DESIGN OF A SWING-BRIDGE

BY

RICHARD BIRD KETCHUM, B. S. '96

THESIS
FOR DEGREE OF CIVIL ENGINEER

COLLEGE OF ENGINEERING
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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Richard Bird Ketchum, B.S., '95,

ENTITLED Design of a Swing-Bridge

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE OF

Civil Engineer.

HEAD OF DEPARTMENT OF Civil Engineering.
INTRODUCTION

With the hope of learning the essential points concerning the practical design of a swing bridge, the writer has undertaken this design. Hence we may state at the beginning that our purpose has been to learn and not to teach. With this idea in mind, we have chosen a structure which we believe represents the best modern practice in this line of work, and using this structure as a guide we have designed a similar bridge under similar conditions. For a beginner, we believe this method of study superior to that of making independent designs with no practical check. With the strain sheet for reference and the existing structure convenient for observation, stresses as well as details have been compared and studied from actual practice.
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1. TYPE OF STRUCTURE

The type of structure to be used in any given case depends almost entirely upon the conditions for which the design is made. We have presented illustrations of some of the various types of draw spans now in use in the city of Chicago and although the conditions are in some cases quite similar, the general design of the bridges vary greatly. Although these bridges have been designed by our ablest engineers, it may be said without hesitation that some of the designs have serious faults, as shown by the practical test to which they are subjected. For example, the Halstead Street lift bridge has proven itself to be a very expensive bridge to maintain and operate. The Northern Pacific bridge was very difficult to build owing to the detail of the curved runway for the counter weights. This portion was all finished by hand as no machine could be arranged to do the work. The swing bridges make a center pier necessary and this is a very important item where navigation is interfered with. The Sherzer type has given some trouble in its operation, the foundations having settled in some cases sufficiently to cause the halves to bind or interfere with each other. Without considering further the merits of the various types we will confine ourselves to the case in question.

As to the type of bridge to be used in this particular case, no other than the swing bridge could be considered. No type of bridge could compare with the swing bridge as to cost and as the
channel is wide the center pier is allowable.

II GENERAL DIMENSIONS

(a) SPAN: The importance of avoiding curves in the alignment of the railroad has no doubt been the reason for placing this bridge in its present position. When we remember that this bridge was paid for by the Sanitary District, it is not difficult to understand why it crosses the canal at such a slight angle thus making a good alignment for the railroad, although the cost of the bridge is very much greater than it would be were the alignment of the railroad changed to cross the canal at a greater angle. Without further consideration we may say that the span is fixed by the alignment desired and the width of the canal.

(b) WIDTH: The width is fixed by the distance from center to center of the tracks with the required clearance. The first is generally between thirteen and fourteen feet. The last varies between seven and eight feet measuring from center of track to nearest point on the truss. For the bridge in question, the distance from center to center of tracks is 14 ft. 0 in.; the distance from center of track to truss is 7 ft. 0 in. For 27 in. plates on the end posts, this gives us 30 ft. 0 in. center to center of trusses.
(c) HEIGHT: The general appearance of the bridge depends largely upon the variation in height from the ends to the center of the span. Recent designs vary from old ones considerably in the general outline of the trusses. In modern designs the curved top chord is often used, especially in long spans. For spans of medium length, the top chord is commonly made straight from the top of the end post to the top of the center post over the pier. The old designs with parallel chords on each arm do not seem to be favored in the most recent designs.

There is no method of obtaining the exact theoretically economic depth for a swing bridge, but we have certain precedents established which have given good proportions; and, having the height of the truss fixed at the end by the minimum allowable clearance, we decide the height at the center so as to best proportion the chord section at the same time keeping in mind the general appearance of the bridge. Since the shear in the truss is the same for any height, the weight of the web members varies almost directly with the height. The chord sections vary inversely with the height. For single track bridges, the question of stability is important in deciding the center height. We have chosen the same heights as were used for the C. M. & N. bridge; but we believe the clear height is smaller than that of best modern practice, although this statement should be modified for special cases. The curve of the top chord of our design is a parabola, the same as on the C. M. & N. bridge.
(d) NUMBER OF PANELS: The advantages of long panels over short ones are; first, smaller number of pieces; second, greater weight of each piece offering greater resistance to impact of train; and third, less number of joints required in truss. The advantages of the short panels are; first, less bending stresses in members of the chords and in stringers due to their own weight; second, less depth required for floor system; and third, easier to handle in shop and in field. Although the advantages are in general about as stated above, we have other considerations which often decide what we should use. Let us take the case in question. Consider seven panels. Their length would be over 32 ft. To use the style of truss which we have chosen with seven panels would necessitate impracticable lengths. Consider eight panels each 28 ft. The great objection to this number is the fact that one floor beam would be in the center of the truss. The same objection would enter in the use of ten panels, and with eleven panels the inclination of the end posts would be at an undesirable angle, besides, the number of pieces and connections would be greatly increased. Hence it seems that nine panels is the proper number to use in this case.

(e) LENGTH OF CENTER PANEL: In our design we have increased the length of the center panel from 18 ft. 3-1/2 in. to 20 ft. The Sanitary District was required to keep the diameter of the center piers under a given size, hence the drum and the center panel were made smaller than they would have been made without limitations.
We believe this length, 20 ft., is in accordance with the best pract-
and more in harmony with the general design. The center panel must
be as short as possible but must give sufficient bearing to insure
the stability of the bridge. See page for table showing general
dimensions of some existing bridges.

III LOADS

(a) DEAD LOAD: The dead load to be used for any given case
may be very accurately estimated from the weights of existing bridges
taking into account the difference in live loads and the difference
in design. In our calculations we used 5300# per lineal foot of
bridge, or 66250# per panel per truss. We here present the shop
weights for some existing bridges.
Weight of the Louisiana Draw Bridge of the Chicago & Alton Ry. Over the Mississippi River.

Span 442 ft. Clear Width 15 ft. 9 in. Height center to center of pins at Center 50 ft. Height at end 30 ft. Length of center panel 20 ft. Panel Length 26 ft. 4-1/2 in. Center to center of Trusses 18 ft. 8 in. Built by the Lassig Bridge & Iron Works in '97-8. Designed by August Ziesing as Chief Engineer for Lassig Bridge & Iron Works. Clear Height 22 ft. Loading as per C. & A. Ry. '97 Specifications. See Diagram below.

<table>
<thead>
<tr>
<th>No. of Pieces</th>
<th>Description</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>built sections of top chord</td>
<td>101,859 lbs.</td>
</tr>
<tr>
<td>16</td>
<td>eye bars, top chord</td>
<td>62,104</td>
</tr>
<tr>
<td>18</td>
<td>sections of bottom chord</td>
<td>274,044</td>
</tr>
<tr>
<td>4</td>
<td>end posts</td>
<td>33,759</td>
</tr>
<tr>
<td>4</td>
<td>center posts</td>
<td>36,174</td>
</tr>
<tr>
<td>24</td>
<td>intermediate posts</td>
<td>145,237</td>
</tr>
<tr>
<td>12</td>
<td>hanger posts</td>
<td>32,944</td>
</tr>
<tr>
<td>28</td>
<td>struts</td>
<td>44,863</td>
</tr>
<tr>
<td>2</td>
<td>portals</td>
<td>6,904</td>
</tr>
<tr>
<td>18</td>
<td>floor beams</td>
<td>91,798</td>
</tr>
<tr>
<td>68</td>
<td>stringers</td>
<td>287,235</td>
</tr>
<tr>
<td>13</td>
<td>sections of drum girders</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>cast center</td>
<td>176,713</td>
</tr>
<tr>
<td>2</td>
<td>girders under engine room</td>
<td>4,450</td>
</tr>
<tr>
<td>10</td>
<td>drum struts and x-frames</td>
<td>5,852</td>
</tr>
<tr>
<td>6</td>
<td>power brackets, diaframs and frames</td>
<td>7,641</td>
</tr>
<tr>
<td>8</td>
<td>sway braces</td>
<td>2,909</td>
</tr>
<tr>
<td>62</td>
<td>top and bottom laterals</td>
<td>33,571</td>
</tr>
<tr>
<td>33</td>
<td>stringer x-frames</td>
<td>8,306</td>
</tr>
<tr>
<td>74</td>
<td>stringer brackets and braces</td>
<td>3,552</td>
</tr>
<tr>
<td>26</td>
<td>shaft supports and strut brackets</td>
<td>4,965</td>
</tr>
</tbody>
</table>

