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MODAL AUXILIARY VERBS AND CONTEXTS

BY

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DISSERTATION

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Modal auxiliary verbs, such as could, might, must, would, and others, have different readings depending on the context in which they occur (Kratzer 1981[45]). The sentence ‘Jess could fry the fish’ can mean that, in a time previous to the utterance of the sentence, Jess had the ability to fry the fish, or it can mean that, at the time of the utterance, Jess frying the fish is a possible event.

Modal auxiliary verbs often create intensional environments, leading the events described by the second verb to be understood to be non-actual events. When the readings are described as being determined by a context, it is often a broad notion of non-linguistic and extra-sentential linguistic context that is the focus of the interpretation. For example, descriptive pragmatic constraints are used in Lewis 1973[54] and Kratzer 1981[45] to characterize types of accessibility relations and types of orderings of worlds.

A large part of the meaning of modal auxiliary verbs, however, centers around how the events described by the second verb are situated relative to the time at which the sentence containing the modal auxiliary is used. Information about the temporal situation of an event is conveyed through the linguistic context in which a modal auxiliary verb occurs, including, but not limited to, lexical properties of the linguistic expressions describing the event in the scope of the modal auxiliary, lexical properties of the modal auxiliary itself, and temporal and aspectual marking on linguistic expressions in the verbal projections.

In order to provide a framework for representing the interactions of tense, aspect, and modality, a fragment of English is given in a Multi-Modal Combinatorial Categorial Grammar (Baldridge & Kruijff[6], Steedman 2012[93]). Modal auxiliaries are given verb-like lexical entries in the grammar using lexical entries that combine features from Villavicencio 2002[101] and standard attribute value matrices of Head Driven Phrase Structure Grammar (Pollard & Sag 1999[77], Sag, Wasow, & Bender 2003[85]). Modal auxiliaries have default lexical arguments with which they combine, and they combine
with temporal and aspectual meaning that is sometimes morphologically manifested through grammatical tense and aspect. Portions of the combinatory methods are based on Bach 1983[4], who argued for less constrained combinatorial rules and unification of features in order to represent modal auxiliaries.

The notion of event semantics (Davidson 1967[22]) plays an important role in the formulation of the compositional semantics due to the way in which event times are related to aspectual meaning. The grammar uses a Neo-Davidsonian approach (Parsons 1990[74]) to representing the arguments of the verb and builds on the work of Champollion 2015[13]. The temporal component is very important in this work and uses portions of the temporal and event ontology proposed in Muskens 1995[68], 2003[67].

Two paradigms of modal auxiliaries are proposed: Tense-bearing modal auxiliaries and non-tense-bearing modal auxiliaries. Within each paradigm, readings are shown to have differing semantics with respect to the semantic roles with which they combine and the temporal and aspectual readings that they can have. Differing results with respect to their behaviour in describing various states of affairs are addressed, as is their behaviour in expressing past tense, sequence of tense contexts (Abusch 1997[1]), and the distribution of perfect aspect.

The formal grammar distinguishes parts of the meaning of sentences with modal auxiliary verbs that can be represented in terms of composition of temporal and aspectual expressions with modal auxiliary verbs or composition of a modal auxiliary verb with its arguments, on one hand, from parts of the meaning that are constrained by a broader notion of context, on the other hand.

The notion of a broader context is not, however, neglected in the treatment. The English language fragment presented in the grammar is interpreted in a relativist semantic model, motivated by the assessment-sensitivity of epistemic modal auxiliaries (MacFarlane 2011[56], Lasersohn 2005[51], Lasersohn 2015[52]). Readings that do not require assessment sensitivity are given truth conditions according to those given for monadic truth in Lasersohn 2015[52].

The interaction of readings with their grammatical distribution provides additional theoretical insights into the linguistic contexts that are conducive to assessment sensitivity, actuality inferences, and counterfactual readings. Most notably, it is shown that assessment sensitivity is only present in modal auxiliaries that are in the non-tense-bearing paradigm.

Parts of the theoretical treatment presented in this work have been applied in areas
of experimental semantics and corpus linguistics for automated classification of modal auxiliary verbs (Moon 2011[64], Moon 2012[65], Moon et al. 2016[66]). These works show that temporal, aspectual, and argument structure information can be used to determine the most likely reading of a modal auxiliary, at the sentence level.
TABLE OF CONTENTS

CHAPTER 1 INTRODUCTION ........................................... 1
  1.1 Modal Auxiliaries Have Multiple Readings; Those Readings Can
      Have Different Temporal Properties .......................... 1
  1.2 Related Research on Modal Auxiliary Readings .................. 3
  1.3 Research Questions ........................................... 4
  1.4 Landscape of the Dissertation ................................. 9
  1.5 Applications and Future Research .............................. 10

CHAPTER 2 BACKGROUND ON MODAL AUXILIARY VERBS .............. 11
  2.1 Taxonomic Categories of Modal Auxiliary Readings ............. 11
  2.2 Necessity and Possibility .................................... 13
  2.3 Epistemic Modals ........................................... 20
  2.4 Non-Epistemic Modals ....................................... 22
  2.5 Priority Modals ............................................ 23
  2.6 Dynamic Modals ............................................ 29
  2.7 Temporal Uses ............................................. 34
  2.8 Other Topics Related to Modal Auxiliary Taxonomies .......... 35

CHAPTER 3 MODAL LOGIC (MONTAGUE’S INTENSIONAL LOGIC) .... 39
  3.1 Syntactic Categories of English ............................... 39
  3.2 Syntax of IL ............................................... 43
  3.3 Semantics of IL ........................................... 45
  3.4 Translation Rules .......................................... 54

CHAPTER 4 MODAL LOGIC AND MODAL AUXILIARY VERBS: THE
            STANDARD THEORY ....................................... 60
  4.1 Introduction .............................................. 60
  4.2 Propositions and Worlds .................................... 60
  4.3 Modal Logic and Modality: Lewis 1973[54] ..................... 61
  4.4 Expansion of Lewis’ Ideas ................................. 70
  4.5 Kratzer’s Analysis ....................................... 71
  4.6 Conclusion .............................................. 79
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>TENSE AND ASPECT</td>
<td>80</td>
</tr>
<tr>
<td>5.1</td>
<td>Taxonomies of Tense and Aspect</td>
<td>80</td>
</tr>
<tr>
<td>5.2</td>
<td>Tense</td>
<td>81</td>
</tr>
<tr>
<td>5.3</td>
<td>Taxonomies of Aspect</td>
<td>82</td>
</tr>
<tr>
<td>5.4</td>
<td>Aktionsart</td>
<td>85</td>
</tr>
<tr>
<td>5.5</td>
<td>Perfectivity and Imperfectivity, also called Viewpoint Aspect</td>
<td>90</td>
</tr>
<tr>
<td>5.6</td>
<td>Perfect Aspect</td>
<td>94</td>
</tr>
<tr>
<td>5.7</td>
<td>Imperfect Progressive and Aktionsart</td>
<td>95</td>
</tr>
<tr>
<td>6</td>
<td>FORMAL GRAMMAR</td>
<td>98</td>
</tr>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>98</td>
</tr>
<tr>
<td>6.2</td>
<td>MM-CCG for a Fragment of English</td>
<td>101</td>
</tr>
<tr>
<td>6.3</td>
<td>Type Theory, Logical Language, and Interpretation</td>
<td>110</td>
</tr>
<tr>
<td>6.4</td>
<td>Features Values and Unification: The Role of the Lexicon</td>
<td>123</td>
</tr>
<tr>
<td>6.5</td>
<td>Translation and Interpretation of Nouns and Non-Finite Verbs</td>
<td>132</td>
</tr>
<tr>
<td>6.6</td>
<td>Review and Conclusions</td>
<td>148</td>
</tr>
<tr>
<td>7</td>
<td>TENSE AND ASPECT IN A FORMAL GRAMMAR</td>
<td>149</td>
</tr>
<tr>
<td>7.1</td>
<td>Introduction</td>
<td>149</td>
</tr>
<tr>
<td>7.2</td>
<td>Times in the Grammar</td>
<td>154</td>
</tr>
<tr>
<td>7.3</td>
<td>Grammatical Aspect</td>
<td>155</td>
</tr>
<tr>
<td>7.4</td>
<td>Tense</td>
<td>161</td>
</tr>
<tr>
<td>7.5</td>
<td>Summary and Examples</td>
<td>162</td>
</tr>
<tr>
<td>7.6</td>
<td>Translation and Interpretation of Closure Operators</td>
<td>167</td>
</tr>
<tr>
<td>7.7</td>
<td>Review and Conclusions</td>
<td>173</td>
</tr>
<tr>
<td>8</td>
<td>EXTENDING THE GRAMMAR TO MODALITY</td>
<td>174</td>
</tr>
<tr>
<td>8.1</td>
<td>Motivating Two Grammatical Paradigms</td>
<td>175</td>
</tr>
<tr>
<td>8.2</td>
<td>Control and Raising in Modal Auxiliary Verbs</td>
<td>187</td>
</tr>
<tr>
<td>8.3</td>
<td>Tense and Aspect with Ability can</td>
<td>197</td>
</tr>
<tr>
<td>8.4</td>
<td>Tense, Aspect, and Epistemic might</td>
<td>211</td>
</tr>
<tr>
<td>8.5</td>
<td>Conclusions</td>
<td>223</td>
</tr>
<tr>
<td>9</td>
<td>THE LANDSCAPE OF MODAL AUXILIARY VERBS</td>
<td>225</td>
</tr>
<tr>
<td>9.1</td>
<td>Introduction</td>
<td>225</td>
</tr>
<tr>
<td>9.2</td>
<td>Dynamic Modal Auxiliaries Readings</td>
<td>227</td>
</tr>
<tr>
<td>9.3</td>
<td>Priority Readings</td>
<td>239</td>
</tr>
<tr>
<td>9.4</td>
<td>Paradigm B Raising Modal Auxiliary Verbs: Speculative and Epistemic Readings</td>
<td>252</td>
</tr>
<tr>
<td>9.5</td>
<td>‘Future’ WOLL</td>
<td>258</td>
</tr>
<tr>
<td>9.6</td>
<td>Perfect Aspect and Modal Auxiliary Verbs</td>
<td>262</td>
</tr>
<tr>
<td>9.7</td>
<td>Subjunctive Modal Auxiliaries</td>
<td>266</td>
</tr>
<tr>
<td>9.8</td>
<td>Grammar of Conditional Clauses with Modal Auxiliary Verbs</td>
<td>268</td>
</tr>
<tr>
<td>9.9</td>
<td>Summary and Conclusions</td>
<td>279</td>
</tr>
</tbody>
</table>
1.1 Modal Auxiliaries Have Multiple Readings; Those Readings Can Have Different Temporal Properties

Modal auxiliary verbs permit various readings. For example, sentence (1) below allows readings in which a ‘Jess-fry-the-fish’ event occurred as well as readings in which there was a potential ‘Jess-fry-the-fish’ event that did not occur:

1. *Pat was surprised that Jess could fry the fish.*

Readings on which the ‘Jess-fry-the-fish’ event occurred are said to have *actuality inferences* or *actuality presuppositions* (e.g., Bhatt 1999[8], Hacquard 2006[34]).

Various readings are elucidated, to some degree, in the potential elaborations on sentence (1) illustrated in sentences (2-4) below:

2. *Pat was surprised that Jess could so deliciously fry the fish for yesterday’s dinner.*

3. *Pat was surprised that Jess could fry the fish, but, thankfully, she didn’t.*

4. *Pat was surprised that Jess could fry the fish at the picnic tomorrow.*

When a modal verb, such as *could*, occurs in the complement of a past tense matrix verb, sequence of tense phenomena occur (Ogihara 1995[72], Abusch 1997[1]):

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1Hacquard 2006[34] shows a clear distinction between uses of French modal verbs that have actuality entailments when used with morphology indicating the perfective aspect and uses of other French modal verbs that have actuality presuppositions. The same type of distinction cannot be applied to English since English modal verbs do not morphologically encode aspect analogously to French. For now, this monograph will refer to the ‘inference’ that an event actually occurred in the English instances of the phenomena.

2Actuality inferences that human subjects draw with sentences such as sentence (2) above were tested experimentally in Moon 2012[65], and it was found that making actuality inferences was positively correlated with past tense embedding verbs. Intensifying adverbs such as *so deliciously* did not show a significant correlation.
embedded clause’s ‘Jess-fry-the-fish’ event can be read as occurring in the same reference time in which the ‘Pat-be-surprised’ event is situated.\(^3\) In such a reading, the event of the embedded clause and the event of the embedding clause are simultaneous. On this reading, both events occur before the speech time. The events can also be read as occurring sequentially. For instance, Pat could be surprised at a time prior to the speech time that precedes the ‘Jess fry-the-fish’ event. The ‘Jess-fry-the-fish’ event could either be past relative to the speech time, or in the future relative to speech time.

In sentences (2-3), the possible event that is referred to in the embedded clause precedes the speech time. Both sentences allow either a simultaneous or sequential reading. On the simultaneous readings of sentences (2) and (3), the potential ‘Jess-fry-the-fish’ event occurs at the same time as the ‘Pat-be-surprised’ event referred to in the matrix clause. On the sequential reading of sentence (2), Pat felt surprised after the time span in which Jess had the potential to fry the fish. On the sequential reading of sentence (3), Pat felt surprised before the time span in which Jess had the potential to fry the fish.

Both the sequential and simultaneous readings of sentence (1) allow readings with actuality inferences, but sentence (2) is presented with the intention of convincing readers that the reading with an actuality inference exists, whereas sentence (3) is presented with the intention of convincing readers that the inference that Jess actually fried the fish can be canceled. In both the sequential and simultaneous readings of sentence (2), it seems likely that a ‘Jess-fry-the-fish’ event actually occurred. Sentence (3) allows the same two temporal readings described for sentence (2), but, in contrast, it explicitly denies that a ‘Jess-fry-the-fish’ event occurred.

<table>
<thead>
<tr>
<th></th>
<th>Simultaneous</th>
<th>Sequential</th>
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<tbody>
<tr>
<td>Actuality Inference (+)</td>
<td>(1), (2)</td>
<td>(1), (2)</td>
</tr>
<tr>
<td>Actuality Inference (−)</td>
<td>(1), (3)</td>
<td>(1), (3), (4)</td>
</tr>
</tbody>
</table>

Table 1.1: Two by two matrix of actuality inferences and sequence of tense phenomena with sentences illustrating each indexed: (1) Pat was surprised that Jess could fry the fish. (2) Pat was surprised that Jess could so deliciously fry the fish for yesterday’s dinner. (3) Pat was surprised that Jess could fry the fish, but, thankfully, he didn’t. (4) Pat was surprised that Jess could fry the fish at the picnic tomorrow.

In sentence (4), the use of the temporal adverb, *tomorrow*, forces an interpretation in which the event referred to in the matrix clause precedes the speech time while the

\(^3\)The use of terminology such as *reference time* is, at this point in the exposition, informal and does not, for instance, indicate the use of Reichenbach’s 1974[82] theory of tense.
event referred to in the embedded clause follows the speech time, illustrating a clearly sequential reading without an actuality entailment where the event in the matrix clause precedes the event in the embedded clause.  

Table 1.1 indexes the example sentences in a two by two matrix of actuality inferences by sequence of tense phenomena. The parameter of speech time is omitted in Table 1.1, but would distinguish sentence (4) from other uses.

The possible temporal ordering of events and presence or absence of actuality inferences that have been discussed so far have been restricted to a particular linguistic context: One in which the modal verb *could* occurs in an embedded clause as the complement of a matrix clause containing a particular propositional attitude verb, *surprised*, in a particular tense, past, and with a durative telic event, *cook the fish*, as the complement of the modal verb. Each of these features can be varied resulting in subtle changes in meaning and different sets of interactions with the temporal adverbs and additional clauses in sentences (2-4). Not only do temporal and aspectual interpretations vary based on linguistic features of the sentence, but the type of inferences compatible with a given modal verb reading vary.

As illustrated by the various readings of sentence (1) above, readings can have different and even incompatible temporal and event entailments. The difficulty of modal use and interpretation in child language learners, second language acquisition, as well as in automated systems attests to the complexity of the task. Nevertheless, modal verbs pervade natural language text and discourse, illustrating their importance for communicative fluency.

### 1.2 Related Research on Modal Auxiliary Readings

Formal Semantic work on modal auxiliary verbs is roughly divided into two closely related approaches. The first type of approach focuses on the way in which the semantics of modal auxiliary verbs is represented, often using modal logic and possible worlds as a theoretical framework (Lewis 1973[54], Kratzer 1981[45]). These works show how that framework is constrained or modified in order to restrict the quantification over

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4A pilot experiment for Moon 2012[65] provided evidence that subjects can interpret sentences such as (4) in such a way that *tomorrow* is indexical to a time preceding their reading of the sentence. On this reading, they perhaps consider the reference time of the reported prediction to have already passed and often infer that the prediction was fulfilled, resulting in an actuality inference.
possible worlds in a way that models the intuitive meaning of modal auxiliary verbs (Schulz 2007[87], Lassiter 2011[53], MacFarlane 2011[56]). Questions of grammar and compositionality are generally placed in the background in order to focus on the modal logic component of meaning.

The second type of approach seeks to provide a compositional analysis of linguistic components of sentences containing modal auxiliary verbs. Common linguistic components that form the basis for such analyses are grammatical components of conditional sentences (Iatridou 2000[38], Condoravdi 2002[19]), other forms of embedding clauses (Henderson 2011[36]), grammatical components of teleological sentences (von Fintel & Iatridou 2007[103]), perfect aspect (Bhatt 1999[8], Condoravdi 2002[19], Hacquard 2006[34]), and Aktionsart (Iatridou 2000[38], Condoravdi 2002[19]). These analyses situate the semantic composition of tense and aspect in modal logic analyses of the modal auxiliary verb, usually using standard notions of conversational backgrounds and ordering sources (Kratzer 1981[45]).

1.3 Research Questions

Data such as the ‘Jess could fry the fish’ sentences show that a given modal auxiliary does not necessarily have a uniquely determined reading when it is considered in isolation from the text, discourse, or non-linguistic context in which it is used; rather, it can have more than one reading. The data also show that the set of readings that are available for a given sentence is limited. Furthermore, the data indicate that there are readings that involve differences in the temporal location of the events referred to in the sentence.

The notion of modal auxiliary readings from Kratzer 1981[45] involves differences in meaning that are not described in terms of their temporal or aspectual properties. The traditional notion of context in discussions regarding modal auxiliary verbs includes the conversational background in which the modal auxiliary is used, and the ordering source. Where the conversational background provides accessible worlds for the evaluation (or sometimes the assertion) of the proposition expressed by the sentence containing the modal, and the ordering source is involved in ranking the worlds in terms of preferences or similarity to other worlds.

The modal force, which determines whether a modal requires satisfaction relative to
all accessible worlds (given an ordering) or at least one, seems to be a lexical feature of
the modal auxiliary, in so far as a given modal either involves necessity or possibility.
The proposition often appears to be defined as some representation of the eventuality
being described in the sentence containing the modal auxiliary verb, similar to the ‘Jess-
fry-the-fish’ event mentioned previously. The modal logic operator used to represent the
denotation of a modal auxiliary takes scope over the proposition and determines its truth
value relative to a context.5

There are several notions of context that play a role in modal auxiliary meaning,
two of which are the focus of this study. First, there is the linguistic semantic con-
text, which involves the temporal and aspectual morphology of the expressions oc-
curring with the modal auxiliary. Second, there is the notion of context that figures
into Kratzer’s 1981[45] definition of a conversational background and ordering source.
Kratzer 1991[48] makes a distinction between when this notion of context is linguisti-
cally present and when it is not. When it is linguistically presented, it is characterized
by an ‘in view of’ phrase, as in sentences (5-6) below:

5. In view of what is known about Jess, she must run a lot.
6. In view of what the doctor commands Jess for the sake of her health, she must run a
   lot.

1.3.1 Context in Other Contexts

Lasersohn 2015[52] proposes a relativist semantic model that assigns denotations to
the content of linguistic expressions in the context in which they are used. Parts of
the framework follow Kaplan 1989[40]. The theory is motivated by various linguistic
expressions, with a central focus on predicates of personal taste, and an example is
given of how to characterize epistemic modality in the theory, but the framework is not
extended to readings of modal auxiliary verbs more generally, as they fall outside the
scope of Lasersohn’s 2015[52] work.

The assumption is made that a sentence generally has only one content in a given
context. Representing modal auxiliary verbs in this framework involves determining the
answers to important research questions: (1) which features of a use of a modal auxiliary

5 The gradability of modal expressions such as likely seems to be lexically determined as well (Kratzer
1981[45], Kratzer 1991[48].
verb are part of the content of that use, (2) which features of a use of a modal auxiliary verb are part of the context of that use, (3) which readings of modal auxiliary verbs are the same regardless of the context (e.g., readings that are strictly associated with the grammatical features of linguistic expressions in the sentence) (4) which readings of modal auxiliary verbs are fixed by a context, (5) which readings of modal auxiliary verbs require a relativist analysis.

Temporal and aspectual meaning is considered to be part of the content of sentences with modal auxiliary verbs; however, as section 1.1 demonstrated, the temporal and aspectual meaning can vary based on the reading. What determines the reading, in examples with multiple readings, is part of the context of use. Different contexts lead to different contents, so the temporal and aspectual expressions can have different denotations depending on the context of use.

