Recently, we carried out the perturbation analysis of $C_2$ spectra and identified forbidden singlet-triplet intersystem transitions,\(^a\) which aroused further interest in other $C_2$ spectra for the many low-lying electronic states of this fundamental molecule. In 1988, the $B^1 \Delta_g - A^1 \Pi_u$ and $B'^1 \Sigma_g^+ - A^1 \Pi_u$ band systems were discovered by Douay et al.,\(^b\) who observed eight bands of the $B^1 \Delta_g - A^1 \Pi_u$ system with $v$ up to 5 for the $B^1 \Delta_g$ state and six bands of the $B'^1 \Sigma_g^+ - A^1 \Pi_u$ system with $v$ up to 3 for the $B'^1 \Sigma_g^+$ state in the Fourier transform infrared emission spectra of hydrocarbon discharges. In the work presented here, we identified twenty-four bands of the two systems, among which the $B'^1 \Sigma_g^+ v = 4$ and the $B^1 \Delta_g v = 6, 7 \text{ and } 8$ vibrational levels involved in nine bands were studied for the first time. A direct global analysis with Dunham parameters was carried out satisfactorily for the $B^1 \Delta_g - A^1 \Pi_u$ system except for a small perturbation in the $B^1 \Delta_g v = 6$ level. The calculated rovibrational term energies up to $B^1 \Delta_g v = 12$ showed that the level crossing between the $B^1 \Delta_g$ and $d^3 \Pi_g$ states is responsible for many of the prominent perturbations in the Swan system observed previously.\(^c\) Nineteen lines of the $B^1 \Delta_g - a^3 \Pi_u$ forbidden transitions were identified and the off-diagonal spin-orbit interaction constant $A_{dB}$ between $d^3 \Pi_g$ and $B^1 \Delta_g$ was derived as $8.3(1) \text{ cm}^{-1}$. For the $B'^1 \Sigma_g^+ - A^1 \Pi_u$ system, only individual band analyses for each vibrational level in the $B'^1 \Sigma_g^+$ state could be done satisfactorily and Dunham parameters obtained from these effective parameters showed that the anharmonic vibrational constant $\omega_v x_e$ is anomalously small (nearly zero). Inspection of the RKR potential curves for the $B'^1 \Sigma_g^+$ and $X^1 \Sigma_g^+$ states revealed that an avoided crossing may occur around $30000 \text{ cm}^{-1}$, which is responsible for the anomalous molecular constants in these two states.\(^d\)


