Acknowledgment

The heroes of this story are the students and visiting scholars whose researches are described. One of the most satisfying experiences I have had was witnessing how fearlessly they pursued new knowledge.
Preface

After completing work on MS and PhD degrees at the Ohio State University and Princeton University, I joined the faculty of the Department of Chemical Engineering at the University of Illinois at Champaign-Urbana in 1953. Over the period 1953-2007, I had the opportunity to work with 78 MS students, 77 PhD students, and 23 Postdoctoral students/visiting scholars. This manuscript gives an account of their accomplishments.

The work was motivated by a desire to improve fundamental understanding of basic problems in fluid dynamics, which are central to understanding chemical engineering operations. Two new techniques which greatly expanded the scope of research during this period were the use of supercomputers to do direct numerical simulations of turbulent fields and the use of optical techniques which provided measurements of velocity fields without interfering with the flow. Collaborations with Prof. John McLaughlin, Clarkson University, and Ronald Adrian, University of Illinois, allowed us to adopt these approaches more gracefully into our research.

Progress was greatly aided by the development of several large-scale facilities to test and motivate theory. We are particularly grateful to the Departments of Chemical and Mechanical Engineering (Figures 62, 72) at the University of Illinois for supplying the space, which made this possible. Financial support from the Shell Development Company, the Department of Energy, and the Office of Naval Research is appreciated.

An important aspect of this research is that it impacted the education of undergraduate and graduate students in engineering. See, for example, Paper 46 for a description of a graduate course in fluid dynamics. The research described in this manuscript enabled 47 undergraduate theses (Figure 77) and, at least, an equal number of undergraduate projects, which were carried out by groups of two students over a period of one semester.

A wide range of topics is covered. The writing is, to a large extent, motivated by a desire to present a more cohesive picture of what was accomplished, than would be obtained by glancing over the list of publications that is presented. I hope I have been successful in doing this.
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Summary

This account has been organized into twenty Sections listed in the Table of Contents.

Section 1 describes the invention of electrochemical probes which allow the measurement of the velocity field closer to the wall than had previously been possible. This provided new opportunities to study boundary layers and the behavior of turbulence in the immediate vicinity of a wall. The technique involves the measurement of the rate of mass transfer to a small electrode embedded in a wall and the use of the mass balance equation to relate the mass transfer rate to the velocity field. The use of an inverse mass transfer approach greatly expands possible usage to reversing flows.

Section 2 exploits Lagrangian methods to provide a more rigorous way to describe turbulent heat and mass transfer. The scalar field is described as resulting from a distribution of sources and sinks, whose behavior can be described by stochastic methods. Direct simulations and laboratory measurements of the temperature field are utilized.

Section 3 describes a major advance in our understanding of turbulent heat transfer or mass transfer at large Prandtl or Schmidt numbers, that was enabled by the use of electrochemical methods to measure the local time-averaged and fluctuating mass transfer rates. The theoretical challenge is to describe how the concentration/temperature field is related to the velocity field. A remarkable result for a cylindrical mass transfer section is that fluctuations at two locations separated by a circumferential distance of 4.6 mm show time variations, which are almost completely independent of one another.

Gas transfer to a liquid is controlled by the behavior in a very thin concentration boundary layer in the liquid close to the interface, since it is characterized by large Schmidt numbers. Section 4 describes work that relates the rate of mass transfer to the velocity fluctuations in this layer and, thus provided the first major advance over the concept of a Nernst diffusion layer. Waves enhance the rate of mass transfer by inducing spatial variations of the gas phase shear stress acting on the interface. These stresses are directly related to the slope of the interface. The equation describing transfer rates is roughly unchanged for order of magnitude changes in the system size because it depends on wave slope rather than wave height. A method was developed to provide measurements of the wave slope. Fluorescence techniques were invented which allow measurement of the concentration variation in the very thin boundary layer.
Liquid metals can provide increased heat transfer rates by increasing the direct contribution due to molecular diffusivity, but they decrease turbulent diffusion by the eddies transporting heat. Section 5 uses a direct numerical simulation to provide a theory for the latter behavior. Correlations for the effect of Peclet number on the turbulent diffusivity and the heat transfer coefficient are developed. Diffusion of heat from a point source in a homogeneous isotropic turbulence is also considered.

Section 6 describes results on the structure of a turbulent shear flow. The tools that were used were electrochemical probes, particle-image velocimetry (PIV), laser-Doppler velocimetry (LDV), and direct numerical solutions (DNS) of the Navier-Stokes equation for flow in a channel and for planar Couette flow. The determination of the origin of the vortices that dominate the behavior close to the wall and the identification of the "superbursts" that dominate the outer flow are of interest.

Section 7 describes experiments in which we used laser-Doppler velocimetry to study the effect of polymers on turbulent flow in a channel. Important results are the identification of changes of turbulence structure with increasing polymer concentration and the demonstration that at large polymer concentrations the Reynolds stress vanishes. In this situation turbulence is produced by average polymer stresses. The interaction of a polymer molecule with fluid turbulence was studied by using a bead-spring chain to represent a polymer molecule and a DNS to produce the turbulent field. A surprising result is that degradation need not be associated with changes in the molecular weight. Suppression of turbulence is also noted in the flow of suspensions of spheres and accelerating boundary layers. This finding has lead to the definition of a unifying concept for this phenomenon. Small, but measurable drag reduction can be realized for imposed asymmetric oscillations.

The work in Section 8 was initiated by a study of air-water flow in a 1x12 in enclosed channel, which identified waves that are playing a central role in gas-liquid flows. Theoretical explanations are given which connect the appearance of these different wave patterns with wave-induced variations of gas phase pressure and shear stress at the interface. The influence of waves on the turbulence in the liquid and gas is discussed.

Section 9 considers the effect of small amplitude spatial (flow over solid waves) and temporal (imposed oscillations) disturbances on turbulence. The results are used to explain wavelike dissolution patterns and wave generation. This work exploited electrochemical techniques to obtain measurements of the shear stress along a solid wavy surface. These were used to test a theory that includes wave-induced variations of the turbulent
viscosity. Of interest, is the establishment of a kinship between flow over a solid wave and an imposed oscillation.

Section 10 uses wall shear stress probes, one and two-component laser Doppler velocimetry, particle-image velocimetry, and direct numerical simulations to provide the most extensive study of flow over large amplitude solid waves. The interest lies in their appearance in a number of applications and in their possible usefulness in testing turbulence theory. Linear non-separated, nonlinear non-separated and separated flow regimes are defined. Four zones are identified for separated flows: The outer flow can be described by linear inviscid theory. Zone 2 is the turbulent boundary layer in the region close to the wall, which is bathed by a forward moving flow. Zone 3 is the separated region. Zone 4 is the shear layer formed when the boundary layer separates. At no time does a circulation pattern exist in the separated region. Turbulence in the separated region is associated with an interaction between the flow close to the wall and a turbulent shear layer. Studies were also carried out with a wavy surface for which the ratio of the wavelength to the height of the channel is small. A comparison of results for flow over smooth and roughened walls shows a universal behavior.

Section 11 describes extensive studies of the behavior of particles in a turbulent field and of their rate of deposition on a wall. The motivation was to obtain an understanding of gas-liquid annular flow. Axial-viewing photography was exploited to determine the relationship of particle turbulence to fluid turbulence. An important contribution is the demonstration that the use of a Lagrangian framework is not only more versatile than an Eulerian framework but also more correct. Direct numerical calculations of the behavior of a suspension of particles show that a drastic suppression of turbulence can be realized with a very small fraction of particles. The development of a stochastic method to represent the fluid turbulence seen by the particles allowed calculations to be made for a much larger range of conditions than would be possible with a DNS.

Section 12 describes our work on gas-liquid annular flow in a vertical tube, for which a portion of the liquid flows along the wall as a film, and a portion, as drops. The fraction of the liquid flowing as drops, the entrainment, is treated as resulting from a balance between the rate of atomization of the liquid film and the rate of deposition of drops. The pressure drop is controlled by the drag on the interface, which is roughened by waves. The latter is directly related to the thickness of the film. We have developed new or improved relations for all of these quantities. Of central importance is the knowledge of drop size. A considerable effort was put into
the measurement and prediction of the distribution function describing drop diameter.

Our work on annular flow in horizontal pipes is described in Section 13. The critical issue is the description of the spatial distribution of liquid (in the core and in the wall film) and of the influence of these asymmetries on the rate of atomization, the rate of deposition and the entrainment. We suggest that a transition from stratified to annular flow occurs when drops reach the top of the pipe in sufficient quantity to form a stable continuous film. This idea is exploited to produce results, which could be important in large diameter natural gas/condensate pipelines.

Section 14 describes our studies of the flow regimes for gas-liquid flow in horizontal or slightly inclined pipelines and of their initiation. Annular flow exists at high gas velocities. Other regimes are stratified flow, slug flow, plug flow, pseudo-slug flow. Our demonstration that polymer additives can cause dramatic changes in the flow pattern is noteworthy because it suggests that one can alter the behavior of gas-liquid systems. The flow pattern can change dramatically in slightly inclined pipelines. Experiments and theory were carried out to obtain an understanding of these changes.

Slug flow may be pictured as a sequence of aerated turbulent units that travel at approximately constant velocity over a stratified layer. Section 15 describes our efforts to model this regime. Experiments involved measurement of the time variation of the liquid holdup at several locations along a pipe. Pressure variations associated with the passage of a slug were determined with a piezoresistive transducer mounted flush with the wall. The theoretical goals were to predict the frequency of slugging (or the distribution of slug lengths) and the pressure profile over individual slugs. A stationary hydraulic jump can be formed by introducing a disturbance into a fluid flowing under a gate located in a pipe that is inclined downward. If air is allowed to bleed into the space behind the gate, the jump can reach a stationary state for which the liquid behind the jump will have a gradually sloping tail and will not fill the pipe behind the jump. Studies in this system provided an opportunity to test theories on slugs.

A model of stratified flow is considered in Section 16. A simple geometric configuration, which considers the interface to be flat, is used. Our main contribution is the modeling of the stress at the interface. We show that this stress scales with the ratio of the wave height to the pipe diameter. By use of a semi-theoretical approach, we developed an expression for the interfacial stress. A discussion of a stratified/annular configuration is given.

Most theoretical work on waves uses the assumption of small amplitudes. Section 17 describes our effort to understand the influence of
nonlinearities. Waves were generated by a dipper in a shallow stationary liquid. At dipper frequencies between resonance conditions, we observed multiple peaks on the interface (as many as five). These waves were not stable in that they continued to change their form as they propagate downstream. We developed a numerical approach by simply calculating the change with time of an interfacial displacement described as the sum of a finite number of propagating harmonics. A wave of permanent form is then the special case where the phases of the different harmonics are not changing with time. Of more relevance to gas-liquid flows are finite amplitude waves which occur in concurrent flow of two fluids. Our efforts in this area lead to the prediction of a bifurcation at a lower gas velocity than is predicted by linear theory and to the development of a relation for wave amplitude. The first of these is significant in that it suggests a mechanism for the initiation of slugs which could be important in very large diameter pipes. The second is used to develop a relation for interfacial stress in stratified flows.

Heat transfer with liquids having a viscosity close to that of water creates density changes which can alter the flow field and even cause premature instabilities. Section 18 describes our work aimed at understanding this phenomenon. The motivation is to improve predictions of the rate of heat transfer. Both vertical and horizontal pipes were considered. Visual studies which show the distortion of the mean velocity profile are of particular interest. The work is also of interest because it shows the influence of inflection points in the mean velocity field on the initiation of an unsteady flow. Experiments were also carried out to examine natural convection caused by mass transfer in an electrochemical system.

Section 19 describes our studies with fluidized, sedimenting and packed beds. Relations between the relative velocity (of the particles and the fluid) and the volume fraction of solids were developed. These were found to be the same for fluidized and sedimenting beds. The transition to turbulent flow in a packed bed was studied. The distinguishing aspect of our studies in packed beds is the focus on the flow around single spheres in the bed. Over a large range of Reynolds numbers, boundary-layer theory is applicable. Methods for carrying out such a calculation were developed; these use an electrolytic tank to determine the external flow. A method for correlating mass transfer rates between the spheres in a packed bed and the fluid is developed. Temperature measurements in a packed bed show an apparent discontinuity at the wall which can differ with different types of packing. This was investigated.
The modified insitu process for obtaining oil from shale involves the creation of rubblized bed by judicious use of explosives. The top is ignited and a combustion wave moves through the raw shale. This creates hot inert gas, which moves ahead of the combustion wave to decompose the kerogen in the raw shale. Oil produced by this pyrolysis is carried from the hot zone as a vapor, which condenses on the cold shale and flows out of the bottom of the bed as a liquid product. Section 20 describes work which we have done to model this combustion wave. The importance of this work is that it suggests operating conditions that optimize the yield.
1. Electrochemical diagnostics

(a) Invention of electrochemical wall shear stress probes

Early work in our laboratory (4) explored the notion that a theoretical treatment of heat or mass transfer between a wall and a turbulently flowing fluid would profit by considering the wall as an array of point sources and sinks. This prompted an attempt to study the behavior of small wall sources. An attractive approach was to introduce a foreign chemical by carrying out an electrochemical reaction on a small circular electrode embedded in the wall (so that the flow was not disturbed). Initial experiments (18) showed unexpected large current fluctuations (mass transfer rates) when a constant voltage was maintained on the electrode. These were interpreted as resulting from velocity fluctuations in the so called "laminar sublayer", where the variation of mean velocity with distance from the wall is the same as would be predicted for a steady laminar flow. This indicated that the velocity in the immediate vicinity of the wall is dramatically different from the notion of a laminar flow, in agreement with the hypothesis put forward in (3). In (26) and (32) we showed that the mean and the fluctuating mass transfer rate could be related to the mean and fluctuating shear stress at a location on the wall. These experiments opened the possibility of measuring the velocity field closer to the wall than had been realized by any known technique.

Heyrovsky and his co-workers at Charles University in Prague developed the polarographic method which uses measurements of the current-voltage relation in an electrolysis cell for qualitative and quantitative analyses of ions in solution. (This work was recognized with the awarding of the Nobel Prize.) In their experiments, one electrode (usually the cathode) is much smaller than the other electrode in the electrochemical circuit, so the current is determined by what is happening at this test electrode. A specie in the solution reacts if the applied voltage exceeds a certain value. At high enough voltages, the reaction proceeds so fast that the current is controlled by the rate of mass transfer and the concentration of the reacting specie at the electrode is negligibly small. This polarization condition is identified by a plateau in the current-voltage curve.

Analytical chemists used this method under carefully controlled hydrodynamic conditions to determine the composition of a solution. The voltage at which polarization occurs identifies the presence of a specie and the magnitude of the current is directly proportional to the concentration.
The techniques pursued by our research group are the inverse of this approach in that the chemistry is carefully controlled so that the current flowing in the circuit is a measure of the properties of the flow field. An advantage of using electrodes mounted flush with the wall is that the flowing fluid tends to keep surfaces clean. Typical thicknesses of the concentration boundary layer are 0.02 to 0.003 mm. The measured current is thus related to the velocity field in this region. The behavior of the probes can be calculated, so a calibration is not needed.

Paper 63 presents a review of our studies (prior to 1969) in which we used electrochemical probes to study flow fields.

(b) Application to velocity boundary layers

One of the main results from boundary-layer theory is the prediction of wall shear stress variation around a body. Yet, the boundary layer in most flows is so thin that conventional methods cannot be used to check the theory directly. The experiments described in (26) and (32) suggested that electrochemical probes could be a valuable tool in this area of research. Consequently, two water tunnels were constructed to study flow around cylinders. One (See Fig. 1) used a gravity feed (37) for measurements at Reynolds numbers of 40, 200, 500. The other (See Figs. 3, 8) used a forced circulation to study flow at Reynolds numbers of 5000 to 10^5 (44). A sandwich-probe of two 25.4 x 0.127 mm wall electrodes separated by 0.05-0.075 mm of insulation was invented to detect the instantaneous direction of flow (44), (See Fig. 2) One of these was the test electrode. The current flowing to it gives the local wall shear stress. If the activation of the other electrode causes a sudden change of the signal, the test electrode is in the wake of the second electrode. By using this technique it was possible to determine the separation location for flow around a 1.9 cm cylinder to within 1°. Measured wall shear stresses were in good agreement with boundary-layer theory up to the separation point. A surprising result was the discovery of a steady separation bubble. Just beyond this bubble the velocity close to the wall is positive. This eventually gives way to a haphazard behavior.

Speculation about the flow pattern is presented in (44). The thicknesses of the boundary layers studied in (44) were in the range of 0.17 to 0.08 mm. The difficulty with using classical measuring techniques and the value of electrochemical probes become evident. Until the publication of (44), measurements of the variation of the velocity gradient around the wall of a cylinder used Preston tubes and were confined to a Reynolds number range
of $10^5$ to $10^6$. Clearly, the electrochemical shear stress probe provided the opportunity for breakthrough studies.

The measurements presented in (44) showed unsteady behavior. We wanted to compare these results to a steady solution of the Navier-Stokes equations. Steady state numerical solutions had been obtained for flow around a cylinder only for Reynolds numbers of 50 or less. In order to avoid numerical instabilities, the above approach was extended to larger Reynolds numbers in Paper 43 by obtaining finite difference solutions of the time dependent equations of motion for Re = 50, 200, and 500. The flow field never became steady since the calculated wake continues to grow. Therefore, the calculations were terminated at sufficiently large times that the velocity field near the wall, the pressure at the wall and the length of the wake bubble closely approached the values they would assume at infinite time.

The success of the experiments described above lead to the use of wall electrodes to study flow around a sphere in beds of spheres (38, 39, 65) and turbulent flow over wavy surfaces (75, 78, 84). To facilitate the studies of packed beds an electrode design was developed that could give the direction of the wall shear stress as well as the magnitude in a three-dimensional boundary layer (38, 56).

(c) Application to turbulence

One of the more important applications of electrochemical wall probes is in the study of turbulence. A limitation is that the concentration boundary layer acts as a capacitance whereby the mass transfer rate does not exactly follow temporal changes of the flow over the probe. A correction which accounts for the effects of this frequency response was considered in (32) and (55).

The use of circular or rectangular electrodes (with the long side perpendicular to the flow) allowed measurements of the fluctuating velocities in the direction of mean flow. A method for measuring fluctuations in the spanwise direction involved the use of a chevron arrangement of rectangular electrodes (32, 34, 45, 47, 49, 50), (See Fig. 4). The publication of an account of this work has also appeared in Acta Mechanica Sinica (128).

All of the analyses of probe behavior used in the references cited above were developed by assuming small amplitude fluctuations. This put a limitation on the application of wall probes. Thus, the assumption is not exact for turbulent flows, where the root-mean-square of the streamwise
velocity fluctuations is close to 0.4. The neglect of nonlinear effects can lead to qualitative, as well quantitative, errors in studies of flows with imposed oscillations. Furthermore, this treatment of the electrode measurements is not useful in haphazard flows such as exist downstream of separation in boundary-layer flows. The remediation of this problem recognizes that the mass transfer surface is well defined and that the probe is small enough that the flow is uniform over its surface at any given time. The equations and boundary conditions describing the concentration boundary layer under these circumstances can be written without using any empirical constants. Numerical finite difference methods can then be used to calculate the time varying mass transfer rate if the time varying velocity gradient at the wall is known.

The problem of interest is the inverse of this, in that the time-varying rate of mass transfer is measured and the time-varying velocity field is calculated. Papers 154 and 159 describe an iterative method by which this inverse problem can be solved. In (154) it is shown that the application of this method provides improved measurements of the amplitude distribution and the frequency spectrum of the streamwise component of the fluctuating velocity gradient at the wall, for a fully-developed flow. Values of rms level, skewness and flatness of 0.37, 0.96, 4.2 were obtained.

The approach outlined above is valid only if the flow over the wall probe does not reverse direction. In (172) it is shown how the inverse mass transfer method can be used to study reversing flows, by considering the difference of the mass transfer rate to two rectangular electrodes separated by a thin layer of insulation. These works expand the possible use of electrochemical wall shear stress probes to situations which cannot be studied by any other known method. They also offer an improved way to analyze the behavior of hot film probes in that heat losses to the substrate can be included in the analysis. Except for the research described in (184), we have not exploited this opportunity. (See Section 7c.)

References 98 and 190, which contain reviews of methods used to measure wall shear stresses, provide a generous discussion of electrochemical probes.

(d) Symposia

On the occasion of the 100th anniversary of the birth of Heyrovsky a CHISA meeting in Prague sponsored a symposium on electrodiffusion flow diagnostics. Thomas J. Hanratty was pleased and honored to give a keynote
address. This was published in the Journal of Applied Electrochemistry (158).

Hanratty was also invited to make introductory remarks to a technical session on "Electrochemical Methods of Turbulence Measurement" held at the Fourth Symposium on Turbulence in Liquids held at the University of Missouri-Rolla, September 22-24, 1975. This talk (68) provides a summary of advances made during the ten years since the invention of the electrochemical wall shear stress probe and of the challenges presented. Noteworthy is the detailed discussion of the fabrication of probes with different configurations by using machining techniques. The paper mentions that techniques used to make printed circuits offer attractive alternative possibilities. The substitution of twin wall electrodes consisting of two semi-circles, rather than a chevron arrangement, was discussed as offering a more compact arrangement. Work exploring the possibility of using wire electrodes to measure velocity fluctuations in the bulk fluid is also described. (Bieg, K., "Measurement of turbulence properties with a mass-transfer analogue of a hot film anemometer", MS thesis in chemical engineering, University of Illinois, Urbana, 1974).
2. Mass and heat transfer

(a) Wall heat/mass transfer; Lagrangian methods

Early studies dictated several themes in the area of heat and mass transfer. During my research as a graduate student at Princeton University, I became intrigued with the notion that turbulent heat and mass transfer might be better understood if Lagrangian methods were used. These involve describing the change of the spatial location of a fluid particle or a marker with time, rather than following changes in a fixed volume in space (the Eulerian method).

A basic element of this approach is Taylor's description of dispersion from an instantaneous point source in a homogeneous, isotropic turbulence. A diffusivity is defined as $E = (1/2)(d\langle X^2\rangle/dt)$, where $X$ is a coordinate of the particle at some time after being admitted to the field. The brackets represent an average for a large number of events. The analysis shows that $E$ increases with time until a constant value, $E_{\infty}$, is reached at large times. This limiting value of the eddy diffusivity is defined as $E_{\infty} = \nu^2\tau^L$, where $\langle \nu^2\rangle$ is the mean-square of the $X$-component of the velocity fluctuations and $\tau^L$, the Lagrangian time constant. The parameter $\tau^L$ is a measure of the time interval over which the motion of the particles is highly correlated.

We carried out an analysis for heat transfer from the walls of a channel to a plug flow of a homogeneous turbulent fluid (4, 10) by considering the walls to contain arrays of sources and sinks of heat markers, whose behaviors are described by Taylor's theory.

The classical description of turbulent heat transfer involves an Eulerian approach whereby changes in a fixed volume in space are considered and the rate of transfer across a plane is given as $-\varepsilon(dT/dy)$. The spatial variation of the Eulerian turbulent transfer coefficient, $\varepsilon$, found in these analyses, is usually interpreted as due to a spatial variation of the turbulence. The turbulence in the above analysis (4) was considered to be homogeneous. Yet, a spatial variation of the Eulerian diffusivity was calculated. This suggested that variations observed in Eulerian measurements are, to a large extent, due to the time-dependence of the turbulent diffusivity defined in Taylor's analysis. Thus, in the calculations, the Eulerian diffusion coefficients observed close to the wall can be smaller than in the center regions of the channel because, on average, the diffusing specie has been in the field a shorter period of time. This work, along with
(3), were recognized with the Colburn Award of the American Institute of Chemical Engineers in 1957.

The Lagrangian approach outlined above was also used to calculate the velocity profile that is realized for a fully-developed flow in a pipe (8, 10). In this case, the pressure gradient is represented by sources of momentum embedded in the fluid. The wall is represented as a series of ring-shaped sinks of momentum. For a fully-developed condition, the sum of the contributions of the wall sinks equals the contribution of the sources representing the pressure gradient. The theoretical problem in carrying out this calculation is the description of the behavior of a single ring-shaped sink. The approach outlined in (21) was used to address this problem.

(b) Experiments on turbulent heat transfer

The mean and fluctuating temperature fields for air flowing through an electrically heated pipe (constant heat flux at the wall) were measured both for developing and fully-developed temperature fields (20, 21, 25). These experiments were facilitated by the design of a special device that avoided the difficulties of bringing probes through the wall of the heating section. (See Fig. 5) A traversing mechanism, which could move in three dimensions, supported a thermocouple or an impact tube. It entered through the exit of the pipe. These studies produced a widely used set of data on the temperature field for turbulently flowing air. The motivation was to test ideas that consider the temperature field as resulting from a distribution of sources and sinks of heat, and to explore the relevance of the measurements of dispersion from point sources located at the center of the pipe.

Far downstream, a fully-developed condition is established for which the shape of the temperature profile is not changing. In this region the bulk temperature and the wall temperature are increasing linearly with distance downstream at a rate dictated by the heat flux at the wall. From the center of the pipe to about 0.02 of a radius from the wall, measurements of the temperatures are given by

$$T^+ - T_c^+ = 7.6\left[\frac{(r/a)^2}{12(r/a)^4}\right],$$

where $T^+ = (T_W - T)\rho c_p v^*/q_W$, $q_W$ is the heat flux at the wall and $T_W$ is the wall temperature.

If an Eulerian approach is used the heat balance equation for the fully-developed region is

$$\bar{U} \left( \frac{\partial \bar{T}}{\partial x} \right) = (1/r) \frac{\partial}{\partial r} \left[ r(k + \varepsilon_h) \left( \frac{\partial \bar{T}}{\partial r} \right) \right],$$

where $\partial \bar{T}/\partial x$ is independent of $r$ and $\varepsilon_h$ is the Eulerian turbulent diffusivity. The eddy diffusivity profile can be calculated from the temperature measurements by
using this equation. It shows a maximum at \( r/a \simeq 0.5 \) and a minimum at the center of the pipe. It is zero at the wall. If the eddy conductivity, \( \varepsilon_h \), is represented by a spatially averaged value, the above temperature defect equation is predicted quite accurately by the integral of the heat balance equation if \((\varepsilon_h + \kappa)/av^*\) is set equal to \(8.1 \times 10^{-2}\).

A surprising aspect of this result is that turbulent diffusivities obtained from measurements of diffusion from centrally located sources (11) are roughly one-half of the diffusivity given above. This result is not fully understood. One possible explanation is that dispersion from a centrally located source is characterized by a smaller turbulent diffusivity than is the temperature profile since diffusion from a centrally located source is strongly affected by the diffusivity close to \( r \approx 0 \). Another possible explanation, cited in (20), is that the skewness of the r-component of the velocity fluctuations is negative. This could be reflected by a larger spread of material toward the center of the pipe than away from the center.

Measurements of temperature profiles in the heat transfer entry section are presented in (21). These provided an opportunity to test theories, presented in the literature, which suggest that the temperature profiles consist of a temperature boundary layer of thickness \( \delta_h \) and a constant temperature core. They picture \( \delta_h \) as increasing with distance downstream until a fully-developed condition is reached where \( \delta_h = a \). The temperature field in the boundary layer is assumed to be the same as found for the fully-developed profile.

Paper 21 agrees with this picture in that the temperature profile near the wall is the same as found in the fully-developed profile (Region 1) and that they show a plug profile at the center for small \( x/a \) (Region 3). However, the approach is flawed in that a gradual transition between these behaviors is observed (Region 2) and the length of the entry region is not defined when \( \delta_h = a \). Region 3 disappears downstream so that a large portion of the temperature field contains only Regions 1 and 2.

Measurements of temperature fluctuations were determined in the equipment described above (25). A 0.000015 inch tungsten wire 0.044 inches long was used for temperature measurements. These were determined from the fluctuating electric current needed to keep this wire at a constant temperature. Measurements of the ratio of the root-mean-square of the temperature fluctuations to the bulk temperature in the fully-developed region were found to be similar to measurements of the ratio of the root-mean-square of the streamwise fluctuations to the mean velocity.
In the beginning of the heat transfer section, the temperature fluctuations may be broken into three regions by examining the recorded signal: a quiet zone in the center of the pipe, a zone of fluctuating temperature close to the wall and an intermediate zone which is intermittently quiet and fluctuating. Farther into the heat transfer section there is no quiet zone and still farther the region of intermittency is not seen.

(c) **Lagrangian analysis of the temperature field**

It was apparent that a simple Eulerian analysis, for which eddy diffusivities obtained from the fully-developed temperature field are used, is not appropriate. This prompted the exploration of a Lagrangian approach that assumes homogeneous turbulence and a uniform velocity field. The temperature field is described as resulting from a series of instantaneous ring sources with areas of $2\pi a \Delta x_i$ and magnitudes of $2\pi a q_w \Delta x, dt$, where $x_i$ is the location of the source. It can vary between 0 and $x$. An analytical expression was developed for the temperature field created by one of these sources. The temperature field at $t = x/\bar{U}$ is calculated by summing up the contributions from the differential sources at $t' = x'/\bar{U}$, where $x'$ varies between 0 and $x$. The Lagrangian time scale, $\tau^L$, and the mean-square of the radial velocity fluctuations, $\overline{\nu^2}$, were specified.

Calculations of $\overline{T^+}$ versus $y^+$, where $y$ is the distance from the wall, for different $x^+$ are in surprisingly good agreement with measurements. Values of the Eulerian eddy conductivity obtained from the calculated fully-developed temperature profile are roughly constant from $r = 0$ to $r/a = 0.6$ and decrease rapidly with increasing $r$ for $r/a > 0.6$, even though a homogeneous turbulence is considered. Again, the spatial variation can be explained because of the time dependency of the turbulent diffusivity and because particles close to the wall have, on average, been in the field for shorter periods of time.

(d) **Computer studies of dispersion**

The Lagrangian analysis discussed above assumed a homogeneous turbulence. A first attempt at addressing this problem is described in Paper 62, which examines the effect of the spatial variation of the mean velocity on point source dispersion. However, a major advance in this direction was the development of computer solutions of the Navier-Stokes equations.
DNS) for turbulent flow in a channel. This opened the possibility of studying point sources in a real turbulent field.

A program was developed to calculate the paths of fluid particles. In principle, this would be very simple since it involves the use of the instantaneous velocity to calculate the displacement over an interval of time. However, the instantaneous position of a particle does not, in general, coincide with a mesh point. A critical aspect of the calculations was, therefore, the evaluation of fluid particle velocities by interpolating the known values of the Eulerian velocity fields at mesh points. A scheme to do this which involves an economical use of computer time was developed in (170). When a heat marker is considered, the influence of molecular diffusion is simulated by imposing a random walk on the particle motion. This is added to the convective part of the motion after each time step, \( \Delta t \), and takes values from a Gaussian distribution with zero mean and a standard deviation of \( (2D\Delta t)^{1/2} \), where \( D \) is the molecular diffusivity.

The application of this algorithm to calculate dispersion from wall sources is given in (188). Very close to the wall, the thermal markers move by molecular diffusion. The distance from the wall at which turbulence becomes important decreases with decreasing \( D \).

Paper (188) also presents results for a continuous wall source and for an array of sources. These were used to confirm the conclusion in (4) regarding wall sources in homogeneous turbulence: That is, the spatial variation of the Eulerian diffusion coefficient is understood by recognizing that, in a fully-developed field, the temperature gradient at a given location has contributions from sources that have been in the field for different times and that the principal contributions are made for a range of times which varies with distance from the wall.

Results for dispersion from point sources located in the body of the fluid are given in (231). These were obtained in a DNS of turbulent flow in a channel. Time scales were defined by using a best fit of the Lagrangian correlation with an exponential function.

(e) *Theoretical description of a point source; Use of stochastic methods*

From the above discussion, the issue involved in pursuing a Lagrangian approach is the development of a theoretical description of a point source in a nonhomogeneous field. This is done in (209) and (231), where a stochastic method based on a modified Langevin equation (See reference 234.) is used to represent the fluid velocities.
The trajectory of a fluid particle in a homogeneous, isotropic turbulence has been described by assuming the change of a component of a fluid velocity over time interval dt is given by the Langevin equation. The displacement is \( \frac{dx_i}{dt} = \overline{U}_i + u_i \), and \( \tau \) is a timescale approximately equal to the Lagrangian timescale, \( \tau^L \). The term \( d\omega \) is a sequence of random numbers, with variance \( <d\omega^2> = dt \), which are uncorrelated in successive time intervals (a Markovian assumption). The brackets indicate an ensemble average. The application of this equation to a homogeneous turbulence is not exact in that it predicts the Lagrangian correlation coefficient to be \( R^L = \exp(-t/\tau^L) \).

A number of researchers have examined how this simple equation can be adapted to describe dispersion in nonhomogeneous fields. Papers 209, 234 take advantage of these studies (in particular, the work of Wilson) to model particle dispersion in a channel flow. A spatial variation of \( \tau_i, \overline{u_i^2} \), and of the random forcing function had to be supplied. The velocity \( u_i \), in the Langevin equation, is replaced with \( u_i/\sigma_i \), where \( \sigma_i = \left( \overline{u_i^2} \right)^{1/2} \). The use of spatially varying \( \sigma_i \) and \( \tau_i \) causes an average, non-physical drift of the fluid particles. This is taken into account by including an average force on the right side of the modified Langevin equation. Paper 209 uses random forcing functions for the normal and streamwise velocity fluctuations, which are uncorrelated and have skewness & flatness. The use of a joint Gaussian function in later papers greatly simplified the calculations and provided more accurate results.

This approach is used (234) to describe the dispersion of heat markers, by considering that the markers are displaced both by convection and by a random walk associated with molecular diffusion. Thus \( \frac{dx_i}{dt} = (u_i + \overline{U}_i) dt + \sigma_m w_i \), where \( w_i \) is a Gaussian distribution function and \( \sigma_m \) is the root-mean square displacement associated with molecular diffusion.

(f) Computer studies of temperature/concentration field

The use of supercomputers to solve both the time-dependent Navier-Stokes and the heat balance equations for turbulent flow in a heated channel was begun in the 1980's. Our initial effort in this area is described in a thesis by Steve Lyons. A program developed by John McLaughlin was implemented. These computer experiments opened new areas and made many laboratory experiments unattractive.
We carried out studies of turbulent heat transfer in a channel at a Prandtl number of one and a Reynolds number (based on the friction velocity and the half-height of the channel) of $Re = 150$. (See Fig. 6). These provided calculations of the three components of the velocity and the temperature at one million points. Their accuracy was checked by experiments described in (148) and (207).

These calculations greatly improved knowledge about the fluctuating temperature. However, the most striking result is the revelation of the structure of the field. A comparison of the correlation coefficient of the fluctuating temperature with the fluctuating velocity demonstrates that the mechanisms for transporting momentum and heat at $Pr = 1$ are closely related. These papers take a step in addressing the theoretical problem of predicting the mean and fluctuating scalar fields from a knowledge of the velocity field.

Calculations for $Pr = 0.3, 1, 3, 5, 10$ are given in Paper 208. The interesting aspect of the results is the demonstration that the spectral density functions of the temperature fluctuations can extend to much larger wave numbers than do the velocity fluctuations for $Pr > 1$. (This is explained in Paper 208 as resulting from the increase in temperature fluctuations because of decreasing dissipation due to molecular diffusivity.) Thus the temperature field shows "phantom turbulence". This phenomenon is demonstrated by a decrease in the correlation of temperature and velocity fluctuations with increasing Prandtl number. However, the turbulent diffusivity appears to be independent of $Pr$ for $Pr > 1$, since it depends on large scale fluctuations.

The increase in the range of wave numbers characterizing the temperature fluctuations with increasing $Pr$ means that it becomes more difficult to study the behavior of wall sources in a direct numerical simulation (DNS) of the turbulent temperature field. One way to get around this difficulty is to study the behavior of wall sources or sinks in a direct numerical simulation of the velocity field. This is done in (188) and (200). The mean temperature field for a heated channel is calculated by representing the walls as being composed of arrays of sources and sinks. In (200) it is demonstrated that the mean thermal field can be calculated in this way for an arbitrarily large $Pr$. Comparisons of these calculations with Eulerian DNS at $Pr \leq 10$ are very good.

A result of particular interest in (188) and (200) is the confirmation of the experimental result in (71) that the mass transfer coefficient varies as $Sc^{-0.704}$, rather than the widely used dependency of $Sc^{-2/3}$, for $Sc=631-37,200$. The use of a $Sc^{-2/3}$ dependency is usually explained by an Eulerian analysis.
which assumes that the turbulent diffusivity varies as $y^3$ through the concentration boundary layer. The explanation for the failure of this approach is that the region where eddy transport varies as $y^3$ is confined only to a small part of the concentration boundary layer close to the wall, not to all of it. This interpretation is substantiated in (215), which used the DNS data described in (208) to explore the behavior of the temperature field in the immediate vicinity of the wall.

(g) Experimental studies; Diffusion from a point source

Several experiments have been carried out to test the theory of Taylor (11). These involved turbulent dispersion of potassium chloride solution in water and of $CO_2, H_2$ in air. The dispersing substances were injected into the flow at the center of a pipe. Samples were removed downstream of the source and analyzed. Thus, concentration profiles could be determined for different diffusion times, given as $t = x/U_c$, where $x$ is the distance downstream and $U_c$ is the mean fluid velocity at the center of the pipe. Two systems with diameters of 3 in and 3.15 in were used.

The measured concentration profiles could be represented with a Gaussian function, which is characterized by a mean square displacement of the injected material, $<X^2>$. Thus, the profiles at different distances from the injector provide the variation of $<X^2>$ with time. These measurements agree with Taylor's theory if the Lagrangian correlation coefficient is given as $\exp(-t/\tau^\varepsilon)$. Turbulent diffusivities, $E_\infty = (1/2)d <X^2>/dt$, obtained for the three diffusing species are in agreement.

(h) Dissipation of a heat sheet in a turbulent field

Paper (30) describes a study of the dissipation of a single heated sheet of air (formed by a heated wire) for a turbulent air flow in a 3 in pipe. Average temperatures, temperature fluctuations and spatial correlations of the temperature fluctuations were measured in the wake of a heated 0.0126 in wire. The behavior of this plume can be described by the centrode, $\overline{y}_c$, and the mean-square of the displacement about this centrode, $(y - \overline{y}_c)^2$, which is a measure of the average thickness of the plume. The rate of increase of $(y - \overline{y}_c)^2$ with distance downstream represents the rate of fine scale mixing. It is analogous to the measure of heterogeneity defined by
Obukhov, $P(0) = \int_{-a}^{a} T^2 dy$, where $T$ is measured with reference to the ambient temperature. A number of authors used the notion of a regular plume whereby $\bar{y}^2$ increases with time; the plume is described by a continuous function, not a probability relation. Measurements presented in (30) are not consistent with this model in that fine-grained turbulence is playing an important role in dissipating the sheet (even for very small times).
Fig. 1 Gravity flow water tunnel.

Fig. 2 Sandwich electrode, two 0.11 mm x 0.93 mm electrodes separated by 0.024 mm of insulation.

Fig. 3 Forced flow water tunnel.

Fig. 4 Chevron electrodes which measure two components of the wall shear stress.
Fig. 5 Traversing mechanism, which can move probe in three dimensions.

Fig. 6 DNS of temperatures and velocity vectors in a plane perpendicular to the direction of mean flow. The height of the channel is 300 wall units.

Fig. 7 T. J. Hanratty with Fraser Johnstone at Illini Grove, circa 1953-1954.

Fig. 8 Jaime Son and Hanratty examine the test section for studying flow around a cylinder.
3. Transport to a wall at large Schmidt/Prandtl numbers

(a) Relation of concentration/temperature field to the velocity field

The goal of a theory of heat or mass transfer is to relate the temperature or concentration field to the velocity field.

Heat/mass transfer for large Schmidt/Prandtl numbers differs significantly from what is found for Pr, Sc=1. All of the spatial variation of the average temperature/concentration occurs within the viscous sublayer. The thickness of the concentration boundary layer decreases with increasing Schmidt number, so that the concentration field is dominated by velocities closer to the wall as Sc increases.

The diffusing species involved in the electrochemical reactions, discussed in the section on electrochemical diagnostics, have small molecular diffusivities, so they have been found useful to look at the effect Schmidt number on turbulent mass transfer rates to a wall at large Schmidt numbers. Measurements of mean transfer rates to a wall indicate that the effect of velocity fluctuations is different from what would be expected from studies of the mean velocity field. The need to understand these results prompted the use of electrochemical methods to measure the fluctuating rate of mass transfer to the wall, as well as the average.

The basic approach is to apply a large enough voltage to an electrochemical cell that the current is controlled by the rate of mass transfer and the concentration of the reacting specie is zero at the surface of the test electrode. Thus, if a section of pipe is a cathode, the electric current is proportional to the mean rate of mass transfer over the whole section. Local time-mean and fluctuating mass transfer rates can be determined by measuring the current flowing to a small electrode embedded in a large cathode surface. [This approach was tested by measuring the variation of the local mass transfer rate around a cylinder which is exchanging mass with a flowing fluid (37).]

The system considered was the wall of a pipe. At the entry, the fluid has a uniform concentration of reacting species. Transfer causes the development of a boundary layer in which the concentration changes from the bulk value to zero at the wall. The thickness of this boundary layer increases with distance downstream. For a long enough mass transfer section, it fills the whole pipe and, eventually, reaches a fully-developed condition for which there is no further change in the concentration profile. The thickness of the concentration profile close to the wall and, therefore, the rate of transfer decreases with distance downstream. In the fully-
developed region the rate of transfer reaches a constant value, and the concentration profile is not changing shape. The length of the transfer section needed to attain a fully-developed condition is called the entry length.

Our first experiment using electrochemical methods considered this entry section (24, 34). The approach was particularly advantageous since it was possible to isolate very short lengths of pipes. The 1 in test section was the cathode of an electrolysis cell. It had nine electrodes ranging in length from 0.0179 to 1.96 in separated by 0.25 in thick Lucite disks held together by six tension bolts. In this way, the effect of length on the mass transfer rate could be studied. The Schmidt number was 2400 and the Reynolds number varied from $4.8 \times 10^5$ to $7.5 \times 10^5$. Measurements of the dimensionless average mass transfer coefficient, $\langle \bar{K} \rangle^+ = \langle \bar{K} \rangle / \nu^+$, versus $x^+$ were obtained, where $\nu^+$ is the friction velocity. The mean velocity profile shows a region of thickness $\delta_v$ close to the wall, where the variation of the velocity is described by the equation for rectilinear laminar motion. All of the data presented in (24) represent situations for which the concentration sublayer was thinner than this viscous sublayer. Thus, turbulent velocity fluctuations which were too small to affect changes in the mean velocity profile were controlling mass transfer to the wall.

For very small $x^+$, the concentration field is described by the equations for laminar motion since turbulent fluctuations are less effective in transferring mass than molecular diffusion. The fluid turbulence increases with distance from the wall. Its importance, therefore, increases with distance downstream since the thickness of the concentration boundary layer increases.

This behavior can be captured by using the following equation to define the concentration field: $(\tau_w y / \mu) \left( \partial \bar{C} / \partial \tilde{x} \right) = \partial \bar{C} / \partial \tilde{y} \left[ (D + \varepsilon) \left( \partial \bar{C} / \partial \tilde{y} \right) \right]$, where $(\tau_w y / \mu)$ is the variation of the mean streamwise velocity within the viscous sublayer, $D$ is the molecular diffusivity and $\varepsilon$ is the turbulent diffusivity. For small $x^+$, transport occurs mainly by molecular diffusivity, $D \gg \varepsilon$. For large $x^+$, $\partial \bar{C} / \partial \tilde{x} = 0$ and the equation for the fully-developed region is obtained. The differential equation is solved using the boundary condition at the wall, $-D \left( \partial \bar{C} / \partial \tilde{y} \right)_w = \overline{K}$.

This Eulerian interpretation is pursued further in (35), which includes a discussion of data from other laboratories. The finding that $\overline{K}$ is a function of $x^+$ implies that $\varepsilon / \nu$ is a function of $y^+ = y \nu^+ / \nu$, where $\nu$ is the kinematic molecular viscosity. Paper (35) shows that $\varepsilon / \nu$ varies as $(y^+)^n$. 

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This implies that $K^+ \propto Sc^{-(n-1)/n}$. In order to select an exact value for $n$, measurements of the concentration field or of the effect of Schmidt number on $K^+$ are needed. Thus, measurements of the influence of $Sc$ on $K^+$ are helpful in determining the spatial variation of $\varepsilon$.

Mass transfer fluctuations show dramatic changes with time reminiscent of surface-renewal models (3). A striking feature is that the frequency decreases with increasing $Sc$. That is, the fluctuations do not respond to high frequency velocity fluctuations. The concentration boundary layer may be looked upon as a filter in that the concentration field does not respond to high frequency velocity fluctuations. A theoretical explanation for this behavior is given in (27).

Another striking result, presented in (27, 49, 208), is that mass transfer fluctuations are weakly correlated in the spanwise direction. That is, measurements from two wall electrodes separated by a circumferential distance of 4.6 mm show time-variations which are almost completely independent of one another. (See figure 2 of reference 72.) However, correlations in the flow direction show scales which are, at least, two orders of magnitude larger than those in the spanwise direction. Thus, (27) describes the structures that are controlling mass transfer as being "spaghettilike".