Total - - - - 1,364,882
<table>
<thead>
<tr>
<th>No. of Pieces</th>
<th>Description</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount carried forward</td>
<td>1,364,882 lbs.</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>9 in. &quot;I&quot; beams</td>
<td>4,272</td>
</tr>
<tr>
<td>12</td>
<td>sections curved angles</td>
<td>2,977</td>
</tr>
<tr>
<td>192</td>
<td>bent plates, splice plates and fill's</td>
<td>930</td>
</tr>
<tr>
<td>76</td>
<td>eye bars</td>
<td>134,480</td>
</tr>
<tr>
<td>40</td>
<td>counter bars and rods</td>
<td>24,572</td>
</tr>
<tr>
<td>60</td>
<td>spider rods and washers</td>
<td>7,724</td>
</tr>
<tr>
<td>62</td>
<td>cast steel wheels</td>
<td>21,202</td>
</tr>
<tr>
<td>14</td>
<td>cast track segments</td>
<td>18,893</td>
</tr>
<tr>
<td>14</td>
<td>cast rack segments</td>
<td>19,264</td>
</tr>
<tr>
<td>2</td>
<td>top cap and spider ring (4 bolts)</td>
<td>9,234</td>
</tr>
<tr>
<td>68</td>
<td>pins with collars</td>
<td>19,077</td>
</tr>
<tr>
<td>34</td>
<td>rail plates and guides</td>
<td>6,706</td>
</tr>
<tr>
<td>8</td>
<td>bed plates and racks</td>
<td>8,322</td>
</tr>
<tr>
<td></td>
<td>operating machinery</td>
<td>36,793</td>
</tr>
<tr>
<td></td>
<td>rivets, bolts, washers, etc.,</td>
<td>16,178</td>
</tr>
<tr>
<td></td>
<td><strong>Total Weight</strong></td>
<td><strong>1,695,511</strong></td>
</tr>
</tbody>
</table>

Total weight per foot of bridge, 3,840
THIRD AVENUE HARLEM RIVER BRIDGE

Span 300 Ft. Width over all 87'6"

Designed by Thomas Clarke.

Built by Phoenix Bridge Company.

Four lattice trusses carrying three 20' roadways with two 9' sidewalks on outside of outer trusses. Trusses have curved outlines. Depth of trusses at center 38 ft. Buckle plate floor. Turntable 60 ft in diameter. 80 - 24" wheels under turntable. Largest turntable ever built. Depth of trusses at ends 18 ft. Operated by a 50 H. P. engine. Opens in two minutes. Traffic 5000 vehicles per day. Total weight of bridge above turntable 5,000,000#.

HARLEM RIVER BRIDGE OF N. Y. C. & H. A. R. R.

Four Track Swing Bridge. Span 389 ft.

Three Trusses 26'0" c. to c.

Drum 50 ft in diameter.

Height at Center over Drum 64 ft. c. to c. pins.

Height at Center Hip 46 ft. c. to c. pins.

Height at End Hip 25 ft. c. to c. pins.

Length of Panels in Arms 23' 1-3/8"

Length of Center Panel 19' 2"

Ballasted track carried over bridge on solid floor. Operated by 2 - 10 x 7 oscillating engines 50 H. P. Total weight 5,000,000#.
SEVENTH AVENUE DRAW SPAN OVER HARLEM RIVER

Span 408'6" c. to c. pins.

Width c. to c. of Trusses 43'6"

Height at center over Drum 62'6" c. to c. pins.

Clear Height 25'0"

Roadway 40 ft. between curb lines.

INTERSTATE BRIDGE AT OMAHA, NEB

Double Track R. R. Bridge with two roadways at sides.

Span 520 ft. c. to c. pins.

Height over Drum 95 ft. c. to c. pins

Height at Center Hip 50 ft. c. to c. pins.

Height at End Hip 25 ft. c. to c. pins.

Length of Panels in Arms 35 ft.

Length of Center Panel 30 ft.

R. R. Tracks 13 ft. c. to c.

Trusses 30 ft. c. to c.

Loading for floor and primary truss members, Waddell's Class X for tracks and 100# per square foot for roadways and sidewalks.

Loading for chords and web members of truss L. I. 9600# per lineal foot with one arm loaded or 8000# per lineal foot with both arms loaded.

Wind load 600# per lineal foot, on lower chord and 280# per lineal foot on upper chord.

The dead load was assumed at 6100# per lineal foot.
Operated by a 40 H. P. Waddell's engine motor. Drum 5'0" deep. 34'0" in diameter. Total weight about 3,000,000#

TRENT VALLEY CANAL SWING BRIDGE AT NASSAU, ONTARIO.

Single Track Railroad Bridge.

Span 217'6", height at Center over Drum 40 ft. c. to c.
Panel Length 25 ft. Length of Center panel 17'6"
Tops of Center posts joined together and double Pin Joint at center to insure continuity.

C. B. & Q. DRAW SPAN OVER MISSOURI RIVER AT ALTON, ILL.

Span 450 ft. Center Panel about same length as other panels.
15 Panels in all. Top chord is straight from center to end. First two diagonals at ends are stiff, members running from end toward the center. There are no counters in the bridge.

Plate Girder Draw, 120'0" Span, Norfolk & Western R. R. Bridge over Dismal Swamp Canal.

Double Track Deck Plate, 4 Girders Spaced 9'9" c. to c., Arms unequal, 1 - 76'8" and 1 - 43'6" web 7'0" deep. Operated by Hand.

From the above descriptions we see that with one exception the panel lengths are from 20 ft. to 30 ft. in length. The Interstate Bridge seems to be an exceptional design in this particular. The length of its members are all much longer than has been used in
similar designs. The ratio of the span to the center height for these bridges varies between 7.8 and 5.3. The average is about 6.0. For our design we have used 6.8. We notice that the C. M. & N. Bridge is exceptional in the size of the drum.

The Third Avenue Harlem River Bridge is exceptional in outline and in construction. The Kinzie Street Draw of the C. & N. W. Ry. is somewhat similar in construction. (See Photograph)
(b) LIVE LOAD FOR TRUSSES: In our calculations we have used 5300# per lineal foot on near track and 4200# per lineal foot on far track; or, \( \frac{22}{30} \times 5300 + \frac{8}{30} \times 4200 = 4787 \# \) per lineal ft. per truss. Panel load equals \( 25 \times 4787 = 120,000 \# \).

The question of choosing the live load is very important and many of the best engineers have made great mistakes in choosing loads too small. The steady increase in the live load for railroad bridges is well illustrated by the following comparison:

**Engine Loads Used on the Chicago and Alton Ry. Since '86**

<table>
<thead>
<tr>
<th>Eq. Uniform Loads for Spans of</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 ft.</td>
</tr>
<tr>
<td>Typical engines used prior to 1886</td>
</tr>
<tr>
<td>Typical engines used from '86 to '88</td>
</tr>
<tr>
<td>Typical engine used from '88 to '97</td>
</tr>
<tr>
<td>Typical engine used from '97 to '99</td>
</tr>
<tr>
<td>Typical engine adopted in 1899</td>
</tr>
<tr>
<td>Heaviest engine used prior to 1899</td>
</tr>
<tr>
<td>Heaviest engine in use at present</td>
</tr>
</tbody>
</table>

The C. & A. Ry. like many other railways has lately replaced many bridges on its line by stronger ones. Although the bridges were otherwise good they were designed too light and could not be strengthened economically. The limit of this increase in the weight of engines and trains is not known but we can hardly conceive of as great changes in the future as we have had in the past. It would seem that the bearing limit for the rail must nearly be reached and
in fact this is evident in some tracks at present. The equivalent uniform load for the present loading specified by the C. & A. Ry. for a span of 225 ft., the length of one arm of our bridge, is about 5400# per lin. ft. of track. Or, considering that when one track is loaded to the maximum the other track will not have the maximum, as is done in our example of the C. M. & N. bridge, we see that the live load for the truss is very closely what we would have if designing under the C. & A. Specifications.

