THESIS.

Photographing Alternating Current Curves

BY

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Preface.

It is the object of the following investigation to study the applicability of a new method for obtaining alternating current curves, and to make a mathematical interpretation of the forms of current waves obtained from circuits, with respect to the relative values of resistance, self-induction, and capacity, when both direct and alternating & M. F.'s are suddenly impressed. The author not being the originator of this method has endeavored in the first part of the work, to confirm the results obtained by other experimenters, giving them credit for their ideas.
In carrying out the second part, he has taken, as a basis, Bedell and Chisholm's "Alternating Currents" for the mathematical discussion of the experimental results. The various cases under these latter conditions are as follows:

1st. Resistance and Self-induction, direct & M.F.

2nd. Resistance and Self-induction, harmonic & M.F.

3rd. Resistance and Capacity, direct & M.F., charge and discharge.

4th. Resistance and Capacity, harmonic & M.F.

direct E.M.F., charge and discharge.

6th. Resistance, Selfinduction and Capacity harmonic E. M. F.

All of these conditions were experimented upon, but owing to the limitations of time, the last four cases were not completed, but there is no reason why the difficulties in the way cannot be surmounted and the curves obtained.

The author takes this opportunity to acknowledge the assistance so ably rendered by Mr. F. J. Foote '93 in the construction of apparatus, and in the experimental work.

E. V. Cooper, B.S.

University of Illinois, May, 1896.
Introduction.

The method of obtaining the form of current waves by the aid of photography was first experimented upon by Messrs. H. J. Hotchkiss and F. E. Milli of Cornell University, the apparatus used being a modified form of M. Blondell's oscillograph. The results of their investigations have been published in two separate articles.*

In the first article, the authors confine themselves principally to the study of the


apparatus, for the purpose of determining the
best form and size of the various parts for
application to different lines of work. Several
very interesting plates are given showing
curves from different sources of alternating
E.M.F. The most conspicuous of these
was a curve taken from a Westinghouse
'Pony' alternator, having a toothed type of
armature. This current presents a remarkable
variation of the current at the crest of the
wave. Fig. (1) on the next page is a
reproduction of this curve and shows its
general form.

A curve taken from a similar machine
having a smooth core armature, has a
smooth sinusoidal form. The conclusion
drawn is that the effect noticed can be attributed to the teeth of the armature core. To prove conclusively that it was due to this, and not to a defect in the apparatus, a curve taken by the instantaneous contact method shows the same effect.

A second plate shows how the current dies away when the E. M. F. is removed from a circuit which contains resistance and self-induction. This case corresponds to the current equation when it contains an exponential term. There is given no direct-
comparision between the observed curve and the corresponding theoretical curve, but the general form would suggest that such a comparison would be very interesting. Fig (2) is a reproduction of the one under consideration.

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**Fig 2.**

If the E. M. F. had been impressed upon the circuit, we would obtain a similar curve only reversed in direction as shown in Fig (3) on the next page. The curve begins at A and after a short interval of time approaches infinitely near 78C, AB.
Another interesting curve is one taken from a synchronous motor when operated by a transformer. It possesses very sharp points at the crests, while the lines between are nearly straight. The conclusion to be drawn is that the method was found entirely satisfactory as far as it was applied. The second article was published by...
Mr. F. E. Miller and in his investigation he has applied the method to the study of the effect of self-induction in circuits when a harmonic E. M. F. is suddenly impressed. Curves are shown for cases where the E. M. F. is introduced, when removed and for both cases in one plate. In this work a specially constructed piece of apparatus was employed to close or open the circuit at a particular point in the phase. The plates obtained showed that the curve of establishment had for its axis the curve of dying away of the current, which confirms exactly with the theory. Fig. (4) shows the curve under discussion.

This may be considered as a very
important result as it again shows the accuracy of the method. The author concludes with expectation to continue the research into circuits containing capacity. The apparatus used in this work, which will now be described, was constructed from the drawings given in the above articles.
Part I

Description of Apparatus.

