MECHANICAL DESIGN OF HIGH FREQUENCY, HIGH POWER DENSITY ELECTRIC MACHINE

BY

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THESIS

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Urbana, Illinois

Adviser:

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ABSTRACT

In recent years, electric vehicles have demonstrated great economic and environmental advantages in the transportation industry due to the advance of battery and power converter technologies. However, large-scale commercial aircraft electrification is hindered by the technology gap of electric machines. This thesis presented a high frequency, high power density (> 13 kW/kg), MW level electric motor design for the application to augment the power of turbo engines on future 737 class hybrid-electric aircrafts.

The thesis will focus on the mechanical design for the innovative motor architecture, in order to facilitate an interdisciplinary design optimization. A permanent magnet type motor with inside-out configuration was chosen because of its advantage of high peak efficiency and compactness in combination of state-of-the-art materials and technologies, such as Halbach array magnets and airgap windings. Combining these novel ideas imposed challenges onto the mechanical design of the motor, mainly because the design was optimized to thin radial builds for weight reduction, while the motor structural integrity needed to be assured for its high speed operation. The works presented in this thesis will aim to tackle the critical mechanical challenges for a proposed motor design. The challenge for static structural deformation includes the rotor radial expansion at high rotational speed, and the static deflection of external rotor due to the effect of gravity. The thin radial builds of the design with high frequency operation also made the motor subject to vibration challenge. Resonant vibration modes analysis was done to both the stator and rotor. In all the studies mentioned, the design challenges were first expressed by analytical calculations, and then confirmed by high fidelity finite element analysis. The mechanical design would mitigate the risks of failure while providing interdisciplinary design insight to achieve an overall high power density for the motor.
ACKNOWLEDGMENTS

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Chapter 1

INTRODUCTION

After remaining relatively quiet for more than a century, the electrified vehicle is undergoing a remarkable worldwide renaissance. Transportation electrification has met great opportunities in recent years due to environmental and economic considerations. Environmental impacts include alleviating global warming and reducing toxic NO\textsubscript{x}, CO and unburned hydrocarbon gases. Economic benefits have resulted from better fuel economy in electric or hybrid-electric vehicles, not to mention the price increase of diminishing petroleum supplies. Compared to combustion engine type vehicles, electrified vehicles have the advantages of high efficiency in energy conversion and energy transportation.

This thesis will present the design of a high frequency and high power density (sometimes known as specific power) electric motor that will advance the powertrain electrification for Boeing 737 class commercial airplanes. Since the high speed operation will introduce difficulties for the high power density electric motor which is designed to achieve light weight, this thesis mainly focused on the interdisciplinary design and optimization to tackle mechanical challenges such as vibration, static deformation and stresses, in order to maintain the integrity of the motor in harsh conditions. For each challenge, the performance parameters will be expressed analytically with the motor structural parameters, in order to provide cross-functional insight when designing the motor structure. Then, the theoretical approximations of the design model will be confirmed by finite element analysis.

1.1 Motivation

The automobile industry has realized the environmental and economic benefits of using the most efficient form of energy – electricity, so auto manufacturers are trending toward electric or hybrid-electric vehicles. Examples are the Tesla Model S and Toyota
Prius. However, it has also been some time since the first jet engine was developed in 1903. The aviation industry has also expected the electrification in air transportation. Even though the aviation industry might be relatively conservative in terms of technology revolution mainly due to the concerns of safety and cost, there are still merits in developing hybrid-electric propulsion, which are supported by the data presented in Tables 1.1 and 1.2.

<table>
<thead>
<tr>
<th>Year</th>
<th>Passengers (Billion)</th>
<th>Goods (Million ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>2.4</td>
<td>40</td>
</tr>
<tr>
<td>2050</td>
<td>16</td>
<td>400</td>
</tr>
</tbody>
</table>

Table 1.2: Jet Fuel Consumption in the United States [2]

<table>
<thead>
<tr>
<th>Year</th>
<th>US Jet-Fuel Consumption (Billion gallons/year)</th>
<th>Price ($/gallon)</th>
<th>Total Cost (Billion dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>16.4</td>
<td>3.05</td>
<td>50</td>
</tr>
<tr>
<td>2012</td>
<td>16.1</td>
<td>3.16</td>
<td>50.8</td>
</tr>
<tr>
<td>2013</td>
<td>16.5</td>
<td>3.03</td>
<td>50</td>
</tr>
</tbody>
</table>

The International Air Transport Association (IATA) estimated that in the year of 2010, there were about 2.4 billion passengers and 40 million tons of goods transported through air transportation. Based on the past data projections, IATA also predicted that by the year 2050, the amount of international air transportation activity will increase about ten times [1].

The economic benefits could be the main driver for the development of high fuel economy drivetrain technology in the aviation industry. As shown in Table 1.2, the current level of expense for jet fuel consumed every year in the United States is about 50 billion dollars [2]. With the efficient operation using electric machines as augmented power in aircraft propulsion, the air transportation could be more sustainable in the future.
As the needs and main drive of technology are recognized, research for air transportation electrification has gained a broad interest in both the government and industry. Research for enabling technologies of electrified aircraft includes power converters, electric machines, and batteries. However, in spite of recent significant advances in battery storage technologies, it is still not economical to use batteries as the energy storage devices on an airplane, given the non-comparable advantage of energy density of jet fuel. The energy density for jet fuel is 46 MJ/kg, which is more than an order of magnitude larger than the energy density of rechargeable batteries (normally 0.875 MJ/kg). Therefore, it is not necessary to currently aim for developing all-electric aircraft with batteries as storage devices, which was also proved by internal research conducted by NASA; instead, hybrid or turbo-electric propulsion might seem to be a better choice given the battery technology today.

1.2 Problem Definition

Similarly to energy density of battery storage, in consideration of the maximum airplane takeoff and landing weight, the electric motor used for the turbo-electric propulsion should also be designed to achieve light weight or high power density (power-to-weight ratio). Weight is a key factor in aircraft operation not only because of the fuel economy but also the consideration of safety in high speed landings. However, most of the current electric machine technologies do not meet the power density requirement to be equipped on a large commercial aircraft. Therefore, the development of a high power density electric machine is critical to advance the future hybrid-electric airplane technologies.

![Electric Machine Development Roadmap for Hybrid-Electric Aircraft](image)

Figure 1.1: Electric Machine Development Roadmap for Hybrid-Electric Aircraft [3]
NASA has investigated the megawatt class electric machine power density that will be beneficial to the development of future hybrid-electric aircraft, as shown in Figure 1.1. The proposed electric motor design in Chapter 2 is aiming at achieving the 13 kW/kg power density by 2020 with a technology readiness level (TRL) 4. Since the proposed electric motor is categorized as non-cryogenic (non-superconducting) machine, it will have twice the power density ratings of the current state-of-the-art electric motor as shown in Figure 1.2 [4]. As a reference, the induction traction motors Tesla used for their vehicles have a power density of 4.5 kW/kg, which is one-third the power density target of the motor design proposed in this thesis.

The challenge of designing such a high power density electric motor will greatly fall into the mechanical design aspect. Since the structural members, which are normally the non-active components in an electric machine, need to be thin and light, it becomes extremely difficult but also critical to ensure the mechanical integrity when the motor is operated at high speed. The work in this thesis will primarily focus on tackling the mechanical challenge for the high speed, high power density electric motor.
Chapter 2

PROPOSED SOLUTION

In order to address the weight concern of the electric motor for hybrid electric aircraft, a high frequency, high power density motor was proposed to serve as augmented power to the aircraft turbo engine, as shown in Figure 2.1. The motor produces 1 MW of output power but weighs only about 70 kg, thus resulting to a power density of 14.7 kW/kg. Key parameters are shown in Table 2.1. The two strategies to achieve this high power density design can be divided into increasing power and reducing weight, which are presented in details in the following sub-sections. Since the proposed motor is intended to provide augmented power to the jet engine when the airplane is cruising, the motor will be mostly operated at steady state around the rated speed. The design is therefore heavily optimized around rated speed operation.

Figure 2.1: Proposed Solution for Aircraft Turbo-Electric Propulsion
### Table 2.1: Proposed Motor Key Performance Ratings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Density</td>
<td>14.7 kW/kg</td>
</tr>
<tr>
<td>Efficiency</td>
<td>98.60%</td>
</tr>
<tr>
<td>Number of Poles</td>
<td>20</td>
</tr>
<tr>
<td>Rated Speed</td>
<td>14000 rpm</td>
</tr>
<tr>
<td>Output Power</td>
<td>1.04 MW</td>
</tr>
<tr>
<td>Weight</td>
<td>70.6 kg</td>
</tr>
</tbody>
</table>

Induction machines (IM) and permanent magnet (PM) machines are the most popular types of machines in vehicle traction application because of their high efficiency competency. Examples are the PM motor in the Toyota Prius and the induction motor in the Tesla Model S. Even though the properly designed induction motor can have higher overall efficiency across a wide range of operation speeds, the PM motor has higher peak efficiency than induction motor [5]. Also, the PM motor is known for its compactness and light weight mainly because of the high energy product in rare earth permanent magnets. Therefore, in the application of augmenting the jet engine power at cruising, it is more favorable to use PM motors.

#### 2.1 Power Increase

The first strategy to achieve high power density is to increase the output power of the motor. To achieve this goal, the design incorporates high speed operation, because power equals torque times speed. The proposed design has a rated speed of 14,000 rpm. As mentioned previously, the motor will be working mostly around the rated speed during the aircraft cruising, so the motor design may not require a wide speed range capability for constant power operation. In this case, the surface mounted permanent magnet (SPM) motor architecture is a more suitable choice than the interior permanent magnet (IPM) motor architecture, mainly because the SPM motor is more power dense due to the need for less rotor iron.

However, a direct challenge for high speed operation of the SPM motor is that the rotor retaining ring thickness becomes essentially large. The retaining ring is a non-magnetic structural member at the outermost of the rotor. It is used to retain the underneath magnets from flying apart due to the large centrifugal force when the rotor is spinning at high speed. For traditional internal rotor electric machine configuration, this
non-magnetic retaining ring structure will take up space in the airgap and thus increase the effective airgap, which in turn decreases the electromagnetic performance of the motor.

The solution to the enlarged airgap is to utilize the inside-out configuration, where the magnets are placed at the inner diameter of the external rotor, as shown in Figure 2.1. Therefore the retaining ring will not occupy space in the airgap, resulting to an achievable 1 mm airgap for the design.

2.2 Weight Reduction

To achieve high power density for the motor, designs for weight reduction should also be elaborated. The first approach is to increase the pole count of the design. The proposed motor design has a total of 20 poles, or 10 pole pairs. For a constant airgap flux density, increasing the number of poles can reduce the flux in each pole. The magnetic circuit in an electric machine shows that all the flux from one pole must go through the stator back yoke to reach the other pole. With the reduction of flux per pole, the flux density in the stator back yoke can also be reduced. Thus, the increased pole count requires a thinner stator back yoke without saturation, reducing the weight of the motor design.