Some examples do not allow multiple readings. Presumably, the context does not change the content of these expressions. A sentence with a modal auxiliary verb could contain a personal pronoun or some other linguistic expression that depends on the context for its content, but, if the reading of the modal auxiliary verb does not depend on the context of use, then the content of the temporal and aspectual linguistic expressions occurring in the sentence with the modal auxiliary verb compose in the grammar to determine the content. Linguistic environments in which modal auxiliary verbs can only have one reading, however, are more rare than linguistic environments that depend on the context of use for their content.

Epistemic modal auxiliary verbs have been argued to require a relativist semantics (Egan et al. 2005[26], MacFarlane 2011[56]; Lasersohn 2015[52]). Uses of modal auxiliary verbs that require a relativist semantics do not have a definite truth value depending on the context in which they occur. Rather, their truth values are assigned relative to, in the case of epistemic modal auxiliary verbs, the person assessing their truth value.

Placing modal auxiliary verbs in this framework leads to interesting insights regarding their semantics and how temporal and aspectual expressions are characterized in the grammar. The decisions made in the proposed grammar are compared to the existing literature on the topic or, if they have not been modeled in the literature, an approach is presented.
1.3.2 Grammatical Features in the Scope of this Study

Uses of modal auxiliaries can occur with a variety of temporal and aspectual properties. A modal auxiliary can occur with a perfect marker and a perfect verb or it can occur with the base form of a verb. It can occur with perfect and progressive marking or with just progressive marking, or without any marking. Beneath all the layers of grammatical aspect, there are a variety of lexical aspects with which a modal auxiliary can occur, also sometimes changing the meaning.

In order to peel off all of the layers of variation and determine the contributions of the component parts, pair-wise comparison of a huge number of factors is required.

<table>
<thead>
<tr>
<th>Modal Auxiliaries</th>
<th>(8)</th>
<th>would, will, could, can, must, might, may, should</th>
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<tbody>
<tr>
<td>Perfect</td>
<td>(2)</td>
<td>present OR absent</td>
</tr>
<tr>
<td>Progressive</td>
<td>(2)</td>
<td>present OR absent</td>
</tr>
<tr>
<td>Aktionsart</td>
<td>(4)</td>
<td>telic/atelic, durative/punctual,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>stative/dynamic, semelfactive/iterative</td>
</tr>
</tbody>
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Table 1.2: Some Basic Parameters in Modal Auxiliary Reading Determination: 
(8 × 2 × 2 × 4 = 128)

There are 128 such pairs of features, given very basic parameters. The temporal adverbs, tense of extra-clausal verbs, and lexical items that could be varied to tease-out meaning differences are not even included, nor is discourse information.

One way to reduce the number of factors under consideration is to separate variations in the temporal location of eventualities that are present due to the meaning of temporal and aspectual features of the sentence from those that are due, in part, to the presence of the modal auxiliary verb.

The grammar of modal auxiliaries developed here is built upon a theory of how eventualities are situated relative to speech and event times, given the temporal and aspectual features that characterize them. For some modal uses, the modal auxiliary contributes to the meaning of the sentence without causing the temporal location of the eventuality relative to speech and reference time to be different than it would be in a sentence without a modal auxiliary verb. In other cases, the presence of a modal auxiliary verb or the context of use of the modal auxiliary verb results in an interpretation of the temporal and aspectual expressions that differs from the way they are interpreted in sentences without modal auxiliary verbs.

There are several uses of auxiliaries that have tense marked on the modals. These
are *can* and *will* that have past forms *could* and *would*, respectively. One of the major contributions of this work is that it provides supporting evidence for representing two paradigms of modal verbs in the grammar. The first paradigm, Paradigm A, forms the past tense by changing the form of the modal. The second paradigm, Paradigm B, refers to past events by means of perfect marking on the verb that follows the modal auxiliary.

### 1.3.3 Theoretical Framework

The formal grammar of modal auxiliary verbs is presented in the tradition of Montague Grammar (Montague 1973[63]), but with substantial additions. Event semantics is added (Davidson 1980[23]) using the variation of neo-Davidsonian event semantics (Parsons 1990[74]) proposed in Champollion 2015[13]. The underlying ontology of events is discussed to the degree necessary for the exposition, primarily following Smith 1991[89].

Tense and grammatical aspect are treated in detail. The syntactic interaction is influenced by that presented in Muskens 1995[68], 2003[67], but is not in the same framework.

Modal auxiliary verbs are added to the grammar in a novel way, according to the main proposals of this work. They receive lexical entries that determine if they are tense-bearing modal auxiliaries or not. Each form composes with tense, grammatical aspect, and event types in the way required to accurately model the grammar of English.

The semantic interpretation of modal auxiliary verbs follows the relativist framework presented in Lasersohn 2015[52]. A similar framework has been shown to be required for an accurate representation of epistemic readings of modal auxiliary verbs (MacFarlane 2011[56]) in order to handle assessment-sensitivity.

The framework in Lasersohn 2015[52] uses standard notions of modal logic in the interpretation of modal auxiliary verbs in that the modal operator ranges over different worlds (operates with different accessibility relations) depending on which type of reading is present.

The modal auxiliary verb interpretation is developed further in this work, encoding more parameters in the context and having a more involved interaction with tense and aspect, in order to account for the observed differences in the grammar of various readings.
1.3.4 Methods

Applied work on this topic has used corpus data (Moon 2011[64], Moon et al.2016[66]) and experimental methods (Moon 2012[65]). Corpus data and experimental results are cited when relevant, but the primary focus of this work is the theoretical model behind the applied work. Both methods, however, continue to influence and sharpen one another. The applied work conducted simultaneously with this research project greatly influenced the outcome of the theory and, likewise, the further development of the theoretical model guides present applied projects, as well as presenting new ideas for future research.

The majority of the data presented and discussed in this work are from the literature that was reviewed and from native speaker intuitions, most of which have been recognized in other works.

The conclusion brings up considerations for how the distributional data could be incorporated into the formal model in the future, which has long been a driving goal behind the research questions.

1.4 Landscape of the Dissertation

Chapter 2 provides a background on traditional taxonomies of modal auxiliary verbs, defining the space of meanings that have been recognized as contributed by modal auxiliaries. Chapter 3 reviews approach to model-theoretic formal semantics in the tradition of Montague 1970[62], 1973[63]. Readers with a background in Formal Semantics who are already familiar with this approach can skip this chapter. Chapter 4 presents formal semantic approaches to modal auxiliary verbs in the model-theoretic, modal logic tradition, giving a background on Kratzer’s 1981[45] influential theory, which remains the standard theory of modal auxiliary verb semantics to date. Chapter 5 provides a background on theories of tense and aspect.

Chapter 6 introduces the grammar in which tense, aspect, and modal auxiliary verbs are to be modeled. Chapter 7 provides the grammar and analysis including an analysis of tense and aspect. Chapter 8 shows the analysis of modal auxiliary verbs in the proposed grammar. It discusses how modal auxiliaries interact with tense and aspect and what readings result. Chapter 9 discusses the analysis of the remaining modal auxiliaries and how they interact with tense and aspect. The concluding remarks are found in Chapter
1.5 Applications and Future Research

The results of this work have applications in a number of areas. In separate work, the grammatical features have been used to limit the number of readings available for a given use of a modal in automated tasks (Moon 2011[64]) and they have been the topic of experimental work regarding the inferences human subjects draw from various linguistic features of a sentence (Moon 2012[65]). The work is useful in treating modal readings as an automated word sense disambiguation task. Work in this area is ongoing (Moon et al. 2016[66]).

Future theoretical research plans involve using a larger text window for determining the reading of a modal auxiliary verb and incorporating contextual features of an extended text or discourse, as well as modeling the role that prosodic information plays in determining the reading of a modal auxiliary verb in spoken uses of modal auxiliary verbs. They also involve using distributional information to build a probabilistic model in which linguistic features that do not fully determine meaning are weighted in such a way as to predict the most likely reading of a modal auxiliary verb in a given use.
CHAPTER 2

BACKGROUND ON MODAL AUXILIARY VERBS

This chapter provides a background on taxonomies of modal auxiliary verbs. It sets the stage for formal theories of modal auxiliary verbs by explaining the variety of interpretations for which formal theories seek to account.

2.1 Taxonomic Categories of Modal Auxiliary Readings

The notion of modality covers various natural language expressions. The most common forms of modality are modal auxiliary verbs such as *could* and *would*; phrasal modals such as *ought to* and *have to*, sometimes called semi-modals; and modal adverbs such as *possibly* and *necessarily*. Some works include the verb *wish* in treatments of modals. Most theoretical and pedagogical texts on modality discuss adverbs, phrasal modals, and modal auxiliary verbs. The linguistic forms behave quite differently grammatically and functionally, and, since this monograph provides a comprehensive theory of grammar and use, it focuses on only one of the major types of modal expressions, modal auxiliary verbs.

There have been many taxonomies of modals proposed. Among them, certain distinctions are prevalent. This chapter first discusses the distinction between necessity and possibility modals. Modals have different degrees of strength. For instance, something that *must* be done is likely to have more severe or more directly enforced consequences if it is not done than something that *might* be done. Similarly, if a person *can* enter through the northeast door of a building, it is not the same scenario as when they *must* enter through the northeast door of the building. Each modal has a sort of strength such that, whatever the modal expresses, it expresses it more strongly if it is a necessity modal than if it is a possibility modal. The notions of necessity and possibility are also present in modal adverbs, most obviously in the use of the adverbs *necessarily* and *possibly*.

In addition to the force that a modal auxiliary verb has, there is a particular type of
meaning it has in relation to the context in which it is being considered. There are three major categories of modal meaning that dominate most taxonomies: Epistemic modals, priority modals, and dynamic modals.

Epistemic modals are modals that highlight the fact that the speaker does not have full knowledge of a situation. When someone says that Tennessee *might* be in the Southern part of the United States, they are using the modal auxiliary verb to convey that they do not know for certain where Tennessee is situated in terms of how the region of the Southern United States is defined.

Priority modals are modals that reflect priorities of a speaker or of a community. When someone says that children *should* be in bed by ten o’clock, they are expressing the way in which they or some community prefers for things to be. Priority modals often do not reflect the way things are, but rather the way some person or group of people would like for things to be.

The set of modal readings considered under the label of dynamic modals are the most heterogenous. Among dynamic uses are ability uses of the modal auxiliary verb *can*. When someone says that Jess *can* swim five miles, they often mean that Jess is able to swim ten miles. Another dynamic use is often called *quantificational*. In quantificational readings, a trait of the subject is described, as in the sentence *A lion can bite.*. It is clear that dynamic modals are different from epistemic and priority modals: They do not express either lack of knowledge or how someone wishes the world to be. What is not clear in most taxonomies is what traits dynamic modals have in common with each other.

The three dominant meaning categories of epistemic, priority, and dynamic provide more fine-grained categories than taxonomies that separate modals into binary categories of epistemic or root modals. Different types of modals in the category of root (i.e., non-epistemic) modals have been argued to have very little in common beyond not being epistemic. The use of the category of dynamic modals at least creates a smaller class of heterogenous forms than the category called *root*.

The root and epistemic distinction is still prominent in literature on modality as a way to separate epistemic readings from other readings, even when authors are aware that the set of root modals is, in itself, quite heterogenous. The distinction between epistemic and what are sometimes called *ontic* or *metaphysical* readings has been discussed in Schulz 2007[87] as evidence motivating the use of different models for modal auxiliary interpretation based on the reading. The distinction is used in Condoravdi 2002[19] to
characterize difference in the interpretation of temporal and aspectual expressions based on the reading of the modal auxiliary verb.

The distinction between epistemic modals and other types of modals has also been useful as a simplifying generalization in work on determining modal readings such as in Hacquard & Wellwood 2012[35] and Rubinstein et al. 2013[84]. Coates 1983[16] draws many taxonomic distinctions among modal auxiliary readings, but still finds the distinction among epistemic and root modals to be a useful in discussing some of the generalizations.

Portner 2009[80] supports the three-way distinction and notes that some authors keep the root distinction and then sub-divide it into priority (including deontic) and dynamic. In this taxonomic presentation, the three-way distinction is used because, later in this work, it is useful to talk about epistemic and priority modals as having similar properties.

Among modal auxiliary verbs, there are types of uses that are often absent from taxonomies or mentioned only peripherally. These uses are generally considered to be primarily temporal. One example is the use of *would* to talk about states of affairs that took place repeatedly in the past, used in sentences like, ‘When I was a child, I would walk five miles to school every day’. Uses of *will* are sometimes treated as future markers and not at modal auxiliaries at all. In this work, each of these uses are treated as a type of modal and they are not excluded from the taxonomy or discussion. Because tense and aspect end up being shown to be an integral part of modal auxiliary meaning for all readings, modals that are recognizably temporal in their meaning are not seen as being a different type of expression.

2.2 Necessity and Possibility

Many taxonomies divide modal verbs into the categories of possibility modals and necessity modals (e.g., Lyons[55], van der Auwera and Plungian 1998[97], Portner 2009[80]). Necessity is often associated with modal verbs such as *would* or *must* as well as modal expressions such as adverbs like *necessarily*. Possibility is often associated with modal verbs such as *could* or *might* and adverbs like *possibly*.

When someone wants to express that something is necessarily true, for instance, a law of nature in the world, one common way to do it is with modal verbs (Quotations
Add water into a mantle peridotite and it will melt at a lower temperature because the bonds in the minerals that make up the rock will be disrupted by the water molecule.

This time we have to use another method to melt that peridotite – we need to decompress it at constant temperature.

The author uses the modal auxiliary verb will as well as the semi-modal need to to express how the physical properties of rock behave in the world.

The author also explains what is possible in the natural world using weaker modal auxiliaries such as can and should:

Under normal conditions, mantle rock like peridotite shouldn’t melt in the Earth’s upper mantle – it is just too cool. However, by adding water you can lower the melting point of the rock. Alternatively, by decompressing the rock, you can bring it to a pressure where the melting point is lower.

The idea of using necessity and possibility in modal taxonomies has been related to the use of the terms in logic. In modal logic, as used by linguists, necessity and possibility have particular semantic interpretations, but the notions themselves stem from older work. Lyon’s (1977:791-792[55]) discusses the origin of the notions in modal logic having to do with aethically necessary propositions, propositions that are necessarily true, and aethically possible propositions, propositions that are not necessarily false. Lyons does not apply the notion of possibility and necessity to all modal types, only to alethic modalities, but notes that alethic modality had been, until the 20th century, the dominant type of modality discussed.

The contrast, however, between modals like can, which are often associated with possibility, and will, which are often associated with necessity, in some readings, has little to do with alethic necessity and possibility. For instance, sentence (7) can describe a situation in which the water does not have the same temperature at all moments:

7. The water can be hot.
Sentence (7) can be used in a situation where the water is sometimes hot and sometimes not hot. In contrast, sentence (8) can be used to describe a situation in which the water is always hot:

8. The water will be hot.

On one possible reading, it is not merely a contrast between the absolute truth of the water being hot versus the absolute truth of the water not necessarily being cold that is conveyed. Rather, it is the internal structure of the water’s temperature state over time. In uttering sentence (7), the speaker is claiming that the temperature of the water fluctuates whereas, in uttering (8), the speaker is claiming that it is consistently hot.

The notion of something like logical necessity and possibility being expressed by the modals could, however, be one way to describe the contrast between sentence (7) and (8), as is perhaps clearer in sentences (9) and (10) below:

9. It is possible that the water is hot.

10. It is necessary that the water is hot.

The logical possibility and necessity readings are compatible with the reading expressing the speaker’s awareness of a fluctuating versus a continuous state of the water’s heat, respectively.

Considering other possible nuances of meaning, it is possible that the distinction between sentences (7) and (8) reflects speaker certainty of the state of the water rather than making a statement about the fluctuating versus constant temperature of the water in the world. Sentence (7) could be expressing that the speaker thinks the water is hot, but is uncertain, and sentence (8) could be expressing that the speaker is certain the water is hot. On such a readings, there is no implication that the water has a more constant state of being hot in one case or the other. For instance, if it is a fact that the water is always hot in the actual world, sentence (8) is compatible with a situation in which the speaker is aware of the constant heat of the water. In contrast, sentence (7) is compatible with a situation in which the speaker does not know that the water is constantly hot, but believes that it is not necessarily going to be temperate.

Whether more or less certainty is expressed has something to do with how the world is as well as what knowledge the speaker has of the world. For instance, standing next to bubbling water in a natural hot spring, a parent might tell their child that the water will be hot. The adult knows from her knowledge of hot springs and the appearance of
the water (maybe, for instance, steam is coming off of the water) that the water is very likely to be hot. An example of a less certain case would be water heated in a home that they are visiting for the first time. It is unknown how high the thermostat is set on the water heater, but, having been burnt before by water from faucets, the parent issues a warning to test the water first. The compatible scenarios differ based on the reading that a sentence receives.

It is not possible to know from an isolated sample sentence into which of many possible scenarios the author intends it to fit. There are many scenarios in the world that could be described by a single sentence. While the author of a sentence could have in mind a particular scenario, it is not guaranteed that the same scenario will be understood by all listeners, nor is the same interpretation necessary in terms of which actions a listeners choose to take. If someone warns that the water can be hot, many listeners will take caution without further clarification. In the case described above, the child might ask further, ‘Do you know that?’ which would lead to a distinction between the reading with speaker certainty of a fluctuating temperature versus speaker uncertainty about an unknown temperature.

Attempts to model modality have tried to separate components of meaning that are present no matter in which scenario they are used from components of meaning that vary depending on the context in which the sentence is used. The consistent part of meaning has been modeled by notions like logical necessity and possibility and the part of meaning that fluctuates with the context has been modeled by restrictions to particular scenarios. The reading on which the water has a fluctuating temperature is a type of dynamic modality, which expresses the possibility of the water being hot (that is, the water is not necessarily temperate).

The reading on which the speaker is uncertain about the temperature of the water is a type of epistemic modality that is represented as logical possibility, restricted to what is known (in this case, what is known to the author). The author does not necessarily know that the water is always temperate.²

While the typical model theoretic formal semantic approach forces a categorical divide between the dynamic reading about a state of water in the world and the epistemic reading about a speaker’s knowledge of that state of the world, some cognitive and func-

²Professor Tania Ionin brings up that the reading of either limited knowledge or knowledge of a fluctuating state seems available for may as well as can. My intuitions agree with that judgment. Although may is not always listed in examples of dynamic (quantificational) modal auxiliaries, placing it there is compatible with the paradigms proposed in this paper.
tional work, on the other hand, would go so far as to deny that there is any significant
distinction at all between the two readings.

Works such as Nuyts 2005[71] have remarked on the inadequacy of possibility and
necessity as labels for modal uses, including those describing a participant’s abilities.
Sentence (11), with the possibility modal could, can indicate that the speaker’s grandfa-
ther did cultivate a hectare of maize for $50:

11. In the 1930s, my grandfather could cultivate a hectare of maize for $50.

Sentence (12), with the necessity modal would can mean the same:

12. In the 1930s, my grandfather would cultivate a hectare of maize for $50.

There is, however, a distinction in that the actuality inference can be denied in (11)
but not in (12):

13. In the 1930s, my grandfather could cultivate a hectare of maize for $50, but he
never did.

14. In the 1930s, my grandfather would cultivate a hectare of maize for $50, *but he
never did. 3

In order to express a use of would with a denial of the actuality of the event, the past
perfect is needed, as in sentence (15) below:

15. In the 1930s, my grandfather would have cultivated a hectare of maize for $50, but
he never did.

The use of perfect is commonly associated with counterfactual uses of modal verbs
and signifies a change in use rather than a paraphrase of sentence (12). So, considering
examples (11)-(14), with parallel morphology, there is a question concerning why the
actuality of the statement with could can be denied, but the actuality of the statement
with would cannot. The notion that the modal verbs would and could convey neces-
sity and possibility, respectively, provides one means of characterizing the meaning of
potential ability, in one case, and habitual behavior, in the other.

There is an additional reading on which both sentences describe scenarios in which
the author’s grandfather did in fact cultivate maize as described. What varies, on this

3In a narrative describing the future from a past perspective, sentences with would can express a past
event that did not happen, as in the sentence, ‘After school Jess would arrange to meet his brother’s
murderer. But, as it turned out, he never did’.
reading, is the consistency of repetition of cultivating a hectare of maize for $50. Sentence (12) conveys that the scenario happened more regularly than in sentence (11). On these readings, the use of necessity and possibility in an interpretation represent the consistency of the repetition of an event over a time frame. Consider the restriction to be each spring represented in the time frame as well as each hectare. With the necessity sentence, each hectare each spring is cultivated for $50. With the possibility sentence, it is sufficient if it is not necessarily the case that he failed to cultivate a hectare of maize for $50 each spring. Of course, many uses of necessity modals allow exceptions. It seems that sentence (12) would be totally felicitous in a scenario in which the author’s grandfather cultivated most hectares for $50 most springs over the time span. Also, with sentence (11), it would be felicitous to use if the author’s grandfather only once cultivated a hectare of maize for $50, but it is probably more common in a scenario where it happened more than once.

Total overlap of meaning with respect to both the actuality of the event as well as the number of repetitions between sentences (11) and (12), however, is possible. For instance, sentence (11) does not exclude a reading in which the speaker’s grandfather did indeed cultivate a hectare of maize for $50 in every single relevant instance in the past time span to which the speaker is referring. On this reading, sentences (11) and (12) show a meaning overlap, except that the use of could puts some slight focus on the participant’s abilities. For example, only sentence (11) can be paraphrased with a form of be able to as in (16):

16. In the 1930s, my grandfather was able to cultivate a hectare of maize for $50.

The fact that sentence (11) can be paraphrased thus illustrates the lexical effects of the modal verb could, which go beyond temporal repetition to convey additional information about the participant’s abilities.