To be able to record the pulsations of an electric current, theoretically a galvanometer needle should be used which possesses no mass. Under such conditions the needle would respond instantly to any change of the current and would have an infinitely small period. In practice these conditions cannot be realized, but they can be approached sufficiently near, so that error may be neglected. A needle fulfilling the above conditions, properly mounted and placed in a magnetic field forms a simple galvanometer. The current to be
studied is made to pass through a coil which surrounds the needle, the latter responding to any change in the value of the current. The movements of the needle are recorded upon a moving photographic plate, by a spot of light reflected from a mirror mounted on the needle. The curve obtained upon development of the plate, shows all the changes of the current with respect to the time.

The construction of the galvanometer is as follows. Plate 1 shows the working drawings of the field magnet. In its construction the best Swedish iron was used for the cores, while the yoke and pole pieces were made of common wrought iron.
All the joints were ground until they fit perfectly. The core was wound with 785 turns of No. 14 copper wire. The pole piece was made diamond pointed in order to concentrate the magnetism and give a uniform field. Plate II shows the $B + H$ curve for the yoke, core, and airgap.

As might be expected the coefficient of leakage was very large. The current for $H = 75$ was 15 amperes, but the airgap curve shows that little is gained after $H = 30$ or $C = 6$ amperes, which was the normal working current.

The construction and mounting of the needle is shown in detail in Plate III. One end of a soft iron wire was ground
Plate I
Diagram of
Electro Magnet
Scale 1" = 4"
Plate II

B. H. Curves from different parts of the magnet.

Area of:
- Cores: 113 sq.cm.
- Air gap: 2.64
- Yoke: 30.0

Coef. of Leakage: 115 for H = 40

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Jan. 31 76

Inductions

Magnetizing Force

Air gap

Yoke
down to extreme thinness on an emery wheel, the width when finished being \( \frac{1}{2} \) millimeter. The mirror used was cut from a microscopic cover glass which was silvered in the following way. To a solution of potassium sodium tartrate was added a solution of silver nitrate. Ammonia was then added until the precipitate was just dissolved. The glass, being cleaned by nitric acid and alcohol, was then dropped on the surface of liquid. After standing from 10 to 20 hours a fine deposit of silver was formed on the underside. The minute mirrors were fastened to the soft rim by shellac. When dry they were cut off with a pair of scissors, thus leaving a bit of soft rim attached to the back of each.
The quartz fibre used for suspensions was drawn by the author, an oxyhydrogen flame being employed. In diameter they varied from .0008 to .0013 of an inch. The needle was fastened upon the fibre in the following manner. The fibre was first attached to the upper bug A of the frame and then to the spring S, by use of dry shellac which was melted by a hot wire. The frame being clamped in a vise, the needle was laid upon a strip of brass which in turn rested upon a double wedge. By sliding the wedges the needle could be brought directly under the fibre until contact was made. A
minute piece of shellac was then dropped upon the back of the mirror from a pointed glass rod. By holding a hot wire near the shellac it was melted and the needle soldered to the fibre. The whole operation after a little practice could be performed in twenty minutes.

The deflecting coils were made elliptical in form and fitted snugly over the brass lugs that support the needle. Seven coils were used ranging from a coil of four turns, four No. 14 wires in parallel using twenty amperes, to a coil of No. 36 wire with a resistance of forty ohms using 1/10 of an amper. The amplitude of the vibration set up at the plate was about
six centimeters. If the amplitude was greater than this, the heating effect due to the current was likely to melt the shellac connections. On plate IV is given a photograph of the complete galvanometer.

The apparatus by which the movements of the photographic plate were controlled consisted of a light-light box, within which works a plate holder of special design. A drawing of the holder is shown in plate V. It is constructed of pine wood and painted black. The spring clip together with the legs 1 and 2 hold the plate firmly in position. The tongues on each side fit into corresponding grooves in the box preventing all side motion.
Diagram of Needle.

Plate III

Side view

Front view.

full size.
The box, which is shown in Plate VI, is constructed of pine wood and painted black inside and out. Fig (2) shows a section and Fig. 3 shows a section of the slit where light is admitted. R is a trigger which releases the plate and at the lower end is a rubber tube which serves to take up the impact of the fall. The distance from R to the slit is sufficient to give the plate a mean velocity of ten feet per second. The whole box is fitted to a base in such a manner that it can be easily detached to drop the plate back to the upper end. Three leveling screws serve for the adjustment. The interior view is shown by the second photograph in
plate IV. Plate VII shows a view of it mounted upon the base.