Because of the small scale of the fluctuations in the spanwise direction, averaging occurred over the surface of a wall electrode. Thus the fluctuations in the local mass transfer coefficient could not be directly measured. Instead, point values were estimated by using measurements with electrodes of different sizes and extrapolating to zero. (See Fig. 10 in this book.) The studies described in (27) were carried out in a 1 inch pipe. An addition to Roger Adams Laboratory included (at the urging of Prof. James Westwater, Head of the Chemical Engineering Department) a five-level tower facility for use in my fluid mechanics research. We took advantage of this by using a specially designed vertical flow loop which had a test section with a diameter of 7.625 inches. This provided us the opportunity to avoid the difficulty of spatial averaging over electrode surfaces. The advantage of doing this was immediately evident. The root-mean square of the mass transfer coefficient was estimated to be 0.47 times the mean mass transfer coefficient in a one inch pipe (27) for a Schmidt number of 2400. The measurements in a 7.625 inch pipe (49) gave a value of 0.29 for this ratio.

The capacitance effect of the concentration boundary layer can be characterized by the frequency spectrum of the mass transfer fluctuations. The development of a theory relating these frequency spectra to the frequency spectra of the velocity field close to the wall (measured with
electrochemical wall shear stress probes in an inert wall) occupied our
attention in several studies. Measurements of spectra over a range of
Schmidt numbers of 631 to 37,200 were made. This was done by using two
electrochemical systems and by varying the viscosity. An interesting aspect
of these experiments is the determination of the effect of Schmidt number on
the time-averaged mass transfer coefficient, reported in (71).

The classical approach to explain the effect of Schmidt number on the
average mass transfer rate at large Schmidt numbers is to assume an analogy
between momentum and mass transfer. This lead to the assumption that the
eddy diffusivity varies with the cube of the distance from the wall
throughout the concentration boundary layer and a prediction that the mass
transfer rate varies with the Schmidt number raised to the -2/3 power. The
experiments described in (71) produced the controversial result that the
exponent on the Schmidt number is minus 0.704 for Sc=631-37,200. The
argument that the eddy diffusion coefficient varies as the cube of the
distance from the wall, close to the wall, is correct but the assumption that
this holds throughout the concentration boundary layer is not correct (as
already noted).

At large Schmidt numbers the concentration boundary layer is thin
enough that the fluctuating velocity field is represented by the first terms in a
Taylor series for which \( w = s_z y \), \( u = s_z y \) and \( v = \beta y^2 \), where \( y \) is the distance
from the wall. Arguments are made in (49) and confirmed in a thesis by
Vassiliadou (Vassiliadou, E., "Turbulent mass transfer to a wall at large
that velocities in the flow direction are having a negligible effect so that the
mass balance equation considers only \( \beta \) and \( s_z \) (which can be related to \( \beta \)
through the continuity equation). Paper (49) argues that the solution of the
linearized mass balance equation gives the correct behavior of the spectral
function of the mass transfer coefficient, \( W_k(\omega) \), at large \( \omega \), where \( \omega \) is the
circular frequency. This has been confirmed in our experiments.

Thus, linear theory provides an explanation of the filtering effect of
the concentration boundary layer: The time-change of the concentration field
at large frequencies varies with the product of the velocity normal to the wall
and the gradient of mean concentration. As the Schmidt number increases
the concentration gradient at the wall increases. This tends to give an
increase in the frequency of the fluctuations of the rate of mass transfer.
However, since the thickness of the concentration boundary layer decreases
with increasing \( Sc \), the velocity normal to the wall in the boundary layer, \( v \),
decreases with the square of the thickness of the concentration boundary
layer. The net effect is that the velocity needs to persist for larger periods to cause changes in the concentration field. It is of interest to note that if the velocity normal to the surface varies as \( y \), rather than \( y^2 \), the thinning of the concentration boundary layer has a smaller effect on the velocities normal to the wall. The concentration boundary layer does not have a filtering effect under these circumstances (90, 91, 100).

However, a complete solution of the linear mass balance equation (91), valid for all frequencies, provides values of \( \bar{K} \) and the mean-square of the fluctuating mass transfer coefficient, \( k^2/\bar{K}^2 \), which disagree with experiment. Nonlinearities, which result from mixing caused by spanwise velocities, must be considered in order to relate correctly the concentration field to the velocity field. Initial attempts to do this used the observation that velocities close to the wall are dominated by elongated structures with spatial correlation coefficients in the spanwise direction that are close to those observed for the mass transfer fluctuations. This lead to an assumption that a pseudo-steady state solution can be used to describe low frequency fluctuations. This approach failed in that the mean and fluctuating mass transfer coefficient are predicted to be too large (49)(79).

Numerical solutions of the unsteady mass balance equation, formulated in (49), which use measured velocity fluctuations at the wall were, therefore, explored. A chevron arrangement of electrodes in an inert wall was used to measure the velocity component at the wall in the spanwise direction, \( w = s_x y \) (47, 50). By using two chevron pairs, \( ds_y/dz \) could be approximated. From the continuity equation, the velocity normal to the wall, \( \beta y^5 \), was then calculated.

The initial solution ignored the direct effect of spanwise velocity fluctuations. The randomly varying \( \beta \) produced random mass transfer fluctuations with a much lower frequency than the velocity fluctuations (90, 94, 95, 137). The same result was obtained when the velocity used in the mass balance equation was filtered so that approximately 90 per cent of the energy was removed. Thus, the influence of Schmidt number on the mean mass transfer coefficient, and on the mean-square of the fluctuations, \( k^2 \), in the mass transfer coefficient was explained. However, the calculations of \( k^2 \) were of incorrect magnitude.

This showed that spanwise mixing needs to be considered. A calculation that allows the normal velocity to vary harmonically in the spanwise direction (with a single wavelength) produced excellent agreement between the calculations and experiment. A summary of these calculations is presented in (137), which explored a large range of Schmidt numbers, \( 10^2 \) to \( 10^3 \).
and presented calculated concentration profiles. The conclusion was reached that $K$ was varying with $Sc^{-2/3}$ at small $Sc$ and as $Sc^{-3/4}$ at very large $Sc$. Thus laboratory studies could represent an "intermediate" between these two extremes. The recognition that $K$ varies as $Sc^{-3/4}$ as $Sc$ approaches infinity could be of importance in considering deposition of aerosol particles.

The response to the goal of a theory on mass transfer, as formulated in the first sentence of this Subsection, is presented in (95, 119, 137) for large Schmidt numbers. Mass transfer is controlled by velocity fluctuations normal to the wall in the immediate vicinity of the wall. The frequency spectrum of $\beta$, $W_{\beta}(\omega)$, has a constant value at very small $\omega$, $W_{\beta}(0)$. The frequency spectrum of the mass transfer fluctuations, $k$, lies almost entirely within the region where $W_{\beta}(\omega)=W_{\beta}(0)$. That is, the mass transfer rate is controlled by very low frequency velocity fluctuations which contain very small amounts of the kinetic energy. Thus, the important hydrodynamic variable is $W_{\beta}(0)$. The best way to determine this quantity is to use linear theory and measurements of $W_{\beta}(\omega)/\bar{K}$ at large frequencies. Equations have been developed (95, 119, 137) which relate $\bar{K}$ and $k^2$ to $W_{\beta}(0)$. Thus for $Sc=631-37,200$, $k^2/\bar{K} \propto W_{\beta}(0)^{0.06} Sc^{-0.06}$. At $Sc>100,000$, $\bar{K} \propto W_{\beta}(0)Sc^{-0.75}$, $k^2/\bar{K}^2 = \text{constant}$. For small $Sc$, $\bar{K} \propto \left(\beta^2\right)^{1/6} Sc^{-2/3}$. Here, $\bar{K}$ and $W_{\beta}(0)$ are made dimensionless by using the friction velocity, $v^*=(\tau_w/\rho)^{1/2}$, and the kinematic viscosity to provide length and velocity scales.

(b) Measurement of normal velocity fluctuations at the wall

The work outlined above shows that mass transfer is related to the velocity field through the limiting behavior, close to the wall, of the velocity component normal to the wall, given as $v = \beta v^*$ for a solid boundary (where all quantities are made dimensionless with local wall parameters, that is, the friction velocity and the kinematic viscosity). This prompted the development of a method to measure $\beta$, described in Paper 115.

The region where this limiting behavior is valid is too thin to allow the use of standard probes. Electrochemical techniques, which recognize that conservation of mass gives $\beta = -1/2(\partial s_z/\partial x + \partial s_z/\partial z)$ at $y \to 0$ were used. Thus $\beta$ was determined by using measurements of the spatial variation of $s_z$ and $s_z$ that employed arrays of chevron and circular electrodes arranged in the
circumferential and streamwise directions. The measurements gave \( \overline{v^2} = 0.0048v^2 \). (Again, all terms are made dimensionless by using wall parameters.) This study shows that the \( \partial \beta / \partial x \) term is making only a 5 percent contribution to \( (\beta^2)^{1/2} \).

Thus, the time variation of \( \beta \) could be approximated by using two pairs of closely spaced chevron electrodes-- justifying the numerical calculations (90, 94, 95, 137) described above.

(c) *Influence of drag-reducing polymers on mass transfer*

A number of investigators had established that drag-reducing polymers can cause a reduction in the rate of turbulent heat transfer between a wall and an aqueous solution in the range of \( \text{Pr} = 5-10 \). Paper (76) describes a study of the influence of a drag-reducing polymer, polyacrylamide, on mass transfer at \( \text{Sc} = 950-1260 \). Electrochemical techniques were used to measure the average mass transfer rate.

The percent decrease in the fully-developed mass transfer rate at a given volumetric flow was found to be greater than the decrease in the pressure gradient (as had been found for heat transfer) and in the turbulence close to the wall. The presentation of the results was approached by using the relation derived in (71), \( \overline{K}/v^* = B\text{Sc}^{-0.704} \) for Newtonian fluids, where the friction velocity, \( v^* \), would be smaller for a drag-reducing polymer solution. Values of \( B \) were smaller for a drag-reducing solution. This indicates that the observed decrease in \( \overline{K}_\infty \) cannot be explained by considering only changes in the friction velocity. Thus, detailed information about the changes in the turbulent velocity field was needed. The results of such a study are given in Paper 119, which presents measurements of the mean, \( \overline{K} \), and fluctuating mass transfer coefficient, \( k \).

The purpose of (119) was not only to explain the influence of drag-reducing polymers on mean mass transfer rates but also to supply a confirmation of the theory presented above for Newtonian fluids. Both the mean mass coefficient, \( \overline{K} \) and the mean-square of the fluctuations, \( k^2/\overline{K} \) were found to decrease with increasing drag-reduction.

Measurements of the influence of drag-reducing polymers on the frequency spectra of the mass transfer coefficient show the same behavior at large frequencies as do Newtonian fluids, so the spectral density function at zero frequency can be evaluated by using linear theory. (See Section 2c.) For Newtonian fluids the dimensionless \( W_\beta(0) \) is a constant, independent of
Reynolds number. For drag-reducing fluids $W_\beta(0)$ is found to decrease with increasing drag-reduction. The equation $\bar{K} = 0.09\left[200W_\beta(0)\right]^{0.21}\text{Sc}^{-0.7}$ calculated from the mass balance equation (Section 2c) agrees with measurements for Newtonian fluids at Sc=631-37,200, where $W_\beta(0)$ is the dimensionless spectral function. This equation also agrees with measurements for drag-reducing polymers. However, the calculated relation for $\overline{k^2}$ for Newtonian fluids (Section 3a) underpredicts the decrease with increasing drag-reduction [or decreasing $W_\beta(0)$]. The spanwise scales used in the computer model to mimic spanwise mixing apparently are not appropriate for drag-reducing solutions.

Thus, the equations relating mass transfer to the turbulent field for a Newtonian fluid are successful in describing influences of polymers on $\bar{K}$. However, some qualitative effects are not captured. This is emphasized in the discussion of mass transfer fluctuations in Paper 119. The amplitude and the frequency of the fluctuations are greater for the Newtonian solvent. In addition, the variation of the mass transfer coefficient with time is smoother and does not show the large positive spikes indicated by measurements for a Newtonian fluid. The probability density function for a polymer solution is Gaussian. The Newtonian solvent shows clear-cut non-Gaussian behavior, in that large positive deviations are more likely than large negative deviations. This is evidenced by positive skew factors.

Later studies, described in Section 7i, provide further discussion of the qualitative differences of fluid turbulence found for polymer solutions.
4. Gas Transfer

(a) *Definition of the problem*

Transfer of components from a flowing gas to a flowing liquid layer is of central interest to many chemical engineering and environmental problems. An engineering representation is that the rate of mass transfer per unit interfacial area, \( N \), is given as \( N = K(C_i - C_B) \), where \( C_i \) is the concentration at the interface, \( C_B \) is the bulk concentration of the species which is being transferred and \( K \) is the mass transfer coefficient. Our interest is for an absorption process in which the specie being transferred does not react with the liquid. Usually, mixing in the gas is much more rapid than in the liquid. Thus, the concentration in the liquid at the interface is the equilibrium value, \( C_i = C^* \). The theoretical problem is to predict \( K \). The classical approach is to use the concept of a Nernst diffusion layer, which suggests the existence of a non-turbulent layer of thickness, \( \delta_c \), in the liquid at the interface, so that \( K = D/\delta_c \), where \( D \) is the molecular diffusivity. The work described below relates the mass transfer rate to the velocity field close to the interface. It represents the first major advance in this area since the presentation of the Nernst theory.

In 1978, most of the fundamental studies of gas absorption involved a consideration of transfer from a stagnant gas to a liquid flowing as a film down a vertical wall. Much larger absorption rates can be realized when the gas is flowing relative to the liquid and waves exist at the interface. An understanding of this phenomenon prompted a number of studies in this laboratory. The goals were to quantify this effect and to define the controlling mechanism. The rate was shown to be related to the wave slope, rather than the wave height. A method was developed to provide measurements of the two components of the slope. Fluorescence techniques were invented, which allow a determination of the concentration profile in the thin boundary layer in the liquid close to the interface.

(b) *Measurements of the effect of gas flow on absorption*

Measurements of the rate of absorption of oxygen, contained in air, into water were carried out in two systems (83,100,103,152). One used a horizontal enclosed rectangular channel through which air and water were flowing concurrently. The other involved air and water flowing downward in
a vertical pipe. In both cases, the velocity of the gas was low enough that atomized drops were not contained in the air.

Measurements with no air-flow in a vertical pipe were correlated as 
\[ \overline{K} = 0.077 \sqrt{m^*} \], where \( \overline{K} = K / v^* \), \( Sc = \mu / \rho D \) and \( m^* = m v^* \rho / \mu \) is the dimensionless film height. The friction velocity is defined with a characteristic stress \( \tau_c = 2/3 \tau_w + 1/3 \tau_i \), where \( \tau_w \) is the stress at the wall and \( \tau_i \) is the stress at the interface. The above relation agrees with most previous measurements. Studies with air flows show values of \( \overline{K} \) which are several times larger than the value predicted by the above equation. A theoretical explanation was not available.

Both (83) and (103) indicate that the increase is associated with interfacial waves generated by the air flow. (See Fig. 11). Paper 103 suggests that the waves induce velocity fluctuations in the gas flow which cause shear stress variations at the interface. These, in turn, create cellular motions in the liquid which enhance normal velocity fluctuations relative to the interface. Since the amplitude of the shear stress fluctuations in the gas should scale with wave slope, rather than wave height, it would be expected that waves of shorter wavelength would be more important.

The Schmidt number characterizing the gas absorption process is large. Therefore, insights gained from our consideration of turbulent mass transfer to a solid boundary (Section 2) were exploited. The main feature is the recognition that all of the concentration change occurs close to the interface and that transfer is controlled by normal velocity fluctuations in the liquid perpendicular to the liquid interface. For an immobile interface (such as a solid wall), these fluctuations vary quadratically with the distance from the interface. As shown in Section 2, a consequence of this is that the concentration boundary layer acts as a filter in that low frequency velocity fluctuations have a dominant influence. This filtering action increases with Schmidt number.

A gas-liquid system, with a clean interface, is fundamentally different in that the interface is mobile and the normal velocity fluctuations relative to the moving interface vary linearly with distance from the interface. As shown in (100,103), this leads to a situation in which the concentration boundary layer does not act as a filter and \( \overline{K} \) varies as \( Sc^{-0.5} \). This result ruled out the idea expressed by a number of researchers that the equation describing the rate of gas absorption is the same as found for turbulent mass transfer to solid surfaces.
Numerical experiments

Numerical solutions of the mass balance equation of the type described in Section 2 and in (94, 95) were used to study gas absorption at a mobile interface (121). Since the concentration boundary layer is so thin, the first terms in Taylor series expansions were used to describe the velocity field. Since concentration changes in the direction of flow, x, are quite small, the influence of the velocity component in the direction of flow can be ignored. The velocity field in the liquid is, therefore, given as $w = w(z,t), v = v(z,t)y$. It was felt that the most important term influencing mass transfer rates is $v$, since concentration gradients in the y-direction are much larger than in the z-direction. However, solutions that neglect $w$ are not well behaved at large $y$. Some mixing in the z-direction needs to be included. The simple assumption that a single harmonic can be used so that $\beta(t) = 2^{1/2} \beta(t) \cos(2\pi/\lambda_z)$. From mass conservation $w(z,t) = -2^{1/2} \beta(t)(\lambda_z/2\pi) \sin(2\pi/\lambda_z)$. Thus, the specification of $\beta(t)$ and $\lambda_z$ provides a velocity field. The term $\beta(t)$ was assumed to be a random function of time. Because of the lack of data, its spectral function was assumed to have the same form as is found for a solid surface.

The calculations were found to be relatively insensitive to the choice of $\lambda_z$ in that values of $\lambda_z \nu'/\nu = 50 - 800$ produced roughly the same results. They directly demonstrated that both high- and low-frequency velocity fluctuations contribute to mass transfer. The dimensionless mass transfer coefficient, $K/\nu'$, was found to be given by the equation $K^* Sc^{1/2} = 0.71(\beta^{2/3})^{1/4}$, where $\beta$ is made dimensionless with the friction velocity and the kinematic viscosity. Thus, the effect of liquid turbulence on the mass transfer rate is defined through $\beta$ and the effect of gas flow is manifested through its effect on $\beta$.

Universal relation for gas absorption

The works described above captured the attention of the environmental community. We were invited to give a keynote address to the Second International Symposium on Air-Water Mass Transfer at Minneapolis, MN in 1990. A copy of this address is given in (152). Our data on gas transfer in a horizontal channel are presented as a plot of $K^* Sc^{1/2}/\nu_i^*$ versus $Re_L = 4m\bar{U}/\rho/\mu$ (117,152), where $\nu_i^*$ is based on the interfacial stress.
and \( m \) is the height of the liquid layer. These showed that \( \frac{KSc^{1/2}}{v_i^*} = 0.12 \), provided \( m^* \) is greater than 40. Studies by Tsacoyanis with much thicker layers are described by \( \frac{KSc^{1/2}}{v_i^*} = 0.12 \) if \( \tau_i/\tau_w > 0.3 \). These seem to suggest that, when air flows are controlling gas transfer, that \( \frac{KSc^{1/2}}{v_i^*} = C_k \). Surprisingly, results for air-water flows in a pipe, given in (83), are best understood by plotting \( \frac{KSc^{1/2}}{v_i^*} \) versus the Reynolds number of the liquid film. These show an upper asymptote for large gas velocities of 0.15 (117, 152).

Turbulence in a gas flow can directly increase turbulence in the liquid by exerting a fluctuating interfacial stress. In (117) and (152), direct evidence for this type of interaction is presented. From conservation of mass, one finds, for a mobile interface, that
\[
-2y\beta = \partial u_i/\partial x + \partial w_i/\partial z, \quad \text{where} \quad u_i \quad \text{and} \quad w_i \quad \text{are the fluctuating x- and z-components of the velocity in the interface caused by gas phase velocity fluctuations.} \]
However, the scales of the gas phase turbulence are too large to produce gradients of the interfacial velocity which are sufficient to make a significant contribution to \( \beta \). Thus, it appears that one should consider the enhancement of \( \bar{K} \), due to gas flow, as associated with the generation of waves, as suggested in (117) and (83).

In (152), the equation developed above is compared with studies of gas absorption in large facilities. The Hamburg data are an example. The wave tank was 1.5 m wide, 1 m high and 15 m long. The height of the water was 0.5 m (to be compared to 0.5 cm in our experiments). At low gas velocities, the interface was smooth. A rapid increase of the mass transfer coefficient was observed with increasing gas velocity when waves appeared on the interface. These give \( C_k \approx 0.22 \). Tests in other large facilities give \( C_k \approx 0.15, \quad C_k \approx 0.12, \quad C_k \approx 0.17 \).

The interesting aspect of the above results is that \( C_k \) shows a relatively small variation over a very large range of facility sizes. This suggests that there is an approximately universal behavior when waves are present on the interface. In (117) and (152) the influence of a single harmonic on the normal velocity fluctuations at the interface was determined by solving the linear momentum equations. The equation developed for \( \beta \) has two leading terms. One considers the direct contribution of wave motion to the mixing. The second considers mixing caused by wave-induced variations of the gas phase shear stress at the interface (discussed in Section 9b). Both are found to vary linearly with the wave slope. Discussions in (152) indicate that both could be important. Thus, smaller wavelength waves
are expected to play a critical role. These could have a more universal character than wave height.

The research outlined above has shown that the rate of gas absorption is controlled by a thin concentration boundary layer in the liquid. The rate is related to the root-mean square of the velocity fluctuations, normal to the interface, that are induced by small wavelength waves. Thus, the theoretical problem is to predict the rate of mass transfer from a knowledge of the wave structure. This prompted studies of wave slopes since calculations indicated that they could be of first order importance. We were encouraged to undertake this enterprise because measurements in the laboratory could be shown to be relevant to rate processes in large bodies of water.

An optical technique was developed to measure the two components of the time-varying slope (189). It was applied in our wave facility which used the concurrent flow of gas and liquid in a horizontal rectangular channel with a height of 2.54 cm. An important finding is that the spectrum of wave slopes and the root-mean square slope are the same as found in large facilities. This could explain the "universal" validity of the equation $(\bar{K}/v_\ast)(Sc^{1/2}) = C_k$. A possible physical explanation, given in (189), is that the maximum magnitude of the wave slope is about $20^\circ$, since waves with slopes greater than about $20^\circ$ would be unstable.

A method for measuring the spatial distribution of wave slopes was developed in theses by Pasmore (Pasmore, Mark E., "A Technique for Determining Wave Slope at an Air-Water Interface", BS thesis in Chemical Engineering, University of Illinois, Urbana, 1994) and by Rathsack (Rathsack, Benjamin M., "Spatial Variation of Slopes for Wind Generated Waves", BS Thesis in Chemical Engineering, University of Illinois, Urbana, 1996). The horizontal channel used in our gas-liquid studies was made from transparent plastic. A grid placed on the bottom wall of the channel was photographed from above. (See Fig. 12) No distortions of the grid were observed when the liquid layer flowing along the bottom wall was smooth. Measured distortions provided a means to calculate the spatial variation of slope that existed at a given time. This technique proved to be successful but, unfortunately, we did not exploit it.

(e) Development of a method to measure the concentration field close to an interface

The rate of gas absorption at a wavy interface is dependent on the behavior in a concentration boundary layer in the liquid of the order of 10-100 microns for a wavy interface and of the order of 1 mm for a smooth
interface. Direct studies of this layer present a challenge because of its thinness and because it could be moving. We explored the possibility of using a fluorescence technique that employs a tracer (pyrene butyric acid) whose fluorescent intensity is inversely related to the oxygen concentration. The fluorescence of the tracer is recorded on a CCD camera so that spatial resolution is preserved.

Initial studies (166) exposed stagnant water, that had been stripped of its oxygen, to air in a 7.6 cm x 7.6 cm x 2.5 cm chamber. The tracer was added to the water and a beam from a nitrogen laser was passed upward through the chamber. The camera, which was horizontal, captured the light from the fluorescing liquid and light reflected in the interface. The position of the interface was located as a minimum in the intensity of the image captured by the CCD camera. The absorption process was followed by measuring the change of the concentration profiles with time. The concentration boundary layer had a thickness of about 2 mm. The rate of absorption at a given time was calculated as the product of the diffusion coefficient for oxygen and the concentration gradient at the interface. The decrease of the rate with time, calculated in this way, agreed closely with measurements of the change of the bulk concentration obtained with an oxygen probe. It is of interest that the shape of the profiles indicate the existence of turbulence caused by natural convection. To our knowledge, paper (166) presents the first direct measurements of the concentration boundary layer for gas absorption.

Studies were also carried out in a horizontal enclosed channel through which the gas and the water were flowing concurrently (181). The oxygen concentration in the water was controlled by bubbling nitrogen into the holding tank. Measurements with a smooth interface, at a gas velocity of 1 m/s, showed diffusion layers with thicknesses of about 1 mm. (See Fig.13) Again rates of transfer obtained from the slope of the concentration profile at the interface agreed with rates calculated from changes of the bulk concentration of oxygen over a given length of channel. The concentration profiles have a shape characteristic of turbulent transfer. Fluctuations around the ensemble average were of the same magnitude as would be seen in single phase turbulence.

An increase in the gas velocity from 1.5 m/s to 2.0 m/s caused a change in $\overline{K}$ from $0.227 \times 10^{-3}$ cm/s to $1.11 \times 10^{-3}$ cm/s. This large increase is associated with the appearance of a 3D stochastic wave pattern. (See Fig. 14) Measurements of instantaneous concentration profiles show a surprising behavior, quite different from what is seen in classical turbulence. For example, the four profiles shown in (181) indicate diffusion layer
thicknesses of 0.5 mm, 1.25 mm, 0.2 mm, less than 50 microns for a fixed
gas and liquid flow. A large number of profiles showed concentrations at the
interface which are close to the bulk value.

In order to obtain an understanding of how to interpret these findings,
Duke (Duke, Steve Richard, "Air-Water Transfer at Wavy Interfaces", PhD
thesis, University of Illinois, Urbana, 1996) modified the fluorescence
quenching technique so as to make simultaneous measurements in the x-y
plane, rather than along a line perpendicular to the wall. (See Fig.15a, b.)
This was done by sending the laser beam through a cylindrical lens to form a
laser sheet. Also, a more systematic method for locating the interface was
developed since a goal was to relate changes in the concentration field to
changes in the wave field.

The most striking result for 3-D wavy interfaces was that single
pictures show small changes in the general characteristics of the
concentration boundary layer along a large length in the direction of mean
flow. However, large differences were observed among different pictures.
This suggested that a cellular secondary flow (Langmuir cells?) was
imposed on the liquid flow. Some preliminary results are given in (191).

(f) Langmuir cells

Visual studies were carried out by Duke (described in his thesis) and
by Monsen (Monsen, Kristen, "Tracking Wave Patterns; Langmuir
Circulations", Chemical Engineering 390 Research Report, 1997) to test this
idea. Spent tea leaves were scattered over the interface. A reflector lamp
illuminated a sheet of white paper attached to the transparent bottom wall.
When the lamp was placed below the channel, the tea was visible but the
wave structure was faint. The leaves were observed to line up in the
interface to form streaks in the mean flow direction when three-dimensional
waves were present. For $v_i^* = 1.18 \text{ cm/s}$ to $1.53 \text{ cm/s}$, the spacing of the
streaks in the spanwise direction varied from 32 mm to 11 mm, where $v_i^*$ is
the friction velocity based on the interfacial stress. These spacings appeared
to scale roughly with the spanwise wavelength, which varied from 22 mm to
11 mm. The wavelength, made dimensionless with $v_i^*$, varied from 390 to
200.

In a second set of experiments the liquid was seeded with 0.5 mm
glass spheres. They sank to the bottom of the channel and formed streaks.
The spacings were the same as for the tea leaves, supporting the idea that
circulations extended over the whole liquid height.
Methods similar to those outlined in Section 2c could be modified to include the effects of Langmuir cells. The expectation is that Langmuir circulations would increase mixing but that the basic relation
\[ \frac{K}{\nu} S_c^{1/2} = 0.71 \left( \beta^2 + 1 \right)^{1/4} \]
would be unchanged. This suggestion needs to be examined further.
Fig. 9 Fluctuations in the mass transfer coefficient from two electrodes with circumferential spacing of 0.0457 cm ($z^+ = 18.1$). Note the large difference in periods for mass transfer and velocity fluctuation.

Fig. 10 Effect of electrode size on the measured mass transfer fluctuations. Term $2a$ is the electrode diameter made dimensionless with wall parameters.

Fig. 11 Top view of the three-dimensional waves that exist in the gas transfer experiments.

Fig. 12 Distortion of the grid below the waves can give local wave slopes.
Fig. 13 Instantaneous measurements of concentration profiles by fluorescence techniques. Smooth interface. $Re_G = 1900$, $Re_L = 300$. Line is calculated from the measurement of the mean absorption rate.

$C_b = 0.13 \text{ mMol/l}$

$C_b = 0.17 \text{ mMol/l}$

Fig. 14 Instantaneous concentration profile by fluorescence. Three-dimensional waves are present.

Fig. 15a The instantaneous two-dimensional concentration field measured by Duke with fluorescence techniques $u^* = 1.17 \text{ cm/s}$.

Fig. 15b Two-dimensional concentration fields obtained by Duke with fluorescence techniques; $u^* = 1.01 \text{ cm/s}$, $1.25 \text{ cm/s}$, $1.59 \text{ cm/s}$.
5. Heat transfer to liquid metals

The use of a liquid metal, such as sodium or mercury, as a heat transfer medium has attracted much attention because of its large conductivity. This property directly enhances heat transfer. However, it has a detrimental effect in that it decreases turbulent transport. This occurs because it causes heat to leak out of eddies, making them less effective as a means of transport. Thus, a consideration of the use of liquid metals opens the theoretical challenge of predicting this effect.

This prompted Robert Johnk (Johnk, Robert Ernst, "Development of temperature profile for turbulent heat exchange in a pipe, PhD thesis, University of Illinois, Urbana, 1961) to analyze the effect of molecular diffusivity of heat, \( D \), on turbulent dispersion from a point source of heat in a homogeneous, isotropic turbulence. Because of the large difference of scales characterizing molecular and turbulent diffusion, the contributions are additive even though turbulent mixing is greatly reduced. From Taylor's theory, dispersion is dictated by a Lagrangian correlation coefficient characterizing the behavior of fluid particles, \( R^L \). Taylor's theory is applicable to heat markers, for which a Lagrangian correlation coefficient, \( R^L_d \), is defined. The theoretical problem is to predict the ratio \( R^L_d / R^L \).

For very small time Saffman has suggested that \( R^L_d \) equals
\[
1 - \left( \frac{5D}{\lambda^E} \right) - \left( \frac{\lambda^L}{\lambda^E} \right)^2 t^2,
\]
where \( \lambda^L \) is the Lagrangian time microscale and \( \lambda^E \) is the Taylor Eulerian microscale. It is noted that if molecular diffusivity is zero the Lagrangian correlation decreases as \( (1 - t^2) \). This indicates that the function describing \( R^L \) is a parabola in the neighborhood of \( t = 0 \). When the molecular diffusivity is large enough, the second term dominates, except for small times. The correlation would then decrease linearly with time. This prompted Johnk to assume that the correlation is given by an exponential and used the properties of a Markovian process to propose that \( R^L_d \) equals
\[
\exp(-t(1/\tau^L + 5D/\lambda^E^2)),
\]
where \( \tau^L \) is the Lagrangian macroscopic timescale. This produces the result that \( E/E_d \) equals \( \left( 1 + 5D/\lambda^E^2 \right)^{-1} \) at very small times. Similar results have been reported in Hinze's book on "Turbulence". Johnk's work was never published but interest in the problem continued in our group.

An analysis presented in (173) uses Corrsin's suggestion, that Lagrangian correlation \( R^L_d \) can be calculated as the spatial average of the Eulerian space-time velocity correlation, to calculate the effect of molecular diffusion on the Lagrangian correlation. (An iterative method of solution...
was used.) The result is similar to what was suggested by Johnk in that \( \frac{R_d}{R^l} \) decreases monotonically with decreasing Peclet number, \( \text{Pe} = \text{Re} \text{Pr} \), for all times. Furthermore it shows that Gaussian correlation functions change to exponential functions when effects of molecular diffusion are considered. It has a limitation that the assumed Eulerian correlation has only one length scale.

Laboratory experiments with liquid metals are challenging because of their nasty properties. However, experiments in a direct numerical simulation (DNS) provide an alternative. This was done in (182) for a study of dispersion from a point source located in the center of a channel with a half-height of \( H \). The Reynolds number was \( \text{Re}_r = 150 \), where \( \text{Re}_r = \frac{H v^* \rho}{\mu} \), \( v^* \) is the friction velocity. The results are consistent with what was presented in (173). Of particular interest is the finding that the shape of the Lagrangian correlation function changes with decreasing \( \text{Pe}_r = \text{Re}_r \text{Pr} \). At very small \( \text{Pe}_r \) the Lagrangian correlation coefficient has a parabolic shape at small times. At values for which molecular diffusion has a measurable effect on the turbulent diffusivity, the correlation function changes its shape so that the curvature disappears at small times and the shape is described by an exponential function.

A study which is closer to application is presented in (230). A DNS of the fully-developed velocity and temperature fields is developed for flow in a channel in which the two walls are kept at different temperatures. (See Figure 16) Thus, if only molecular conduction were operative the fully-developed temperature field would show a linear variation from wall to wall and the heat flux would not vary with distance from the wall. For a turbulent field the temperature profile would not show a linear relation. Temperature gradients would be larger near the walls because the influence of turbulence on transport is smaller than in the center of the channel. However, the heat flux (which is the sum of turbulent and molecular contributions) does not vary with distance from the wall. This behavior makes the interpretation easier. The calculations were done for \( \text{Re}_r = 150 \) and for Prandtl number (designated by \( \text{Pr} \)) equal to 1.0, 0.3, 0.1, 0.05, 0.025. The Prandtl number is defined as \( \frac{\mu}{\rho D} \), so it decreases with increasing molecular diffusivity.

For \( \text{Re}_r = 150 \) the Peclet number (\( \text{Pe} = H v^*/D \)) equals 150 for \( \text{Pr} = 1 \). The turbulent diffusion coefficient, \( \varepsilon_T \), is approximately equal to 0.08\( H v^* \) in the central region of the channel for the case that molecular diffusion is not having an effect on its value. At \( \text{Pr} = 1 \) the ratio of the molecular diffusivity to \( \varepsilon_T \) equals about 0.08. Thus \( D \) is having a small effect on turbulent
mixing. At Pe=0.025 (Pr=0.025) the molecular diffusivity is nine times
greater than the turbulent diffusivity in the absence of any effect of molecular diffusivity. The temperature profile is very close to what would be
realized in a laminar flow. That is, turbulence is having no effect. Plots of
the ratio of $\varepsilon_T$ to $\varepsilon_L$, in (230), show that, for a given Pe, the ratio of the local
turbulent diffusivity to what would be realized at Pr=1 is roughly constant
over the whole cross section of the channel. This ratio is shown to be a
function only of Pe by doing calculations for two different $\tau_{Re}$. These
findings provide a simple approach to calculating heat transfer in liquid metals.

Calculations of the spectra of the temperature fluctuations (230) show
that a large change in shape occurs when molecular diffusivity starts to have
a significant effect on the Lagrangian correlation. Consistent with the loss of
the parabolic shape at very small times, there is a drop in the contribution of
large wave numbers. The spectrum assumes a shape characteristic of what is
expected for an exponential form for the correlation function. The works
outlined above indicate that considerable progress has been made in
providing an understanding of the influence of molecular diffusivity since
the work of Johnk. They agree with the assumption that that the correlation
is given by an exponential function. The observation that the decrease in the
Lagrangian correlation caused by molecular diffusivity is a function of Pe,
independent of Pr, suggests that when the effect of molecular diffusion on the
correlation is significant, the microscale of turbulence is not playing a
role. The approach outlined in (173) for relating Lagrangian to Eulerian
correlations with an exponential form in an isotropic homogeneous field
offers an attractive theoretical analysis on the effect of molecular diffusion
on turbulent transport.

Paper 230 uses a relation developed for the influence of Peclet
number on turbulent transport and a classical Eulerian approach to develop
an equation for the rate of heat transfer. In (234) a stochastic method is used
to calculate turbulent temperature fields. This analysis requires the
specification of the influence of Peclet number on the Lagrangian timescale.
6. Structure of turbulent flows

(a) Prologue

The main objectives of a theory on turbulent shear flow are the understanding of how turbulence is produced and of how to use this understanding to develop predictive methods. One approach presumes that a knowledge of the structure of the turbulence is needed. This has been an important thrust in my studies---particularly with respect to turbulence generated at a wall. A summary of our accomplishments in this direction is presented in (202).

Measurements of mean velocity show that there are three regions: Close to the wall a viscous wall layer exists in which viscous effects are playing a role in determining the variation of mean velocity. It extends from the wall to $y^+ = 40$, where the plus superscript indicates that the distance from the wall has been made dimensionless with wall parameters, the kinematic viscosity, $\nu$, and the friction velocity, $v^*$, equal to $(\tau_w/\rho)^{1/2}$. Here $\tau_w$ is the drag on the wall and $\rho$ is the fluid density.

In the outer flow, the variation of mean velocity depends on large eddies which scale with the size of the field; e.g., the pipe diameter or the half-height of a rectangular channel. This region extends outward from about 20 percent of the distance from the wall to the centerline. The intermediate region, called the log-layer, is one in which the variation of the average velocity does not depend on the size of the field or on the viscosity.

(b) The viscous wall-layer

The viscous wall layer has been divided into two parts: the laminar sub-layer in which the variation of mean velocity is the same as would exist for a laminar flow ($y^+ < 5$) and the buffer region ($5 < y^+ < 40$). Measurements of concentration fields for large Schmidt numbers and visual observations of particle motion indicated that a laminar flow does not exist in the immediate vicinity of the wall. Yet, textbooks in the 1940's did not acknowledge this behavior. This motivated a study in 1956 which pictured that the turbulence right at the wall could be represented with a surface renewal model where large scale motions periodically replace the fluid at the wall (3, 19).

The development of electrochemical methods opened the opportunity to study flow in the viscous sub-layer. Reiss (18, 26) and Mitchell (32)
clearly demonstrated the highly unsteady behavior in the viscous sub-layer. They showed large fluctuations in the velocity gradient at the wall, which have a roughly Gaussian distribution, and reported values of $\left(\frac{s_x}{S_x}\right)^2$ equal to 0.31 (34). [Later results of Mao (154), which used much more advanced methods to correct the probes for frequency response, produced values of $\left(\frac{s_x}{S_x}\right)^2 = 0.35 - 0.36$.] A comparison of frequency spectra of the pressure at the wall with frequency spectra of $s_x$ shows that the velocity fluctuations observed at the wall do not represent a passive response to pressure fluctuations, as had been suggested by several investigators (34). The use of pairs of electrodes in a chevron configuration (45, 47) showed large fluctuations in the spanwise component of the velocity gradient at the wall, $\left(\frac{s_z}{S_z}\right)^2 = 0.11 - 0.12$. These measurements contradicted the notion that the viscous sublayer can be pictured as an oscillating Couette flow.

Arrays of wall probes were used to determine spatial correlations for the streamwise and spanwise components of the velocity gradient at the wall (67,69,87). Structures which are elongated in the flow direction and have a spanwise size of about 50 wall units were indicated. Measurements of cross-correlations between $s_x$ and $s_z$, at a given streamwise location, for different spanwise spacings revealed that the streamwise fluctuations at the wall were caused by eddies with strong spanwise fluctuations. Measurements with time delays supported the notion that the streamwise turbulence in the viscous wall layer is caused by flow structures which bring fluid with high streamwise velocity to the wall, exchange momentum with the wall and eject low velocity fluid from the wall.

A central research result on wall turbulence was the discovery by Kline that dye injected at the wall forms a streaky structure with a spacing of about 100 wall units. If one assumes that the behavior at the wall, on average, can be represented by pairs of eddies of the type described above, drafts of low momentum fluid away from the wall would be separated by a distance of 100 wall units (consistent with the streaky structure). This interpretation is consistent with observations in (67) which used a circumferential array of wall electrodes to provide the time-varying $s_x$ and $s_z$ simultaneously at 20 locations.

Turbulence measurements made in the 1950's show that both the production and dissipation of turbulent energy are quite large, and close to one another, in the viscous wall region. However, the difference of these two large numbers is such that there is a net positive production which supplies
energy to the outer region where local dissipation is the same as local production (in the log-layer) or greater (in the core). Thus, for flow over a smooth wall the viscous wall region may be considered to be the engine that drives the turbulence. This provided a motivation to use electrochemical methods to study further the relation of the pattern of wall velocity gradients to flow at locations away from the wall in the viscous wall region. These were carried out in a 19.37 cm pipe that contained arrays of electrodes embedded in the wall both in the circumferential and streamwise directions. (See Fig. 17) Electrochemical probes were used in the fluid to relate spanwise fluctuations in the fluid at different distances from the wall to patterns of the velocity fluctuations at the wall. Conditional sampling could be made for situations for which the wall electrodes indicated strong flows in the spanwise direction, strong flows toward the wall, or strong ejections from the wall (96). Combined measurements of time-varying wall patterns with instantaneous profiles of the streamwise velocity were made with a specially designed rake with seven probes (89,99). (See Fig.18) 

A summary of these measurements and of their theoretical implications is given in (150). The main point is that the $\lambda^+=100$ structures (suggested from measurements with wall probes) are strong contributors to the Reynolds stress and extend through the wall layer. They can be identified close to the wall from the spanwise variation of the streamwise and spanwise velocity fluctuations. At the edge of the viscous wall layer they can be related to normal velocity fluctuations with $\lambda^+=100$. However, at this location they are not so easily related to the streamwise and spanwise velocities since there is a large contribution to these components by eddies with $\lambda^+=200-700$ (which could be closed patterns with centers above $y^+=40$). The extent of the wall eddies was found to be about 300 wall units in the flow direction. (They have an aspect ratio of about 6.)

These results prompted the use of a $2^{1/2}D$ model to represent the wall eddies (77, 80, 134, 150). This involves the solution of the three-dimensional Navier-Stokes equations between the wall and the edge of the viscous wall layer for a flow which is homogeneous in the flow direction. These models calculate how the flow responds to an imposed boundary condition at the edge of the viscous wall layer. They progressed from a pseudo-steady state assumption which ignores the effect of transient terms in the Navier-Stokes equations (77) to one which considers the time-varying term and represents the flow with one wavelength (80).
This approach qualitatively captured the behavior in the viscous wall layer, particularly for small $y^+$. (It was adopted by several investigators). However, it gave an unreasonable picture of the turbulent energy balance for $y^+ > 15$. This lead to the realization that larger wavelengths need to be considered in order to capture the behavior of the streamwise and spanwise components of the velocity fluctuations throughout the viscous wall layer. (Paper 134 and a thesis by C. Nikolaides, "A study of the coherent structures in the viscous wall region of a turbulent flow", PhD thesis, University of Illinois, Urbana, 1984). The main conclusions coming from this analysis is that the $\lambda^+ = 100$ eddies are the main contributors to the velocity fluctuations normal to the wall and to the Reynolds shear stress. This prompted a consideration of why the scale of the stress producing eddies assumes a wavelength of $\lambda^+ = 100$. Calculations with the $2^{1/2}$ D model show that net production of turbulence in the viscous wall layer is negative for small $\lambda^+$ and positive for large $\lambda^+$. The production is sufficient to match the net dissipation in the outer flow if $\lambda^+ \approx 100$.

(c) **Suppression of boundary-layer turbulence in favorable pressure gradients**

Turbulence in boundary layers with favorable pressure gradients are found to show a decrease in turbulence and even a transition to laminar flow. It is generally accepted that the principal influence of a favorable pressure gradient on turbulence is in the region close to the wall, the viscous wall layer. Scaling arguments and the $2^{1/2}$ D model described above were used to provide a physical interpretation of this phenomenon and to predict the pressure gradient at which laminarization occurs (133,134). The basic physics that comes out of this analysis is as follows: In the outer flow, the pressure gradient is balanced by a mean acceleration. As the wall is approached, the mean velocity decreases and the mean acceleration decreases. The shear stress imposed on the boundary layer is then composed of turbulent & viscous shear stresses and the portion of the pressure gradient which is not balanced by mean acceleration. Thus the turbulent stresses decrease with respect to what would exist in a boundary layer with zero pressure gradient. This decrease in the Reynolds stress is accompanied by a decrease in the production of turbulence.
The introduction of direct numerical solutions (DNS) of the Navier-Stokes equations into our research efforts had an enormous impact. This can be understood by pointing out that our most advanced laboratory experiments obtained simultaneous incomplete measurements at about twenty locations. Our initial computer experiments obtained complete information, simultaneously, at one million points. (The limitation in computer experiments is that simple systems and low Reynolds numbers need to be considered. The availability of more powerful computers is overcoming these limitations. However, the need to handle very large amounts of data becomes limiting.) Laboratory studies have been an enormous help in doing computer experiments. They are used to check the accuracy of the computations and to provide guidance as to the questions that need to be asked when a converged solution is obtained.

Initial work was done for turbulent flow in a channel with a half-height of $H$ (145, 156,165). The Reynolds number was $Re = \frac{HV}{\nu} = 150$. Periodicity in the streamwise and spanwise directions was assumed, with periodicity lengths of 1900 wall units and 950 wall units. The main results suggested by studies with electrochemical probes were confirmed. Closed eddies with a size of about 50 wall units and streamwise dimensions of 400-500 wall units were found to be the main contributors to the Reynolds stress in the viscous wall layer (145).

By a tedious direct examination of the velocity and vorticity vector patterns in the $y$-$z$ plane for a number of locations downstream of one another, these eddies were shown to be vortices which start upstream as small vortices that are attached to the wall. They grow in size in the downstream direction and, eventually, lift from the wall (145,178). By examining vector fields at different times, these wall vortices were found to originate from attached vortices of small extent in the flow direction. An important finding is that the wall vortices regenerate themselves by a process that appears to be weakly dependent on the outer flow. This involves the enhancement of streamwise vorticity at the wall, of opposite sign, at a location where a stress-producing eddy lifts from the wall.