(c) LIVE LOAD FOR THE FLOOR SYSTEM: Let us first consider the stringers. The maximum L. L. moment as used on the C. M. & N. bridge is

\[
\text{C. M. N.} = 336500 \text{ Ft. lbs.}
\]

\[
\text{do for C. & A. '99 Specifications} = 360000 '
\]

\[
\text{do for heaviest actual C. & A. Engine} = 308000 '
\]

From the above we notice the C. M. & N. bridge stringers were designed for a loading about midway between the C. & A. typical and C. & A. actual engines. Let us compare the end shear for the stringers. The end shear for the above loadings are respectively:

\[
\text{C. & M. N.} = 59800#
\]

\[
\text{C. & A. '99 Specifications,} = 68500
\]

\[
\text{C. & A. actual engine} = 58500.
\]

The shear for the actual engine is very near that for which the C. & M. N. bridge was designed.

In our design we have used the C. & A. '99 Specifications for the stringers. Although this loading is some heavier than that used
The dead load for the stringers will consist of the stringer and the track. The stringer will weigh seven times its length per foot, and the track will weigh about 200# per foot per stringer. This makes a total of 9375# per stringer or 46375# end shear for dead load as compared with 5000# as used on the C. M. & N.

Let us now consider the floor beam. The maximum live load end shear on the intermediate floor beams as used on the C. M. & N. is 165200#. For the C. & A. '99 Specifications we have 176000#; and for the C. & A. 154-5 ton engine we have 148000# as the maximum L.L. shear for the intermediate floor beams. In the above we have considered both tracks fully loaded. This is not a fair assumption, however, for there is but little probability of both tracks being loaded to a maximum at the same floor beam. If this should occur occasionally we might be justified in using a higher fiber stress for this condition. For example, we have a similar problem to deal with in double track bridges where three girders are used. The center girder is seldom designed twice as heavy as the outer one, but instead it is usually designed 1-1/2 to 1-3/4 times the strength of the outer girder. Let us assume one track fully loaded and the other carrying 85% of the full load. The maximum end shears are 95% of the above or 167000# and 141000# for the C. & A. '99 loading and 154-5 ton engines respectively. Hence it seems that the former load-
ing is quite near the loading used on the C. M. & N. We have used the C. & A. '99 loading in our design.
(d) WIND LOAD: Bridge specifications generally state what the wind load shall be as so much per lineal foot for each lateral system. It would seem proper to specify a certain allowable wind pressure per square foot of exposed surface for a bridge of such unusual dimensions as the one in question. As to what this unit pressure shall be the authorities differ widely. Fifty pounds per square foot of vertical surface is about the maximum value in use and thirty pounds is about the minimum value which should be used. In the following design we have considered forty pounds per square foot of vertical surface and in addition to the surface of the truss we have considered a moving train on the bridge ten feet high. We used for the top laterals 300# per lineal foot, and for the bottom laterals a uniform load of 200# and a moving load of 400# per lineal foot. In Waddell's design of the Interstate Bridge at Omaha, Neb., 280# per lineal foot uniform load was used on upper laterals and 600# per lineal foot for the lower laterals. On the C. M. & N. Bridge 350# per lineal foot was used on the top laterals and only about 400# per lineal foot for the bottom laterals.
II DETERMINATION OF STRESSES

Without attempting to cover the entire field of analysis of all the various types of swing bridges, we will consider immediately the case involved in our design. Where the center panel is not braced strong enough to carry all the shear over the center, the continuity is only partial and the analysis must be based upon this fact. In our design we have followed the assumptions made for the C. M. & N. bridge. The bridge is considered as a continuous beam over four supports with equal moments at the center. Since the theory of the analysis is based upon the "theorem of three moments" we present the derivation of these fundamental formulae leading to the derivation of the formulae for the reactions.
THEOREM OF THREE MOMENTS

(a) UNIFORM LOAD: Since \( M \) at any section of a beam equals the moment at any other section plus the shear into its arm plus the moment of all the intervening forces about a point in the section in question, we have

\[
M_x = M_r + 3rx - \frac{1}{2} \ell_r x^2
\]  

(1)

From the equation of the elastic curve we have,

\[
\frac{d^2y}{dx^2} = \frac{M}{EI}
\]

(2)

Substituting \( M \) as given in (1) in equation (2) we have,

\[
\frac{d^2y}{dx^2} = \frac{m_y - 3rx + \frac{1}{2} \ell_r x^2}{EI}
\]

Integrating,

\[
\frac{dy}{dx} = \frac{m_y x + \frac{3}{2} S_y x^2 - \frac{1}{6} \ell_y x^3 + C_1}{EI}
\]

Integrating,

\[
y = \frac{1}{2} \frac{m_y x^2}{EI} + \frac{1}{2} \frac{S_y x^3}{EI} - \frac{1}{24} \frac{\ell_y x^4}{EI} + C_1 x + C_2
\]

when \( x = 0 \), \( y = 0 \) and \( C_2 = 0 \)

\[
\begin{align*}
& x = l_r, \quad y = 0 \quad \Rightarrow \quad C_1 = \frac{1}{2} \frac{S_y \ell_y^3}{8 EI} - \frac{1}{24} \frac{m_y \ell_r^4}{EI} = \frac{1}{6} \frac{m_y \ell_r^4}{EI} - \frac{1}{24} \frac{m_y \ell_r^4}{EI} = \frac{1}{24} \frac{m_y \ell_r^4}{EI} \\
& dy = \frac{12 m_y (2x - 2y) + y S_y (3x^2 - l_y^2) - 8y (y x^2 - l_y^2)}{2 EI}
\end{align*}
\]

(3)

Hence,

\[
\frac{dy}{dx} = \frac{12 m_y (z_x - x_{+1}) + 3S_y (3x^2 - l_y^2) - 8y (y x^2 - l_y^2)}{2 EI}
\]

(4)

Similarly we take the origin at the next support and we get,

Make \( x = 1 \) in (3) and \( x = 0 \) in (4) and equate these values of \( \frac{dy}{dx} \) then,

\[
12 m_y l_r + 3S_y l_y^2 - 3S_y l_y^2 = -12 m_y l_{+1} - 3S_y l_{+1} - 8y l_{+1}^2
\]

Substituting the values of \( S_y \) and \( S_{y+1} \) in terms of \( M_r, M_{r+1} \) and \( M_{r+2} \) we get

\[
M_y l_r + 2 m_y l_{+1} (l_y + l_{+1}) + m_{r+2} l_{+1}^2 = \frac{P_x l_y^2}{4} - \frac{P_{y+1} l_{+1}^2}{4}
\]

which is the Theorem of Three Moments for uniform loads.
(b) CONCENTRATED LOAD: From equation (1) we have, 
\[ m_x = m_y + s_{xy} - \frac{\partial r}{\partial e}(x-a) \]  
(1)

From equation (2) we have 
\[ \frac{d^2u}{dx^2} = \frac{m_y}{c_1} \]  
(2)

Make \( x = l_y \) in (1) and we get \( S_y \) in terms of the moments at the supports 
\[ S_y = \frac{m_y l_y - m_y}{l_y} - \frac{\partial r}{\partial e}(l_y - a) \]  
(3)

Substitute \( M_x \) from (1) in (2) and we get, 
\[ \frac{d^2u}{dx^2} = \frac{m_y + s_{xy} - \frac{\partial r}{\partial e}(x-a)}{c_1} \]  
(4)

Equation (4) can be integrated between the limits \( x \geq 0 \) and \( x \), with the condition that \( x \geq a \), or the point in question is always to the right of the load \( P \). When \( x = 0 \), \( a \) must equal zero also, hence \( (x - a) \) must be integrated simultaneously between the limits \( x = a \), and \( x \). In this way we get the tangent of the angle which the tangent to the elastic curve makes with the horizontal to be, 
\[ \frac{dy}{dx} = \frac{m_y + \frac{1}{2} s_{xy} - \frac{1}{2} \frac{\partial r}{\partial e}(x-a)^2}{c_1} + c \]  

