigned component values, are presented in Figure 4.8.
Next, loss models are generated for each HP-2 subcircuit. In this example, all observed resistances increase when the model is placed in the Wheeler cap, in which the radiation resistance is not present. Therefore a parallel loss model is appropriate, and all $R_{\text{rad}}$ and $R_{\text{loss}}$ can be determined accordingly. The free space and Wheeler cap subcircuits are then combined to form a single equivalent circuit (Figure 4.9) for the antenna.

![ Equivalent circuit for 2.4 GHz PIFA ](image)

<table>
<thead>
<tr>
<th>circuit</th>
<th>$R_{\text{rad}}$ (Ω)</th>
<th>$R_{\text{loss}}$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>238k</td>
<td>15.5k</td>
</tr>
<tr>
<td>B</td>
<td>254k</td>
<td>1.07M</td>
</tr>
<tr>
<td>C</td>
<td>307k</td>
<td>6.53k</td>
</tr>
</tbody>
</table>

Figure 4.9: Equivalent circuit for 2.4 GHz PIFA, with resistances separated by loss effects

The relative powers dissipated in each resistor in the model are computed
at each frequency point. Summing the total power dissipated in the radiation resistances and dividing by the total power dissipated in all resistances, the total efficiency is calculated. The efficiency estimate, included in Figure 4.10, demonstrates good agreement with the efficiency values computed by HFSS.

Figure 4.10: Efficiency estimate comparison: modified method vs. HFSS computation
The modified Wheeler cap method is well-suited for efficiency estimation for a wide range of antennas. In this chapter, efficiency values are produced for a number of common microstrip antennas, including two rectangular patch antennas and a quarter-wave patch antenna. S-parameter datasets are obtained from HFSS-simulated antennas and measurement of specially constructed physical antennas.

The microstrip patch antenna [1, 4] is a highly popular design. This type of antenna consists of a layer of dielectric material with a metal coating deposited on both sides. They are popular due to the relative ease with which they can be manufactured. Designs can be mass-produced to exacting specifications using low-cost printed circuit board fabrication technology. Furthermore, microstrip patch antennas are mechanically robust, have a low profile form factor, and can be installed in a wide range of locations. The use of substrates with higher dielectric constants allows the physical sizes of radiating structures to be reduced, which can result in designs with lower radiation efficiency [26]. Because of this phenomenon, and because the various substrates can introduce loss, radiation efficiency measurement of microstrip patch antenna designs is particularly important. As noted previously, conventional radiation efficiency measurement techniques often provide poor radiation efficiency estimates for microstrip patch antennas [11, 12, 13, 18].

5.1 Rectangular Microstrip Patch Antenna

The rectangular microstrip patch antenna is the archetypical microstrip antenna. To create a rectangular patch, the metallization on the top side of the patch is reduced in size through a milling or etching process. The resulting patch should have one dimension with an effective electrical length of approx-
imately $\lambda/2$ in the substrate medium [4]. One popular feed configuration is through a coaxial cable partially threaded perpendicularly through the dielectric. This is referred to as a probe feed. The shielding conductor of the coaxial cable is terminated at the ground plane, while the center conductor is threaded through the dielectric and soldered to the top metallization of the patch.

To demonstrate the application of the modified Wheeler cap method, we configure HFSS simulations of a standard microstrip patch antenna. The first simulation generates the S-parameters of the free space; the second produces the S-parameters as measured in the Wheeler cap. Figure 5.1 depicts the antenna model in its free space configuration. The HFSS model for the corresponding Wheeler cap simulation is illustrated in Figure 5.2.

Figure 5.1: 1 GHz microstrip patch antenna, free space simulation
From the simulated S-parameter results, we can directly estimate the efficiency using the conventional Wheeler cap model. The results, shown in Figure 5.3, are disappointing. Numerous efficiency dropouts are visible near the operating frequency of the patch. Neither the series nor the parallel model is applicable over the entire 0.9 to 1.1 GHz band.

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Microstrip Patch Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0</td>
</tr>
<tr>
<td>0.95</td>
<td>0.1</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>1.05</td>
<td>0.3</td>
</tr>
<tr>
<td>1.1</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Figure 5.2: 1 GHz microstrip patch antenna, Wheeler cap simulation

Figure 5.3: Conventional Wheeler cap efficiency results
The same S-parameter data is then used in the modified Wheeler cap algorithm. After vector fitting, cavity mode rejection, HP-2 subcircuit identification, and final efficiency calculation, we obtain the efficiency results found in Figure 5.4.

![Microstrip Patch Efficiency Graph](image)

**Figure 5.4:** Calculated efficiency using modified Wheeler cap algorithm

Excellent agreement is observed between the efficiency curves at the intended frequency of operation of the patch. Away from this region, the use of the HP-2 subcircuit model results in slightly elevated efficiency estimates.

### 5.2 Reduced Profile Rectangular Patch Antenna

The design principles employed in the previous section can be used to develop physically smaller antennas. The antenna’s ground plane—the largest part of the previous patch antenna—often must be limited in size due to form factor constraints. This can affect the performance of the antenna in a number of ways. As noted in [27], reducing the size of the ground plane causes the operating frequency of the patch to increase. Perhaps the most important consequence for radiation efficiency, however, is that a ground plane of reduced size has reduced ability to terminate surface currents on the antenna structure. Surface currents may then travel along the outside of the
connector cable. Any attached cable can therefore become an integral part of the radiating system. We have designed a 2.8 GHz rectangular microstrip patch antenna with a reduced-size ground plane for efficiency testing.