These data have illustrated that the notions of necessity and possibility are associated with certain sets of modal expressions and that the association has an intuitive basis. They have also illustrated that the categories of necessity and possibility are associated with a variety of modal auxiliary readings. The class of modal verbs thought of as necessity modals differ from the class of modal verbs thought of as possibility modals in many other ways. For example, the former can convey more speaker certainty about events in the world being referenced, the temporal repetitions in the time span that the speaker references can be less consistent with possibility modals than with necessity
modals, and some minimal pairs allow denial of actuality inferences in the case of possibility modals but not in the case of necessity modals. Furthermore, aspects of modal meaning, such as the focus on a participant’s abilities in some uses of could, cannot be insightfully explained in terms of notions of possibility and necessity.

In summary, an interpretation of these data in terms of logical possibility and necessity is compatible with certain types of meaning, but a characterization of modality as being equivalent to logical necessity and possibility ranging over all possible worlds is too strict, in Lewis’ 1973 terminology, to characterize actual uses of modal auxiliaries. In order to capture the readings that exist, the worlds that the modal logic operators range over need to be meaningfully restricted. Several theories have been proposed regarding how to capture the similarity that necessity and possibility modals have to logical necessity and possibility without requiring total equivalence of the two notions.

Both necessity and possibility modals are typically divided into several categories that characterize their meaning. Following aspects of the taxonomies in Nuyts 2005, Portner 2009, and Lassiter 2011, for example, the three major categories are epistemic modals, priority modals, and dynamic modals. In each category, there are modals that express a particular degree of modal force (Kratzer 1981). For example, as mentioned in the introduction, there is more modal force in the claim that Tennessee must be in the Southern part of the United States than there is in the claim that Tennessee might be in the Southern part of the United States. While both statements express a lack of speaker certainty, the statement with must expresses more certainty than the statement with might. The expression of uncertainty is common among modals with epistemic meaning. The degree of uncertainty is characterized in terms of a different parameter, the parameter known as modal force. The modal force is similar to necessity and possibility, but it is intended to allow something more like degrees of certainty rather than a polar distinction between absolute logical necessity and logical possibility.

With another type of modals, priority modals, rules or expectations can be stated. As mentioned in the introduction, it is more serious to be told that you must do something than that you should do something. Both statements would inform you of an obligation or expectation, but the modal force of a statement with must is generally stronger than the modal force of a statement with should.

Treating necessity and possibility as a separate parameter that interacts with the modal meaning allows the relative strength of a modal use to be explained relative to multiple
modal types. The strength of a modal is often described in connection with the notion of \textit{gradability}. Gradability refers to the way in which modals can be used to express stronger or weaker propositions. Gradability is especially noticeable in comparisons, such as when someone says that one thing is more necessary or more likely than another as in sentences (17) and (18) below:

17. \textit{It is more necessary to have an umbrella than a rain-hat.}

18. \textit{It is more likely that it will rain than that it will snow.}

Comparative gradability is more easily expressed in a single sentence by modal adverbs, as above, than it is by modal auxiliary verbs. But is it possible to compare two sentences with modal auxiliary verbs. For instance, when thinking about what to do next, a person might have the two thoughts:

19. \textit{I should brush my teeth, but I have to leave right away in order to catch my bus.}

In their mind, the person considers that, while brushing their teeth is important, it is more important to catch the bus. Of two desirable states of affairs, one is more desirable than the other. Such comparisons can be expressed through grades of modality. Feeling that one \textit{should} do something is weaker than feeling like one \textit{has to} to do something. Intuitively, if the two priorities are in competition, the stronger one will be chosen.

2.3 Epistemic Modals

Epistemic modals are a type of modal use that has been recognized in nearly every modal taxonomy (e.g., Lyons 1977[55], Coates 1983[16], Palmer 1986[73], Portner 2009[80]). Uses of modals such as (20) below are epistemic:

20. \textit{You must have gotten your hair cut this morning.}

The prominent reading of the sentence is one in which the speaker is saying that, based on what she observes, the subject has gotten her hair cut. Various modals have epistemic readings and are thought of as encoding degrees of certainty. The modal verb \textit{could}, for instance, can convey weak epistemic meaning. Sentence (21) has a reading in which the speaker is weakly claiming, based on epistemic knowledge, who she suspects of being last to use the toilet paper:
21. *Jess could have been the last one to use the toilet paper.*

Epistemic uses of modals are uses in which the information in the scope of the modal is supported by the facts of which an individual, often the speaker, has knowledge. What is meant by *the information in the scope of the modal* is best illustrated by example. In sentence (21) above, the information in the scope of the modal *could* is the information that can be loosely paraphrased as ‘*Jess was the last one to use the toilet paper*’. The presence of the modal auxiliary indicates that the speaker is not certain, but only surmises that there is some likelihood that Jess was the last person to use the toilet paper.

The same sentence with *could* replaced by *must* displays stronger certainty, or could be said to have more modal force. The use of *may* or *might* would convey less certainty. Because epistemic modal uses express degrees of certainty based on facts, many theories have considered them to have probabilistic meaning, or to be best characterized in terms of their relative position on a scale (e.g., Lyons 1977[55], Nuyts 2000[70], Lassiter 2011[53]).

Theoretically, epistemic modals have, in the bulk of the cognitive and functional literature, as well as traditional taxonomies (e.g., Lyons 1977[55]) been characterized in terms of probabilities or degrees of likelihood based on the speaker’s knowledge. In the formal semantic tradition, which will be described in more detail below, epistemic modals are treated as quantifiers over possible states of affairs. The role of speaker knowledge is represented as a limitation on which states of affairs are mentally available to the speaker, given his or her state of knowledge at the time of utterance (or possibly time of reference) (e.g., Kratzer 1981[46]). Alternatively, epistemic knowledge is represented in other formal semantic theories by partial information (e.g., Veltman 2005[99]).

In Section 2.2, necessity and possibility were discussed. Epistemic modality illustrates one non-binary way in which necessity modals and possibility modal are used. No matter what the modal force of the modal auxiliary that is being used epistemically, the meaning is that the claim is supported, to a certain degree, by the speaker’s knowledge. With the strongest epistemic use, expressed with the necessity modal auxiliary *must*, it is not the absolute necessity of the information in the scope of the modal that is being expressed:

22. *Jess must have been the last one to use the toilet paper*

Rather, sentence (22) expresses that the speaker has narrowed the possibilities of who
used the toilet paper last down and feels certain, based on the evidence, that it was Jess. Different people have different beliefs about the truth of epistemic modal auxiliaries depending on what is known for them. For example, if I am watching the scenario of the empty toilet paper roll unfold, and I hear Pat say that Jess must have been the last person to use the toilet paper, I may be snickering knowing that, unknown to Pat, I left the empty toilet paper role. Sentence (22) is not assessed the same from my perspective as it is from Pat’s, because I have knowledge that she is not privy to. If I start to laugh and tell Pat that it was me, it is unlikely that Pat will continue to defend the truth of sentence (22). So whether or not a statement with an epistemic modal auxiliary is true in the estimations of a particular individual is dependent on their knowledge state at the time at which they assess the sentence as true or false.

Epistemic modals have many interesting properties and have been researched extensively. One point that has been the subject of recent debate is the fact that epistemic modals seem to be assessed differently depending on the time at which they are assessed and the knowledge of the person who is assessing them (MacFarlane 2011[56]).

### 2.4 Non-Epistemic Modals

Section 2.5 discusses priority modals. Traditionally, both priority and dynamic modals have been considered to be root modals. The term *root modals* is often used synonymously with the term *non-epistemic modals*. Certain proposed taxonomies, however, such as Nuyts’ 2005[71], Hacquard’s 2006[34], and Portner’s 2009[80] distinguish among major categories of non-epistemic modals. Portner 2009[80] and Nuyts 2000[70], 2005[71] claim that modal uses traditionally grouped under the category of root modals have little in common besides being non-epistemic and, therefore, they do not use the category of root modals in their taxonomies.

Hacquard 2006[34] refers to *true deontic* modal auxiliaries. They are the type of performative deontic use that places an obligation on an addressee. She does not consider them to be a sub-type of root modal, but says that they form a different category. Nuyts 2005[71] also distinguishes among deontic uses in terms of whether they place an obligation on the addressee or describe the first participant in relation to a set of rules or expectations.

In all of the taxonomies above, modal uses such as those included under the labels
epistemic, priority, and deontic have been considered to exhaust the modal readings that occur. Necessity and possibility are, as described above, a parameter that represents the degree of strength a particular modal has, for example, the strength of certainty with epistemic modals or the strength of obligation with certain priority modals.

Uses that are often considered to be temporal are excluded in modal taxonomies, but, in this monograph, they are considered to be modal auxiliary readings. Many readings discussed in the next section on priority modals are shown to have aspectual properties that make them more similar to the readings that are usually considered to be temporal than most works observe. Nuysts 2005[71] is an exception in that he considers dynamic modals to be largely aspectual, however, unlike this work, he does not explicitly include temporal readings in his discussion of modality.

2.5 Priority Modals

Priority modals is a term used in Portner’s 2009:140[80] taxonomy to describe deontic, bouletic, and teleological modalities.

Deontic modality has to do with the obligations, commands, or expectations of an individual or a community. Deontic modality is most stereotypically represented by phrases like you should or you ought to as in sentence (23):

23. You ought to wash your hands of those chemicals before touching the food.

According to the expectations of the speaker, it is preferred that the addressee wash her hands. In this case, it might be in the best interest of the addressee as well.

Bouletic modality has to do with the internal desires of an individual or group.

Teleological modals are often called goal oriented modals. They tell the means by which something is accomplished as in sentence (24):

24. In order to get to the bathroom, you have to walk through the kitchen.

Sentence (24) describes a path from one place to another, which is the direction a goal must take. Teleological modals have been shown to have interesting properties. They are more about the means of reaching a desired state than an ordering of desired states.

Each of these modal types is considered to be a type of non-epistemic modal in binary classifications. The three readings, however, are not necessarily grouped together under a single sub-category, such as ‘Priority Models’ category, in all taxonomies.
The general binding thread among priority modals is that they involve a sort of ranking among what is preferred.

2.5.1 Deontic Modals

A deontic modal is a priority modal that expresses an obligation or command, as in the reading of sentence (25) below in which the doctor is commanding Jess to jog a lot.

25. You must jog a lot.

Hacquard 2006[34], and others (e.g., Lyons 1977[55], Feldman 1986[27], Ninan 2005[69], Nuyts 2005[71]), have proposed an additional distinction among deontic modals: Those that state the comparative desirability of a state of affairs (in terms of the rules, laws, or obligations of an individual or community), and those that, in saying these more desirable states of affairs, also perform a speech act placing an obligation on some participant to cause the more desirable state of affairs to come about or giving some participant permission to make them come about.

Sentences such as (25) are considered to be addressee-oriented in that they place an obligation on the addressee. Other uses are considered to be participant-oriented in that they refer to the obligations of the first participant (usually the grammatical subject) of the sentence. The distinction between these two readings is brought out in contexts in which there is an overt clause stating the purpose of the clause containing the modal expression as in sentence (26) below (similar data is also discussed in von Fintel & Iatridou 2007[103]):

26. In order to lower her blood pressure, Jess should jog a lot.

Although it is subtle, there is a difference in that sentence (25) constitutes a performative act of the speaker obligating Jess to jog a lot whereas sentence (26) does not. Rather, sentence (26) describes a goal of lowering blood pressure as well as the means by which the goal can be achieved.

In English, this distinction often goes unnoticed, and would be hard to characterize in terms of grammatical features. In fact, as Hacquard 2006:41[34] points out, about any performative modal use can be made into a goal-oriented, non-performative use by adding a purpose clause to the surrounding discourse.

These data are important for Hacquard’s 2006[34] investigation, however, because, in French, as well as other languages, the two types of deontic readings differ with
respect to actuality entailments. A deontic statement that carries the expectation that
the addressee perform the act cannot occur with perfect morphology on the modal verb.
In contrast, a deontic statement about how the world ought to be, that ‘describes an
obligation on the subject’ (Hacquard 2006[34]), can occur with the perfect.

The relevant data from Hacquard 2006 (repeated below) presents a scenario in which
there is a child, Kitty, and her mother tells the babysitter, who is the addressee, that
Kitty must do her homework. She shows that, upon coming home, the mother can use
the imperfect form of the modal but not the perfect form of the modal, to congratulate
the babysitter on fulfilling her obligation. Cases in which Kitty did not do her homework
and the congratulations are sarcastic are included to show that the data is not sensitive
to whether the event occurred or not.

27. Kitty devait faire ses devoirs, et elle les a fait. Bravo! (data from Hacquard
(2006:42[34]))
   Kitty must-imp do her homework, and she did it. Congratulations!

   Kitty must-imp do her homework, but she didn’t. Congratulations!

29. ??Kitty a dû faire ses devoirs, et elle les a fait. Bravo!
   Kitty must-perf do her homework, and she did. Congratulations!

30. ??Kitty a dû faire ses devoirs, et mais elle ne les a pas fait. Bravo!
   Kitty must-perf do her homework, but she didn’t. Congratulations!

Hacquard (2006:43[34]) concludes that the perfect form of the modal is not possible
when an obligation is placed on the addressee, because it conveys that the doing or not
doing homework event has already passed and “putting an obligation on an addressee
after the fact is simply pointless.”(Hacquard 2006:43[34]). Hacquard (2006:42[34])
shows that similar data hold for the French verb phrase that she translates as be supposed to. Apparently, then, the readings in sentences (27) and (28) are somehow stating
a purpose rather than a command, which manages to be conveyed in spite of the con-
gratulatory remark.

An example that Hacquard (2006:67[34]) gives of the perfect being used with a de-
ontic reading that is non-performative is given below, for contrast:

---

4The author does not align these French and English data in the original document.
Jane had to take the train to go to Paris.

Other authors, such as Nuyts (2005:9[71]), propose that deontic uses that are not performatives express a scale of desirability or moral necessity. The uses that are performatives ‘involve an intention to instigate or to (not) hinder another person’s actions or positions . . . pertaining to the [(un)desirable] state of affairs, in view of the judgment of its degree of acceptability’ (Nuyts 2005:9[71]).

What both analyses have in common is that they recognize the fact that uses that place some obligation on the first participant (or addressee) differ from those that do not. As Ninan 2005[69] points out, however, when the illocutionary act of placing of obligation is performed, the speaker is in a state of non-factuality with respect to the act he or she wishes to see performed. That is to say, performatives or true deontics are infelicitous if the person performing the illocutionary act either knows for certain that the act has been performed already or knows for certain that the act will not be performed. For example, if a speaker says sentence (32) to Jess’s care-giver, it is infelicitous if he or she knows Jess has already completed the homework packet, as in (33).

Jess must complete his homework packet before ten o’clock.

Jess must complete his homework packet before ten o’clock, and I know that he already has.

It is also infelicitous if the speaker knows that Jess will not complete his homework packet before ten o’clock:

Jess must complete his homework packet before ten o’clock, but I know he won’t.

Especially when considering effects of discourse or context, the distinctions among types of deontic modal verb readings are relevant in English, since there is, to some speakers, no infelicity in the non-performative versions of sentences (33) and (34):

Jess must complete his homework packet before ten o’clock in order to get a prize, and I know that he has already completed it (so here is is his prize).

Jess must complete his homework packet before ten o’clock in order to get a prize, but I know that he won’t complete it.

The author’s intuition is that sentence (35) is a bit infelicitous and would be more likely to be stated with the perfect form must have. Sentences (35) and (36) are considered to be very close in meaning to teleological modals.
2.5.2 Teleological Modals

The previous section discussed that some deontic uses of modals are very similar to goal-oriented or teleological modal uses. Goal-oriented modals, as in von Fintel and Iatridou 2007[103], are also called *teleological modals*. A teleological modal expresses goals as in sentence (37) below and in sentences (35) and (36) above.

37. *Pat called so that you would know he was coming late.*

While teleological uses of modals differ from deontic uses in that they cannot express obligations on others, they are similar in that they involve preferences. Teleological uses, however, are oriented to the preferences of the first participant, also called subject-oriented. In the previous sub-section, it is possible to notice this change of perspective based on the use. For example, in sentences (33) and (34), the obligation for Jess to finish his homework can be placed on either Jess himself or on the addressee (e.g., his care-giver). The same does not hold for sentences (35) and (36): The goal is being set for the first participant, Jess. There is no way to interpret the prize as being rewarded to the addressee. The addressee might come to the conclusion that there is some reward for him or her, if Jess reaches the goal set for him, but the goal stated in the sentence can only hold of the first participant.

In Hacquard’s 2006[34] taxonomy, there is little distinction between teleological modal verbs and non-performative deontic verbs. While her decision is based on evidence of the interactions of both uses with morphological aspectual marking on the modal verb, her intuitions hold in other languages as well.

Teleological modals are often in used in multi-clausal contexts in which there is some expression like *in order to*:

38. *In order to be admitted to the program, you must have taken all of the prerequisite courses.*

At this point, it is an open question whether or not teleological modals are different from deontic modals or merely a special environment in which deontic modals occur.

2.5.3 Bouletic Modals

A *bouletic modal* expresses the desires of some individual or community as in sentences (39) and (40) below:
39. *I must have another one of Pat’s cookies (because they taste so good).*

40. *The residents will use the ramp rather than the steps.*

What bouletic readings have in common with deontic and teleological modal readings is that they express the preferences of participants. In some ways, bouletic modal uses are also similar to volitional modals in that they refer to an internal need or desire of the participant. Palmer (1986:115)[73] discusses the notion of boulomaic modality only briefly and notes that it might not form a strict category of modality.

Nuyts (2005:26-27[71]), however, gives the notion of ‘boulomaic attitude’ full status in his categorization of modal expressions. He posits that boulomaic attitudes are related to volition and that, although they do not have a concise lexical category with which they can be associated, they constitute a category of speaker attitude towards a proposition that is distinct from deontic and other uses.

Part of the taxonomic complexities that all of the preceding authors acknowledge, is that, in practice, it is often hard to tell a deontic use from a boulomaic use of a modal. The complexity lies in the fact that boulomaic uses express the desires of a participant and deontic uses can also express the desires of a participant, but are not limited to such desires: Deontic uses can also describe the desires and preferences of larger groups, such as a community. Furthermore, the desires of the participant in deontic uses are associated with obligations and norms, in some ways, lending them more credence than individual desires, which are often exclusively subjective. Boulomaic modals are considered to be a subtype of deontic modal auxiliaries.

2.5.4 Summary

The use of the term *priority* to describe these modal verb uses reflects the fact that they describe an event that is more desirable or ideal, given how the world actuality is and how it could be. Theories of interpretation have used the idea of ordering possible states of affairs according to the preferences of the speaker or how things would be in an ideal world, for example. Certain states of affairs, intuitively, take priority over others in the context in which a sentence with a priority reading of a modal verb is uttered.

When discussing priorities, deontic uses of modals that place some obligation on a participant, as in (25), indicate the preference of the speaker with respect to certain planned or proposed events. Teleological readings of modal verbs, as in (37) express
the priorities or goals of the subject. Similarly, bouletic readings of modal verbs, such as (39) indicate which desires are a priority for the relevant participant.

<table>
<thead>
<tr>
<th>Priority</th>
<th>Deontic Law</th>
</tr>
</thead>
<tbody>
<tr>
<td>deontic law</td>
<td>The pawn can move forward.</td>
</tr>
<tr>
<td>obligation</td>
<td>Jess should think of Syd’s feelings.</td>
</tr>
<tr>
<td>permission</td>
<td>You can have a popsicle.</td>
</tr>
<tr>
<td>teleological</td>
<td>Jess bought a popsicle so that Syd could eat it.</td>
</tr>
<tr>
<td>bouletic</td>
<td>Syd has to eat a popsicle.</td>
</tr>
</tbody>
</table>

Table 2.1: Table of Priority Modal Auxiliary Verb Readings

As with epistemic modals, priority modals have been treated in the majority of the literature on language meaning in connection with a scale of how desirable or necessary a state of affairs is, given the norms, requirements, views, or laws of an individual, community, deity, or other component of social organization. As with epistemic modals, the scale is polar in that there are desirable and undesirable states of affairs about which interlocutors speak. In formal semantic approaches, priority modals are evaluated relative to hypothetical states of affairs in which the laws, views, or requirements are upheld (e.g., Kratzer 1981[46]). In some models, the notion of ordering priorities is given more prominence (e.g., Portner 2007[79]) and the requirements placed on the first participant are also modeled.

2.6 Dynamic Modals

Dynamic modals comprise the most elusive category of modal meaning. Very in-depth studies have been done on dynamic modal auxiliaries (e.g., Bhatt 1999[8], Hacquard 2006[34]), however, the studies focus on particular dynamic readings rather than on what defines the category as a whole.

Portner’s 2009[80] taxonomy splits dynamic modals into the subcategories of volitional modals and quantificational modals. Other authors, however, have not been satisfied with the term volitional since ability uses of modal verbs, which are often included, are intuitively different from expressing a participant’s will, as the term implies. In what follows, the set of modal uses called dynamic will be subdivided into ability modal uses, volitional uses, and quantificational uses.
2.6.1 Ability Modals

*Ability* modals, such as the use in sentence (41) are classified differently in various taxonomies.