By using a vortex identification scheme, spanwise vortices in the viscous wall layer were observed (216). Streamwise vortices create shear layers by pumping low velocity fluid away from the wall. One or more spanwise vortices are formed on the top of this lifted shear layer. These grow in size and rotate to form streamwise vortices. The self-generation of wall vortices by the processes described in (145) and (178) gives rise to
vortices which tend to align with one another (as shown in reference 145). Thus, the long streaks of negative velocity at the wall could have contributions from a number of wall vortices. A summary of these self-sustaining mechanisms is given in (202).

The development of a DNS for a larger Reynolds, \( \text{Re}_r = 300 \), allowed an examination of the effect of Reynolds number. The simulation used the same resolution as the one at \( \text{Re}_r = 150 \) and required a much larger grid, 256x129x256. It was tested for accuracy by LDV measurements in our flow channel (207). This new DNS clearly showed that the vortices in the viscous wall layer are much smaller at \( \text{Re}_r = 300 \) than at \( \text{Re}_r = 150 \). However, the sizes made dimensionless wall scales are the same (221).

The spectra describing the velocity fluctuations normal to the wall for \( \text{Re}_r = 150 \) and \( \text{Re}_r = 300 \) are the same in the viscous wall layer if they are scaled using wall parameters. They also seem to be close in value for all four quadrants of the Reynolds shear stress. However, spectra for the spanwise and axial velocity fluctuations have larger scales (made dimensionless with wall parameters) at larger Reynolds numbers (202). Thus, the DNS calculations confirm the result from electrochemical measurements that there are large contributions from outer flow eddies to the streamwise and spanwise velocity fluctuations in the viscous wall layer.

(e) Outer flow; DNS and PIV studies

A major happening in our laboratory was the design of a special channel flow facility in which optical studies could be performed. (See Fig. 19) Initially, laser-Doppler velocimetry (LDV) was used to measure the time varying velocity at a given location. This optical technique had the advantage over probes in that measurements could be made under conditions that the flow is reversing direction (flow over large amplitude wavy walls) and that the measurement does not interfere with the flow. Professor Ron Adrian provided important inputs to this study. Over this period, Prof. Adrian was developing particle-image velocimetry (PIV) techniques to measure the fluctuating velocity components in a plane at a given instant of time. We decided to use these techniques in our turbulent flow channel.

The PIV technique is simple in principle. The fluid is seeded with small (5 microns) solid particles. A plane of the flow is illuminated with a pulsed laser and photographed. Thus, two images of the particles were obtained on the photographic plate. By measuring the distance between the two images, the velocity components can be obtained from the time interval
between pulses. In order to obtain the turbulent velocities, mean velocities were subtracted from the measurements. This involved the determination of the difference between two large numbers, so the measurements had to be quite accurate to obtain a meaningful picture of the structure of turbulence. I was surprised by the quality of the results. This is a testament to the ingenuity of Prof. Adrian.

Validation of the method was obtained by measuring the instantaneous velocity field in an x-y plane of the channel, where x is the coordinate in the direction of flow and y is the coordinate perpendicular to the wall (155). Eleven PIV photographs, taken at statistically independent times, were interrogated at spots separated by 0.5 mm in the x- and y- directions to generate 12,000 vectors that agreed with LDV measurements and DNS. Paper 198 shows how the instantaneous velocity field in the x-z plane can be measured by interrogating a plane parallel to the wall. These studies differed from the few previous published works of this type in that dense seeding of much smaller particles was used. A finer spatial resolution of the velocity field could be realized. Thus, instantaneous flow structures could be observed.

In subsequent work, we found that the flow was characterized by intermittent flow surges and that these large scale events were the principal locus for quadrant-2 Reynolds shear stress events in the outer flow. This work is described in full in Papers 217, 225. However, it was completed earlier than 2001 and appeared in bits and pieces before that date. For example, see Papers 155, 160, 186, 187.

It appears that the understanding of the outer region of a turbulent flow resides in providing more information about the large scale motions discussed above. This prompted the use of the DNS at \( \text{Re}_\tau = 150 \) and at \( \text{Re}_\tau = 300 \) to examine the flow vectors in a plane perpendicular to the direction of flow (202, 221). In this plane the large structures appear as eruptions from the viscous wall region. A number of researchers have observed the breakup of low speed streaks in the region \( y^+ = 30-50 \) that they called "bursts". In (221) the large scale eruptions in the outer flow are called "superbursts" since they can extend beyond the center of the channel.

Robert Moser provided us with results from a DNS at \( \text{Re}_\tau = 950 \). This calculation is of particular interest since it is at a large enough Reynolds number that a log-layer is present. (This is not the case for \( \text{Re}_\tau = 150, 300 \).) An account of this study is given in (255). Observations of vectors in the y-z plane reveal that embedded structures creating large Reynolds shear stresses in the viscous wall layer are the same at \( \text{Re}_\tau = 950 \) as was found at lower
Reynolds numbers. The direct influence of the outer flow on the velocity fluctuations at the wall increases with increasing $Re$. However, these large scales are affecting only the spanwise and streamwise velocity fluctuations in the viscous wall layer.

The main stress producing motions in the log-layer were found to be quasi-streamwise vortices which increase in size with increasing distance downstream.

Large scale motions in the outer flow, which are large quadrant 2 contributors to the Reynolds shear stress, are visible as intermittent surges of negative velocity fluctuations in the x-y plane. By examining fluctuations in the y-z plane, information about their 3-D structure is obtained. In this framework, they appear to be eruptions of low momentum fluid from the log-layer to the center of the channel. (See Fig. 20) The flow in these eruptions is more complex at $Re = 950$ than at smaller Reynolds numbers. An examination of the vector pattern suggests that x-vortices located outside the log-layer are lifting the fluid in these eruptions.

Much of what is found from this study about the log-layer and the outer flow could be surmised from the literature. However, the contribution is that visualizations of the log-layer and outer flow are supplied. The vortices in the log-layer could originate from vortices that emerge from the viscous wall layer. One can also speculate that the superbursts result from an instability in the vortical pattern in the log-layer.

Studies with DNS and PIV of turbulent flow over smooth and rough surfaces (239, 240) suggest that the turbulence in the outer flow is approximately the same for smooth and rough surfaces. Differences in the eddy structure are taken into account by setting the level of the friction velocity.

(f) *Turbulent Couette Flow*

Plane Couette flow in a fluid is caused by the motion of two infinite parallel planes in directions opposite to each other. It is often considered to be the simplest shear field. There is no mean pressure gradient, and the driving force is a constant shear stress transmitted through the fluid from one plane to the other. It appears to be attractive for producing a theory for turbulent shear flow. This prompted the development of a DNS of this flow (202, 203). We were surprised to find that the system is more complicated than we thought. The mean flow was not unidirectional. Rather, a secondary flow exists, consisting of roll cells which extend from wall to wall. They
have a spanwise dimension of the same magnitude as the distance between the planes and extend over the whole length of the Couette system. Thus, the goal of this study changed to one of discovering the properties of this secondary flow and of understanding why it exists.

An examination of the velocity vectors in a plane perpendicular to the direction of flow shows that they look similar to what would be observed for a turbulent Poiseuille flow. However an average of the vectors in this plane over the whole length of the field for 17 realizations clearly shows roll cells. An understanding of the flow is facilitated by representing the mean velocity as the sum of an average in the x,z-directions, \(< U_x >\), and the secondary flow, \(u'_x, u'_z\). The first of these averages is a function of y; the second is a function of y and z. The turbulent velocities, \(u'_x, u'_y, u'_z\), are functions of x, y, z, t. The secondary flow pattern (over most of the field) is described as an inviscid rotational flow. This assumption is supported by results from the DNS.

Equations for the kinetic energy of the secondary flow and of the turbulence can be developed. They contain terms which represent an interchange of energy between the turbulence and the secondary flow. For example, \(-\rho u'_x u'_y < U_x >/dy\) represents a transfer of energy from the turbulence to the mean flow, \(< U_x >\). Terms \(-\rho u'_x (\partial u'_x / \partial z)\), \(-\rho u'_z (\partial u'_z / \partial z)\)

represent an interchange between the y-component of the kinetic energy of the turbulence and the secondary flow. Terms \(-\rho u'_y (\partial u'_y / \partial y)\),

\(-\rho u'_z (\partial u'_z / \partial y)\)

represent the interchange between the z-component of the kinetic energy of the turbulence and the secondary flow. The secondary flow can be sustained if the net contribution of these terms is positive and if they balance the dissipation of the secondary flow by viscosity.

Thus, the mechanism governing the transfer of kinetic energy from the turbulent fluctuations is different from what is usually found in flows that are approximately unidirectional. The common situation is that, on average, kinetic energy is extracted from the mean flow by turbulence through the Reynolds shear stress. In the case being considered, the secondary flow affects the turbulence and modulates the Reynolds stresses in a way that they can transfer energy to the secondary flow. Thus, small scales of motion supply kinetic energy to the larger scales, contrary to the usual notion of an energy cascade.

Couette flow differs from Poiseuille flow in that the velocity profile is asymmetric. In the Couette system, flows outward from the bottom wall that extract energy from the mean flow (a quadrant one event), continue to do so.
when they move past the center of the channel. The opposite is the case for
turbulent Poiseuille flow. Consequently, conditions are favorable for the
enhancement of turbulent structures that extend from wall to wall.

For these structures to develop into roll cells, they must affect
turbulence in a special way. The secondary flow introduces new Reynolds
stresses and creates a nonhomogeneous field in the spanwise direction. The
computations show that the Reynolds stresses defined with reference to the
3-D flow are associated with a net transfer of energy to the secondary flow.
This is an example of Prandtl's second kind of secondary flow, a stress-
induced pattern.
Fig. 16 Turbulent flow in a channel with a heated bottom wall and a cooled top wall. Sections perpendicular to the mean flow are shown.

Fig. 17 Hans Hogenes, Kailung Lau, Hanratty at the 8 inch pipe used for turbulence measurements.

Fig. 18 Rake of electrochemical probes used to measure instantaneous profiles.
Fig. 19  Forward scattering laser Doppler velocimeter used to study turbulent flow in a channel. The transmitting and receiving optics, located on the opposite sides of the channel, were mounted on a beam that was located on a specially designed table that could move in three directions.

Fig. 20  Velocity vectors in a plane perpendicular to the direction of mean flow that were obtained with a direct numerical simulation. The horizontal lines indicate the outer boundaries of the viscous wall layer and the log-layer. A super burst is identified by arrow (2).

\[z / H = 5.7 \text{ to } 8.2.\]
7. Drag-Reduction/Turbulence suppression

(a) Polymer and surfactant additives

One of the most spectacular findings with turbulent flows is that the addition of very small amounts of a high molecular weight polymer can dramatically change the turbulence so that drag on a wall is reduced. The understanding of this phenomenon is a challenge to theories of turbulence. Two difficulties encountered by studies in this area are that polymers can degrade in flowing through pipes & pumps and that probes in the flow can give erroneous measurements of the velocity field because the flow around the probe is changed from what would be experienced by a fluid without polymers.

This prompted us to use electrochemical probes mounted flush with the wall to study changes in the flow field, since they do not interfere with the flow. Difficulties were encountered in that smaller drag reductions were realized in the electrolyte solutions used with the electrochemical methods and that larger degradation was realized. These were overcome by using higher concentrations of polymer. Experiments were done in a once through Couette device and in a 2.54 cm pipe in a flow loop which used a Moyno pump. These are described in (52, 60, 61, 77). The reduction in the velocity gradient at the wall was found to be the same as the reduction in the wall shear stress, indicating that the viscosity of the drag reducing solution at the wall is the same as for the solvent. The measured fluctuations in the axial and spanwise velocity gradients at the wall were not intermittent and decreased even when compared at the same friction velocity.

The most striking result was the finding that the scale in the spanwise direction increased with increased drag-reduction. This was interpreted by arguing that the size of the eddy structures in the viscous wall layer (the streak spacing) increases with increasing drag-reduction (151). Support was, therefore, provided for the conjecture that the phenomenon could be understood within the context of Newtonian behavior by considering how polymers increase the size of wall eddies and, therefore, the thickness of the viscous wall layer. However, there were some contradictions. At large drag-reductions the structure of the spatial correlation function is different from what is suggested by an eddy model for the viscous wall layer. Furthermore, the spectral density function showed a strong damping of high frequency fluctuations. There were also aspects of the measurements that suggested the flow at very large drag-reductions is fundamentally different from what is observed for low drag reductions.
A notion that was presented by a number of investigators is that coiled high molecular weight chains unravel in the presence of turbulence in the buffer layer between the viscous sublayer and the log-layer. Dissipation, therefore, increases and a thickening of the viscous wall region occurs. We wanted to test this idea by measuring the configuration of the polymers in a flow field. A laboratory study seemed very difficult in a large flow system. We, therefore, decided to carry out the study in a direct numerical simulation (DNS) of turbulent flow in a channel (180, 201).

The polymer molecule was represented by the FENE-P bead-spring model. A particle tracking code (170) was used to track the paths of fluid particles and to record the changes of the components of the velocity gradient tensor that they see. These would represent the velocity field that would be seen by a polymer molecule since its radius of gyration is smaller than the smallest eddy in the flow. The use of the time variation of the DNS velocity field allowed a calculation of the evolution of the average configuration of a FENE bead-spring as it moves about the field. Of particular interest were the conditions under which unraveling occurs.

Preliminary computational experiments were also done in a laminar Couette flow, a pure extensional flow and an extensional flow with different amounts of rotation. The size of the calculation increased exponentially with the number of beads. It was found that five beads were needed to capture the physics displayed in rheological laboratory experiments.

The chain unraveled if the product of the relaxation time and the wall velocity gradient is large enough. Bead-spring chains introduced close to the wall for a channel flow unraveled to about 87% of the fully-extended length while they were still in the sub-layer (contrary to ideas that were prevalent). When they move into the buffer region, the stretching and orientation of the bead-springs change with time. However, the amount of stretching was always significant. Beyond the buffer layer, very small unraveling was observed.

In (201) it is shown that this unraveling is accompanied by polymer stresses which can have time-averaged, as well fluctuating values. The time-averaged polymer stress can lead to the existence of polymer shear stresses such that the force on a control volume due to pressure gradient is balanced by a viscous shear stress, a Reynolds shear stress and a polymer stress. The fluctuations in the polymer stress can give rise to a transfer of energy between the fluid and the polymers (that is, a "dissipation"). Large positive values of this dissipation were not found to be associated with dissipative hydrodynamic motions. Thus, they cannot be described simply by increasing the viscosity. Furthermore, they are not associated pure elongational flows in
the viscous wall layer, as was commonly believed. However, it was found that large values of this dissipation are associated with flow oriented vortices. Thus, in agreement with the suggestions made in Papers 60,61, this study shows that intermittent large added dissipation due to the presence of unraveled polymer chains can result in an increase in the size of the wall vortices.

A criticism of the calculations in (180) and (201) is that only single bead-springs were studied. The presence of other polymer molecules causes changes in the turbulence which are not taken into account. Thus, the calculations, strictly speaking, are valid only for concentrations close to zero. Furthermore, they consider only single molecules and do not consider the effects of aggregation. Laboratory studies of the changes in the fluid velocity fluctuations in the presence of high molecular weight polymers were needed.

The development of laser-Doppler velocimetry techniques (LDV) offered the opportunity to carry out such studies without interfering with the flow. This was one of the reasons for constructing our facility for optical studies of turbulence. Reference 212 describes a study which used a 5.08 cm x 61 cm channel with a length of 3 m. It could operate over a Reynolds number (based on the half-height of the channel and the bulk velocity) range of 5100-48000. A master solution of Percol 727 (a copolymer of polyacrylamide and sodium acrylamide) was prepared in a 1000 gallon tank by injecting the powder into the tank through a commercial mixing device that insures that individual particles are wetted. The solution was agitated at 30 rpm for 4 hours with a four finned impeller. The concentrated solution was fed by gravity and injected into the channel flow through a wall slot. The concentration of the injected polymer was varied from 50 to 2000 ppm. Dye experiments did not indicate a breakup of the injected polymer into visible threads for concentrations of 1000 ppm or less.

The fluid was circulated with a 5 hp centrifugal pump that had undersized rotor blades. The concentration of the polymer increased with time but it was noted that the pressure gradient remained constant over a period of 5-7 minutes. After 7 minutes, injection was stopped and the liquid was circulated through a 60 hp pump at a high flow rate for 15 minutes. The 5 hp pump was then turned on and the pressure drop was found to be the same as for water. The injection process was repeated until the final concentration of degraded polymer was 3-150 ppm.

Previous studies with LDV were done at the University of Michigan and Purdue University. However the experiments described in (212) covered a wider range of concentrations. As a result, they have been widely quoted.
A momentum balance on a differential volume in the fully-developed region of a channel or pipe flow reveals that the forces on the volume due pressure gradient in a water flow are balanced by viscous shear stresses and Reynolds shear stresses (which are equal to the negative of the average of the product of the velocity fluctuations in the direction of mean flow and perpendicular to this direction). Measurements with LDV in a Newtonian fluid agree with this equation. However, in a polymer solution this is not the case. The difference is explained by assuming the existence of a polymer stress, which has not been measured directly. [However, the calculations in (180) and (200) provide a possible physical interpretation.] In (212), polymer stresses were obtained, indirectly, by a momentum balance.

All of the experiments were carried out at a constant volume flow. A range of drag-reductions of 10-69% was studied. Noteworthy is the observation of a drag-reduction of 29% with a concentration of only 0.28 ppm. For drag reduction less than 35%, the measurements of mean velocity agreed with what had already been suggested. A log-layer was identified whose slope is the same as observed for a Newtonian fluid. The thickness of the buffer layer (that is, the effective slip) increased with increasing drag-reduction. Average polymer stresses were indicated only in the viscous wall layer. A damping of velocity fluctuations was observed at high frequencies. These results show that striking changes in the turbulence close to the wall are associated with a thickening of the viscous wall layer and the appearance of added mean stresses. They are consistent with the suggestion in (180) and (201) that wall vortices interact with the polymers in such a way that local large positive values of $\varepsilon_p = \tau_{ij}^p (\partial u_i / \partial x_j)$ are generated. (See Section 6.) These would have the effect of introducing large local dissipation which would increase the size of these vortices (151).

Perhaps, the most dramatic results were obtained for experiments with drag-reductions of 64% and 69% (See Fig. 21). These results show that that the Reynolds shear stress was close to zero throughout the flow even though the fluid showed a turbulent behavior. These suggested that this turbulence is qualitatively different from what is observed for a Newtonian fluid in that it is produced by polymer stresses and not by Reynolds shear stresses. The magnitude of the fluid turbulence was found to decrease with increasing polymer concentration and the relative contribution of large frequency fluctuations was reduced. Another surprising result was that drag-reduction increases with increases in the concentration of the injected solutions. These results suggest that the injected polymer solution contains aggregates whose size increases with the concentration. Aggregates, apparently, were not completely destroyed in the flow channel.
Particle-image-velocimetry was used to measure the fluctuating velocities in the x-y and x-z planes. The studies gave further insights into changes in the structure of turbulence (219). These experiments were carried out in the same system used in (212). Large drag-reductions of 41% and 55% were explored. The most striking effect was a decrease in small scale fluctuations. The greatly decreased activity of the wall in creating turbulence at high drag-reductions is demonstrated by an observed decrease (or, even, elimination) of ejections from the wall. The measurements are consistent with the suggestion in (211) that, at maximum drag-reduction, the wall is not creating turbulence. This gives rise to a zero value of the Reynolds shear stress even though streamwise and wall-normal velocity fluctuations are not zero. (Turbulence is created by fluctuating polymer stresses.) PIV measurements, presented in (248), also show qualitative changes in the velocity patterns at maximum drag-reduction.

A study (246) with a channel for which the bottom wall contained 600 sinusoidal waves with a wavelength of 5 mm and an amplitude of 0.25 mm was also carried out in the flow loop described in (212). The procedure and the polymer were the same. A fluorescein dye was mixed with the polymer particles so that the existence of polymer filaments could be observed. Drag-reduction and spectra of the velocity fluctuations were found to be quite similar for smooth and rough walls even though processes at the wall which are creating turbulence are different. This clearly indicates that the focus on changes in the eddy structure close to a flat wall cannot provide a complete explanation of polymer drag-reduction. The fluorescein studies revealed the existence of polymer threads under certain conditions. Larger drag-reductions were realized at the same mixed concentration if threads were present. As pointed out in (246), it is quite likely that small aggregates of polymer molecules exist even when they are not observed visually.

Reference 248 describes experiments which use PIV to study the breakup of polymer filaments. The filaments are found to behave as if they are solid bodies which break up in high shear regions close to the boundary. The breakup process provides an explanation of why filaments are not observed close to the wall and offers the possibility of providing a heterogeneous distribution of small aggregates of polymers which could be more effective than uniformly distributed molecules, as had been suggested by a number of previous investigators.

The prevalent notion was that degradation of the effectiveness of polymer additives is due to the breakage of polymer molecules by the fluid turbulence. This was investigated in a series of experiments in which molecular weight distribution was determined as degradation (as evidenced
by pressure drop measurements) proceeded (242). The surprising result was obtained that no changes in the molecular weight distribution were detected for concentrated solutions of hydrolyzed polyacrylamide. The conclusion was that small clusters of polymer molecules were present and that degradation occurred because of the break-up of these clusters. Clearly, more work with other polymer solutions is needed.

The equipment described above was used to study the behavior of surfactant solutions which produced 62-70 percent drag-reduction (210). The turbulence was found to be the same as for a polymer solution under maximum drag-reduction. The Reynolds shear stress was zero over the whole cross-section of the channel and large frequency fluctuations were damped.

(b) *A general approach for interpreting turbulence suppression/drag-reduction*

A study (250) aimed at understanding the sharp decrease of the deposition coefficient of drops, in annular gas-liquid flows, with increasing drop concentration was carried out in a DNS of turbulent flow in a channel. The particle size and density ratio were \(d_p^+ = 1.9\) and \(\rho_p/\rho_f = 1000\). (The plus superscript indicates that the quantity has been made dimensionless with the friction velocity in the absence of particles, \(v_0^+\), and the kinematic viscosity.) Gravitational effects were not considered. The inertial time constant was large enough that particles only approximately followed the fluid turbulence. This gives rise to local slip velocities between the spheres and the fluid whereby particles can exert positive or negative forces on the fluid.

The particle Reynolds numbers were small enough that vortices were not shed. Particles were injected at the walls. They move around in the fluid and eventually deposit on the walls. The rate of deposition increases with particle concentration. A stationary state is possible for which the rate of injection equals the rate of deposition. The fluid velocity was calculated by solving Navier-Stokes equations which include point forces that represent the force of the fluid on the particles. The calculations agree with measurements of deposition coefficients in annular flow. They show significant reductions of the fluid turbulence at volume fractions as low as \(10^{-4}\). For a volume fraction of \(3 \times 10^{-3}\), the fluid turbulence disappeared. (See figures 22, 23)

These results can be understood if one recognizes that the modified Navier-Stokes equations equate the acceleration of fluid particles (the
substantial derivative) to the pressure gradient forces, viscous forces and particle forces (the negative of the force of the fluid on the particles). The time-average of this equation for a fully-developed turbulent flow at a given distance from the wall of a channel produces an equation which equates \( \rho_j \overline{uv} \) to the sum of the averages of the pressure gradient, the force of particles on the fluid in the volume between the center of the channel and the location under consideration and viscous forces, where \( \rho_j \overline{uv} \) equals the turbulent contribution to the acceleration. (It is the negative of the Reynolds shear stress.) Thus, for a constant pressure gradient particle forces can give rise to a decrease in the Reynolds shear stress, \( -\rho_j \overline{uv} \). This causes a decrease in the production of turbulence.

In (256) we explored whether this concept represents a generalized interpretation of drag reduction/turbulence suppression:

Paper 212 shows that the flow of a solution of a copolymer of polyacrylamide and sodium acrylate shows marked decreases in the drag on the wall and in the fluid turbulence at concentrations as low as 1.26 to 3 ppm. A finding of several investigators is the existence of "added stresses" or "polymer stresses". The research community has commented on this interesting phenomenon, but has focused on other findings in attempts to explain drag-reduction. We feel that an explanation is more easily found by considering the "added stresses".

The fluctuating field exerts forces on the polymer molecules that cause them to unravel and reorient (180, 201). The existence of "added stresses" suggests that the forces on the polymers have a positive mean, as well as fluctuating values. Thus, on average, polymer molecules exert a negative mean force on the fluid in the direction of flow. Their influence can be represented by including a mean value of the derivative of a polymer stress, \( \partial \tau^p_i / \partial x_i \), in addition to the contribution of viscous stresses, \( \partial \tau^l_i / \partial x_i \).

Section 6c discusses laminarization of turbulent boundary layers in accelerating flows. The boundary-layer equation is given as follows:

\[
\frac{d}{dx} \left( \frac{\overline{U_1'}}{\partial x_1} + \frac{\overline{U_2'}}{\partial x_2} \right) = -\left( \frac{1}{\rho_j} \frac{d \overline{P}}{dx_1} + \nu \left( \frac{\overline{U_1'}}{\partial x_1^2} + \frac{\overline{U_2'}}{\partial x_2^2} \right) - \frac{\partial \overline{u_x}}{\partial x} \right).
\]

where the pressure gradient, imposed by the outer flow, is independent of \( x_1 \) and \( x_2 \) is the distance from the wall. The terms on the left side can be pictured as describing a distribution of point sources, designated by \( f_{at} = -\frac{D \overline{U_1'}}{Dt} \), where \( f_{at} \) is negative in an accelerating flow. The integration of the boundary-layer equation from \( x_2 \) to the edge of the boundary layer, \( \delta \), gives
\[-u_i u_z = -\left(\frac{1}{\rho_f}\right)\left(\frac{d\bar{P}}{dx_i}\right)(\delta - x_z) + \int_{x_z}^{\delta} f_{d1} dx_2\] in the region where viscous stresses can be neglected. It is seen that the Reynolds shear stress and, therefore, the turbulence decrease with increasing acceleration.

A damping of turbulence by injecting bubbles into the flow is well known. However, the mechanism has not been established. It is quite possible that the conditions needed to realize this behavior can be obtained by treating bubbles as point forces. This notion has not been investigated.

(c) Effect of imposed oscillations on the mean drag at a wall

In Section 1c the influence of imposed temporal sinusoidal oscillations on turbulent flow in a pipe is discussed. The works described there are limited to linear behaviors, where the mean wall shear stress is not affected and the oscillating shear stress has only one harmonic. The principal measuring method was the electrochemical wall probe. The development of inverse mass transfer methods (described in Section 1c) allowed us to expand the application of electrochemical probes to consider large amplitude oscillations and reversing flows.

This approach is pursued in (111), where the influence of large oscillations on turbulent flow in a pipe is studied. The goal was to examine whether the mean drag on a wall can be affected. Experiments with imposed large amplitude sinusoidal oscillations were found to produce average drags on the wall that range from 1.00 to 1.03 times the value that would be realized in fluids without polymers. However, two experiments in which reversed flows appeared at the wall for appreciable periods of time showed drag-reductions of 7 and 13 per cent.

Imposed nonsinusoidal oscillations were also studied. These had periods of favorable pressure gradient about twice longer than that for unfavorable pressure gradients and two sudden changes in the pressure gradient. Drag-reductions of 10-15 per cent were realized. Paper 111 suggests that this phenomenon could be associated with the speed at which a flow adjusts to sudden changes in the pressure gradient. However, the proposed interpretation of drag-reduction in accelerating boundary layers in Section 6c could also be applicable.
8. Wave generation by a gas flow

(a) Prologue

Hanratty spent the summer of 1954 with the chemical engineering group of Shell Development Company, in Emeryville, California. He was impressed with the central position of multiphase flows in industrial processes. This provided a motivation to understand interfacial instabilities in a gas-liquid flow, and the role of waves in increasing gas drag.

A simple flow system was constructed to observe happenings. (See Fig. 73) This produced a concurrent flow of air and a liquid in a transparent 1 inch x 12 inch horizontal enclosed channel, which was long enough to have a fully-developed flow in the downstream part of the channel (7). The liquid was admitted at the bottom wall. At very low gas velocities the interface was smooth. With increasing gas velocity two-dimensional waves appeared at gas velocities of 4.4 to 15.1 ft/s (measured at the center of the channel) and film heights of 0.16 to 0.32 in. (See Fig. 24) At gas velocities of 9-19.9 ft/s the two-dimensional waves became unstable and broke into a three-dimensional pebbled structure. (See Fig. 25) At a gas velocity about five times that needed to generate regular waves on water, roll waves appeared on the interface. (See Fig. 26) Droplets were torn from water surface at gas velocities of 100-200 ft/s. The roll waves may be pictured as liquid surges which are moving much faster than the average velocity of the liquid layer. Reference 7 likens them to waves coming on to a beach.

Measurements of the mean velocity in the gas showed a distortion whereby the maximum appeared above the center of the gas space, defined as the region between the average location of the interface and the smooth upper wall (7). This distortion resulted from the existence of a larger drag at the wavy interface than at the top wall. From measurements of the pressure gradient and the location of the velocity maximum, stresses at smooth and wavy interfaces could be compared. This approach presumes the Reynolds shear stress is zero at the location of the maximum velocity. This need not be the case, so comprehensive measurements of the Reynolds shear stress were made by Miya (Miya, Masayoshi, "Effect of waves on turbulence", MS thesis, University of Illinois, Urbana, 1966). These showed that, for the system considered, the location of the zero shear stress is the same as the maximum, within experimental error.

Two methods were developed to measure the time-varying height of the liquid layer. In initial studies, we passed a small chopped beam of light through the transparent channel and the liquid layer on to a photomultiplier.
tube. By dissolving methylene blue dye in the liquid, absorption was increased. Variations in the thickness of the layer resulted in variations in the intensity of the light impinging on the photomultiplier tube (16, 31). The measurement of the time-varying resistance between two parallel platinum wires (with diameters of 0.003 in and a spacing of 0.10 in) which extend through the liquid (57, 59) proved to be a more flexible method to measure the time-varying height. In (59), 0.001 in wires were used to capture the behavior of small capillary waves. This approach cannot be used with extremely thin layers because of capillary rise on the wires. In these cases, electrodes flush with the wall were used (114, 175).

(b) Regular waves

The problem of predicting the gas velocity at which waves will appear at the interface has attracted the attention of researchers for a number years. Air flowing over a wavy surface experiences spatial variations of the gas phase pressure and shear stress acting on the interface. The classical approach is inviscid. It predicts that instability occurs when suction at the wave crest overcomes the forces of gravity and surface tension. The calculated critical gas velocity is too large to explain the initiation of waves at an air-water interface.

A viscous approach was taken in (28). The stability of the gas-liquid interface was explored by imposing a propagating wave with a wavelength in the flow direction, $\lambda_x$, and a complex velocity, $c$, on the interface. The disturbances were considered small enough that the linearized Navier-Stokes equations could be used. The influence of the gas flow was considered by introducing a wave-induced pressure and shear stress variation along the wave interface. The Reynolds number, based on the wavelength and the wave velocity, is large enough that viscosity is having a negligible effect on the flow field except for thin viscous boundary layers in the liquid at the wall and at the interface.

Damping is much greater in the viscous boundary layer near the wall. Waves with wavelengths small enough that disturbed velocities at the wall are unimportant are more easily generated. Thus the wavelengths observed in (7) and (28) tended to scale with the height of the liquid flowing along the wall.

The sheltering hypothesis was explored in (28). The assumption is made that there exists a pressure component which is in phase with the wave slope, $P_t = as \rho_g \alpha (\bar{U}_g - c)^2$, where $s$ is the sheltering coefficient, $\rho_g$ is the gas
density, $\alpha$ is the wave number, $a$ is the wave amplitude and $c$ is the wave velocity. Instability occurs when energy fed to the wave by pressure is balanced by viscous dissipation. The physical notion behind this equation is that the gas flow separates behind the wave. Yet, at transition the waves have too small an amplitude for separation to occur. This led Ursel to introduce a review of the problem with the statement that waves are generated by an air flow by processes which cannot be regarded as known. The carefully controlled experiments described in (7) provided an opportunity to address this problem.

Reference 28 shows that the sheltering hypothesis with $s = 0.3$ gives a good prediction of the critical velocity but underpredicts the critical wavelength. In 1959, quasilaminar solutions of the gas phase Navier-Stokes equations were developed by Benjamin and by Miles to calculate pressure variations in the gas phase. (Turbulence effects were considered only insofar as they were used to calculate the mean velocity in the undisturbed flow.) These theories were of interest because they predicted sheltering without the occurrence of separation. Good agreement was realized. Similarly, the prediction of the transition to a pebbled wave structure obtained with the quasilaminar analysis agrees with observation. The work described in Papers 7,28 provides a basic physical understanding of the generation of waves and showed sufficient agreement with the quasilaminar approach to encourage further work which would include turbulence effects in the gas phase.

One of the results of this work is that the critical gas velocity needed to generate waves increases with liquid viscosity because of the increased damping in the liquid. Reference (141) shows (perhaps, for the first time) that at high enough liquid viscosities the critical gas velocity for the appearance of waves is predicted by an inviscid Kelvin-Helmholtz instability!

(c) Roll Waves

Roll waves had been observed frequently for inclined liquid flows in water runways and in drainage problems. Reference 15 adapts theories used in these situations to examine their initiation by air flows, for which the liquid is moved by gas drag at the interface, rather than by gravity. (See figure 26)

The critical conditions for the initiation of roll waves are predicted by introducing a small interfacial disturbance into a stratified flow. The waves are assumed to have very long lengths compared to the heights of the gas and liquid spaces, so pseudo-steady state assumptions may be made for the
disturbed flow. References 15, 29, 97 use integral equations to formulate the laws of conservation of mass and momentum. Since an enclosed channel is considered, spatial changes in the gas bulk velocity are introduced by the waves. These are accompanied by changes in pressure which have a minimum at the crest of a wave. Instability occurs when this suction overcomes the restoring force of gravity. This theory correctly predicts observed critical gas velocities, for liquids with different viscosities and surface tensions (except for extremely thin layers).

Of interest is the observation in (15) that the addition of a surface active agent dampened small amplitude waves so that roll waves are initiated on a flowing liquid with a smooth interface. The critical gas velocity is increased. An attempt to explain this difference by changing the relation for the interfacial drag was not successful. We now recognize that surfactants can dampen turbulence so that the flow in the liquid could have been laminar. This was not considered and probably explains the observed behavior.

Small wavelength waves introduce variations in the shear stress at the interface because of induced variations of the gas velocity and the boundary roughness along the interface. The component of this stress variation in phase with the wave slope can be destabilizing. More complete analyses presented in (57, 97) show that these are not so important in dictating instability as suction forces caused by the wave-induced pressure variation (except for very thin layers).

(d) Description of a fully-developed roll wave

Stability theory indicates that roll waves are initiated as a long wavelength sinusoidal wave. At gas velocities above the critical, regular waves develop into highly asymmetric waves with gradually sloping backs and a sharp decrease in height at the front. These are observed to move intermittently over a base film of constant height. A striking feature, described in (40) and (57), is that this base film is the same as would be observed at transition. Measurements of the height of the wall layer, of the shear stress at the wall (by using flush mounted hot-film probes) and of pressure variation along roll waves were used to develop a model. The interfacial shear stress was assumed to vary along the wave interface because of changes in the small wavelength waves that exist. Roll waves in gas-liquid flows were found to be fundamentally different from what is found for flow down inclined planes in that the front of the wave cannot be assumed to be an hydraulic jump. The interfacial drag and the resistance at
the wall increase with increasing height. A momentum balance shows that the height increases if the resisting stress at the wall is greater than the drag at the interface. It is suggested that these stresses are equal at the crest and that a sharp decrease in the height accompanies a laminarization of the wall layer (a decrease in the wall shear stress).

The work cited above was extended to cases in which appreciable entrainment is occurring (64). The approach just outlined needs to be modified to include momentum increases due to deposition and atomization. It is noted that, under these circumstances, a sudden decrease in height at the front of the wave could be associated with a decrease in atomization.

(e) Atomization

At gas velocities much higher than that needed for the initiation of roll waves, drops are removed from the surface of flow surges which resemble the so-called "disturbance waves" found in vertical flows. The mechanism for this happening was studied by obtaining high speed photographs (3500-6000 frames per second) with a camera located beneath the transparent bottom wall (41). The flow surges were found to be covered with ripples. "One of these ripples suddenly accelerates and moves to the front of the disturbance or roll wave. The central section of the ripple is lifted by the air stream leaving one or both ends connected to the flowing liquid film. As the detached region of the apparent tube of liquid is lifted, it is blown into an arc which narrows as it stretches downstream until it ruptures into a number of pieces. Though difficult to count, as many as twenty droplets can be sent streaming off into the flowing air after the filament rupture. Finally the end attached to the water film lies down ahead of the roll wave and is swept under the passing disturbance." (See figure 27)

Paper 41 suggests that drop removal occurs because of an imbalance between pressure variations in the air flowing over the wavelets and the stabilizing force of surface tension. That is, it occurs by a Kelvin-Helmholtz instability. The equation for a KH instability was derived for gas flow with uniform velocity, so a direct comparison with observations could be troublesome. The pressure variation over a wavy interface in a boundary layer (with a spatially varying velocity) was considered by Cook (Cook, G.W., "Shear stress and pressure variations over small amplitude waves", MS thesis, University of Illinois, Urbana, 1967). The linearized momentum balance equations (the Orr-Sommerfeld equations) were solved numerically (by using a computer program which removes parasitic errors) to obtain the pressure and shear stress variation along a wavy surface. The results of this
analysis were used in (41). The calculated gas velocity needed to remove waves from the interface agrees with observation for a wavelength that was assumed to be five times the peak height of the roll waves. This choice is consistent with visual observation.

The method for estimating the critical wavelength, outlined above, was open to criticism so Zanelli carried out a study to make a firmer choice of a mechanism. (Zanelli, Severino, Relation of Entrainment to Wave Structure on a Liquid Film, presented at the International Symposium on Two-Phase Flow Systems, chaired by Gad Hetsroni, Technion City, Haifa, Israel, Aug. 21-Sept. 2, 1971). The wave height measurement technique described in (59) was used. However, the circuitry was modified so that higher frequency (small wavelength) waves could be studied. When atomization was not occurring, the disturbances on top of the waves were observed to move at the same velocity and to be identifiable with probes separated by a distance of 4 inches. This was not the case when atomization was occurring. The wave structures on the tops of the roll waves changed after a distance of 4 inches.

Observations at constant gas velocity produced the following picture:
At low liquid flows (small film heights) the waves generated on the top of roll waves had too small a wavelength to be removed by a KH instability. With increasing liquid flow, the waves increased in wavelength and became unstable. The observed liquid flow at which atomization was first observed agrees with the KH analysis developed by Cook.

(f) Wave heights and spectra

The experiments with 3-D "pebbled" wave patterns suggested that they produced a well defined system in which to study the effect of waves on processes such as interfacial drag and gas absorption. Waves can be characterized by measurements of the deviation of the height of the liquid layer from the average, $\Delta h$. Root-mean square values of $\Delta h$ and frequency spectra are reported in (16, 36) and in a thesis by Cohen (Cohen, L.S., "Interaction between turbulent air and a flowing liquid film", PhD thesis, University of Illinois, Urbana, 1964). The deviations were found to be random and described by a Gaussian function. Characteristic lengths in the streamwise and spanwise directions were approximately equal. The spectral functions describing the deviations show a maximum at a frequency which is roughly independent of the friction velocity, $v_f^*$. The value is equal to about one-half of the number of waves per second, calculated from the frequency
with which the time varying height of the liquid layer crosses the average location of the interface.

Drag on a roughened surface can be characterized as equivalent to what would be measured for a surface covered by sand grains with a height of $k_S$. Velocity profiles over sand-roughened surfaces were plotted as the ratio of the velocity to the friction velocity versus the logarithm of the distance from the wall, made dimensionless with the friction velocity and the kinematic viscosity. For sand roughened surfaces the slope of such a plot is the same as would be found for a smooth surface, except that it is displaced downward by an amount $\Delta u/\nu^*$. This displacement velocity can be related to the sand roughness for solid surfaces, $k_S$. Studies of flow over water waves have been interpreted in this way, so that the wavy surface can be characterized by a sand roughness (17). Careful measurements, reported in (36), show that the slopes of the semi-logarithmic plots of velocity over water are larger than what is found for sand roughened surfaces (that is, the von Karman constant is smaller). In (36), this difference is interpreted by showing that the turbulent energy balance changes when water waves are present because there is a direct transfer of energy at the interface, manifested by the correlation between pressure fluctuations and the movement of the interface in a direction normal to the interface.

Thus, in (36), correlations of friction factors (in the literature), rather than gas velocity profiles, were used to evaluate the sand roughness characterizing the interface. It is of interest to compare the roughness obtained in this way with wave heights. For a sinusoidal wave, the height is defined as the distance between the trough and the crest, that is, as twice the amplitude. The mean-square of displacement, $\Delta h$, of a sinusoidal wave is given as $2a = 2\sqrt{2(\Delta h^2)^2}$. This is a measure of the average wave height. In (36), the equivalent sand roughness of water waves is shown to vary between $2\sqrt{2(\Delta h^2)^2}$ and $3\sqrt{2(\Delta h^2)^2}$.

(g) Film heights

The presence of 3-D pebbled waves introduces random disturbances into the liquid. Reference 31 explores a conjecture, made by a number of researchers that the profile of average velocities in the liquid is the same as would be measured for a turbulent single phase flow past a wall. The consequence of this concept is that $m^+ = m^* \rho/\mu$ is an unique function of the
liquid film Reynolds number. This is confirmed in (31) so long as a 3D wave pattern exits.

(h) *Symposium on waves*

Opportunities for basic studies of waves was recognized by a symposium entitled Waves on Fluid Interfaces, held at the University of Wisconsin and chaired by Richard E. Meyer. Hanratty was pleased to be one of the fifteen invited speakers. His paper on Interfacial Instabilities Caused by Air Flow Over a Thin Liquid Layer, along with the contributions of the other participants, appear in a book on Waves on Fluid Interfaces published by Academic Press in 1983. See Reference 97.
Fig. 21 Influence of polymers on Reynolds stress, which is associated with the production of turbulence.

Fig. 22 Direct numerical calculations of turbulence for flow of a suspension spheres. Note that the Reynolds stress is close to zero at a volume fraction, $\alpha$, equal to $3 \times 10^{-3}$.

Fig. 23 Calculated instantaneous velocity field in the center plane of a channel for a suspension flow. Note the drastic decreases in the turbulence for volume fractions of $4.9 \times 10^{-4}$ and $3 \times 10^{-3}$. 

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Fig. 24  Initiation of waves on a smooth interface.

Fig. 25  At a higher gas velocity a three-dimensional pattern appears.

Fig. 26  At a still higher gas velocity roll waves appear.

Fig. 27  Ripples being lifted and shattered by a 60 ft/s air flow.
9. Flow over small amplitude wavy surfaces

(a) Prologue

Quasi-laminar solutions of the linearized Navier-Stokes equations by Benjamin and Miles for fluid flow over small amplitude waves have had an important impact on efforts to specify the wave-induced variations of pressure and shear stress along a wavy interface. (See Fig. 28) These involve the assumption that turbulence can be taken into account simply by using a turbulent mean velocity profile. The need to understand what are the roles of wave-induced variations of the turbulence and the choice of a coordinate system in which to describe the gas flow motivated studies of turbulent flow over solid wavy surfaces. The principal tool was the use of electrochemical techniques to measure the shear stress variation along small amplitude waves. (See Fig. 28, 29)

The theoretical challenge is to understand how the turbulence responds to small amplitude spatially-induced disturbances. The effect of temporally-induced small amplitude oscillations is a related problem, so studies of this type were also undertaken.

Applications of these results include mass transfer to wavy boundaries, wavelike dissolution patterns and wave generation. Our work on these problems is summarized.

(b) Shear stress measurements over small amplitude solid waves

References 78, 93 present results on experiments in a 2 in x 24 in rectangular channel through which an electrolyte was flowing. Trains of ten sinusoidal waves with wavelengths of 2 in were machined on a 1.25 in x 24 in x 24 in slab of Lucite and located on the bottom of the channel in a region where the flow was fully-developed. The two trains that were studied had amplitudes of 0.0114 in and 0.0125 in. The variation of the shear stress along the wave surface was measured by using electrochemical techniques. (See Fig. 29) The function describing this variation had only a single harmonic and the average shear stress was the same as would be measured on a flat plate. These results supported the assumption that the waves were small enough for a linear response to be realized.

Measured values of the amplitude and the phase of the spatial variation of the shear stress were compared with theories which use different models for the wave-induced variation of the Reynolds stress (93). The momentum equations describing the flow field were formulated in a
boundary-layer coordinate system which has its x-coordinate parallel to the wave surface and its y-coordinate perpendicular to it. The time-averaged velocities, the turbulent stresses and the pressure gradients are assumed to be composed of an average over the wavelength at a constant value of y and a component with a periodic variation of the same wavelength at the surface.

A model (78, 93) which successfully describes the data is the mixing-length relation of Loyd, Moffat and Kays, who used a modified form of the van Driest equation whereby the mixing-length \( \ell = \kappa \sqrt{\frac{1}{2} - \exp\left(-\frac{\gamma^2}{4A^2}\right)} \), \( \kappa \) is the von Karman constant and all terms are made dimensionless with wall parameters. The argument of the exponential is a damping function which represents the influence of viscosity in the buffer layer. The parameter A is a measure of the thickness of the viscous wall layer. This is taken to be a function of the pressure gradient so that turbulence is damped and the thickness wall layer is increased in favorable pressure gradients. For situations in which the pressure gradient is varying (such as would be experienced for wavy walls) an effective pressure gradient is used since the flow does not respond immediately to a change. Term A is now described with a linear differential equation which contains a relaxation constant, \( k_r \).