When \( x = 0 \), \( c \) equals differential \( y \) over differential \( x \) which equals tangent \( \alpha \) 

Hence, 
\[ \frac{dy}{dx} = \frac{m_y + \frac{1}{2} s_{xy} - \frac{1}{2} \frac{\partial r}{\partial e}(x-a)^2}{c_1} + \tan \alpha \]  
(5)

Integrating we get the equation of the elastic curve, 
\[ y = \frac{\frac{1}{2} m_y x^2 + \frac{1}{4} s_{xy}^2 - \frac{1}{2} \frac{\partial r}{\partial e}(x-a)^3}{c_1} + \tan \alpha x + c \]  

Taking the origin at the left support \( y = 0 \), when \( x = 0 \) and \( c = 0 \), 

Hence, 
\[ y = \frac{\frac{1}{2} m_y x^2 + \frac{1}{4} s_{xy}^2 - \frac{1}{2} \frac{\partial r}{\partial e}(x-a)^3}{c_1} + \tan \alpha x \]  

Substituting for \( S_y \) its value in (3) we get, 
\[ y = \frac{\frac{1}{2} m_y x^2 + \frac{1}{4} \left[(m_{xy} - m_y) e_{yx} \frac{\partial r}{\partial e}(x-a)^2 - \frac{\partial r}{\partial e}(x-a)^3 \right]}{c_1} + \tan \alpha \]  
(6)

Making \( x = 1 \) and substituting \( kl \) for \( a \), we get,
Substituting this value for \( \tan \theta \) in equation (5) and making \( x = 1 \), we have,

\[
\frac{dy}{dx} = \tan \theta = -\frac{1}{6 \varepsilon} \left[ m_y l_y + 2 m_y l_y + P_y l_y^2 (k - k^3) \right]
\]

Similarly we may write for the \( r \)-th span,

\[
\frac{dy}{dx} = \tan \theta = -\frac{1}{6 \varepsilon} \left[ m_{y-r} l_{y-r} + 2 m_{y-r} l_{y-r} + P_{y-r} l_{y-r}^2 (k - k^3) \right]
\]

Since these equations hold good for all values of \( P \), we may make \( P \) equal to zero in each case then equate (7) and (8) and we have,

\[
m_{y-r} l_{y-r} + 2 m_{y-r} l_{y-r} + m_{y-r} l_{y-r} = -\frac{P_{y-r}}{6 \varepsilon} l_{y-r}^2 (k - k^3) + \frac{P_{y-r}(2k-3)k^3}{2}
\]

which is the equation of Three Moments for concentrated loads. The summation sign is inserted in the equation (9) for more than one load in one span.

Equation (9) is really more directly applicable to swing bridges than the equations for uniform loads, for all through swing bridges carry their loads to panel points where it is transmitted to the truss or girder.

\[
\tan \theta = -\frac{1}{6 \varepsilon} \left[ m_y l_y + 2 m_y l_y + P_y l_y^2 (k - k^3) \right]
\]

\[
\frac{dy}{dx} = \tan \theta = -\frac{1}{6 \varepsilon} \left[ m_{y-r} l_{y-r} + 2 m_{y-r} l_{y-r} + P_{y-r} l_{y-r}^2 (k - k^3) \right]
\]

\[
m_{y-r} l_{y-r} + 2 m_{y-r} l_{y-r} + m_{y-r} l_{y-r} = -\frac{P_{y-r}}{6 \varepsilon} l_{y-r}^2 (k - k^3) + \frac{P_{y-r}(2k-3)k^3}{2}
\]
FORMULAE FOR THE REACTIONS

(a) METHOD #1, (Johnson-Schaub): In Johnson's book on "Modern Framed Structures" p. 194, the derivation of the formulae for the reactions for our case is given but the results given here are all wrong. (See Eng. News Vol. 34, No. 17) Mr. Schaub, the author of that portion of the book, admits the error and corrects it in the above reference. The following is the correct derivation by one method:

In equation (9) p. make \( r = 2 \) and \( l_1 = l_2 = l \) and we have,

\[
2m(l + l) + m_2 l_2 = -P_1 l^2(k - k^3) - P_2 l^3(2k - 3k^2 + k^3)
\]  
(1)

Making \( r = 3 \) in the same equation we have

\[
m_2 l_2 + m_3 (l + l) = -P_3 l^2(k - k^3) - P_4 l^3(2k - 3k^2 + k^3)
\]  
(2)

Adding (1) and (2) we have,

\[
m_2 (2l + 3l_2) + m_3 (2l + 3l_2) = -P_1 l^2(k - k^3) - P_3 l^3(2k - 3k^2 + k^3)
\]

Considering load on but one arm only \( P_2 \) and \( P_3 \) equal zero, then

\[
m_2 (2l + 3l_2) + m_3 (2l + 3l_2) = -P_1 l^2(k - k^3)
\]  
(3)

Making \( M_2 \) equal to \( M_3 \), we get,

\[
m_2 = -\frac{P_1 l}{2l + 3m} (k - k^3) \quad \text{when} \quad m = \frac{l_2}{l}
\]  
(4)

Taking moments we have also,

\[
m_2 = R_2 l - P(l - k) = R_1 l - P(1-k)
\]  
(5)

Equating (4) and (5) and solving for \( R \) we get,

\[
R_1 = \frac{P}{l} \left( \frac{1}{1 - \frac{1}{2m}} - k^3 \right)
\]  
(6)

\[
R_2 = \frac{P}{l} - R_1
\]  
(7)

\[
R_3 = \frac{P}{l} \left[ \frac{1}{2m} \left( \frac{1}{l^2} - k^3 \right) \right]
\]  
(8)
(b) METHOD #2 (DuBois): Let

the length of end span equal l, and

the length of center span equal nl.

We make \( M_1 \) equal \( M_3 \) then,

\[
S_1 = \frac{m_1}{l} + \frac{q'(l-\gamma)}{l}, \quad S_2 = \frac{m_2}{l} + \frac{q_1}{l}, \quad S_3 = \frac{m_3}{l} = S_x \tag{1}
\]

From these equations we can get the reaction at any support for any position of \( P \) if \( M_2 \) is known.

Let \( S \) equal the panel length, \( a \) the area of chord section, \( v \) the lever arm for any chord member, and \( x \) the distance of \( P \) from the left support.

Now if \( v \) be the lever arm for any chord member and \( M \) the moment at the center of moments for that member, then the stress in the member equals \( M + v \). Since the modulus of elasticity of the member is by definition \( \frac{\text{unit stress}}{\text{unit deformation}} \) we have,

\[
E = \frac{s}{\Delta} \tag{1}
\]

The work of the internal stress gradually applied equals \( \frac{1}{2} \lambda S \) \( \Delta \) \( S \).

Substituting the value of \( \lambda \) from (1) we have work equals \( \frac{1}{2} S = \frac{s^2}{2a} \).

Substituting this value of \( s \) in (3) we have work equals \( \frac{1}{2} S \).

The total work of straining all the members is then \( \frac{1}{2} S \) and this must be a minimum.

Now for any distance \( x \) from \( A \) between \( A \) and \( P \) we have

\[
M = -S_1 x = \frac{m_1 x}{l} - \frac{q'(l-\gamma)}{l} \tag{3}
\]

For any point between \( P \) and \( B \) we have \( m = -S_1 x = q(x-\gamma) = \frac{m_1 x}{l} - \frac{q_1 l}{l} \).
For any point between B and C we have \( M = M_2 \).