It was necessary in some of the experiments to have the circuit automatically closed by the falling plate. This was accomplished as follows: A block of wood two inches long and $\frac{1}{2}$ inch square was sawed nearly through lengthwise. At one end two pieces of brass C and D were driven into the block as shown in the cut.

To these were soldered strips of spring brass.
which extended the length of the block along the sides. A bit of spring brass wire was arranged in the slot so that the end came between C and D, making contact with C. The part B was extended high enough above so that the plate holder in passing would press the spring in contact with D. The end A was allowed to trail out behind. A groove was cut in the back of the slide box, the bottom and sides of which were lined with brass which led to three binding posts. The block could now be pressed into the groove and moved to any desired point. The plate in falling would then open or close the circuit depending on the connections.
Plate V

Diagram of plate holder.

Scale 1/2
In carrying out these experiments it was necessary to work in a dark room. A stereopticon arc lantern, enclosed in a box, furnished the light by which the photographs were taken. The lamp allowed the adjusting of the position carbon behind the negative and in this way the most intense light was made to fall upon the mirror of the needle. A photograph of the lamp is shown in plate VIII.

The apparatus just described was sufficient to obtain the most simple curves, but in the more complex cases, a few additions were necessary, which will be described later.
Part II

Preliminary Test

The apparatus being arranged as shown in plate IX, a test was made to determine the period of the needle. The galvanometer was placed in series with a storage battery and the circuit closed by the falling plate, the needle vibrating under the impulse. Plate X,a shows the results obtained, the number of double vibrations being 2700 per second. Figure 6 shows how the needle behaved when acted upon by an alternating E.M.F. It is evident at once that the cause of the sharp points are due to the natural period of the needle, and this
is confirmed by counting the number of points in a given horizontal distance, which is found to equal the number of vibrations for the same distance on Fig. a. It is at once apparent that the needle possesses too much weight and a smaller one must be used. After a few trials a needle was mounted which made 5200 double vibrations in one second, and later one which made 7200. The needles used by Messrs. Hotchkiss and Millis made about 3500 double vibrations per second. It was found, that in order to eliminate the effect of the natural vibration, the needle should vibrate over 3000 times per second.
Diagram of arrangement of apparatus.
Plate I

Natural period of needle
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Plate II

Current 6 Frequency 133 Pony Alternator
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Curves from Different Sources of E.M.F.

It was thought interesting to make a study of curves from different sources of E.M.F. in order to compare their peculiarities. In these experiments the needle used possessed a natural vibration of 5200.

Plate XI. a shows the results obtained from a Westinghouse “Ping” alternator having a toothed core armature. It will be seen at once that this curve presents the same general appearance as that described in the introduction. The result confirms the conclusion already drawn that the effect is due to the armature teeth. A large number of curves were taken from this machine and all presented
the same general characteristics. Fig. 6 on the
same plate shows curves taken from arc
lighting machines. No. 1 is a curve from
a ten-light Brush dynamo and No. 2 from
a three light T. H. Dynamo. The variation
of the current from the average value is
slightly over one ampere.

Plate XII shows curves taken from the three-light
T. H. Dynamo when run as a three-phase
alternator, the armature windings being connected
as shown in the figure.
Figure a shows the curve taken from the single coils, while Fig. b shows the form for two coils in series. These curves were taken with the fields excited by a ten-ampere current. Curves taken by the instantaneous contact method with a current of 1.8 amperes in the field showed that the curve for the case when the two coils are in series was nearly a true sine wave. Further investigations should be made before conclusions could be drawn.

The next three plates show curves taken from the following transformers, the primary and secondary curves being given side by side.

T. H. Plate XIII Fig. a
Diamond
Elkhart  Plate XIV  Fig. a
Wood    "    "   b
Packard "   XV "   a
Diamond (1876 type) " "   b.

The transformers were excited by a T.H. alternator and the curves are characteristic of that machine. There is no perceptible difference between the primary and secondary curve, even for wide changes of load.

Plates XVI and XVII show curves taken from a Westinghouse Pony alternator when run as a synchronous motor by a similar machine. The first is taken with the motor running on no load and no added self-induction in the circuit. The current
varied periodically from a very small value up to 25 or 30 amperes. The cycles succeed each other at periods of about one second, and in order to obtain the best results a continuous photographic film should be used. A curve under these conditions would have a general form as shown in fig. (7).