However, high pole count motor design with high speed operation means its electrical frequency will also become high. The electrical frequency in the stator winding of a 20-pole motor running at 14,000 rpm is 2.3 kHz, which results to much higher AC losses in the copper winding compared to a conventional motor with only four or eight poles.
The solution of minimizing AC copper losses is to utilize Litz wire shown in Figure 2.2 as the armature conductor in the stator. As opposed to DC copper loss, which conventionally is just the $I^2R$ loss, AC copper losses are the skin effect and proximity effect caused by the high frequency alternating current, increasing the effective resistance in the conductors. The skin effect is the self-induced eddy current mostly opposing the current flowing in the center of the conductors, caused by the alternating magnetic field. Therefore, it reduces the effective conducting area and increases the conductor resistance. The usage of Litz wire reduces the skin effect by dividing a large conductor into smaller strands of parallel wires so that the skin depth is usually larger than the individual strand radius. The proximity effect is the current crowding in a smaller region of a cluster of parallel conductors, caused by an alternating magnetic field from the nearby conductors. Similar to skin effect, the constrained conducting area increases the effective resistance. With the Litz wire strands twisted and woven in a certain angle, the skin effect can be reduced. The trade-off here is the relatively higher cost for Litz wire conductors, as well as the difficulties in forming the winding turns rigidly during manufacturing.

The second weight reduction approach is to totally remove the rotor back iron by placing the permanent magnets in a special arrangement called the Halbach array. The proposed design utilizes the Halbach array with six segments of magnets per pole, thus giving a total of 120 discrete segments of magnets in the rotor. Halbach array magnets will produce a sinusoidal magnetic field on one side, while canceling the field to nearly
zero on the other side. This is done by arranging magnets in a spatially rotating pattern. An example is shown in Figure 2.3. Since there is no need for soft magnetic materials to shield the field on the back of the magnets, the rotor back yoke can be eliminated. Because the magnetic back yoke consists of heavy materials, normally steel, its elimination reduces a considerable amount of weight of the motor. The trade-off is that the Halbach array magnets would require slightly more magnet materials than the direct magnetized configuration. Also, the segmented magnets would increase the manufacturing costs and difficulties in rotor assembly.

![Halbach Array Magnetization Pattern](image)

**Figure 2.3: Halbach Array Magnetization Pattern [6]**

The third approach to reduce weight is to utilize the air-gap windings for the stator armature. The air-gap winding eliminates the stator teeth, thus reducing the weight as well as the stator iron loss. The air-gap winding design also facilitates the usage of Litz wire conductors because of their low formability and difficulties in winding into stator slots. Even though there are minor drawbacks in the motor design using air-gap windings, such as accounting for weaker coil flux linkage, weaker heat dissipation, and worse structural support, the University of Illinois design team of the NASA motor project was able to overcome the challenges and prove the benefits in power density augmentation over those drawbacks.

The last approach is to make use of the high performance materials for the purpose of weight reduction, as shown in Table 2.2. High performance can be concluded to have a high strength-to-weight ratio and high magnetic permeability. The rotor retaining ring, which is a main structural member, will be made of carbon fiber. Titanium is chosen for the rotor shell. The rotor shell provides a thin layer between the carbon fiber retaining ring and the magnet segments, making it easy to join them together during assembly. The magnets are high grade NdFeB, which is a rare-earth permanent magnet material with
high energy product. The stator core is laminated cobalt iron, which possesses a high magnetic saturation limit and strong mechanical properties. The heat sink is made of aerospace grade aluminum, because aluminum is known for its light weight and excellent thermal conductivity.

<table>
<thead>
<tr>
<th>Table 2.2: Bill of Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part</td>
</tr>
<tr>
<td>Retaining Ring</td>
</tr>
<tr>
<td>Rotor Shell</td>
</tr>
<tr>
<td>Magnet</td>
</tr>
<tr>
<td>Winding</td>
</tr>
<tr>
<td>Stator Core</td>
</tr>
<tr>
<td>Heat Sink</td>
</tr>
<tr>
<td>Ground Cylinder</td>
</tr>
</tbody>
</table>

2.3 Mechanical Challenges

With all the ideas to increase motor power density introduced in Sections 2.1 and 2.2, it is not hard to foresee a relatively high cost and challenges in manufacturing such a motor. However, to push the boundary of electric machine technology, NASA, the sponsor of the Fixed-Wing project, encouraged the University of Illinois design team to use the state-of-the-art materials and technologies to develop a proof of concept. To realize such a high power density goal put a burden on the mechanical design of the motor, because the material usage should be minimized, while ensuring the motor integrity at all operation conditions.

To achieve the ultimate goal of a high power density design as discussed in this chapter, there are three main mechanical design challenges of the proposed radial flux inside-out motor:

- Structural deformation
- Vibration
- Bearing system

Structural deformation includes the radial expansion of the rotor when it is rotating at high speed, and the bending of the rotor when it is sitting horizontally. The vibration will
happen on both the rotor and the stator. Therefore the design needs to consider both the stator resonance modes and the rotor dynamics. In the bearing system design, not only the difficulty in assembling the bearings due to the special mounting of the external rotor needs to be taken care of, but also meeting the challenge to accommodate thermal growth in the system requires design attention.

**Figure 2.4:** Bearing System Design with Angular Contact Ball Bearings

Since the focus of this thesis is the mechanical design for a motor to achieve high power density, the bearing system design will only be briefly introduced here as it is not related to the electrical performance of the motor. For the application of inside-out motor architecture in this research, several configurations of bearing system design were explored, such as the usage of duplex bearing, and the two bearing configuration consists of a deep groove ball bearing and a cylindrical roller bearing. However, the final proposed design chose to use two angular contact ball bearings for the system as shown in Figure 2.4. One advantage of angular contact ball bearing is that it has loading capabilities in both the radial and axial directions. Also, the bearing system with angular bearings can be applied with axial preload, thus increasing system stiffness, which is beneficial to the rotor dynamic performance. The effective bearing force direction due to the axial preload is shown as dash lines in Figure 2.4. In the proposed bearing system design, a wave washer installed at the non-drive end bearing is used as a spring to
accommodate thermal growth of the system during operation. Since the difference of temperature distribution between the rotor shaft and the stator bearing housing, as well as the materials difference of the stator and the rotor, the thermal growth in the bearing system needs to be absorbed by the spring. Careful design and selection of the wave washer is done to obtain the largest preload available, in order to ensure overall high system stiffness at all times while preventing excessive thermal stress in the structures or the bearings.
Chapter 3

INITIAL SIZING

The first thing most electric machine designers will do when designing a motor is to roughly estimate the size of the motor for required loadings. With specified input and output of the motor, the size of key electrical components like stator windings and rotor magnets can be determined. In addition to the prescribed electrical loading and magnetic loading requirements, this chapter will present an initial sizing investigation from an interdisciplinary standpoint for the proposed solution outlined in Chapter 2. The main goal of the discussion in this chapter is to provide the optimal selections for the motor aspect ratio and the rotor tip speed, in order to come up with a baseline motor as the first iteration design for future detailed optimization. In each section, trade-offs among motor parameters will be examined toward the ultimate design goal – motor power density.

3.1 Diameter and Length

An initial sizing for the motor diameter and length was done in the early stage of the research to study their effect toward power density of the inside-out design. Starting from the baseline dimensions, four different combinations of motor diameters and lengths were investigated, as shown in Table 3.1. The baseline design in the early stage of the research had a diameter of 12.2 in and active length of 9 in. Since the rated speed for the baseline design is 18,000 rpm, it had a rated torque of 530.5 Nm, thus giving a torque per rotor volume (TRV) value about 45 kNm/m$^3$, which falls into the aerospace class of machine design [7]. The TRV value provides a good empirical reference for the initial sizing design of different classes of electric machines. Even though it differed from the proposed solution shown in Chapter 2 by some small mechanical design features, the effect on weight by diameter and length variations still hold similarly because the structural architecture is the same.
<table>
<thead>
<tr>
<th>Rotor OD (in)</th>
<th>Active Length (in)</th>
<th># of Poles</th>
<th>Total Weight (lbm)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.2</td>
<td>9</td>
<td>20</td>
<td>143.34</td>
<td></td>
</tr>
<tr>
<td>15.86</td>
<td>6.605</td>
<td>26</td>
<td>156.78</td>
<td></td>
</tr>
<tr>
<td>19.52</td>
<td>5.217</td>
<td>32</td>
<td>172.28</td>
<td></td>
</tr>
<tr>
<td>24.4</td>
<td>4.075</td>
<td>40</td>
<td>196.73</td>
<td></td>
</tr>
</tbody>
</table>

In this study, there were five basic assumptions for the varied diameter and length models:

- Constant electric loading
- Constant magnetic loading
- Constant airgap flux crossing area
- Constant rotor tip speed
- Constant radial thickness for all components
The electric loading is the linear current density in the armature winding; the magnetic loading is the flux density in the airgap. By keeping them constant, all the designs in the above cases can have similar design requirements for armature windings and rotor magnets, similar copper and iron loss, and more importantly, similar shear stress in the airgap as well as the TRV value as shown by the following equation.

\[ TRV = 2\sigma \propto (A \times B) \]  

(3.1)

where \( \sigma \) is the airgap shear stress, \( A \) is the linear current density in the armature, and \( B \) is the airgap flux density.

By keeping the airgap flux crossing area constant, together with the constant shear stress for all the designs, the total force provided in the airgap will also be the same for all cases. Since power equals force times velocity, by further assuming constant rotor tip speed, all the varied designs will have similar output power.

Figure 3.1: Component Weight Break-Down for Varied Designs

For the first-order consideration, the above assumptions indicated that the designs with larger diameter and shorter length will not change the machine output power. However, considering the second-order effect, if the design uses a larger diameter, the centrifugal force seen by the rotor will be smaller. Therefore, the radial expansion when
the rotor is spinning at high speed can be reduced. Moreover, the diameter increase is associated with the rotor length reduction, so increasing the diameter will also be beneficial to the bending deformation of the rotor. The rotor bending can be thought of as a cantilever beam bending due to gravity, since the rotor is supported by the end plate only on the drive end. With the second-order effects, it is mechanically better to pursue larger diameter designs. Detailed trade-off discussions will also be presented in the following chapters.

Even though there are underlying structural benefits for choosing designs with larger diameter and shorter length, the associated weight increase for such designs defeats the purpose of increasing the baseline design diameter. The weight increase will reduce the power density of the motor. The detailed component weight break-down chart in Figure 3.1 shows that most of the weight increase came from the rotor shell, ground cylinder and armature windings. As the diameter increased, the vertical end plates in the rotor shell and the ground cylinder became larger. Also, the total volume of end windings increased. These components added more weight to the motor without contributing to its electromagnetic performance. The radial flux inside-out motor with a large diameter was not weight efficient design.

Therefore, the design in this research was optimized around the baseline sizing, with the rotor outer diameter close to 12 in and rotor length close to 11.5 in. This resulted to an aspect ratio close to 1:1, at which most motors were designed. Detailed design optimizations were made on the selected baseline based on the motor electromagnetic, thermal and structural performance.

3.2 Tip Speed and Retaining Ring Thickness

Starting from a more recent optimized baseline design, it is necessary to select a rotor tip speed for rated operation, which will optimize the power density of the design with considerations for its mechanical integrity. Since the inside-out motor operates at high speed, the radial expansion of the rotor will be the main performance and reliability concerns. Thus, thickness of the main structural member to restrain the radial expansion,
the carbon fiber retaining ring thickness, will also be selected accordingly for designs with different rated tip speeds.