41. *Mayra can see the mountains from her window.*

For classifications such as that in van der Auwera & Plungian (1998:35[97]), they are considered to exemplify participant-internal modality, since an ability is about what a certain participant is capable of doing. For Portner (2009:135-140[80]), ability modals are a type of circumstantial modal that are considered to belong to the *volitional* subcategory of dynamic modals along with *opportunity* and *dispositional* uses. The classification is based on the fact that circumstances ‘affect the actions available to a volitional individual’ (Portner 2009:135[80]).

In Nuyts’ 2005[71] rendering, ability modals are not restricted to readings that are participant-internal, but rather held to have participant-external variants, most clearly exemplified by inanimate first participants as in sentence (42) below.

42. *The political climate in the area could cause further unrest.*

Dynamic modals are agent-oriented (Nuyts 2000:25) in that they describe the abilities or volitions of the first participant of the sentence, as in the ability modal examples that have been discussed, such as sentence (11) repeated below:

43. *In the 1930s, my grandfather could cultivate a hectare of maize for $50.*

Nuyts 2005[71] also extends the category to examples that would be considered teleological in other taxonomies, such as in sentence (44) below:

44. *To get into the garden you must pass through the kitchen (there’s no other way).* (Nuyts 2005:8[71])

Nuyts 2005[71] claims that such examples express an ability even though it is limited by external circumstances. He also extends the notion to examples with inanimate objects such as sentence (45), due to their intuitive meaning and syntactic similarity to animate uses.

45. *It can snow in winter.*
Ability uses of modal verbs are important since they have been closely associated with actuality entailments in other languages (e.g., Bhatt 1999[8], Hacquard 2006[34]). In English, sentences such as (41) above also have an actuality inference. Given sentence (41), it is highly likely that it follows that Mayra did see the mountains. The most central meaning of ability modals are such examples expressing what the first participant of a sentence is capable of doing.

2.6.2 Volitional Modals

Although ability uses of modals have been considered to be volitional (e.g., Portner 2009[80]), the term is reserved here for other volitional uses of modal verbs. One example includes uses of will to express desire as in sentence (46) below:

46. I will be the first professional female football player!

Other volitional uses, describing traits of a person, as in sentence (47) below, might be called dispositional.

47. Sydny would wear those pants.

The line, however, between dispositional uses and the quantificational uses of the next section will be difficult to discern in many instances.

2.6.3 Quantificational Modals

Portner 2009[80] argues that quantificational modals are a category that needs to be recognized due to its exceptional properties. Quantificational modals are characterized by their similarity to existential and universal quantification. An example was used above, when comparing necessity and possibility, in sentence (7), repeated as (48) below:

48. The water can be hot.

The meaning of the sentence is similar to an existential quantifier over times such that the water is hot at some of those times. Correspondingly, the sentence stating that the water will be hot is like a universal quantifier over all times the water exists predicating that it is hot.

Another example is in sentence (49), for instance, which expresses that all SUVs get good gas mileage (ironically, only when they are not moving):
49. An SUV will get good gas mileage sitting in the driveway.

But when a modal associated with possibility, such as can, is used, there is a stronger actuality inference:

50. An SUV can get good gas mileage sitting in the driveway.

It seems that the SUV does get good mileage sitting in the driveway.

Quantificational modals have been recognized as quantifying over individuals (Carlson 1977, Heim 1982, Brennan 1993, cited in Portner 2009:136). Sentences (49) and (50) quantify over the set of SUVs. In the case of sentence (49), getting good gas mileage sitting in the driveway is claimed to be a property of the entire set of SUVs. The situation with sentence (50) is more complicated. It does not seem to say that the property of getting good gas mileage sitting in the driveway is a property of some SUVs, rather, that it is an occasional property of all SUVs. Portner (2009:136,213-214)[80] proposes that quantificational modals quantify over situations, rather than worlds or individuals.

Calling only some modals quantificational is a bit problematic since quantification is involved in several types of modal uses. Some uses seem to quantify over times, others contribute iterative or frequentative aspect (e.g., Nuyts (2005:20[71])), and, in some accounts, as mentioned, quantification is over individuals.

Summary

Nuyts 2005[71] considers dynamic modals to be considerably different than other modal uses. He claims that they primarily involve a type of aspectual quantification over events rather than the type of scale he proposes for other uses. Lassiter 2011[53], when discussing the future direction of presenting a unified approach to the interpretation of modal expressions, expresses skepticism that ability uses of modal verbs will ever be characterizable in terms of the scales he proposes. Together, ability, volitional, and dynamic modals form a category whose elements have little in common beyond not being priority or epistemic modals. The working general description is that they are first-participant-oriented.

Dynamic modals as well as bouletic modals are all participant-internal in van der Auwera & Plungian’s (1998:35) taxonomy, but bouletic modals are considered participant-internal necessity, and dynamic modals are participant-internal possibilities. Participant-
external modalities include deontic modals and certain root modals best associated with goal-oriented modals.5

Formal theories have interesting things to say, or avoid saying, as the case may be, about dynamic modals. In Kratzer’s 1981[46] account of modals, it is assumed that dynamic modals are treated the same as other uses, but ability uses, for instance, are only discussed in a general sense. The fact that actuality inferences in English and actuality entailments in other languages are stronger with ability modals led to interesting studies in the formal semantic tradition in which the seeming lack of need for possible worlds was discussed (Bhatt 1999[8]) or rescued (Hacquard 2006[34]). Lassiter 2011[53] states in his conclusion that he is uncertain whether his analysis could extend insightfully to dynamic modals. Nuyts 2005[71] further acknowledges their difference from other modal types claiming that they are rather quantificational aspect markers. Intuitively, ability uses, for instance, do not mark the degree of ability one has, but rather the time frame over which the ability held as well as the nature of the repetitions.

<table>
<thead>
<tr>
<th>Dynamic</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ability</td>
<td>Jess <em>can</em> swim twenty miles.</td>
</tr>
<tr>
<td>volitional</td>
<td><em>I will</em> swim twenty miles in the competition today.</td>
</tr>
<tr>
<td>circumstantial</td>
<td><em>Winds can</em> reach up to ten miles per hour in this region.</td>
</tr>
<tr>
<td>quantificational</td>
<td><em>Domestic dogs can</em> still bite friendly humans.</td>
</tr>
</tbody>
</table>

Table 2.2: Table of Dynamic Modal Auxiliary Verb Readings

The more fine-grained the taxonomy becomes, the more difficult it is to form discrete categories on which the majority of authors agree. This is not a shortcoming, but rather the nature of taxonomic work: It is important to find a balance between being fine-grained enough that useful generalizations are captured and not being so fine-grained that no generalizations can be stated. The categories of epistemic, priority, and dynamic modals have been a helpful way of grouping modal uses according to their special traits.

5Hacquard’s 2006[34] treatment of modality also uses a notion similar to participant-internal versus participant-external modals, with deontic crossing the two categories: Performative deontics being participant-internal and other deontics being participant-external. This distinction is treated on par with de dicto and de re meaning.
2.7 Temporal Uses

As previously mentioned, the focus in synchronic classification is often on the variety of meanings that certain modal forms can have. In spite of the focus on forms, however, some uses of modal auxiliaries are not given adequate attention and considered to be only temporal markers. For instance, there is a use of the English modal would that seems to be merely temporal as in sentences (51)-(52) below:

51. When Sydny was a kid, he would walk to school every day.

52. During classes, Jess met the professor who would later become his thesis adviser.

In sentence (51), the use of would is one with the meaning of a habitual past action. Sentence (52) is a future in the past use. In both cases, there is a non-defeasible inference that the event actually occurred.

53. #When Sydny was a kid, he would walk to school every day, but Sydny did not walk to school as a kid.

54. #During classes, Jess met the professor who would later become his thesis adviser, but she didn’t become Jess’s thesis adviser.

Sentences (53) and (54) show that it is odd to continue with a contradiction of the inference that the event occurred in the past.

The dual use of the English modal would as a past marker and as a modal has been attested for centuries. Ziegeler 2000[104], for instance, shows that the distinction was present in Old English texts. Providing a uniform semantics for past uses of would and modal uses runs the risk of reducing their similarities to the point of positing homophonous ‘woulds’. Yet, as different as the two forms are in meaning, individual uses still permit multiple readings. For example, if the phrases When Sydny was a kid and every day were omitted from sentence (51), the clause, Sydny would walk to school licenses various readings.

Probably the most commonly debated modal verb is will, which some take to be only a temporal marker and not a modal (Portner 2009[80]); however, even synchronic corpus work (e.g., Romer 2004[83]) shows that both future and volitional uses still exist.

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6See also Comrie 1985[18] regarding English will.
2.8 Other Topics Related to Modal Auxiliary Taxonomies

2.8.1 Historical Development of Modal Auxiliaries

The three major categories of modal auxiliaries are epistemic, priority and dynamic, which were discussed above. They have an interesting historical relationship with each other. Historically, there was a shift in modal auxiliaries in English and many other Indo-European languages from non-epistemic meaning to epistemic meaning (e.g., van der Auwera & Plungian 1998[97]). The epistemic meaning became more speaker-centered (sometimes called *subjective*) over time (Traugott 1989[96]). Historically, priority modals, particularly deontic modals, often emerge as an intermediary stage when a verb transitioning from a main verb to an epistemic auxiliary modal (Traugott 1989[96]).

One way in which this historical trend has left a mark on contemporary English can be seen in the fact that dynamic and priority uses of modals often have a prominent lexical meaning, expressing abilities, desires, and goals. For example, *can* often refers to an ability and occurs in environments where it can be replaced with the appropriate form of the phrase *be able to*.

55. Jess can play the piano.

56. Jess is able to play the piano.

The modal *would* is often associated with volitions:

57. Jess would like a piece of cake.

58. Jess wants a piece of cake.

Epistemic meanings expressed by modals, in contrast, vary only in degree. The differences among the sentences below are largely related to speaker certainty:

59. Jess might be sick today.

60. Jess may be sick today.

61. Jess could be sick today.

62. Jess must be sick today.
Intuitions vary, but generally, *must* is considered to express stronger certainty than the other forms.

Another way in which epistemic modals differ from other modals is in the pervasiveness of their use. While one, two, or at most three lexical modals can express a particular dynamic meaning, epistemic meaning can be expressed by the majority of modal auxiliaries, as shown in sentences (59)-(62) above. Epistemic meaning is a more general category and the historical process could be described as bleaching (on the notion of bleaching see Hock & Joseph 2009[37]). With bleaching, a form loses part of its lexical meaning but retains other parts, often causing multiple forms to appear to be duplicating each other in the semantic space. The way in which many speakers of North American English do not distinguish *may* from *might* in epistemic readings is such an example.

### 2.8.2 Communicative Function of Modals

In pedagogical texts, modals are often divided into taxonomies according to their communicative function. One communicative function of modals is for making polite requests, such as in sentence (63) below:

**63.** *Would you like another glass of tea?*

The use of modals in polite requests is one of the first forms non-native speakers learn in acquisition and modals in polite requests appear, in my estimation, being a language learner, to play a much more prominent role in day to day, face to face, communication than other uses.\(^7\)

There is sometimes a connection between communicative functions of modals and the taxonomic categories of epistemic, priority, and dynamic modals to which they belong. The communicative function of stating an obligation, for instance, is often managed by deontic modals. Polite request uses of modals often use ability modals (e.g., *‘Can you (Are you able to) come over tomorrow?’*).

But the connection between meaning and communicative function is not exact. An ability modal can be used, for instance, to express a command as in sentence (64):

**64.** *Could you (are you able to) shut the window?*

---

\(^7\)Corpora do not support the prevalence of polite requests, but that is largely due to where the spoken and written corpora come from.
In formal semantics, the communicative function is treated as being separate from the meaning of the modal. Many of the examples above involve asking questions. Questions are presumed to have a different interpretation than declarative sentences. Part of the way in which modals are used is due to the semantics of interrogative sentences.

Formal semantic models of communicative interaction have lagged behind models of declarative sentences. One reason for the lag is that the complexity of representing interactions precisely appears to involve modeling speaker intentions. Another is because questions and commands, for instance, bring up complicated questions about meaning. Meaning in formal semantics is often associated with truth values. While a declarative sentence can be true or false compared to a model of the actual world, it is an open question what the truth value of a question or an command is, or if they even have truth values.

2.8.3 Modality and Subjective Speaker Attitudes

It has been argued by some authors (e.g., Bybee et al. 1994:176[11]) that modals encode the subjective attitudes of speakers. As has been shown in the previous sections, some modal uses express speaker attitudes, and other uses do not.

Priority modals can be thought of as expressing speaker attitude, in the broader sense of expressing a desire. It is hard to think of the degree of certainty someone has about the occurrence of a state of affairs as being an attitude, but epistemic modals are intended to be included.

There is a problem with claiming that modal auxiliary verbs express speaker sentiment because, for many uses, it is not the sentiment of the speaker, but the sentiment of the first participant that is being presented. In sentence (65), it is Jess’s inner compulsion that is encoded by the modal, not the speaker’s subjective attitude towards his compulsion.

65. Jess has to sneeze

Other times, modality can express both speaker sentiment and first participant sentiment. For example, sentence (66) can express the sentiment of both Pat and the speaker:

66. Pat might go to the cinema tonight

In the case in which the speaker is merely reporting Pat’s attitude, it is more of the first participant’s attitude than the speaker’s that is being related. Speaker sentiment
does not play an obligatory role in dynamic modals, nor does it play a role in temporal uses. It could be argued to play some role in epistemic and priority uses, but does not completely explain their meaning.

2.8.4 Conclusion

This chapter has provided a background on taxonomies of modal auxiliary verbs and discussed the various readings that modal auxiliaries can have.
This chapter reviews model theoretic formal semantics. The most common variety of modal logic used by linguists grows out of that presented by Kripke 1963[50]. This logic involves the use of possible worlds, and accessibility relations are used for restricting the worlds over which the modal operators range.

The particular type of Kripke-style logic most commonly used in formal semantics, as done by linguists, is Montague’s Intensional Logic, IL (Montague 1970[62], 1973[63]), a variety of which is presented below. IL is a typed higher-order logic. As such, it has greater expressiveness than first order logic, but lacks the soundness and completeness proofs.\footnote{Montague also presents a grammar of English, which is also included here. It is important to remember, however, that IL could be used in conjunction with other grammars of English or any target language.} Montague also presents a grammar of English, which is also included here.

3.1 Syntactic Categories of English

Montague represents the syntactic categories of English with a categorial grammar.

**Definition 1.** The set of categories of English syntax, Cat, is defined as follows:

(i) $e, t \in \text{Cat}$.

(ii) If $a \in \text{Cat}$ and $b \in \text{Cat}$, then $(a/b)$ and $(a//b) \in \text{Cat}$.

Where $e$ and $t$ are labels for distinct sets.

Definition 1 is a recursive definition. Categories are built up from atomic categories $e$ and $t$ by combining the licit categories recursively. Examples of possible categories

\footnote{For discussion of first-order re-formulations of natural language semantics, see Fox & Lappin 2005[30].}
<table>
<thead>
<tr>
<th>Syntactic Category</th>
<th>Abbreviation</th>
<th>Common Syntactic Name</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>((t/t/e))</td>
<td>T</td>
<td>Proper Nouns</td>
<td>John</td>
</tr>
<tr>
<td>((t/e))</td>
<td>IV</td>
<td>Intransitive Verbs</td>
<td>sleeps, runs</td>
</tr>
<tr>
<td>((t/e)/(t/(t/e)))</td>
<td>TV</td>
<td>Transitive Verbs</td>
<td>hits, kicks</td>
</tr>
<tr>
<td>((t//e))</td>
<td>CN</td>
<td>Common Nouns</td>
<td>bear</td>
</tr>
<tr>
<td>(t)</td>
<td>t</td>
<td>Sentences</td>
<td>the bear sleeps, John kicks the bear</td>
</tr>
</tbody>
</table>

Table 3.1: Chart of some syntactic categories from Montague’s Intensional Logic, their label in more standard syntactic systems, and a natural language example of each category.

in IL are \(e, t, ((e/t)//t), (e/t)(e/t), \) etc., where the parenthesis are used only to show the order in which categories are combined.

Some common categories, and the perhaps more familiar name of the kind of syntactic expression that they label, are listed in Table 3.1 below.

Each lexical expression of a given category \(\alpha\) in a fragment is considered a basic expression and is a member of the set \(BE_{\alpha}\) of all basic expressions of the fragment of English. The basic expressions of the fragment used for illustration in Table 3.1 are listed below:

\[
BE_{(t/(t/e))} = \{John\}
\]

\[
BE_{(t/e)} = \{sleeps, runs\}
\]

\[
BE_{(t/e)/(t/(t/e))} = \{hits, kicks\}
\]

\[
BE_{(t//e)} = \{bear\}
\]

\[
BE_A = \emptyset \text{ (the empty set) if } A \text{ is any category other than those mentioned in the previous definitions.}
\]

Note that no expressions of category \(e\) or expressions of category \(t\), are in the basic expressions in this fragment. Rather, sentences, which are labeled by the category label \(t\), are built up from basic expressions.

Montague 1973[63] introduces the set \(P_A\) for the phrases of category \(A\), which include and are built up from the basic expressions.

**S1.** \(BE_A \subseteq P_A\) for every category \(A\).
Syntactic rule S1 states that the basic expressions are a subset of the phrasal categories of the language.

Determiners, such as *the*, are introduced by functional rules.

**S2b** If $\zeta \in P_{(t/e)}$, then $F_1(\zeta) \in P_{(t/(t/e))}$, where $F_1(\zeta) = \zeta$.

Syntactic rule S2b states that if an expression is a common noun (category $(t/e)$), then there is a function, $F_1$ that applies to that expression and results in a form of the common noun with the definite article *the*.

$$F_1(\textit{bear}_{(t/e)}) = \textit{the bear}_{(t/(t/e))}$$

Montague proposes similar rules for other articles and quantifiers such as as S2c for *every*:

**S2c.** If $\zeta \in P_{(t/e)}$, then $F_2(\zeta) = \textit{every} \ \zeta$

Montague 1973[63] lists a number of rules of functional application, describing the way in which categories combine. Due to the small size of the fragment presented here, only the one general rule of functional application above is necessary. Rule S3′ states that phrasal categories combine with other phrasal categories in a principled way.

**S3′: Rule of Functional Application:** For all expressions $\alpha, \beta \in P_A$, if $\alpha \in P_{a/b}$ and $\beta \in P_b$, then $F_4(\alpha, \beta) \in P_a$ where $F_4(\alpha, \beta) = \alpha \beta$.

$$F_4(\textit{the bear}_{(t/(t/e))}, \textit{sleeps}_{(t/e)}) = \textit{the bear sleeps}_t$$

The alternative notation in parenthesis in Rule S3′ is to account for expressions formed with a double slash, such as common nouns of category $(t/e)$. The additional clause is only present in order to deal with the fact that Montague wanted to distinguish common nouns from intransitive verbs, since both would be category $(t/e)$ in a grammar with only the single slash. In S3′, the category of the item on the left is such that it requires an item of the category on the right to reduce categories.

---

2Montague’s 1973[63] additional rules dealt with pronouns, which the fragment here does not contain, and with matching the third person plural form of verbs. The verbs in this fragment are already in the third person, matching the available terms. If the terms *John* and *the bear* were conjoined in a subject noun phrase, then a rule changing the verb form (e.g., *sleeps to sleeps*) would need to be added.
Montague 1973[63] also gives syntactic rules for the conjunction and disjunction of sentences, verb phrases, and terms. The rules are given below.\(^3\)

**S11.** If \(\phi, \psi \in P_t\), then \(F_8(\phi, \psi), F_9(\phi, \psi) \in P_t\), where \(F_8(\phi, \psi) = \phi\) and \(\psi, F_9(\phi, \psi) = \phi\) or \(\psi\).

**S12.** If \(\gamma, \delta \in P_{(t/e)}\), then \(F_8(\gamma, \delta), F_9(\gamma, \delta) \in P_{(t/e)}\)

**S13.** If \(\alpha, \beta \in P_{(t/(t/e))}\), then \(F_9(\alpha, \beta) \in P_{(t/(t/e))}\).

Rule S11 states that, for two expressions labeled as category \(t\), there is a function, \(F_8\), which writes the expressions with the conjunction *and*, and a function \(F_9\) which writes the expression with the disjunction *or* and the output of the function is also of category \(t\).

Rules S12 and S13 state that, when the functions apply to intransitive verbs and terms, the resulting expression has the same category as the conjuncts. And example of the application of the functions to each category is shown below:

\[
F_8(\text{the bear sleeps}_t, \text{John kicks the bear}_t) = \text{the bear sleeps and John kicks the bear}_t
\]

\[
F_9(\text{sleeps}_{t/(t/e)}, \text{runs}_{t/(t/e)}) = \text{sleeps or runs}_{t/(t/e)}
\]

\[
F_9(\text{John}_{t/(t/(t/e))}, \text{the bear}_{t/(t/(t/e))}) = \text{John or the bear}
\]

Montague then presents rules of quantification (S14-S16 in Montague’s 1973[63] paper).

Rules of ‘tense and sign’ (S17) were also presented in Montague 1973[63]. These rules deal with morphological negation and tense by rule. For the small fragment presented here, these rules are not necessary.

Making trees with the category combinations is simple, given the application of rule S3’ in 3.1.