Fig. 7

This accounts for the two curves being of different amplitudes since the plate would only record a very small fraction of this long wave.
When self-induction is introduced into the circuit, the effect is to cut the current down to only $\frac{1}{4}$ of its original value, as shown by Fig. a. To make the amplitude larger, a finer coil was placed around the needle and the curves shown by Fig. b were obtained. Plate XVI-6 shows the generator curve and it is the same as those observed previously. The needle in this case made 7200 double vibrations per second, being the highest yet used.

It would seem from the results of the experiments, that this method of obtaining the forms of current waves, would admit of wide application, both in experimental and practical work. That it is reliable will be shown in subsequent experiments.
Plate XIII

Plate XI

T. H. Transformer. Sec.

Fig. a

Plate XIV

Diamond Transformer Sec.

Fig. b
Part III

The Effect of the Exponential Term.

We will now proceed to apply this method to the study of the effect of the exponential term in the equation for current when the E.M.F. is suddenly impressed upon a circuit containing resistance, self-induction, and capacity. The first case discussed will be the most simple, where a direct E.M.F. is impressed on a circuit containing resistance and self-induction.

From Bedell and Cukow we have for the equation of electo-motif forces, when only self-induction is considered,

\[ e = Ri + L \frac{di}{dt} \]
which is a differential equation. In any case under consideration the E.M.F. is some function of the time thus:

\[ e = f(t) \]

Substituting this in the equation we obtain,

\[ L \frac{di}{dt} + Ri = f(t) \]

or

\[ \frac{di}{dt} + \frac{R}{L} i = \frac{f(t)}{L} \]

which is a linear equation of the first order. The solution of this equation is,

\[ i = C e^{-\frac{Rt}{L}} + \frac{1}{L} \int e^{-\frac{Rt}{L}} f(t) \, dt \]

which is an equation for current with respect to
the time.

In the particular case under consideration $f(t)$ is constant and equal to $E$. Substituting in equation (3) we have

$$i = C E^{\frac{-Rt}{L}} + \frac{i}{L} E^{\frac{-Rt}{L}} \int E \frac{Rt}{L} E dt,$$

which when reduced equals

$$i = C E^{\frac{-Rt}{L}} + \frac{E}{R}$$  \hspace{1cm} (4)

The constant $C$ is determined by the conditions that when $t=0$ $i=0$ and substituting in equation (4) we get

$$0 = C + \frac{E}{R}$$

or

$$C = -\frac{E}{R}$$
Putting in equation (4) the value of this constant it becomes

\[ i = \frac{E}{R} \left( 1 - e^{-\frac{Rt}{L}} \right) \]

The locus of this equation would be a logarithmic curve of the form shown in Fig. 8, which may be determined by a graphical method.

![Graph](image)

**Fig. 8**

The experiment to illustrate this equation was carried out in the following manner. A self-induction of 0.754 henry's was connected in series with
a resistance and the ratio of the two so
adjusted that the time constant of the circuit \( \frac{L}{R} \)
was equal to .02. The plate in its fall
closed the circuit and the results obtained are
shown in Plate XVIII. The constants of the
circuit are,

\[
\frac{E}{R} = 1.43
\]

\[
L = .0754.
\]

The equation for \( i \) then becomes upon
introduction of these constants,

\[
i = 1.43 \left(1 - e^{-\frac{t}{0.00833}}\right)
\]

and the dotted line on the plate is the locus
of this equation, one inch of the x axis being
taken equal to .00833 seconds.

The two curves agree remarkably well
when we consider the many ways in which
error could enter. If the E. M. F. had been
removed from the circuit the equation would
have been

\[ i' = \frac{E}{R} e^{-\frac{Rt}{L}} \]

which would have given a similar curve to
the above except that it would be reversed. Fig 6
shows a curve under these conditions though it
was taken as a building up and not as a
dying away effect.
Harmonic E.M.F.

