DESIGN, CONSTRUCTION AND TEST

OF A

10 K. W. TRANSFORMER

BY

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Design, Construction and Test

of a

10 K. W. Transformer.

Introduction.

The purpose of this transformer is to transform an alternating current at a potential of 440 volts and a frequency of 60 cycles per second to an alternating current at either 1100 or 2200 volts and the same frequency in any quantity up to its full capacity of 10,000 watts.

The transformer will be used as an intermediate step up transformer between the Westinghouse alternators which have recently been installed at the University and a number of transformers designed for primary voltage of 1100, thus making it possible to operate the latter on the low voltage (440) machines at a frequency of 60 cycles per second.

The conditions under which the transformer is assumed to be used are those of commercial lighting, the object in design being a high all day efficiency with good regulation.

The empirical method of design is here followed. A study was made of the best existing transformers, particularly Westinghouse transformers of 4 and 7 1/2 kilowatt capacity. Curves and equations were thus obtained showing the relations between the quantities most important in the design. From these relations and equations the principle dimensions of the new transformer are found. This method gives an efficiency equal to those studied and is in fact an interpolation between standard sizes, except where the constant entering into the design were altered to suit the new conditions.
The most important quantities determined in this way were $B_{\text{max}}$, the maximum magnetic density in the iron, and $A$, the current density in the conductors, these quantities were however somewhat changed.

There being no machines at hand for punching core plates, Westinghouse Standard 10 K. W. transformer punchings are used and the design made accordingly.
Fundamental Formulae.

The relation between number of turns $S$, impressed electromotive force $E$, and maximum magnetic density $B_{\text{max}}$ is found as follows:

From Faraday's law we have for the E.M.F. to overcome the self induction of a coil,

$$e = S \frac{dN}{dt} = S A \frac{d(B \sin \omega t)}{dt} = S AB \omega \cos \omega t.$$  

The maximum value of the E.M.F. is accordingly,

$$E_{\text{max}} = S AB_{\text{max}} \omega$$

and the square root of mean square value is,

$$E = \frac{1}{\sqrt{2}} S AB_{\text{max}} \omega$$

$E$ and $B$ are in C.G.S. units; expressing $E$ in volts and putting $2\pi n$ for $\omega$,

$$E = 10^{-8} 2\pi n S AB_{\text{max}}$$

$$= 4.45 \times 10^{-8} n S AB_{\text{max}}$$

$$S, A = \frac{10 E}{4.45 n B_{\text{max}}}, \quad (1)$$
General Discussion of Design.

This is a fundamental formula in transformer design. For a given frequency and electromotive force and assumed maximum value of the induction, the number of turns on the primary may be computed, or rather the product of the number of turns by the area of the magnetic circuit.

The relation between $S$ and $A$ may be anything we please provided the product has the proper value and the proportioning of $S$ and $A$ is largely a question of judgement and experience.

In a transformer the relation between the primary turns ($S_1$) and the secondary turns ($S_2$), is the same as the ratio of the primary to the secondary E.M.F.,

$$\frac{S_1}{S_2} = \frac{E_1}{E_2} \quad (2)$$

A proper value of $S_1$ being chosen this fixes the value of $A$ since the product $S_1A$ is known.

Since the transformation ratio and power are given we may consider that approximately the full load primary current will be $\frac{W}{E_1}$ amperes and the full load secondary current $\frac{W}{E_2}$ amperes. \( (3) \)

The currents and number of turns in the coils now being known it remains to fix the size of conductor required. This is determined from considerations regarding the heat loss ($CR$) and the allowable drop ($CR$) due to the resistance of the secondary.

In practice an allowance of from 1000 to 2000 circular units per ampere is made.
The dimensions and proportions of the iron core are next determined. The cross section $A$ of the tongue being already fixed we will now find a suitable relation between its length and breadth. The figure shows the form of plate most used.

$$A = 2bc.$$ It remains to find $C$, the "built up" length.

Let $r$ denote $\frac{C}{2b}$.

In the figure the coils occupy the space $a \times ya$. Now suppose that $A$ remaining constant, we vary $r$. If $r$ is small $b$ is large and the weight of iron and iron loss is large, while the amount of copper and copper loss are small. If however $r$ is made large, the opposite conditions hold true. In the best practice $r$ is given a value from 2 to 5. The constant $y$ is given a value from considerations based on magnetic leakage. The dotted arrow shows the leakage path, consequently the reluctance of this path can be increased or decreased by giving various values to $y$. The next figure shows the leakage paths in the arrangement of coils used in the following design.