---

\(^3\)It must be noted that the rules of functional application do not account for conjoined subject terms in the tiny fragment above, since the verbs are all in the third person singular form and no rule changing the morphology is provided.
John sleeps

\[
\frac{\text{John} \quad \text{sleeps}}{t(\text{t(e)}) \quad (\text{t(e)})}
\]

(3.1)

Using a transitive verb is similar, by applying \(S3'\) to kicks and a bear as in equation 3.2, and to John and kicks a bear in equation 3.3. The node from bear to a bear is derived by function \(F_1\).

John kicks a bear

\[
\frac{\text{John} \quad \text{kicks a bear}}{t(\text{t(e)}) \quad (\text{t(e)})}
\]

\[
\frac{\text{kicks} \quad \text{a bear}}{t(\text{t(e)}) \quad (\text{t(e)})}
\]

\[
\frac{\text{bear}}{t(\text{t(e)}) \quad (\text{t(e)})}
\]

(3.2)

(3.3)

The syntactic categories are mapped to semantic types, which are then given an interpretation. In the next sections, it will be shown how an interpretation is assigned to the syntactic structures.

3.2 Syntax of IL

In Montague’s original 1973[63] paper, an artificial language was constructed and an interpretation was assigned to that language. Montague then showed how English was translated into the artificial language. In subsequent applications of Montague Grammar, the artificial language is frequently omitted and an interpretation is assigned directly to the grammar of English (or some other target language) for which the author is providing a semantic interpretation.

The syntax of IL is The set of types in IL is defined as follows.
**Definition 2.** *The set $Y$ of types is the smallest set such that:

(i) $e, t \in Y$.

(ii) If $a \in Y$ and $b \in Y$, then $<a, b> \in Y$.

(iii) If $a \in Y$, then $<s, a> \in Y$.

Montague defines the sets of variables as follows:

**Definition 3.** *(from Montague 1973:23)* If $n$ is any natural number and $a \in Y$, then $v_{n,a}$ is the $n^{th}$ variable of type $a$.

The set $\text{Con}_a$ is the set of constants of type $a$.

Similar to the set $P_a$ of basic expressions and phrases licit in the syntax of English, there are *meaningful expressions* in the syntax of IL.

**Definition 4.** *The set of meaningful expressions of IL is defined recursively as follows, where $V_a$ is the set of variables of type $a$, and $C_a$ is the set of constants of type $a$:

(i) Every variable and constant of type $a$ is in $\text{ME}_a$.

(ii) If $\alpha \in \text{ME}_a$ and $u$ is a variable of type $b$, then $\lambda u \alpha \in \text{ME}_{<b,a>}$. 

(iii) If $\alpha \in \text{ME}_{<a,b>}$ and $\beta \in \text{ME}_a$, then $\alpha(\beta) \in \text{ME}_b$.

(iv) If $\alpha, \beta \in \text{ME}_a$, then $\alpha = \beta \in \text{ME}_t$.

(v) If $\phi, \psi \in \text{ME}_t$ and $u$ is a variable, then $\neg \phi, [\phi \land \psi], [\phi \lor \psi], [\phi \rightarrow \psi], [\phi \leftrightarrow \psi], \forall u \psi, \exists u \psi, \Box \psi, \Diamond \psi \in \text{ME}_t$.

(vi) If $\alpha \in \text{ME}_a$, then $[^{\uparrow} \alpha] \in \text{ME}_{<s,a>}$. 

(vii) If $\alpha \in \text{ME}_{<s,a>}$, then $[^{\downarrow} \alpha] \in \text{ME}_a$.

(viii) Nothing else is in any set $\text{ME}_a$.

Rule (4i) states that the typed constants and variables of IL are well formed expressions. Rule (4ii) explains that the functional types result in well formed expressions of the rightmost type, after functional application.

Rule (4iii) states that the lambda operator $\lambda$ can be combined with any well formed expression resulting in another well formed expression. The typed lambda calculus is
an important part of Montague’s IL. Lambda extraction allows the formal language to characterize properties of entities, for example. Lambda reduction is licensed when lambda expressions are combined with a variable or constant of the appropriate type. For example, in an expression of the type $\lambda v_1.e[v_1, e, t](v_1, e)$, the lambda operator is applied to a variable $v_1$ of type $e$. $\lambda$-reduction can happen, in a syntactic derivation, for instance, when $\lambda v_1.e[v_1, e, t](v_1, e)$ combines with a constant of type $e$ such as $j$. By $\lambda$-reduction, $\lambda v_1.e[v_1, e, t](v_1, e)(j)$ substitutes a constant of the same type as the variable for the variable, resulting in the expression $v_1, e, t(j)$. The usefulness of lambda expressions is that they allow generalizations to be represented and allow substitution of any variable or constant with a matching type. For a full background on the lambda calculus that Montague used, see Church 1940[14],1941[15].

Rules (4iv & v) describe operators that lift between extensions and intensions. For example, if an expression $\alpha$ is of type $<e, t>$, then $^\wedge \alpha$ gives intension the intension of $\alpha$, which is of type $<s, <e, t>>$. Similarly, if there is an intensional expression $\beta$ of type $<s, <e, t>>$, then $^\gamma \beta$ gives the extension, or the corresponding expression of type $<e, t>$, restricted to a particular world of interpretation.

At the present point in the exposition, no interpretation of IL has been given. There is a syntax defined, and it has been shown how the typed basic expressions of a fragment of English combine to form phrases. The meaning of those phrases is intuitive, because they are words and phrases of English. If the basic expressions had been arbitrary sequences of symbols, it would only be described, so far, how, by various functions, those symbols concatenate, change form slightly, and how other symbols are added.

### 3.3 Semantics of IL

An interpretation is also known as a model. A model in IL is an intensional model.

**Definition 5. Interpretation:**

An interpretation $\mathcal{M}$ for the formal language IL is a 4-tuple $<U, W, T, F>$ where

(i) $U$ is a non-empty set (construed of as entities, sometimes called the universe of $\mathcal{M}$),

(ii) $W$ is a non-empty set (construed of as a set of possible worlds),

(iii) $T$ is a non-empty set (construed of as a set of temporal points), and
(iv) \( F \) is a function (construed as an interpretation function of \( M \)) assigning an interpretation to each constant in \( ME_a \).

Every expression in IL it interpreted relative to the parameters defined in the model. There are domains defined for expressions.

**Definition 6. Notation for Parameters:**
\( D_{a,U,W,T} \) is the set of possible denotations of type \( a \), given the parameters \( U, W, T \).

The domain of the two simple types \( e \) and \( t \) are as follows:

**Definition 7. Simple Type Domains:**

(i) \( D_{e,U,W,T} = U \)

(ii) \( D_{t,U,W,T} = \{0, 1\} \)

Definition 7i says that the domain of expressions of type \( e \), relative to the parameters \( U, W, T \), is the set \( U \) of entities. Definition 7ii states that the domain of expressions of type \( t \), relative to the parameters \( U, W, T \) is the set \( \{0, 1\} \). The type \( t \) is associated with propositions. As will be described in more detail below, propositions that map to 0 are false relative to the model and propositions that map to 1 are true relative to the model.

The types of IL, however, were recursively defined so, while Definition 6 covers a simple type, it does not tell what the domain of each complex types is. If \( X \) and \( Y \) are sets, then \( X^Y \) is the set of all functions that map from \( Y \) to \( X \). The superscripted set is the domain and the other set the co-domain. The Cartesian product of \( X \) and \( Y \), written \( X \times Y \) is the set of all ordered pairs \( <x, y> \) such that \( x \in X \) and \( y \in Y \).

**Definition 8. Functional Type Domains:**

For any type \( a \) in \( M \),

(i) \( D_{<a,b>,U,W,T} = D_{a,U,W,T}^{b,U,W,T} \)

(ii) \( D_{<s,a>,U,W,T} = D_{s,U,W,T}^{a,U,W,T} \)

Definition 8(i) states that a functional type \( <a, b> \) is interpreted as a function from the domain of \( a \) to the co-domain of \( b \). For example, an extensional intransitive verb
of type \(<e,t>\) denotes a function from the domain of entities to the domain of truth values.\(^4\)

A function from a domain \(A\) to the range \(\{0, 1\}\) is a \textit{characteristic function} in that it can be described in terms of the elements \(B \subseteq A\), that map to 1. It is called the characteristic function of \(B\) over \(A\). The subset of elements that map to 1, in the example case, are the entities of which an intransitive predicate holds.

\textbf{Definition 9.} \textit{Given any} \(X \subseteq Y\), \textit{the characteristic function of} \(X\) \textit{over} \(Y\), \(f_X\) \textit{is that function from} \(Y\) \textit{to} \(\{0, 1\}\) \textit{such that for all} \(y \in Y : F_X(y) = 1\) \textit{if, and only if} \(y \in X\). And, if \(f_X\) \textit{is a function from} \(Y\) \textit{to} \(\{0, 1\}\), \textit{then} \(X = \{y | f_X(y) = 1\}\). \textit{The characteristic function of a set} \(X\) \textit{and the set} \(X\) \textit{are interchangeable} (Gamut 1991:84[32]).

For example, the grammar introduced above had the entities \textit{John} and \textit{the bear}. It is helpful to imagine a scenario that we might like to characterize in a world where John and the bear live. The world parameter is set to some world, say the actual world in which we live, it could be labelled \(w_1\), and the universe \(U\) contains John and the bear, and there is some time, like midnight when, in the real world, John is sleeping and the bear is awake. The situation described can be characterized as the predicate \textit{sleeps} applying to John and not the bear. More formally, in IL, there is a function specific to the intransitive verb \textit{sleeps} that maps \textit{John} to 0 and \textit{the bear} to 1, shown in Figure 3.1.

The example given in Figure 3.1 does not deal with the way in which intensional types are handled. The interpretation domains are relative to the fixed parameters of \(w_1\) and \(t_1\). Intensional types are not necessary in the fragment presented above, but IL ads

\(^4\)The mapping of intransitive verbs to types is premature, since the mapping is not defined at this point in the exposition. It will be shown that the mapping is more complicated than indicated here. The example does, however, simply illustrate the general means of interpreting extensional intransitive verbs in model theoretic semantics.
Figure 3.2: The function representing intension of the intransitive verb *sleeps*.

Montague 1973[63] adds intensionality by the following rule for the interpretation function $F$:

**Definition 10.** $F$ is a function having as its domain the set of all constants, and whenever $a \in Type$ and $\alpha \in Con_a$, $F(\alpha) \in S_{a,U,W,T}$.

Senses are functions from the Cartesian product of worlds and times to typed domains. For example, the intension of the intransitive verb *sleeps* is a function from world-time pairs to functions from entities to truth values, as shown in Figure 3.2.

Figure 3.2 shows a mapping from world and time pairs to the domain of entities to the domain of truth values. The figure only gives a partial mapping. In a full mapping, there would be a pair for every world and time in $W$ and $T$ and there would be mappings to all possible scenarios. What Figure 3.2 shows is that in world one, at time one, John sleeps and the bear does not sleep. At world one time two, both John and the bear sleep. At world two and time one, the bear sleeps and John is awake.
If a more complete fragment of English with tenses and modal auxiliaries were presented, we could characterize situations like the following based on the model in Figure 3.2: John was sleeping and the bear was not sleeping, then later the bear and John were both asleep. Such a scenario is the case based on Figure 3.2, if we consider time two to be later than time one. Both scenarios happen in world one, but at different times. This scenario shows how the temporal parameter can be used to describe various temporal states in, for instance, the actual world.

Let world one be the actual world, at time two, John remembers that he was asleep and the bear was awake at time one and feels disturbed. He reflects on the past and thinks, ‘I was awake in the past while the bear was asleep, but it could have been the case that the bear was awake and I was asleep’. Such a scenario is characterizable in terms of Figure 3.2, because world two contains a non-actual scenario that is a permutation of what happened in world one (the actual world) at time one.

The symbol $s$ is a special symbol that does not label a type of natural language expression. Rather, $s$ allows the construction of types for intensions, which, as mentioned, are functions from possible worlds to extensions.

The notions of extensions and intensions are not easy to characterize quickly, but might be somewhat intuitive if described informally first. One of Frege’s 1892[31] examples was of two names for the planet Venus as the morning star and the evening star. People did not always realize that they were seeing the same planet in both cases. Therefore, sentences (67) and (68) do not necessarily have the same truth value:

67. John believes that he has seen the morning star.

68. John believes that he has seen the evening star.

It is possible for John to believe that he has seen the morning star but not believe that he has seen the evening star, because he perceives them as being different from each other. The extension of the morning star is the same as the extension of the evening star: Both terms refer to the same object in the world. But the information that they convey is different. When John learns that the morning star and the evening star are the same object, it feels, intuitively, like he gained new information. 5

Intensionality is important in modeling the semantics of natural language. Examples of the need for intensionality, pointed out by Frege 1892[31], include embeddings under

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5Fitting 2012[29] in The Stanford Encyclopedia of Philosophy entry on intensional logic presents an informative introduction to the issues of intensionality.
propositional attitude verbs, as illustrated in (67) and (68). Propositional attitude verbs (Karttunen 1973 [41], Karttunen 1974[42]) are a type of verb that take an argument that is a sentence and that express an attitude towards the content, or intension of, a sentence. For example, believe above takes a first participant (i.e., subject) argument, John, and a sentential argument, he has seen the morning star. The embedding clause John believes [that], expresses a mental relation to the embedded sentence, he [John] has seen the morning star. The mental relation, expressed by the speaker is that John stands in a belief relation to the sentential argument, he [John] has seen the morning star.

Consider a scenario in which the speaker knows that the morning star is the evening star, but John does not know that they are the same star. The speakers can say, John believes that he has seen the evening star, but not the morning star. The reason why the speaker can make this statement, knowing that the stars are the same, is because it is phrased in terms of John’s beliefs. In contrast, in the same scenario, it would be odd for the speaker to say, John has seen the morning star but not the evening star.6

Relating these data to the theory of IL, two expressions, α and β are extensionally equivalent if they refer to the same element in the set of things labeled by a sequence of e’s and t’s in the semantic type theory.

In a model without intensionality, sentences (67) and (68) would have the same interpretation. Giving the two sentences the same interpretation does not capture the fact that John could believe that he saw the morning star without believing that he saw the evening star. Intensional models allow the morning star and evening star to have the same extension (or reference) but different intensions (or senses).

Intesionality is captured through the use of possible worlds (or world-time pairs, as presented here). Sentence (67) is true in all of those world-time pairs in which John believes that he has seen the morning star, and sentence (68) is true in all of those world-time pairs in which John believes that he has seen the evening star. The idea behind the solution is that there is a way in which the world is, and it is such that the morning star and the evening star are the same thing. There is also a way in which the world is perceived. It is possible to imagine worlds in which the way John perceives the world to be and the way in which the world is are the same, with respect to the morning star and the evening star. But this possible world-time pair is not the actual world at the

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6The conditions of use are not as simple as stated here. For example, a speaker can say John has seen the morning star but not the evening star with a gesture like a wink or a smile to someone who also knows that the two stars are the same. The speaker can also say John believes that he has seen the morning star but not the evening star while, like John, not knowing that they are the same.
time of utterance. It may be that, in the actual world, at a future time, John learns that the two stars are the same, but at the world-time pair coinciding with the utterance of the sentence (on the scenario being supposed), the way in which things are and the way in which John speaks of them do not match.

In the mapping of English expressions to IL, extensional equivalence and intensional equivalence are determined differently. Two expressions are extensionally equivalent if one can be substituted for the other (in an extensional environment) and the truth value is the same. Because the morning star and the evening star are mapped to the same thing in the model, the two English terms are extensionally equivalent. In contrast, the set of world-time pairs in which John believes that he has seen the evening star are not the same as the set of world-time pairs in which John believes that he has seen the morning star. If one term in the English language were substituted for the other, the meaning would not be the same.

Frege’s 1892[31] idea, which inspired Montague’s 1974[63] account, was that, cognitively, when someone says $a = a$, there is no new information, but when someone $a = b$ there is information gain. The difference between what is said, in each case, is significant, even if what is meant in both cases is to show an equivalence. It is possible for various linguistic expressions to refer to the same thing but have many different senses in which they can be used. The morning star and evening star senses are one example.\(^7\)

So far, the interpretation domain of constants has been described, but an interpretation domain for variables must also be presented.

**Definition 11. Variable Assignment:**

A function $g$ is a $M$-assignment of values to variables where

(i) $M$ is an interpretation.

(ii) $g(u) \in D_{a,U,W,T}$ when $u$ is a variable of type $a$,

Definition 11 explains how superscripts are used to encode various ways of evaluating an expression in the intensional model of IL. Generally, if the meaning of an expression

\(^7\)Some authors feel that expressions that are logically equivalent should have some difference of meaning encoded in the model, since some logical truths are easier to grasp than others. Authors such as Fox and Lappin 2005[30] offer possible directions in which these difficulties could be solved, for instance, by making changes to the typing system or equivalence operator, but some authors feel that problems with how to model intensional expressions persist.
is not dependent on a certain parameter, than that parameter is not shown in the superscripts. For example, by leaving out the $U$ superscript, indicating the universe, it is implicit that the interpretation holds regardless of what universe of individuals is described. If the examples in the figures had contained a domain including *Mary* and *the cat*, for instance, rather than *John* and *the bear*, it would not change the basic mapping of the intransitive verb *sleeps*.

In general, the nature of the entities in the domain to which rules of interpretation apply does not affect the rules of interpretation. However, if the model were to contain some axioms about the lexical semantics of entities, the meaning of an expression would be dependent on the domain of entities, and the $U$ superscript would need to be present. There are other cases when the universe of individuals is involved as a dependent parameter, such as relativist accounts, that will be discussed in Chapter 6.

Definition 11(i) says that the denotation of a variable of any given type is in the domain of that type. Definition 11(ii) states that the superscript of the interpretation $M$ and the variable function $g$ are present with a meaningful expression, what is being described is the meaning of the expression given the model and a variable assignment. The information in the superscript is necessary for the proper interpretation of the expression. Other non-listed parameters are not essential for the truth definition and could be other than they are without changing the meaning. Therefore, for instance, if an expression contains no variables, only constants, it is not essential to list $g$ in the superscript.

It is important to note that these conventions are not necessarily followed carefully in most expositions. There are often independent parameters present in the superscripts when they are not necessary there.

Definition 11(iii) says that, if an expression is evaluated extensionally, rather than intensionally, then the meaning is dependent on the world and time in which it is evaluated. In this case, the expression is evaluated in a constant model, with a particular variable assignment, and relative to a particular world and time. In Figure 3.2, which shows an intensional model, it can be seen how fixing the interpretation to a particular world and time changes to whom the predicate *sleeps* applies. If the meaning is dependent on $w_1, t_1$, then the state of affairs is one in which John sleeps and the bear does not sleep. If the meaning is dependent on $w_2, t_1$, then the bear sleeps and John does not sleep. As mentioned previously, the model is not dependent on the universe of entities, and, as a result, the extension of *sleeps* does not inherently mean that some particular individual sleeps. The fact that some entity $e_1$ is mapped to *John* in the particular model
is not part of language meaning, in general.

**Definition 12. Interpretation Function:**
For each meaningful expression of IL, the intensional interpretation function $F$, represented by the double brackets $[\ ]$ and $[\ ]$, assigns an interpretation as follows:

(i) For each $\alpha \in V_a$, $[\alpha]_{M_{w,t,g}} = g(\alpha)$.

(ii) For each $\alpha \in C_a$, $[\alpha]_{M_{w,t,g}} = F(\alpha)$.

(iii) If $\alpha \in ME_a$ and $v \in V_b$, then $[\lambda v \alpha]_{M_{w,t,g}}$ is that function $n$ from the domain $D_{b,U,W,T}$ such that for all $x \in D_{b,U,W,T}$, $n(x) = [\alpha]_{M_{w,t,g}}$. Where $g'$ is that function exactly like $g$ except that $g'(v) = x$.

(iv) If $\alpha \in ME_{<a,b>}$, and $\beta \in ME_a$, then $[\alpha(\beta)]_{M_{w,t,g}} = [\alpha]_{M_{w,t,g}}([\beta]_{M_{w,t,g}}).

(v) If $\alpha \in ME_a$, then $[\wedge \alpha]_{M_{w,t,g}}$ is that function $n$: such that for all $w' \in W, t' \in T$, $n(w', t') = [\alpha]_{M_{w,t,g}}$.

(vi) If $\alpha \in ME_{<a,d>}$, then $[\forall \alpha]_{M_{w,t,g}} = [\alpha]_{M_{w,t,g}}(< w, t >)$.

(vii) If $\alpha, \beta \in ME_a$, then $[\alpha = \beta]_{M_{w,t,g}} = 1$ iff $[\alpha]_{M_{w,t,g}} = [\beta]_{M_{w,t,g}}$ and 0 otherwise.

If $\phi$ and $\psi$ are $ME_a$, then:

(viii) $[\neg \phi]_{M_{w,t,g}} = 1$ iff $[\phi]_{M_{w,t,g}} = 0$.

(ix) $[\phi \to \psi]_{M_{w,t,g}} = 1$ iff $[\psi]_{M_{w,t,g}} = 1$ or $[\phi]_{M_{w,t,g}} = 0$ or both and 0 otherwise.

(x) $[\phi \lor \psi]_{M_{w,t,g}} = 1$ iff $[\psi]_{M_{w,t,g}} = 1$ or $[\phi]_{M_{w,t,g}} = 1$ or both and 0 otherwise.