In this study, the rotor tip speed was varied from 150 m/s to 340 m/s (Mach 1) with an increment of 10 m/s. For all the variation models, the following assumptions were held:

- Constant electrical loading
- Constant magnetic loading
- Constant retaining ring safety factor
- Constant radial expansion

Similar to Section 3.1, both electrical loading and magnetic loading are assumed to be constant in order to provide a constant shear stress in the airgap, as indicated by Equation (3.1). To ensure mechanical integrity of the design, stress in the structural member, which consists mainly of the hoop stress as will be shown in Chapter 4, also needs to be constant in all tip speeds. Similarly, the radial expansion needs to be held constant in order to keep the airgap reluctance in the machine magnetic circuit for all tip speeds. The restrained stress and expansion can be translated as the requirement for thickness of the carbon fiber retaining ring, as shown in the calculation in this section.

The hoop stress in the retaining ring is a result of the centrifugal force provided by the components underneath the ring. This is illustrated in Figure 3.2, which shows the upper half of a circular cross section. The solid area is the retaining ring; the area enclosed by dash-line is the rest of the rotor components, which includes magnet segments and rotor shell. By considering the force balance of the free body diagram of retaining ring, the sum of forces in vertical direction must be zero. Thus, the hoop force in the retaining ring and the centrifugal force acting on the inner surface of the retaining ring have the following relationship.

\[
\int \sin \theta dF_c = 2F_h
\]  
\[
(3.2)
\]

where \( dF_c \) is the representation of infinitesimals of the centrifugal force, and \( F_h \) is the hoop force.
The centrifugal force for a point mass is:

\[ F = m \frac{v^2}{r} \]  \hspace{1cm} (3.3)

where \( m \) is the point mass, \( v \) is the linear velocity of the point mass, and \( r \) is the distance to the rotation center.

Thus, the centrifugal force infinitesimals can be found by

\[ \frac{dF_c}{d\theta} = \frac{v^2}{r} \frac{dm}{d\theta} + m \frac{d\left(\frac{v^2}{r}\right)}{d\theta} \]

Since \( v \) and \( r \) are not functions of \( \theta \), meaning \( \frac{d\left(\frac{v^2}{r}\right)}{d\theta} = 0 \), then

\[ \frac{dF_c}{d\theta} = \frac{v^2}{r} \frac{dm}{d\theta} = \frac{v^2}{r} \left( \rho L \frac{IR^2 - (IR - \delta)^2}{2} \right) \]  \hspace{1cm} (3.4)

where \( \rho \) is the average mass density for the components below the retaining ring, \( L \) is the axial length, \( IR \) is the inner radius of the retaining ring, and \( \delta \) is the depth of the components below the retaining ring.

By plugging Equation (3.4) back into Equation (3.2),
\[ \int_{0}^{\pi} \sin \theta \frac{v^2}{r} \left( \rho L \frac{IR^2 - (IR - \delta)^2}{2} \right) d\theta = 2F_h \]

we can solve to get the hoop force as

\[ F_h = \frac{v^2}{r} \left( \rho L \frac{IR^2 - (IR - \delta)^2}{2} \right) \]  

(3.5)

For the ease of calculation, the linear velocity at the location of the retaining ring inner radius will be used to approximate the force calculations. This will also ensure the extra capability for the calculated retaining ring thickness. Then, the hoop stress can be calculated by dividing the hoop force with retaining ring cross-sectional area (perpendicular to the hoop direction).

\[ S_h = \frac{F_h}{tL} = \frac{v^2_{IR}}{tIR} \left( \rho \frac{IR^2 - (IR - \delta)^2}{2} \right) \]  

(3.6)

where \( t \) is the retaining ring thickness.

To obtain the radial expansion, the ring extension due to the tension in the hoop direction must be obtained first. By Hooke’s law, the extension can be calculated as:

\[ \Delta l = \frac{S_h l}{E} = \frac{S_h (2\pi IR)}{E} = \frac{v^2_{IR} \pi \rho}{tE} \left( IR^2 - (IR - \delta)^2 \right) \]  

(3.7)

where \( l \) is the perimeter of retaining ring, \( E \) is the Young’s modulus of the retaining ring material.

Then, the radial expansion can be obtained from the extension.

\[ \Delta r = \frac{\Delta l}{2\pi} = \frac{v^2_{IR} \rho}{2tE} \left( IR^2 - (IR - \delta)^2 \right) \]  

(3.8)

From both Equation (3.6) and Equation (3.8), it can be observed that, in order to ensure constant hoop stress and constant radial expansion, the \( \frac{v^2}{t} \) ratio needs to be kept constant. Therefore, it is reasonable to say the required retaining ring thickness is roughly proportional to the square of rotor tip speed. For this study, to obtain the required retaining ring thickness for each rotor tip speed, calculation from a baseline reference value is needed.
With the \( \frac{v^2}{t} \) ratio, \( k \), for a more recently selected baseline design,

\[
\frac{v_{IR}^2}{t} = k = 4113286.7 \text{ m/s}^2
\]

the carbon fiber can reach a safety factor about 7, and a radial expansion about 0.4 mm when the rotor is operating at 14,000 rpm. Since the rotor tip speed and the linear speed at retaining ring IR has the following relationship,

\[
\frac{v_{tip}}{IR + t} = \frac{v_{IR}}{IR}
\]  

(3.9)

the required retaining ring thickness for each rotor tip speed variation model can be calculated using the following equation.

\[
\left(\frac{v_{tip}}{IR + t}\right)^2 = k = 4113286.7 \text{ m/s}^2
\]  

(3.10)

With the corresponding carbon fiber thickness calculated for each tip speed, the power density of the design can be then calculated. First, the power of a design can be expressed as:

\[
P = F_{ag}v_{ag} = \sigma A_{ag}v_{ag}
\]

\[
= \sigma A_{ag}v_{tip}\frac{R_{ag}}{IR + t}
\]  

(3.11)

where \( \sigma \) is the shear stress in the airgap, \( A_{ag} \) is the airgap flux crossing area, and \( R_{ag} \) is the radius to the airgap.

Noticeably in Equation (3.11), only \( v_{tip} \) and \( t \) change for each design variation. The shear stress, \( \sigma \), in the airgap is constant if the electric loading and the magnetic loading are kept constant.

The mass of the design is expressed as:

\[
m = m_o + m_{cf}
\]

\[
= m_o + \rho_{cf}L_{cf} \pi [(IR + t)^2 - IR^2]
\]  

(3.12)
where \( m_o \) is the mass of components other than carbon fiber in the design, \( \rho_{cf} \) is the carbon fiber density, \( L_{cf} \) is the axial length of the carbon fiber.

Then, the power density of a design can be expressed as:

\[
pd = \frac{P}{m} = \frac{\sigma A_{ag} v_{tip} R_{ag}}{m_o + \rho_{cf} L_{cf} \pi [(IR + t)^2 - IR^2]}
\]

(3.13)

Using Equation (3.10) and Equation (3.13), the retaining ring thickness and the power density of the design for each tip speed can be plotted, as shown in Figure 3.3.

![Figure 3.3: Effect of Rotor Tip Speed on Required Retaining Ring Thickness and Motor Power Density](image)

Even though the figure shows that power density of motor design can be further increased by using higher rotor tip speeds, this approach is not always viable if considering from the machine loss standpoint. The total loss of the machine can be divided mainly to copper loss, iron loss and mechanical loss (mostly windage loss).

\[
P_{loss} = P_{cu} + P_{fe} + P_{windage}
\]

(3.14)

Both the iron loss and windage loss have a strong dependency on the rotor speed. The eddy current loss component in iron loss is known to be proportional to the square of
motor electrical frequency, so it will have a faster growth than the power density with increasing rotor tip speed. Similarly, the windage loss happening on the OD of the rotor will increase much faster than power density. Because the surface drag force is proportional to velocity square, the windage loss is then proportional to velocity cube.

Moreover, design with tip speed close to or above Mach 1 needs to be avoided. Otherwise, the mechanical design of the motor also needs to pay special attention for the surrounded air compressibility effect and aerodynamic heating [8], in order to ensure the safety of the aircraft system and the performance of the motor.

Lastly, even though the mechanical performance of the retaining ring can be compensated with increased thickness at higher rotor tip speed, the thickness of non-structural members stays the same, meaning the internal stress in the magnets will increase due to the increase of centrifugal force, compressing the magnets. Therefore, a rotor tip speed about 247 m/s was chosen for the design as a compromise of the motor power design in order to mitigate the mechanical risks. With the chosen tip speed, the design power density is about 14.4 kW/kg, still exceeding the 13 kW/kg design goal.
Chapter 4

DESIGN FOR STATICS

This chapter will discuss design strategies to tackle the mechanical challenges for inside-out type motors during steady-state operating conditions. They include the motor spinning at high speed as well as under the effect of standard gravity. In each section, theoretical models and equations will be derived in order to quantify the structural performance of the proposed motor, which includes stress and deformation. The theories will be followed by corresponding Finite Element Analysis (FEA) results and comparison. Discussion of the results can provide essential insights for designing structurally sound inside-out electric motors while achieving a high power density.

4.1 Radial Expansion

Since the proposed motor is a radial-flux inside-out design, the rotor radial expansion needs to be restrained when the motor spins at high speed. Otherwise, the radial expansion will increase the airgap length, which in turns will increase the airgap reluctance of the motor magnetic circuit. To ensure the motor electromagnetic performance and the high power density, it is necessary to understand the trade-offs between relative parameters in the rotor structure design when the radial expansion at high speed rotation needs to be restrained to an acceptable level.

4.1.1 Theoretical Preparation

To understand what causes the rotor radial expansion when it is spinning at high speed, it is helpful to first look at how the retaining ring actually behaves. Figures 4.1 - 4.3 show the stresses in different directions within the retaining ring structure obtained through FEA simulations. As shown, the average stress at the outer surface of the carbon fiber retaining ring in the radial direction is about 1 MPa; the average stress in hoop direction is about 400 MPa; and the average stress in axial direction is about 4 MPa. The hoop stress is two orders of magnitude greater than the stresses in the other two directions.
Figure 4.1: Retaining Ring Stress in Radial Direction

Figure 4.2: Retaining Ring Stress in Hoop Direction

Figure 4.3: Retaining Ring Stress in Axial Direction
Since the stress in the retaining ring is mainly in the hoop direction, the problem of rotor radial expansion can be treated to minimize the elongation of the retaining ring structure in the hoop direction. What really happens when the rotor is spinning at high speed can be understood from the force balance diagram of the retaining ring, by Figure 3.2 shown in Section 3.2. The centrifugal force acting on the retaining ring, which is caused by the other rotor components underneath of the retaining ring, translates to the effect of hoop force within the retaining ring structure.

In order to express the elongation of the retaining ring in the circumferential direction, the retaining ring can be treated as a beam by unfolding the retaining ring to a flat structure. Then, from the equation for Young’s modulus,

\[ E = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta l/l} \]  \hspace{1cm} (4.1)

the elongation in the circumferential direction is

\[ \Delta l = \frac{Fl}{AE} = \frac{Fl}{tLE} \]  \hspace{1cm} (4.2)

where \( F \) is the hoop force within the retaining ring, \( l \) is the equivalent perimeter of the retaining ring, \( A \) is the cross-sectional area perpendicular to the hoop direction which equals the retaining ring thickness \( t \) times the active region axial length \( L \), and \( E \) is the Young’s modulus of the retaining ring structure.