For any point between C and D - distance \( x \) from C we have

\[
M = m_2 - S_3 x = \frac{m_2(l - x)}{l}
\]

We have then for the work of straining all the chord members from equation (5)

\[
\text{work} = \sum \left[ \frac{m_2 x - \rho(l - x)x^2}{2 \varepsilon a v} \right] \frac{s}{x^2} + \sum \left[ \frac{m_2 x - \rho(l - x)x}{2 \varepsilon a v} \right] \frac{s}{x^2}
\]

Since the work is here given in the terms of \( M \) and known quantities and the work must be a minimum we place the first derivative equal to 0.

\[
\frac{d}{dM} \left( \text{work} \right) = 0 = \sum \left[ \frac{m_2 x - \rho(l - x)x^2}{2 \varepsilon a v} \right] \frac{s}{x^2} + \sum \left[ \frac{m_2 x - \rho(l - x)x}{2 \varepsilon a v} \right] \frac{s}{x^2}
\]

Hence

\[
M_2 = -\frac{\rho \left[ \frac{l \rho (l - x)x^2}{a v^2} + \sum \frac{l \rho (l - x)x^2}{a v^2} \right]}{2 \sum \frac{l \rho (l - x)x^2}{a v^2} + \sum \frac{l \rho (l - x)x^2}{a v^2}}
\]

Reducing we get

\[
M_2 = \rho \left[ \frac{l \rho (l - x)x^2}{a v^2} - \sum \frac{l \rho (l - x)x^2}{a v^2} + \sum \frac{l \rho (l - x)x^2}{a v^2} \right]
\]

Now assume \( a \) as constant, chords parallel and small panels, then \( dx = s \) and we have,

\[
M_2 = \int_0^l x^2 dx - \frac{\rho}{2} \int_0^l x^2 dx + \frac{l}{2} \int_0^l x dx - \int_0^l x dx - \frac{\rho}{2} \int_0^l x^2 dx
\]
Integrating,
\[ \int \frac{dy}{y} = \int \frac{x^2}{2} dx = \frac{x^3}{6} + C \]

Reducing,
\[ m_z = \rho \left( \frac{\frac{1}{6} (\frac{y^2}{3} - \frac{y^2}{2})}{\frac{1}{3} (x^2 + 3m)} \right) \]

Let \( k = \frac{z}{l} \),
\[ m_z = \frac{\rho l}{2} \left( \frac{k - k^3}{z + 3m} \right) \]

Substituting the value of \( M \) in (1) and we get,
\[ S = -\frac{\rho}{2} \left( \frac{k - k^3}{z + 3m} \right) + \frac{\rho l}{d} (d - y) \]

which is the same as equation (6) Method #1. Equations (7) and (8) may be derived by putting the above value of \( M \) in equations (1) and (2). These equations (6), (7) and (8) Method #1 have been used in getting the reaction as given on our diagrams.

We believe the above methods as given represent about all that exists in literature on the derivation of the formulae for reactions for the case in question.

One feature noticed in the comparison of the above methods should perhaps be emphasized. Methods #1 and #4 use only the principles of statics, while the other two methods the principles of least work are used.
Since there are four unknown reactions, four conditions are necessary to determine them. The principles of statics furnish us three of the required conditions, while the principles of the least work give us the fourth. From statics we have, the sum of the reactions equals the total load or
\[ R_1 + R_2 + R_3 + R_4 = P \tag{1} \]
Also the sum of the moments of the reactions about any point equals the moment of the load, or taking moments about A,
\[ R_y (2l + u l) + R_3 (l + u l) + R_y l = P k l \tag{2} \]
And the third condition, since no shear is transmitted across the center panel,
\[ R_1 + R_2 = P \tag{3} \]
and the fourth condition is that the total internal work in the truss is a minimum. From principles of least work, the total internal work of the stresses is proportional to the sum of the squares of all the bending moments. Now \( R_x \) is the bending moment at any point on the left of the load. \( R_1 x - P(x - kl) \) is the bending moment at any point between \( P \) and point \( B \); also \( R_3 l \) is the bending moment in the middle span, and \( R_3 x \) is that for any point in span \( C - D \). Then,
\[ \int_0^{l/2} R_i x^2 dx + \int_0^l (R_i x - P x + P k l) dx + \int_{l/2}^l R_3 x^2 dx + \int_R^r R_3 x^2 dx \]
equals a minimum \( \tag{4} \)
is a mathematical expression of the fourth condition. These four equations give the four unknown reactions as on the previous page.
(d) METHOD #4

Referring to the figure let BS, and BS be drawn normal to AB and BC respectively and \( \theta \) equal the angle \( S_{BS} \).

Similarly construct the angle \( \phi_3 \). Draw BA tangent to AB at B. Let BA be the curve assumed by AB when deflected from the position AB by the load P. Draw CD tangent to CD at C. Now \( \phi_2 - \phi_3 \) equals the angular deflection of the tangent at C of a beam whose moment of inertia equals that of the truss BC fixed at B and strained by the moment \( M \).

Now from the theory of flecture,

\[
\phi_2 - \phi_3 = \frac{m_2 l_2}{E I} = -\frac{R_3 l_3 l_2}{E l_1} \tag{1}
\]

Since \( R_3 \) and \( M_2 \) are negative,

\[
\phi_3 = \frac{E D}{l_3} = -\frac{R_3 l_3}{E l_1} \tag{2}
\]

\( DD' \) being the deflection of CD due to \( R_3 \). But \( A' A'' \), the deflection of AB due to P, equals

\[
\frac{P}{E l_1} \left[ \frac{(l_2 - \frac{1}{3})^3}{3} + \frac{2}{3} (l_2 - \frac{1}{3})^2 \right] \tag{3}
\]

\[
A' A'' = l_1, \phi_2 \tag{4}
\]

\( AA'' \) equals the deflection of AB due to \( R, = \frac{R_1 l_3^3}{3 E l_1} \tag{5} \)

\( M_2 = M_3 = R _1 l_3 = R, l_1 - P(l_1 - z) \tag{6} \)

Combining (3), (4) and (5), we have,

\[
\frac{R_1 l_3^3}{3 E l_1} - l_1, \phi_2 = \frac{P}{E l_1} \left[ \frac{(l_2 - \frac{1}{3})^3}{3} + \frac{2}{3} (l_2 - \frac{1}{3})^2 \right] \tag{7}
\]

Substituting the value of \( \phi_2 \) from (1) and (2) and multiplying both members by EI we get,

\[
\frac{R_3 l_3^3}{3} + l_1, k_3 l_2^2 (l_2 + l_3) = \frac{P}{E l_1} \left[ \frac{(l_2 - \frac{1}{3})^3}{3} + \frac{2}{3} (l_2 - \frac{1}{3})^2 \right] \tag{8}
\]

Substituting the value of \( R \) from (6) and reducing and solving for \( R_3 \) we have,

\[
R_3 = \frac{-P (l_2 - \frac{1}{3})^2}{2 l_1 l_3 (l_1 + l_2 + l_3)} \tag{9}
\]

Make \( z = kl \) then \( M_2 = M_3 = -\frac{P l_2^2 (k - k^3)}{2 (l_1 + l_2 + l_3)} \)

Let \( l_1 = l_3 \) then \( M_2 = \frac{P l_2^2 (k - k^3)}{2 (l_1 + l_2 + l_3)} \tag{10} \)
DIAGRAM NO. 4
LIVE LOAD ON ONE ARM, CONTINUOUS. SCALE 1"=100,000
DIAGRAM NO. 8

LIVE LOAD COMING ON BRIDGE SIMPLE SPAN

R₁ = 106700
R₂ = 200000
R₃ = 280000
R₄ = 346700
R₅ = 400000
R₆ = 740000

SCALE 1" = 100000"
Diagram No. 9

Live Load coming on one arm while other arm is fully loaded. Con.
IV ANALYSIS OF TRUSSES

(a) DIAGRAM #1: Condition, dead load only, ends touching. This diagram differs from #1a only in the distribution of dead load. In #1 the whole load is considered as acting at the bottom chord. In #1a one third the load is considered as acting at the top chord except at U-6 and U-8. The only difference in the results of the two methods is in the stress in the posts near the end. We have used #1a for our post stresses as will be seen by the strain sheet.

Check: stress in L-5--L9 equals \( \frac{66250 \times 6 \times 87.5 - 33125 \times 175 - 66250 \times 25}{50} \)

equals \( \frac{38884000}{50} \), equals \( 777,700 \# \)

(b) DIAGRAM #1a: Condition same as #1.

Check: stress in L-5--L9 same as above.