The next case to be considered is when the direct E.M.F. is replaced by a harmonic E.M.F. and \( e = \sin \omega t \). Substituting in the general equation for current we have

\[
i = \frac{E}{L} e^{-\frac{Rt}{L}} \int_{-\frac{Rt}{L}}^{\frac{Rt}{L}} \sin \omega t \, dt + c, e^{-\frac{Rt}{L}}
\]

Solving

\[
i = \frac{E}{L} \left( \frac{R}{L^2} \sin \omega t - \omega \cos \omega t \right) + c, e^{-\frac{Rt}{L}}
\]

which may be simplified by the use of the formulas

\[
A \sin \Theta + B \cos \Theta = \sqrt{A^2 + B^2} \sin (\Theta - \tan^{-1} \frac{B}{A}),
\]
whence,

\[ i = \frac{E}{\sqrt{R^2 + L^2 \omega^2}} \sin(\omega t - \tan^{-1} \frac{L \omega}{R}) + \epsilon e^{-\frac{Rt}{L}}. \]

After the circuit has been closed for a few seconds the exponential term may be neglected and the current becomes a simple harmonic function of the time. Just what is the effect of the exponential term will now be shown.

Let \( \psi = \theta - \tan^{-1} \frac{L \omega}{R} \)

and \( I = \frac{E}{\sqrt{R^2 + L^2 \omega^2}} \).

The equation then reduces to

\[ i = I \sin \psi + \epsilon e^{-\frac{Rt}{L}}. \]

Now let the circuit be closed at any time \( t \), \( \psi \) at that time being equal to \( \psi_i \). Then \( t = t_i \), it is evident that \( i \) must be zero and we
have

\[ 0 = I \sin \psi + Ce^{-\frac{Rz}{L}} \]

or

\[ C = -I \sin \psi, e^{-\frac{Rz}{L}}. \]

Substituting in equation (2) we get,

\[ i = I \sin \psi - I \sin \psi, e^{\frac{R(z-z)}{L}}. \]

This equation shows that the current curve is the resultant of two curves whose equations are

\[ i' = I \sin \psi \]

\[ i'' = I \sin \psi, e^{\frac{R(z-z)}{L}}. \]

To represent these equations a particular case is solved as shown in plate XIX. Here \( \psi = 45 \), and it is evident since the current must always be zero when the circuit is closed, that the exponential term would have a maximum effect when \( \psi = 90 \) and no effect when \( \psi = 0 \). The plate shows
that after a length of a second the effect of the exponential term has disappeared.

To obtain this curve experimentally would require the use of a device by which the circuit could be closed at any particular phase angle. This was accomplished in the following manner. The trigger which released the plate holder allowing it to fall was replaced by an electromagnetic device, the armature of which on being attracted would release the plate. Wires were led from this into the dynamo room. A wooden disk was firmly attached to the armature which carried near the periphery a brass pin. A lever was so arranged that it could be thrown in the path of this pin, which when struck would release a second lever that closed
the circuit through the electromagnet, thus releasing the plate. The plate thus released at a particular position of the armature, would arrive at the automatic trigger at a particular phase, which would be the same in all cases, provided the time of fall and the speed of the alternator were constant. A trial plate showed that these conditions were nearly fulfilled, and the automatic trigger was adjusted to close the circuit at 90°. The results are shown in plate X. This curve presents all the characteristics of the theoretical case only in a more marked degree. The time constant was so small that in 0.5 seconds the effect of the exponential term had disappeared.
Plate XIX

\[ r = 40 \quad L = 2 \quad I = 10 \]

\[ \psi_1 = 45 \]

by

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Time in sec.
Conclusion.

Thus still remains a large field for research in this line of investigation, and therefore it will not be safe to base conclusions too strongly upon the preceding experiments. The most important factor however that ought to be established is the reliability of the method, and the proof of this rests upon the experimental and mathematical evidence. The only weak point in the experimental evidence is in the case of the T. H. arm dynamo when run as a three phase alternator, but the discrepancy may be due to the fact that a higher field strength causes a distortion of the wave, otherwise the results of the experiments.
see to be very accurate. The harmony existing between the mathematical deductions and the experimental results prove the accuracy of each other. Therefore we may conclude that these investigations are sufficient to establish the reliability of the method.

There is no doubt that this method will find a wide application in all kinds of experimental work. It has been applied to find the variation of the temperature in the cylinder of a steam engine through out a revolution.

As this work is not completed, the author hopes that someone will continue these investigations, and that new facts may be added to the knowledge of science.