Noticeably, in Equation (4.2), it is a good approximation by considering the Young’s modulus effect of the retaining ring material only, because the components underneath it are not meant to be structural members to restrain expansion. The magnets are segmented into 120 discrete pieces. The resin is put between the magnet segments to work as adhesives. Thus, the magnets will not restrain themselves in place under the effect of centrifugal force. Also, the titanium rotor shell between the magnets and the retaining ring will not provide many structural benefits, because it is designed to be a thin layer for the purpose of ease of joining the magnets and the ring during assembly. The Young’s modulus of titanium is low compared to even the NdFeB magnet materials.
4.1.2 Discussion and Analysis

The first thing that can be observed from Equation (4.2) is that the elongation can be reduced by decreasing the hoop force. The hoop force is directly related to the centrifugal force effect and is proportional to the square of rotational velocity, as shown in Equation (3.5) in Chapter 3. Furthermore, the rotor radial expansion can be approximated using Equation (3.8), which shows that the radial expansion is also proportional to the square of rotational velocity.

The results of the theoretically approximated radial expansion and the simulation results are presented in Table 4.1 and Figure 4.4. The rotor retaining ring thickness used

Table 4.1: Rotor Radial Expansion Calculation and Simulation Results with Baseline Retaining Ring Thickness and Varied Rotational Speed

<table>
<thead>
<tr>
<th>RPM</th>
<th>Theoretical @ IR</th>
<th>Simulation @ ID</th>
<th>Simulation @ OD</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>0.1891</td>
<td>0.2756</td>
<td>0.1838</td>
</tr>
<tr>
<td>12000</td>
<td>0.2724</td>
<td>0.3969</td>
<td>0.2646</td>
</tr>
<tr>
<td>14000</td>
<td>0.3707</td>
<td>0.4675</td>
<td>0.3601</td>
</tr>
<tr>
<td>16000</td>
<td>0.4842</td>
<td>0.7057</td>
<td>0.4705</td>
</tr>
</tbody>
</table>
for the calculations and simulations is 0.5 in. In the FEA simulations, the Young’s modulus of both the magnets and rotor shell are reduced by 100 times of their original values. Doing this will make those components very soft, so that they will not share any stress from the carbon fiber retaining ring, acting like a non-structural members.

Figure 4.4: Graphical Comparison of Rotor Radial Expansion Calculation and Simulation Results with Baseline Retaining Ring Thickness and Varied Rotational Speed

As shown in Figure 4.4, the theoretical calculation results approximate closely to the simulated radial expansions at the outer diameter of the rotor. This is reasonable because the calculation using Equation (3.5) gives the hoop stress at the inner diameter of the retaining ring, which results that the radial expansion output from Equation (3.8) to be very close to the outer diameter of the retaining ring. It can be observed clearly that both the calculation results and the simulation results at rotor OD have second-order polynomial trend lines. The radial expansion is growing with the square of the rotor rotational speed. However, the maximum radial expansion is in the inner diameter of the rotor, at the inner surfaces of the magnets. The radial expansion should be uniform across the depth of the rotor if all the components are considered as rigid bodies in the radial direction. This is not the case because the presence of one of the mechanical properties in the components – Poisson’s ratio. Poisson’s ratio describes how much a material will
contract in one direction when it is stretched in the perpendicular direction, due to the conservation of mass.

\[ v = -\frac{\varepsilon_{\text{trans}}}{\varepsilon_{\text{axial}}} \]  

(4.3)

where \( \varepsilon_{\text{trans}} \) is the strain in transverse direction, and \( \varepsilon_{\text{axial}} \) is the strain in the axial direction (perpendicular to transverse).

Underneath the carbon fiber retaining ring, the NdFeB magnets have a Poisson’s ratio of 0.24; the titanium rotor shell has a Poisson’s ratio of 0.342. Thus, when those components are stretched in the hoop direction, they contract in the radial and the axial directions. Since the components are becoming thinner in radial direction during rotation, the radial expansion measured at the inner diameter of the rotor is larger than that measured in the rotor outer diameter. Figure 4.4 shows all the points for the expansion at rotor ID are shifted upward.

The effect to radial expansion from another parameter that can also be observed in Equation (4.2) is the effect of the retaining ring cross-sectional area. The radial expansion can be reduced by increasing the carbon fiber cross-sectional area, which equals to the ring thickness, \( t \), times the active region axial length, \( L \). However, it only makes sense to increase the retaining ring thickness, because increasing the axial length will also increase the weight of the magnets, resulting to a larger hoop force.

Similar to speed variation, the effect of retaining ring thickness can also be investigated using Equation (3.8). The equation shows that the rotor radial expansion is proportional to the inverse of retaining ring thickness. The calculated approximations and the FEA simulation results are compared in Table 4.2. The rotor in all these cases is running at the rated speed 14,000 RPM, with four variations of retaining ring thickness.
### Table 4.2: Rotor Radial Expansion Calculation and Simulation Results with Varied Retaining Ring Thickness at 14,000 RPM

<table>
<thead>
<tr>
<th>Retaining ring thickness (mm)</th>
<th>Radial Expansion (mm)</th>
<th>Screen shot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theoretical</td>
<td>Simulation @ ID</td>
</tr>
<tr>
<td>6.35</td>
<td>0.704</td>
<td>0.705</td>
</tr>
<tr>
<td>12.7</td>
<td>0.352</td>
<td>0.5403</td>
</tr>
<tr>
<td>19.05</td>
<td>0.235</td>
<td>0.4675</td>
</tr>
<tr>
<td>25.4</td>
<td>0.176</td>
<td>0.4325</td>
</tr>
</tbody>
</table>

Starting from the retaining ring thickness of $t = 12.7$ mm (0.5 in) in the baseline design, the retaining ring thickness is varied to be $0.5t$, $1.5t$ and $2t$ as shown in Table 4.2. In Figure 4.5, the calculation results are observed to better approximate the simulation results at the outer diameter locations of the rotor. The reason is the same in the cases at the speed variation study. Using Equation (3.8), the calculation is for the approximated expansion at the retaining ring OD, which should be very similar to the expansion at its ID. The calculated expansion at the baseline thickness is closest to the simulated expansion, because the calculation is calibrated at the baseline point. Also, the expansions at rotor ID in simulation are higher than the expansions at rotor OD because of the Poisson’s ratio effect in the magnets and the rotor shell, similar to that for the
speed variation cases. Those parts are becoming thinner during rotation, leading to differences in the expansion measurements.

![Graphical Comparison of Rotor Radial Expansion Calculation and Simulation Results with Varied Retaining Ring Thickness at 14,000 RPM](image)

Figure 4.5: Graphical Comparison of Rotor Radial Expansion Calculation and Simulation Results with Varied Retaining Ring Thickness at 14,000 RPM

However, unlike the speed variation cases, for the thickness variation cases it can also be observed that the errors between the calculation results and the simulation results at rotor OD are not biased in single direction, meaning some errors are positive and some are negative. Even though at the first sight of Equation (4.2) it seems the retaining ring elongation is proportional to only the inverse of thickness, actually the hoop force in the ring is also a function of the thickness because the weight increase of the ring itself will also increase the total centrifugal force effect acting on the ring. But this fact is not captured in Equation (3.5), where the hoop force is the result of the centrifugal force of components below the ring only. Furthermore, the effective ring perimeter \( l \) in Equation (4.2) actually changes slightly with the change of ring thickness. This level of data discrepancy is expected for the reasons of these assumptions in the theoretical calculations.

Furthermore, it can also be observed in Equation (4.2) that by increasing the Young’s modulus of the retaining ring material, the radial expansion can be reduced. However, since the Young’s modulus of the chosen IM-7/PEEK carbon fiber composite is already
as high as 172.4 GPa, there are few other choices for the retaining ring material which could have a higher modulus while possessing a high strength-to-weight ratio at the same time.

The last parameter in Equation (4.2) is the circumferential perimeter of the retaining ring. It indicates that, with smaller rotor diameter, the radial expansion can be reduced. However, using a smaller diameter means that the corresponding force in the motor airgap needs to increase in order to maintain the same level of output torque. Torque can be increased by adding the motor magnetic loading or electric loading. Magnetic loading can be increased by making the magnets thicker. However, this will increase the rotor weight. Electric loading can be increased by pumping more current through the stator windings, which will increase the copper loss and bring more burdens of heat dissipation. Since the magnetic loading and the electric loading are optimized to their best, changing the size of the components below the carbon fiber retaining ring may not be a straightforward or effective method.

For the summary of this section, the four basic approaches to reduce the rotor radial expansion for the motor design is to modify the retaining ring hoop force \( F \), the retaining ring perimeter \( l \), the retaining ring thickness \( t \), or the retaining ring material elastic modulus \( E \). To reduce the retaining ring hoop force, which is effectively the same as to reduce the centrifugal force, the motor design can use a lower rotational speed, or smaller rotor diameter. Lowering speed could be a more effective approach because the centrifugal force is proportional to the square of speed. However, doing both will require an increase of either the magnetic loading or the electric loading of the motor design. Since these loadings are directly related to the loss of the motor and therefore the motor thermal management, the proposed design presented in Chapter 2 chose a rated speed of 14,000 RPM and an airgap diameter of 10.892 in. The loadings for the proposed design are optimized in both electromagnetic and thermal disciplines with slight compromise of the motor power density in order to mitigate the mechanical risks. As the Von Mises stress of each component shown in Table 4.3, the carbon fiber retaining ring for the proposed design currently has a safety factor of 7.2; the titanium shell has a safety factor of 3.1; the NdFeB magnets have a safety factor of 1.7.
Table 4.3: Von Mises Stress in Each Rotor Component

<table>
<thead>
<tr>
<th>Components</th>
<th>Stress (MPa)</th>
<th>Screen shot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retaining Ring</td>
<td>403</td>
<td><img src="image" alt="Screen shot of retaining ring stress" /></td>
</tr>
<tr>
<td>Shell</td>
<td>284</td>
<td><img src="image" alt="Screen shot of shell stress" /></td>
</tr>
<tr>
<td>Magnet</td>
<td>48</td>
<td><img src="image" alt="Screen shot of magnet stress" /></td>
</tr>
</tbody>
</table>

Another effective approach to reduce the radial expansion is to increase the retaining ring thickness. However, since the expansion does not have a linear relationship with the ring thickness, the optimal thickness is chosen to be 0.5 in, which is near the knee of the curve shown in Figure 4.5. Even though it might seem beneficial to restraining expansion by further increasing the retaining ring thickness, design trade-offs considering the increase of rotor windage loss and thermal resistance with a thicker retaining ring also need to be taken into account.

### 4.2 Static Deflection

The external rotor configuration for the proposed design also introduces another structural challenge when the motor is mounted horizontally. Figure 4.6 the cross-section view of the motor, where the active region of the rotor is supported only by the end-plate at the drive end. Under the effect of gravity, the rotor will deform even it is not rotating, as the free end will sag in the direction of gravity.

![Figure 4.6: External Rotor Cross Section](image)
Even though most inside-out motor designs eliminate this concern by mounting the rotor in the vertical direction, it is not a viable option for the application of coupling the motor to the turbo engine of an airplane. For the ease of power transmission through a geared system, it is still advantageous to have the motor rotating about the axis parallel to the axis of the turbo engine. Thus, in this section, the discussion of how to reduce the rotor static deflection in the direction of gravity will be presented regarding the horizontal motor operation.