HFSS is used to simulate the patch antenna inside and outside of a Wheeler cap. The models used in simulation are illustrated in Figures 5.5 and 5.6 respectively. A discrete frequency sweep is configured to provide results at 10 MHz intervals from 1 to 4.5 GHz. (This frequency resolution is selected to ensure admittance peaks can be resolved appropriately for accurate RLC fitting.) From the HFSS simulation data, we extract S-parameters to compare the results of the modified method with HFSS’s numerically computed efficiency.

Figure 5.5: 2.8 GHz rectangular microstrip patch antenna with small ground plane and cable, free space simulation
5.2.1 Antenna Construction and Efficiency Measurement

Physical construction of the reduced-size rectangular microstrip antenna allows us to validate the results of our previous simulation and demonstrate the laboratory application of the modified Wheeler cap technique. To form the patch, a sheet of Rogers Duroid 5880 substrate is processed using a T-Tech QuickCircuit milling machine. This desktop CNC machine is used to remove mettallization surrounding the top of the patch and route the patch outline. A Hirose U.FL cable provides the coaxial feed; the inner conductor is threaded through the feed hole drilled in the patch and soldered in place. The outer shielding conductor terminates at and is soldered to the ground plane. A photograph of the constructed antenna is included in Figure 5.7.
Figure 5.7: Constructed 2.8 GHz rectangular microstrip patch antenna

The antenna is attached to a calibrated Agilent PNA series network analyzer via a U.FL to SMA adapter. S-parameters are collected, both in free space and when a Wheeler cap is placed over the antenna, at 801 distinct frequency points over a 1 to 4.5 GHz range. Both sets of measured S-parameters are imported into Matlab for processing. After modal admittance decomposition and filtering of cavity resonance modes using the techniques outlined in Chapter 4, the admittance responses in Figure 5.8 remain.
The genetic algorithm is applied to convert each modal admittance to a HP-2 subcircuit. From the RLC component values generated for the Wheeler cap and free space modal admittances, an equivalent circuit is developed that specifically indicates radiation and loss resistances. Finally, the total efficiency estimate is calculated. In Figure 5.9 we compare this estimate based on our measurements to predicted efficiencies based on simulated data.
5.3 Quarter-wave Patch Antenna with Cable

The quarter-wave patch antenna represents a further reduction in size of the rectangular microstrip patch antenna. To create a quarter-wave patch, a traditional rectangular patch is cut in half along its width dimension, leaving the top metallization with an electrical length of approximately $\lambda/4$ in the substrate. The top metallization is then shorted to the ground plane. The result is an electrically smaller, less directive patch antenna with a form factor more appropriate for mobile devices. A quarter-wave patch antenna based on the previous rectangular patch antenna is designed and simulated in HFSS. The free space model can be viewed in Figure 5.10. The Wheeler cap simulation model is illustrated in Figure 5.11.
The antenna is constructed using the milling techniques and coaxial attachment procedures outlined previously. The short is implemented using a strip of 3M #1181 copper tape that wraps around the edge of the antenna, overlapping the top metallization and ground plane. A photograph of the resulting antenna is included in Figure 5.12.
Using the network analyzer, S-parameter values are collected over a frequency range from 1 to 4.5 GHz. These values are processed in Matlab according to the modified Wheeler cap method. After the vector fitting and cavity resonance rejection steps, the resulting filtered admittance responses are shown in Figure 5.13.

The admittance responses are used to produce equivalent circuits based on the HP-2 model. Figure 5.14 illustrates the final efficiency estimate. Again the modified Wheeler cap algorithm generates accurate efficiency results for all frequencies of interest.
Figure 5.13: Processed free space and Wheeler cap measured admittance responses

Figure 5.14: Efficiency comparison for quarter-wave patch antenna
CHAPTER 6
CONCLUSIONS

The Wheeler cap method is a valuable technique that can be used to quickly and accurately estimate the radiation efficiency of small antennas. However, the limitations of the method for large structures and multiresonant antennas must be recognized. Accurate broadband measurements of radiation efficiency are difficult to obtain. The equivalent circuit models most commonly used in Wheeler cap algorithms are inadequate for representing antenna input impedance over a broad band. Higher-order circuit models hold promise for broadband impedance modeling of antennas, and the Wheeler cap technique can be adapted to accommodate these models. The traditional Wheeler cap method as well as many of its subsequent refinements are subject to disturbance by cavity resonance modes. The new Wheeler cap technique helps remove the effects of this interference when it occurs. The resulting procedure is more computationally intensive but offers a number of improvements over the original technique.

For microstrip patch antennas with small ground planes, an attached coaxial cable can become part of the radiating system. The greater electrical size of the antenna and the additional potential for excitation of cavity resonance modes poses a unique challenge for Wheeler cap methods. Under these conditions, the performance of the new method is very good. Over a series of simulated and constructed antennas, in Wheeler caps of various sizes, efficiency aligned well with results obtained through traditional methods. In particular, excellent agreement between simulated and experimentally processed data is observed at the intended operating frequency of each antenna.

We intend to continue development of this method, as further challenges must be resolved in the decomposition of input impedance into subcircuits that correspond to an antenna’s radiating and non-radiating mechanisms. Enhancements of this type can be expected to deliver more accurate efficiency estimates far away from the operating frequencies of the antenna under test.
An ideal Wheeler cap algorithm should be easy to implement, capable of producing accurate results for a wide range of antenna types, and rooted in electromagnetic theory. Our approach maintains these principles of the original Wheeler cap method while providing improved performance for broadband, resonant, and multiresonant antennas. The use of higher-order models based on characteristic mode theory helps to deliver improved efficiency results.
REFERENCES