(xi) $[\phi \land \psi]_{M_{w,t,g}} = 1$ iff $[\psi]_{M_{w,t,g}} = 1$ and $[\phi]_{M_{w,t,g}} = 1$ and 0 otherwise.

(xii) $[\forall \phi x]_{M_{w,t,g}} = 1$ iff for all $x \in D_{a,U,W,T}, [\phi]_{M_{w,t,g}} = 1$, where $g'$ is that function exactly like $g$, except that $g'(v) = x$, and 0 otherwise.

(xiii) $[\exists \phi x]_{M_{w,t,g}} = 1$ off there exists some $x \in D_{a,U,W,T}$ such that $[\phi]_{M_{w,t,g}} = 1$ where $g'$ is that function exactly like $g$, except that $g'(v) = x$, and 0 otherwise.

(xiv) $[\Box \phi]_{M_{w,t,g}} = 1$ iff $[\phi]_{M_{w,t,g}} = 1$ for all $w' \in W$ and $t' \in T$

(xv) $[\Diamond \phi]_{M_{w,t,g}} = 1$ iff $[\phi]_{M_{w,t,g}} = 1$ for some $w' \in W$ and $t' \in T$. 

53
Rules 12(i & ii) state that the interpretation of variables and constants in $M$ is whatever the functions $g$ and $F$ assign to those constants given a particular model and variable assignment.

Rule 12(iii) describes the interpretation of expressions with $\lambda$ operators. $\lambda$ abstraction is a function that involves a particular model, world, time, and variable assignment. The variable assignment, however, is what the $\lambda$ function changes.

Rule 12(iv) explains that the interpretation of functional types is a function from the domain of the leftmost expression to the range of the rightmost expression.

Rules 12(v & vi) state that $^\wedge \alpha$ denotes the intension of $\alpha$ and $^\vee \alpha$ denotes the extension of $\alpha$. The $^\wedge$ operator adds a function from world times pairs to the (either simple or functional) expression that it precedes. The $^\vee$ operator precedes an intensional type, and it removes the outer-most function in the interpretation of that expression, which, for an intensional type, is a function from world and time pairs to either other functions (in the case of a functional types) or to a type domain (for simple types).

Rules 12(vii-ix) define the logical operators for disjunction, conjunction, and conditionals. Rule 12x defines the universal quantifier $\forall$ and states that, the $\phi$ is true in a particular world and time if there is a satisfying variable assignment for all possible assignments of the variable. Rule (12xi) states that the existential quantifier $\exists \phi$ is true in a particular world and at a particular time if there is at least one variable assignment in which $\phi$ is true.

Finally, the modal operators $\Box$ and $\Diamond$ are defined. The box operator holds if the proposition $\phi$ in its scope holds in all worlds and times. As will be discussed later, the worlds that are accessible for valuation can change. The term with the diamond operator is true if there is at least one world and one time in which the proposition in the scope of the operator holds.

### 3.4 Translation Rules

In order to translate English into IL, some basic rules are given.

**Definition 13.**

$f(e) = e,$

$f(t) = t,$
Table 3.2: Chart of some intensional semantic types from Montague’s Intensional Logic, their corresponding label in the typed syntax, and a natural language example of each syntactic type.

\[
f(A/B) = f(A // B) = << s, f(B) >, f(A) > \text{ whenever } A, B \in \text{Cat}.\]

Definition 13 states that the function \( f \) takes a syntactic type \( e \) or \( t \) and translates it to the semantic type with the same label. For complex types, \( f \) adds a level of intensionality, by means of the intensional operator \( s \). As a result, each time a type combines by functional application, the intensional type is used. The syntactic type \( (t/e) \), for example, becomes \( << s, e >, t > \) in the translation from English to IL. Some common intensional semantic types are in Table 3.2.

Montague’s Definition 13 ensures that functional types are interpreted intensionally. The reason why Montague does this is because he wants to account for verbs like believe, which create intensional contexts. Because the partial fragment of Montague’s fragment presented here only has expressions with extensional meanings, the full use of the theory will not be demonstrated. Rather, for extensional uses, such as those in our fragment, Montague makes a series of rules necessitating the use of the \( \vee \) operator in order to interpret extensional verbs.

Because Montague’s combinatoric rules in 13 complicate the semantic types, sometimes expositions simply use extensional types, such as \( < e, t > \) for expressions of the syntactic type \( (t/e) \) in order to avoid confusion. That is not the exposition presented here because IL is an intensional logic, and being aware of the intensionality is relevant, even though the fragment presented introduces intensionality only to later change the meaning to an extension, by rule. The theory is, without the rules for extensional readings, intensional.

Montague provides a set of translation rules corresponding to the syntactic rules.

\begin{align*}
\text{T1a} & \quad \text{If } \alpha \text{ is in the domain of } h, \text{ then } \alpha \text{ translates to } h(\alpha). \\
& \quad \text{Where } h \text{ is an arbitrary function assigning translations to constants.} \\
\text{T1d} & \quad \text{If } \alpha \in ME_e, \text{ then } \alpha^* \text{ is to be } \lambda P^\vee [P_{<t,<<s,e>,t>>}(\vee \alpha)], \text{ and John translates to } j^*.
\end{align*}
Table 3.3: Chart of variables abbreviating the $\nu_{n,a}$ format.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Semantic Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u, v$</td>
<td>$e$</td>
</tr>
<tr>
<td>$x, y$</td>
<td>$&lt; s, e &gt;$</td>
</tr>
<tr>
<td>$M, N$</td>
<td>$&lt; s, &lt; e, t &gt;$</td>
</tr>
<tr>
<td>$T, V$</td>
<td>$&lt; s, &lt; e, &lt; e, t &gt;$</td>
</tr>
<tr>
<td>$P, Q, R$</td>
<td>$&lt; s, &lt; s, e, t &gt;$</td>
</tr>
<tr>
<td>$P, Q, R$</td>
<td>$&lt; s, &lt; s, &lt; s, e, t &gt;, t &gt;$</td>
</tr>
</tbody>
</table>

T2 If $\zeta \in P_{(t/e)}$ and $\zeta$ translates to $\zeta'$, then the $\zeta$ translates into

$$\lambda P_{<s,<s,e>,t} \exists x_{<s,e>}[\forall y_{<s,e>}[\zeta' x_{<s,e>} \leftrightarrow x_{<s,e>} = y_{<s,e>}] \land P_{<s,<s,e>,t>}^{(\nu_{<s,e>})}]$$

T4' If $\delta \in P_{(a/b)}, \beta \in P_{b}$, and $\delta, \beta$ translates into $\delta', \beta'$, respectively, then $F_{4}(\delta, \beta)$ translates into $\delta'(\beta')$.

Definition 3.4: T1a states that variables are translated as whatever the function $h$ assigns them to be. Translation rule T1d states that expressions of type $e$ translate to a complex type. The proper name $John$ is such a constant. Rule T2 describes the translation of the definite article the. Rule T3' is a general rule that tells how two types combine.

Rules T11-T13 provide an interpretation for conjunction of sentences and verb phrases.

T11 If $\phi, \psi \in P_{t}$ and $\phi, \psi$ translate into $\phi', \psi'$ respectively, then $\phi$ and $\psi$ translates into $[\phi \land \psi]$. $\phi$ or $\psi$ translates into $[\phi \lor \psi]$.

T12 If $\gamma, \delta \in P_{(t/e)}$ and $\gamma, \delta$ translate into $\gamma', \delta'$ respectively, then $\gamma$ and $\delta$ translates into $\land\gamma'(x) \lor \delta'(x)]$.

T13 If $\alpha, \beta \in P_{(a/(t/e))}$ and $\alpha, \beta$ translate into $\alpha', \beta'$ respectively, then $\alpha$ or $\beta$ translates into $\land P[\alpha'(P) \lor \beta'(P)]$.

The rules of quantification in T14-T16 describes how to combine expressions of different types.

It is helpful to introduce distinct variables for each to, abbreviating the $\nu_{n,a}$ format introduced. A list of variable conventions appear in Table 3.3. For perspicuity, the type labels remain with the variables in the exposition.

As previously mentioned, the theory is an intensional theory. The following axioms, among other things, ensure an extensional reading for non-intensional uses.
Definition 14. Axioms of IL

(i) \( \exists \alpha \Box [v_e = \alpha] \), where \( \alpha \) is \( j \).

(ii) \( \Box [\delta(x_{s,e})] \leftrightarrow \exists a_e x_{s,e} = \wedge v_e \) where \( \delta \in BE_{(t/e)} \).

(iii) \( \exists M_{s,e} \forall x_{s,e} \Box [\delta(x_{s,e})] \leftrightarrow \gamma M_{s,e} \forall (\gamma x_{s,e}) \), where \( \delta \in BE_{(t/e)} \).

(iv) \( \exists T_{s,e} \forall x_{s,e} \forall Q_{s,e} \Box [\delta(x_{s,e})] \leftrightarrow \gamma Q_{s,e} \forall (\gamma x_{s,e}), \forall y_{s,e} \), where \( \delta \in BE_{(t/t/e)} \).

Axiom 14(i) ensures that proper names denote the same individual regardless of the world and time parameters. Because there is only one proper name, \( John \), in this fragment, the rules could have been stated for the constant \( j \) rather than more generally, but the general definition is used when multiple proper names are present.

Axiom 14(ii) ensures that common nouns are the same individual concepts regardless of the intension.

Axiom 14(iii) forces the interpretation of intransitive verbs to be extensional. For Montague’s fragment, which contained intensional expressions, he included in the rule an exception for intransitive verbs that were to be interpreted intensionally.

Axiom 14(iv) forces transitive verbs to be interpreted extensionally. As with intransitive verbs, Montague’s fragment included verbs such as \( seeks \) among the exceptions.

Montague 1973[63] also included additional axioms ensuring that intensionality was correctly handled.

Montague also included a number of additional rules, some of which are merely notational conventions, but that are necessary to present and discuss before examples are given.

Definition 15. Other Rules and Notational Conventions:

(i) If \( \phi \in ME_{s} \), then \( \phi \) is a formula and \( \mathfrak{f}^{M_{s,e}} = 1 \) if, and only if \( \mathfrak{f}^{M_{s,e}} = 1 \) for every assignment \( g \).

(ii) \( \Box [\delta(x_{s,e})] \leftrightarrow d_\ast(x_{s,e}) \) if \( \delta \) translates any member of \( BE_{(t/e)} \) or \( BE_{(t/t/e)} \).

(iii) \( \Box [\delta(x_{s,e}, P_{s,e})] \leftrightarrow P_{s,e} \exists (\lambda y_{s,e} \delta_\ast (\gamma x_{s,e}, \forall y_{s,e})) \) if \( \delta \) translates any member of \( BE_{(t/e)/(t/t/e)} \).
Rule 15(i) defines a formula as being true relative to a world and a time if, and only if, it is true for any possible variable assignment.

Rule 15(ii) ensures the extensionality of intransitive verbs that are not intensional.

Rule 15(iii) ensures the extensionality of transitive verbs that are not intensional.

3.4.1 Examples of Sentence Translation and Interpretation in IL

Natural language expressions in IL are mapped into a syntactic algebra of the object language that is then mapped to the translation language, which is then mapped into an intensional semantics. The mapping from the syntactic algebra of the object language to the translation language is a homomorphism as is the mapping from the syntactic algebra of the translation language to the intensional semantics. Because of the composition of homomorphisms, the syntactic algebra of the object language can be mapped directly to the intensional semantics.\(^8\)

Putting together all of the rules, axioms, and levels of translation in Montague Grammar can be tedious, but all of the parts necessary for interpretation are present and only have to be used appropriately.

The sentence *John sleeps* is translated from the basic expressions in the syntax:

\[
\text{John} \left( t \right) \quad \text{sleeps} \left( t \right)
\]  

It was shown in the syntactic tree how the sentence is combined syntactically, given the syntactic types.

Semantically, *John* is translated to the constant *j*, which is of type *e*. However, several rules change the semantic types of proper nouns. First, by axiom, *j* is ensured to be a rigid designator: referring to the same individual regardless of the world or time of interpretation. Second, *j* is translated as follows by translation rule T1d:

\[
 j^* \rightarrow \lambda P_{<s,<<s,e>,t>>}[P_{<s,<<s,e>,t>>}(\forall \land j)]
\]  

The verb *sleeps* gets its semantic type from Definition 15:

\[
\text{sleeps}_{<s,<<s,e>,t>>}
\]

\(^8\)A concise description of the benefit of homomorphic mappings is covered in Dowty (1979:28-31[25]).
By function $F_4$, described in translation rule T4', the verb *sleeps* has an additional level of intensionality when combining with a noun phrase:

$$j^* \rightarrow AP_{<,<<,>,>}[P_{<,<<,>,>}(\forall^j e)(\forall^j e)](\forall^j e)(3.7)$$

By standard application of $\lambda$ reduction, the form of *sleeps* combines with the semantics of *John*:

$$sleeps_{<,<<,>,>}(\forall^j e)(3.8)$$

By Axiom 14i(ii) for the extensionality of non-intensional verbs and rule 17iv, we get:

$$sleeps_{<,<<,>,>}(\forall^j e)(3.9)$$

Without types, the end result is $sleeps_j$, and, by rule 17iv, asserts that John has the property of sleeping.

What has been covered so far is the use of an intensional logic to provide a semantic interpretation for extensional expressions. Intensional contexts associated with the embedding verbs, such as *believe*, were discussed, but not modeled in the fragment. Montague’s theory did many additional things besides what have been illustrated here. It also covered de dicto and de re readings of indefinite noun phrases. For instance, in Montague’s IL, the sentence *John seeks a unicorn* could be interpreted as a state of affairs in which John sought a particular unicorn or a case in which he sought any unicorn. The grammar also could interpret natural language quantifiers with various scope options.
CHAPTER 4

MODAL LOGIC AND MODAL AUXILIARY VERBS: THE STANDARD THEORY

4.1 Introduction

This chapter reviews the history of treatments of the semantics of modal auxiliaries that focus on what type of model-theoretic interpretation is best for representing modal auxiliary meaning.

4.2 Propositions and Worlds

The interpretation of a formula $\phi$, where $\phi$ is a meaningful expression of type $t$ in IL, yields a function from the set of world-time pairs $W \times T$ to the set $\{0, 1\}$. Any function that maps elements of a set to $\{0, 1\}$ is a characteristic function, meaning, it characterizes a subset of the domain, call it $W' \subseteq W \times T$, which is the set of elements mapping to 1. In the case of IL, the meaning of the number 1 is truth. The world-time pairs that map to 1 are the world-time pairs in which the formula is true. The worlds-time pairs in the domain that map to 0 are the world-time pairs in which the formula is false. Sometimes the meaning of a formula is defined as being the set of worlds-time pairs in which the formula is true. For example, imagine a model with three worlds, one time, and two formulas, $\phi$ and $\psi$. The following interpretations are given (by stipulation):

\[
\begin{align*}
F(\phi)(w_1)(t_1) &= 1 \\
F(\phi)(w_2)(t_1) &= 0 \\
F(\phi)(w_3)(t_1) &= 1 \\
F(\psi)(w_1)(t_1) &= 1
\end{align*}
\]
\[ F(\psi)(w_2)(t_1) = 1 \]
\[ F(\psi)(w_3)(t_1) = 0 \]

In this particular model containing only two propositions, each proposition denotes the set of world-time pairs that map to 1.

\( \phi \) can be associated with the set of worlds such that \( F(\phi)(w)(t) = 1 \), so \( \phi \) is associated with \{ \( < w_1 t_1 >, < w_3 t_1 > \} \)

\( \psi \) is associated with \{ \( < w_1 t_1 >, < w_2 t_1 > \} \)

Although a proposition can be characterized as the set of world-time pairs in which that proposition is true, sometimes, it makes more conceptual sense to talk about a world-time pair corresponding to the set of propositions that are true at that world-time pair.\(^1\)

It is possible, given the meaningful expressions \( \phi \) and \( \psi \) and the possible world-time pairs \( < w_1, t_1 >, < w_2, t_1 >, < w_3, t_1 > \), to describe the propositions that hold in each world-time pair. Looking at the examples, it is clear that \( < w_1, t_1 > \) supports the propositions \( \phi \) and \( \psi \), that \( < w_2, t_1 > \) supports the proposition \( \psi \) but does not support the proposition \( \phi \), and that \( < w_3, t_1 > \) supports the proposition \( \phi \) but does not support the proposition \( \psi \).

In what follows, the standard way of thinking of propositions as sets of world-time pairs will be used, but sometimes world-time pairs are referred to by the propositions they support. For instance, in the example above \( < w_1, t_1 > \) is a \( \phi \)-supporting world and a \( \psi \)-supporting world. In the discussion of Lewis 1973[54], the time will be omitted, following the author’s presentation.

### 4.3 Modal Logic and Modality: Lewis 1973[54]

Lewis’ 1973[54] work on counterfactuals was foundational in later treatments of modal auxiliaries, which are not necessarily in counterfactual constructions, so it will be discussed first.

So far, in the discussion of \( IL \), it has been implicit that, whenever a modal operator \( \Box \) or \( \Diamond \) quantifies over worlds, it ranges over all \( w \in W \). It has been mentioned that the

---

\(^1\) For an example of a theory that takes propositions as primitives, see Thomason 1980[94].
standard formal semantic theory of modals characterizes their interpretation in terms of quantification. In a very simple theory, assuming necessity and possibility is part of the meaning of natural language modals, modal expressions in the target language (English, in this case) could be translated to logical necessity and possibility. The operators $\Box$ and $\Diamond$, as defined above, have a proposition in their scope and the proposition is evaluated in all possible worlds. If the formula is $\Box \phi$, then $\phi$ is evaluated relative to every world in the model and $\Box \phi$ is found to be true if $\phi$ holds in each of the worlds. If the formula is $\Diamond \phi$, then $\phi$ is evaluated relative to every world in the model and $\Diamond \phi$ is found to be true if $\phi$ is true in at least one of those worlds.

There are many reasons why such an analysis would need to be tweaked to deal with natural language modality. When a necessity modal is used, as in the sentence *You must go to bed!*, it is not the case that the addressee goes to bed in every possible world. In fact, it might well be the case that the addressee does not go to bed in the world in which the statement is uttered. Rather, the meaning is more close to something to the effect that the addressee must go to bed in all ideal worlds, according to the speaker’s ideal of people following his (or his community’s) rules. Similarly, with an epistemic possibility modal use, as in the sentence *John might be the murderer*, it is not the case that John is the murderer in every possible world that can be imagined, rather it is the case that John is the murderer in some world (possibly the actual world in which the utterance was spoken) in which all of the evidence available to the speaker is exactly as it is in the actual world. Natural language modality rarely, if ever, is well represented as quantifying over all possible worlds in a model.\(^2\)

Accessibility relations limit the worlds over which a given use of natural language modal ranges in the interpretation model. Accessibility relations constrain the worlds over which any expression that quantifies over possible worlds ranges.

Accessibility relations can be represented by a two-place accessibility relation $R$, which takes $w_1, w_2 \in W$ as arguments such that $Rw_1w_2$ reads, ‘$w_2$ is accessible from $w_1$’.

The model of IL is augmented accordingly such that $R$ is included as well as an ordering relation $\leq_R$, which will prove useful for determining which sets of worlds are

\(^2\)The one exception in which simple quantificational modality could work is one in which the domain and grammar of the model was sufficiently small that ranging over all worlds made no difference, but these would be very small models indeed, each with their own grammar and domain, and there are, perhaps, better ways of tackling the problem that would allow a shared model for many different scenarios and many different uses of natural language modals.
Definition 16. An interpretation $\mathcal{M}$ for the formal language $\text{IL}$ is a 6-tuple $< U, W, T, \leq, R, \leq_R, F >$ where

(i) $U$ is a non-empty set, construed as a set of entities (sometimes called the universe of $\mathcal{M}$),

(ii) $W$ is a non-empty set, construed as a set of possible worlds,

(iii) $T$ is a non-empty set, construed as a set of temporal points and intervals, and

(iv) $\leq$ is a partial order on the set $T$.

(v) $R$ is a two-place accessibility relation on $W$, and

(vi) $\leq_R$ is a partial order on $W$.

(vii) $F$ is an interpretation function having as its domain the set $U$, and whenever $a \in \text{Type}$ and $\alpha \in \text{Con}_a$, $F(\alpha) \in S_{a,U,W,T}$.

The set of possible worlds $W$ and accessibility relation $R$ of a model together define a Kripke Frame. There are various systems defined by accessibility relations, developed by Kripke[50] and subsequent researchers. The system a set of accessibility relations fits is standardly described in the philosophical and logic literature. One example is whether or not the accessibility relations in the model are symmetric or not. If, for every $w, w'$, $R(ww') \leftrightarrow R(w'w)$, then the accessibility relation is symmetric.

For example, a model $\mathcal{M}$ might allow the accessibility relation $R$ to be reflexive. So, for every $w \in W, R(w, w)$, that is, every world is accessible to itself.

The types of relations used for characterizing natural language modality in the formal semantic literature have been more descriptive than formal, for example, deontic modals have been characterized as ranging over ‘all the worlds in which the laws are obeyed’. Nothing about the interpretation model helps with determining which worlds these are or how they might be determined. Much of the literature on modal interpretation has involved constructed scenarios as a proof of concept that the model works. Such examples involve the author selecting which facts they believe to be relevant to the evaluation and representing those facts as holding in certain worlds.