4.2.1 Theoretical Preparation

To understand what determines the deflection magnitude, it is important to first see where in the rotor structure has the greatest deformation. In this case, the rotor can be dissected into basically two sections – the transverse section and the vertical section. The transverse section basically consists of components in the active region, which include carbon fiber retaining ring, cylindrical shell layer, and magnet, as shown in the left of Figure 4.7; the vertical section mainly consists of the vertical end plate of the shell, as shown in the right of Figure 4.7. In order to tackle the rotor static deflection problem, structural members in either the transverse section or the vertical section can be reinforced. For efficient material usage, the section with more deformation should be structurally reinforced.

Figure 4.7: Cross-Section View of Rotor Transverse Section (on the left) and Vertical Section (on the right)
In order to find out which section contributes more to the total deflection of the rotor, structural FEAs are done on the baseline motor design. Table 4.4 summarizes the FEA results of the entire rotor deformation, the deformation of the transvers section only, and the deformation of the vertical section only.

Table 4.4: Deformation at Different Rotor Location

<table>
<thead>
<tr>
<th>Case</th>
<th>Deflection (mm)</th>
<th>Percentage of total deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire rotor</td>
<td>0.04072</td>
<td>100.000%</td>
</tr>
<tr>
<td>Transverse section</td>
<td>0.00027</td>
<td>0.663%</td>
</tr>
<tr>
<td>Vertical section</td>
<td>0.04039</td>
<td>99.190%</td>
</tr>
</tbody>
</table>

Noticeably, the deflection of the transverse section and that of the vertical section did not perfectly add up to the total deflection of the entire rotor. This is due to the intrinsic problem of setting up the FEA for each section separately. To obtain the deflection of only the transverse section, the end plate is set to be fixed when the effect of standard earth gravity is applied to all parts. To obtain the deflection of only the vertical section, the effect of gravity is removed from the setup; instead, the effect of rotor sagging is represented by an equivalent moment, $M$, applying on the vertical end plate, as shown in Figure 4.8.
Figure 4.8: Equivalent Moment Relation Diagram

The moment can be calculated using the following equation.

\[ M = Fd = m_{\text{tran}}gd \]  \hspace{1cm} (4.4)

where \( F \) is the equivalent force as a result of the weight of transverse section, \( m_{\text{tran}} \) is the mass of transverse section, \( d \) is the distance of equivalent force to end plate center.

As shown in Table 4.4, over 99% of the total rotor deflection came from the deformation of end plate. Therefore, in order to reduce the rotor static deflection, it will be more effective to reinforce the structural design of the end plate rather than the transverse section.

The deformation of end plate is analogous a beam bending deformation. To understand what parameters determine the bending deformation, the maximum deflection of a cantilever beam is used [9].

\[ \delta_{\text{max}} = \frac{FL^3}{3EI} \]  \hspace{1cm} (4.5)

where \( F \) is the force acting on the end of the beam, \( L \) is the length of beam, \( E \) is the beam material Young’s modulus, and \( I \) is the area moment of inertia of the beam cross section.

The force \( F \) in the above beam bending equation is proportional to the weight of the rotor transverse section in the case of rotor. Also, the beam length \( L \) in the equation has a linear relationship with the plate diameter. Even though the \( I \) in equation is the area
moment of inertia of a beam with constant cross section, it is still informative if it is compared to the cross section of the end plate. The cross section of the end plate cutting through the center is shown in Figure 4.9, with \( D \) being the plate diameter, and \( t \) being the plate thickness. For the bending direction shown in Figure 4.9, the area moment of inertia for this cross section is calculated as the following [10].

\[
I = \frac{Dt^3}{12} \tag{4.6}
\]

![Figure 4.9: Rotor End Plate Cross Section through Plate Center](image)

Since the rotor static deflection is mainly the effect of bending of the rotor end plate, it can be deduced by using Equations (4.5) and (4.6) that the rotor static deflection has the following approximated relationship with the motor geometric parameters.

\[
\delta_{\text{max}} \propto \frac{FL^3}{3EI} = \frac{m_{\text{trans}}g \left(\frac{D}{2}\right)^3}{3E \frac{Dt^3}{T^2}} \propto \frac{m_{\text{trans}}D^2}{Et^3} \tag{4.7}
\]

### 4.2.2 Discussion and Analysis

From Equation (4.7), it can be observed that increasing the end plate thickness is an effective approach to reduce the static deflection, because the deflection is inversely proportional to the cube of end plate thickness. To confirm this theory, structural FEA's were performed for designs with different end plate thickness. The deflection results are shown in Table 4.5 and Figure 4.10.

**Table 4.5: Rotor Static Deflection of Designs with Different End Plate Thickness**

<table>
<thead>
<tr>
<th>( t ) (mm)</th>
<th>Deflection (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.54</td>
<td>1.2569</td>
</tr>
<tr>
<td>5.08</td>
<td>0.21438</td>
</tr>
<tr>
<td>10.16</td>
<td>0.04072</td>
</tr>
<tr>
<td>15.24</td>
<td>0.01749</td>
</tr>
<tr>
<td>20.32</td>
<td>0.010134</td>
</tr>
<tr>
<td>30.48</td>
<td>0.0051786</td>
</tr>
</tbody>
</table>
A data trend line is obtained for the FEA results, as shown in Figure 4.10. However, according to the trend line equation, the rotor deflection is not inversely proportional to the exact cube of end plate thickness. The discrepancy between the FEA results and the theoretical approximations might come from multiple sources. The most obvious source of error is the oversimplified assumption of approximating the rotor deflection as a simple cantilever beam bending deformation. The underlying assumption for Equations (4.5) and (4.6) is that the bending object must have constant cross section. However, the cross section of the end plate is becoming larger as it moves closer to the plate center. What is more, the actual deformation of the end plate is not pure bending. Even though it could be thought as bending when viewing the rotor from above, there is also some degree of torsional deformation involved if viewing the rotor horizontally (from left or right). Due to the complicated geometry of the rotor design, the above discussion in this section is intended to provide more qualitative analysis and guidelines for future motor designers.

However, for a specific motor design, a quantitative numerical relationship between rotor static deflection and key geometrical parameters is still achievable using the FEA trial method. For the proposed design in this thesis, the relationship between static deflection and end plate thickness is obtained from the data trend line of the FEA results, both in the unit of [mm].
\[ \delta_{\text{max}} = \frac{8.6}{t^{2.233}} \] (4.8)

The optimal end plate thickness should be chosen as the point near the knee of the curve in Figure 4.10, because it effectively restrained the rotor static deflection without using excessive materials. The chosen rotor end plate thickness for the proposed motor design is 10.16 mm. It is not exactly on the center point of the knee because there is still a weight margin for the motor design. Even the power density will reduce by a small factor, the risks such as unbalanced rotor rotation and electrical imbalance can be reduced.

Other approaches to reduce the rotor static deflection can also be observed from Equation (4.7):

- Use material with higher elastic modulus for the rotor shell
- Reduce the mass of the rotor transverse section
- Reduce the rotor diameter

The elastic modulus of titanium is about 114 GPa. Steel will have a higher modulus than titanium, but even for high strength steel like Inconel, its strength-to-weight ratio is still lower than titanium. The strength-to-weight ratio of the shell material will not only play a role in the power density of the motor design, but will also affect the rotor radial expansion during rotation. Furthermore, the general availability of those high strength steel is more limited than titanium. Thus, the proposed design picked titanium as the rotor shell. A trade-off is that titanium cannot be welded, which will increase difficulties in manufacturing.

As the numerators in Equation (4.7), even though the mass of the rotor transverse section and the rotor diameter can be reduced in order to decrease the deflection, they will not be discussed explicitly in this thesis, because the magnetic loading and the electrical loading of the motor design are already optimized in terms of the motor electromagnetic and thermal performance. Changing the mass of magnets or the stator winding current to compensate for a smaller rotor diameter will only be the last resort if excessive deflection cannot be solved with structural member reinforcement.
Another intuitive approach to reduce the rotor deflection can be observed in Equation (4.4). The equivalent moment \( M \) acting on the rotor end plate is proportional to the distance \( d \), which in turn is proportional to the motor active length \( L \), as shown in Figure 4.8. Decreasing motor active length can reduce the rotor deflection, because the effective force applying distance gets smaller for bending deformation. Similar to the reason for parameters like the rotor transverse section mass and rotor diameter, change of \( L \) will imply change of electric or magnetic loading requirements, so it will not be discussed in this section.

Considering beyond using a rectangular shape for the end plate cross section, another approach to reduce rotor deflection is to employ structural stiffeners to reinforce the rotor end plate. Stiffeners like ribs can be added to the corners near the inner shaft or the rotor transverse section, with examples circled in Figure 4.11. Adding stiffeners actually increases the effective area moment of inertia of the end plate cross section, thereby reducing the deflection. The reason for stiffener usage is similar to the wide range of structural I-beam usage in constructions and buildings. However, the proposed motor design in Chapter 2 did not have reinforcing stiffeners, mainly because the exact motor mounting schematics on the turbo engine is not yet available. It will be easier to add stiffeners during the integration phase of the entire power train system.

![Figure 4.11: Rotor End Plate Stiffeners Example](image)

Lastly, the deflection of the entire motor should also include the deformation of the stator. The structural design for stator can follow similar design guidelines of the rotor design presented in this section. The stator static deflection can be much more accurately
modeled as cantilever beam bending using Equation (4.5). Because the total motor deflection contributed by the stator is very small, it will not be presented in this thesis.
Chapter 5

DESIGN FOR DYNAMICS

During dynamic operation of an electric machine, there are numerous vibrations at different locations, frequencies and amplitudes. Basically, two major components to look at for vibration are the rotor and the stator. Typical effects of a vibration problem in the electric machine is that it will affect the performance by consuming excessive power; it will also decrease the reliability because cyclic force will tend to increase the possibility of fatigue failure on the parts, not to mention that the unstable vibration might cause instantaneous damage to the parts [11]. Also known as noise, vibration in a way is also an environmental problem, because it could potentially cause hearing loss of operators near the machines.

In this chapter, resonant vibrations will be investigated for the stator and the rotor separately. Even though there could be unavoidable sources of vibrations in the motor, the goal is to design the structure that will prevent the vibrations happening at resonant frequencies, because vibration at resonant frequency is unstable vibration which could result to immediate damage to the motor. To understand what parameters could change the resonant frequencies of a structure, both proposed the stator and the rotor are approximated to simplified structural elements in the theory preparation. The established calculations are then confirmed by FEA, which is a 3-D modal analysis for the stator, and a 1-D rotor dynamics analysis for the rotor.

5.1 Stator Resonance Modes

Unlike normal electric motors which usually have thick steel back yokes, comparably the motor proposed in this thesis seems relatively flimsy because many components are thin. They are optimized to achieve light weight. The effect of thin radial builds together with the high speed operation could lead the motor to exposure of serious vibration issues. For the stator, this could mean severe failure modes when vibration occurs around the resonance frequency of the structure, which is also known as the natural frequency.
Therefore, in this section, the discussion mainly focuses on how to design the stator structure to avoid the vibration sources meeting at the stator resonance frequencies, while paying attention to achieving a high power density of the motor design.

5.1.1 Theoretical Preparation

To tackle the stator vibration problem, the design goal is to make sure the stator resonance frequency does not coincide with the cyclic force frequency that will potentially excite the corresponding stator resonance mode. This could be achieved by eliminating the sources of cyclic force seen by the stator, or by designing the stator structure such that its resonance frequencies will be far away from the frequencies of the excitation forces. Usually there are many cyclic forces presented in an electric machine, but they will not cause severe instant damage to the machine if they are not happening at the resonance frequencies. However, if a source of cyclic force does present at the resonance frequency of the structure and its direction will potentially excite the corresponding resonance mode pattern, the vibration of that source will be an amplified vibration, having vibration amplitude much larger than the non-resonant vibrations.