The calculations produced good agreement with the amplitude and phase of the wave-induced variations of the wall stress measured as a function of the wave number, \( \alpha \), made dimensionless with the friction velocity and the kinematic viscosity. This could be varied by changing the flow rate. Thus, as the flow rate decreases dimensionless \( \alpha \) increases and the changes are so rapid that wave-induced variations of the turbulence can be ignored; that is, a quasi-laminar model can be used. As the flow rate increases, dimensionless \( \alpha \) decreases. For very small dimensionless \( \alpha \), the flow approaches an equilibrium condition where relaxation is not a factor. The theory predicts that at intermediate dimensionless wave numbers a sharp increase in the phase angle of the wave-induced variation of the wall shear stress occurs (signaling a change from an equilibrium condition to a quasi-laminar condition). The studies in (78) used a pump which could not produce large enough flows to see if the predicted relaxation is correct. The use of larger wavelength waves was not attractive because of possible influences of the top wall on the flow near the wavy surface.

This lead to the installation of a new pumping system that used a 60 hp motor, that is, a power five times larger than was being used. A new electrical system for the laboratory was needed. The results of this study are presented in (110). Again, the waves had a small enough amplitude that a linear response was obtained. The relaxation phenomenon was demonstrated
by a sharp change in the phase angle of the wall shear stress variation over
the same range of wave numbers as predicted as predicted in (78,93).

(c) Effect of imposed temporal oscillations

The experiments described above provided the first demonstration of
relaxation in wave-induced changes of turbulence. In order to document this
phenomenon further, the response of turbulence to temporally imposed
oscillations was studied in (122). This case is somewhat simpler because it
does not involve the use of a wavy boundary which raises concerns about the
selection of a coordinate system in which to analyze the behavior.

Experiments were done in a 19.4 cm pipe. The main flow was
generated by a centrifugal pump. Imposed small amplitude oscillations were
introduced by a piston. The frequency of oscillation was fixed at 0.325 or
0.625 Hz by adjusting the circular frequency of the motor driving the piston.
Electrochemical methods were used to measure the time-varying shear stress
at the wall. This could be considered to be the sum of an ensemble average,
a wave-induced variation, and turbulent fluctuations. The average was the
same as would be obtained for a flow without imposed oscillations. The
time-varying pressure gradient was also measured. At the center of the pipe,
viscous and turbulent stresses are small, so the time variation of the velocity
at that location can be calculated from the time variation of the pressure
gradient.

The wave-induced variation of the shear stress at the wall was fitted
with a sinusoidal curve. The amplitude and phase, relative to the time-
varying centerline velocity, were plotted versus the circular frequency of the
imposed oscillations, \( \omega \), made dimensionless with wall parameters. A
relaxation of the type shown in our study of flow over wavy walls was
obtained. In order to make a more direct comparison, it was assumed that the
variation can be pictured as a spatial variation which is transported
downstream with the convection velocity (made dimensionless with the
friction velocity) of the turbulence in the viscous wall layer.

Thus comparisons between the two experiments could be made by
equating \( \alpha \) to \( \omega/15 \). Relaxation was observed at the same values of \( \omega \) and
\( \omega/15 \). Furthermore, the model of Loyd, Moffat and Kays for mixing length,
used to describe spatial variations introduced by solid waves, successfully
correlates the temporally-induced oscillations.

This work was extended in (136) by carrying out experiments in a
5.08 cm pipe. These captured, more fully, the relaxation from a quasi-steady
behavior at low frequencies to a quasi-laminar behavior at high frequencies
and tested the scaling suggested in (122). Furthermore, measurements of wall shear stresses at an array of electrodes arranged in the circumferential direction allowed measurements of the time variation of the spanwise turbulence scale in the viscous wall layer. These supported the notion that imposed oscillations are changing the dimensions of the wall vortices and, therefore, the thickness of the viscous wall layer. These results are consistent with those presented in (122).

One of the goals of the studies in (122,136) was to obtain a better understanding of the turbulence in the viscous wall layer by examining how it responds to imposed spatial or temporal disturbances. It was found that variations of the pressure gradient accompanying these disturbances causes cyclical changes in the size of the wall vortices and, therefore, the thickness of the viscous wall layer. An unanticipated effect observed in (122,136) was that, when the imposed frequency of the temporal oscillations is close to the characteristic frequency of the turbulence, a complex interaction is observed, which is not understood. Clearly, this matter deserves more attention.

(d) *Wavelike dissolution patterns*

The presence of wavelike dissolution patterns is of considerable interest. They are observed on the underside of ice in rivers and canals, in limestone caves and glaciers. They are believed to originate from small sinusoidal waves formed when the material begins to dissolve or ablate into a flowing fluid. The prediction of the onset of this instability provided an interesting application of our studies of flow over small amplitude wavy solid waves. A necessary condition for the growth of a disturbance is that the maximum in the mass transfer rate occurs somewhere in the trough. It will propagate forward or backward, depending on the location of the maximum in the trough. Therefore, we sidestepped the usual approach of directly observing a dissolving surface. Instead, mass transfer rates were measured along a small amplitude wavy surface over which a fluid is flowing turbulently (85, 86). (See Fig. 30)

The experiments were carried out in a 5.08 cm by 61 cm rectangular channel. A brass insert with a train of ten waves with an amplitude and wavelength of 0.03175 cm and 5.08 cm was located in a section of the bottom wall at a place where the flow was fully-developed. This insert was plated with platinum and used as the cathode of an electrolysis cell. An electrochemical reaction was carried in such a way that the cathode was polarized and the current is proportional to the rate of mass transfer.
Local values of the mass transfer rate were determined by measuring the flux to wires, with a diameter of 0.064 cm, inserted through holes in the cathode and ground flush with the wave surface. These were glued in place and insulated from the rest of the cathode by the epoxy glue. The variation of the wall shear stress along the wave could be determined by measuring the current flowing to the wire electrodes under conditions that the main body of the cathode was deactivated. (See Fig. 29) The wave amplitude was small enough that a linear response was realized, in that the average was the same as would be observed with a flat surface and deviations around the average showed a sinusoidal behavior.

These wave-induced variations of the mass transfer rate were compared with a solution of the linearized mass balance equation which assumes that the wave-induced variation of the eddy diffusivity is proportional to the wave-induced variation of the eddy viscosity (that is, the analogy). Good agreement between the measured and calculated mass transfer rates was realized by using the eddy viscosity model described in Section 9(b).

Paper 85 uses these results in an analysis of the stability of a dissolving surface. The phase for the wave-induced mass transfer variation, $\theta$, is described as the degrees by which the maximum precedes the wave crest. It is shown that $90^\circ < \theta < 270^\circ$ for growing waves. A dimensionless wave number, $\alpha$, is defined using the friction velocity and the kinematic viscosity. In the mass transfer experiments described above, the effect of $\alpha$ was explored by varying the flow rate rather than the wavelength.

Maximum growth occurs at $\theta = 180^\circ$, for which the maximum in the mass transfer rate coincides with the wave trough. The mass transfer measurements described in (86) show that phase shifts of this magnitude are possible. Calculations which use the relaxation theory, outlined in Section 9(b), to calculate the wave-induced variation of the eddy viscosity, predict that dissolution rates are possible for $\alpha^+ < 0.003$ and that the maximum growth occurs at $\alpha^+ = 0.001 - 0.002$ for a Schmidt number of 729. Observations of the wavelength of the dissolution pattern caused by water flowing over limestone give a value of $\alpha^+ = 0.0028$. These are in close agreement with what would be expected from the mass transfer measurements presented in (86) and just slightly smaller than calculations which use eddy viscosity concepts.

Calculations for melting ice, which is characterized by a Prandtl number of 13.7, show that dissolutions patterns are possible for $\alpha^+ < 0.004$ and that a maximum growth rate is realized at $\alpha^+ = 0.002$. This is in excellent agreement with observations in controlled experiments. Furthermore, the
observed propagation velocity of the dissolution waves agrees closely with predictions.

Thus (85, 86) support the notion that wavelike dissolution patterns observed on the underside of river ice and in caves could have originated from a dissolution instability. This work attracted the attention of the mechanics community so that Hanratty was invited to make a contribution to Annual Reviews of Fluid Mechanics (88).

(e) Generation of waves on very thin liquid films

At arbitrarily low liquid rates and high gas velocities, a liquid film over which the gas is flowing is covered by long-crested slow moving ripples having a steep front and a low ratio of the amplitude (10-20 microns) to wavelength (2-3 millimeters). (See Fig. 28) Between ripples the surface is smooth and the flow appears to be laminar. At sufficiently high liquid flow rates and high gas velocities "disturbance" or "roll waves" appear on the film. These have a much larger velocity than the ripples and a very large spacing. For flow in a small diameter pipe, the roll waves appear as frothy rings which cover the whole pipe circumference, and the distance between successive waves is several diameters.

Paper (175) presents an analysis which shows that the capillary ripples are associated with the component of the interfacial drag force in phase with the wave slope, which overcomes the stabilizing effect of surface tension. The approach taken is to solve the linear momentum equations to determine the growth rate of small two-dimensional wavelike disturbances at the interface. It is argued that the fastest growing wave is the precursor of the ripples. In carrying out the analysis it is convenient to consider separately the gas and liquid flows. In this framework the gas flow is found to affect the stability through the imposition of pressure and shear forces on waves that appear at the interface. The prediction of the pressure and shear stress variation along a wavy surface over which a turbulent gas is flowing then becomes the central problem in analyzing the growth of small amplitude waves.

The thickness of the films on which these ripple waves occur is quite small. For example, at a liquid Reynolds number of 36 and a gas Reynolds number of 78,000, a water film on the wall of a 4.2 cm pipe would have an average thickness of 147 microns. This allows a simplification of the analysis which considers wavelengths which are long enough for a shallow liquid assumption to be made, yet small enough that surface tension effects are important. The analysis considers that the growth of small amplitude
waves is governed by an imbalance between the destabilizing effects of inertia, the component of shear stress variation in phase with the wave slope, the component of surface pressure in phase with the wave height and the stabilizing effects of surface tension. The wavelength of the fastest growing two-dimensional waves were compared to measurements.

This comparison leads to the conclusion that nonlinear processes cause two-dimensional unsteady waves to develop into a three-dimensional wave pattern with a distance between crests equal to twice the wavelength of the initial instability, since ripple spacing calculated in this way is very close to measurements.

(f) More on the generation of regular waves

This subsection is a continuation of the material presented in Section 8(b), which explains the initiation of regular waves as due to pressure variations in phase with the wave slope. Initiation occurs when energy fed to the waves by these variations balances dissipation.

The analysis differs from that described in Section 9e in that the flowing liquid layer is much thicker, so a pseudo-steady state approximation cannot be made. A linearized form of the Navier-Stokes equations (the Orr-Sommerfeld equation), with boundary conditions that include wave-induced variations of the pressure and shear stress, is used to describe the wavy liquid. Reference 28, discussed in Section 8(b), presents measurements of the initiation of waves in a horizontal enclosed channel and explores the use of a quasi-laminar model for the gas phase to calculate wave-induced variations of the interfacial pressure over small amplitude waves in a Cartesian coordinate system. The agreement was sufficiently close to encourage us to pursue this approach further. Problems associated with this analysis are errors associated with the use of Cartesian coordinates and the neglect of wave-induced variation of the turbulence in the gas phase.

These were addressed by utilizing the shear stress variation over small amplitude waves. See K. A. Frederick ("Wave generation at a gas-liquid interface", MS thesis, University of Illinois, Urbana, 1982) used a boundary-layer coordinate system and the relaxed mixing length model (See Section 9b) were used. The linearized equations, describing the wave-induced disturbances in the gas flow, differ from those used for flow over solid waves in that the wave surface propagates with a velocity $c_R$. Two constants appear in the turbulence model. One, $k_1$, gives the effect of pressure gradients on the thickness of the viscous layer under equilibrium conditions.
The other, $k_L$, recognizes that the pressure is varying too rapidly for an equilibrium condition to be realized. In order to avoid criticism, Frederick used the constants recommended by Loyd, Moffat and Kays, $k_i = -55$ and $k_L = 3000$, from measurements in turbulent boundary layers. Calculations were done in a curvilinear coordinate system without (Model A) and with (Model D*) a consideration of wave-induced variations of the turbulence in the gas phase.

The analysis presented in (28) predicts critical gas velocities which are about 25 per cent higher than observations. The predictions of Frederick agree within experimental error and correctly predict the effects of a change of viscosity. Furthermore, the predicted fastest growing wavelength at gas velocities above the critical agrees with observations. The effect of wave-induced variations of the turbulence was shown to be small for these thick liquid layers, as predicted by theory.
10. Flow over large amplitude solid wavy surfaces

(a) Prologue

Flow over large amplitude solid waves offers a far more complicated pattern than what is observed for flow over small amplitude waves because of the appearance of nonlinearities and the possible occurrence of flow separation behind the waves. Several studies were carried out in this area. They have been of interest to the engineering community because of the occurrence of flows of this type in many applications. However, our chief interest was that they offer a well-defined flow field that can be used to test computational models for turbulent flow and turbulent heat/mass transfer.

Initially, electrochemical techniques were utilized to measure the variation of the shear stress along a wavy surface up to the separation location. Unfortunately the methods for analyzing haphazard flows with electrochemical probes, described in (154), were not available at that time, so that wall shear stresses in the separated region could not be studied. Some velocity measurements in the main flow were made with hot film probes.

The development of laser-Doppler velocimetry enabled the measurement of velocities when a flow is changing direction. The implementation of this optical technique greatly enhanced our efforts in this area. The more recent availability of direct numerical solutions of the Navier-Stokes equations (DNS) provides even greater opportunities. We developed a DNS code to describe turbulent flow over a wavy wall and demonstrated its usefulness. Clearly, this is the direction to go if we are to provide a complete understanding of this flow problem.

Studies of flow over solid waves provide a well defined system to define the influence of roughened walls on fluid turbulence. A study with a long train of sinusoidal 5 mm waves support the notion that the outer flow has a universal behavior.

(b) Flow regime map

Papers (75) and (84) presented the most comprehensive study of the effect of wave amplitude and flow rate at the time of their publication. They give the first mapping of flow regimes for large amplitude waves. The studies were carried out in a 5.08 x 60.96 cm enclosed rectangular channel. A train of ten waves was located on the bottom wall at a place where the flow is fully-developed. Wave trains with ratios of height to wavelength ($2a/\lambda$) of 0.0125, 0.03125, 0.05, 0.125 and 0.200 and a length of 5.08 cm were
used. This $\lambda$ was chosen because it is the largest for which disturbances introduced by the waves do not interact with top flat wall. The wave trains were thus characterized by the height to wavelength ratio, a wave number, made dimensionless with the friction velocity and the kinematic viscosity, $\alpha = 2\pi v/\lambda \nu^*$. The dimensionless wave number was varied by changing the flow rate.

The wall shear stress, in the forward moving direction, was measured by using electrochemical methods. The location at which the mean flow reverses direction was determined by observing dye injected at the surface and by using the sandwich electrode, developed in (44). (See Fig. 31) A Fourier analysis of the stress variation provides a quantitative measure of nonlinearities.

For small enough amplitudes, separation does not occur. However, nonlinearities can exist, as indicted by the appearance of second harmonics in the variation of the wall shear stress. A criterion that $av^*/\nu$ must be less than 27 in order to have a linear behavior was established. Reference (93) interprets this as a requirement that the wave amplitude needs to be small compared to the thickness of the viscous boundary layer that forms close to the surface of the wave.

As the amplitude increases, induced flow variations become so large that a reversed flow exists close to the wave. Surprisingly, the condition for separation to occur can be closely approximated by using linear theory (75). On the basis of these observations a flow regime map was developed in (84).

Pressure profiles over the wavy surface are described by a single harmonic, even when the shear stress profiles show a highly nonlinear behavior or when a very small mean reversed flow exists close to the surface. However, as the size of the separated region increases the pressure profile becomes more complicated. It can eventually show two maxima, located close to where reversed flow starts and ends.

These results suggested that a zonal approach could be used to analyze the flow that exists when a large mean separated region exists (84). Zone 1 would be the outer flow. This could be calculated by using linear inviscid theory to describe the flow over a composite surface which consists of the wave and the outer boundary of the mean separated region. Zone 2 would be a boundary layer which exists close to the wave surface between the locations where the mean flow attaches to and separates from the surface. Zone 3 would be the separated region. Zone 4 would be the shear layer formed when the boundary layer separates.
(c) One-component LDV measurements

Clearly, mean velocity measurements were needed to obtain a better picture. Our first efforts produced measurements of the mean and fluctuating velocity components in the flow direction. Laser-Doppler velocimetry depends on the principle that the frequency of light scattered from small particles is shifted in frequency because of the velocity of the particles. In collaboration with Prof. Ron Adrian, a LDV forward scattering system was designed. The beam from a He-Ne laser was split in two. An optical system combines these two beams inside the channel to form an illuminated volume with dimensions of 0.035, 0.035, and 0.38 mm in the x-, y-, and z-directions. The water was seeded with 0.5 micron latex spheres in an amount such that roughly one particle was present in the measuring volume at any given time. Light scattered by particles in the measuring volume was collected on a photomultiplier tube on the opposite side of the channel. The transmitting and receiving optics were attached to an aluminum beam located under the channel. A special traversing mechanism was designed to move the support beam precisely in two directions. Reliable measurements could be made as close to the surface as 0.13 mm. The Doppler signals were frequency-shifted by 200 kHz to permit measurements of negative velocities.

Initial studies, reported in (101,109) and in the thesis by Buckles (Buckles, John Jeffrey, "Turbulent separated flow over wavy surfaces", PhD thesis, University of Illinois, Urbana, 1983) were made with trains of sinusoidal waves that had wavelengths of 50.8 mm and amplitudes of 2.54 mm \((2a/\lambda=0.125)\), 5.08 mm \((2a/\lambda=0.2)\). Comprehensive results on the separated regions were obtained.

The results for \(2a/\lambda=0.2\), presented in (101), will be discussed first. Measurements of the time-mean velocities showed that the flow separated at \(x/\lambda_c=0.14\) and reattached at \(x/\lambda_c=0.69\), where \(x=0\) corresponds to a crest. Streamlines were constructed from the measurements of mean velocity. Thus an average recirculation zone could be defined. It had a maximum thickness of 6 mm (to be compared with the wave height of 10.16 mm) at \(x/\lambda_c=0.4\). The most striking aspect of this study is the demonstration that at no time did the instantaneous flow pattern show a recirculation zone such as suggested by the mean velocities. Thus, the intermittency, \(I\) (the fraction of the time that the velocity has plus values) had a value of 0.50 in the center of the recirculation zone and values between 0.1 and 0.5 at the outer boundary of the separated region. The locations where \(I=0.1\) lie above the outer boundary of the separation zone. The intermittency is quite high at the separation and reattachment locations. Fluid motions in the recirculation
region show large excursions in time and velocities as great as the average velocity.

The reattachment location has as its signature a maximum (of large magnitude) of the root-mean square of the pressure fluctuations. At reattachment, new boundary layers form along the wall in both the upstream and downstream direction. The boundary layer in the upstream direction is not well defined and could progress only a short distance. The boundary layer in the downstream direction is quite thin and turbulent, as evidenced by the existence of a maximum of the turbulent intensity within it. This boundary layer progresses to the separation location on the next wave, where it moves away from the surface as a free shear layer. This shear layer spreads quite rapidly downstream of separation owing to the high intensity of the turbulence and the divergence of the mean streamlines. A maximum in the intensity of the turbulent velocity fluctuations occurs in the free shear layer, approximately at an inflection point of the mean velocity profile. The location of the shear layer can, in fact, be defined more precisely through this maximum than it can through the average velocity profile, making it possible to map the trajectory of each shear layer.

In this way, we could detect a layering of the flow consisting of a primary shear layer and other shear layers which were formed at upstream crests. The primary shear layer is different from what is found for a free shear layer in that its rate of spread is greater and it interacts with a wave surface.

Visualization studies of separated flows, described in (84), show the separated shear layer rolling up into vortices which fill the entire trough. In contrast, if this region behaved as a free shear layer, one would expect more isolated eddy structures with a passive fluid in the reverse-flow zone separating the shear layer from the wall. These results suggest that turbulence in the separated region could be interpreted as resulting from the interaction of the shear layer with the wall. Photographs in (84) show columnar motions carrying dye between the wave surface and the outer flow (reminiscent of the "superbursts" described in Section 6(e).

Measurements of the mean velocity for 2a/\(\lambda\)=0.125, reported in (101,109), were found to be similar to those obtained for 2a/\(\lambda\)=0.20 at the same flow rate. Separation and reattachment occurred at x/\(\lambda\)=0.16 and x/\(\lambda\) =0.62 for 2a/\(\lambda\) =0.124. These are close to the values, x/\(\lambda\)=0.14 and x/\(\lambda\) =0.69 found for 2a/\(\lambda\)=0.20. However, the thickness of the time-averaged separated zone is smaller for the smaller height.

Measurements of the pressure variation along the surfaces provided an interesting comparison of the flow behavior for 2a/\(\lambda\)=0.125, 0.20. Both
show a peak slightly downstream of reattachment. However the peak is higher for the smaller amplitude wave! A region of constant pressure at the wall is displayed in the central part of the recirculating zone for $2a/\lambda=0.2$ is not observed for $2a/\lambda=0.125$. As shown in (75), a decrease of $2a/\lambda$ to 0.05 produces a very small region of reverse flow at the wave surface and a sinusoidal variation of the pressure. It is clear that the understanding of pressure profiles depends on our ability to predict the trajectory of the shear layer formed shortly after separation, in that the pressure field can be calculated by applying inviscid flow theory to a wave which has a shape defined by a composite of the wave surface and the separated region. This idea is pursued in a thesis by Buckles (Buckles, John Jeffrey, "Interpretation of pressure measurements for separated flow over a wavy surface", MS thesis, University of Illinois, Urbana, 1979).

Further studies with one-component LDV are reported in (132, 138) and in theses by Kuzan (Kuzan, John David, "Velocity measurements for turbulent separated and near separated flow over solid waves", PhD thesis, University of Illinois, Urbana, 1986; Kuzan, John David, "Separated flow over a large amplitude wavy surface", MS thesis, University of Illinois, Urbana, 1983) and by Frederick (Frederick, Kenneth Arthur, "Velocity measurements for turbulent nonseparated flow over solid waves, PhD thesis, University of Illinois, Urbana, 1986"). These cover a wide range of conditions and agree with the flow regime map developed in (82). Studies in which Reynolds number (or dimensionless wave number) was kept constant and the wave height was varied are discussed above.

The influence of Reynolds number (or dimensionless $\alpha$) for wave trains with a constant value of $2a/\lambda=0.20$ was investigated in studies summarized in (138). For $Re=4,080$ ($\alpha=0.0118$), a very large recirculation region was observed. Separation and reattachment occur at $x/\lambda=0.12$ and $x/\lambda=0.70$. An increase of the Reynolds number to 10,000 produced a thinner recirculation zone with separation and reattachment occurring at $x/\lambda=0.12$ and $x/\lambda=0.70$. A dramatic decrease in the thickness of the separated region is observed with an increase in the Reynolds number to 30,000. Separation and reattachment occurred at $x/\lambda=0.23$ and $x/\lambda=0.63$. Presumably, the recirculation zone would not be observed at a larger Reynolds number (a smaller dimensionless wave number). The observed decrease in the size of the separated region with decreasing wave height is consistent with intuition. This might not be the case for decreasing dimensionless wave number. A possible explanation is that the cyclical change in the viscous boundary layer can be too rapid at large Reynolds numbers for a separation condition to develop.
The LDV techniques described in (101) were used in (132) to study flow with \( 2a/\lambda = 0.0312 \) (Re=6400, dimensionless \( \alpha = 0.008 \), \( av^*/\nu = 12.3 \)) and with \( 2a/\lambda = 0.05 \) (Re = 38,800, dimensionless \( \alpha = 0.00165 \), \( av^*/\nu = 95.2 \)). These clearly show (as predicted by inviscid theory) that the distance from the wall at which disturbances introduced by the wavy boundary are felt decreases with decreasing wave number. Neither of these waves produced time-averaged reversed flows. Both showed a sinusoidally varying pressure at the wave surface. Wall shear stresses measured in the experiment with \( av^*/\nu = 12.3 \) showed a linear response. A nonlinear response was observed when \( av^*/\nu = 95.2 \). It is of particular interest to note that shear layers can be identified by maxima in the magnitude of the turbulent velocity fluctuations at inflection points in the mean velocity profile, even though a mean separated flow is not indicated.

(d) **Nonlinear models**

A consideration of an analytical approach to describe flow over large amplitude waves required a different framework than was used for waves that were small enough that the linearized momentum equations could be used. Therefore, a modification of the boundary-layer analysis of McLean was explored. Flow in a two-dimensional channel with a wavy bottom wall and a flat top wall was studied. An orthogonal transformation was used to map the physical domain to a rectangular grid.

A two-dimensional flow was considered. The mean velocities in the \( x \)- and \( y \)-directions, \( \overline{U} \) and \( \overline{V} \), are calculated with the Reynolds averaged momentum equations. These contain Reynolds stress terms, \( \overline{uv}, \overline{u^2}, \overline{v^2} \), where \( u \) and \( v \) are fluctuating velocity components in the \( x \)- and \( y \)-directions and the overbar indicates a time-average. The theoretical problem is to develop a representation of these turbulence terms. An isotropic eddy viscosity model was used, whereby the stress components are related linearly to the rate strain components. The proportionality constant is the eddy viscosity.

The equations of motion and the turbulent stress equations are formulated in a Cartesian coordinate system. McClean showed how these equations are written in the transformed coordinate system. These are solved numerically using no slip conditions at the top and bottom boundaries. The eddy viscosity relation of Cess for flow in a channel was used. It contains the vanDriest damping parameter, \( A \), which dictates the thickness of the viscous wall layer. The model of Loyd, Moffat and Kays (Section 9(b)) was
used to define $A$. The $y$-coordinate appearing in this equation is taken to be the distance along a vertical line to the wave surface in the bottom half of the channel and as the distance from the top wall in the top half of the channel. Calculations of the shear stress variation along the wavy wall for $2a/\lambda = 0.001$ agree with correlations developed for small amplitude waves.

Comparisons of the calculations with experiments under conditions that the flow was very close to separation ($2a/\lambda = 0.03125, \alpha^+ = 0.008$; $2a/\lambda = 0.05, \alpha^+ = 0.00165$) showed approximate agreement with measurements of the variation of the wall shear stress and the streamwise mean velocity profiles along the wave surface (132). Comparisons were also made for flows which showed the beginning of a separated region ($2a/\lambda = 0.125, \alpha^+ = 0.000135; 2a/\lambda = 0.05, \alpha^+ = 0.000624$) (138). Again, rough agreement was realized. However, a closer examination of the results reveals flaws.

The good results, to a large extent, are a reflection of the fact that the calculations exactly represent inertia effects and the viscous contributions to the stresses. However, a closer examination shows flaws in the use of an eddy viscosity model. It leads to a poor prediction of nonlinear effects, as evidenced by the ratio of the second harmonic to the first harmonic, obtained from a Fourier analysis of the periodic variation of the velocity in the flow direction. Some concerns are the use of a friction velocity defined with the wall shear stress for flow over a flat surface and the prediction of fluctuations in the Reynolds shear stress which are too large at large distances from the surface. (In this region the turbulence might be frozen in that the Reynolds stresses respond sluggishly to wave-induced variations of the flow field.)

Data for a wave train with a large separated region are given in the PhD thesis of Kuzan, cited above. The conditions were $2a/\lambda = 0.20$ and $\alpha^+ = 0.0118$. The calculated pattern of streamlines is given in figure 19c of Paper (138). The measured streamlines are given in the Kuzan thesis. It is not surprising that calculations are poorer than was realized for non-separated flows. However, what is surprising is that the nonlinear analysis captures the essential features of the flow in a semi-quantitative way. For example, the calculated separation and reattachment locations agree closely with measurements. However, the calculated thickness of the separated region is much larger than the measured thickness. Clearly, the eddy viscosity model is a poor representation of what is happening.

An extensive visual study, involving judicious use of dye injection techniques was carried out by Kuzan. These provided insights that would be useful in modeling. The boundary layer separated from a location near the
crest to form a shear layer which has a number of vortical structures. Some of them flowed over the downstream crest. Others interacted with the wall. These were believed to provide the turbulent mixing in an otherwise "stagnant" region. Kuzan carried these ideas further by using a vortex dynamics model to represent the flow field.

(e) New turbulence models; Direct numerical simulations

It is clear that the simplified eddy viscosity model does a reasonable job in calculating flow over large amplitude waves. However, improvements are needed in specifying the Reynolds shear stress. More recent results support the notion that a modification of the zonal model, suggested in (84), would be a useful approach. Five zones are identified:

1. A forward moving boundary layer close to the wall starts at a location ahead of the crest and extends to separation. Classical methods appear to work.
2. A turbulent shear layer forms in the separating boundary layer, that dictates the level of the Reynolds shear stress. The modeling of this shear layer is of primary importance.
3. Turbulence in the separated region is probably governed by the intermittent interaction of the shear layer with the wall. One or two bursts travel from the wave trough to the outer flow. The origin of these bursts is not understood.
4. Görtler vortices form in the region above the crests.
5. The outer flow is best described with a frozen turbulence model. That is, the wave-induced variations of the flow are occurring too rapidly to create wave-induced variations of the Reynolds stress.

In order to obtain more direct knowledge about the Reynolds shear stresses and their relation to flow structures, two investigations were inaugurated. Laboratory experiments were carried out by using an LDV system to measure simultaneously both the streamwise and wall normal velocity components. It consisted of a 35 mW Spectra Physics He-Ne laser and TSI optics which use polarization rotation to achieve channel separation. The three beam, two component system was operated in the forward scatter mode. Flow reversals were detected by the use of a Bragg (acousto-optic) cell with electronic downmixing. The system studied was water flow over a train of sinusoidal waves of length 5.08 cm and height to wavelength ratio of 0.1. The flow rate was chosen so as to provide a separated region. That is, the Reynolds number, based on the half height of the channel and the bulk velocity, was 3380. The results presented in (222) greatly improve the
physical picture developed from one component measurements in (101). In particular, the locations of regions of turbulence production are clearly defined. Turbulence near the boundary is demonstrated to be fundamentally different from what is found for flow over a flat plate in that production is not associated with the flow oriented vortices described in (178) and other places. However, the turbulence quantities in the outer flow are the same as for a flat plate, if scaled with an appropriate friction velocity. This suggests that turbulence in the outer flow could have a universal character independent of how it is produced at the wall!

The wave-induced variations of the mean streamwise and normal velocities complicates the scaling in that they contribute to a transfer of momentum. Thus, at a given distance from the wave surface the net transfer of momentum has contributions from the time-averaged velocity components, Reynolds shear stresses and viscous stresses. It is found that a proper scaling is obtained by using a friction velocity defined with the Reynolds shear stresses.

Measured contours of the Reynolds shear stress and of the production of turbulence provide interesting insights into the need to model the shear layer and the boundary layer. The curve representing maxima in the Reynolds shear stress at different locations along the wave surface trace the passage of the shear layer. The contours of the production of turbulent kinetic energy also trace the front part of the shear layer. However, the largest contributions are found in the boundary layer where the mean velocity gradient is quite large.

The Reynolds number characterizing the experiments in (222) was small enough that a direct numerical simulation was feasible. This involved the application of spectral element methods of Patera and Karniadakis to a channel with a smooth top wall and a wavy bottom wall (205, 239). This approach expanded the scope of the inquiry in that one can relate Reynolds stress to flow structure. The DNS calculations are consistent with laboratory measurements described in (194).

The streamlines calculated from mean velocities show separation at $x / \lambda = 0.15$, reattachment at $x / \lambda = 0.62$ and a small separated region. The ridge of the contours of Reynolds shear stress suggests a shear layer where there is a large production of turbulence. (See figure 32) After reattachment, a boundary layer forms. This is also a locus of large turbulence production. Figure 3 of Paper (239) shows contours of fluctuating streamwise velocity in an x-z plane just above the wave crest. No evidence of a streaky structure is offered. Figure 4 shows contours of regions in the x-y plane where large contributions to the Reynolds shear stress are being made intermittently.
These are large scale events which originate close to the wave surface and extend large distances into the flow field.

Figure 6 in (239) shows vortical structures (which have been identified by a scheme developed by Chong). These form on the upstream side of the wave and disappear as they progress over the crest. (See Fig. 33). They, therefore, appear to be the result of a centrifugal instability.

Figure 7 in (205) presents the velocity vectors in the separation region at different spanwise locations. The flow showed a haphazard motion and no hint of a separation bubble. As had already been observed in dye studies, large bursts extending from the surface to about \( y/h = 0.3 \) (where \( h \) is the half height of the channel) were observed. Vector plots at different spanwise positions show that they are uncorrelated over distances as small as \( 0.3\lambda \).

The use of DNS and information about this flow which has been gained through the series of experiments described above offer the tools needed to develop a theory for flow over wavy boundaries.

(f) *Effect of roughened walls on turbulence-Universality of the outer flow*

Studies with a wavy wall in a channel with a half height of \( H \) offered a good opportunity to obtain an understanding of the influence of roughened walls on turbulence. This involved the use of waves with small enough dimensionless amplitudes, \( a/H \), that wave-induced mean flows are making a direct contribution to the momentum transfer only over a very small fraction of the channel section, close to the wavy boundary. For a fully-developed flow with a roughened wall, the momentum flux at a plane which is a short distance from wavy surface can, thus, be represented by the Reynolds shear stress, alone.

These types of considerations prompted a study in a 5 cm high channel in which the bottom wall had a long train of sinusoidal waves with a wavelength of 5 mm (one-tenth as large as considered in investigations described above) and a height of 0.5 mm. Thus \( 2a/\lambda \) was of the same magnitude as used in the studies with larger wavelength waves. The dimensionless wave number was varied by changing the flow rate. A straight line extrapolation of measured Reynolds stresses to the wall produced the characteristic wall stress used to define the friction velocity.

Measurements of the pressure drop over a length of channel and of the mean velocity suggest that that the wavy wall may be characterized as having an equivalent sand roughness, \( k_s' \), equal to 3.44 \( a \), where \( a \) is the wave amplitude. This is to be compared with the \( k_s = 3.95 \ a \), characterizing
the wavy wall, considered in the DNS described in the preceding section. Studies were made at Reynolds numbers of 3200, 11,000 and 46,000. Results for the largest flow rate were considered in (239). These are characterized by a dimensionless wave amplitude of 30.2, a dimensionless wavelength of 605 and a dimensionless equivalent sand roughness of 104, where the measured friction velocity and the viscosity are used as the scaling factors. Thus, the surface may be considered as completely rough, where the measured friction velocity and the kinematic viscosity are used as the scaling factors. The goal in examining these data was to test the suggestion that turbulence in the outer flow is universal if the friction velocity is used to scale turbulent velocities. The expectation was that turbulence for flow over a roughened surface would be larger; however, the increase would be accompanied by an increased friction velocity.

This was substantiated by the measurements in that scaled root-mean-squares of the streamwise and normal velocity fluctuations were found to be the same as found for a smooth wall. Similar agreement was realized for measurements of spectra. The only disagreement was with the quadrant analysis of the Reynolds shear stress, which showed values of the ratio of the fourth and second quadrants that are about 25 per cent larger than found for a smooth wall.

Particle image velocimetry (PIV) was used in wavy wall studies to measure turbulent vector components in a plane perpendicular to the wall and parallel to the direction of flow (241). These are compared with PIV measurements made over a smooth wall (223). No major differences were evident. In both cases, large quadrant 2 Reynolds shear stresses are embedded in large scale motions that can extend over a large fraction of the channel cross section. Calculated contours of two-point correlations are similar to what is found for flow over a smooth wall, which showed an orientation of 6-8 degrees to the wall for the streamwise correlation. A value of 9 degrees was calculated from PIV measurements with a wavy wall.

Thus, measurements with a completely rough wall support the notion that turbulence in the outer flow shows a universal behavior. The small differences reflect a memory of how turbulence was created at the wall.

Studies at two smaller Reynolds numbers for which the surface may be considered as having an intermediate roughness and as being hydraulically smooth are described in (241h). Differences in the turbulence from what is realized for a smooth wall were observed. These could not be explained--suggesting that their characterization from pressure drop measurements is simplistic.
Fig. 28  Ripples observed on a 20 cP liquid at large gas velocities and an arbitrarily small liquid flow.

Fig. 29  Variation of the mass transfer rate over a wavy surface, measured with electrochemical methods, at $Re_{\lambda} = \lambda U_f / \nu = 20,900$, Schmidt number = 729.

Fig. 30  Variation of the wall shear stress, measured with electrochemical probes, along a wavy wall located on the bottom of a rectangular channel.

Fig. 31  Dye study for $2a/\lambda = 0.125$. A separated region extends from $x/\lambda = 0.1$ to $x/\lambda = 0.6$. 
Fig. 32 Velocity vectors calculated by direct numerical simulation for planes at $z/\lambda = 0$ and $z/\lambda = 0.5$. Lower curve is the outer edge of the separated region, indicated by the time-averaged flow. Upper curve is the location of the shear layer, indicated by the time-averaged flow.

Fig. 33 Vortical structures obtained with the vortex identification scheme of Chong, Perry, and Cantwell are believed to evolve from a Görtler instability.
11. Behavior of particles in a turbulent field

(a) Prologue

Section 2 describes research on the behavior of fluid particles, molecular species or thermal markers in a turbulent field. The present section summarizes results for particles which have a different density from the turbulent fluid in which they are entrained, so that they do not follow fluid velocity fluctuations exactly. Simplifying assumptions are made that the particles are solid and spherical, that the drag coefficient describing the force of the fluid on a particle is the same as would be experienced in a steady flow and that the concentration is very small. The motivation is the understanding of annular gas-liquid flows for which part of the liquid flows along the wall as a film and part as drops entrained in the gas flow.

The foci are the prediction of how particles distribute in a turbulent flow and of the rate of deposition on a boundary. Eulerian doiffusion models of the type used in Section 2 were explored as a framework for describing particle distributions. It was found that the use of a Lagrangian approach, whereby the concentration field is described as resulting from a distribution of point sources, is not only more versatile but also more correct. In this framework the description of the behavior of a point source becomes the fundamental issue. The development of a stochastic method to represent fluid turbulence seen by the particles enabled the study of point source dispersion over a wide range of conditions.

The use of a direct numerical simulation allowed a study of the effect of particle collisions and feedback. A surprising result is that the presence of particles can alter the fluid turbulence at very small volume fractions. This, in turn, results in a decrease in the rate of deposition.

A paper by Reeks was a pioneering study on turbulent motion of particles in a homogeneous, isotropic field. Work aimed at providing improvements on this analysis is also described.

(b) Mechanisms for particle deposition

The most important parameter characterizing the behavior of a particle in a turbulent fluid is the relaxation time, \( \tau_p \). A particle injected into a stagnant fluid with velocity \( v \) will stop after it travels a distance \( S \). The time constant, \( \tau_p \), is defined as \( S/v \). For situations in which Stokes law describes the fluid drag, \( \beta = 1/\tau_p = 18\mu_f / d_p \rho_p \). The rate of deposition per unit
area is usually expressed by the equation \( R_D = k_D C_B \), where \( C_B \) is the bulk concentration and \( k_D \) is the deposition coefficient, with units of velocity. A review of the literature prior to 1977 (74) produced the following results:

For \( \tau_p^* = \tau_p v^* / \nu < \text{ca} \ 0.15 \), particles closely follow the turbulent fluid motion. Under these circumstances deposition occurs by Brownian motion and \( k_D \) is defined in the same way as for fluid particles. (See Section 2c.) When \( \tau_p^* > \text{ca} \ 0.15 \), particles do not follow fluid turbulence over the whole field; they impinge on the wall by an inertial mechanism, called a "free-flight".

For \( 0.15 < \tau_p^* < 20 \), particles disengage from the turbulence in the viscous wall region where the magnitudes of the turbulent velocities are decreasing rapidly as the distance from the wall decreases. Thus, the average position from which a particle starts a free-flight to the wall increases as \( \tau_p^* \) increases. Since the turbulence increases rapidly with distance from the wall, the rate of deposition increases with increasing \( \tau_p^* \). Paper (74) shows that available data indicate that \( k_D / \nu^* = 3.25 \times 10^{-4} \tau_p^{*2} \). Thus, for \( 0.15 < \tau_p^* < 20 \), there is a remarkably strong increase in \( k_D \) with air velocity and with particle diameter; that is, \( k_D \) varies as \( \nu^* \) and as \( d_p^4 \).

For \( \tau_p^* > \text{ca} \ 20 \), particles start a free-flight to the wall from a region outside the viscous wall layer where fluid turbulence is not varying rapidly with distance from the wall. Thus, \( k_D / \nu^* \) is not strongly affected by \( \tau_p^* \). For extremely large \( \tau_p^* \), particles move in unidirectional paths, so that deposition is strongly dependent on the velocity with which they entered the field (254).

(c) Deposition of aerosols

Aerosols are very small particles (of the order of 10 microns or less) which tend to follow the fluid turbulence. They are usually characterized by \( \tau_p^* < 20 \). For \( \tau_p^* < 0.15 \), where Brownian motion is controlling deposition, \( k_D^* \) varies with \( \text{Sc} = \nu_f / D \). Since the diffusivity, \( D \), for Brownian particles is quite small, the deposition process is characterized by very large Schmidt numbers. The usual practice is to assume that \( k_D^* \propto \text{Sc}^{-2/3} \). However, our work on turbulent transfer, described in Section 2c and in (95, 119, 137), indicates that \( k_D^* \propto \text{Sc}^{-0.7} \) for \( \text{Sc} = 631 - 37,200 \) and as \( \text{Sc}^{-3/4} \) for \( \text{Sc} > 100,000 \) (characteristic of aerosols). These results need to be investigated more thoroughly by examining Brownian deposition for a wide range of particle sizes.
The theory that was commonly accepted for inertial deposition of aerosols for $0.15 < \tau^+ < 20$ pictures that particles are transported to a certain distance from the wall by turbulent diffusion and then thrown to the wall in a free flight. The velocity with which the particles are thrown to the wall is assumed to scale with the root-mean-square of the velocity fluctuations at the location at which the free-flight is initiated.

The development of direct numerical simulations for turbulent flow in a channel provided an opportunity to study the trajectories of solid and fluid particles and, therefore, to test concepts regarding inertial deposition. This prompted the studies described in (167, 185, 252). An examination of the local fluid velocity along the path of a solid particle reveals that the particle lags the fluid and eventually becomes "detached" to begin a free-flight trajectory. Several new or revised concepts arose from these studies:

(1) At a given distance from the wall a small fraction of the particles, with the highest velocities perpendicular to the wall, start a free-flight. Thus, the probability distribution of particle velocities, rather that the root-mean-square of the velocity fluctuations, is needed. This explains why early theories on inertial impaction predicted that particles strike the wall with much smaller velocities than is observed.

(2) Particles start their free flight to the wall from a number of locations.

(3) Particles that start a free flight can stop before they hit the wall. This can lead to their being trapped in a region close to the wall where fluid turbulence is small. This explains the observation, by a number researchers, of a large concentration of particles close to the wall where the fluid turbulence is small.

(4) Particles are defined as striking the wall when their centers are at a distance of one radius from the wall, where fluid turbulence is very small—but, not zero. Although turbulent deposition is a possible fate of trapped particles (as suggested by several researchers), the most likely scenario is that these particles are caught in a large outward flow to regions away from the wall, where they have another chance to experience a free flight to the wall.

The theoretical challenge is to relate rates of deposition to Eulerian properties of the turbulent velocity field. A method for doing this which uses insights outlined above is presented in (167). The first step in such an analysis is relating particle turbulence to fluid turbulence.
Point source dispersion; Particle turbulence

Our earliest work on particle dispersion involved point sources in homogeneous, isotropic turbulence. In (131) drops with diameters of 50-150 microns were injected downward at the center of a pipe at a velocity close to the mean fluid velocity. A piezoelectric crystal, placed in the supply line, created disturbances in the jets, emerging from orifices with diameters of 25.4, 50.8, 76.3 microns, that caused it to break into uniform droplets. Special attention had to be given to the design of the injector since it was desirable to insert drops at high velocities through tiny orifices without producing any directional bias. [See (131) and Lee, M.M., "Droplet dispersion in vertical turbulent pipe flow", MS thesis in Chemical Engineering, University of Illinois, Urbana, 1984.]

Droplet fluxes were determined from light scattered by drops moving through a laser beam directed along a diameter. At fixed distances from the injector, drop concentration profiles, measured in this way, showed a Gaussian distribution with a mean-square distribution of $\langle X^2 \rangle$. For small times, the change of $\langle X^2 \rangle$ with time gives the mean-square of the particle velocity fluctuations in the radial direction, $\langle v_r^2 \rangle = \langle X^2 \rangle / t^2$. For large times, a turbulent diffusivity can be calculated as $\varepsilon_p = \left( \frac{1}{2} \right) \left( \frac{d \langle X^2 \rangle}{dt} \right)$. Measured $\langle v_r^2 \rangle$ were shown to be less than the mean square of the fluid velocity fluctuations $\overline{u_r^2}$. The ratio $\left( \frac{\langle u_r^2 \rangle^{1/2}}{\overline{u_r^2}^{1/2}} \right)^{1/2}$ decreases with increasing particle diameter and with increasing gas velocity, as would be expected, because the particles cannot follow the fluid velocity fluctuations exactly.