On this form of stamping while the length of the leakage path is even less than the other dimension of the window, the arrangement of the coils when thus sandwiched in is such that the magnetomotive force tending to set up leakage lines is so subdivided so that the lines are set up along four different paths as shown in the figure.
MAGNETIC FLUX

showing leakage paths.
The leakage is therefore only one fourth what it would be if there was only one primary and one secondary coil wound side by side. In general then the leakage effect for this arrangement is inversely as the square of the number of pairs of coils.

(In the figure above there is only 2 pairs of coils as far as leakage is concerned, the additional primary coils making no difference).

This arrangement is advantageous in large Units and is used in recent Westinghouse transformers.

The coils are easy to wind and often on account of their depth require no insulation between layers. The ventilation and cooling qualities are good if a slight space is left between coils, the ends of which can be pried apart giving free access to the air. The tongue being split, does not rise to a higher temperature than the sides.
Design.

\[ B_{\text{max}} = 6500, \quad A_o = 32.76 \text{ cm}^2 = 211.3 \text{ cm}^2 \]

\[ E_1 = 440, \quad E_2 = 1100, \quad n = 60. \]

Using formulae (1),

\[ S_i = \frac{10^8 E}{4.45 B_{\text{max}} A_o n} = \frac{10^8 \times 440}{4.45 \times 60 \times 6500 \times 211.3} \]

\[ = 120. \]

By (2)

\[ S_2 = \frac{E_2 S_i}{E_1} = \frac{1100 \times 120}{440} = 300. \]

Assuming a current density of 1500 circular units per ampere we find the cross section of conductor required as follows:

By (3)

\[ \frac{W}{E_1} = C_1, \quad \frac{10,000}{440} = 22.5 = C_1 \]

\[ \frac{W}{E_2} = C_2, \quad \frac{10,000}{1100} = 9. = C_2 \]

Total circular milage = \( 1500 \times C_1 \text{ primary} = 1500 \times 22.5 = 33750 \text{ (c.m.)} \)

A single conductor of this cross-section would be rather stiff to wind, if in the form of an ordinary cylindrical wire, the eddy current loss also in the wire of this size would be appreciable.

Ribbon conductors could be here used to advantage, as ribbon winds easily and is very compact. On account of the difficulty of obtaining ribbon, the primary will be wound with two wires in parallel. Two number 8's B.& S. have a combined cross section of 32768 c.m. and being the nearest size they are chosen for the primary conductor.

Cross section of secondary is 1500 x 9 = 13500 c.m.
This would correspond to a # 9 B & S wire, but partly for reasons above stated and partly for flexibility of connections for various voltages the secondary conductor is made to consist of two wires in parallel.

Two # 12's have a combined cross section of 13182 c.m. and being the nearest size they are selected for the secondary.

Double cotton covered wire is used in accordance with common practice.

Following Westinghouse practice the fine wire is wound in four coils and the large wire in two, the arrangement being as shown in Plate # (2).

As already stated a Westinghouse standard punching is used, its form and dimensions being shown in Plate # (3). The next step is to find the size of the coils, the number of layers of wire and the number of turns per layer, such numbers being selected that there shall be proper space for insulation between and around the coils.

This is entirely a matter of trial, different depths of coils being tried for both primary and secondary until an arrangement is found which gives a good clearance all around the iron and yet gives sufficient space for good insulation between the coils.

In this case the best arrangement is found to be as follows:-

Primary - - 2 coils in parallel each 3.5 x 3/4" consisting of 24 layers of 5 turns each.

Secondary. - - 4 coils in parallel series (see diagram of connections) each 3" x 1/2" and consisting of 30 layers and 5 turns each.
Stampings,

for

10 kw. transformer.

Scale $\frac{1}{2}$.

Wilson & Brockway.
A section of the coils showing insulation and clearance space is shown in Plate #( ).

The size of the coils being fixed the mean length of turn was found graphically to provide sufficient length for an easy curve around the ends, and a slight addition 1/3" was made to the known length of tongue to allow for irregularities in the winding.

The mean length of turn is 41 1/4" which gives for the total length of primary,
\[ \frac{2 \times 120 \times 41.25}{12} = 840' \]
and secondary,
\[ \frac{4 \times 150 \times 41.25}{12} = 2100' \]

The resistance of 840' ft of #8 is at 70° F, .54 ohms since the coils are in parallel, the working resistance is 1/4 x .54 ohms or .13 ohms. The resistance of 2100' of #12 is 3.36 ohms, the working resistance of secondary is as connected for 1100 volts,
\[ \frac{1}{4} \times 3.36 \text{ ohms} = .84 \text{ ohms} \]

The weights are computed from wiring tables are,

- primary ...... 50.4#
- secondary ... 50.4#

(Insulation = .2 weight of copper).