Therefore, the first step would be to identify the possible sources of cyclic force in the stator that would result in vibration – their causes and frequencies. Quickly pinpointing the sources of vibration problem is essential to maintain electric machine performance and reliability. The sources of vibration mainly include, but are not limited to [12]:

- Electrical imbalance
- Mechanical unbalance – motor, coupling, or driven equipment
- Mechanical effects – looseness, rubbing, bearing, etc.
- External effects – base, driven equipment, misalignment, etc.

Since the vibrations due to mechanical unbalance, mechanical effects and external effects are mostly related to the operation of the motor in system level, it is not intuitive to incorporate the discussion of those sources in this thesis. The detailed coupling of motor to aircraft turbo engine and the behavior of aircraft system are not specified in the research. Even though a more comprehensive list of electrically and mechanically
induced vibrations is presented in [13], only key sources of vibration due to electrical imbalance will be introduced in this section.

For stator vibration, it is important to first look at the sources of cyclic forces around twice the frequency of the motor electrical power supply. This can be understood if looking at a fix point on the stator: the electromagnetic attracting force between the stator and rotor is maximum when the flux at that point is maximum in magnitude, meaning at either a positive or a negative peak of the flux waveform in the airgap, as shown in Figure 5.1.

![Figure 5.1: One Period of Flux Wave and Magnetic Force Wave [11]](image)

Since the motor electrical frequency is the same as the frequency of its rotating airgap flux, this cyclic magnetic pull force will always present in an electric machine with twice the electric frequency, which is also shown in the next equation [11].

\[ F \propto \frac{B^2}{d} \]  

(5.1)

where \( B \) is flux density in airgap, and \( d \) is airgap length.

The common causes for the electrical imbalance in twice the line frequency are (1) elliptical stator, and (2) non-symmetrical airgap. The fundamental electromagnetic force of the motor will attempt to deflect the stator into elliptical shape, or to the shapes with a number of nodes that corresponds to the number of motor poles. This deflected shape will further increase the unbalance as a result of the magnetic pull force in Equation (5.1) being an unbalanced force seen by the stator at twice the line frequency. Similarly, if the
airgap between the stator and rotor is not symmetrical, the twice line frequency vibration will also increase significantly. In this case, there is a point where the airgap is always smaller than anywhere else. This structural unbalance will increase the effect of electrical imbalance.

There is also vibration in the stator that happens at the rotor rotational mechanical frequency. Although it might not be as important as the twice line frequency vibration mentioned above because the rotor mechanical frequency is relatively low, it is prevalent in electric machines. This type of vibration is usually caused by the source of unbalance that rotates with the rotor.

The common causes for vibration happening at rotor rotational frequency are (1) eccentric rotor, and (2) broken rotor bar. In the case of eccentric rotor, there will be a net unbalanced magnetic force seen by the stator, since the force at the minimum gap is greater than the force at anywhere else in the airgap. This net unbalanced force will rotate with the eccentric point of the rotor, therefore causing vibration at the rotor mechanical frequency. The vibration due to a broken rotor bar works in a similar mechanism as the eccentric rotor case, except there is a point where the force is smaller than anywhere else. For an induction machine, a broken rotor bar will have smaller or even no current flowing through it, resulting to a net unbalanced force seen by the stator. For a permanent magnet machine, especially the ones with a large number of magnet segments like the proposed motor with Halbach array, the broken magnet or rotor bridge will also cause stator vibration at rotational speed.

The last source of vibration happens at relatively high frequency, usually known as rotor bar passing frequency. A point on the stator will see force harmonics which fluctuate every time as a rotor teeth or slot passes by. Thus, the frequency of this cyclic force equals to the rotor rotational frequency times the number of slots on the rotor. For the proposed motor using Halbach array magnets, although there is visually no rotor slot or tooth, the discontinuity between segments will also essentially have similar effects. However, this type of vibration usually is not the main concern for reliability issue because its magnitude is much smaller than other vibrations [12].
After obtaining the excitation frequencies of potential sources of cyclic force, the second step would be to investigate the resonance modes for a given stator geometry, including frequencies of the resonance modes and their corresponding mode shapes. To determine whether the designed motor will experience a resonant vibration, it is necessary to obtain information from both steps, because a resonant vibration will happen only when both of the following criteria are met:

1. The frequency of the cyclic force is at or near a stator resonant frequency.
2. The cyclic force has a pattern to deform the structure to the particular mode shape at that resonant frequency.

During design of electric machines, the resonance modes of a structure are usually obtained through FEA approach, FEA modal analysis in particular. However, for the purpose of interdisciplinary optimization, it is necessary to be able to analytically express the resonant frequency for a given mode shape, so that the trade-offs between design parameters can be used to achieve the ultimate goal of a high power density motor design.

Figure 5.2: Modelling the Stator with Fourth Mode of Vibration as Dissected Beams
Similar to the method presented in [11], the inner stator of proposed motor can be represented as a flat beam structure by unrolling the cylindrical stator. Because it is easy to calculate the fundamental frequency for beam bending, the unrolled stator structure is then dissected to shorter beams for higher vibration modes, which is explained in the Figure 5.2.

The length of a dissected beam is equal to the circumferential length for one-half the mode wave length at the mean diameter of the stator. Thus, the relationship between the effective length of the beam and the mode shape can be expressed as:

\[ L = \frac{\pi D}{2M} \]  

(5.2)

where \( D \) is the mean diameter of the stator structure, and \( M \) is the mode index, which corresponds to the number of nodes for a given mode shape.

If further approximating the unrolled beam structure to a simply supported beam with fixed-fixed end condition, the natural frequency of the stator can be calculated using the equation for fundamental frequency of a beam [14].

\[ f = \frac{1}{2\pi} \left( \frac{22.373}{L^2} \right) \sqrt{\frac{EI}{\rho}} \]  

(5.3)

where \( E \) is the material elastic modulus, \( I \) is the area moment of inertia of the beam cross-section, and \( \rho \) is the linear density of the stator.

By plugging Equation (5.2) into Equation (5.3), the resonant frequency of the stator for a selected mode shape can be analytically obtained as follows:

\[ f = \frac{2M^2}{\pi^3} \left( \frac{22.373}{D^2} \right) \sqrt{\frac{EI}{\rho}} \]  

(5.4)
5.1.2 Discussion and Analysis

The stator with the following geometry is investigated by FEA modal analysis method to obtain resonance modes of the structure.

![Stator for Modal Analysis, Isometric View (left) and Section View (right)](image)

Figure 5.3: Stator for Modal Analysis, Isometric View (left) and Section View (right)

The modal analysis was run to find all the resonance modes of the structure up to a frequency of 6,000 Hz. However, only the mode shapes with radial nodes pattern were summarized in Table 5.1. Other mode shapes, such as those with warping deformation along axial length or with torsional deformation, were not shown here, because they could not be excited by the sources of cyclic force as discussed above. In the results, the mode index number corresponds to the number of nodes of the radial patterns.

Using Equation (5.4), the stator resonance mode frequencies can be analytically obtained as sanity check for the FEA simulations. In order to compare the theoretical approximations to the FEA simulations, calculations were done to find the frequencies of the same modes. The results comparison is shown Table 5.2.
Table 5.1: Stator Resonance Modes Obtained by FEA Simulations

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1363.6</td>
<td><img src="image1.png" alt="Shape 1" /></td>
</tr>
<tr>
<td>3</td>
<td>2150</td>
<td><img src="image2.png" alt="Shape 2" /></td>
</tr>
<tr>
<td>4</td>
<td>2929.4</td>
<td><img src="image3.png" alt="Shape 3" /></td>
</tr>
<tr>
<td>5</td>
<td>3916.9</td>
<td><img src="image4.png" alt="Shape 4" /></td>
</tr>
<tr>
<td>6</td>
<td>5105.3</td>
<td><img src="image5.png" alt="Shape 5" /></td>
</tr>
<tr>
<td>10</td>
<td>11352</td>
<td><img src="image6.png" alt="Shape 6" /></td>
</tr>
</tbody>
</table>

Table 5.2: Comparison of Stator Resonance Mode Frequencies between Theoretical Calculations and FEA Simulations

<table>
<thead>
<tr>
<th>Modes</th>
<th>Theoretical f (Hz)</th>
<th>Simulation f (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>709.26</td>
<td>1363.6</td>
</tr>
<tr>
<td>3</td>
<td>1595.84</td>
<td>2150</td>
</tr>
<tr>
<td>4</td>
<td>2837.04</td>
<td>2929.4</td>
</tr>
<tr>
<td>5</td>
<td>4432.88</td>
<td>3916.9</td>
</tr>
<tr>
<td>6</td>
<td>6383.35</td>
<td>5105.3</td>
</tr>
</tbody>
</table>

It can be observed from Figure 5.4 that the simulation results also have a second-order polynomial trend line similar to the trend of the theoretical calculations. As a sanity check, it is good enough to observe the resonance mode frequency of the
Figure 5.4: Graphical Comparison of Stator Resonance Mode Frequencies between Theoretical Calculations and FEA Simulations

Simulation also has a dependency of the square of mode number. However, it can also be seen that the simulation results have a lower-order term in the trend line. This discrepancy comes from the over-simplified approximation of the complicated stator structure using a simple beam model. The unrolled stator has a non-constant cross section. This can be visualized if looking the stator in the circumferential direction. The cavities in the stator heat sink make the simplified beam structure with changing area moment of inertia and linear density. What is more, Equation (5.4) is intended to calculate the natural frequency for a beam made of only one material. The stator assembly should be solved as a beam with composite materials to obtain accurate representation of the effective elastic modulus.

Back to the concerns of whether the design will have resonant vibrations, it is necessary to determine whether the sources of cyclic force in the motor will match the structural resonance modes. The vibrations at one time rotor rotational frequency can quickly be excluded from the risks of resonance vibration. For the proposed design, the highest speed of normal operation is 14,000 RPM, which gives a rotational frequency of 233.33 Hz. However, the frequency of the lowest resonance mode, the two-node mode, from either the theoretical calculation (709 Hz) or the FEA simulation (1364 Hz) is much
higher than the rotor rotational frequency. Also, the vibration at rotor teeth passing frequency is not a concern here because its magnitude is generally too small to be observed [12].

The main concern, which is also the main motivation for this study, is the vibration source at twice the line frequency. It can be thought of as the electromagnetic force interaction peak happening every time a pole is passing by. This excitation force has a frequency of 4,667 Hz, which is relatively high compared to that of the normal four-pole or eight-pole motors with lower operational speed. However, even though the frequency of this electromagnetic excitation force increases with the number of motor poles, the risks for resonance vibration does not necessarily increase. The resonance mode shape for a 20-pole motor should have radial pattern of 20 nodes. As can be seen in Table 5.1, the resonance mode with the 10-nodes pattern already has a resonant frequency about 11,352 Hz. The expected resonant frequency for the 20-nodes mode, which is about 36,256 Hz, will be much higher than the pole passing frequency. It can be therefore concluded that the stator will not experience resonant vibration.