But the types of logical relations that hold among accessibility relations do matter: Describing the family of logics to which a particular logic belongs helps other
researchers understand what axioms hold in the theory and, consequently, how the system will behave given the particular phenomena to which it is being applied. It is just not clear how certain logical properties relate to the descriptive accounts of natural language modality. A complete summary of well-known logic families will not be presented here, but the details can be found in SEP[33].

Lewis’ 1973[54] account of modality in the context of counterfactual conditionals was foundational in the literature that was to follow. Kratzer’s 1977[44] work was the first linguistic approach to extend Lewis’ basic idea of modality to modal expressions more generally, rather than only in the context of counterfactual conditionals. Kratzer’s ideas have been the most prominent in formal semantics as practiced by linguists and continue to be refined and developed. Other researchers have contributed important innovations or discovered parallel approaches, especially where conditionals are concerned (e.g., Pollock 1976[78], Veltman 1976[98]). Some important variants include the auto-epistemic model proposed in Veltman 2005[99] and the use of preferences in ordering commands and deontic expressions in Portner 2007[79].

Strictness

Lewis defines a sphere of accessibility for the interpretation of the modal operators ◻ and ◇. The spheres of accessibility are sets of worlds accessible from a given world as defined in 17 below:

**Definition 17. Sphere of Accessibility**

Let us have an assignment to each world w, a set $S_w$ of worlds, called the sphere of accessibility around w and regarded as the set of worlds accessible from w.(Lewis 1973:7[54])

The modal operators range over an accessibility sphere, when evaluated from a particular world. Accessibility spheres are a way of characterizing the different kinds of possible accessibility relations that could exist.

The accessibility spheres correspond to accessibility relations as follows:

**Definition 18. $S_w =_{def} \{w' \in W | R(ww')\}$**

Definition 18 states that a system of spheres $S_w$ centered around a world $w$ is, by definition, all worlds $w'$ in the set $W$ of worlds such that $w'$ is accessible from $w$. 
The presence of systems of spheres would require Montague’s IL definition to be modified as follows:

**Definition 19. Rule 5xii and xiii with accessibility relations:**

\[
[\Box \phi]^{M_{w,t,g}} = 1 \text{ if, and only if, } [\phi]^{M_{w',t',g}} = 1 \text{ for all } w' \text{ such that } R(ww') \text{ (or throughout } S_w). \\
[\Diamond \phi]^{M_{w,t,g}} = 1 \text{ if, and only if } [\phi]^{M_{w',t',g}} = 1 \text{ for some } w' \text{ such that } R(ww') \text{ (or somewhere in } S_w). 
\]

Lewis gives a definition for the operators when they are specifically applied to a conditional:

**Definition 20. Lewis’ Rule for Modalized Conditionals:**

\[
[\Box (\phi \rightarrow \psi)]^{M_{w,t,g}} = 1 \text{ if, and only if } [\phi \rightarrow \psi]^{M_{w',t',g}} = 1 \text{ for all } w' \text{ such that } wRw' \text{ (or throughout } S_w). \\
[\Diamond (\phi \rightarrow \psi)]^{M_{w,t,g}} = 1 \text{ if, and only if } [\phi \rightarrow \psi]^{M_{w',t',g}} = 1 \text{ for some } w' \text{ such that } wRw' \text{ (or somewhere in } S_w). 
\]

Lewis’ definitions for modalized conditionals are an intermediate step in his theory. He shows counterfactuals to require a more complicated semantics, which will be described in the next paragraphs, but, first, an explanation of strictness is necessary. If the sphere of accessibility is such that the modal operator \( \Box \) ranges over all possible worlds, then the strictest form of modality is used. It corresponds to logical necessity – things that are true in any set of possible worlds. The least strict modal is one that only ranges over the world of evaluation (the actual world, for instance). Lewis calls this type of modality fatalistic necessity.

The interpretation of most uses of natural language modal auxiliary verbs, intuitively, lies somewhere between these levels of strictness. Some such descriptive categories Lewis gave were physical necessity where the accessible worlds are those in which the laws of nature that hold in the world of evaluation also hold, inevitability at time \( t \), in which accessible worlds are those just like the world of evaluation up to a time \( t \), and necessity given a certain set of facts is a type of modality that ranges over worlds in which that set of facts holds. Lewis also mentions deontic necessity as a type of
modality that ranges over morally perfect worlds and does not include the world of evaluation being accessible to itself.

Determining the strictness of a particular modal auxiliary use implies that there is some structure to the types of accessibility relations. In order to determine whether a given use of a modal is stricter than another, Lewis gives the following criteria:

**Definition 21. Comparative Strictness (Lewis 1973:8[54]):**
Suppose we have two necessity operators \( \Box_1 \) and \( \Box_2 \) corresponding to the assignment to each world \( w \) of spheres of accessibility \( S^1_w \) and \( S^2_w \) respectively. Then the strict conditional \( \Box_2(\phi \rightarrow \psi) \) is stricter at world \( w \) than \( \Box_1(\phi \rightarrow \psi) \) if, and only if, \( S^2_w \) properly includes \( S^1_w \).

Strictness can be compared, in some cases, and described as set containment, but not all modals are comparably strict since some spheres could be disjoint sets or have disjoint subsets. For counterfactuals, it is important to be able to compare the strictness of various spheres, therefore, a slightly more involved model is used.

Lewis 1973[54] proposes a single logical operator \( \Box \rightarrow \) to represent counterfactual meaning as in sentence (69):

69. *If kangaroos didn’t have tails, they would topple over.* (from Lewis 1973[54])

The \( \Box \rightarrow \) operator abbreviates the conditional grammar and the modal verb *would*. Lewis takes the modal verb *would* to encode necessity with *could* and *might* encoding possibility. According to the logic he develops, the possibility counterfactual \( \phi \rightarrow \psi \) is equivalent to \( \neg(\phi \rightarrow \neg\psi) \).

With sentence (69), Lewis 1973[54] points out that people do not take the consequent clause of sentence (69) to hold in all worlds, for example, it is not evaluated in a world where kangaroos have no tails but where gravity does not exist. Such a world is, intuitively, too different to be evoked by the modal use. For such reasons, Lewis introduced a definition of the ordering \( \leq_R \) based on similarity of worlds so that counterfactual interpretation could be restricted to the most similar worlds. In Lewis’ model, the consequent clause of sentence (69) is evaluated in the world most like the actual world where the minimal difference is that kangaroos do not have tails.\(^3\)

---

\(^3\)Subsequent literature in philosophy and artificial intelligence has dealt with the fact that it is impossible, many times, to find a unique minimally different world (e.g., see McCarthy 1980[59] on circumscriptio).
The means by which Lewis 1973[54] models the relations is through what he calls a system of spheres. The system ensures that a set of worlds is used that results in concentric containment of spheres. First the system of spheres will be introduced, then additional motivations for it will be discussed.

**Definition 22. System of Spheres (Lewis 1973:13-14[54]):**

Let $S$ be an assignment to each possible world $w$ of a set $S_w$ of sets of sets of possible worlds. Then $S$ is called a (centered) system of spheres, and the members of each $S_w$ are called spheres around $w$, if, and only if, for each world $w$, the following conditions hold:

(i) $S_w$ is centered on $w$; that is, the set $\{w\}$ having $w$ as its only member belongs to $S_w$.

(ii) $S_w$ is nested; that is, whenever $S$ and $T$ belong to $S_w$, either $S$ is included in $T$ or $T$ is included in $S$.

(iii) $S_w$ is closed under unions; that is, whenever $P$ is a subset of $S_w$, and $\bigcup P$ is the set of all worlds $w'$ such that $w'$ belongs to some member of $P$, $\bigcup P$ belongs to $S_w$.

(iv) $S_w$ is closed under non-empty intersections; that is, whenever $P$ is a nonempty subset of $S_w$, and $\bigcap P$ is the set of all worlds $w'$ such that $w'$ belongs to every member of $P$, $\bigcap P$ belongs to $S_w$.

Where $S$ and $T$ are variables for sets of worlds and $P$ is a variable for sets of sets of worlds.

Definition 22i states that the world around which the system is centered is also a member of the system. It is the ‘nucleus’ of the system.

Definition 22ii states that the sets in the sphere form concentric subsets around the center, so each inner sphere is less strict than the spheres that contain it strict with the center world being the only element in the least strict sphere.

Definition 22iii states that the union of any subsets in the sphere is also in the sphere and 22iv states that the intersection of any non-empty subsets of the sphere is also in the sphere. The closure rules ensure concentric sets.

Given the system of spheres, Lewis 1973[54] provides truth conditions for counterfactuals.
Definition 23. $\phi \square \rightarrow \psi$ is true at a world $w$, according to a system of spheres $S_w$, if, and only if, either

(i) no $\phi$-world belongs to any sphere $S$ in $S_w$, or

(ii) some sphere $S$ in $S_w$ does contain at least one $\phi$-world, and $\phi \rightarrow \psi$ holds at every world in $S$.

The first part of Definition 23 is reminiscent of the false antecedent condition of material conditionals where, if the antecedent is false, the conditional is true, regardless of the value of the consequent.

The second part is for when the antecedent is true. In this case, the sphere contains a world supporting the consequent as well, and both are true in that world. The same condition holds if the antecedent is not supported in any world in the system of spheres against which the proposition is evaluated.

Let us assume an accessibility relation $R$, which makes accessible the worlds that are part of the largest (most strict) sphere in the system centered around the world of evaluation, $w$. The antecedent of a counterfactual is evaluated relative to each concentric circle of worlds, starting with the least strict and going out to more strict. If the antecedent comes up false for every world in the range of the modal operator (that is, in ever world accessible to the world of evaluation), then the counterfactual is true due to the false antecedent condition.

If, however, a world is found, that is accessible to the world of evaluation, and that supports the antecedent, then the evaluation of the consequent is further restricted to the sphere containing the world in which the antecedent holds. In that sphere, the conditional $\phi \rightarrow \psi$ is evaluated and it must hold, per the rules of a material conditional, in every world in the sphere, either by the false antecedent condition (it is a non-antecedent-supporting world) or by the truth of both the antecedent and the consequent in that world.

4.3.1 Non-monotonicity

Lewis 1973[54] also noted that the logic of counterfactuals is non-monotonic, that is, sequences of counterfactuals result in contradictions when more information is added. For instance, the second conditional in sentence (70) has an opposite consequent of the previous conditional:
70. If Anna had come, it would have been a lively party, but if both Anna and Otto had come, it would have been a dreary party. (based on Lewis 1973:10[54]).

The logic would be something like:

\[(\phi_1 \square \rightarrow \psi), (\phi_1 \land \phi_2 \square \rightarrow \neg \psi)\]  

(4.1)

Information added to the antecedent changes the value of the consequent. When information added to the antecedent of a conditional changes the truth value of the consequent, then the model is considered to be non-monotonic. Monotonic models preserve truth values when information is added. It should be the case, in a monotonic model, that whenever \(a \rightarrow b\) holds, if \(a \land c\) also holds, so should \(a \land c \rightarrow b\). As sentence (70) illustrates, monotonicity is not a property of counterfactuals in natural language.

Lewis’ system of spheres handles the problem of non-monotonicity by making counterfactuals variably strict, meaning that the strictness can change sub-spheres in the evaluation of each counterfactual. It might be the case that the \(n^{th}\) sphere from the center of \(S_w\) has \(\phi_1\) worlds but no \(\phi_2\) worlds, and \(\phi_1 \square \rightarrow \psi\) comes out true relative to that sphere. In order to evaluate \(\phi_1 \land \phi_2\), there might be a stricter sphere, \(n + 1\) from the center in which there is a \(\phi_1 \land \phi_2\)-supporting world. The second counterfactual is evaluated relative to the more strict sphere. In this way, counterfactuals are considered to be variably strict in Lewis’ system: Their strictness can change when information is added.

Tichy’s Problem

One enduring problem was pointed out with Lewis’ system of spheres by Tichy 1976[95]. Tichy set up a puzzle in which there is a man named Jones who always wears a hat when the weather is bad, but who either does or does not wear a hat when it is sunny. Given a day on which the weather is bad, what is the truth value of the counterfactual below?

71. If the weather had been sunny today, Jones would have been wearing his hat.

In the world most like the actual world, (71) is true, because it is most like the actual world not to change the fact that Jones has a hat on. Therefore, Lewis’ system predicts that the sentence is true and one in which the consequent says, ‘Jones would not have been wearing his hat’ is false, because there is a world more like the actual world in which he is not. Intuitively, however, it seems that Jones not wearing his hat, if it had
been sunny, is equally likely. There are variations on Tichy’s problem, but each deals with the problem of determining a unique, most similar world. Subsequent treatments of counterfactuals, such as Veltman 2005[99], when showing how they improve on Lewis’s 1973[54] theory, show their ability to solve Tichy’s Problem and allow either consequent to result in a true counterfactual.

4.4 Expansion of Lewis’ Ideas

Kratzer’s 1981[45] approach expands Lewis’ 1973[54] account of counterfactuals to modal auxiliaries and modal adverbs in general and further characterizes the accessibility relations and orderings needed to account for modal meaning. In fact, Kratzer’s 1981[45] work can be seen as a reformulation of Lewis’s 1973[54] theory with the following additions: It extends the account to all modal uses, not just those in counterfactual conditionals, it re-formulates spheres in terms of conversational backgrounds, modal bases, and ordering sources, and it re-defines gradability. Kratzer’s subsequent work in 1989[47] added additional structure to the propositions supported in accessible worlds by adding ‘lumps of thought’ and situations to the model.

Lewis 1973[54], only dealing with counterfactual sentences, described an ordering in which worlds were ordered solely by their similarity to the actual world. He describes many types of strictness related, for example, to deontic and epistemic uses, but does not develop these types of strictness, as they are not as clearly applicable to the target data he wishes to investigate. Kratzer 1981[45] proposes the use of multiple axes on which modal meaning in German is classified. The methods introduced have since been applied to English as well as other languages. The three axes are accessibility relations among possible worlds, ordering relations on possible worlds, and the modal force of some grade of necessity or possibility. Each lexical modal is characterized by the three axes with variation arising from the parameter setting within each axis. Unlike Lewis’ theory, there are many ordering sources that can be relevant, such as deontic ordering sources that order by what is commanded and epistemic ordering sources that order based on what is known.
4.5 Kratzer’s Analysis

In Montague’s 1973[63] system, linguistic expressions of category $t$ were sentences, and, in IL, they had the interpretation domain of the set $\{0, 1\}$. When a sentence of English is translated into IL, the translation is considered to be a (well formed) formula. Given how functional types and interpretations were defined in Chapter 3, a formula is interpreted as a function from world-time pairs to the set $\{0, 1\}$. As stated above, and repeated in Definition 24 below, there is a characteristic function that can be defined over the set of world-time pairs. Characteristic functions are an important part of Kratzer’s definition of propositions.

**Definition 24.** Given any $X \subseteq Y$, the characteristic function of $X$ over $Y$, called $f_X$, is that function from $Y$ to $\{0, 1\}$ such that for all $y \in Y : F_X(y) = 1$ if, and only if $y \in X$. And, if $f_X$ is a function from $Y$ to $\{0, 1\}$, then $X = \{y | f_X(y) = 1\}$. The characteristic function of a set $X$ and the set $X$ are interchangeable (adapted from Gamut 1991:84[32]).

The definition below is one way of describing the characteristic function that characterizes a proposition as the set of world-time pairs in which it is true:

**Definition 25.** Given any $P \subseteq W \times T$, the characteristic function of $P$ over $W \times T$, $f_P$ is that function from $W \times T$ to $\{0, 1\}$ such that for all $<w, t> \in W \times T : F_p(<w, t>) = 1$ if, and only if $<w, t> \in P$.

According to Definition 24, it can be stated equivalently:

**Definition 26.** If $f_P$ is a function from $W \times T$ to $\{0, 1\}$, then $P = \{<w, t> | f_P(<w, t>) = 1\}$. The characteristic function of a set $P$ and the set $P$ are interchangeable.

By these definitions, a proposition is the set of worlds in which it is true. It is sometimes conceptually helpful to speak of the set of propositions that are true, given a world-time pair:

**Definition 27.** Given a world-time pair, $<w, t>$, $Q$ is the set of all $P$ such that $<w, t> \in P$.

Kratzer’s definitions will make use of quantification over possible worlds in order to interpret modality.

In the definitions that follow, $[\phi]^M_{w,t,g}$ is used to mean that $M$ provides an interpretation for $\phi$ relative to $w, t, g$.
A proposition \( \phi \) is a logical consequence of a set of propositions \( \Gamma \) if, and only if, \( \phi \) is true in all \( w \in W \) such that all the propositions in \( \Gamma \) are true.

**Definition 28. Logical Consequence:**
\( \phi \) is a logical consequence of \( \Gamma \) if, and only if \( \models^{M, w', t, g} \phi = 1 \) for all \( w' \in W \) and all \( t' \in T \) such that, for all \( \psi \in \Gamma, \models^{M, w', t, g} \psi = 1 \).

Where \( \Gamma \) is a set of propositions.

Consistency is defined on sets of propositions. A set of propositions is consistent if it can be modeled in \( M \).

**Definition 29. Consistency:**
The set of propositions \( \{ \Gamma \cup \phi \} \) is consistent if, and only if there exists at least one \( w' \in W \) and at least one \( t' \in T \) such that, for all \( \psi \in \Gamma, \models^{M, w', t, g} \psi = 1 \).

A set of propositions \( \Gamma \) is consistent if, and only if, there is a world in \( W \) and time in \( T \) where all propositions of \( \Gamma \) are true.

Logical compatibility is defined as the ability to add a proposition to a set in order to form a consistent set.

**Definition 30. Compatibility:**
\( \phi \) is compatible with the set \( \Gamma \) of propositions if, and only if, there exists a \( w' \in W \) such that, for all \( \psi \in \Gamma, \models^{M, w', t, g} \psi = 1 \) and \( \models^{M, w', t, g} \phi = 1 \).

Each of these fairly standard logical notions plays a central role in defining how the more complicated parameters are defined.

Conversational Backgrounds

Kratzer’s accessibility relations involve a *conversational background*. In the previous chapter, it was mentioned that Kratzer proposed a conceptual account of modal auxiliaries in 1977 [44] in which she described each modal as taking two arguments: The first argument is a proposition and an the second is an ‘in view of’ phrase. The in-view-of phrase described the information that was relevant for the modal. For example, an epistemic modal is interpreted in view of what is known. The conversational background is a more formal way of characterizing the in-view-of phrase.

The conversational background function \( \text{Conv} \) can be reformulated as in Definition (31) below as a function from a world \( w \) to the powerset of powersets of worlds:
**Definition 31.** *Conversational Background:*

Conv : \( W \rightarrow \wp(\wp(W)) \) such that \( \forall w [ w \in \bigcap \text{Conv}(w) ] \)

Since Conv is a total function on \( W \), each world is assigned a set of propositions that are true in it.

Kratzer’s conversational backgrounds relate to accessibility relations as follows:

**Definition 32.** *If Conv is a conversational background, then the set of worlds that are accessible in a world \( w \) with respect to Conv is \( \bigcap \text{Conv}(w) \). (Kratzer 1981:297[45])*

The worlds that support every proposition in the set of propositions designated by the Conv function are the accessible worlds. The intersection of the members of each set of worlds results in a set of worlds in which every proposition in the conversational background is true. Unlike Lewis’ system of spheres, the conversational background does not form a centralized, concentric set of sets of worlds. Rather, there is a common core of worlds (supporting all relevant propositions) with no internal structure. The propositions have other worlds in their representation that do not support all of the other propositions and that, therefore, are not accessible. There is, in Kratzer’s 1981[45] work, no additional structure to the intersection of worlds in terms of the general definition of a conversational background: It only designates a set of propositions relative to which a modal is evaluated.

Necessity and possibility in Kratzer’s 1981[45] theory are defined as logical consequence and consistency, respectively, relative to a conversational background.

**Definition 33.** *Simple Necessity:*

\[ [\phi]_{M,w,t,g} \text{ is a simple necessity with respect to Conv}(w) \text{ if, and only if it follows from Conv}(w). \]

**Definition 34.** *Simple Possibility*

\[ [\phi]_{M,w,t,g} \text{ is a simple possibility with respect to Conv}(w) \text{ if, and only if it is compatible with Conv}(w). \]

There are obviously many possible assignments of propositions to a world for the function Conv. Some of Kratzer’s 1981[45] conversational backgrounds are more formally specified than others. The totally realistic and empty conversational backgrounds, for instance, constrain the possible assignments given by Conv in terms of the intersection worlds of which the propositions consist.
Definition 35. A totally realistic conversational background is a function Conv : \( W \to \varphi(\varphi W) \) such that \( \forall w \in W : \bigcap Conv(w) = \{ w \} \).

Definition 36. An empty conversational background is that function Conv : \( W \to \varphi(\varphi W) \) such that \( \forall w \in W : \bigcap Conv(w) = \emptyset \).

The totally realistic conversational background ensures that the only world that all propositions share is the actual world. It constitutes a unique characterization of the actual world. There is, however, still room for variation among assignments since there are many sets of propositions each of which has the actual world as its only common member, that is, there are various unique ways of characterizing the actual world.

In the empty conversational background, it is ensured that the set of propositions is incompatible. There are, again, many possible sets of incompatible propositions, but the important point of the definition is that the empty conversational background is incompatible. The empty conversational background, by its inconsistency, removes the influence of the conversational background as an axis of semantic interpretation.

Usually, a totally realistic conversational background is not used, but other sorts of ordinary conversational backgrounds. Two of these that Kratzer 1981\[45\] defines are realistic and stereotypical conversational backgrounds.