A novel method for measuring directly particle turbulence is described in (131). The key ingredient is to view the field axially. (See Fig. 35) In this way, the $r$- and $\theta$-components of the velocity, that control mixing, are directly measured. The much larger streamwise velocity is effectively filtered. The system studied was downward flow in a 5.08 cm pipe. Drops were injected in the center of the pipe by using the system described in (131). The pipe cross-section is illuminated by flashes at three locations. A camera located at the bottom of the pipe views upward into the pipe and captures images in the illuminated cross-sections. Timing of the flashes was synchronized so that three images of the particles were obtained as they progressed down the pipe. Velocity magnitudes were calculated as the quotient of the distance between images and the time interval between
flashes. The direction of travel was determined by defocusing the image on the last light sheet.

The average value of the tangential component obtained from a large number of photographs was zero. The average value of the radial component gives a direct measurement of the average flux caused by diffusion. (To our knowledge, this is the first time that such a direct measurement was made.) A diffusivity can be calculated as \( \varepsilon_p(r) = \bar{V}_{pr}C / (-dC/dr) \). This is believed to be a more reliable result than obtained in by measuring \( d\langle X^2 \rangle/dt \) in the limit of large times, since the particles might not have been in the field for a long enough time to reach the limiting behavior. Papers 131, 139 show that \( v_{pr}^2 \) and \( v_{p\theta}^2 \) are roughly constant from the pipe center to \( r = 0.9R \). A semi-theoretical correlation is presented which relates these quantities to the ratio of the Lagrangian time scale of the fluid to the inertial time scale of the particle, \( \tau_L/\tau_p \). Values of \( \left( v_{pr}^2 \right)^{1/2} / \left( u_r^2 \right)^{1/2} \) as low as 0.3 were observed, where \( u_r \) is the r-component of the fluctuating fluid velocity. The ratio of the diffusivity of the fluid to \( \varepsilon_p \) was found to be close to unity, provided the ratio of the terminal velocity of the particles, \( v_T \), to \( \left( v_{pr}^2 \right)^{1/2} \) is less than ca 0.8. The data for large \( v_T/\left( v_{pr}^2 \right)^{1/2} \) show a decreasing \( \varepsilon_p \), which is believed to be due to Yudine's "crossing of trajectories effect".

(e) Crossing of trajectories, turbophoresis, trapping at the wall

Papers 163, 164 present an extension of the axial viewing technique which allows measurements of the mean radial flux of particles, the mean-square of the velocity fluctuations and the mean-square of the fluctuations of the acceleration. The pipe cross-section was illuminated with programmed flashes that have different colors at five axial locations. This allowed measurements of the accelerations as well as the velocities of particles. Water flowed downward in a 5.08 cm pipe. Slurries of glass (2.42 specific gravity) and stainless steel (8.0 specific gravity) spheres, with a diameter of 100 micrometers, were injected through a small centrally located tube at the same velocity as the flowing water.

The inertial time constants characterizing these studies suggest that particle velocity fluctuations were the same as fluid velocity fluctuations. This was confirmed by the measurements.
One of the experiments, Run 4 in (164), with stainless spheres was characterized by a large enough ratio of the free-fall velocity to the particle turbulence that the Yudine crossing of trajectories effect could be important. This was substantiated by a measurement of the particle turbulent diffusion coefficient, which was 0.55 times the fluid diffusion coefficient. The mean particle acceleration was the same for all runs. However, the mean-square of the fluctuations of the acceleration was found to be larger for Run 4. The multiple-image photographs demonstrate this difference, in that a substantial portion of the trajectories showed sudden changes in direction. These sudden changes indicate that high instantaneous accelerations occur when a particle moves from one region of highly correlated flow to another. Thus trajectory crossing is accompanied by a decrease in the Lagrangian time scale of the particles and no change in the root-mean square of the velocity fluctuations.

The measurements of mean accelerations for four of the runs agree with one another. All show a change from a negative to a positive particle acceleration at $y^+ \approx 40$ where the root-mean square of the radial velocity fluctuations is a maximum. From a consideration of the equation of particle motion, it is shown in (164) that the existence of a mean acceleration indicates the existence of a mean force on the particle which causes a mean drift velocity, $\bar{V}_r$, in the radial direction; that is, $\beta \bar{V}_r = \left(\frac{dV_r}{dt}\right) - \frac{v_r^2}{\rho} / r$, where $\beta$ is the reciprocal of the inertial time constant. Furthermore, one can show that $\left(\frac{dV_{pr}}{dt}\right) = \left(\frac{\partial v_{pr}^2}{\partial r}\right) + \frac{v_{pr}^2}{r}$, so that a change in sign of the mean acceleration is consistent with the notion of turbophoresis, which pictures particles as having a tendency to drift from regions of high turbulence to regions of low turbulence. Thus, (164) provides an interpretation of the notion of turbophoresis espoused by several researchers.

The measurements of the concentration field for stainless steel spheres, which have a large free-fall velocity compared to the root-mean square of the velocity fluctuations indicate a net transport of particles to the wall (164). This matter was investigated further in (163). An examination of photographs, at $z = 352 cm$, of the particles revealed that a large number are trapped at the wall. For example, 80 per cent of the stainless particles were trapped at the wall. This number starts to decrease at $Re = 50,000$. No trapped particles were observed at $Re = 73,000$. For glass particles, 40 per cent of the particles were trapped at $Re = 22,000$. These trapped particles move very slowly as necklaces at a distance of less than one particle diameter from the wall. They have an average circumferential spacing which is the same as the streaky
structures observed close to the wall in turbulent flows. The suggestion is made in (163) that this behavior is observed when the Saffman lift force toward the wall overcomes the ability of fluid turbulence to mix the particles. It is also suggested that the location of the particles is dictated by a balance between the Saffman lift force and a wall-induced force associated with the displacement of fluid as a particle moves parallel to the wall.

(f) Diffusion models for deposition of particles with large inertial time constants

As indicated in Section 11b, depositing particles in annular flows for which the dimensionless time constants, \( \tau^*_P \), are greater than ca 20 start their free-flight to the wall from locations outside the viscous wall layer. For such particles, the nonhomogeneities close to the wall are not important, so the turbulent field can be approximated as being homogeneous. A particle-fluid system, in which the particles are injected at the center of a pipe, was considered in (144).

A technique developed in (130) was used to measure deposition rates for downward flow in a 5.1 cm pipe. Dye was added to the injected water. When drops hit the wall, they formed streaks which evaporated and left a dye powder behind. After a fixed time the run was terminated and the nine part test section was disassembled. Washing the different sections revealed the number of drops which had deposited. In this way a plot of the fraction of the injected drops which deposit over a length of pipe was determined. The drops were introduced at the velocity of the gas. Their motion is influenced by the gas phase turbulence. Eventually, their behavior in planes perpendicular to the direction of mean flow is completely controlled by the gas phase turbulence.

In the region where deposition occurs, drops have been in the field a long time so that they have reached their asymptotic behavior. Thus, their dispersion was calculated from the diffusion equation with a constant diffusion coefficient. The solution requires the specification of a boundary condition at the wall. The handling of this problem could represent the most important contribution in (144). The collision of drops with the wall results in their coalescence with the wall. (See paper 130.) Thus the wall may be considered to be a perfect absorber.

For molecular diffusion this would dictate the assumption of zero concentration at the wall. This is not the case for droplet deposition. This can be seen by using a radiation boundary condition at the wall; that is, \( R_D = \)
\[-\varepsilon_p \left( \frac{d\overline{C_p}}{dr} \right)_{r=a} = \overline{V_D \overline{C_p}(a)} \], where \( R_D \) is mass transfer rate per unit area and \( V_D \) is the velocity with which the droplets are carried to the wall. If \( \varepsilon_p \) is represented as the product of a velocity and a length, \( V \) and \( L \), then \( \varepsilon_p \) \( \equiv \overline{VC_p}(a) \). It can be seen that if \( L \) is very small and (as is the case for molecular diffusion) and \( \left( \frac{d\overline{C_p}}{dr} \right)_w \) is finite, then \( \overline{C_p}(a) \) is zero. However, if \( L \) is of the order of the length scale characterizing \( \left( \frac{d\overline{C_p}}{dr} \right) \), \( \overline{C_p}(a) \) is a finite number. This is the case for particle deposition.

Thus, the boundary condition at the wall is \( R_D = -\varepsilon_p \left( \frac{\partial \overline{C_p}}{\partial r} \right)_w = A \left( \overline{\nu_{pr}} \right)^{1/2} \left( \overline{C_p} \right)_w f \), where \( V_D \) is assumed to be proportional to the root-mean square of the radial velocity fluctuations. For example, if \( v_r \) is described by a Gaussian distribution with zero mean, \( A = (2/\pi)^{1/2} \). Term \( f \) is the fraction of the particles at the wall which are moving toward the wall and \( \left( \overline{C_p} \right)_w \) is the average concentration of particles at the wall. Solutions of the diffusion equation with the above boundary condition produced results in agreement with measurements for particle sizes varying from 50 microns to 150 microns and Reynolds numbers varying from 36,000 to 52,000. In these calculations \( f \) was set equal to \( 1/2 \) and \( \overline{\nu_{pr}} \), \( \varepsilon_p \) were given values indicated earlier in this manuscript. If the mass flow of particles to the wall is defined by the rate equation \( R_D = k_D \left( \overline{C_{pb}} - \overline{C_{pw}} \right) \), it can be shown that the overall resistance is the sum of a resistance due to diffusion to the vicinity of the wall and a resistance associated with free-flight to the wall. For the point source experiments reported in (144), both resistances are important.

The success of the diffusion model encouraged its application to vertical annular flow, for which part of the liquid flows along the wall as a film with a highly disturbed interface and part as drops entrained in the gas (153). The entrainment is determined as a balance between the rate of removal of liquid from the wall layer and the rate of deposition of drops. Paper 153 considers the prediction of the rate of deposition and the droplet concentration profile. The droplet field is described to be the result of a number of ring shaped sources on the wall. The theoretical problem is to predict the behavior of one of these sources. The diffusion equation was used to do this. Droplets injected into the flow disperse in the flow and eventually deposit on the wall film (which is considered to be a perfect absorber). Eventually all of the droplets from a single source deposit. The droplets at any given cross-section originate from different wall sources and, therefore, have been in the field different lengths of time.
The diffusion equation is solved to give the behavior of a single ring source. The boundary condition is the same as used for a source located at the center of the pipe. However, the fraction of the drops close to the wall, which are moving toward the wall, \( f \), is allowed to be a function of time. For \( t \to 0 \), \( f = 0 \) since all of the particles are moving away from the ring source at the wall. For large diffusion times, \( f = 1/2 \). The diffusion coefficient also varies with time. It is a constant at large diffusion times and varies linearly with time at small \( t \).

The fully-developed concentration profile is calculated by adding the contributions of all of the wall sources. It is essentially flat and shows a slight maximum near the wall, in agreement with experimental measurements. The resistance to mass transfer is different from what is found for a centrally located source, in that diffusion to the neighborhood of the wall is fast enough that the deposition constant is controlled by the free-flight boundary condition. Thus, for annular flows, deposition is calculated with the boundary condition; that is, \( k_D = (1/2\pi)^{1/2} \left( \frac{v_{pr}^2}{v_{pr}^2} \right)^{2/3} \), where the root-mean square of the particle velocity fluctuations, \( \left( \frac{v_{pr}^2}{v_{pr}^2} \right)^{2/3} \), can be related to fluid turbulence by using results described earlier in this section.

A method commonly used to determine \( k_D \) in vertical annular flows is to measure the buildup of a film downstream of a section in which the liquid film is completely removed from the wall. The film reforms by depositing drops and removed again at various distances downstream of the first removal section. An interesting finding was that the deposition coefficient decreases sharply immediately downstream of the film removal location and approaches an asymptotic value far downstream. It was found that this effect can be attributed to a change in the concentration profile to a more diffuse shape.

One should point out several restrictions to the validity of the calculations outlined above: (1) The description of the concentration as consisting of contributions from a number of wall sources implies that the concentration is small enough that particle-particle interactions are not important; (2) The particles are large enough that free-flight starts outside the viscous wall layer, \( \tau_p > 20 \); (3) They are not so large that they move to the wall on unidirectional paths from the source.

For point sources at the center of a pipe, the particles rapidly become entrained in the turbulence. When the scale of the turbulent motions close to the source is comparable to the pipe radius, the droplets can move in unidirectional paths to the wall. (See, for example Paper 82.) Clearly, under
these circumstances the eddy diffusion coefficient does not assume the value characterizing long time diffusion. In the context of the diffusion analysis, drops originating from a wall source will move away from the wall with velocities characteristic of the particle turbulence, $\left(\frac{v_{pp}^2}{2}\right)^{1/2}$. Again, these could reach an opposite wall with unidirectional motions if the scale of the particle turbulence is comparable to the pipe diameter.

Papers (161, 171, 174, 176) extended the approach described above to horizontal annular and sediment flows where particle trajectories are affected by gravity. This was done by including the term $V(t-t')(\partial C/\partial y)$ in the diffusion equation where $V(t-t')$ is defined by a determinate equation that gives the trajectory of a particle caused by gravitational forces. Reasonable results were obtained, but the method is open to question.

(g) Stochastic model for dispersion of solid particles

Several shortcomings exist with calculations that used a diffusion equation: Only a limited range of variables could conveniently be investigated. There were uncertainties about how to expand the model so as to cover extraneous effects such as gravity and turbophoresis. These shortcomings motivated a series of papers which used a stochastic method to model fluid turbulence.

Our studies of the mixing of fluid particles or of molecular species in a turbulent field are outlined in Section 2. One of the tools that was used is stochastic modeling (Section 2c.) of the trajectories of fluid particles or thermal markers in a turbulent field. This approach has been used to describe the path of a solid particle in a turbulent field. The equation of motion of a particle contains a random forcing function dictated by the random velocity of the fluid with which it is in contact. In (237) this is specified by using a modified Langevin equation. The calculated behavior of solid particles in a channel flow was found to be the same as obtained in experiments with a DNS (243). This encouraged further exploration of this approach.

Paper (247) considered flow in a vertical channel. An idealized annular flow is defined whereby particles are injected at the wall and removed from the field when the center of the particle is at a distance from the wall of one radius. The calculations were done by considering the field to result from contributions of sources located on the two walls of the channel, which inject particles at $(V_{px}^{0+}, V_{py}^{0+}, V_{pz}^{0+}) = (15, 1, 0)$ at the bottom and at $(15, -1, 0)$ at the top, where the superscripts indicate that the velocities are
made dimensionless with the friction velocity. Only the fully-developed condition was considered. The outcome is the most extensive study of particle turbulence available. Only one Reynolds number was considered, $Re_e = 590$, for which Eulerian fluid turbulence quantities could be obtained from a DNS.

Two sets of calculations were done. In one of these, the dimensionless particle diameter was kept constant, $d^*_p = 0.368$, and the particle time constant, $\tau^*_p$, was varied from 0.977 to 20,500 by varying $\rho_p/\rho_f$, where $\tau_p$ is the average of $\tau$ over the cross section of the channel. In the second, $\rho_p/\rho_f$ was kept constant, at a value of 1000, and the particle time constant was varied by changing $d^*_p$. In this way all of the deposition regimes discussed previously were studied in a single numerical experiment. The calculations capture the region where deposition is controlled by free-flight to the wall from locations in the viscous wall layer, $ca3 < \tau^*_p < ca30$.

Deposition for the region $\tau^*_p = 30$ to $\tau^*_p = 1000$ roughly agrees with the relation $k_D = \left(\frac{1}{\sqrt{2\pi}}\left(V^2_{pw}\right)^{\frac{1}{2}}\right)$, suggested for the regime where deposition occurs by free flights from locations outside the viscous wall layer. Brownian motion affected deposition for $\tau^*_p < 3$. An estimate was made that Brownian motion starts to contribute to deposition somewhere between $\tau^*_p = 0.1$ and $\tau^*_p = 1$ ($d_p = 1-3\mu m$ for water drops in air under atmospheric conditions). The calculations also show a decrease in particle velocity fluctuations with increasing particle inertia and provide concentration fields that help in interpreting laboratory studies. The calculated mean particle acceleration provided an evaluation of the turbophoretic velocity.

At $\tau^*_p > ca1000$, drops deposit by a trajectory mechanism whereby they move in free-flight from one wall to another. At $\tau^*_p > 3000$ particles deposit at the opposite wall at the same velocity with which they were injected into the field. The importance of the trajectory mechanism for particle deposition is dependent on the size of the flow channel. All of the works cited above were done with $Re_e = 590$ so they could not completely capture this behavior. Paper (253) uses the stochastic analysis to examine deposition by unidirectional trajectories over a range of $Re_e$. The possible importance of this mechanism in annular flows has been cited by a number of researchers. This work is one of the more thorough studies of this phenomenon that has been carried out.
The trajectory regime in a vertical conduit was found to exist for \( \tau_{pb} V_0^0/2H \) greater than 1.2, where 2H is the height of the channel and \( \tau_{pb} V_0^0 \) is the stopping distance. For \( \tau_{pb} V_0^0/2H \) greater than ca. 10, the deposition coefficient is constant and the average velocity of the depositing particles equals the average velocity of the injected particles. A transition region exists for \( 1.2 < \tau_{pb} V_0^0/2H < ca. 10 \). In this region the mean velocity with which the particles deposit is less than \( V_0^0 \) because the particles are affected by fluid turbulence and because particles moving in a uniform trajectory are slowed by fluid resistance.

It is to be noted that the saltation regime, observed in a horizontal conduit, is similar to the trajectory regime in vertical flows, in that particles injected at the bottom boundary are not affected by fluid turbulence.

(h) *Horizontal flows of particle-fluid suspensions*

Horizontal gas-liquid flows exist in a number of applications. Yet, they are poorly understood because additional complications are introduced. Gravity can strongly affect deposition and the concentration distribution of particles. The effect of gravity on transport has been given more attention in sediment transport than in gas-liquid flows. Theoretical analyses of the behavior of this system have, for the most part, used Eulerian methods, which could be flawed. This prompted the analysis presented in (249,252). It differs from most previous studies in that it uses a Lagrangian analysis and a stochastic method to model fluid turbulence seen by the particles. The Langrangian method considers the particle field as resulting from the contributions of a large number of wall sources. The analysis differs from that for vertical flows in that it includes the force of gravity in the equation describing the trajectories of particles. The influence of gravity can be characterized by \( v^* T = v_T/v^* \) or by \( g^* = g\mu/v^*3 \rho \), where \( v_T \) is the terminal velocity. It can affect the behavior of the system, directly, by providing an additional mechanism for deposition and, indirectly, by altering the concentration distribution of particles.

An infinitely wide channel for which H is the half-height was considered. The friction Reynolds number was \( Re_T = 600 \). Particles were injected from the bottom with velocity \( V_0^+ = 15, V_{0v}^+ = 1, V_{0z}^+ = 0 \). Particles were removed from the field when they were a distance \( d_p/2 \) from a wall. Calculations were done for \( \tau_p^+ = 3 - 40 \). The deposition coefficients were characterized by defining three regimes: (1) For \( v_T^* < ca.0.01 \) particles are
distributed symmetrically, as observed for vertical flows, even though the ratio of the rates of introduction of particles from the top and bottom walls, $R_{AT}/R_{AB}$, may not equal unity. However, the rate of deposition is enhanced because gravity provides a mechanism by which particles trapped in the viscous sublayer at the bottom wall can deposit. (2) For $g^+ > ca.4 \times 10^{-2}$, a saltation region exists. For the conditions explored in (249,252), the saltating particle does not reach the top wall. (3) An intermediate region, which includes situations for which particles do not reach the top wall, can be defined for $v_x^+ > ca.0.01$ and $g^+ < ca.4 \times 10^{-2}$. Calculated velocities of depositing particles help in providing a physical understanding of the mechanism of deposition. For $g^+ < 10^{-2}$, particles reach the top wall, so annular flow can exist. This calculation is of particular importance in many practical operations where wetting of the top wall could be highly desirable.

Calculations of concentration profiles are given in (249). These show how gravity causes asymmetries and how these are connected with increases in deposition.

(i) Evaluation of the Eulerian approach

Fully-developed vertical and horizontal flows, treated in (247,249) provide a simple system to analyze in an Eulerian framework. For a vertical flow, $V_p C_p + v_p^2 c_p = 0$, where the first term is the mean transport in the $x_2$-direction due to the turbophoretic velocity and the second term is the average of the product of the fluctuating velocity and the fluctuating concentration. It represents the contribution of turbulent mixing. If a diffusion model is used, $\overline{v_p^2 c_p} = -\varepsilon_p \left( \partial \overline{C}/\partial x_2 \right)$. Thus, transport due the turbophoretic velocity is balanced by turbulent mixing. Close to the wall, $V_p$ is negative (toward the wall). However, in the outer flow $V_p$ is positive since the turbulence is decreasing with distance to the wall. Thus a negative $\varepsilon_p$ is needed in the outer flow to balance the equation. This is nonphysical.

For horizontal flows, $\partial (C_p V_p)/\partial x_2 + \partial (v_T C_p)/\partial x_2 + v_p^2 c_p = 0$, where $v_T$ is the terminal velocity of the particles. Again, unrealistic values of $\varepsilon_p$ are needed if $\overline{v_p^2 c_p}$ is represented by a diffusion equation.

Our interpretation is that $\overline{v_p^2 c_p}$ is affected by large scale motions which cannot be captured by using diffusivity concepts.
All of the work described in the preceding subsections makes the assumption that the suspension is dilute enough that particle-particle interactions are not important and that the fluid turbulence is not altered. Measurements of the deposition constant agree with calculations at small concentrations. However, with increasing concentration $k_p$ is found to decrease. This prompted the study in a direct numerical simulation described in (250).

Vertical flow in a channel with a half-height $H$ was considered. The Reynolds number, $Re_z$, defined with the half-height of the channel, the friction velocity without particles, and the kinematic viscosity of the fluid, was 150. Effects of gravity were neglected. Particles were injected from sources at both walls and removed from the field when they were one radius from the wall. Disperse flows with different volume fractions were realized by varying the rate of injection.

Since the particle and fluid velocities are not equal, the particles can exert a force on the fluid. The velocity field of the fluid was calculated by solving Navier-Stokes equations that include a distribution of point forces representing the fluid-particle interactions. Both elastic and inelastic collision models were included in the equation of motion of the particles.

The surprising aspect of these results is that the particles decrease the fluid Reynolds stresses at volume fractions as small as $10^{-4}$. Of particular interest, in Paper 250, is the presentation of complete velocity fields. Changes are observed similar to what is found with solutions of drag-reducing polymers as the drag on the wall decreases. Similar studies in another laboratory, which were done in a large eddy simulation (LES), did not show such a large effect. We believe that the reason for this is the inability of LES to capture small scale events. The agreement of our calculations with those of McLaughlin & Kontomaris is encouraging.

This result could have more general interest, in particular, regarding the observation of a damping of turbulence in solutions of high molecular weight polymers. In this situation, the unraveling of polymer molecules is associated with the existence of mean stresses in the fluid. (See Section 7.) An interpretation is that the consequence of the forces of the particles on the fluid is to require a lower pressure gradient to maintain the fluid flow at a constant rate. This is associated with a decrease in the Reynolds shear stress and, therefore, the production of turbulence.
In (250), elastic collisions are found to increase strongly the particle turbulence. Inelastic collisions cause small decreases in particle turbulence.

There is a kinship of this study to work in my PhD thesis (5), in which I used measurements of diffusion from a point source to examine how the presence of solids in a fluid bed increases fluid turbulence. In (13) this technique was used to study turbulence in the downward flow of water and solid particles in a three inch vertical pipe. Glass spheres ($\rho = 2.20$) with average diameters of 380 & 100 microns and copper spheres ($\rho = 8.92$) with an average diameter of 200 microns were used. The particle fractions were larger than considered in the calculations described in (250), 0.005 to 0.023. The ratio of the slip velocity to the centerline velocity varied between 0.021 and 0.29. The fluid turbulence increased if the slip velocity and volume fraction were large enough.

One of the surprising results in (13) is that it is possible to have a situation for which turbulence is not greatly affected at volume fractions as large as 0.015. Another is a sudden increase of turbulence with increasing concentration in some of the experiments. Visual observations showed a transition in the flow pattern of the solids from one in which the particles are acting independently of one another to one in which they are undergoing large scale coherent motions. It resembles a transition from laminar to turbulent flow. An explanation is not given in (13), but particle-particle interactions and particle slip must be playing important roles.

Thus, this study produced different results from what is found for gas-liquid annular flows. The chief difference in the conditions prevailing in the two studies is that the density of the fluid (water) used in (13) is much larger than the density of the fluid used in (250). Clearly, the scaling of the feedback mechanism needs more attention.

One possible explanation of differences seen in (13) and (250) is that the Reynolds stresses characterizing fluid turbulence are much larger in (13)--but this is speculative.

(k) Studies in homogeneous isotropic turbulence

One of the important contributions to our understanding of particle turbulence is given by Reeks. The basic problem in understanding the influence of fluid turbulence on particle turbulence is that one needs to know the time-variation of the fluid velocity seen by a particle as it moves around the field and not as seen by a fixed observer (the case for turbulence measurements in the laboratory). This was considered as unsolvable. However, Reeks provided a solution for a homogeneous, isotropic field.
An iterative procedure was used, whereby the first approximation to the time variation of the fluid turbulence is what would be seen by a fixed observer. The particle turbulence was found to decrease relative to the fluid turbulence as the inertial time constant of the particles increased; yet, the diffusivity of the particles remains relatively constant. Thus, the scale of the particle turbulence increases.

We explored further an iterative analysis for a homogeneous, isotropic field in two papers. Reeks limited his analysis of particle motion by using the steady-state form of Stokes equation. In (157) the influence of time-varying motion on the drag was considered by including the effects of the Basset force. It was found that the Basset force has virtually no influence on the structure of the fluid velocity fluctuations seen by the particles or on particle diffusivities. The crossing of trajectories associated with gravitational forces, however, tends to enhance the effect of the Basset force on particle turbulence.

In Paper 204, the analysis of Reeks for homogeneous, isotropic turbulence is extended so as to include a nonlinear drag law. The principal issues were the evaluation of the inertial time constant and the mean slip. Unlike what is found for Stokesian drag, the time constants are functions of the slip velocity and are anisotropic.
12. Gas-liquid annular flow in a vertical pipe

(a) Prologue

Gas-liquid flows are ubiquitous in industrial applications and in the environment. Yet, they are poorly understood. The central issue is the prediction of how the phases distribute. This, in turn, requires a consideration of local interfacial interactions. That is, large scale motions are controlled by small scale phenomena. Considerable progress has been made in a number of laboratories over the past forty years, so that multiphase flow is emerging as an important discipline.

This Section describes our contributions to understanding vertical annular flow. In this regime, part of the liquid flows as a film along the wall and part, as drops entrained in the gas. The interfacial stress varies with the flow rate of the film. Thus, the pressure gradient depends on the fraction of the liquid flow, E, entrained as drops. An attractive approach is to view E as resulting from a balance between the rate of atomization of the liquid film, $R_A$, and the rate of deposition of drops, $R_D$. Thus, measurements of $R_A$ and $R_D$ are a priority. A knowledge of drop size is of major importance since it is needed to predict drop turbulence and the influence of gravity on the motion of drops.

Annular flows are encountered in many applications, for example, in nuclear reactors and in petroleum pipelines. Their importance is manifested by the publication of "Annular Flow" (authored by G.F. Hewitt and N.S. Hall Taylor) and the sponsorship of a series symposia on the topic. The first of these was an International Symposium on Two Phase Annular and Dispersed Flow, held in Pisa, Italy, June 24-29, 1984, which I helped to organize (108,118).

(b) Film height and interfacial stress

The mass flow is designated as $W_L$; then $W_L - W_{LF} = EW_L$, where E is the entrainment and $W_{LF}$ is the mass flow in the liquid film. The pressure drop is directly related to the interfacial drag, $\tau_i$. For single phase flows, $\tau_i = f_s (1/2) \rho U_G^2$. The friction factor for an annular flow, $f_i$, can be much larger than $f_s$ because the boundary presented by the liquid film is roughened by the presence of waves.
Early studies focused on predicting $f_i$ and the film thickness for a given $W_{LF}$ and gas flow. Two ideas were found to be useful: One is the assumption that the profile of the mean velocity in the film is the same as would be experienced for turbulent single phase flow. The other is that $\tau_i/\tau_s = f_i/f_s = f(m/d_i)$. These relations presume that the interface may be considered as completely rough because of the presence of waves. The heights of these waves are pictured to scale with the height of the film, so that $m/d_i$ is proportional to the ratio of an equivalent sand roughness to the pipe diameter.

Paper 70, published in 1976, summarizes experimental studies and uses the above assumptions to develop direct equations for $m/d_i$ and $f_i/f_s$. The implementation of the first assumption involves the use of eddy viscosity relations for single phase flow and the integration of the momentum balance for a fully-developed flow. The stress level in the film is dictated by the interfacial stress, $\tau_i$. However there can be a significant variation of $\tau$ because of gravitational forces in the film. Paper 70 derived the relation $m^+ = \gamma(Re_{LF})$ where the friction velocity is defined using a characteristic stress, $\tau_C = (2/3)\tau_w + (1/3)\tau_i$. The film Reynolds number is defined as $Re_{LF} = 4W_{LF}/\mu_L P$, where $P$ is the pipe perimeter and $\mu_L$ is the liquid viscosity. The term $\gamma(Re_{LF}) = \left[ (0.705 Re_{LF}^{1/2})^{2.5} + (0.0379 Re_{LF}^{0.9})^{2.5} \right]^{0.40}$ is a composite of solutions for steady laminar flow and a fully turbulent film. Relations $f_i/f_s = 1 + 1400F$ and $m/d_i = 6.59F/(f_i/f_s)^{1/2}$ were derived, where $F = \left[ \gamma(Re_{LF})/Re_{G}^{0.9} \right] \left[ \mu_L/\rho_L \right] \left[ \mu_G/\rho_G \right]^{1/2}$ is a modified Martinelli parameter. These relations proved to be useful. However they only roughly predicted the effects of pipe diameter and did not consider the influence of changes in the wave pattern.

At low gas velocities the film is covered with asymmetric capillary ripples. These exist for arbitrarily small value of the film Reynolds number. They are described in Section 9(e). For large gas velocities there is a transition to a highly disturbed pattern, at a critical $Re_{LF}$, which is characterized by the intermittent appearance of flow surges, called roll waves in (15). (Hewitt called them "disturbance waves"). (See Fig. 37 and 38)

Paper (106) improves the correlations in paper (70) by presenting new measurements which expanded the range of variables and by examining the difference in behavior that occurs with a change of wave pattern. Conductance techniques were developed to measure film thicknesses, which
ranged from 30-200 microns. By using glycerine-water solutions, low \( \text{Re}_{LF} \) could be explored.

The principal difference from the analysis presented in (70) was that the characteristic length in the gas phase should be taken to be \( \nu_G (\tau_i / \rho_G)^{1/2} \), rather than \( d_i \). Thus \( \tau_i / \tau_s \) is related to \( m_G^+ = m_G (\tau_i / \rho_G)^{1/2} / \nu_G \), rather than to \( m / d_i \). Furthermore, from a consideration of available data, the relation for \( m^+ \) when roll waves are not present is \( m^+ = 0.34 \text{Re}_{LF}^{0.6} \), rather than the steady laminar relation \( m^+ = 0.705 \text{Re}_{LF}^{0.5} \). This is explained in (106) from a consideration that films with ripple waves may be considered to be unsteady laminar flows. For \( \text{Re}_{LF} > ca300 \), roll waves appear. Then film heights, both for upflows and downflows, are represented by \( m^+ = 0.19 \text{Re}_{LF}^{0.7} \).

For gas velocities greater than \( 25 \text{m/s} \), \( f_i / f_s - 1 = 0.045 (m_G^+ - 4) \). Thus, for \( m_G^+ < 4 \) the amplitudes of the waves on the interface are small enough that the interface may be considered to be hydraulically smooth. Measurements for downflows show that \( f_i / f_s - 1 = 0.045 (m_G^+ - 5.9) \). Relations such as these, along with measurements of \( E \), are used in (106) and in (108) to describe the pressure drop and the average film height.

(c) \textit{Atomization; Drop size; Roll waves}

More details about roll waves are given in (40) and (41). In (40) a special technique was developed to measure the height of the film, \( h_0 \), which exists between roll waves. This is observed to be same as exists just before the transition to an intermittent wave pattern. Thus, the roll waves at low film flows may be pictured as surges moving over a base film of height \( h_0 \). Increases in liquid flow are accommodated by increasing the volume of liquid carried by the roll waves. Flow in the base film is unchanged if the liquid flow is not too large. At high gas velocities, characteristic of vertical annular flows, \( h_0^+ = h_0 \sqrt{\tau_i / \rho_L / \nu_L} = ca12 \).

Atomization is initiated with the appearance of roll waves, which are a collection of capillary ripples. High speed photography (40, 29) was used to show that atomization occurs through the removal of these wavelets by a Kelvin-Helmholtz mechanism. As discussed in Section 8e, the removed wavelets break into a number of drops. Thus the size of the drops is not proportional to the wavelength of the unstable waves, as was presumed by a number of researchers. The unstable waves are highly distorted so that before final breakup, the bulk of the liquid is contained in an arc-shaped tube.
of liquid. In Paper 73, the argument is made that the drop size scales with the diameter of this tube. Through a series of theoretical conjectures and the use of the small amount of data that were available, the following relation for volume median drop diameter, $d_{vμ}$, was obtained:

$$\left(\frac{d_{vμ}}{d_i}\right)^2 \approx 1.6 \times 10^{-2}$$

where $\sigma$ is the surface tension, $d_i$ is the pipe diameter and $f_s$ is the friction factor for flow over a smooth surface.

The suggestion that the drop diameter increases with pipe diameter raised to the one-half power was at variance with some correlations that had been proposed. The resolution of this issue is very important since it affects how we use data from laboratory studies to predict behavior in pipes with diameters of several feet, such as used to transport natural gas/condensate.

Our original work on drop size, described in (73), used an electric probe in a horizontal channel. A wire charged to a high voltage is placed in the flow stream. When a conducting droplet strikes the probe, its surface charges to the probe potential, thereby causing a pulse in the electric current which depends on drop diameter. This technique has a number of shortcomings associated with grounding and with the detection of smaller drops. Therefore, other approaches were considered. In (197), straightforward photographic techniques were used. In (206), a new photographic technique which employs backlighting was developed. The film was removed from the wall just before the test section, which had two arms. A strobe light source shined through one arm. A CCD sensor captured images of the shadows of the drops. Experiments were carried out for upward flow of air and water in a 4.2 cm pipe at a gas velocity of 30 m/s. In (224, 232) a laser-diffraction technique was used in horizontal pipes with diameters of 9.53 cm and 2.54 cm. A modified version of the immersion technique, developed by Okada, was used in a 9.53 cm pipe (228). Drop samples were collected in a high viscosity oil and photographed. Advantages of the immersion method are that the drops are spherical when photographed and that the measurement does not require the removal of film from the wall. Measurements of Sauter mean diameters agree with those obtained with laser diffraction (Paper 224) in the same system. This technique appears to offer promise for testing in large diameter pipes used in field operations.

Several results from these measurements are worth mentioning: (1) The effect of gas velocity and pipe diameter on drop size supports the equation suggested in (73). (2) The drop size distribution appears to be best represented by a log-normal equation. (3) The liquid flow rate has a small but significant effect on drop size. (4) By averaging the results for different liquid flows, the following relation for the volume median drop diameter
was obtained: \((d_{v_d}/d_i)(\rho_u u_d^2 d_i/\sigma)^{1/2} = \text{const}\). Surprisingly, this agrees with the equation presented in (73).

(d) Rate of atomization

Our initial effort to determine \(R_A\) was to bring a gas flow in contact with a liquid flowing along the bottom of an enclosed rectangular channel. (Tatterson, D.F., "Rates of atomization and drop size in annular two-phase flow", PhD thesis, University of Illinois, Champaign-Urbana, 1975; Alonso, G.F., "The effect of liquid viscosity on the rate of atomization in concurrent air-liquid flow", MS thesis, University of Illinois, Champaign-Urbana, 1975). The increase in entrainment with distance downstream of the entry was measured. The rate of atomization was determined by fitting the change in entrainment close to the mixing section with a linear relation. Paper 81 proposes the equation,

\[
(\Gamma - \Gamma_0)^n = \frac{G A A U k R}{\rho_0 \rho G},
\]

where \(\Gamma_0\) is a dimensional constant, \(\Gamma\) is the mass flow per length, \(\Gamma_0\) is the critical film flow for the initiation of roll waves, and \(\tilde{R}_A = \frac{R_A}{\rho_u (\rho_u \rho_l)^{1/2}}\). For \((\Gamma - \Gamma_0)\) less than zero, \(R_A\) is zero. Paper 81 used this equation with \(n = 1\) to correlate measurements from several laboratories, that employed the same technique to measure \(R_A\). A surprisingly good agreement is realized. Of particular importance was the inclusion of the concept of a critical flow rate, \(\Gamma_0\).

The technique used in these experiments had the disadvantages that it assumed that deposition in the entry section is close to zero and that the wave structure is the same as found in a fully developed condition where \(R_A = R_B\). Therefore, we explored the tracer technique developed by Quandt. A continuous supply of salt solution was fed into the liquid film flowing along the wall of a pipe under conditions that the flow is fully-developed. The concentration of salt in the film is measured at different distances downstream from the point of injection. Mass balances can then be used to relate these measurements to the rate of interchange between film and the initially unsalted droplets in the gas (147). Measurements for air and water in 2.54, 4.24, 5.71 cm circular pipes, presented in (147), were correlated with the above equation using \(n = 0\). The result that \(\tilde{R}_A\) did not vary with \(\bar{U}_G\) was perplexing in that it was at variance with the relation suggested in (81).

However, recent work by Bertodano suggests that \(n=1\) and that the above equation be changed to \(\tilde{R}_A = k_A \bar{U}_G (\Gamma - \Gamma_0)/\sigma\), where \(\sigma\) is the surface tension. This is an attractive equation since \(k_A\) is a dimensionless parameter.
(See Paper 226, where all available measurements of $R_A$ and of $E$ are reviewed. These cover a wide range of variables.)

The use of a linear equation with a critical film flow, first proposed in (81), is useful because of its simplicity and because it introduced, in a formal way, the notion that the liquid in the wall layer cannot be fully entrained. The rate equation given above differs from that used in (81) because the effect of surface tension is introduced directly.

Measurements for $R_A$ at large $W_{LF}$ are given in (193, 226). These show that the linear relation between $R_A$ and the excess film flow rate breaks down at large liquid flows. They seem to suggest a constant value of $R_A$ at large $W_{LF}$, consistent with Taylor's theory for thick layers. As explained in (193), the increase in deposition at large liquid flows tends to increase the average thickness of the base film and, therefore, to decrease the excess film flow rate for a given $W_{LF}$; that is, the assumption of a constant flow rate in the base film breaks down.

Early studies (41, 51) in this laboratory suggested that atomization of ripples contained in the roll waves are removed by a Kelvin-Helmholtz mechanism. Taylor considered a thick viscous layer covered uniformly with capillary ripples having a wavelength of $\lambda$. He assumed that the gas flow can be calculated with inviscid theory; he argued that the volume removed per unit area scales as $\lambda^2/\lambda^2$ and that $R_A$ scales as $R_A \propto \lambda/t_G$, where $t_G$ is the time constant characterizing the growth of unstable waves. For water, the effects of liquid viscosity can be ignored, so $\lambda = f(We)$, where the Weber number is defined as $\rho_G \left( U_g \right)^2 \lambda/\sigma$. If the unstable wavelength is that which gives the most rapid growth, the above equation simplifies to $\lambda = \text{constant}$, contrary to the measurements which show a dependency on the liquid flowrate in the wall film.

This theory was revisited in the thesis by Tatterson, cited above. This involved a consideration of the effect of the height of the liquid film. The waves on the base film cannot be removed because they are dominated by wave-induced variations of the interfacial shear stress (which causes them to tumble rather than to atomize). The role of disturbance waves in the atomization process is that they intermittently present thick films for which waves are dominated by wave-induced gas phase pressure variations. Tatterson suggested that the wavelengths of the unstable ripples scale with the height of the roll waves; that is, $\lambda = f(We_{rw})$, where $I$ is the fraction of the time disturbance waves are present and $We_{rw} = \rho_G \left( U_g \right)^2 h_{rw}/\sigma$. 
This idea was explored in (143), where measurements of $I$ and $h_{RW}$ are presented for air-water flows. An approximately linear dependence of $\bar{R}_A$ on $We_{bw}$ was discovered. Both $I$ and $h_{RW}U_G^2$ were found to increase with the excess film flow rate, $\Gamma - \Gamma_0$. Thus, the equation derived from Kelvin-Helmholtz theory is consistent with $\bar{R}_A = k_A U_G (\Gamma - \Gamma_0)/\sigma$, which was derived empirically.

(e) Rate of deposition

The rate of deposition is described by $R_D = k_D C_B = k_D (W_{LE}/Q_g S)$, where $k_D$ is the deposition coefficient, $S$ is the ratio of the drop velocity to the gas velocity and $Q_g$ is the volumetric flow of the gas. Mechanisms for drop deposition are outlined in Section 11b. The size of the drops in annular flow are such that the dimensionless inertial time constant is large, $\tau_p > 20$. Thus, the dominant mechanism involves drops starting a free-flight to the wall from a region outside the viscous wall layer. From results cited in Section 11f, we suggested the use of $k_D = (1/2\pi)^{1/2} \left( \overline{v^2}_{pr} \right)^{1/2}$, where $\overline{v^2}_{pr}$ is the root-mean square of the radial component of the particle velocity fluctuations outside the viscous wall layer. The validity of this equation was explored in (193). Measurements of $k_D$ obtained in our laboratory (147) and in other laboratories were used.

The testing of the proposed relation required the evaluation of the particle turbulence $\overline{v^2}_{pr}$. As summarized in Section 13d, we modified a theory developed by Reeks (131,139) to obtain the following relation between particle and fluid turbulence: $\overline{v^2}_{pr}/u_r^2 = \left( \tau_{LF}/\tau_p \right)^{0.7 + \left( \tau_{LF}/\tau_p \right)^{-1}}$ where $u_r$ is the mean-square of the fluid velocity fluctuations, $\tau_{LF}$ is the fluid Lagrangian time constant and $\beta = 1/\tau_p$. The evaluation of $\tau_p$ requires a knowledge of the drop diameter, $d_p$. The experiments described in (197) provided measurements of the drop flux profile, the spatial variation of the fluid velocity, and the drop size.

Good agreement between the measured and calculated $k_D$ for very dilute concentrations was realized. However $k_D$ was found to decrease with increasing concentration (as had been found previously by Govan, Namie & Ueda and Andreussi & Zanelli). The suggestion is made in (197) that at very large concentrations, $k_D$ varies as $C^{-1}_B$. This behavior is still not completely
understood. Paper 197 rules out increases of drop size with increasing concentration and presents calculations which suggest that the increase of drop interaction with increasing concentration could be accompanied by decreasing particle turbulence. However, a later study of suspension flow (250, 254) shows that a strong damping of fluid turbulence can occur because of forces of the particles on the fluid. It is quite likely that this is the principal cause of the decrease of $k_D$ at large concentrations.

(f) **Entrainment**

For a fully developed flow, $R_A = R_D$. The following equation for $E$ results if linear relations for the rate process are used:

$$(E/E_M)/(1 - E/E_M) = k_A d_i U_G^3 \sigma (\rho_G \rho_L)^{0.5}/4k_D \sigma ,$$

where $E_M = (W_L - W_{LFC})/W_L$ is the maximum entrainment and the rate of atomization is given by

$$\tilde{R}_A = k_A U_G (\Gamma - \Gamma_0).$$

Paper 81 compares these equations only with data for air-water flow in small diameter tubes, because that was all that was available in 1979. The data for air and water are correlated quite well if $k_D$ is assumed to be approximately independent of gas velocity. From the discussion in Section 12e, an increase in gas velocity is accompanied by an increase in the fluid turbulence and a decrease in the drop diameter. These would have counterbalancing effects on the deposition coefficient.

A study was initiated in order to expand the data base (108). The experiments involved upward flow of air and water-glycerine solutions in 2.29 and 4.2 cm pipes. The liquid viscosity was varied from 1.1 to 4.6cP. Local droplet fluxes were measured by withdrawing liquid through impact tubes. The flux profiles were integrated to obtain the mass flow of drops, $W_{LE}$. The film flowrate was obtained by subtracting this from the total liquid flow, $W_{LF} = W_L - W_{LE}$. The entrainment was calculated as $E = W_{LE}/W_L$. For a given pipe diameter the entrainment measurements could be fitted by the above equation by selecting an appropriate constant value of $k_A/4k_D$.

The few results available for fluids with small surface tensions indicated larger values of $k_A/4k_D$ than was found for air-water.