The copper losses at full load are,

- primary \( C_1 = 23.5 \) \( R_1 = .13 \) \( C_1 R_1 = 65.8 \text{ watts} \)
- secondary \( C_2 = 9 \) \( R_2 = .84 \) \( C_2 R_2 = 68.04 \text{ watts} \)

Total ................ 133.8 watts.

Now since the copper losses vary as the square of the load, we have,
\[ \sum C^2R = K(W) , \quad K = \text{constant} . \]
Using full load values,

\[ K = \frac{\sum C^2 R}{(w)^2} = \frac{133.8}{10000} = 0.00001338. \]

The iron losses are due to hysteresis and eddy currents.

The hysteresis loss in watts is estimated from Steinmetz formula.

Ergs per cu. cm. lost per cycle = \( \eta \cdot B_{\text{max}} \) where \( \eta \) is a coefficient depending on the particular iron used; a good value for transformer iron is 0.024.

\( B_{\text{max}} \) is in G. G. S. lines per sq. cm. if \( n \) = frequency then,

\[ \text{Ergs lost per sec.} = \text{Vol} \times n \times \eta \cdot B_{\text{max}}. \]

or Power lost in hysteresis = \( \frac{\text{Vol}}{10} \times n \times \eta \cdot B_{\text{max}} \) watts.

Vol. = volume in cubic centimeters,

\[ = \frac{\text{weight}}{0.28} = \frac{242}{0.28} = 860 \text{ cu. in.} = 14104 \text{ cu. cm.} \]

Hyst. loss = \( \frac{14104}{10} \times 60 \times 0.024 \times 6500 \)

\[ = 259.1 \text{ watts.} \]

The form factor of the E.M.F. wave of the machines on which this transformer is to be used not being known, Still's empirical eddy current is used.

Power lost in eddy currents; watts per lb.,

\[ W_e = \frac{1.4 t \cdot n \cdot B_{\text{max}}}{10} \]

\[ = \frac{1.4 \times 0.13 \times 60 \times (6500 \times 6.45)}{10} \]

\[ = 1502 \]

Total eddy current loss = 242\# \times 0.1502 = 363 \text{ watts.} \]
Efficiency.

The efficiency is calculated from the formulae,

\[
E = \frac{W}{W + W_h + W_c + K(W)^2}
\]

\(W = \) net output.
\(W_h = \) watts lost in hysteresis.
\(W_c = \) watts lost in eddy current.
\(K(W)² = \) loss due to resistance.

<table>
<thead>
<tr>
<th>Load</th>
<th>Copper Watts</th>
<th>Loss</th>
<th>Total Loss</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.333</td>
<td>296.7</td>
<td>77.20</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>2500</td>
<td>10.3</td>
<td>305.7</td>
<td>89.10</td>
</tr>
<tr>
<td>50</td>
<td>5000</td>
<td>33.45</td>
<td>323.8</td>
<td>94.07</td>
</tr>
<tr>
<td>75</td>
<td>7500</td>
<td>74</td>
<td>369.4</td>
<td>95.29</td>
</tr>
<tr>
<td>100</td>
<td>10000</td>
<td>133.8</td>
<td>429.2</td>
<td>95.88</td>
</tr>
<tr>
<td>125</td>
<td>12500</td>
<td>207.8</td>
<td>503.2</td>
<td>96.12</td>
</tr>
</tbody>
</table>

All day efficiency,

Copper loss 5 hrs. = 5 x 133.8 = 669.0 watt hrs.

Iron " 24 " = 24 x 295.4 = 7089.6

Useful watt hours ...... 5 hrs. = \( \frac{50000}{37748.6} \)

All day \( E = \frac{50000}{37748.6} = 86.8 \% \).
The coils for this transformer were wound on strong hard-wood formers whose sides were slotted to facilitate the binding and removal of a finished coil. Drawings of the formers with dimensions are shown in Plate #(4).

The former was mounted on a shaft, and the winding performed by hand. Considerable tamping was necessary in order to keep the turns of the heavy wire in place. The number of turns were counted on an automatic register. The coils were thoroughly shellaced and allowed to dry under pressure, giving them a rigid form and increased insulation. They were then wound with tape leaving only terminals exposed. The primary and secondary terminals were brought out from opposite ends of the transformer.