(c) DIAGRAM #2: Condition, live load, simple span. This condition combined with #6 gives maximum stresses in nearly all the chords.

Check: stress in L9-M2 equals \( 480,000 \times 1414 \), equals \( 677,700 \# \)

(d) DIAGRAM #3: Condition, full live load on both arms, continuous. This condition gives maximum stresses in U7-U9-U10 and L9-L10 when combined with #1.

Check: stress in U9-U10 equals \( \frac{120000 \times 8 \times 112.5 - 360000 \times 225}{70} \)

equals \( 385000 \# \)
(d) DIAGRAM #4: Condition, live load on one arm only, continuous. This diagram is for the loaded arm and gives no maximums where all the cases are considered as we have done for this bridge. However, this condition is sometimes taken for maximum chord stresses instead of our #2. This is the method used in "Johnson's Framed Structures" and of course it gives smaller chord stresses than those of our #2. Condition #4 seems to be nearer the practical condition especially with a very large bridge for the "simple span" will never be realized on account of the enormous end lift that would be necessary to produce it.

Check: stress in L9-L10 equals $\frac{120000 \times 8 \times 112.5 - 421800 \times 225}{70}$

Equals 187000#

(e) DIAGRAM #5: Condition, live load on one arm only, continuous. This diagram is for the unloaded arm. This condition combined with #1 gives minimum stresses for all the chords and end posts except at the center.

Check: stress in L9-L10 is the same as in #4.

(f) DIAGRAM #6: Condition, dead load ends raised, continuous. As noted under #2 this condition with #2 gives maximum chord stresses

Check: stress in L9-L10 equals $\frac{112.5 \times 530000 - 225 \times 41000}{70}$

equals 710000#

(g) DIAGRAM #7: Condition, live load moving off of bridge, simple span. This condition gives us the maximums for the diagonals from center toward the end.

Check: Since this is a simple span, the drafting checks itself by
(h) DIAGRAM #8: Condition, live load coming on bridge, simple span. This condition is similar to #9 but is not so severe hence we get no maximums. #9 combined with #1 gives maximums in diagonals from end toward the center and exceed #1 with #8.

Check: See #7.

(i) DIAGRAM #9: Condition, live load coming on one arm while other arm is fully loaded, continuous. See #8.

Check:

stress in U9-U10 for R1 equals \[
\frac{46500 \times 225 - 120 \times 2000}{70} = 193000 \text{#}
\]

" " " " R2 " \[
\frac{133.2 \times 225 - 240 \times 187.5}{70} = 215000 \text{#}
\]

" " " " R3 " \[
\frac{205.7 \times 225 - 360 \times 175}{70} = 240000 \text{#}
\]

" " " " R4 " \[
\frac{261.7 \times 225 - 480 \times 162.5}{70} = 274000 \text{#}
\]

" " " " R5 " \[
\frac{305.7 \times 225 - 600 \times 150}{70} = 305000 \text{#}
\]

" " " " R6 " \[
\frac{336.7 \times 225 - 720 \times 137.5}{70} = 335000 \text{#}
\]

It may be noted here that the stresses in the primary members of the truss are not taken from the above diagrams. It would be inconsistent to design the floor for one load and these primary members for a different load. Hence we have simply the maximum end shear of the one floor beam for the maximum stress in U1-L1, M1-L6 and
Top Lateral System

1. 300# per lin. ft. Bridge swinging
2. Closed

Bottom Lateral System

1. 200# per lin. ft. Deadload, 400# per lin. ft. Train load
M2-L8, and for the stresses in U7-L7, M2-L7 and M1-L7, we use the floor load direct.

(j) STRESSES IN PINS: The pins cannot be figured until we determine the sections on the members. See p.

V STRESSES IN LATERALS AND PORTAL BRACING.

(a) TOP LATERAL SYSTEM: As stated before, we use 300# per lineal foot on the top chord and get maximum stresses in all panels except U1 U2 by using condition (1). (See diagram of stresses p. )

We notice that the stresses given for the C. M. & N. Bridge are much greater than ours for the top lateral system while the stresses for the lower system are much lower. However, the difference in the allowable unit stresses, as will be explained later, gives sections which do not differ greatly from ours. The wind load used on the C. M. & N. Bridge is 350# per lineal foot.

(b) BOTTOM LATERAL SYSTEM: Our uniform load here is 200# per lineal foot and our moving load 400#. As will be noticed from the stress diagram, our stresses are much higher in the end panels than on the C. M. & N. Bridge but we are using heavier loads and get all our maximums by considering each arm as a simple span while in the C. M. & N. the maximums are greater when the bridge swings free.

No doubt the designer of the C. M. & N. Bridge has considered that the stiff floor will take a large amount of wind stress through the connections. It must also be remembered that stiff laterals take compression stresses and for this reason the sections may be slight-
ly reduced, especially when the laterals are riveted to the stringers as is often done.

(c) END PORTALS: Although our wind load is some lighter than that used on the C. M. & N. Bridge, our stresses in the struts of the end portal are higher. We do not know the assumptions made here in the design of the C. M. & N. Bridge, but probably allowance has been made for the portion of the wind load carried to the bottom chord by the intermediate braces. (See p. for further treatment of portal)

(d) CENTER PORTALS AND INTERMEDIATE SWAY BRACES: By "center portal", we mean the portal at U7 L9. We have analyzed this portal as a simple "cross" portal paying no attention to the bending stress due to the deep struts. An exact theoretical analysis of the portal is of very little value in the practical design. The analysis we have used gives us an approximate idea of what is necessary and we find that in order to get the necessary rigidity we must exceed any section which would be required by the stresses. In fact, all bridge portals must be designed more or less by empirical rules rather than theoretical deductions. We see an illustration of this statement in the C. M. & N. Bridge. While the stresses are given to the nearest one hundred pounds, the sections given for all the struts in the center portals are the same.

The intermediate sway bracing is designed simply for rigidity and not according to any computed stresses. The same is true of the bracing at U8 L8.

(e) BRACING IN THE CENTER PANEL: We have analyzed this portal on the assumption that all the wind load of the upper chord is taken
to the top of the tower. This, of course, is improbable but gives us a basis to work from as to proportions of the various members. However, what has been said in this paragraph applies to the design of this bracing. Theoretical stresses are not taken as the only factor in determining the sections.

### III Determination of Sections

#### I Allowable Unit Stresses

We have followed the unit stresses as used on the C. M. & N. Bridge. These are about as specified by Cooper but they are much higher than are used by the C. & A. Ry. The stresses used are as follows:

<table>
<thead>
<tr>
<th>Member</th>
<th>Unit Stress L.L.</th>
<th>Unit Stress D.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0 U1</td>
<td>9300</td>
<td>9300</td>
</tr>
<tr>
<td>U1 U5</td>
<td>9700</td>
<td>9700</td>
</tr>
<tr>
<td>U5 U7</td>
<td>9686</td>
<td>9686</td>
</tr>
<tr>
<td>U7 U9</td>
<td>9500</td>
<td>16000</td>
</tr>
<tr>
<td>U9 U10</td>
<td>9500</td>
<td>19000</td>
</tr>
<tr>
<td>L0 L9</td>
<td>9000</td>
<td>18000</td>
</tr>
<tr>
<td>L9 L10</td>
<td>9820</td>
<td></td>
</tr>
<tr>
<td>U1 L1</td>
<td>6000</td>
<td>6000</td>
</tr>
<tr>
<td>U2 L2</td>
<td>9390</td>
<td>9390</td>
</tr>
<tr>
<td>U3 L3</td>
<td>9205</td>
<td>92055</td>
</tr>
<tr>
<td>U4 L4</td>
<td>9205</td>
<td>9205</td>
</tr>
<tr>
<td>U5 L5</td>
<td>9174</td>
<td>9174</td>
</tr>
<tr>
<td>U9 L9</td>
<td>9630</td>
<td>9630</td>
</tr>
<tr>
<td>Diagonals</td>
<td>9500</td>
<td>19000</td>
</tr>
</tbody>
</table>
DRUM: 1-WEB 7½ x 1/2
4-LD 6" x 6" x 12"/16

CHORD BEAM: 1-WEB 7½ x 3/4
4-LD 6" x 8" x 5/8
4-COV. PL. BOT. 14" x 3/4
4-TOP

SECTIONS OF TRUSS MEMBERS

INT. FL. BEAM: 1-WEB PL. 39¾ x 3/8
4-LD 6½ x 6½ x 11/16
4-PLATS 6" x 3/4
1-COV. PL. BOT. 14½ x 11/16

END FL. BEAM: 1-WEB PL. 59¾ x 3/8
4-LD 6½ x 6½ x 11/16
4-PLATS 6" x 3/4
2-COV. PL. BOT. 14½ x 11/16

STRINGER: 1-WEB PL. 31½ x 3/8
4-LD 6½ x 4½ x 5/8

SPANELS @ 25° - 6° = 27.5°
We give below the unit stresses required by the C. & A. Ry., which vary a large percentage from the above.