Paper 226 reviews measurements of $E$ that were made in a number of laboratories. In particular, it was desired to capture effects of surface tension which had not been handled as well as would be desired in previous work. Part of this endeavor was the development of a relation for the critical gas velocity $d_i^{0.5} U_G (\rho_G \rho_L)^{0.25}/\sigma^{0.5} \approx 40$ needed for the initiation of annular flow. In order to capture the effect of surface tension, the Bertodano modification
to the equation presented in Paper 81 was used. Thus, two equations were developed

\[
\frac{E}{E_M} \left[1 - \frac{E}{E_M}\right] = k_d \frac{d}{U_G S(\rho_G \rho_L)^{0.5}} / 4k_d \sigma
\]

and

\[
\frac{E}{E_M} \left[1 - \frac{E}{E_M}\right] = k_d \left(\frac{U_G - U_{GC}}{U_G}\right)^2 \frac{U_G S(\rho_G \rho_L)^{0.5}} {4k_d \sigma} \]

. The second of these, which is particularly useful for gas velocities close to the critical, directly recognizes that \( E = 0 \) for \( \overline{U}_G < \overline{U}_{GC} \). Two approaches were taken to evaluate \( k_d \). One substitutes \( \overline{U}_G \) for \( S/k_d \). The other sets \( S = 1 \) and uses relations developed in Section 12e to calculate \( k_d \). Paper 226 critically analyzes measurements of \( E \) and uses acceptable results to test the above equation to see if it captures the effects of gas density, surface tension, pipe diameter, direction of flow and gas velocity. Good comparisons were realized--but there is room for improvement.

The studies described in (147) provided measurements for the effect of liquid film flow rate on the rate of atomization, \( R_d \). They also provided results on the influence of drop concentration on the rate of deposition, \( R_{dp} \). These show a linear dependence of \( k_d \) on the bulk concentration, \( C_B \), at low \( C_B \). However, surprisingly, \( k_d \) was found to decrease with increasing \( C_B \) at large concentrations. A possible interpretation of these results is presented in Section 11(j), which describes studies which show that the decrease in \( k_d \) is caused by a damping of the gas phase turbulence, due to feedback. As seen in the equations for \( E \), presented above, a decrease in \( k_d \) should result in an increase in \( E \), not seen in the measurements (147). This surprising result is discussed in (226, 193).

Paper (193) presents a summary of data related to this problem. It shows that that the decrease in \( k_d \) is accompanied by a decrease in \( R_d \). Thus, a sharp change in the entrainment is not observed. However, it is shown that the decrease in \( k_d \) is having a slightly greater impact than the decrease in \( R_d \). Clearly, this is an area which needs more attention.

\((g)\) Drop bouncing; Critical heat flux

All of the works on particle deposition discussed above and in Section 10 assumed that the boundary is a "perfect absorber" in that droplets or particles striking it are removed from the flow. This issue is addressed in Paper 130, which describes experiments in which 50 micron drops were injected into an air flow through a centrally located tube into a 50.8 mm pipe. Three wall conditions were investigated: (1) grounded brass, (2) Plexiglas, and (3) Plexiglas wetted with a relatively smooth water film, having a thickness of 155 microns. Deposition rates were about the same for
brass and Plexiglas walls. The presence of the water film on the wall was found to impede deposition greatly, possibly by promoting droplet bouncing.

There is no evidence that droplet bouncing is a factor in annular flow for which the film has large amplitude waves. However, there are a number of circumstances where this finding could have an impact. Many measurements of deposition rates in annular flow were made by sucking liquid from the wall film so that it is thin enough that atomization is not occurring. The buildup of the film downstream of the film thinning unit was used to calculate the deposition rate. Drop bouncing in these instances could influence the reliability of the results.

A more interesting application of these results is to heat transfer studies with phase change, in which a critical condition is reached where there is a sharp decrease in the heat transfer rate. (See paper 130.) The interpretation has been offered that the heat flux causes droplet deposition to be suppressed by vapor flowing normally from the remaining film. The experiments described in (130) offer an alternate interpretation. In the heat transfer region where the critical phenomenon is observed, the wall film is sufficiently thin that roll waves do not exist, and droplet bouncing could inhibit droplet deposition from replenishing the film.

(h) **Separated flow**

A symposium (Third International Workshop on Two-Phase Flow Fundamentals) sponsored by the Department of Energy was held in London, June 15-19, 1992). Its purpose was to define needed areas of research. We were honored by an invitation to make a presentation on separated flows, which include annular and stratified flows. This paper contains a summary of work, described in this Section and in Sections 13 & 15, that had been done before 1992.
Fig. 34  DNS of turbulence in a plane perpendicular to the direction of flow. Particles are represented by point forces: (a) single phase flow, (b) volume fraction of particles equals $4.9 \times 10^{-4}$, (c) volume fraction of particles equals $3.0 \times 10^{-3}$.

Fig. 35  Axial viewing photography of paths of particles in a turbulent flow.

Fig. 36  Axial photography of particles trapped at the wall.
Fig. 37  Vertical annular flow. Note the intermittent disturbance waves.

Fig. 38  Decreasing gas velocity causes a breakdown of annular flow to a churn flow, with up and down motions at the wall.

Fig. 39  Hanratty, David Siegwarth, Gerald Cook, Ted Wegner at research seminar, circa 1967.

Fig. 40  Ted Wegner, Harry Dimopoulos, circa 1967.

Fig. 41  Dimitri Hatziavramides, Henry Chan, Hanratty, Larry Chorn, circa 1977.

Fig. 42  PhD students Nikolaos Andritsos, Eleni Vasiliadou, Ken Frederick, John Vames, Larry Williams, circa 1985.

(a) Prologue

The description of annular flows in horizontal pipes encounters a number of difficulties not present in vertical flows. The layer of liquid flowing along the wall is asymmetrically distributed because the gravitational force on the wall film causes the liquid to drain downward. Similarly, gravity can cause asymmetries in the drop flow, whereby larger concentrations exist at the bottom part of the pipe. To a first approximation, the liquid distributions may be described as a balance between gravitational settling and turbulent mixing. The understanding of the asymmetries are a central problem. Yet, experimental studies are sparse.

Annular flow is usually defined as a wetting of the top of the pipe. Thus, a flow pattern can exist in which the liquid flows mainly along the bottom of the pipe and entrained drops do not reach the top of the pipe.

(b) Droplet concentration profiles

Film flows and, therefore, entrainments were measured for air and water flowing in horizontal 2.54 & 5.08 cm pipes by withdrawing the film through a length of pipe with a porous wall (105). Local measurements of droplet flux, $F_{LE}$, in a 9.53 cm pipe are reported in (192). (See Fig. 43, 48). These were obtained with a sampling tube. Local drop concentrations, $C_p$, were calculated from these measurements by using the relation $C_p = F_{LE} / S U_G$, where slip, $S$, is the ratio of the drop velocity to the local average gas velocity, $U_G$.

Measured concentrations were approximately constant along horizontal chords. Thus, the concentration field could be represented by a variation along a diameter parallel to the direction of gravity. The concentration is largest at the bottom of the pipe. This model is not perfect. Profiles in the vertical direction, at intermediate gas velocities and high liquid rates, show a complicated behavior, that suggests the existence of a secondary flow which is downward at the wall and upward along the vertical centerline. The horizontal concentration profiles show a small bump in concentration at the center under conditions that the vertical concentration and velocity profiles suggest the existence of a secondary flow.

The above observation is surprising. A number of investigators have suggested the existence of a secondary flow that is due to the variation of the...
roughness around the circumference, caused by the presence of waves. This would lead to a secondary flow upward at the wall and downward at the center. These might exist, but the dominant secondary flow was found to be in the opposite direction to that induced by wall roughness.

These results motivated a study (183) in which measurements of the gas phase velocities were made with an impact tube. (See Fig. 44 and 45). The test section could be rotated so that traverses could be carried out along different diameters. In this way, the velocity could be mapped over the whole cross section of the pipe. Lines of constant velocity (isotachs), constructed from these measurements, were used to detect secondary flows.

For gas-liquid flow in an infinitely wide rectangular channel, secondary flows do not exist and the maximum in the velocity is located above the center of the gas space because the wavy liquid flowing along the bottom wall is associated with large interfacial drag. For stratified flow in a pipe without entrainment, the maximum is located below the center of the gas space. This can be explained by the existence of a secondary flow which transports drops downward at the center of the pipe. When large numbers of drops are present, a secondary flow exists which is downward at the wall and upward at the center. The maximum velocity then occurs below the center of the gas space.

Thus, Papers 183,192 support the notion that the presence of drops in the gas phase can cause a flow which moves downward at the wall and upward in the center of the pipe. Close to the walls, a relatively large concentration and, therefore, a large mixture density exists. Variations of this density can cause secondary flows analogous to what is observed when natural convection is added to a mean flow.

Only an approximate model for the drop distribution is available. This was obtained by recognizing that the concentration varies mainly in the vertical direction, so that one can represent the concentration field by a series of chords extending over the breadth of the pipe on which the concentration is constant. Thus, it is only necessary to calculate the concentration profile along the vertical diameter bisecting the pipe. Distances along this line are defined by the symbol y. Thus, the change over a distance dy is considered to be caused by a balance among gravitational settling, \( v_r \overline{C}_p \), turbulent mixing, represented by \(- c \frac{d \overline{C}_p}{dy}\), and the net contribution to the field due to atomization and deposition. This balance can be greatly simplified by neglecting the contributions due to atomization and deposition. The equation
\[
\overline{C}_p / \overline{C}_{p0} = \exp\left(-2v_r y / \xi_\text{drop} \nu' d_t\right),
\]
which uses the assumption that \( \varepsilon = \xi_\text{drop} (d_t/2) \nu' \) is then obtained. Term \( d_t \) is the tube
diameter, $v^*$ is the friction velocity and $v_T$ is the terminal velocity of the drops. The concentration at the bottom of the pipe ($y=0$) is designated by $C_{p0}$.

A comparison of the exponential function with measurements in 5.08 cm and 9.53 cm pipes (227) shows a rough agreement which improves with increasing pipe diameter. The chief departure from the semi-logarithmic relation is at the top of the pipe, where mixing due to secondary flow and the influence of wall fluxes could be dominant. Obviously, more can be done with the theory. Some encouragement was also obtained in (249), where annular flow in a rectangular channel is considered. In this situation, a secondary flow does not exist and wall flux terms exist only at the top and bottom walls. Excellent agreement with the semi-logarithmic relation, cited above, was realized. These results, as well as experiments in pipes suggest that the semi-log equation is a good starting point for predicting the concentration profile, especially in large diameter pipes.

(c) **Entrainment**

Entrainment measurements were made for annular flow in horizontal pipes with diameters of 2.31 cm, 5.08 cm and 9.53 cm (105, 192). The usual approach in calculating entrainment had been to use correlations for vertical pipes. This failed, both qualitatively and quantitatively. The most glaring difference is that entrainment in horizontal pipes at low and moderate gas velocities is much smaller and increases more rapidly with increasing gas velocity. The reason for this is that deposition rates can be larger because of direct contributions of gravitational settling and because gravity causes a distortion of the concentration profile. These issues are addressed in (227). The approach taken for vertical flows was used, in that entrainment is based on the development of equations for the rates of atomization and deposition.

The rate of atomization varies around the circumference of the pipe because of the variation of film thickness. The assumption is made that, locally, the equation for the rate of atomization is the same as found for a vertical pipe. Thus, the average around the circumference is given as

$$< R_A > = \left( k_A \overline{U_G} \rho_G \rho_L \sigma / \nu \right) \left( \Gamma_{LF} \right) \left( \Gamma_0^* \right) \left( \theta = \pi / \pi \right) \left( (\Gamma_0 - \Gamma_{LF}) \right) d\theta$$

where $\theta_c$ is the angular location at which $\Gamma = \Gamma_0$. The rate of deposition varies around the circumference because both $k_D$ and $C_{pw}$ vary. The local rate of deposition can be defined as $R_D = (k_D C_w / C_b) (W_{LE} Q_G S)$. Equating
< \( R_A \) > and < \( R_D \) > gives the following relation for entrainment:

\[
\left( \frac{E}{E_M} \right) = 1 - \left( \frac{E}{E_M} \right) = k_d \left( \frac{\rho L \rho G}{C_W} \right)^{1/2} k_d \left( \frac{C_W}{C_B} \right) \sigma \text{, where}
\]

\( E_M = 1 - \left( \frac{\pi d_c \Gamma_c^*}{W_L} \right). \) Paper (227) substitutes the settling velocity, \( v_T \), for \( k_d \left( \frac{C_W}{C_B} \right) \) in order to obtain a simple relation for low gas velocities:

\[
\left( \frac{E}{E_M} \right) = A_d \left( \frac{\rho L \rho G}{C_W} \right)^{1/2} \sigma v_T \text{.}
\]

At large gas velocities gravitational effects are negligible and the correlation for vertical flow can be used.

Surprisingly, this simple approach provides a good first approximation for entrainment for a wide range of operating conditions.

An attempt was made in (227) to improve this correlation by using results presented in Section 13b for predicting concentration profiles and by modifying theoretical constructs for \( k_d \) in vertical pipes, as indicated below.

(d) Deposition in horizontal annular flow

For drop sizes that exist in annular flows, deposition occurs by free-flight from locations outside the viscous wall layer. The deposition rate is then described as the product of the average velocity of the particles hitting the wall, \( V_w \), and the local concentration. For a vertical flow \( V_{pW} = \left( v_{pr}^2 / 2\pi \right)^{0.5} \) . This result was extended to horizontal flow by including the effect of gravity.

The distribution of velocity fluctuations is assumed to be Gaussian, so that

\[
p(x) = \frac{1}{(2\pi)^{1/2} \sigma_p} \exp \left\{ - \frac{(x - \mu)^2}{2\sigma_p^2} \right\}, \quad x = \mu + v_{pr}, \quad \sigma_p = \left( v_{pr}^2 \right)^{1/2}
\]

where \( \mu = \) the average velocity. For a vertical flow, \( \mu = 0 \); for a horizontal flow, \( \mu = v_T \cos \theta \). The local \( k_d(\theta) \) is then given by the equation

\[
V_{pW} = \int_0^\infty \exp(x) dx.
\]

Values of this integral are given in (227). To my knowledge, this represents the first prediction of the circumferential variation of the deposition rate in a horizontal pipe.

(e) Film distribution in horizontal annular flow

The distribution of film height, \( \tilde{h}(\theta) \), in a horizontal annular flow has been the focus of a number of investigators. Here, \( \theta \) is the angle measured from the bottom of the pipe. The following observations about air-water flow in a horizontal pipe at atmospheric conditions are given by L. A. Dykhno in a 1996 memorandum.
At low superficial gas velocities \((U_{SG} = 5-9 \, \text{m/s})\), a stratified flow is observed. At 12m/s, waves increase in amplitude with increasing gas velocity and have steep spilling fronts. These are the roll waves described by a number of authors. The spreading of waves up the pipe wall is observed with increasing gas velocity. At approximately \(U_{SG} = 17 \, \text{m/s}\), the gas flow starts to atomize the waves on the liquid interface. Droplets deposit on the pipe wall and, in this way, greatly enhance the wetting of the wall at locations above the pool of liquid on the bottom of the pipe. At \(U_{SG} = 24 \, \text{m/s}\), \(U_{SL} = 0.09 \, \text{m/s}\), a thick layer with roll waves exists at the bottom of the pipe and a continuous film covered the upper part of the pipe. The formation of a continuous film is slightly dependent on the liquid velocity. Thus, at a superficial liquid velocity of 0.02 m/s it occurs at a greater superficial gas velocity, 30m/s. As the height of the wall film increases, disturbance waves of the type seen in vertical annular flows exist over part of the film. These are haphazard packets of capillary waves which could be considered patches of turbulence. Atomization occurs over these disturbance waves. This enhanced the amount of liquid entrained in the gas. Eventually, at high enough gas velocities, disturbance waves covered the whole circumference.

Dye injected into the film was observed to move downward (due to gravitational pull) when no disturbance waves were present. However, when the dye encountered a disturbance wave, it was observed to disperse explosively in the circumferential direction, as would be expected in a turbulent patch.

The prediction of the circumferential variation of the height of the wall film has received the attention of a number of researchers because of its central role in understanding horizontal annular flows. Several mechanisms have been proposed. The height of the wall film, \(h(\theta)\), has been measured in our laboratory with conductance probes by Laurinat (Laurinat, J. E., "Studies of the effects of pipe size on horizontal two-phase flows". PhD thesis, University of Illinois, Urbana-Champaign, 1982) and by Williams (Williams, L. R., "Effect of pipe diameter on horizontal two-phase flow". PhD thesis, University of Illinois, Urbana-Champaign, 1990). These measurements are presented in Papers 116,192. The first study used the momentum balance in the circumferential direction to develop an equation for \(\bar{h}(\theta)\). The goal of this effort was to develop an equation which incorporates all of the mechanisms that had been explored in the literature for redistributing the liquid in the wall film.
We have expanded the approach in (116) by also using the equation for conservation of mass. The following result was obtained:

\[ \dot{\varepsilon}_h (d\delta L/dx) = (\tau_{ss}/\rho_L) I_1 + (1/\rho_L) (\partial \tau_{xs}/\partial x) I_2 - g(\sin \theta + \cos \theta \partial \delta \theta /\partial x) I_3 - \dot{\Gamma}_x \]

(1) \hspace{1cm} (2) \hspace{1cm} (3) \hspace{1cm} (4) \hspace{1cm} (5)

where \( x = a \theta \) is the distance along the circumference. Terms \( I_1 \) and \( I_2 \) are integrals defined in (116). Term (1) represents turbulent mixing. Term (2) represents the influence of interfacial drag in the \( x \)-direction. This can be associated with secondary flow in the gas or it could represent the \( x \)-component of the drag on waves that are not oriented perpendicular to the direction of mean flow. Gradients of the turbulent stress in the liquid in the circumferential direction can exist because of the variation of film height around the circumference. Term (3) represents the contribution of this effect. Term (4) represents the force of gravity in the circumferential direction. The first part of (4) is dominant. The second part of (4) can be important only very close to \( \theta = 0 \). The average volumetric flow in the circumferential direction, per unit length of pipe in the circumferential direction, is defined as \( \dot{\Gamma}_x \). From conservation of mass \( \dot{\Gamma}_x = \int_0^x (R_{D}/\rho_L - R_{A}/\rho_L) dx \). When \( \theta = \pi \), \( \dot{\Gamma}_x = 0 \), since the flow in the film is symmetric. Thus, Term (5) represents a redistribution of the film caused by atomization and deposition.

A number of investigators have argued that film transport can occur by atomization and deposition of drops and used (4) and (5) to describe the film distribution. The notion that the distribution is caused by gas phase secondary flows (or interfacial stresses in the circumferential direction) is represented by (4) and (2). One can look upon (1) or/and (3) plus (5) as representing a spreading mechanism.

Paper (116) considered (2), (3), (4) and (5). To my knowledge, Term (1) had not been fully explored. Its inclusion was motivated by the visual observations described at the beginning of this subsection, which suggest a turbulent spreading in the disturbance waves similar to what is associated with turbulence. Under circumstances where disturbance waves cover the whole circumference, most researches (including Paper 116) have argued that the variation of the height is weakly dependent on atomization, deposition and gas-phase secondary flows. This suggests that the distribution is given by a balance between Terms (1) and (4), that is, between turbulent mixing and drainage due to gravity. (Thus, the mechanism would be similar to what has been used to describe droplet concentration profiles.)
solution of the resulting equation produces $\bar{h}(\theta)$ in agreement with experiment. However, a theoretical expression for $\varepsilon_n$ is not available.

When disturbance waves do not occupy the whole circumference, the film distribution at the top of the pipe is not affected by terms (1) and (3), so that droplet deposition and interfacial stresses are the important mechanisms for bringing liquid to the top of the pipe.

(f) *Holdup in the liquid film*

The holdup in horizontal gas-liquid annular flows is the volume occupied by the drops and by the film. The latter is the chief contributor. In a horizontal annular flow it would be given as $2\pi am$, where $m$ is the average height around the circumference. Paper 102 adopts results obtained for vertical annular flows to develop a relation for holdup.

For vertical annular flows it was found that the film height could be predicted if it is assumed that the velocity profile is the same as would be obtained for a single phase turbulent flow. It is assumed that this relation holds locally at a given location on the circumference. The integration around the circumference produces a relation for $m$ that is similar to what is found for $h$ in vertical annular flows. That is $m^+ = f(Re_{LF})$, where $Re_{LF} = 4\Gamma/\nu_L$ and $\Gamma$ is the mass flow in the film divided by the pipe circumference. The friction factor used in the definition of $m^+$ was based on the average value of $\tau_i$ around the perimeter. Measurements for air-water flowing in 2.54 cm and in 5.08 cm pipes are correlated quite well with the equation. However, the values of $Re_{LF}$ are less than was observed for a vertical pipe. This arises from the asymmetry of the film in horizontal flows in that $< h^n > \neq m^n$.

Not so much success was experienced in correlating results on the interfacial friction factor. This is because the wave patterns at low and high gas velocities are different in that the liquid configuration at low gas velocities looks more like a stratified flow. The best approach is to represent the behavior at large and at low gas velocities as, respectively, being close to a vertical and to a stratified flow. A fuller discussion of stratified-annular flow is given in Section 16e.
(g) Transition to annular flow in a horizontal pipe; Wetting the top of the pipe

The transition from a stratified flow to an annular flow in a horizontal pipe is promoted by droplets wetting the top of the pipe (123), for pipes with diameters of 2.54 cm and larger. This lead to the supposition that annular flow should be initiated close to the gas velocity at which atomization occurs. Experiments for air and water flowing in 2.52 cm and 9.53 cm pipes (129) show that the transition from a stratified to an annular flow occurs at gas velocities which are roughly twice as large as what is required to atomize the liquid in the stratified flow.

Data for larger pipes, especially those used to transport natural gas, show transitions considerably different from what is suggested in (129). The condensate in these systems has a lower surface tension than water. Furthermore, the vapor has a larger density than air at atmospheric pressure. Thus, the initiation of atomization occurs at smaller gas velocities. This lead to the exploration, in (235), of a better criterion; that is, the ability of drops to reach the top of the pipe in sufficient quantity to form a stable continuous film.

The top of the pipe was defined as \( \theta = 0 \), rather than \( \theta = \pi \). The thickness of the film in a region close to the top was calculated. Disturbance waves would not exist in this region at the transition to an annular pattern, so the flow in the film could be considered as laminar and atomization is not occurring. Thus, liquid is brought to the top of the pipe by deposition. It drains down the pipe walls by gravity. To a first approximation, the height of the liquid at the top of the pipe can be calculated from (4) and (5). The inclusion of term (2) would decrease the drainage, but this was not considered in the calculations reported in (235).

The height of the film at the top of the pipe is then calculated as

\[
h = \left(3R_D a \sqrt{\frac{\nu_T}{g \rho_L}}\right)^{1/3}.
\]

The rate of deposition at the top of the pipe is given as

\[
R_D = k_D C_w,
\]

where \( k_D \) is calculated with the approach used in Section 12(d) and \( C_w \) was estimated with the simplified semi-logarithmic relation for the concentration field given in Section 13(b).

Calculations for air and water in a 9.53 cm plastic pipe showed a negligibly small film thickness for \( U_{sg} < 20 \text{ m/s} \). Transition to annular flow was reported in (123) to occur at \( U_{sg} \approx 32 \text{ m/s} \). This corresponds to a calculated film thickness at the top of the pipe of 20 microns. Similar results were obtained for a 2.54 cm pipe. Calculations for a 20.32 cm pipe carrying natural gas/condensate showed much larger film thicknesses than 20 microns.
at transition to annular flow. This is explained in (235) as reflecting the larger wall roughness in a commercial pipeline; that is, a film thickness of, at least, the size of the roughness elements would be needed.

The principal result of this work is that the formation of a film on the top wall provides a better criterion for transition to annular flow than the initiation of atomization. The calculations for the formation of a continuous film around the pipe circumference is of considerable importance to the practicing engineer because of its possible impact on understanding pipe corrosion and thermal stresses in the pipe wall in heat transfer applications.
14. Flow regimes and transitions for gas-liquid flow in horizontal and inclined pipes

(a) Prologue

Before summarizing our accomplishments in understanding gas-liquid flows, it would be remiss not to mention the impact of international meetings held at Exeter, 1965, and at Waterloo, Canada, 1968, in setting a new tone for multiphase research. Interactions with Abe Dukler (196), with the Institut de Mecanique des Fluides in Toulouse, with Geoff Hewitt, and with Theo Theofanous have added to the fun of pursuing work in this area.

Our work on waves and on vertical annular flows led to a participation in the Design Institute for Multiphase Processing, sponsored by the American Institute of Chemical Engineers. This involvement, plus support from Shell Corporation, prompted the development of a special flow loop, which was housed in the Mechanical Engineering Laboratory of the University of Illinois (See Figs. 62, 72). The use of this space allowed the study of air and a liquid flowing in pipes that varied in diameter from 2.54 cm to 9.53 cm. It had a working length of 2.6 m. Air velocities up to 100 m/s in the 9.53 cm pipe were available. This special facility allowed us to test ideas that we had on the design of gas-liquid systems and enabled a number of discoveries. (I am grateful to the Mechanical Engineering Department for giving us this opportunity.)

Much of the work described in this Section and in Section 11 was summarized in the Plenary Lecture given at the Third International Conference on Multiphase Flow, Lyon, France, June 8-12, 1998 (Paper 213) and in an invited presentation on Separated Flow Modeling & Interfacial Transport Phenomena at a Conference on Computational Fluid Dynamics at Apeldorn (162).

A workshop, organized and chaired by T. J. Hanratty, held at the University of Illinois, outlined scientific issues whose resolution will help advance and define the field of multiphase flow. The following statement is contained in the summary of the workshop findings (238, 244, 245): "The reason why multiphase flows are much more difficult to analyze than single phase flows is that the phases assume a large number of complicated configurations. Therefore, it should not be surprising that the understanding of why the phases configure in a certain way is the principal scientific issue. Research is needed which identifies the microphysics controlling the organization of the phases, develops physical models for resultant multiscale
interactions and tests their validity in integrative experiments/theories that look at the behavior of a system."

(b) Flow regime maps

Thus, the first order goal in analyzing the behavior of gas-liquid flows is the prediction of the configuration of the phases, called a flow regime. This information is usually presented in two-dimensional maps. Our initial effort was the definition of flow regimes for air-water flows. The test sections were transparent so visual observations were possible. We immediately became aware that this approach could be misleading in differentiating between slugs and pseudo-slugs. Slug flow is defined as the intermittent appearance of slugs of liquid that fill the whole cross-section of the pipe and move over a stratified layer of liquid. They usually entrain gas. At large gas velocities, slugs are highly aerated and have short lengths. Therefore, they look like large amplitude waves that touch the top of the pipe (which we call pseudo-slugs). This is why visual observations have difficulties. We, therefore, developed an instrumental method to identify slugs (127), by measuring the time-varying pressures at two locations. We were able to identify slugs as causing large pressure pulses, that are coherent over large lengths of pipe, and move at a velocity close to that of the gas flow.

In Paper (124), we examined the effect of a change of pipe diameter from 2.54 cm to 9.53 cm on the conditions needed for a given flow regime to exist. Mandhane plots which use the superficial gas velocity, $U_{SG}$, as the ordinate and the superficial liquid velocity, $U_{SL}$, as the abscissa were made (See Fig 47) The study was of interest in that flow behavior was studied in a larger pipe than usually employed in laboratory experiments. Two striking new results were obtained: The pseudo-slug regime is quite large in a smaller pipe. The superficial liquid velocity needed for the initiation of an intermittent flow is much larger in larger pipes.

The effect of liquid viscosity on the flow regime map is explored in (141).

(c) Transition to slug flow

The prediction of the transition from stratified to intermittent flow had received the attention of a number of researchers in 1986. A popular approach was to associate it with the instability of a stratified flow to long
wavelength disturbances. Inviscid plug flow was assumed for both the gas and the liquid. Instability occurs when the wave-induced pressure suction on the crests of waves overcomes the stabilizing effect of gravity. The predicted gas velocity for a given height of the stratified layer was found to be too large by a factor of about two.

This problem was revisited in Papers 120, 126. The chief difference from the prevailing theory was the inclusion of viscous effects in both the gas and the liquid. Inviscid theory predicts a wave velocity which is equal to the liquid velocity and, therefore, no effects of liquid inertia. Viscous long wavelength theory provides an equation for the wave velocity which is similar to what is predicted by kinematic wave theory. That is, the wave velocity is not equal to the liquid velocity. As a consequence, the liquid inertia is destabilizing and the transition to an intermittent flow is predicted to occur at smaller heights of the stratified layer. Good agreement with experiments was realized for air-water flow at \( U_{SG} < ca3.3 \text{m/s} \).

At large gas velocities, the transition to intermittent flow was recognized in (124) as occurring by a coalescence of large amplitude irregular waves. Thus, the use of a theory for transition based on the growth of small disturbances for these situations should not be expected to be operable. Researchers hypothesized that, at large gas velocities, transition is governed by a necessity that the ratio of the height of the stratified flow to the pipe diameter, \( h/d_i \), be larger than 0.5. Paper 142 provides theoretical definitions of two necessary conditions for the existence of a slug. The second of these is based on a material balance. As the slug moves downstream it consumes liquid at a rate \( q_F = (c_F - U_{L1})A_{L1} \), where \( U_{L1} \) and \( A_{L1} \) are the velocity and area of the liquid in the front of the slug, and \( c_F \) is the velocity of the front of the slug. Liquid is shed from the back of the slug at a rate \( q_B \). For \( q_F > q_B \), the slug will grow in length. A necessary condition for a slug to exist is, therefore, \( q_B \leq (c_F - U_{L1})A_{L1} \).

Paper 142 argues that the back of a slug may be considered to be a bubble and uses Benjamin's analysis for reversible flow past a cavity to represent \( q_B \). The necessary condition is then calculated to be \( h_L/d_i = 0.562 \) and \( \left[ (c_F - U_{L1})/(gd_i) \right]^2 = 4A_{L1}/\pi d_i^2 = 0.542 \). This analysis was consistent with available data. Its shortcoming was that it neglected the effects of aeration.

This lead to the study in (195) which provides measurements of the shedding rates of slugs that were obtained by a judicious use of conductance probes. These clearly show that the representation of the tail of a slug by a Benjamin bubble is valid only for very small gas velocities. Over most of the
velocity range, it is found that that the critical height of a stratified layer is given by \( h/d, \approx 0.20 - 0.25 \). From conservation of mass one can show that the velocity at the rear of the slug, \( c_{B1} \), is given by \( q_B = (c_{B1} - U_{L3})(1 - \alpha)A \), where \( U_{L3} \) and \( \alpha \) are the velocity and void fraction in the body of the slug.

The most important result coming from (195) is the demonstration that necessary condition 2 predicts the transition from stratified to slug flow at large gas velocities where slugs form by coalescence of large amplitude irregular waves. Furthermore, this analysis shows that necessary condition 2 is not dependent on gas density. The long wavelength viscous analysis predicts that the critical gas velocity decreases with increasing gas density. As a consequence, one expects that a consideration of the necessary condition for the existence of slugs will be operative at low gas velocities when the gas density is large.

Viscous long wavelength instability theory gives the picture that a long wavelength wave grows until it touches the top of the pipe. This idea was explored in (177). Air-water flowing a 0.095m horizontal pipe was studied at atmospheric conditions. A specially designed entrance box was used. Conductance probes along the test section and visual observation allowed a study of how slugs form. They were found to evolve from waves, with a length of about 0.085m, that are generated by a Jeffreys sheltering mechanism (See Section 8b.), whereby energy fed to the liquid by the wave-induced pressure variations in phase with the wave slope is dissipated by liquid viscosity. These waves grow in height and eventually double in wavelength by a nonlinear resonance mechanism. (See Paper 149 and Section 14c.) Depending on the height of the liquid, the growth can lead to a breaking wave or to waves that fill the whole pipe cross section. (See Section 14e and Paper 241 for a discussion of how the presence of polymers can affect this doubling of wavelength.) Calculations are presented in (177) to determine whether the bifurcated wave will break before it reaches the top of the pipe. These used the suggestion of Banner & Phillips that wave-breaking occurs when the velocity of the liquid at the crest equals the wave velocity.

In view of the above observations, the success of viscous long wavelength theory in predicting the appearance of intermittent flows in the air-water system at low gas velocities is mystifying. In this context, it is worthwhile to point out that a small amplitude very long wavelength disturbance, that was observed prior to the appearance of 0.085 m waves, could be a trigger.
The effect of liquid viscosity on the initiation of an intermittent flow was studied in (141) in 2.52 and 9.53 cm pipes. As discussed in Section 8b, Jeffreys waves are generated when energy fed to disturbances by wave-induced pressure variations in phase with the wave slope is larger than viscous dissipation. At viscosities of 20 cp and larger, dissipation is so great that Jeffreys waves do not appear and transition to intermittent flow occurs with a stratified flow which has a smooth interface. Calculations and observation show that the initiation of small wavelength waves is the precursor to the appearance of slugs in viscous liquids. With a slight change of flow conditions, large amplitude isolated disturbances grow out of these regular small wavelength waves. A possible mechanism for the rapid evolution of these ripples is proposed in (149, 168) and in Section 14(d), whereby the initiation is triggered by a Kelvin-Helmholtz instability.

If the liquid layer is thick enough, the disturbances touch the top of the pipe and form slugs. In thin layers that occur for \( U_{sg} > 4 m/s \), these disturbances evolve into irregular waves. Paper (141) provides measurements which show that inviscid Kelvin-Helmholtz stability theory predicts the appearance of slugs on thick layers with a high liquid viscosity and irregular waves on thinner layers. At \( U_{sg} > 4 m/s \) transition is predicted by considering necessary conditions for the existence of slugs.

The bottom line is that there is no single theory to predict the transition from a stratified to an intermittent flow. Three mechanisms are considered above: a viscous long wavelength instability, a Kelvin-Helmholtz instability and a necessary condition for the existence of slugs. A methodology for predicting transition which uses these three mechanisms, is outlined in (229).

Papers 149, 168 speculate that a subcritical KH instability predicted for waves with finite amplitude could trigger the formation of slugs on low viscosity liquids in large diameter pipes. This has not been confirmed as yet.

(d) Pseudo-slug flow

Paper (229) shows that a viscous long wavelength stability analysis does a good job in predicting the critical height for the transition to a slug flow for air-water at low gas velocities and atmospheric pressure. However, the critical liquid flow is often poorly predicted. This, to a large extent, is due to the presence of large amplitude roll waves (See Section 8c.) on the stratified flow at the transition from a pseudo-slug flow to a slug flow.
A study was carried out in (236) to resolve this paradox. The experiments were done with air-water flow in a 2.54 cm horizontal pipe, for which a large pseudo-slug region exists. Time-varying liquid holdup was measured with conductance probes. The techniques described in Section 14b were used to differentiate between slugs and pseudo-slugs which, in (236), are identified as large amplitude roll waves (described in Section 8c). Changes in the holdup pattern and the static pressure fluctuations were studied at a fixed superficial gas velocity, $U_{SG}$, as the superficial liquid velocity, $U_{SL}$, was increased.

These showed that viscous long wavelength stability theory (120) describes the initiation of pseudo-slug flow. (See Section 14c.) At very low gas velocities, this theory predicts the transition to slug/plug flow. At somewhat larger gas velocities, the analysis predicts the appearance of roll waves on a stratified layer which is not thick enough for the existence of stable slugs. As the liquid flow increases beyond this (at constant gas flow) it was found that the average height increases only moderately, principally because the velocity of the roll waves increases to accommodate the increased liquid flow. At very high liquid flows, a transition to slug flow occurred, principally through coalescence of roll waves. Transition to slug flow is predicted by considering the stability of slugs. (See Section 14b.)

An important observation in (236) is the very large gas drag which is caused by the appearance of large amplitude roll waves. For example, the friction factor at transition to slug flow can be 30 times larger than would exist at a smooth interface. This has also been noted in (218), which considers the transition to slug flow in inclined pipes. (See Section 14g.)

(e) Effect of drag-reducing polymers

Several researchers had studied the effect of drag-reducing polymers on pressure drop in gas-liquid flows. Our efforts in this area were different in that we looked at their influence on the configuration of the phases, which is so important in understanding the performance of gas-liquid flows. This provided an interpretation of pressure drop measurements but, more important, it suggested new opportunities for controlling and optimizing the performance of gas-liquid flow systems.

A study of the effect of additions of drag-reducing polymers to air-water flow in a horizontal 9.53 cm pipe produced the striking result of a change of the flow pattern from annular flow to stratified flow (Paper 220). This was observed with concentrations of only 10-15 ppm of a co-polymer of
polyacrylamide and sodium acrylate. The change was accompanied by a drag-reduction of 48 per cent. The mechanism involved the damping of disturbance waves (which are similar to a turbulent flow). This, in turn, reduces the rate of atomization and the ability of the liquid to spread upward along the wall. A secondary effect is a damping of the waves on the stratified flow that finally results.

A second study (223) in a horizontal 2.54 cm pipe allowed an examination of the influence of a drag-reducing polymer on an annular flow at larger gas velocities for which the film on the wall is evenly distributed and the disturbance waves form a ring around the circumference, similar to what is observed in vertical flows. Again, the disturbance waves were destroyed, the flow pattern was changed and a large decrease in drag was observed.

The data for a 2.54 cm pipe suggest that drag-reducing polymers destroy disturbance wave. It is speculated in (223) that, in vertical pipes, this would lead to small atomization, thicker wall films and possibly flooding.

The influence of a drag-reducing polymer on pseudo-slugs, interfacial drag and transition to slug flow was examined for air and water flowing in a 2.54 cm horizontal pipe (233). The addition of polymers was found to increase the critical $\frac{h}{d_i}$ and the critical $U_{sl}$ for initiation of slugs & roll waves and to increase the range of superficial liquid velocities over which pseudo-slugs exists. The addition also causes a decrease in the interfacial friction coefficient, which can be particularly large in the pseudo-slug regime. Because of the decrease in $f_i$, a pronounced increase in $\frac{h}{d_i}$ is observed. The decrease in $f_i$ also causes a decrease in the frictional pressure drop. However, this is counterbalanced because of an increase in the gas velocity that is associated with an increase in the liquid holdup.

We interpret these results by arguing that the polymers destroy turbulence in the slugs and dampen waves at the interface. We suggest that laminarization of the slug is associated with an increase in the velocity of the bubble behind the slug. (See Section 14c.) This causes an increase in the shedding rate and, therefore, a destabilization of the slug. If one argues that the pseudo-slug regime is terminated when the roll wave velocity is approximately equal to the slug velocity, then the increase in slug velocity, caused by the addition of polymer, could also be associated with the lengthening of the range of $U_{sl}$ over which pseudo-slugs exist.

The addition of polymers does not diminish the importance of roll waves. On the contrary, the liquid holdup of these waves increases. We suggest that the decrease in $f_i$ is associated, mainly, with damping of small
wavelength waves on the roll waves and on the stratified flow between the roll waves.

Paper 241 examines the effect of drag-reducing polymers on a stratified flow in a 9.53 cm pipe. Large reductions in drag (as indicated by decreases in the pressure drop) were realized. These could be associated with decreases in the turbulence in the liquid and in the waves located at the interface. Section 14c discusses the evolution of waves into slugs (177). Particularly noteworthy is the discovery that waves generated in the early part of the pipe double in wavelength (by a resonance mechanism) and that these waves present the possibility of a transition to slug flow. A puzzling result in Paper 241 is that this doubling in wavelength is inhibited by the presence of polymers.

(f) Transition to plug flow

A slug is defined as a highly aerated mass of liquid moving over a stratified layer. Gas is entrained at the front of the slug, which resembles a turbulent hydraulic jump. For a constant liquid superficial velocity and decreasing gas velocity, slugs have been reported to change into "plug flow", which is defined as long bubbles moving along the top of the pipe. This transition is explored in (146).

The experiments were performed for air and water flowing in a 0.0953m pipe. The first effect noted as the gas velocity decreased (at a constant superficial liquid velocity) was the change of the front of the slug to a staircase configuration (See Fig. 51b), where the liquid level increased in two steps. This occurred at a superficial gas velocity of 0.62 \( \text{m/s} \). The Froude number characterizing this transition is

\[
Fr = \left( \frac{c_F - U_{1l}}{gd_i} \right) \left( \frac{gd_i}{c_F} \right)^{0.5} = 2,
\]

where \( c_F \) is the translational velocity of the front of the slug and \( U_{1l} \) is the velocity in the liquid carpet. With a further decrease in superficial gas velocity, the level of the liquid in front of the first hydraulic jump increases and the height of the second stage decreased. At \( U_{sg} \approx 0.1 \text{m/s} \) (\( Fr \approx 1.2 \)), the front of the slug took the shape of a Benjamin bubble and aeration of the liquid was negligible (See Fig 51c). This was defined as the transition to plug flow.

In (142) the first of the two necessary conditions for the existence of a slug was that the front of a slug is defined as an irreversible hydraulic jump. The critical Fr defined from the first necessary condition agrees with experimental observations. Further discussion of the transition from slug flow to plug flow is given in (169), which considers stationary slugs.
Paper (229) outlines how the above criterion can be used to present the transition in a Mandhane plot (See Figs. 47, 49).

(g) Inclined pipes

Transition to slug flow is greatly enhanced in pipes that have an upward orientation. Consequently, careful consideration has to be given, in the design of pipelines, so as to have local inclinations which are as small as possible. This motivated a number of researches in our laboratory aimed at obtaining an understanding of how pipeline inclinations affect the mechanism of slug formation (218). (See Fig. 50)

A specially designed system was used (Figs 48, 62) The racks holding the pipeline were mounted on a 27m beam which was supported by a series of five lifting stations and a stationary pivot at one end of the beam. Each lifting station was comprised of a supporting bed, a pair of screw jacks and a pair of rollers. The rollers allowed the point of contact between the bed and the beam to change. Each lifting station was synchronized by a gearing system so that the beam remained straight as it was inclined. The mechanism, which was driven by a 0.25 HP motor, could tilt the pipeline between \(-2^\circ\) and \(+2^\circ\). An electronic revolution counter was used to determine the inclination from a horizontal position. Each degree of inclination corresponded to 3435 revolutions of the main drive shaft. The angle of inclination could, therefore, be obtained with a high accuracy.

The gas and liquid were combined in a tee section at the beginning of the pipeline. Studies of the upward flow of air and water in a 0.0763 m pipe are described in Paper 218. When an upward inclination is used, gravity opposes the flow and the height of a stratified flow is larger at the same superficial gas and liquid velocities. An important contribution in (218) is the determination of a critical superficial gas velocity, \(U_{SGc}\), below which the drag of the gas on the interface is not sufficient to maintain an upwardly flowing stratified layer. Values of \(U_{SGc} = 3\, m/s, 5\, m/s, 7.1\, m/s\) were obtained for inclinations of \(0.2^\circ, 0.4^\circ, 1.2^\circ\). A theory for predicting \(U_{SGc}\) is presented in (218) (See Fig. 50)

It is shown that, for \(U_{SG} > U_{SGc}\), the mechanisms defining the transition to slug flow are the same as found for a horizontal pipes. The critical \(h/d_i\) was found to be independent of orientation. For a given \(U_{SL}\) and \(U_{SG}\), the height of the stratified flow increases with increasing inclination angle because of the opposing force of gravity, as noted above. Thus, inclination increased the instability. One of the difficulties in predicting the critical
condition is that the waves and, therefore, the interfacial drag increase as transition is approached. A procedure for calculating transition is given in (229).

The most important contribution of (218) is the description of the formation of slugs for $U_{SG} < U_{SGc}$. Below $U_{SGc}$, slugs must carry liquid out of the pipe to allow for a positive $U_{SL}$. Liquid at the inlet forms a pool whose height changes with time. Slugs appear very rapidly when the pool reaches a critical height. Paper (218) suggests that this occurs because of a Kelvin-Helmholtz instability. The Kelvin-Helmholtz wave formed at this time propagates downstream as a slug. The height of the pool suddenly decreases and another slug is formed when the liquid at the inlet builds up to a critical condition. The result is a periodic appearance of slugs which move over a stratified layer which is flowing backward toward the inlet. The calculation of the period between slugs should be possible by using the notion of a Kelvin-Helmholtz instability. However, some consideration needs to be given to the fact that the slug flow regime in upwardly inclined pipes differs from what is found for horizontal pipes, in that the stratified flow is moving in a direction opposite to that of the slug.

Results for the effect of downward inclinations of $-0.2, -0.5, -0.8$ degrees for air-water in a $0.0763m$ pipe are presented in (214). In these cases gravity aids the flow. Thus the liquid in a stratified flow moves at a larger velocity than in a horizontal pipe for the same superficial gas velocity and the same liquid height. An interesting feature of these studies is the damping of Jeffrey waves. Thus, at low $U_{SG}$, the interface was smooth. (Similar behavior had been observed by Andreussi & Persen.) The reason for this damping has not been established. However, we believe that it is associated with the increased wave velocity observed in downwardly inclined flows. The critical layer in the gas phase (where the gas velocity equals the wave velocity) moves farther away from the interface. This would have an effect on the wave-induced variation of pressure in the gas at the interface.

The transition to slug flow was shown in (214) to be predicted by the viscous long wavelength analysis presented in (120). However, the mechanism for transition is quite different from what is observed for horizontal pipes (177). Very long wavelength waves grow and the gas velocity above the crests increases. Eventually, it is large enough that a Kelvin-Helmholtz instability causes a closure of the air gap and the formation of a slug. The slug frequency was shown to be equal to the frequency of the long wavelength waves.
Fig. 43  Revolving test section that allowed a mapping of the local droplet flux and the local streamwise gas velocity.

Fig. 44  Curves of constant streamwise velocity at low drop concentrations. The dotted curve indicates the locations of wave peaks.

Fig. 45  Curves of constant streamwise velocity at high drop concentrations.

Fig. 46  Curves of constant drop concentration. Conditions are the same as for Fig. 45.

Fig. 47  Flow regimes for air and water flow in horizontal pipelines.