However, it is reasonable to suspect potential vibration that can cause resonance modes with the integer factor of 20-nodes, which in this case are 2-nodes, 4-nodes, 5-nodes, and 10-nodes mode. Among them, only the 2-nodes, 4-nodes, and 5-nodes modes fall below the rated 4,667 Hz twice line frequency. For the proposed design, the motor should be operated with caution when the twice line frequencies are around those resonant frequencies, which, if converted to rotor speed, are 4091 RPM, 8188 RPM and 11750 RPM.

Lastly, in order to obtain a design insight of how to change the resonant frequency of the stator structure, Equation (5.4) can be examined qualitatively. Generally, the best way to eliminate the risks of resonant vibration is to move the resonant frequencies of the structure above the range of the motor operation speed. To increase a resonant frequency of the structure, the equation indicates that it could be done by decreasing the stator diameter or the stator linear density (in circumferential direction), or by increasing the material elastic modulus or the cross section area of moment of inertia. It is intuitive to make the structure “stiffer” against vibration by making the stator as a smaller diameter
cylinder or using stiffer materials. However, the attempt to increase resonant frequency by modifying the stator ring cross section should be done with extra attention. Even though the area moment of inertia, \( I \), in the equation can be increased by making the cross section larger, the linear density, \( \rho \), would also increase with larger cross section. Therefore, structural design decision should be made to increase the \( \frac{I}{\rho} \) ratio as a whole, instead of looking at just the effect of a single parameter.

### 5.2 Rotor Dynamics

Similar to concerns for instability in the stator during dynamic operation, the rotor is also subject to vibration issues because the radial builds of the rotor are thin and it operates up to 14,000 RPM, which is relatively high for its size. For vibrations of the rotor, a special branch of engineering that studies the lateral and torsional vibrations of rotating shafts is called rotor dynamics, which is more complicated and harder to predict the vibration using 3-D FEA method than the resonant vibrations in the stator with zero rotational speed. The principle components of a rotor-dynamic system are the shaft or the rotor body, the bearings, and the seals. The shaft or the rotor is the rotating component of the system. The bearings steadily support the rotating components with certain amount of stiffness and damping. The seals prevent undesired fluid leakage flow into or out of the machines. Although the seals have rotor-dynamic properties that can cause large rotor vibrations during interaction, they are not discussed in this thesis because the proposed motor employs air-cool schematics and the system level integration schematics are unclear at this stage of the research.

Today, most of the high performance machines normally operate above its first critical speed, generally considered to be the most important mode in the system, although they still avoid continuously operating at or near the critical speeds. Maintaining a critical speed margin of 15\% between the operating speed and the nearest critical speed is a common practice in industrial applications [15]. Because the application of the motor is to augment turbo engine power during cruising of aircraft, the operating speed of the motor is fixed at the rated speed for most of the operation. Therefore, in this section, discussion will be mainly about how to design the rotor structure with large enough margins between the rated speed and the structure critical speed, as well as precautions of
crossing critical speed during speed ramp-ups, while paying attention to the design trade-offs in other disciplines.

5.2.1 Theoretical Preparation

In designing the rotor structure, in order to mitigate the rotor vibration risks with dynamic operation up to a high rotational speed, it is important understand what parameters and how those parameters affect the vibration modes of the rotor-dynamic system. Even though numerical analysis can accurately estimate the location of the resonance mode of the rotor-dynamic system, it cannot give an analytical relationship between the mode frequency and, for example, the damping and stiffness of the system. Therefore, analytical expressions obtained for simple cases are used to approximate the proposed inside-out rotor design so that qualitative design intuition for the rotor structural parameters can be used in interdisciplinary design.

Due to the complexity of the nonlinear analysis in rotor dynamics analysis, the nonlinear models of the system are frequently treated as simple ones. Common approaches include consideration of a symmetrical rigid rotor [16] or a Jeffcott rotor [17]. However, the theoretical calculations presented in section will be following mostly [15], where detailed derivations were done with considerations of gyroscopic effect. Calculations considering with gyroscopic effect have the advantage of factoring in the stiffness of the rotor supports into the mode frequencies. The analytical expression that will be presented in this sub-section will demonstrate the dependency of the actual critical speed of rotor-dynamic system to the system geometry and rotating speed of the rotor. Also, the gyroscopic effect on the rotor can increase or decrease the critical speeds related to some system modes as a function of the rotational speed.

For the ease of discussion, the rotor of the proposed motor design can be considered as a simplified geometry as shown in Figure 5.5. Section 1 is the rotor shaft where the bearings will be sitting on, which is hollow cylinder structure. Section 2 is consists of three layers of materials, which are carbon fiber retaining ring, titanium rotor shell, and NdFeB magnets, from the outermost to the innermost respectively. It can also be viewed as a hollow cylinder. Section 3 is the rotor end plate, which is a cylinder structure. The
material densities of both section 1 and section 3 are the density of the chosen titanium, \( \rho_t \). The density of section 2 is represented as a composite density with weighted average of the three materials densities, \( \rho_c \). Therefore, the polar moment of inertia and the transverse moment of inertia of the rotor structure can be calculated using parallel axis theorem by adding the moment of inertia of each section.

For polar moment of inertia, since the three sections are rotating about the same axis, the total polar moment of inertia of the rotor is simply the sum of all sections about their center of mass.

\[
J_{tot} = J_1 + J_2 + J_3 
\]  
(5.5)

The polar moment of inertia for sections 1 and 2 can be calculated as hollow cylinders, and for section 3 as a solid cylinder. The moment of inertia equations for the two sections about their center of mass are [18]:

Hollow cylinder:

\[
J = \frac{\pi \rho L}{2} \left( r_o^4 - r_i^4 \right) 
\]  
(5.6)

where \( \rho \) is the section density, \( L \) is the length in axial direction, \( r_o \) is the outer radius, and \( r_i \) is the inner radius.
Solid cylinder:

\[
J = \frac{\pi \rho L r^4}{2} \quad (5.7)
\]

As for transverse moment of inertia, since the bearings are sitting on both ends of section 1, the transverse moment of inertia of the whole rotor is calculated about the axis perpendicular into paper at the \( \frac{L_1}{2} \) location of section 1, as shown in Figure 5.5. Except section 1, neither of the other two sections is rotating about the axis through its center of mass. Thus, the parallel axis theorem is applied for section 2 and 3. The total transverse moment of inertia is calculated as:

\[
I_{tot} = I_1 + \left[ I_2 + m_2 \left( \frac{L_2 - L_1}{2} \right)^2 \right] + \left[ I_3 + m_3 \left( \frac{L_1 + L_3}{2} \right)^2 \right] \quad (5.8)
\]

where \( m \) corresponds to the mass of a section, and \( L \) corresponds to the length of a section.

Similarly, for each section, the transverse moment of inertia about center of mass can be calculated using equations for hollow cylinder and solid cylinder [18].

Hollow cylinder:

\[
I = \frac{\pi \rho L r^2}{12} \left[ 3(r_o^4 - r_i^4) + L(r_o^4 - r_i^4) \right] \quad (5.9)
\]

Solid cylinder:

\[
I = \frac{\pi \rho L r^2}{12} (3r^2 + L^2) \quad (5.10)
\]

An important characteristic of the rotor in this section is the ratio of the polar to the transverse moment of inertia, which is given by

\[
P = \frac{J_{tot}}{I_{tot}} \quad (5.11)
\]

It can be noticed that the value of this ratio is affected by the rotor geometry. Generally, this ratio will be large if the rotor has large diameter but short axial length. If the rotor is
designed to be small in diameter but long in axial direction, the ratio will be small and closer to zero.

The dynamics of a rotating system is actually the interaction between the rotor inertial force and the stiffness/damping forces generated by the lateral deformation of the shaft. This is expressed in the equations of motion of the system [15], which is generally in the vector form as

\[
M \ddot{X} + (G + C) \dot{X} + KX = F
\]  

(5.12)

where \( M \) is the mass/inertia matrix, \( G \) is the gyroscopic matrix, \( C \) is the damping matrix, and \( K \) is the stiffness matrix; \( X \) is the solution vector consisting of the lateral displacements of the rotor center of mass and the angle displacements of the rotor rotational axis, and \( F \) is the forcing function vector.

By assuming that the bearing support system provides small or no damping (which is reasonable because the bearing system design in the proposed motor does not have rubber seals like O-rings, and it is stiff in both axial and radial direction due to large bearing preload), as well as that the unbalanced force in the rotor is negligible, the equation of motion in Equation (5.12) then becomes

\[
M \ddot{X} + G \dot{X} + KX = 0
\]  

(5.13)

For simplification, both bearings are assumed to have the same stiffness, and the center of mass of the whole rotor is assumed to be in the middle of section 1. Thus, the solution to the characteristic equations of the above differential equation will have the similar forms as presented in [15].

The cylindrical mode natural frequency is obtained from solution of the characteristic equations of Equation (5.13) corresponding to the rotor translational or parallel motion, which is found to be

\[
\omega_n = \pm \frac{2k}{\sqrt{m}}
\]  

(5.14)

where the plus and minus signs correspond to the forward and backward modes respectively, \( k \) is the radial stiffness of the bearing, and \( m \) is the mass of the rotor.
The conical mode natural frequency is obtained from the solution of the characteristic equations corresponding to the angular dynamics of the rotor. The angular dynamics of the rotor includes the effect of gyroscopic moment acting on the rotor, which will affect the mode natural frequency as a function of rotor speed. However, it is helpful to first obtain the natural frequency of the conical mode when the rotor is not rotating.

\[
\omega_{n0} = \sqrt{\frac{kL_i^2}{2I_{tot}}} \tag{5.15}
\]

Then, when the rotor is rotating, the conical mode natural frequency is

\[
\omega_n = -\frac{P\omega}{2} \pm \sqrt{\left(\frac{P\omega}{2}\right)^2 + \omega_{n0}^2} \tag{5.16}
\]

where the plus and minus signs correspond to the forward and backward modes respectively, and \(\omega\) is the rotor rotational speed.

Furthermore, the bearing stiffness, \(k\), can be also related to the rotor geometry. The analytical expression for the bearing radial stiffness is shown in Appendix A. The bearing stiffness is proportional to a power of the applied radial force on the bearing. For the chosen angular contact ball bearing for the proposed motor design, in this case, the radial force consists of the rotor weight and a factor of the axial preload of the bearing system. Despite the rotor weight, the radial force can be increased by applying larger axial preload to the bearing system. The limit of the maximum preload of the bearing system is determined by the size of the bearings, which in turns corresponds to the size of the rotor shaft.

### 5.2.2 Discussion and Analysis

For design regarding to the vibration of rotor, the ideal situation is to have all the vibration modes natural frequencies far above the rotor rated speed, so that during speed ramp-ups, the rotor will not meet any resonant vibration. To do that, motor designer should design the rotor structure in the direction of increasing the mode frequencies. However, if the rotor inevitably has to encounter some vibration modes, the designer
should also try to suppress the natural frequencies of those modes, making them further below the speeds at which the motor is most frequently operating.

First, from Equations (5.14) – (5.16), it can be observed that the by increasing the stiffness of the system, the resonance mode frequency can be increased. For both cylindrical mode and conical mode, the natural frequency is proportional to the square root of the bearing stiffness. As discussed above, the stiffness of the bearings support system is proportional to a factor of the applied preload, which in turn is generally proportional to the size of the bearing of its type. However, we cannot always simply choose larger bearings for larger stiffness, because as the bearing becomes larger, its rated speed decreases due to larger bearing friction. Thus, for the rotor design, the bearing should be chosen to its largest size with a rated speed still higher than the rotor rated speed.