In members U2 U3, U3 U4 and U4 U5, eight-tenths of the section required for the compression stress was added to the section required for the tensile stress. This seems rather a severe requirement where the seversal of stress does not occur suddenly but it is usually specified.

II. SECTION OF TOP CHORD

In designing this section, it was very desirable to select general dimensions such that the heaviest portions of the chords could be made up of the same depth plates and the same angles as the lightest portion giving us uniform sections for the side plates and angles throughout the top and bottom chord and in the end posts. The only joint at which the packing was at all complicated was at U7, hence the packing was a secondary consideration.

III. SECTION OF BOTTOM CHORD

See paragraph above.

IV. SECTIONS OF WEB MEMBERS

In designing the posts, the section of the angles was kept uniform throughout. The widths and makeup of the post section had to be such as to offer good opportunity for the floor beam connections. The width of the chords also fixed the transverse dimensions of the post. The section of the tension members was chosen to pack conveniently in the joint and were made as deep as economy
allowed to resist the bending stress due to the weight of the member. The number of pieces was kept at a minimum in order to insure more perfect distribution of the load among the pieces.

V PORTALS AND LATERAL BRACING

In designing the portals, we have not designed the members according to theoretical stresses but have used as great depths as possible and put in sections uniform throughout which offer good connections and simple construction, rigidity being the paramount consideration in deciding the sections.

VI STRESSES IN THE FLOOR SYSTEM

It would be out of place in this thesis to enter into the detail as to the method of getting the moments and shears on the floor beams and stringers. Under the head of loads, we have given all the important stresses.

The following diagrams were used in analyzing the floor:
The dead load shall be proportioned to cover the maximum stresses due to any possible position of the following specified live load, taken together with the stresses due to dead load, without subjecting any member to a greater stress per square inch than herein specified.

The dead load shall consist of the total weight of metal in the structure plus 400 pounds per linear foot of track in the shape of floor, consisting of the ties, rails, guard rails and all spikes and bolts necessary to fasten same.

The live load per track shall consist of two engines and tenders coupled together as given below followed by a train load of 4,000 pounds per linear foot.

### ALLOWABLE UNIT STRESS IN SOFT STEEL AND IRON

**NOT REAMED.**

In soft steel or iron the unit stress per square inch shall in no case exceed the amounts given below.

<table>
<thead>
<tr>
<th></th>
<th>For live load.</th>
<th>For dead load.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>In Tension.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bottom chords and main diagonals</td>
<td>8,000 pounds.</td>
<td>12,000 pounds net section.</td>
</tr>
<tr>
<td>Eye bars</td>
<td>7,500</td>
<td></td>
</tr>
<tr>
<td>Built sections</td>
<td>7,500</td>
<td>11,500</td>
</tr>
<tr>
<td>Counters</td>
<td>7,500</td>
<td></td>
</tr>
<tr>
<td><strong>In Compression.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hip suspenders and floor beam hangers as well as all members subjected to sudden loading</td>
<td>7,000 pounds net section.</td>
<td></td>
</tr>
<tr>
<td>Bottom or tension flanges, of built girders</td>
<td>8,000</td>
<td></td>
</tr>
<tr>
<td>Lateral, vibration and cross bracing</td>
<td>12,000</td>
<td></td>
</tr>
</tbody>
</table>

In allowing for section cut by rivet holes the holes must be figured ⅛ inch larger than the nominal size of rivet.

End posts and vertical posts of pin spans and all main members with two pin ends.

\[
\frac{7,500}{\frac{L^2}{18,000 R^2}} \quad \text{for live load} \quad \frac{11,500}{\frac{L^2}{18,000 R^2}} \quad \text{for dead load.}
\]

These specifications were written by A. Zeising.
End posts and top chords of lattice spans, top chords of pin spans, and all main members with one or both ends fixed not otherwise specified.

\[
\frac{7,500}{L^2} \quad \text{for live load.}
\]
\[
\frac{11,500}{24,000 \ R^2} \quad \text{for dead load.}
\]

Main web members in lattice spans.

\[
\frac{7,000}{L^2} \quad \text{for live load.}
\]
\[
\frac{10,500}{24,000 \ R^2} \quad \text{for dead load.}
\]

Lateral, vibration and cross bracing.

\[
\frac{12,000}{L^2} \quad \text{for live and dead load.}
\]
\[
\frac{14,000}{24,000 \ R^2} \quad \text{for dead load.}
\]

In above formulae:
\[
L = \text{length of column in inches.}
\]
\[
R = \text{least radius of gyration in inches.}
\]

Top or compression flanges of plate girders and floor girders, must have same gross sections as tension flanges.

No compression member shall be more than 50 times its least diameter in length.

In proportioning rollers under moving ends of spans use the following formula:

\[
300 \times d = p.
\]

\[
d = \text{diameter.}
\]
\[
p = \text{greatest allowable pressure per lineal inch of roller.}
\]

The bearing on pins or rivets shall never exceed 12,000 pounds per square inch of bearing surface (diameter times thickness of piece).

The shear on pins or rivets shall never exceed 7,500 pounds per square inch.

The above allowable stresses shall be reduced 25 per cent. for all hand and field rivets. In lateral and vibration connections above allowable stresses may be increased 25 per cent.

The bending moments on pins shall not produce a stress greater than 15,000 pounds per square inch on outer fiber.

The shear in web plates shall never exceed 6,000 pounds per square inch net section across fiber, or direction of rolling, nor more than 5,000 pounds per square inch net section in line with fiber.

All bed plates shall be so proportioned that the bearing on masonry will not exceed 250 pounds per square inch.

In cases where members are subjected to alternate stresses (tension and compression) they must be designed to resist both kinds.

In determining the sectional area in such cases the member must be proportioned for the stress requiring the greater gross section and to this section 80 per cent. of the gross section required by the opposite stress must be added.

ALLOWABLE UNIT STRESSES IN SOFT STEEL REAMED.

In reamed soft steel the unit stresses specified above, for soft steel and iron not reamed, may be increased 7 1/2 per cent.

ALLOWABLE UNIT STRESSES IN MEDIUM STEEL.

In medium steel the unit stresses specified above for soft steel and iron not reamed may be increased 20 per cent. All holes in medium steel must be drilled or punched and reamed.

BALLAST FLOOR.

In cases where ballast floor is used on bridge, above unit stresses may be increased ten (10) per cent. Ballast must be 15" deep.

GENERAL DETAILS.

The kinds of materials to be used in the different parts of the structure will be marked on the plans.

Where possible the structure and details must be so designed that the stress in each member can be accurately computed.
C.S.A. TYPICAL ENGINE 162.5 T.
" ACTUAL " 154.5 T.

MAXIMUM BENDING MOMENTS
FOR SPANS BETWEEN 5' AND 75'.

Bending Moments in Thousands of Foot-Pounds.