Fig. 48  Horizontal gas-liquid flow facility. Pipes are mounted on a beam whose inclination angle is controlled by screw mechanisms along the length.
Fig. 49  Flow patterns for gas-liquid flow in a horizontal pipe. (a) Annular pattern at low liquid flow rate, \( U_{SG} = 120 \text{ ft/s}, U_{SL} = 0.05 \text{ ft/s} \), (b) annular flow with waves wrapping around the pipe, (c) a pseudo-slug at a low gas velocity, (d) plug flow.

Fig. 50  Pattern for 0.4 degree upflow, \( U_{SG} = 0.54 \text{ m/s} \). Slugs are seen to be moving upward over a downward flowing base layer.

Fig. 51  Back of air pockets. (a) Typical slug flow, \( U_{SG} = 1.45 \text{ m/s}, U_{SL} = 0.95 \text{ m/s} \); (b) Staircase observed at the beginning of the transition to plug flow, \( U_{SG} = 1.45 \text{ m/s}, U_{SL} = 0.95 \text{ m/s} \); (c) Symmetric cavity (a Benjamin bubble), \( U_{SG} = 0.1 \text{ m/s}, U_{SL} = 0.95 \text{ m/s} \).
15. Slug flow

(a) Frequency of slugging

A number of investigators have used a model for predicting pressure drop and liquid holdup that pictures slug flow as consisting of a sequence of aerated turbulent units that travel at an approximately constant velocity over a stratified layer. (See Fig. 52) A critical issue in developing these models is the prediction of slug frequency and the distribution of slug lengths. This is important since the frequency of pressure pulses that accompany slugs could affect the operation of a pipeline system.

Papers 211 and 253 investigated the mechanisms governing slug frequency. Experiments were done for air-water flow in a horizontal 0.0763m pipe with a length of 23m. The mixer was a tee at the inlet. The air was forced through an orifice located five pipe diameters upstream of the tee section. The velocity of the air through the orifice approaches the velocity of sound. Consequently, downsteam variations of gas phase pressure caused by the formation of slugs or their discharge from the pipe do not strongly affect the inlet air flow. The variation of the liquid height with time was measured with conductance probes located along the pipe. Pressure patterns associated with the passage of a slug were measured using a piezoresistive transducer mounted flush with the wall.

These experiments showed that available theories on frequency of slugging are not correct. The central feature of the results is that a number of mechanisms are operative, depending on the conditions. Interpretation of the results is helped by a consideration of three heights: $h_o$ is the height of a stratified liquid layer below which slugs cannot be generated; $h_c$ is the critical height of a stratified layer at which slugs appear; $h_e$ is the equilibrium height which would exist for a stratified layer on which slugs are not present. Another important concept is associated with the Froude number, based on the velocity & height of the stratified layer, 

$$Fr = \frac{U_L}{g\bar{h}},$$

where $g$ is the acceleration of gravity. For $Fr > 1$, disturbances at the interface cannot move upstream.

For small gas velocities, the height of the liquid layer at transition to slugging is greater than $h_o$. For small liquid and gas velocities slugging occurs far downstream of the entry. The slugs appear periodically; that is, the interval between the appearance of slugs is constant. The Froude number characterizing the flow in the stratified flow for this case is less that unity. The height of the layer at the inlet was found to be approximately constant...
and equal to \( h_s \) when slugs are present (not \( h_e \), as supposed by several investigators). Thus, information about the slugs is transmitted backward to the inlet, because \( Fr < 1 \).

The mechanism governing the frequency of appearance of slugs for low gas and liquid velocities is given in (211): Slugs evolve from waves at the interface by the process described in Paper 177 and in Section 14 b. These waves grow and eventually touch the top of the pipe to form a slug at a given distance downstream, \( L_p(t) \). The liquid behind the slug collapses and a depression wave moves upstream (because \( Fr < 1 \)). It is reflected from the inlet where the height of the liquid is kept at \( h_s \) by the incoming liquid. Shallow water wave equations are solved to describe this process. Another slug is presumed to form when \( h = h_s \) at \( L_p/t \), at a time after the formation of the slug. Measurements of the time variation of the height profile provide support for this suggested process. Furthermore, calculations of the time interval between slugs agrees with the measured frequency of slugging. (The pipe location at which slugs form, \( L_p \), had to be measured.)

Slugs with regular frequencies were also observed for flow in downwardly inclined pipes where frequency is given by the frequency of the unstable long wavelength viscous waves and in upwardly inclined pipes where the frequency is given by the time needed for the liquid pool at the inlet to reach a height where the gas flow over it has a velocity which promotes a Kelvin-Helmholtz instability. Except for low gas and liquid velocities slugging is stochastic in horizontal pipes. Papers (213) and (252) investigate the properties of this regime. The distribution of the time intervals between slugs, which equals the reciprocal of the frequency of slugging, \( f_s \), is directly related to the distribution of slug lengths. An approximate semi-theoretical relation between the average slug length, \( L_s \), and the frequency of slugging, \( f_s \), is given as \( f_s d_t / U_{sl} = 1.2(L_s / d_t)^{-1} \) in (253). Thus, stochastic slugging can be described as a distribution of slug lengths or as a distribution of time intervals.

If the pipe is long enough, that a fully-developed condition is reached for which slugs with a random distribution of lengths move over a stratified layer of height \( h_0 \). An examination of this distribution function seems to conform to the suggestion of several researchers that the slug must exceed a certain minimum length, \( (L_s)_{min} \), in order to be stable. Our measurements give a value of \( (L_s / d_t)_{min} \approx 5 \). Also, in agreement with other investigators, we find that a log-normal function provides a reasonable fit to the distribution of slug lengths. The two constants in this function can be evaluated by
specifying the mean slug length, \( \bar{L}_S \), and the standard deviation of slug lengths, \( \sigma_{LS} \). Values of \( \bar{L}_S/d_i \approx 20 \) and \( \sigma_{LS}/\bar{L}_S \approx 0.35 \) are suggested in (252). The large value of \( \sigma_{LS} \) reflects the large range of slug lengths observed.

From theoretical considerations, the frequency of slugging, \( f_s \), for a number different systems is correlated by plotting \( f_s d_i/U_{SL} \) versus \( U_{SL}/(U_{SG} + U_{SL}) \). The data suggest a minimum at \( U_{SL}/(U_{SG} + U_{SL}) \approx 0.2 \). However, this minimum is not pronounced and \( f_s d_i/U_{SL} = 0.05 \) provides a rough fit for all the data. It is of interest to note that the product of \( f_s d_i/U_{SL} = 0.05 \) and \( \bar{L}_S/d_i = 20 \) is in agreement with the relation between \( f_s \) and \( L_S \) suggested above.

Papers (211) and (253) suggest that the prediction of the probability function representing the distribution of slug lengths is the principal theoretical problem in describing stochastic slugging. This necessarily requires a knowledge of how slugs evolve along the pipe to form a fully-developed pattern. Details about this process are given in (253): Large amplitude waves appear at the inlet. One of these evolves into an incipient slug (perhaps, aided by wave coalescence). This slug is characterized by a sharp drop in the liquid level in its wake and a larger velocity than the waves. It grows by consuming larger amplitude waves. If it attains a sufficient length to be stable it will propagate downstream. If not, it will decay to form a large amplitude wave (See Fig. 56).

These observations lead to the suggestion of the following model: The height of the stratified layer entering the pipe is close to the equilibrium value, \( h_e \). A development length of at least \( L_D \) is needed for the formation of unstable waves (See Fig. 55) which have the possibility to form a slug. Thus, slugs appear in the unstable region \( L > L_D \). The appearance of a slug at \( t = t_n \) and \( L_F > L_D \) is accompanied by the depletion of the liquid behind the slug where the height is assumed to be \( h_0 \). Because \( Fr > 1 \), a bore is formed where the level of the liquid in the stratified layer changes from \( h_e \) to \( h_0 \). The bore moves downstream with a velocity \( c_{jump} \) and is supplied by the liquid entering the pipe. Slugs have a larger velocity than the bore so the depleted layer increases in length with increasing time. The probability of forming a slug somewhere in the pipe for \( t > t_n \) increases linearly with the length of the unstable region, \( L(t) - L_D \). This assumption results in a higher probability of forming larger slugs, that is, to a long tail in the probability function, such as found in a log-normal distribution. It is further assumed that it is equally probable for a slug to appear anywhere in the unstable region and that there
is a minimum length, \( (L_s)_{\text{min}} \), below which the slug is unstable. If \( L_s < (L_s)_{\text{min}} \) and the stratified flow in front has a height \( h \leq h_s \), the slug collapses. Subsequent slugs can increase their length by adding the liquid deposited by decayed slugs (See Figs. 54, 56).

A computational algorithm based on the above model is developed in (253). Rough agreement with measurements is realized. However, this work is not final. More work is needed.

Two sets of measurements reported for very large petroleum pipelines show values of \( \frac{L_s}{d_i} \) which are an order of magnitude larger than what is observed in the laboratory. This puzzling result is discussed in (253). A number of researchers have suggested that fully developed slugs defined in (211) and (253) change further because slugs coalesce. This is said to occur because the "gas bubbles" between two adjacent slugs move at different velocities when slugs do not have the same length. Measurements of \( c_B \) presented in (253) do not support this notion. Furthermore, values of \( \frac{L_s}{d_i} \) reported from several laboratories for \( L/d_i \) ranging between 200 and 9500 show no systematic variation of \( \frac{L_s}{d_i} \) with pipe length, \( L \).

Possible explanations for the difference between field tests and laboratory measurements are given in (253).

(b) Model for slug flow

The large pressure pulsations that accompany slugs have been used by many researchers to detect their existence. This suggests that there would be a discontinuous increase in pressure over each one of the slugs in a pipeline. These results indicate that a general method to predict the overall pressure drop should be developed from an understanding of the behavior of individual slugs and a prediction of the number of slugs in a given length of pipe. An investigation is described in (179) in which the pressure profiles over individual slugs were measured by using a piezoresistive transducer. The void fraction, the velocity, the length of individual slugs and the liquid height in front and behind the slugs were measured with two pairs of conductance wires stretching across the diameter of a 0.095m horizontal pipe carrying air and water.

The pressure drop over a slug was considered to have the following contributions: \( \Delta P_r = \Delta P_f + \Delta P_f + \Delta P_f \), due to the existence of a hydraulic jump in front of the slug, a frictional pressure drop in the slug, and a pressure change in the rear of the slug due to the change of velocity along the top
Models for these three contributions are developed in (179). (See Fig. 53) The measurements allowed the determination of $\Delta P_r$ and an approximation of the individual contributions from $\Delta P_h$, $\Delta P_f$ and $\Delta P_r$. Good agreement between experiment and the models was realized both for stable and unstable slugs.

A critical issue, which was not considered in enough detail in (179) is the aeration of a slug. At low and moderate gas velocities gas enters by the tumbling in front of the slug and leaves from the back. A special facility was developed to measure rates of aeration (169). A stationary hydraulic jump was formed by introducing a disturbance into the liquid flowing under a gate located in a pipe that is inclined downward. The jump occurred downstream of the gate. If air is allowed to bleed into the space between the jump and the gate, the jump will reach a stationary state and the rate of entrainment of gas by the jump will be equal to that in the downstream collector, which was maintained at constant pressure by a special device. The liquid behind the jump will have a gradually sloping tail and will not fill the whole length of pipe. Thus, the static jump that is formed resembles the rapidly moving slugs observed when gas and liquid flow in a pipeline and is, therefore, called a stationary slug. The rate of entrainment was determined by measuring the rate of flow into the bleed line.

The stationary jump differs from a moving slug in that the wall resistance is negligibly small. Measurements of the pressure changes over the front and back agree with analyses of the type used in (179) to describe moving jumps, where the velocity of the liquid entering the stationary slug is analogous to the velocity of a moving slug relative to the velocity of the liquid in the carpet in front of the slug. Thus, observations of a stationary slug offer the opportunity to study, in detail, the behavior of the front and back of a moving slug.

At $Fr = U_{t1}/(gd_1)^{1/2} = 1.7$, the stationary slug is unaerated and the front part of the tail agrees almost exactly with the inviscid equation describing a Benjamin bubble. The whole tail is represented closely by a composite of the rapidly varying inviscid solution and a more slowly varying solution, which uses viscous shallow liquid equations. [The hypothesis in (142) is thus confirmed, in detail, for non-aerated slugs.] For a given $h_1/d_1$, the minimum flow rate at which slugs are formed is characterized by a $Fr$ which increases with decreasing $h_1/d_1$. The tails of these stationary slugs are found to be Benjamin bubbles. For small Fr, the slugs are unaerated. For flow rates larger than the minimum, the aeration increases and the tail can no longer be described as a Benjamin bubble. Bubbles are occluded at the front and rise
to the top of the slug unit. At large enough void fractions, a highly unsteady tail is produced.
16. Stratified flow in a pipe

(a) Prologue

At low flows of a gas and a liquid in a horizontal pipeline, a stratified regime exists whereby the liquid moves along the bottom of the pipe and the gas, concurrently with it. Although this pattern may be considered the simplest for gas-liquid flows, predictions of its behavior are not completely satisfactory. A summary of our studies of this flow pattern is given Paper 199.

The usual practice is to consider the interface to be flat and horizontal. Geometric parameters are, then, the length of the interface, \( S_i \), the area of the liquid, \( A_L \), the area of the gas, \( A_G \), the lengths along the circumference wetted by the gas and the liquid, \( P_G \), \( P_L \), the height of the stratified layer at its center, \( h_L \). If \( h_L \) and \( d_i \) are known, all of the other parameters can be obtained from geometric considerations. Of interest to the design engineer is the prediction of volume fraction of liquid, \( A_L / (A_L + A_G) \), and the pressure gradient, \( dP/dx \).

Analytical approaches assumed a fully-developed flow and used overall force balances for the gas,

\[
-A_G \frac{dP}{dx} - \tau_{WG} P_G - \tau_i S_i = 0,
\]

and for the liquid,

\[
-A_L \frac{dP}{dx} - \tau_{WL} P_L + \tau_i S_i = 0.
\]

Interfacial stress, \( \tau_i \), and wall stresses, \( \tau_{WL}, \tau_{WG} \), vary in space. The simplification of representing them by constant values is utilized. Friction factor relations are used to calculate the stresses:

\[
\tau_i = f_i \left( \rho_G U_{GB}^2 / 2 \right), \quad \tau_{WL} = f_L \left( \rho_L U_{LB}^2 / 2 \right), \quad \tau_{WG} = f_G U_{GB}^2 / 2.
\]

The friction factors are assumed to be given by the Blasius equation or by Poiseuille's law for Reynolds numbers of \( \text{Re}_L = d_L U_{BL} / \nu_L \) and \( \text{Re}_G = d_G U_{BG} / \nu_G \), with hydraulic diameters defined as \( d_L = 4A_L / P_L \), \( d_G = 4A_G / (P_G + S_i) \). (See Papers 123,129.) Furthermore, \( f_i \) was assumed to be equal to \( f_G \). The performance of this model was not satisfactory.

Papers 125,126,129 describe experiments aimed at improving this approach. These were carried out in horizontal pipes with diameters of 2.52 and 9.53 cm. The liquid viscosity was varied between 1 and 80 cP. Conductance probes were used to measure the mean liquid height and wave properties. The gas phase force balance was used to determine \( \tau_i \) from measurements of the pressure gradient and \( h_L \). The liquid momentum balance was used to calculate \( \tau_{WL} \).
The main finding is that the assumption \( f_i = f_g \) leads to serious errors. Improved predictions can be realized by relating \( f_i \) to wave properties at the interface. The use of the concept of a hydraulic diameter for the gas phase was tested (125) by measuring the pressure drop for gas flow in a 2.52 cm pipe with inserts at the bottom of the pipe in the shape of the idealized liquid flow. Good agreement with the friction factor relation for a circular pipe was observed if the Reynolds number is calculated using a hydraulic diameter.

The use of measurements for single phase flow in a pipe to predict \( \tau_{WL} \) is questionable, so additional improvements can be realized by using a different relation for \( \tau_{WL} \).

(b) Wave properties

Wave properties were studied in (129). Four patterns were defined:
Regular 2D waves are associated with wave-induced variations of the pressure in the gas at the interface which are in phase with the wave slope.
Irregular large amplitude waves and atomization are associated with wave-induced pressure variations at the interface in phase with the wave height. At conditions corresponding to the presence of pseudo-slugs, large amplitude waves coalesce to form a collection of larger amplitude waves similar to the roll waves described in (7). Linear stability theory was used to predict conditions for the initiation of these instabilities.

(c) Interfacial stresses

The most striking result is that the ratio of the interfacial friction factor, \( f_i = (1/2) \rho_g U_{GB}^2 \), expressed as \((f_i/f_g) - 1\), scales linearly with the ratio of the wave height to the wavelength, \( \Delta h/\lambda \), over a range of liquid viscosities of 1-80 cP and for pipe diameters of 2.52 and 9.53 cm. There is no general relation for \( \Delta h/\lambda \), so it is instructive to examine the dependency of \( f_i/f_g \) on flow conditions. For a fixed thickness of the liquid layer, \( h_L/d_i \), the friction factor, \( f_i/f_g \), increases with increasing superficial gas velocity. For small gas velocities, where the interface is smooth or has regular waves, \( f_i/f_g \approx 1 \). With the appearance of irregular waves, the friction factor increases strongly with the superficial gas velocity, \( U_{GS} \). The critical condition for this to occur, \( U_{Gr} \), is given by the linear Kelvin-Helmholtz instability. The following
relations are suggested in (125,126): \( f_i/f_g = 1 \) for \( U_{gs} \leq U_{gr} \) and \( (f_i/f_g)_{-1} = 15(h_L/d_i)^{0.5}(U_{gs}/U_{gs,j} - 1) \) for \( U_{gs} > U_{gs,j} \). (See Fig.57)

(d) Prediction of wall stress, \( \tau_{wl} \)

The use of a friction factor relation, \( \tau_{wL} = f_L\left(\rho_LU_{BL}^2/2\right) \), where \( f_L \) is obtained from single phase studies, is useful because of its simplicity. However, some improvement is obtained by recognizing that the liquid layer is sheared and by using eddy viscosity relations. (See Section 8f and Paper 31.) However, a study by Suzanne at the Institut de Mecanique de Toulouse has revealed the existence of secondary flows in the liquid, which can transfer momentum. Paper (125) did not take this into account.

These relations for \( \tau_i \) and \( \tau_{wL} \) are exploited in (125) to develop design procedures to predict the pressure drop and holdup. Good agreement with experimental data was realized. (See Fig. 58)

(e) Stratified-annular (Pool – Annular flow)

Studies of the distribution of liquid around the circumference of a pipe for annular flow reveals a pool configuration at at small gas flows. (See Section 13e.) The group \((\rho_G/\rho_L)^{1/2}Fr\), where \( Fr = U_{sg}/(d_i g)^{1/2} \) is the defining parameter. For air and water at atmospheric conditions, \( Fr < 50 \) is a criterion for a pool model to be appropriate. Most of the wall layer is located at the bottom of the pipe, so the asymmetric model for the film distribution described in Section 13e might not be useful. This lead Williams to explore a pool-annular model (Williams, R.W., "Effect of pipe diameter on horizontal annular two-phase flow", PhD thesis, University of Illinois, Urbana, 1990), where most of the liquid is in a stratified flow and the remainder is a film flowing along the wall.

The simplification is made that the interface is flat in the stratified portion. The height of the liquid at the center of the stratified layer is given as \( h_L \), which is related to the liquid flowing in the pool by using momentum balances for the gas and the liquid. The pool flow and the interfacial stress are calculated by using the approach for stratified flows outlined above.

The following results for air and water flowing in a 0.095 m pipe provide some insights about the pool model: The fraction of the liquid in the pool varies from 0.95 to about 0.85 as the superficial gas velocity increases.
from $30 \text{ m/s}$ to $45 \text{ m/s}$. The fraction of the liquid entrained in the gas as drops increases from 0.12 to 0.50.
Fig. 52  Typical slug observed in a 0.024 m horizontal pipe.

Fig. 53  Pressure and holdup profiles for a slug. Comparison of theoretical and experimental pressure drops due to hydraulic jump at the front, frictional pressure losses in the body and exit. Holdup does not reach 1.0 because of aeration.

Fig. 54  Development of slugs: wave pattern close to inlet, L/D = 4, slugs at L/D = 37, close to a fully developed state, L/D = 200.

Fig. 55  Formation of a slug by coalescence.

Fig. 56  Stochastic formation of slugs. Two large amplitude waves form at inlet. The first encounters a thick enough layer to form a stable slug. The second collapses because it cannot pick up enough liquid to form a stable slug.
Fig. 57  Stratified flows for $D = 2.52 \text{ cm}$,
$m_L = 4.5 \text{ cP}$, $U_{SL} = 0.03 \text{ m/s}$:
(a) $U_{SG} = 4.3 \text{ m/s}, f/f_S = 1.3$
(b) $U_{SG} = 6.4 \text{ m/s}, f/f_S = 1.6$
(c) $U_{SG} = 9.8 \text{ m/s}, f/f_S = 5.5$
(d) $U_{SG} = 12.0 \text{ m/s}, f/f_S = 7.0$
(e) $U_{SG} = 16.0 \text{ m/s}, f/f_S = 9.5$

D = 2.52 cm  $\mu_L = 4.5 \text{ cP}$  $U_{LS} = 0.03 \text{ m/s}$

Fig. 58  Stratified flows for $D = 2.52 \text{ cm}$,
$\mu_L = 4.5 \text{ cP}$, $U_{SG} = 9.8 \text{ m/s}$:
(a) $U_{SL} = 0.002 \text{ m/s}, f/f_S = 1.6$
(b) $U_{SL} = 0.01 \text{ m/s}, f/f_S = 3.5$
(c) $U_{SL} = 0.02 \text{ m/s}, f/f_S = 4.0$
(d) $U_{SL} = 0.03 \text{ m/s}, f/f_S = 5.5$
17. Fundamental studies of nonlinear interfacial waves

(a) Prologue

The central role of waves in understanding gas-liquid flows motivated several fundamental studies of wave behavior.

Our interest in using drag-reducing polymers to change the behavior of gas-liquid flows (Section 14e) was, to some extent, motivated by studies of their effect on the propagation velocity and damping of interfacial disturbances introduced by a wave-maker in a stationary liquid layer. An unexpected observation was the appearance of higher frequency waves than were introduced by the wave-maker. This motivated an investigation, which included a mathematical study of the interaction of several wave components.

Most of the theoretical studies of wave behavior have utilized assumptions which are strictly correct for small amplitudes. Yet, in many cases, the nonlinearity is an essential feature. Examples of our effort to study nonlinear effects are analyses of the interaction of several wave components in a static layer, cited above, and a consideration of the generation of finite amplitude Kelvin-Helmholtz waves by an air flow. An outcome of our work on the second of these is the suggestion of a way for predicting wave height, interfacial drag in stratified gas-liquid flows and another mechanism for the initiation of slugs which might be useful in large diameter pipes.

(b) Effect of drag-reducing polymers on waves

In order to obtain insights about the effect of drag-reducing polymers on wave behavior, Kim (Kim, Young Yuel, "Wave motion on viscoelastic fluids", MS thesis, University of Illinois, Urbana, 1968) examined the behavior of solutions of Polyox WSP-301 and of Polyhall. Experiments were carried out in a tank (40in by 8in by 4in) made of Plexiglas. A plunger introduced waves of a given frequency at the center of the tank. These propagated to both ends of the tank, which contained screens that absorbed the energy of the waves. The surface was illuminated by a stroboscopic lamp. This light was visualized on a Teflon sheet located below the interface. The wave velocity was determined from the measured wavelength and the known frequency. A technique developed by Bernier (Bernier, L., "Wave motion on viscoelastic fluids", MS thesis, University of Illinois, Urbana, 1966) was used to measure the wave height. An array of parallel lines oriented in the same direction as the dipper was placed below the wave
A time-exposure picture with a camera located above the interface produced a series of bands whose thickness depends on the wave slope. The wave height was calculated from this measurement and the known frequency.

Measurements of the relation between the wave frequency and the wavelength were described by inviscid theory; that is, no effect of drag-reducing polymers was noted. Wave damping was observed close to the wave-maker. These experiments can be interpreted by assuming that the polymers form a relatively rigid viscoelastic film at the interface.

(c) **Behavior of waves at "near resonance"**

The evolution of sinusoidal free waves into a pattern with two peaks had been observed by a few researchers. This behavior was explained as occurring when a pair of capillary-gravity waves, with wave numbers \( k = (g/n\sigma)^{1/2} \) and \( k = (ng/\sigma)^{1/2} \), can travel at the same velocity where \( n \) is a whole number different from 1. This is a resonance condition in that the exchange of energy between the pair is enhanced. Phillips showed that resonance could occur at third order (three components interacting) for gravity waves on deep water and at second order (two waves interacting) on shallow water. Kim observed that, by changing the frequency of a wave-maker, one could see a stable dimple riding on gravity waves at sharply defined frequencies, as expected. Between these frequencies, waves with multiple peaks were present. These were not stable in that they continued to change their form as they propagated down the tank.

To my knowledge, this latter behavior had not previously been reported. Because of our curiosity about this phenomenon, the investigation on the effect of polymer additions veered in another direction. We built (59) a more elaborate facility than used in the MS study of Kim. It was a Plexiglas tank, 30 ft long, 1 ft wide and 8 in deep. The tank contained distilled water, which was kept at a depth of 3.5 in. Special precautions were taken to keep the interface clean. Surface elevations were determined by measuring the resistance between two parallel 0.001 in platinum wires. For shallow water experiments a platform, 4 ft long, was placed inside the tank. In steady-state experiments, waves were generated by a half-immersed Plexiglas dipper with a thickness of 3/16 in. In experiments with wave packets, a high potential wave generator was used. A brass bar, of ¼ in diameter, was placed across the tank about 5mm above the water surface. An alternating electric field, with a maximum peak-to-peak voltage of 5 KV, superimposed on a 9 KV steady field was applied between the bar and the water. By modulating
the alternating field, an amplitude modulated wave packet could be introduced on the interface (See Fig.59).

When the dipper operated at 5 cycles/s in a deep water layer the wave profile at different distances from the wave-maker does not change its form dramatically. When the frequency was such that surface tension is important, higher order harmonics appear, which can be large enough that additional crests are observed. These effects are particularly evident close to second order resonance, \( n=2 \), where there is a monotonic transfer from the first to the second harmonic; but, in general, they are observed over a frequency range which depends on the initial wave steepness.

The resonant frequency at which a large monotonic transfer of energy from the first to the second harmonic could be decreased by using a shallow liquid layer. However, if the layer was too shallow, growth of higher harmonics obscured this clear cut transfer from the first to the second harmonic. The large exchange of energy amongst the different harmonics was explored by decreasing the height of the water layer while the frequency of the dipper was held at 5 cycles/s. At a height of about 1 cm very little distortion of the sinusoidal wave was observed. At heights of 0.7 and 0.85 cm, a large growth of second harmonic occurs close to the wave-maker. At greater distances progressively higher harmonics appear in the wave pattern. For example, at a height of 0.79 cm, waves with five crests were observed over the period of the wave-maker. As the height of the layer is further decreased, the number of harmonics that are present in the pattern in significant amounts decreases.

It was not clear how to use available theories (which focused on waves of permanent form) to describe the observations. Thus, a straightforward approach of representing the behavior by considering an interaction of four wave components was used (59). The displacement of the interface from its average location is given as \( \eta = \sum_{n=0}^{N} a_n \cos(n\theta + \gamma_n) \), where \( a_n \) is the amplitude of the \( n \) harmonic, \( \theta = kx - \omega t \), \( k \) is the wave number, \( \omega \) is the frequency, \( x \) is the coordinate in the direction of propagation, \( t \) is the time, and \( \gamma_n \) is the phase angle. The velocity field is described by the equations for an inviscid, irrotational flow. The full nonlinear form of the conditions at the interface was used. Quadratic interactions were neglected. The system is described by calculating the variations of the amplitudes, \( a_n \), and their phases, \( \gamma_n \), with time. Viscous damping was introduced as a second order effect. The calculated variations with time are related to the distance from their wave-maker by using the group velocity. In the context of this formulation, the
energy transferred among the Fourier components observed under certain conditions is explained in terms of the rate of change of the relative phases of the different harmonics. Waves of permanent form are defined as conditions for which $\frac{d\gamma_n}{dt} = 0$.

If the wave-maker operated at 9.86 cycles/s and the height of the liquid layer is decreased, the monotonic growth of the second harmonic observed in deep layers changes to a cyclic growth and decay. As predicted by theory, the observed frequency for resonant growth of the second harmonic decreases with decreasing depth.

A more spectacular growth of higher harmonics could be observed in shallow layers. For example, the operation of the wave-maker at 3.04 cycles/s with a 0.65 cm water layer displayed seven additional crests. The interaction equations would need to use more interacting wave components to capture the behavior of all the experiments with shallow layers. However, good agreement was realized between experiments and calculations for cases in which the use of four harmonics is sufficient.

The work carried out by Kim opened new ways of understanding nonlinear interactions which, regretfully, we did not have an opportunity to pursue.

**(d) Finite amplitude waves in concurrent flows**

The work described above was aimed at understanding free waves; that is, under conditions that there is no flow and the effect of the density of the medium above waves is not having an effect. The waves which play such an important role in two-phase flows, however, are generated by the relative motion of two fluids (not by a wave-maker). The theoretical approach usually taken to understand these waves (Sections 8 and 14) has adopted a linear assumption whereby the waves are of small enough amplitude that the equations defining the system can be linearized. Inviscid Kelvin-Helmholtz waves were found to play a major role in gas-liquid flows. They receive their energy from wave-induced pressure variations which are 180 degrees out of phase with the wave height.

In Papers 135, 140, 149, we investigated the description of periodic waves. (See Fig. 60) The focus was on nonlinear Kelvin-Helmholtz waves. The system studied was concurrent flow of two fluids. The flow was considered to be inviscid and the velocities were uniform in the two phases. The difference of the velocities between the two phases (the current velocity) was designated as $U_r$. 
Saffman & Yuen considered gravity waves on fluids of infinite extent and identified two different factors which limit the existence of a stable solution. The first of these is called a "geometric limit", at which wave profiles become unphysical as the wave height increases. The second is a "dynamical limit". It is encountered when the current velocity is increased, with the wave height fixed. For \( U_r \) larger than a critical, \( U_{rc} \), solutions for the wave height cease to exist. For very small wave heights, the dynamical limit is associated with the Kelvin-Helmholtz instability.

Paper (135) extends the analysis in that it considers the effect of the layer height, \( h \). New theoretical & computational methods were needed. The properties of weakly nonlinear steady waves were obtained by using Whitham's averaged variational principle. A numerical approach based on the boundary integral method allowed calculations to be made over a wide range of conditions.

The increase in the critical current velocity with wave amplitude (based on a dynamical limit) observed by Saffman & Yuen was found to be less pronounced with decreasing depth. For small enough layer heights the effect of increasing wave amplitude is just the opposite of what is observed for unbounded fluids; the critical velocity decreases with increasing amplitude and there is no steady wave solution of given wavelength for current velocities larger than the critical predicted by linear theory.

Results on the geometric limit were somewhat different from what had previously been suggested. The location in the wave profile where a vertical tangent appears moves away from the crest and approaches the trough as the current velocity exceeds the Kelvin-Helmholtz linear critical velocity. A possible interpretation is given in (135).

In Paper 149 we examined finite amplitude capillary-gravity waves of permanent form that are propagating on the interface between two unbounded fluids in relative motion. Chen & Saffman had shown that when free surface waves (zero current velocity) reach a certain height a bifurcation (caused by resonance) is possible whereby a doubling of the wavelength occurs. Paper 149 shows that, in general, the presence of a current velocity increases the height for this bifurcation to occur. A new result is that for waves with lengths shorter than the resonant wavelength and approaching it, the Kelvin-Helmholtz instability becomes increasingly subcritical. An example of this resonance is presented in Section 14c, where the transition to slug flow for air-water flows is described.

We suggest that a Kelvin-Helmholtz instability can trigger a bifurcation. This notion can provide a mechanism for the formation of slugs when air and a viscous liquid (greater than 20cP) flow through a horizontal pipe. For
this situation, capillary-gravity waves appear on a smooth interface at the gas velocity required for a linear Kelvin-Helmholtz instability. These waves rapidly evolve into slugs by a process which is not understood. The calculations presented in (149) show that a bifurcation is possible when these waves grow just slightly in height. The paper suggests that this could be the first step in the development of a slug in viscous liquids.

For liquids with viscosities close to water, capillary-gravity waves are generated (by a sheltering mechanism) at much lower current velocities than are needed for a linear Kelvin-Helmholtz instability. However, since the KH instability can occur at lower gas velocities for finite wave amplitudes, it is possible for a subcritical KH instability to trigger the formation of slugs from waves created by a sheltering mechanism (149, 168). We estimate that slugs would form by this mechanism for an air-water flow at a current velocity of 4.5 m/s. Observations in pipelines with diameters of 2.52-9.53 cm show that slugs appear at lower current velocities. (See Section 8.) A theory based on a viscous long wavelength instability is consistent with the data. However, this theory defines a critical velocity which increases with pipe diameter, so it could predict an unrealistically high critical velocity. Thus, for very large pipes it is possible that the mechanism outlined above could be operative.

(e) Wave height in stratified flow

The discovery in (125) that the friction factor characterizing the interfacial stress in wavy stratified flows is a function of the ratio of the wave height, $\Delta h$, to the wavelength, $\lambda$, prompted an investigation which utilized the studies of finite amplitude Kelvin-Helmholtz waves discussed in Section 17d to provide a relation for the wave height. We could not obtain an analytical expression for $\Delta h$. However, in (140), the analysis is used to suggest the dimensionless groups defining $\Delta h$.

In order to proceed with this study it was assumed that $\Delta h$ correlates with the geometric limit since it is often associated with wave breaking. Paper (140) develops a relation for $\Delta h/\Delta h_{\text{deep}}$ as a function of $kh_L$, where $\Delta h_{\text{deep}}$ is the limiting height of the waves on a deep liquid layer and $k = 2\pi/\lambda$. This relation is represented as $\Delta h/\Delta h_{\text{deep}} = \alpha(kh_L)$.

From dimensional reasoning and experimental data, Paper 140 suggests $\Delta h/\lambda = 0.079\alpha(U_G - U_L)/U_{cl}$, where $U_{cl}$ is the critical relative velocity for the existence of linear waves. There is a similarity between this relation and the empirical correlation suggested in (125) and in Section 16c. The use of these
relations requires a knowledge of the wavelength $\lambda$. Suggestions for approximating this quantity are given in (140).
18. Natural convection effects

(a) Prologue

The Graetz equation, with modifications to account for variations of the liquid viscosity with temperature, does a satisfactory job in predicting heat transfer rates for laminar flow of most fluids. However, it fails miserably for liquids with viscosities close to that of water. This prompted a series of experiments to discover the reasons for this behavior.

(b) Vertical flows

Work was initiated in an undergraduate thesis (Kabel, Robert Lynn, "A study of instabilities in the flow through a tube caused by natural convection", BChE thesis, University of Illinois, Urbana 1955) that examined how heating affects flow patterns for water flowing upward. A glass pipe with a deposit of metal on its outside was used. The metal film was so thin that one could see inside. The pipe was heated by passing an electric current through the metal film. The flow was slow enough that it was laminar when the tube was not heated.

Dye was injected through a capillary tube located at the pipe center below the test section. For isothermal situations, a streamer was observed to move through the pipe. However, when the heating current was turned on, a surprising behavior was exhibited which, to our knowledge, had not been reported in the literature. The streamer moved into the heating section, then stopped and altered direction. Its appearance was similar to what would be observed for flow around a streamlined body.

This behavior was explored further by flooding the tube with dye and observing what happens when the dye is flushed away. (See Fig. 64) In the final stages, the flushing produced a paraboloid of dye located in the center of the pipe. Increasing and decreasing the flow caused the paraboloid to move up and down. (A motion picture was made to catalogue the behavior.) At the downstream end, the boundaries of the paraboloid wavered and eventually took on a turbulent appearance. These observations produced the following interpretation: The lower density of the hot fluid near the wall caused it to move upward relative to the mean flow. A different behavior would be expected for cooling an upflow. The colder fluid near the wall would move downward relative to the mean flow.

A more detailed study of these phenomena is described in (9). Water was circulated through a 2.19 cm glass tube. The tube was heated or cooled
by circulating hot or cold water through a Lucite jacket located concentrically to the tube. The expectation was that heating in upflow and cooling in downflow would respectively behave the same as cooling in upflow or heating in downflow if flow distortions were due primarily to natural convection effects. This was found to be the case. The distorted profiles can become unstable, so that transition to turbulence is possible at much lower Reynolds numbers than for isothermal flows. This could lead to higher rates of heat transfer. The main point of our studies was to understand these distortions and whether they can lead to a turbulent flow. The results have a broader interest in that they test notions about the relation of instability to inflectional velocity profiles.

Two types of heat transfer experiments are conveniently conducted, one using a constant heat flux (Kabel thesis and Paper 22) and the other using a constant temperature wall (Paper 9). For the case of a constant heat flux, if changes in temperature affect only the gravity term in the equations of motion, a condition is attained far downstream in the heat transfer section such that there is no further change in the shape of the velocity profile and such that temperature and pressure are varying linearly with distance downstream. The maximum distortion of the velocity profile occurs in this region of fully-developed flow. For a constant temperature wall the fully-developed condition exists downstream where the temperature of the fluid equals the temperature of the wall. The maximum distortion of the velocity profile occurs somewhere between the inlet and the outlet.

An analytical solution for the fully-developed velocity and temperature profiles for flow through a tube with a constant heating or cooling rate at the wall is presented in (9). (See Fig. 63, 67) The flow is assumed to be laminar and the fluid viscosity is constant. The results mimic observations discussed above. The parameter governing the behavior of the system is the quotient of the Grashof number, \( Gr = a^3 \rho g \beta |\Delta T|/\mu^2 \), and the Reynolds number, \( aU_b \rho / \mu \), where \( a \) is the pipe radius, \( \beta \), the coefficient of cubical expansion, \(|\Delta T|\), the absolute value of the difference between the wall temperature and the centerline temperature.

For heating in upflow (or cooling in downflow), the effect of density variation with temperature is to increase the velocity near the wall and cause a flattening of the velocity profile in the center of the pipe. At \( Gr/Re > 32.94 \), the profiles have a dimple at the pipe center and the point of maximum velocity moves radially toward the wall as the ratio of the heat flux to the flowrate \((Gr/Re)\) is increased. There is a reversal at the center of the pipe for
$Gr/Re$ greater than 319.1. All profiles with $Gr/Re$ greater that 32.94 have points of inflection.

If natural convection is opposed to the direction of forced flow (heating in downflow) the calculated velocity at the wall decreases as the heat flux is increased. For $Gr/Re = 9.87$ there is a point of inflection at the wall and at greater $Gr/Re$ the calculated point of inflection moves radially toward the center of the pipe. At $Gr/Re = 52.2$, the calculated velocity gradient at the wall becomes zero and reversal is predicted to occur at the wall for further increases of the ratio of heat flux to flowrate.

Paper 14 uses boundary-layer theory to obtain an approximate solution for the shape of the velocity profile as a function of axial location in the heat transfer section, when heating was done with a constant temperature wall. At the location of maximum profile distortion the velocity and temperature may be described by the same equations as used for fully-developed flow under constant flux heating. However, for the case of a constant temperature wall it is convenient to use a parameter different from $Gr/Re$.

Experiments with a constant flux are described in (22). Five levels of the Roger Adams Laboratory were used in order to have a vertical test section that was long enough to approach a fully-developed flow. A 60 foot length of 0.787 inch copper pipe was used. The downstream 50 feet was wrapped with heating ribbon and surrounded with a 1 inch thickness of insulation. Both upflows and downflows of water were studied. Transition to an unsteady flow was detected by measuring temperature fluctuations at the outlet and in the wall.

For upflow experiments, it was found that an instability occurs when $Gr/Re = 33$, that is, when the velocity profile first developed inflection points. This instability requires a length of pipe (that depends on Reynolds number) to develop into a noticeable disturbance or turbulence. Thus, the appearance of an irregular flow was first evident for $Gr/Re > 33$.

Contrary to these results, heating in downflow produced a sudden transition once the velocity profiles became unstable. The critical $Gr / Re$ is much larger than the theoretical value of 9.87 required for a symmetric flow to develop inflection points. Thus, it appears that inflection points close to the wall do not lead to an unsteady flow. This, in addition to the observation of a more sudden explosive transition suggested that transition is associated with separation at the wall, which is predicted to occur at $Gr/Re = 52.2$ for a fully-developed symmetric flow. Further insights are obtained from our experiments with a constant temperature wall, where a transparent wall
allowed visual observation of the behavior of dye that is injected into the flow.

A summary and interpretation of the work described in (9, 14, 22) is presented in (12). This suggests that the instability caused by inflection points close to the wall leads to an asymmetric flow (for cooling in upflow). Asymmetry was observed for $Gr/Re = 74$. The transition to an eddying flow occurred at $Gr/Re = 75 - 141$.

The equipment described in (22) was used to determine heat transfer coefficients for vertical flows, defined as $h = q_w / \Delta T$, where $q_w$ is the heat flux at the wall and $\Delta T = T_w - T_b$, which is not varying in the flow direction (since a fully-developed condition is considered). The results of this study are summarized in (23). The departure from laminar theory is interpreted as resulting from distortions of the velocity profile and from the presence of a disturbed flow.

(c) Horizontal flows

Our studies of the effect of natural convection on heat transfer with horizontal pipes were initiated by N. Apostalakis (Apostalakis, N., "Effect of heat transfer upon flow field at low Reynolds numbers in horizontal tubes", MS thesis, Univ. of Illinois, 1957). A jacketed transparent exchanger was used to heat or cool water flows. The path of an injected dye streamer was studied. Unlike what was observed in vertical configurations, an unsteady behavior was not caused by heat transfer. However, a strong secondary flow consisting of two circulation cells was observed at small temperature differences. An understanding of this secondary flow was the central focus of future work.

Since the flow is nonturbulent, the differential equations representing the velocity field are known. However, because of our need to calculate three components of the velocity, a straightforward solution is not available. A central feature of the research is the simplification of the defining equations. Mikesell (Mikesell, Ritchie Dean, "The effect of heat transfer on flow in a horizontal pipe", PhD thesis Univ. of Illinois, 1963) showed that perturbation solutions, that had been explored in other laboratories, are valid at Grashof numbers which are too small to be useful. Measurements of the temperature field suggested a boundary-layer analysis in that a region exists close to the wall where the secondary flow shows a sharp change in the temperature perpendicular to the wall; that is, a boundary layer region exists whose thickness, $\delta_T$, is small compared to the pipe radius. This leads to a
simplification of the equations describing the temperature field and the secondary flow.

However, there is a serious difficulty with this approach, in that the velocity and temperature boundary conditions at the outer edge of the boundary layer are not specified. This is because the boundary layer feeds upon and empties into the core; that is, the fluid outside the enclosed boundary layer. Because the boundary-layer flow and its boundary conditions are not independent of each other, experimentation was needed to clarify the problem. The specification of core conditions is simplified by using the experimental observation that isotherms in the core are horizontal so that the temperature varies only in the vertical direction, Y. The spatial variation of the temperature in the core, \( T_c \), can be calculated by equating the heat convected upward in the boundary layer at a given Y to the heat convected downward in the core. This requires the calculation of the volumetric flow upward in the boundary layer. The specification of this quantity, then, provides the coupling between the boundary layer and the core.

The theoretical background for this approach and the mathematical formulation are given in (42). For simplification, the wall temperature is assumed not to vary around the circumference. The bulk averaged temperature, \( T_b \), is calculated from an overall energy balance. The heat flux at the wall is fixed so that both \( T_w \) and \( T_b \) are increasing linearly with distance downstream.

A Grashof number is defined as \( Gr = a^3 \beta g \Delta T/\mu^2 \). From dimensional reasoning, it can be shown that the dimensionless boundary layer thickness, \( \delta_r/a \), varies as \( (Gr \cdot Pr)^{-1/4} \). Thus, boundary-layer theory is valid for large \( (Gr \cdot Pr) \). Let x and y be coordinates in the circumferential and wall-normal directions in the thermal boundary layer. The temperature balance equation is simplified because molecular transfer is important only in the y-direction and convective transfer, in the axial direction, can be ignored. Thus, the secondary flow controls the rate of heat transfer to the wall. A Nusselt number can be defined as \( Nu = q_w a/\Delta Tk \). It is shown in (42) that \( Nu = C_1 (Gr \cdot Pr)^{1/2} \). The equations defining the secondary flow and the temperature field need to be solved to obtain the constant \( C_1 \).

Prandtl number, Pr, has an interesting effect. Again, from dimensional reasoning, (42) shows that for large Pr the convective term in the equations for the secondary flow does not affect the shape of the profile of axial velocity, even when a vigorous secondary flow is present. Thus, the
streamwise velocity profile retains a parabolic shape in heat transfer situations (except for effects of temperature on the viscosity).

An approximate integral method to calculate the velocity and temperature fields was developed by Siegwarth (Siegwarth, David Philip, "Effect of natural convection on heat transfer in a horizontal tube", MS thesis, Univ. of Illinois, Urbana, 1966) and Readal (Readal, T.C., "Effect of density gradients on the rate of heat transfer for flow through a horizontal pipe, MS thesis, Univ. of Illinois, Urbana, 1966). A constant flux heat transfer for which the temperature was not varying circumferentially was considered. The assumption of a large $Pr$ was used. Results from this effort are summarized in (42). Good agreement with experiment was realized if liquid viscosity is evaluated at the bulk temperature of the fluid. The Nusselt numbers are predicted by the above equation if $C_i = 0.45$.