To see the effect of bearing stiffness on the rotor-dynamic performance of the system, a FEA solver using 1-D beam elements, XLRotor, is used to obtain high fidelity results. The geometry input of the proposed motor in the software is shown in Figure 5.6, which composed of both the stator and the rotor.

By varying the stiffness of the bearing system, $k$, four vibration modes are observed in the proposed motor design with the maximum frequency calculation to 50,000 CPM (Cycle Per Minute). The mode shapes for each resonant mode are shown in Figure 5.7. By observation, Modes 1, 2 and 4 are clearly conical modes, as most of the rotor structure
is displaced by some angles. However, Mode 3 is closer to be a cylindrical mode, because the rotor structure has a certain amount of lateral offset.
The mode frequencies results at rotor speed of 14,000 RPM for different bearing stiffness are shown in the Figure 5.8. The results indicate by increasing the bearing radial stiffness, the natural frequencies of all resonant modes can be increased. It can also be clearly observed that the natural frequencies of Modes 2 and 3 are very sensitive to the change of bearing stiffness at low values, as the curves are steep in the beginning but flatten out later. Since Mode 3 is very close to a cylindrical mode, its trend line has a power close to 0.5, which coincides to Equation (5.14). However, the usage of power type trend line for other modes does not give good approximations. This is expected because, if plugging Equation (5.15) into Equation (5.16), the natural frequency of non-zero speed conical mode is not proportional to the square root of bearing stiffness for conical modes.

A design trade-off for further increasing the bearing stiffness by going with even larger bearing is that the motor rated speed needs to decrease. Depending on how close to the motor rated speed a mode frequency can get, the rotordynamics design to increase the mode frequency should always first consider using the largest bearings possible before altering the motor rated speed. The first-order effect of decreasing motor rated speed is that the motor electric and magnetic loadings need to increase, thus increasing the motor copper and iron losses. This effect on the motor efficiency and power density can be dated back to the discussion in Chapter 3. Noticeably, the uses of larger bearings and therefor larger rotor shaft will also slightly change the polar and the transverse moment of inertia of the rotor. The effect of the change of moment of inertia due to larger shaft needs to be investigated case by case carefully.

From Equations (5.15) and (5.16), it can also be observed that the structure moment of inertia plays a role in the natural frequency of conical modes. Equation (5.15) indicates that the natural frequency of zero-speed conical mode increases as the rotor transverse moment of inertia, \(I_{tot}\), decreases. In spite of the effect of zero speed conical mode frequency, Equation (5.16) shows that the increase of polar to the transverse moment of inertia ratio \(P\) can increase the non-zero speed conical mode frequency. Therefore, it can be summarized that in order to increase the conical mode natural frequency, the rotor polar to transverse moment of inertia ratio needs to increase. This can be translated as
designing the rotor with a favorably larger diameter and shorter axial length. However, the conclusion above is counter-intuitive if considering the design to maximize the stator resonant frequency as mentioned in Section 5.1, because Equation (5.4) implies that smaller diameter design can increase the resonant frequencies for stator modes with radial patterns. Therefore, trade-off between the stator resonant vibration performance and the rotor-dynamic performance needs to be evaluated if the motor diameter is changed.

The mechanical design of electric machines regarding to machine vibration performance is more like a design check to ensure stable normal operation for the motor design range. Because the vibration of the motor does not directly related to its output power at steady-state operation nor to its power density, the design regarding to motor vibration is therefore not the first-order design objective that is used to optimize the motor design in this thesis. The discussion outlined in this chapter is aimed to provide what and how the structural parameters affect the motor vibration performance, in order to enable effective solution to excessive vibration for a specific application case. For the proposed motor, an example below shows how to check whether the motor will experience rotor vibration.

![Figure 5.9: Campbell Diagram of Rotor-Dynamic System in the Proposed Design](image-url)
By looking at the Campbell diagram as shown in Figure 5.9, for speed ramp-up to the rated speed of 14,000 RPM, the designed rotor will only encounter one vibration mode, Mode 1. Since Mode 1 is a backward whirling mode, which means its natural frequency decreases as rotor speed increases, it will not cause serious vibration problem as long as the motor went pass the speed without staying near that speed. Also, it shows that the rotor will not encounter the next mode, Mode 2, until a speed about 16,500 RPM. Thus, it has a speed margin greater than 15%. As for the polar to transverse moment of inertia ratio, the proposed design has a ratio about 1.2. The rotor needs to be designed with this ratio out of the empirical range of 0.8-1.2 [19]. Also, since it is greater than unity, the gyroscopic effect will contribute apparent stiffness to the rotor system, thus raising its forward whirling mode frequencies [20].
Chapter 6

CONCLUSION AND FUTURE WORKS

This thesis has presented a high power density electric motor design for hybrid 737 class commercial airplane application with in depth analysis of its mechanical design. The proposed motor design with rated output power of 1 MW and rated speed of 14,000 RPM has potential to reach a power density as high as 14.4 kW/kg. To maximize the motor power density, the motor needs to have a structure that will efficiently use the materials to provide a structurally sound design. Current state-of-the-art materials and technologies were combined in the proposed motor design in order to increase the motor output power and decrease the motor weight. A permanent magnet type motor with inside-out configuration was chosen because of its advantage of high peak efficiency and compactness compared to other motor types. Because up to this date there are relatively few mature motor designs using the inside-out configuration for such high power level applications, the mechanical design of the proposed motor needs to be studied in a cross-functional manner. Since the motor weight reduction was important in this research, the effect of structural parameters to the motor mechanical performance was coupled to the trade-off analysis in the motor thermal and electromagnetic disciplines.

In order to start the detailed multidisciplinary design optimization, a baseline design was established during the initial sizing study. The baseline design narrowed down a rough range for the motor parameters such as diameter, length and tip speed, in order to provide a region for the most power dense motor design. By assuming proper electrical parameters being constant, the study showed that the motor with an aspect ratio close to 1:1 would yield a high power density while maintaining good structural integrity. The design with larger diameter would have poorer power density because the non-active parts (vertical end plates) of the motor became heavier without contributing to the motor electrical or mechanical performance. Also, a rotor tip speed of about 247 m/s was selected with the considerations of balancing the motor power density and efficiency.
Because of the high rotor rotational speeds and the unconventional inside-out configuration, the motor structural deformation at steady-state operation introduced challenges for the motor in obtaining its desired performance. The major challenges were the rotor radial expansion due to the centrifugal effect of a rotating rotor, and the rotor static deflection due to the effect of gravity. In this thesis, they were both addressed first by theoretical calculations that used simple elements to approximate the complex geometry of the proposed design, then by confirmation of 3-D finite element analysis to obtain a high fidelity results and optimization. The results and analysis concluded that, for selected materials, the radial expansion was proportional to the square of rotor tip speed, the inverse of retaining ring thickness, and the rotor diameter. As for static deflection, the maximum deflection at the end of the rotor is proportional to the active length, the square of rotor diameter, and the inverse of end plate thickness.

The proposed motor design was also subject to structural challenges in terms of vibration issues during dynamic operation, due to high frequency harmonics presented in the motor and relatively thin radial builds if compared to machines in ground applications. The motor vibration was analyzed in two components independently. The vibration analysis for the stationary component was the stator resonance modes. The analysis for the rotating component was the rotor dynamics. Similar to the static deformation problems, analytical calculations were established for the proposed structure in order to provide insight for interdisciplinary design trade-offs, followed by higher fidelity computer solutions using finite element analysis. The stator resonance modes were obtained through 3-D FEA modal analysis for the proposed design. For the rotor resonance modes, a rotor dynamics software package using 1-D beam elements was employed for faster iterative designs. The study showed that stator vibration was inversely proportional to the square of the stator diameter, and directly proportional to the square root of the cross-section area moment of inertia to density ratio. For rotor system dynamics, the mode frequency could be increased with the system stiffness. For conical vibration modes specially, the mode frequency also had a positive relationship with the structure polar to transverse moment of inertia ratio.
Whereas the proof of concept studies in this thesis could provide insight of the motor structure for the interdisciplinary design and mitigate most of the critical mechanical risks for the inside-out type high power density motor, there are future works that could be beneficial to raise the proposed motor design to a higher technology readiness level (TRL). As an ongoing task in this research project, a physical test of the full-sized rotor was still being constructed. The rotor rotating test up to 20% overspeed could prove ruggedness of the design, as well as obtain the deformation and vibration data for the calibration of analytical and computer analysis for more accurate design optimizations in the future. Since vibration was analyzed separately for the stator and the rotor, the interacting effect of the two components was not included in this thesis. For example, an imbalance response analysis in rotor dynamic system due to the electromagnetic interactions in the airgap could be conducted to further ensure the rotor dynamic performance. Even though analytical expressions of motor deformation were difficult to obtain for a very accurate description of the complex geometry of the proposed design, numerical solution method might still be possible to provide more accurate results than the approximated calculations in this thesis, and faster solution than the FEA methods. Furthermore, if the environment of the motor application is more specifically defined, the mechanical design of the motor could be done in a level with more details, for example, the usage of structural stiffeners. The proposed motor could achieve a high TRL if a more comprehensive study is done in the aircraft system level.
Appendix A

SKF BEARING STIFFNESS CALCULATION

SIMPLIFIED BEARING STIFFNESS

DEEP GROOVE BALL BEARINGS & ANGULAR CONTACT BALL BEARINGS

\[ K_r = 1166 \cos^{53} \alpha (i Z)^{23} D_w^{1/3} F_r^{1/3} \]
\[ K_a = 3409 \sin^{53} \alpha (i Z)^{23} D_w^{1/3} F_s^{1/3} \]

CYLINDRICAL ROLLER BEARINGS & TAPERED ROLLER BEARINGS

\[ K_r = 3390 \cos^{19} \alpha (i Z)^{6.9} l^{0.8} F_r^{0.1} \]
\[ K_a = 14430 \sin^{19} \alpha (i Z)^{6.9} l^{0.8} F_s^{0.1} \]

SPHERICAL ROLLER BEARINGS

\[ K_r = 1813 \cos^{74} \alpha (i Z)^{34} l^{1/2} F_r^{1/4} \]
\[ K_a = 6061 \sin^{74} \alpha (i Z)^{34} l^{1/2} F_s^{1/4} \]

SYMBOLS & UNITS:
\( K_r = \) Radial stiffness, N/mm
\( K_a = \) Axial stiffness, N/mm
\( i = \) number of active rows
\( \alpha = \) contact angle, degrees
\( Z = \) number of rolling elements per row
\( D_w = \) rolling element diameter, mm
\( l = \) roller length, mm
\( F_r = \) radial load, N
\( F_s = \) axial load, N
STIFFNESS - Example

For a simplified evaluation of a shaft's dynamic behavior, a Customer wishes to know the radial stiffness of an SKF 6309 SRDGBB. The radial force on the bearing is 1000N. What is the radial stiffness of the bearing?

D_w = 17.462 mm
Z = 8
i = 1
α = 0

K_r = 1166 \cos^{5/3} \alpha (i Z)^{2/3} D_w^{1/3} F_r^{1/3}

K_r = 1166 \cos^{5/3}(0)(8)^{2/3}(17.462)^{1/3}(1000)^{1/3}

K_r = 121,000 N/mm
REFERENCES