Length of Span in Feet.
C & A SPECIFICATIONS
For 1900

End Shear on Stringers & Floor Beam Concentrations in Pounds

Maximum End Shear on Stringer

Maximum Floor Beam Concentration under 111. Poi

Spans for Equivalent Uniform Load Curve

Equivalent Uniform Load

Spans in feet for Floor Beams & Stringers
STRESSES AND SECTIONS OF LOADING BEAM.
VII STRESSES IN THE LOADING BEAM

To arrive at the dead load reaction on the loading beam we have figured the weight of the C. M. & N. trusses from the sections given and have estimated the weight of the floor system from the actual weights of floors in single track spans lately built for the C. & A. Ry. bridge at Glasgow, Mo. Multiplying the weight of the single track floor by 1.85 we get 1400# per lineal foot. Hence we have the following,

| Weight of Two Trusses       | 1,200,000# |
| Weight of Floor (steel only)| 665,000    |
| Weight of Tracks            | 360,000    |
| Total Weight of Bridge      | 2,245,000# |

Using one fourth the total weight for the end load on loading beam, we get the dead load stresses as shown on diagram.

In determining the live load stresses, we have considered a live load all on the bridge, both tracks loaded to a maximum. This gives a maximum in all members except L1 U2 and L2 L2. Our maximum live load concentration is 600,000# which gives stresses as shown on diagram.

VIII STRESSES IN DRUM

Referring to the diagram we notice that we have the load applied at eight points of support on the drum. The drum being supported by wheels which run on the circular track. The supporting beams transmitting the stress to the drum form a chord of the drum.
circle. The drum is subject to compression throughout the entire circumference, hence we must provide for stiffeners at frequent intervals on the web. This provision is easily and efficiently by use of radial trusses in the drum running from a large center casting to the web stiffeners. After providing for the direct compression due to the weight of the bridge, we add a percentage for rigidity and govern ourselves in the design by practical tests of drums in existing structures. The supporting beam or "chord" beam is figured for the maximum concentration in the center and considered a simple beam supported at both ends.

IV DEFORMATION AND CAMBER

I FORMULAE

The equation for the change of length for a member due to a certain stress is

\[ c = \frac{U \times \delta}{E} \]

where

- \( c \) = change in length,
- \( U \) = the unit stress and
- \( E \) = the modulus of elasticity. This equation comes directly from the fundamental relation between the stress and deformation for elastic material. Since \( E = \frac{\sigma}{\epsilon} \), we may write

\[ c = U \times E \times \frac{\sigma}{\epsilon} = l \]

or the total change of length equals the total length multiplied by the unit deformation.

Now in order to find the total deflection of a bridge at any point under a given load, we first find the stresses in the bridge for the given loading then by the above formulae we find the total change in length of each member of the truss. We then find the
stress in each member of the truss due to a weight of one pound or a unit of weight at the point where the deflection is desired. This stress is then multiplied by the total change of length for each member and the algebraic sum of the products gives the total deflection due to stresses in the members.

II PLAY IN PIN HOLES

Usually the pin holes are from 1/32" to 1/62" larger than the pins and this difference adds to the deflection in proportion as it changes the total length of each member and its effect is found as above described for the deflection due to stresses. In our design the deflection due to play in pin holes is a small matter since we have riveted connections at nearly all the joints.

III TEMPERATURE CHANGES

The deflection due to change of temperature is figured for a range of 60 degrees Fahrenheit and the total change of length for this range was taken at 0.00004\% of the length. These changes are multiplied by the coefficient given by the load of unity and the algebraic sum taken as the total deflection.

IV AMOUNT OF CAMBER

The amount of camber for each arm is the same as for simple spans of the same length and we have used 1" per 100'. The total effect of the camber upon the end deflections is found as described above be multiplying the change of length for each member by the co-
efficient for the load unity and getting the algebraic sum.

V END LIFT

The end lift must be so designed as to produce a reaction greater than the greatest possible negative reaction which the loading can produce. Hence, we first find the greatest negative reaction then assuming the end lift equal to or greater than this quantity we find the resulting stresses and deflection. Subtracting this deflection from the total end deflection we find the amount of raise which the end lift must care for.

As the above calculations for deflections are made upon assumed quantities which vary considerably, it is found that the exact deflections of a swing bridge is impossible to compute, hence in the practical design the total deflection is usually assumed from experiments which have been made and the mechanism of the lift so made that a slight variation in the amount of lift may be cared for by the adjustment of the lift.

V DETAILS

I PACKING OF THE JOINTS

The question of packing is very simple except at U7 where we have five members. This joint was so clumsy in the four track draw bridge which is just being erected for the Belt Ry. over the Drainage Canal that two pins were used to simplify the joint. One pin carries all the members except the hanger which is placed upon a separate pin just below the joint.
II JOINTS OF TRUSSES

All posts are rigidly connected to the chords and at LO no pin is used as is shown in detail p. In figuring the size of the pins we have used a material which is exceedingly useful and systematic. To illustrate the method we will take a very simple case as shown in the figure.

Designating the points of application by the numbers 1, 2, 3, and 4 we arrange our forces, shears, and moments as follows for both vertical and horizontal components:

<table>
<thead>
<tr>
<th>Point</th>
<th>Force</th>
<th>Shear</th>
<th>Arm</th>
<th>Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+5</td>
<td>+5</td>
<td>2&quot;</td>
<td>+10</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>+5</td>
<td>2</td>
<td>-120</td>
</tr>
<tr>
<td>3</td>
<td>-10</td>
<td>-5</td>
<td>2&quot;</td>
<td>+10</td>
</tr>
<tr>
<td>4</td>
<td>+5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The moment and shear for any point is then taken at once from the table.

III LATERAL CONNECTIONS

The bottom laterals have riveted connections and the top laterals pin connections.
IV FLOOR

There is no special feature to the detail of the floor.

V DRUM, SPIDER AND TURNTABLE

The drum receives the weight at eight points through the chord beam which is framed into the inside of the drum so that the top of the chord beam is about flush with the top of the drum. The detail of the drum of the Belt Line Ry. bridge above referred to is unique in having only a portion of the radial axles for the wheels run to the center casting. The remainder are short and have a bracing about three feet from the wheel on an auxiliary circular truss inside the track.

VI MACHINERY

We will not take up this question further than to state that the end lift machinery for the C. M. & N. Bridge consists of a wedge which is forced into position by a plunger from a compressed air cylinder just back of the wedge.
No. 1. Chicago, Madison and Northern R.R. Bridge over The Drainage Canal
No. 2. Chicago, Madison and Northern R.R. Bridge
(Photo taken before canal was in use.)
No. 3. Western Avenue Bridge over Drainage Canal.

No. 4. A.T. & S.F.R.R. Bridge at Corwith over The Drainage Canal
No. 5. C.& A.R.R. Bridge at Glasgow Missouri over the Missouri River

No. 6. Kedzie Avenue Bridge over The Drainage Canal C/M.& N.Bridge in the Background
No. 7. A.T.& S.F.R.R. Bridge at Lemont
over
The Drainage Canal
No.8. Northwestern Elevated R.R. and Wells Street Bridge over The Chicago River. (Double Deck)
No. 9. Lake Street Elevated R.R. and Lake Street Bridge over The Chicago River. (Double Deck)
No. 10. The Chicago and Northwestern R.R. Bridge at Kinzie Street over The Chicago River. (Riveted Connections)
No. 11. C.M. & ST. P.R.R. Bridge near Kinzie Street
over
The Chicago River
No.12 North Avenue Bridge over the Chicago River.
No. 13. C.M.&.ST.P.R.R. Plate Girder Draw Bridge over
The Chicago River
No. 14. C.M. & ST.P.R.R. Wooden Bowstring Draw Bridge over
The Chicago River

No. 15. North Halsted Street Jack-knife Draw Bridge over
The Chicago River
No. 16. Canal Street Jack-knife Draw Bridge 
over 
The Chicago River
No. 17. Chicago and Northern Pacific R.R. Bridge over The Chicago River
No.19. Van Buren Street Sherzer Rolling Lift Bridge over The Chicago River
No. 20. Foundations for the Eight Track R.R. Bridge near Western Avenue over The Drainage Canal
Bridge in the Background
No. 22. Halsted Street Lift Bridge over the Chicago River.
No. 23. Eads Bridge at St. Louis over the Mississippi River.
No. 24. Model of Coney's Bridge Patented March, 1900.
(Bridge Open)
No. 25. Same as No. 24. (Bridge Closed)