(There are several errors in Paper 42. In Equation 44, the right side should be multiplied by 2. In Equation 50, the right side should be multiplied by $\pi/2$. The constant in Equation 72 and in figure 7 should be 0.45, rather than 0.471. The numbers in the $\phi_C$ column in Table 1 should be $-0.531, -0.531, -0.528, -0.513, -0.479, -0.416, -0.321, -0.191, -0.031, 0.156, 0.358, 0.562, 0.743, 0.813, 0.922, 0.926, 0.926, 0.926$.)

Results of experiments to test the theoretical results, presented in (42), are provided in Paper 48. The system studied was a 36 ft length of electrically heated 2 1/2 in I.D. pipe through which ethylene glycol was circulated. By using a relatively thick wall, 1 in, and a material of high conductivity, aluminum, the heat flux to the outside was distributed in the pipe wall so that the temperature around the inside wall showed only a small variation even though the heat transfer coefficient changed by as much as 40-fold around the circumference. The temperature and velocity distributions were measured at the outlet of the heat-transfer section. The temperature at the wall was measured along the whole length of the pipe. The bulk temperature, $T_{wb}$, was calculated from the measured power supplied to the heat transfer section and the heat losses through the insulation. Tests were conducted under conditions where $T_{wb} - T_\theta$ was constant over the last 6 ft of the heat transfer section.

The temperature and the streamwise velocity were measured at the outlet with a traversing mechanism that entered through the outlet. The probe holder could pivot as well as rotate so it was located at different radial positions to $\pm 0.005$ in and different angular positions to $\pm 1.0^\circ$. Finite difference solutions of the partial differential equations describing the system are also presented in (48). Calculated velocities and temperatures
provided a check on the measurements. However, the calculations expanded the scope of the study by providing results for the secondary flow.

The studies were carried out for $Pr = 80$. Relatively large secondary flows were observed for temperature differences between the wall and the fluid as low as $0.05^\circ F$. For $(GrPr)^{1/4}$ greater than 30, boundary-layer theory appears to be a good approximation to the temperature field. Again, isotherms in the core are shown to be horizontal. Although the secondary flow had a large effect on the temperature field, it showed little effect on the axial velocity profile, as had been predicted for large Pr. The secondary flow pattern showed relatively large upward velocities near the wall and small downward velocities in the core.

(d) **Electrochemical systems**

Because of our use of electrochemical methods to study flow fields, we had an interest in natural convection caused by reactions at a surface. As a consequence, we studied mass transfer to a cylinder by carrying out an electrochemical reaction on a cylindrical cathode placed in a stagnant fluid (66). The system chosen was the reaction of potassium ferricyanide to form potassium ferricyanide. No supporting electrolyte (such as potassium hydroxide) was used.

The cathode test sections were plated brass cylinders, with a length of 2.5 cm and diameters of 1.2 or 2.5 cm. These were supported on plexiglas or nylon rods located in a plexiglas tank. The anode was a nickel cylinder having a diameter of 30 cm and a length of 17 cm, centered around the cathode. A hole was drilled in each of the cathodes so as to accommodate a 0.075 cm platinum wire that was mounted flush to the cylinder wall. By rotating the cathode, the position of the wire could be changed.

The voltage applied to the electrodes was large enough that a limiting condition was reached whereby the reaction rate is rapid enough that the concentration of ferricyanide at the cathode surface was zero. The electric currents flowing to the cathode and to the embedded electrodes were used to calculate the total mass transfer rate and the local rates around the circumference of the cylinder.

The chemical reaction at the surface creates a density difference from that of the bulk density. This gives rise to natural convection currents. The flow around the surface is characterized by a Grashof number, defined as $a^3g\gamma\Delta\rho/\rho_u^2$. The flow of electric current to the cylinder is the sum of a mass transfer rate and an amount due to migration in the electric field. The
latter quantity and $\Delta \rho$ were calculated by methods given by Wilke, Eisenberg and Tobias.

A mass transfer coefficient, $K$, can be defined as $N = K(C_B - C_w)$, where $C_B$ is the concentration of the reacting specie in the tank and $C_w = 0$ is the concentration at the surface of the electrode. A Nusselt number, $\text{Nu}$, is defined as $ka/D$, where $D$ is the molecular diffusion coefficient. At large enough $(GrSc)^{1/4}$, where $Sc$ is the Schmidt number, the concentration and velocity fields are described by boundary-layer theory, where the external field is the stagnant condition that exists in the tank.

Measurements of the distribution of mass transfer rates around the cylinder were found to confirm this approach. At large enough $Gr$, turbulence and an increase in the mass transfer rate is noted in the rear of the cylinder. At still larger $Gr$ the flow in the front of the cylinder showed an unsteady behavior. This suggested the possibility of a Görtler instability. A local minimum in the mass transfer rate was noted in this region.

The average mass transfer rate to the cylinder was given by $\text{Nu} = 0.42(GrSc)^{1/4}$. This is quite close to the transfer rate observed for heat transfer inside a horizontal pipe, pointing out the similarity of the two problems.
Fig. 59  Development of a wave packet at $n = 2.021$ (981 c/s) with distance from wave maker.

Fig. 60  Large amplitude wave.

Fig. 61  Hisao Kada, PhD 1959, and Hanratty in Tyoko, circa 1995. Kada, a veteran of WW2, was my second PhD.

Fig. 62  Dave McCoy at the two-phase flow facility, circa 1977.

Fig. 63  Nikolaos Andritsos, Anastasius Karabelas, Hanratty at Monastery of Stavronikita, Mount Athos peninsula, circa 1988.
Fig. 64 Visual study of the effect of heating on an upflow of water. The tube was flooded with dye. The paraboloid is the remnant after flushing with water.

Fig. 65 Cooling upflow creates an asymmetric field with downward flow on one side of the pipe. Again dye is purged from a flooded field.

Fig. 66 Theoretical fully developed velocity profiles for downward flow in an electrically heated pipe.

Fig. 67 Theoretical fully developed velocity profiles for upward flow in an electrically heated pipe.
19. Packed, fluidized and sedimenting beds

(a) Prologue

During my employment at Battelle Memorial Institute, in Columbus, Ohio, I was involved with the development of a gas phase catalytic process for the production of the rocket fuel, hydrazine. Most of my effort was devoted to finding the right catalyst, but of equal importance was the design of the reactor. In particular, one needed to pay attention to the decomposition of the desired product. This motivated a parallel study of this decomposition in a silica vessel (1). The decomposition was found to occur on the wall and not homogeneously.

This work on reactor design opened an interest in pursuing a PhD degree in the area. Prof. Richard Wilhelm accepted me into his research group at Princeton University. I became interested in reactors which used packed and fluidized beds (5) and carried this interest into the research I started at the University of Illinois.

Design approaches to packed beds considered the average behavior over a number of particles. My thought was that our understanding of the behavior of this system could be enhanced by examining the flow around single particles in the bed.

The need for such a focus motivated several studies. Equipment was developed to study a cubic (or rhombohedral) arrangement of spheres. Fifteen layers of 3 in Plexiglas spheres were packed in a 6 in x 6 in column made of Plexiglas. Half and quarter spheres were placed next to the wall in alternate layers in order to preserve the geometry of a dense cubic arrangement. This test section was located in a gravity flow facility described in (37). Visual studies in this facility used decalin as the test fluid so as to match the refractive index of the Plexiglas.

(b) Fluidization and sedimentation

My principal interest at Princeton was mixing in fluid beds. Paper (5) describes some of the results from my PhD thesis.

It should not be surprising that one of the first research projects initiated at the University of Illinois was a Master thesis on fluidization and sedimentation. The results of this study are presented in Paper 6. In fluidization, a fluid is passed through a bed of spheres at a large enough velocity that the spheres are lifted from the support and suspended in the
flowing fluid. As the velocity is increased the bed expands. Of interest is the prediction of the void fraction, \((1 - \alpha)\), as a function of the fluid velocity.

For our studies of a sedimenting bed, the flow into an expanded bed was discontinued. Solids at the bottom settle out to form a fixed bed on the support. The top level of the bed settles at a velocity \(U_s\). Both \((1 - \alpha)\) and \(U_s\) remain constant during the sedimentation process. The settling velocity decreases with increasing \((1 - \alpha)\).

The relative velocity between the fluid and the particles, \(U_r\), is related to the settling velocity \(U_s\) by \(U_r = U_s/(1 - \alpha)\). This is to be compared with the relative velocity in a fluid bed \(U_s/(1 - \alpha)\), where \(U_s\) is the superficial velocity which the fluid would have if it were flowing through an empty tube. In both cases, relative velocity \(U_r\) is less than would be realized for a single particle falling through a stationary fluid, \(v_r\). Thus, the presence of other particles cause a decrease of the relative velocity between the solids and the fluid.

A goal of (6) was to measure \(U_r\) for both fluidized and sedimenting beds as a function of \(\alpha\). Particular attention was paid to low Reynolds numbers, \(\text{Re}_p = d_p U_r \rho / \mu\), where Stokes law would be applicable. This required the use of fluids with a greater viscosity than water.

The experiments were carried out in a 4 in column. Steel spheres \((d_p = 0.022\text{in}, \rho_s = 7.43\text{g/cc})\), glass spheres \((d_p = 0.028\text{in}, \rho_s = 2.88\text{g/cc})\) and glycerine-water solutions with viscosities ranging from 1-390 centipoise were used. The particle Reynolds number varied between 0.001 and 58.2. A copper screen provided support for the bed. It was necessary to have a uniform flow in order to obtain meaningful measurements. Thus, copper turnings were introduced below the bed support and baffles were placed in the elbow preceding the column. Special precautions were taken to obtain uniform spherical particles. The particles were sieved through screens and, then, through carefully controlled openings between metal slabs. After sieving, the particles were rolled down inclines with different angles to insure they were spherical.

Measurements were made of the dependency of the relative velocity on \(\alpha\). Measurements of \(U_r\) with fluidized and sedimentating beds agree for \(\text{Re}_p < 0.07\). The relation between \(U_r\) and \(\alpha\) was found to be described exactly with a theory by Hawksley whereby the relative velocity was calculated by considering Stokes law, derived for a single particle settling under gravity in an infinite fluid. However, the properties of the fluid were altered to account for the presence of other solid particles. Thus, the density is considered to be that of the suspension and the viscosity is given by a
relation derived by Vand. The agreement was excellent, but not so good at large Reynolds numbers where a different drag relation from Stokes law was used.

At particle Reynolds numbers below 0.8 the beds had a uniform appearance, with no large mass movements of particles. The particles, however, were continually coming together in small groupings and then dispersing. At Re above 2, mass movements of groups of particles were observed. At the highest Reynolds numbers the particle flow pattern consisted of a random eddying motion. There were large variations in the solids concentration, and the motion was quite similar to that obtained in gas-solids systems.

(c) Packed beds; Flow patterns

Pressure drop measurements for flow in a dumped fixed bed of spheres show a linear increase with velocity at low velocities and an approximately quadratic increase at large velocities, similar to what is found for laminar and turbulent flow in a straight pipe. However, relations describing the influence of fluid velocity on pressure drop, heat and mass transfer in a packed bed do not show sharp changes of the type noted at the transition to turbulence in a pipe. Paper 33 provides some insights into this behavior. The system consisted of a dumped bed of 1 in glass spheres in a 12 in diameter glass column.

For the visual studies, methyl benzoate was circulated through the column. Since the refractive index of methyl benzoate closely matches that of glass, it was possible to view the interstices inside the bed when a strong source of illumination was used. A dye was injected through a small hole at the forwardmost point of a sphere located in the bed. At low flow rates the colored filaments had steady complicated patterns. Once the pattern was established, it remained immobile for very long periods of time. As the flow was increased, the colored streamers were observed to wave gently and at a Reynolds number (based on the diameter of the spheres and the fluid velocity in the empty column) of 300 the pattern was found to be highly turbulent. The transition from a steady to a haphazard turbulent flow was relatively sudden in that it occurred for \( \Re_p = 110 - 150 \). (See figures 68, 69)

Measurements of local values of the magnitudes (not the direction) of the velocity gradients at the surface of a sphere were obtained at fourteen locations around a meridian. The electrochemical techniques described in Section 1 were used. The electric current (proportional to the rate of mass transfer) flowing to the electrodes was steady for low Reynolds numbers and
highly turbulent for large Reynolds numbers. The transition indicated by these measurements occurred for \( \text{Re}_p = 110 - 150 \). The surprising outcome of this study is that the mean velocity gradients at the surface of the spheres show the same functionality with increasing Reynolds number for steady and unsteady flows.

These results indicate that the drag on the spheres in the bed is not strongly dependent on whether or not the flow is turbulent; the change in the direction of flow caused by the presence of the spheres has a much stronger effect than do deviations due to turbulence. This suggested that one should focus on the flow around single spheres in order to develop a model for a bed.

In Paper 39, electrochemical methods described in Sections 1 and 2c were used to measure overall & local mass transfer rates, and local magnitudes of the wall shear stress for a single sphere in a dumped bed. The measurements showed large variations of the local mass transfer coefficient and local wall shear stress around the surface of the sphere, in striking contrast with the assumption of a uniform mass transfer rate which is sometimes used to analyze the performance of packed beds. The distinct difference in the behavior of the front and back portions of a sphere in a packed bed seemed to contradict the notion that a packed may be viewed as a system of interconnected channels. A model which emphasizes the flow around individual particles is more consistent with the measurements, which suggest that the flow over the surface of a sphere could be described as a three-dimensional boundary layer.

The strong influence of contact points suggested that better progress could be made by using regular arrangements of spheres. This motivated the development of equipment (described in the Prologue) to apply techniques developed in (39) to a cubic array of spheres.

In Paper 53 experiments are described in which a test sphere in a cubic array had a number of taps through which a dye solution was injected. (See Fig. 70) The test sphere could be rotated through 360° so that studies with taps covering the whole surface of a sphere were available. The directions of the dye streamers close to surface at different locations were determined. In some tests a large amount of dye was injected and then purged from the field by the flow. The last places from which the dyed fluid disappeared were at separation bubbles.

The flow at a Reynolds number of 82 was steady even though nine regions of reversed flow were observed on the surface of the test sphere. (See Fig. 71) Four of these were located behind contact points at an angle, \( \phi \).
of 45° from the front of the sphere, and four, behind contact points at \( \phi = 90° \). The reversed flow located at the rear of the sphere had four contact points at \( \phi = 135° \) as its forward points. The topology of the limiting streamlines is characterized by a number of singular points. These are identified in (53) for \( \text{Re}_p = 82 \). Transition to an unsteady flow occurred at \( \text{Re}_p = 90 - 120 \). Studies of the topological features of the limiting streamlines at \( \text{Re}_p = 200 \) were more difficult to observe. However, their general features were the same as found at \( \text{Re} = 82 \).

(d) Flow regimes for a regular packed bed

The studies described above suggested that the flow in a packed bed could be analyzed by considering four regimes: (1) creeping flow, \( \text{Re} \to 0 \), (2) steady boundary-layer flow, \( \text{Re} \to \text{Re}_c \), (3) unsteady boundary-layer theory, \( \text{Re} \to \text{Re}_c \), and (4) fully turbulent flow, \( \text{Re} \to \infty \). Stewart and his co-workers at the University of Wisconsin used numerical techniques to solve the creeping flow equations for regular arrangements of spheres. Paper 39 examines the effects of Reynolds number on local velocity gradients on a sphere and concluded that the flow over most of the surface can be described by boundary-layer theory at Reynolds numbers close to \( \text{Re}_c \). It appears that regime 3 can be treated by the same model as regime 2 since no dramatic changes have been found in the pressure drop, the overall mass transfer rate or the pattern of local mass transfer rates on a single packing for steady and unsteady flows. In fact, Paper 54 suggests that boundary-layer theory should provide a good model for \( \text{Re} = 20 - 1000 \), and possibly larger. Some data exist in the literature which suggest that at large \( \text{Re} \) the magnitude of the spatial variation of local time-averaged heat or mass transfer rates becomes much smaller. This would tentatively suggest that as \( \text{Re} \to \infty \) the flow might be approximated as a well mixed turbulent flow.

(e) Boundary-layer model for a packed bed

The boundary-layer model assumes that an inviscid flow approximation is valid over most of the flow field. Close to a boundary a region exists where viscous effects are important. This region is thin enough that the Navier-Stokes equations can be simplified. The solution of these boundary-layer equations requires the specification of the pressure gradient and the velocity at the outer edge of the boundary layer. These are obtained by considering the outer flow to be described by the Laplace equation.
Paper 65 examines the possibility of using boundary-layer theory to describe flow in a close packed cubic array of spheres. An analytical solution of the Laplace equation is not possible because of the geometric complexity of the flow field. Therefore, the outer flow was obtained by using a specially designed electrolytic tank. This approach is described in a PhD thesis by Karabelas (Karabelas, Anastasius J., "Flow through regular assemblies of spheres", PhD thesis, University of Illinois, Urbana, 1970).

It is well known that inviscid, irrotational flow of a fluid is described by the equation $\frac{\partial^2 \phi}{\partial x_i^2} = 0$, where the velocity vector is given by $u_i = -\frac{\partial \phi}{\partial x_i}$, and a repeated index indicates a summation. Term $\phi$ is called the potential function. Lines or surfaces of constant potential are termed "equipotentials". Solutions of the Laplace equation also describe electric fields where $\phi$ corresponds to the electric potential and $u_i$ corresponds to the electric current. This analogy was used to calculate the velocity at different locations on the surface of the test sphere. The method was tested (65) by considering a single layer of spheres located in a rectangular tank. A voltage drop was established over the tank by plate electrodes. The other walls of the tank and the spheres are nonconductors. Embedded electrodes were used to measure the variation of the potential over the surface of a test sphere. Differences in these potentials give the variation of the velocity around the surface.

Paper 65 presents measurements for boundary-layer flow over a sphere in a cubic array of spheres. These include measurements of the magnitude of velocity gradients at the surface and of local mass transfer rates on a sphere that is exchanging mass with the flowing fluid.

Measurements of pressure variations around a single sphere presents a challenge because of the very small differences in pressure. We, therefore, used an air flow at higher velocities. A transducer was invented which is based on measurements of liquid conductivity between two wire electrodes (58). Its main features are its high sensitivity ($2.5 \times 10^{-4} \text{mmHg}$), its efficiency, and the simplicity of its design.

The interpretation of the measurements was facilitated by using the visual studies described in Paper 53 and in Section 19c. This resulted in a map of the limiting streamlines over the surface of a sphere. As mentioned in Section 19c, nine separation bubbles exist. Eight are associated with contact points at $\phi = 45^\circ$ and $\phi = 90^\circ$. A large cloverleaf separated region existed at the rear, which had forward locations at the $\phi = 135^\circ$ contact points. Three types of boundary-layer regions cover the remainder of the surface. These originate from nodal points of attachment at the front of the packing.
at the back sides of the four separation bubbles that originate at $\phi = 45^\circ$ and at the back sides of the four separation bubbles that originate at $\phi = 90^\circ$.

Separation of three-dimensional boundary layers can occur at envelopes of the limiting streamlines, as well as places where the velocity gradient at the surface is zero. Evidence for the existence of these envelopes is given in (65).

The studies described in (65) laid a foundation for applying boundary-layer theory. A finite difference method was developed in (56). A particular problem was avoiding instabilities associated with large cross flows. Example calculations were done for a single layer of spheres. The test sphere in this arrangement has separation bubbles behind four contact points and at the rear of the sphere. The outer inviscid flow was obtained from experiments in an electrolytic tank.

(f) Mass transfer in packed beds

The widespread use of packed beds in the chemical industry has lead to numerous studies of mass transfer. Paper 54 summarizes twenty-eight proposed correlations. A need for a more systematic approach was evident. The proposal was made that this could be accomplished by establishing asymptotic relations for different ranges of variables for a well defined packing. This idea was tested in (54) by using electrochemical methods to measure mass transfer rates to a single sphere in a close-packed cubic array of spheres at a Schmidt number, $Sc$, of 1600.

A Sherwood number is defined as $Sh = kd_P/D$, where $k$ is the mass transfer coefficient, $d_P$ is the diameter of the sphere, $D$ is the molecular diffusivity; a Grashof number is defined as $Gr = d_P g \rho \Delta \rho / \mu^2$, where $\rho$ is the density, $\Delta \rho$, the density change, $g$, the acceleration of gravity, $\mu$, the viscosity; a Schmidt number, defined as $Sc = \nu / D$, where $\nu$ is the kinematic viscosity; a Reynolds number, as $Re = d_P U_s \rho / \mu$, where $U_s$ is the superficial fluid velocity.

For laminar natural convection, $Sh = 0.46(GrSc)^{1/4}$. For forced convection at low $Re$, $Sh = 4.58(Pe)^{1/3}$. For forced convection close to the transition to unsteady flow ($Re_c = 90 - 120$), $Sh = 2.39 Re^{0.56} Sc^{1/3}$. (This equation actually does a good job for $Re = 20 - 1000$. Interpolation formulas were developed for conditions stratiﬁng these asymptotic behaviors. From boundary-layer theory, the exponent on $Re$ would be 0.50, rather than 0.56.
The difference reflects the fact that a boundary layer does not exist over the entire surface of the sphere—and that separated regions are present.

Clearly, this work opens new possibilities for future studies in this area. For example, at very large Re one would expect that laminar boundary layers do not dominate and that flow in the interstices is quite turbulent.

(g) **Wall heat transfer coefficients in packed beds**

Temperature profiles measured in a heat transfer section containing a packed bed show an apparent discontinuity at the wall. Common practice is to represent the temperature by an eddy conductivity model whereby the heat flux at any location is given by the product of an effective conductivity and the temperature gradient. Such calculations need to use the boundary condition 

\[ q_w = h(T_w - T_i) \]

where \( T_i \) is the temperature obtained by extrapolating temperature measurements in the fluid to the wall.

Values of the heat transfer coefficient are found to vary as \( G^{0.77} \) for spherical packing, where mass velocity, \( G \), is the product of the fluid velocity and the fluid density, and as \( G^{0.5} \) for cylindrical packing. Plautz suggested that this difference in behavior exists because spheres create only point contacts with the wall, while cylinders obstruct flow at the wall (Plautz, D., "Transfer rates and wall coefficients in packed beds", PhD thesis directed by Frazier Johnstone, University of Illinois, Urbana, 1953). The \( G^{0.77} \) dependency of \( h \) for spherical packing is understandable if one considers that channeling exists at the wall and that the flow is analogous to turbulent flow in a channel. The results for cylindrical packing were more problematical.

In a brief paper (2), an attempt was made to resolve this paradox. The picture is used that the fluid adjacent to the wall does not flow in a continuous fashion but is sidetracked by packing which is in its path. The heat transfer process at the wall was thought to be similar to a surface renewal model (3, 19) described in Section 6b. Fluid of relatively uniform temperature is thrown against the wall and the fluid adjacent to the wall assumes the wall temperature. Transfer of heat occurs inward in the same manner as if a block of material were placed adjacent to the wall. If the contact time, \( \theta_c \), of the fluid mass with the wall is very small, the heat will penetrate only a small distance inward. According to this picture,

\[ h = 2(\rho C_p k / \pi \theta_c)^{1/2} \]

where \( \theta_c = d_p / CU \), \( U \) is the fluid velocity, and \( C \) is a constant of order unity. Thus, \( (hd_p / k) = 0.97(d_p U \rho / \alpha \mu)^{1/2} \), where \( \alpha \) is the

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void fraction. This relation is in good agreement with measurements for cylindrical packing.
20. Shale oil retorts

(a) Prologue

In the late 1970's and early 1980's I became involved with work at Occidental Petroleum on the modified insitu process for obtaining oil from shale. (See Fig. 74) A rubblized bed is created in a section of a mountain by a judicious use of explosives. The top of the bed is ignited and a combustion wave moves through the raw shale. This creates a flow of (hopefully) inert gas that moves ahead of the combustion wave and decomposes the kerogen in the raw shale. Oil produced by this pyrolysis is carried from the hot zone as a vapor, which condenses on cold shale particles and flows out the bottom of the bed as a liquid product. The pyrolysis leaves behind a bed of particles containing carbonaceous residue (carbon with a small amount of hydrogen), a variety of carbonates, some silicates and a large amount of inert rock. The carbonaceous residue in the processed shale provides the fuel for the combustion. The size of the oil yield depends on keeping the combustion zone separated from the retorting zone, on minimizing the combustion of oil by oxygen which has bypassed the hot carbon residue, and on minimizing coking and cracking of the hydrocarbons that are produced.

The modeling of combustion waves moving through a bed of solids becomes a major consideration. Papers (92, 112, 113) were the first to address this problem.

(b) Combustion of a bed of particles impregnated with carbon

In the first of these papers (92) the combustion of a bed of particles impregnated with carbon was considered. The balance and rate equations are greatly simplified by using a coordinate system which moves at the same velocity as the combustion zone. For large enough times a "fully-developed condition" is reached for which the temperature profile is given by a steady-state solution. A heating zone in which no reaction occurs is given by an unsteady solution. A constant temperature region, between the heating and reacting zones, grows in time.

A parameter $\beta = \frac{\rho_G C_G V_G}{\rho_S V C_S}$, where $C_G$ is the heat capacity of the gas, $\rho_G$, the gas density, $\rho_S$, the shale density, $V_G$, the gas velocity, $V$, the combustion zone velocity, is critically important. For $\beta < 1$, the combustion zone occurs at the front of the thermal wave. For $\beta > 1$, the combustion occurs at the back. For $\beta = 1$, a fully-developed wave cannot exist. Most
experiments with shale retorts are for conditions such that $\beta < 1$. Of particular interest is the calculation of the amounts of oxygen and carbon which are not reacted in the combustion zone. Bypassing oxygen can consume some of the oil product. This leads to a decrease in yield.

The velocity of the reaction zone can be calculated from the gas flow into the bed by a simple mass balance if the fraction of uncombusted oxygen, $\eta_s$, and the fraction of uncombusted carbon, $\psi_s$, are known. The evaluation of $\eta_s$ and $\psi_s$ emerges as the principal theoretical problem. Two kinetic equations for the reaction of the carbon were explored.

The results coming from the analysis are the temperature profiles of the combustion waves and the uncombusted carbon and oxygen, that is, the yield.

(c) Temperature profiles in processed shale

The calculations done in (92) were for the combustion of carbon impregnated particles, so that only exothermal reactions occur. Paper 112 considers processed shale which contains carbonates and silicates that can decompose in the combustion zone. Since these reactions are endothermic the maximum realized temperature will be less than what would be calculated in (92). An important aspect of this analysis is that a separate thermal front moving at its own velocity can be associated with each reaction. The separation or combination of these fronts determines the temperature profile.

(d) Relation of oil yield to the structure of the temperature wave

Paper 113 includes the retorting of kerogen in the analysis of the thermal wave. The hot gases decompose the kerogen into oil, gas and a carbonaceous residue which supplies fuel for the combustion. The goal was to predict the oil yield.

From calculations presented in (92) and (112), operation at $\beta > 1$ appears particularly attractive because the combustion and pyrolysis zones are the most separated and the fraction of oxygen bypassing the combustion zone is zero. The main causes of oil loss are the coking and cracking of the oil. Coking is the degradation of the oil while it is in the liquid phase. Its severity depends on the time it takes to vaporize. This, in turn, is proportional to the heating rate of the solid. Cracking, which is the thermal
degradation of the oil in the gas phase, varies inversely with the rate at which the oil vapor is cooled.

For the case of $\beta < 1$ the gas is not depleted of its oxygen when it enters the pyrolysis zone. (See Fig. 75) The oxygen bypassing the combustion zone is consumed by the combustible gases, the oil vapors and the oil liquids present in the pyrolysis region. This oil combustion is directly related to the fraction of oxygen consumed by oil and gas production. The coking in this case is less severe than in the case $\beta > 1$ because of the higher heating rates.

Calculations are presented for several cases in paper 113. On the basis of the experience gained in these analyses, the recommendation is made that the best conditions to maximize oil yield are $\beta > 1, N_{d3} = \frac{T_{adc}}{T_{di}} > 1$, where $T_{adc}$ is the reaction temperature for the case that all of the dolomite and calcite decompose and $T_{di}$ is the temperature at which dolomite starts to decompose. For $N_{d3} > 1$ the heat released by the exothermic combustion is enough to decompose all of the carbonates and cause the carbonate decomposition zone to move faster than the combustion zone. (See fig. 76) Also for $\beta > 1$, a separation of the combustion, the carbonate decomposition and the solid heating zones occurs.

The conditions $\beta > 1$ and $\frac{T_{adc}}{T_{di}} > 1$ might not be easy to realize with lean shale for which the small amount of carbon requires a very low oxygen concentration in the gas. If this oxygen concentration drops to a certain level no combustion is possible. In this case, adding a fuel gas (which could be recycled exit gas) will increase the fuel concentration. The fuel in the recycle gas will have the same effect as the carbon in the solids.

In order to minimize the ignition time and associated oil losses, paper 113 suggests the use of a special ignition process by which inert gas at a temperature $T_{di}$ is initially passed through the bed. This causes kerogen to decompose and the pyrolysis front to move a small distance into the bed, leaving behind a bed of pyrolyzed shale. This bed can then be ignited by using a cold gas feed containing oxygen.

In modified insitu retorting, the exposure of the roof to hot gases for large periods of time can result in its collapse. This collapse imposes an increase in the required overburden depth and, therefore, in the separation needed between consecutive vertical retorts. This limitation is critical for insitu retorting because it restricts the amount of shale that will be processed. This is a factor penalizing insitu retorting compared to above ground retorting. In the case of $\beta < 1$, where all known retorts had been operated, the
very diffuse thermal region which exits behind the combustion zone could cause prolonged heating of the roof. In the case of $\beta > 1$, the reaction, which exists in front of the long heating zone, has a stiff steady state temperature profile. The absence of backward diffusion of heat could prevent prolonged heating of the roof.
Fig. 68  Dye filament in a dumped bed of spheres.

Fig. 69  Hanratty, James Mitchell, Kenneth Jolls inspect the instrumented sphere to be used in the dumped bed, circa 1965.

Fig. 70  Wake of a sphere in a regular cubic bed.

Fig. 71  Suggested pattern of limiting streamlines for a sphere in a close packed cubic array.

Fig. 72  Larry Williams and Hanratty at gas-liquid flow facility, circa 1989.

Fig. 73  Arnold Hershman (PhD, 1960) at the facility built by James Engen (MS, 1956) to define the waves that appear in gas-liquid flow.
Fig. 74  Sketch of a modified insitu process for getting oil out of shale. Note that the combustion wave consists of several zones.

Fig. 75  Calculated temperature profile in the combustion wave. Conditions are such that combustion occurs in the front close to the raw shale.

Fig. 76  Calculated temperature profile. Operating conditions are such that combustion occurs in the back of the wave, next to the spent shale.

Fig. 77  Prevention of flooding in a sieve tray column studied in a BS thesis by Ryan McLean, 1996. Insertion of a porous plate in the downcomer prevents the creation of bubbles/foam.
Nomenclature

a         Radius of a pipe
a         Amplitude of a wave
A         van Driest parameter which is a measure of the thickness
          of the viscous wall layer
$A_{L1}$  Area of the liquid in the stratified flow in front of a slug
$A_L$     Area of the liquid in a stratified flow
$A_G$     Area of the gas in a stratified flow
$C_P$     Heat capacity
C         Concentration
$\bar{C}$ Temporally-averaged concentration
$C_i$     Concentration at the interface
$C_B$     Spatially-averaged concentration
$C^*$     Equilibrium concentration of a substance in the liquid
          with the concentration in the gas
$C_{P}$   Concentration of particles
$C_{PW}$  Concentration of particles at the wall
C         Concentration fluctuation
$C_p$     Fluctuation in the particle concentration $= C_p - \bar{C}_p$
c         Wave velocity
c_f       Velocity of the front of a slug
c_B       Velocity of the back of a slug
c_jump    Velocity of a hydraulic jump
d_p       Particle diameter
d_p^*     Dimensional particle diameter $= d_p \sqrt{\frac{\rho}{\mu}}$
d_t       Diameter of a tube
d_vµ      Volume median drop diameter
D         Molecular diffusivity
E         Turbulent diffusivity of fluid particles originating from a
          point source
$E_\infty$ Value of E for large diffusion times
$E_d$     Turbulent diffusivity of a molecular species or a thermal
          marker originating from a point source
E         Entrainment of droplets in a gas flow $= \frac{W_{te}}{W_L}$
$E_M$     Maximum possible entrainment $= \left( \frac{W_{LF} - W_{LFc}}{W_L} \right)$
f_S       Frequency of slugging
\( f_S \)  
Friction factor for a single phase flowing over a smooth boundary

\( f_i \)  
Friction factor for flow over a wavy boundary

\( F = \left[ \gamma (Re_{LF})/Re_G^{0.9} \right] (\nu_L/\nu_G)(\rho_L/\rho_G)^{1/2} \)

\( F_{LE} \)  
Drop flux, mass per unit time per unit area

\( Fr \)  
Froude number \( = \overline{U_L}/g\overline{h} \)

\( g \)  
Acceleration of gravity \( g = \frac{g\mu}{\rho v^3} \)

\( Gr \)  
Grashof number \( = a^3 \rho^2 g \beta \Delta T/\mu^2 \)

\( h \)  
Heat transfer coefficient

\( h \)  
Height of a liquid layer

\( \overline{h} \)  
Time-mean height of a liquid layer

\( \Delta h \)  
Displacement of a wavy interface from the mean \( h = \overline{h} \)

\( \Delta h^2 \)  
Mean-squared displacement

\( h_{RW} \)  
Height of a roll wave

\( h_0 \)  
Critical height of a stratified layer below which slugs cannot be generated

\( h_S \)  
Height of a stratified layer at which slugs were observed to appear

\( h_e \)  
Equilibrium height which would exist for a stratified layer on which slugs are not present

\( h_1 \)  
Height of the liquid layer in front of a slug

\( h_L \)  
Height at the center of a stratified layer at the bottom of a pipe

\( H \)  
Half-height of a channel

\( I \)  
Intermittency, the fraction of the time roll waves are present

\( k \)  
Thermal conductivity

\( k \)  
Fluctuation of the mass transfer coefficient

\( k_s \)  
Size of sand grains on an artificially roughened surface

\( k_i, k_L \)  
Constants appearing in the Loyd, Moffat, Kays modification of the van Driest equation for the mixing-length

\( k_D \)  
Deposition parameter \( = R_D/C_B \)

\( k_{D1} \)  
Reciprocal of this quantity is the resistance associated with diffusion to the vicinity of the wall

\( K \)  
Mass transfer coefficient \( = N/\Delta C \)
Local time-mean mass transfer coefficient
\( \langle K \rangle \) Average mass transfer coefficient over a length of pipe
\( \langle K^+ \rangle \) Dimensionless mass transfer coefficient = \( \langle K \rangle / \nu^* \)
\( \ell \) Mixing-length
\( L_D \) Location at which slugs can form
\( L_S \) Slug length
\( \bar{L}_S \) Mean slug length
\( L_{F} \) Location in a pipe where a slug is observed to form
\( (L_S)_{\text{min}} \) Minimum length needed for a slug to be stable
\( m \) Average height of a liquid film
\( m^* \) = \( mv^* \rho / \mu \)
\( m^*_G \) = \( mv^* \rho_G / \mu_G \)
\( N \) Rate of mass transfer at a boundary per unit area
\( P(0) \) Obukhov measure of heterogeneity = \( \int_{-a}^{a} T^2 \, dy \)
\( \Pr \) Prandtl number = \( C_p \mu / k \)
\( Pe \) Peclet number = \( 2aU / D \)
\( Pe_z \) = \( 2av^* / D \)
\( Pe_p \) Peclet number for particle transfer = \( d_p U / D \)
\( P \) Pressure
\( \hat{P}_I \) Pressure component at the interface in phase with the wave slope
\( \hat{P}_R \) Pressure component at the interface that is 180 degrees out of phase with the wave height
\( P \) Perimeter of a pipe = \( \pi d_t \)
\( \Delta P_T \) Total pressure over a single slug
\( \Delta P_h \) Pressure associated with the hydraulic jump in front of a slug
\( \Delta P_f \) Frictional pressure loss in a slug
\( \Delta P_r \) Pressure change at the rear of a slug
\( P_G \) Perimeter of the gas phase in an idealized stratified flow
\( P_L \) Perimeter of the liquid phase in an idealized stratified flow
\( q_w \) Heat flux at the wall
\( q_B \) Volumetric flow of liquid out the back of a slug
$q_F$ Volumetric flow of liquid consumed at the front of a slug
Radial location
$
Re_{LF} \quad \text{Reynolds number characterizing the liquid}
$
film = 4W_{LF}/\mu_L P$
$L^2 \quad \text{Lagrangian correlation coefficient characterizing the}
\text{dispersion of fluid particles}$
$L^D \quad \text{Lagrangian correlation characterizing dispersion of a}
\text{molecular specie or of a heat tag}$
$R_D \quad \text{Rate of deposition per unit area}$
$R_A \quad \text{Rate of atomization per unit area}$
$R_{AT} \quad \text{Rate of atomization from the top wall}$
$R_{AB} \quad \text{Rate of atomization from the bottom wall}$
$R_A^\sim \quad \text{Dimensionless rate of atomization} = R_A/U_G (\rho_L \rho_G)^{1/2}$
$\langle R_A \rangle \quad \text{Average rate of atomization around the circumference of}
\text{a horizontal pipe}$
$Re_p \quad \text{Particle Reynolds number} = d_p U_{SGP}/\mu$
$Re_{pc} \quad \text{Critical particle Reynolds number for a transition to}
\text{unsteady flow}$
$S \quad \text{Ratio of drop velocity to gas velocity}$
$S_i \quad \text{Length of the interface in an idealized stratified flow}$
$Sc \quad \text{Schmidt number} = \mu/\rho D$
$Sh \quad \text{Sherwood number}$
$s_z \quad \text{z-component of the fluctuating velocity gradient at the}
\text{wall}$
$s_x \quad \text{x-component of the fluctuating velocity gradient at the}
\text{wall}$
$t \quad \text{Time}$
$T \quad \text{Temperature}$
$\overline{T} \quad \text{Time-averaged temperature}$
$\overline{T}^2 \quad \text{Mean of the temperature squared}$
$\overline{T}^+ \quad \text{Dimensionless mean temperature} = \left(\overline{T}_W - \overline{T}\right) \rho C_p v^*/q_W$
$T_C \quad \text{Temperature at the centerline or in the core}$
$T_W \quad \text{Wall temperature}$
$U \quad \text{Velocity in the x-direction}$
$\overline{U} \quad \text{Time-averaged velocity}$
$\overline{U}_c \quad \text{Value of } \overline{U} \text{ at the center of a pipe or channel}$
\( U_L \) Mean velocity in the liquid film
\( U_i \) Velocity component in the i-direction, where \( i = 1,2,3 \) corresponds to x, y, z
\( U_G \) Gas velocity
\( U_{SG} \) Superficial gas velocity, equal to the volumetric gas flow divided by the area of the duct
\( U_{SL} \) Superficial liquid velocity, equal to the volumetric flow of the gas divided by the area of the duct
\( U_{L1} \) Velocity of the stratified flow in front of a slug
\( U_{L3} \) Velocity in the body of a slug
\( U_{SGc} \) Critical superficial gas velocity for the initiation of waves in an inclined pipe. See Section 14g
\( U_{BG} \) Bulk-averaged gas velocity
\( U_{BL} \) Bulk-averaged liquid velocity
\( U_{Gt} \) Critical gas velocity for irregular waves to appear at the interface
\( U_r \) Current velocity, equal to the difference of the velocities of the two phases in a stratified flow
\( U_{rc} \) Critical current velocity
\( U_S \) Settling velocity
\( u_i \) Fluctuating velocity in the x-direction at the interface
\( V \) Velocity component normal to the wall
\( V_{pr} \) Velocity component of a particle normal to the wall
\( V_{pW} \) Velocity of particles depositing on a wall
\( V_{tp} \) Turbophoretic velocity
\( v^f \) Velocity component of a fluid particle in a Lagrangian sense
\( \langle v^2 \rangle \) Average value of \( v^2 \) for a large number of particles
\( v^* \) Friction velocity based on the wall stress \( = (\tau_w / \rho)^{1/2} \)
\( v_i^* \) Friction velocity based on the interfacial stress \( = (\tau_i / \rho)^{1/2} \)
\( v_0^* \) Friction velocity in the absence of particles
\( \nu \) Component of the fluid velocity fluctuations in the y-direction
\( \nu^2 \) Mean-square of \( \nu \)
\( v_{pr} \) Component of the fluctuating particle velocity in the \( r \)-direction
\( v_{p\theta} \) Component of the fluctuating particle velocity in the \( \theta \)-direction
\( v_T \) Terminal velocity of particles
\( W \) Velocity in the \( z \)-direction
\( W_k \) Spectral density function of fluctuations in the mass transfer coefficient, \( k \)
\( W_\beta \) Spectral function of \( \beta \)
\( W_L \) Mass flow of liquid
\( W_{LF} \) Mass flow of liquid in a wall layer
\( W_{LE} \) Mass flow of liquid entrained as drops in the gas
\( W_{LFC} \) Critical film flow below which atomization does not occur
\( W_{e\lambda} \) Weber number based on the wavelength = \( \rho_G U_G^2 \lambda / \sigma \)
\( W_{e_RW} \) Weber number based on the height of a roll
\( \text{wave} = \rho_G U_G^2 h_{RW} / \sigma \)
\( w \) Fluctuation in \( W \)
\( w_i \) Value of \( w \) at the interface
\( X \) Coordinate of a fluid originating from a point source
\( \langle X^2 \rangle \) Average value of \( X^2 \) for a large number of events
\( x \) Coordinate in the flow direction
\( x_1 \) Location of a wall source
\( x^+ \) Dimensionless \( x \)-coordinate = \( x \nu / \mu \)
\( x_i \) Coordinate in the \( i \)-direction, \( x_1, x_2, x_3 \) correspond to \( x, y, z \)
\( Y \) Distance in the vertical direction in the core-used in a boundary-layer analysis for natural convection in a horizontal pipe
\( y \) Coordinate in a direction perpendicular to a wall
\( y_c \) Centroid of a plume of heat
\( z \) Spanwise coordinate

**Greek Symbols**
\( \alpha \) Wave number = \( 2\pi / \lambda \)
\( \alpha \) Void fraction
\( \alpha^+ \) Wave number made dimensionless with the interfacial stress and the kinematic viscosity
\( \beta \) Coefficient of cubical expansion

\( \beta \) Reciprocal of the particle time constant = \( 1/\tau_p \)

\( \beta \) First term of a Taylor series representation of the velocity component normal to the wall, \( v = \beta y^2 \)

\( \beta \) Parameter used in shale oil retorts = \( \rho G C_p G / V C_S \rho_S \)

\( \delta \) Thickness of velocity boundary-layer

\( \delta_v \) Thickness of the viscous sublayer

\( \delta_c \) Thickness of a concentration boundary-layer

\( \delta_T \) Thickness of a temperature boundary-layer

\( \varepsilon_p \) Turbulent diffusivity of particles

\( \varepsilon_h \) Turbulent diffusivity of film height

\( \varepsilon \) Turbulent diffusivity defined in an Eulerian framework

\( \varepsilon_T \) Turbulent diffusivity for heat

\( \phi \) Potential function, \( u_i = -\partial \phi / \partial x_i \)

\( \gamma(Re_{LF}) \) Function of film Reynolds number defining the dimensionless film height, \( m^+ = \gamma(Re_{LF}) \)

\( \Gamma \) Mass flow in a wall film per unit circumference length

\( \Gamma_0 \) Critical value of \( \Gamma \) for the initiation of roll waves

\( \langle \Gamma \rangle \) Average value of \( \Gamma \) around the circumference of a horizontal pipe

\( \eta \) Displacement of the interface from its average location

\( \kappa \) von Karman constant

\( \kappa \) Molecular diffusivity of heat = \( k/\rho C_p \)

\( \lambda \) Wavelength

\( \lambda_i \) Component of the wavelength in the i-direction

\( \lambda^E \) Taylor Eulerian microscale

\( \lambda^L \) Taylor Lagrangian microscale

\( \mu \) Viscosity

\( \mu \) Average of a stochastic quantity

\( \nu \) Kinematic viscosity = \( \mu/\rho \)

\( \theta \) Phase angle

\( \theta \) Equals \( \alpha x - \omega t \)

\( \rho \) Density

\( \rho_p \) Particle density

\( \sigma \) Surface tension

\( \sigma \) Root-mean square
\( \sigma_{LS} \) Root-mean square of slug length
\( \sigma_p \) Root-mean square of fluctuating particle velocity
\( \tau_i \) Local shear stress at an interface
\( \tau_W \) Shear stress at a wall
\( \tau_c \) Characteristic stress for a liquid film = \( 2/3 \tau_W + 1/3 \tau_i \)
\( \tau_{ij} \) Force in the j-direction on a plane on a plane perpendicular to the i-axis
\( \tau_{ij}^p \) Stress due to polymers
\( \tau_{ij}^l \) Stress component due to molecular viscosity
\( \tau_{WG} \) Stress at the wall for the gas phase in a stratified flow
\( \tau_{WL} \) Stress at the wall for the liquid phase in a stratified flow
\( \tau_p \) Relaxation time for a particle moving in a flow field
\( \tau_p^+ \) Dimensionless relaxation time = \( \tau_p v_t^2 / \nu \)
\( \tau^L \) Langrangian time constant
\( \omega \) Circular frequency
\( d\omega \) Sequence of random numbers with variance \( \langle d\omega^2 \rangle = dt \)
PUBLICATIONS OF THOMAS J. HANRATTY
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81. With J. C. Dallman and B. G. Jones, “Interpretation of Entrainment Measurements in Annular Gas-Liquid Flows,” Two-Phase Momentum, Heat and


Non-technical publication