The coils were assembled with red fibre insulation between and around them and were suspended in a vertical position. A strong wrought frame (see Plate #(5)) attached to a vertical board by angle irons, was then slipped up around the coils and served as a base on which to build up the stampings. Wooden guides were used to keep the layers in even line.

When the core was completely assembled, bolts were inserted in the iron frame mentioned above and the punchings were securely clamped. Additional insulating material was then inserted around the coils wedging them firmly in place.

The transformer was mounted on a strong wooden frame, by means of angle irons attached to the iron frame.
FORMER

FOR

PRIMARY COIL.

Scale \( \frac{2}{3} \).

Wilson & Breckway
Iron Frame For 10 KW Transformer

Scale 1/8.

Wilson & Brockway.
The binding posts of which there are two for each coil, are arranged on a sill attached to the wooden frame. The terminal wires leading to the posts are insulated by rubber tube.

The transformer being intended solely for inside use, an iron case was not made.
BASE OF TRANSFORMER.
The efficiency of a transformer is the ratio of the power taken from its secondary to the power supplied to its primary. Denoting these by $W_2$ and $W_1$, we have,

\[
\text{efficiency} = \frac{W_2}{W_1}
\]

If $W_0$ denotes the transformer losses, we have,

\[
\text{efficiency} = \frac{W_1 - W_0}{W_1} \quad \text{or} \quad \frac{W_2}{W_2 + W_0}
\]

The most convenient and one of the most accurate methods of measuring power in an alternating current circuit, is by means of a wattmeter.

In order that a wattmeter may give practically correct indications for loads of any character, it is necessary that the self induction of the pressure circuit be very small as compared with its resistance. To this end the coil should be wound with as few turns as possible and put in series with a large non-inductive resistance.

The Weston wattmeter fills this qualification and its indications give the power expended in the circuit in which it is connected without any correction since it is provided with a compensating coil.

For this efficiency, this transformer was connected up for 440 volts primary and 1100 volts secondary. Two wattmeters were used, one in the primary circuit to measure the power supplied to the transformer and one in the secondary to measure the output. The connections were made as shown in Plate #(11).
ARRANGEMENT OF BINDING POSTS.

CONVENTIONAL METHOD OF CONNECTIONS.

Wilson & Brockway.
Some Connections
With Various Ratios.
CONNECTIONS
FOR
440 TO 1100 VOLTS.

PRIMARY.
SECONDARY.

Wilson & Brockway.
Connections ~ Wattmeter Test

- **P** Primary Coil
- **S** Secondary
- **Wₚ** **Wₛ** Weston Wattmeters
- **Vₚ** **Vₛ** Weston Voltmeters
- **Mₚ** **Mₛ** " Multipliers
- **R₁** **R₂** Water Rheostats
- **A** Hoyt Ammeter
- **D** Alternator
The primary voltage was kept as nearly constant as possible, but varied somewhat when other loads were thrown on and off the engine.

The iron losses were measured by reading a wattmeter placed on the primary when the secondary was open-circuited.

The volt meters used in reading the primary pressure and the drop of potential between the secondary terminals, were calibrated with their multipliers by comparison to a correct Weston direct current voltmeter.
Weston Wattmeter Test

at 125 cycles per sec.

Iron losses at normal voltage = 269 watts (at 60 cycles).

<table>
<thead>
<tr>
<th>Watts</th>
<th>%</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>Secondary</td>
<td>Load</td>
</tr>
<tr>
<td>1480</td>
<td>1200</td>
<td>12</td>
</tr>
<tr>
<td>3580</td>
<td>2180</td>
<td>31.2</td>
</tr>
<tr>
<td>6557</td>
<td>6210</td>
<td>62.1</td>
</tr>
<tr>
<td>9864</td>
<td>9410</td>
<td>94.1</td>
</tr>
<tr>
<td>10901</td>
<td>10400</td>
<td>104</td>
</tr>
</tbody>
</table>

Regulation —— 2.34% drop at full load at 125 cycles.

Note. This test was taken under some disadvantages as the switch board, rheostats, etc., were not erected in the new laboratory.
Comparison of Experimental (A) & Computed Efficiency (B).

10 kW Transformer

Per Cent of Load

Per Cent Efficiency

- 100
- 90
- 80
- 70
- 60
- 50
- 40
- 30
- 20
- 10
- 0

10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130
Section of Transformer
Showing Arrangement of Coils
Thos. Brockway & Wicox
May 6th 1858