Definition 37. A realistic conversational background is a function Conv : \( W \to \varphi(\varphi W) \) such that \( \forall w \in W : w \in \cap Conv(w) \).

A realistic conversational background has a common core of accessible worlds in which each of the propositions is true, unlike the totally realistic conversational background, which has a single world at its intersection.

Definition 38. A stereotypical conversational background is a function Conv : \( W \to \varphi(\varphi W) \) such that the propositions that are supported by the common ground constitute the normal course of events for contextually determined individual or community.

A stereotypical conversational background assumes that all things behave as expected for the individual or community under consideration, as determined by the context. There is no formal constraint beyond the general definition of what can be part of a conversational background (i.e., a consistent set of propositions).

Other types of conversational backgrounds, such as epistemic and deontic conversational backgrounds also have a correspondence to intuitive descriptions of the types of propositions over which a modal operator would range:
**Definition 39.** An epistemic conversational background is a function \( \text{Conv} : W \to \wp(\wp W) \) such that \( \forall w \in W : \text{Conv}(w) \) contains all those propositions that are established knowledge in \( w \), (for a contextually determined group, community, etc.).

**Definition 40.** A deontic conversational background is that function \( \text{Conv} : W \to \wp(\wp W) \) such that \( \forall w \in W, \text{Conv}(w) \) contains all those propositions \( \phi \) such that \( \phi \) is commanded in \( w \) (by some contextually determined individual or law, etc.).

Kratzer 1981[45] also suggests, but does not define, other conversational backgrounds such as teleological conversational backgrounds, involving goals, and buletic conversational backgrounds involving wishes. For any reading a modal can have, there is, presumably, a conversational background expressed as an accessibility relation constrained to be things that fit that meaning category.

What Kratzer’s theory proposes are general descriptions of what types of sets of propositions are relevant for interpreting certain modal uses. How to form those sets of propositions or properties shared among those sets of propositions is not part of the theory, but is considered to be either pragmatic or, according to some linguists, beyond the scope of linguistics, in general. It would, no doubt, be useful, as a formal theory, if there were some way to formally designate the type of conversational backgrounds against which particular uses of modals are evaluated or to be able to investigate the properties of the sets of propositions that prove relevant for the evaluation of a sentence with a modal auxiliary verb. The task of providing insight into how to go about specifying such sets of propositions has been considered too vast to approach. However, it is common, in papers on the topic, for authors list portions of such sets of propositions and proceed with the interpretation of a particular modal use as a ‘proof of concept’. It is not possible yet, given state of the art tools, to extract or determine such sets of propositions from texts, partly because it is often world knowledge that is not explicitly stated that forms a significant part of the background premises against which modals are understood. Even more than the daunting task of representing world knowledge, lack of progress in the area of determining the context in which a modal is interpreted has been due to the fact that, in the last forty years since Kratzer’s theory was proposed, it has not been the primary interest of linguists to investigate the degree to which, not only readings, but also information relevant to the context in which a reading is evaluated is present in linguistic expressions in the linguistic context in which a given modal is uttered.
4.5.1 Gradability

One feature that Kratzer’s 1981[45] theory is often cited as having is that of capturing the gradability of modal auxiliary verbs (and modal adverbs). The fact that certain modals seem to convey more certainty than others needs to be captured, it is believed, in the semantic model. Lewis’ idea of comparative strictness was approaching gradability, but notions of gradability were not worked out in his proposal. Kratzer brought the investigation up to a point at which more could be modeled and understood about the notion.

Ordering relations are defined to determine which worlds are considered to be the closest, corresponding to Lewis’ closeness definition for worlds in systems of spheres. The ordering relations are also crucial in dealing with the grading that occurs with modal expressions. Kratzer calls the possible orderings ordering sources and defines ordering sources as follows:

**Definition 41.** $\Gamma$ can induce an ordering $\leq_{\Gamma}$ on the set of all $w \in W$.

The ordering source, $\leq_{\Gamma}$, is defined as follows: For all $w \in W$ and $w' \in W$, $w \leq_{\Gamma} w'$ if, and only if, $\{\phi : \phi \in \Gamma \land w' \in \phi\} \subseteq \{\phi : \phi \in \Gamma \land w \in \phi\}$.

The set $\Gamma$ is called an ideal. A world $w$ is at least as close to the ideal $\Gamma$ as a world $w'$, if, and only if, all propositions of $\Gamma$ that are true in $w'$ are true in $w$ as well. The ordering relation results in something similar to the centered system of spheres: By using the subset relation, it ensures that a world is closer to the ideal if the propositions true in it are a subset of (or the same as) the propositions true in other worlds.

Given the ordering relation, Kratzer introduces more fine-grained modal relations: Human necessity and possibility, slight possibility, and comparative possibility.

**Definition 42.** Human Necessity:

A proposition $\phi$ is a human necessity given a world $w' \in W$, a function $\text{Conv}$, and an ordering relation $\gamma$ if, and only if, the following condition is fulfilled:

For all $w' \in \cap \text{Conv}(w)$ there is a $w'' \in \text{Conv}(w)$ such that $w'' \leq_{\gamma(w)} w'$ and for all $w''' \in \cap \text{Conv}(w)$: If $w''' \leq_{\gamma(w)} w''$, then $w''' \in \phi$.

A human necessity is defined relative to both a conversational background and an ordering source. A proposition is a human necessity if it is true in all accessible worlds that come closest to the ideal. There may be many closest worlds, not a unique one.
Definition 43. Human Possibility:
A proposition $\phi$ is a human possibility given a world $w \in W$, a function $\text{Conv}$, and an ordering relation $\gamma$ (written $\leq_\gamma$), if, and only if, it is not the case that its negation, $\neg \phi$ is a human necessity given the same world, function, and ordering relation.

A human possibility is defined as the dual of human necessity. A proposition is a human possibility if it is true in at least one closest world. If its negation is true in all of these closest worlds, then it cannot be a human possibility.

Definition 44. Slight Possibility:
A proposition $\phi$ is a slight possibility in a world $w$ with respect to a modal base $\text{Conv}$ and an ordering source $\gamma$ if, and only if, $\models_{M,w,t,\gamma} \text{Conv}(w) \cup \{\phi\}$ ($\phi$ is compatible with $\text{Conv}(w)$) and $\neg \phi$ is a human necessity in $w$ with respect to $\text{Conv}$ and $\gamma$.

A slight possibility is something that is not possible in all of the closest worlds, but that, if slightly less close worlds are considered, is possible.

Definition 45. Comparative Possibility:
A proposition $\phi$ is more possible than a proposition $\psi$ in a world $w$ in view of a modal base $\text{Conv}$ and an ordering source $\gamma$ if, and only if, the following conditions are satisfied:

(i) For all $w' \cap \text{Conv}(w)$: If $w' \in \psi$, then there is a world $w'' \in \cap \text{Conv}(w)$ such that $w'' \leq_{\gamma(w)} w'$ and $w'' \in \phi$.

(ii) There is a world $w' \in \cap \text{Conv}(w)$ such that: $w' \in \phi$ and there is no world $w'' \in \cap \text{Conv}(w)$ such that $w'' \in \psi$ and $w'' \leq_{\gamma(w)} w'$.

Comparative possibility allows the comparison of propositions to see which is more possible than the other. Kratzer’s comparative possibility has the property of being transitive and symmetric, and it says that, for every accessible $\psi$-world, there is an accessible $\phi$-world that is at least as close to the ideal, and that there is no accessible $\phi$ world for which there is no accessible $\psi$-world that is at least as close to the ideal. Much more research has been done, since Kratzer’s 1981[45] publication, on how human agents evaluate comparative possibilities.

The following equivalences are true for any modal base $\text{Conv}$ and the empty ordering source $\gamma$:

Definition 46. Simple and Human Necessity:
A proposition is a simple necessity in a world $w$ with respect to $\text{Conv}$ if, and only if, it is a human necessity in $w$ with respect to $\text{Conv}$ and $\gamma$. 
Definition 47. Simple and Human Possibility:
A proposition is a simple possibility in a world \( w \) with respect to Conv if, and only if, it is a human possibility in \( w \) with respect to Conv and \( \gamma \).

When there is no ordering source, there is no difference between human and simple necessity or human and simple possibility.

4.5.2 Accounting for the Data

The conversational background can be used to determine either the modal base (i.e., the set of propositions against which a modal is evaluated) as well as to determine the ordering source, that is, the closeness of the worlds supporting the propositions to some ideal. Different types of conversational backgrounds can be used to determine each. For instance, something can be evaluated in the worlds determined by an epistemic modal base with an ordering determined by a stereotypical conversational background. Such a context would be one in which the information expressed is limited to what the speaker (or assessor) has knowledge of, but, out of what information they have knowledge of, that information that follows the normal pattern of events is closer to the world of evaluation than that information that does not. It is also possible to have accessibility relations defined but ordering sources left empty, or vice versa.

An example that Kratzer gives is a distinction between a circumstantial and epistemic reading. She presents the following data (from Kratzer 1981:303[45]):

72. In dieser Gegend könenn Zwetschgenbäume wachsen.
In this area, plum trees can grow.

73. Es kann sein, daß in dieser Gegend Zwetschgenbäume wachsen.
It may be that plum trees are grown in this area.

Sentence (72) can have either an epistemic or a circumstantial reading, whereas sentence (73) only allows an epistemic reading. On the circumstantial reading, it is the possible to be said of an environment in which plum trees could grow, given the nature of the soil and the temperature. The epistemic reading can, however, be false, in the same scenario, for instance, if it is known to be the case that plum trees will never grow there. For instance, because it is said in a future time when plum trees are unrecoverably extinct. It is not the case, in such a scenario, that sentence (73) can be used felicitously.
4.6 Conclusion

This chapter has discussed a portion of the literature covering model-theoretic interpretations of modal auxiliary verbs. Some modal auxiliary readings have received more attention than others. In particular, counterfactual conditionals and epistemic modal readings have been of interest in demonstrating the need for more complex interpretation models.

There remain open questions in the interpretation of modal auxiliary verbs. In particular, questions remain unresolved around which is the best way to handle gradability and ordering of worlds.

There is a debate regarding the role of epistemic reasoning in modal readings that are not only epistemic. And there is a debate whether to handle epistemic modals in terms of contextual analyses or relativist analyses. However, it is clear that, whatever changes are made to the formal methods, a theory of modal interpretation needs certain components.

These accounts of the interpretation of modal auxiliaries have largely ignored the grammatical environments in which they occur and the individual contributions of linguistic expressions in that context.

One large hole in the coverage of the theories discussed so far is regarding so-called ‘future’ uses of will, ‘future in the past’ uses of would, and other uses of modal auxiliaries that appear to have primarily a temporal meaning.

The notion of context has been used, in the discussion of modal auxiliaries, to include a heterogenous set of things including pragmatic contexts, world knowledge, and reasoning about the beliefs of others on one hand and linguistic context on the other. It is not always possible to separate these types of context. For example, linguistic expressions that introduce intensional contexts and indexical expressions are part of a linguistic context that must be tied to a context that involves world knowledge. However, by making the grammar and model provide interpretations to these expressions, it is possible to describe important distinctions regarding how context (in the general sense) determines the reading of a modal auxiliary, as the next chapters will illustrate.
5.1 Taxonomies of Tense and Aspect

In a non-formal grammar of English, tense and aspect are often broken down into just a few categories. It is common to read in English textbooks that there are three tenses, which are the past, present, and future, and that there are three aspectual markings, which are the simple, progressive, and perfect. If the goal of explaining tense and aspect is to teach a student the names of what parts of speech exist, for instance, these categories might be good enough, at least for speakers with a beginner to early intermediate proficiency in English. The six labels are also a sufficient explanation for proficient people who are learning labels for uses that they already employ. It is possible to give straight-forward examples of each type of tense and each type of aspect without getting into the details of why more explanation is necessary. For instance, a basic grammar might say, English has three tenses, the past, present, and future, and give example sentences:

74. Jess sold cookies. (SIMPLE PAST)

75. Jess sells cookies. (SIMPLE PRESENT)

76. Jess will sell cookies. (SIMPLE FUTURE)

In (74), each tense is given with simple aspect. The other aspects are the progressive and the perfect, permuted below:

77. Jess {was/is} selling cookies. (PAST/PRESENT PROGRESSIVE)

78. Jess will be selling cookies. (FUTURE PROGRESSIVE)

79. Jess {had/has} sold cookies. (PAST/PRESENT PERFECT)
80. Jess will have sold cookies. (FUTURE PERFECT)

81. Jess {had/has} been selling cookies. (PAST/PRESENT PERFECT PROGRESSIVE)

82. Jess will have been selling cookies. (FUTURE PERFECT PROGRESSIVE)

Similarly, for applied tasks, such as the task of creating a part of speech tagger, teaching the basic morphology of temporal and aspectual markers in English is sufficient to get accurate labeling. Part of speech taggers work pretty well with just the information, for instance, that the verbal ending -ing means that the verb is labeled as having progressive aspect. Tense and aspect get complicated, however, when they are no longer characterized in terms of connecting a particular morphology to a particular grammatical name for the morphology.

For example, the progressive morphology has a lot of nuances of meaning. The sentence Jess was sneezing can mean that Jess was about to sneeze or that she was sneezing over and over again. The sentence Jess was building a house does not entail that there is a completed house that Jess built. In contrast, the sentence, Jess is jogging does entail that there was an eventuality in which Jess jogged.

When trying to model how multiple states of affairs relate to each other, in terms of starting and ending points, or when events overlap, versus when they are sequential, the system of three tenses and three aspects does not provide enough information. The interactions of tense and aspect with modality create an even more complicated picture.

5.2 Tense

The system of tense developed by Reichenbach 1947[82] used three times to define tense. The utterance (or speech) time is the time when the linguistic example is authored. The event time is the time when the event begin described takes place. The reference time is the time to which the utterance refers.

Reichenbach 1947[82] proposed that many categories of tense and aspect can be characterized by a relation among these three times. For example, simple past tense is characterized by a reference time that precedes the speech time and an event time that equals the reference time. When the reference time precedes the speech time but the event time precedes the reference time, the tense conveyed is still past, but, rather than being simple past, the category is the past perfect.
It was noted explicitly in Klein 1994[4] that Reichenbach’s 1947[82] system handled tense via a two-place relation between the speech time and the reference time. Aspect, in contrast, was handled in terms of a two-place relation between the reference time and the event time.

The types of two-place relation operators used to describe tense and aspect depend on the temporal ontology in place, but typically include some variant of at least the three relations of equality, precedence, and inclusion.

Tense, in general, is characterized as being past, present, or future. The past represents a precedence relation such that the reference time precedes the speech time. The present represents either an equality relation where the speech time and reference time are the same, or, more commonly, an inclusion relation such that the speech time is included in the span of the reference time. The future involves a (possible non-strict) precedence relation such that the speech time precedes the reference time.

5.3 Taxonomies of Aspect

Aspect is a term that is conventionally used to cover a number of heterogenous facets of natural language.

Dik 1997[24] helps clarify the meaning of aspect by breaking the notion up into five sub-categories. The first category refers to the type of state of affairs, which includes the Aktionsart discussed by Vendler 1957[100]. It refers to the type of action the lexical items in the verb phrase encode. The second category is that of Perfectivity or Imperfectivity. The perfective grammatically encodes point of view in which the state of affairs is an indivisible whole. The imperfective aspect encodes an ‘inside point of view as being non-complete or in progress.’ (Dik 1997:221[24]).

The third is phrasal aspectuality, which indicates whether it is the beginning, continuation or end of the state of affairs. Phrasal aspectuality refers to aspectual markers that signify in which phase of development a state of affairs is. Examples, modified from Dik (1991:225[24]), are:

**Ingressive** Jess started picking raspberries.

**Progressive** Jess was picking raspberries.

**Continuous** Jess continued picking raspberries.
**Egressive** Jess stopped picking raspberries.

In English, all forms of phrasal aspectuality, except for the progressive, are marked by lexical verbs indicating their aspect, such as *started*, and *stopped*. Progressive and continuous phrasal aspectuality co-occurs with imperfective aspect (Dik 1997:223[24]), because they describe the internal structure of a state of affairs.

For Smith 1991[89], phrasal aspectuality is conflated with viewpoint. For example, Smith’s definition of perfective viewpoint precludes all of Dik’s categories of phrasal aspectuality. Only progressive and continuous are compatible with her definition of imperfective aspect. The ingressive seems to fit her neutral viewpoints definition. It is not clear how egressive aspect would be treated in her system.

In this model, the ingressive is represented by the initial point of a state of affairs and the egressive by the final point of a state of affairs. Both progressive and continuous uses, as explained in Dik’s phrasal aspectuality model, constitute phases within the relevant state of affairs.

The fourth type is *perspectival aspectuality*, which ‘relate[s] the occurrence of the state of affairs to an outside temporal reference point’ (Dik 1997:221[24]), for example, if the perspective is one of just entering the state of affairs or just following it. The values Dik gives are prospective aspect, immediate aspect, recent perfect, and perfect aspect (not all of which occur as grammatical morphemes in English).

Smith 1991[89] presents a formal theory of aspect and discusses similar categories. Her theory accounts for Aktionsart, which she calls *situational aspect* and *viewpoint aspect*, which includes perfective, imperfective, and neutral viewpoints. Her account of quantificational aspect occurs within her characterization of some Aktionsarten and viewpoint interactions. This monograph will use much of Smith’s 1991[89] theory but also include portions of Dik’s 1997[24] theory as components.

Perspectival aspect refers to the way in which a state of affairs is viewed from an external point of view. Dik (1991:238-239[24]) lists the types of perspectival aspectuality as prospective aspect, immediate aspect, recent perfect, and perfect aspect. The difference between future tense and prospective aspect and between past tense and perfect aspect are not immediately clear. Distinguishing examples are given in the (modified) examples below:

**Future Tense** One day, Jess will pick raspberries.

**Prospective Aspect** Jess is going to pick raspberries.
Past Tense  Last spring, Jess picked raspberries.

Perfect Aspect  Jess has picked raspberries since last spring.

The future tense sentence refers to something Jess will do in the remote future. The prospective aspect, in contrast, is ‘a prediction about what is going to happen in the future on the basis of what information the speaker has now’ (Dik 1997:239[24]). The difference between the past and the perfect is that the perfect continues to have relevance to the present moment.

The fifth category is quantificational aspectuality, which refers to quantification over sets of occurrences of states of affairs (Dik 1997:222[24]). Dik includes iterative aspect, habitual aspect, and frequentative aspect. Aspectual quantification quantifies over states of affairs without referring to the internal structure. It refers to the frequency of a state of affairs. Dik (1997:236[24]) gives the following examples of predication operators signifying quantificational aspect:

Semelfactive Aspect  just a single time

Iterative Aspect  several times

Frequentative Aspect  many times

Distributive Aspect  several times, with different participants

Smith 1991[89] discusses aspectual quantification as a derived feature of certain Aktionarten and world knowledge. Aspectual quantification is significant in modal descriptions. As Nuyts 2005[71] suggested, quantificational aspect was the primary meaning of some modal uses that are generally considered under the term dynamic.

Aspectual quantification has been described in the case of when semelfactives combine with progressive aspect. The reading in which a repeated occurrence takes place is a change from semelfactive aspect to iterative aspect.

Quantificational Aspect is used in Nuyts 2005[71] to refer to the number of repetitions of an event. It is different from qualitative aspect, which describes the internal quality of an event. Modals are claimed to be stative (e.g., see Hacquard 2006[34]), but the qualitative aspect of a modal expression, in this work, is taken to be determined by the verb with which the modal combines. There are some limitations on which modal readings occur with which verbs, apparently due to the qualitative aspect of the main
verb phrase (see Coates 1983[16]). But, in general, qualitative aspect is not considered to be part of modal representation but a feature of the verb phrase that interacts with modal meaning.

Interactions among Types of Aspectuality

Dik (1997:223[24]) states that iterative and habitual readings only occur naturally with the imperfect aspect but can occur with the perfect aspect, resulting in a modified interpretation as conative (interrupted), iterative, or distributive:

83. *Jess tried to pick a quart of raspberries.* (conative)

84. *Jess picked a quart of raspberries several times.* (iterative)

85. *Jess, Pat, and Sydny each picked a quart of raspberries.* (distributive)

The telic picked-a-quart-of-raspberries state of affairs, though closed and complete, is forced to an interpretation by the surrounding linguistic context.

Smith 1991[89] describes a number of derived uses of aspect that occur when, for instance, a temporal adverbial changes the way in which an Aktionsart is understood. In her formal theory, she lists basic and derived meanings for each Aktionsart, which will be outlined.

5.4 Aktionsart

For the purpose of modeling a grammar, it is important that the model be as conceptually clear as possible while also achieving the desired description. It makes sense, therefore, to group eventualities into categories according to their most likely Aktionsart. Since prepositions, plural noun phrases, directional phrases, amount noun phrases, and grammatical aspect have systematic effects on Aktionsart, the changes can be described by rules. The result is that no eventuality has an absolute Aktionsart regardless of the linguistic expressions with which it occurs. But it is possible to describe the Aktionsart of an eventuality in a sentence. It is possible to determine which portion of the eventuality is being referred to and how that eventuality is situated relative to the time at which the utterance occurs, as well as relative to other contextual factors.
The first type of aspectuality mentioned, the Aktionsart was most famously discussed in Vendler 1957[100]. Vendler presented four major categories of Aktionsart, two types of continuous Aktionsarten and two types of discontinuous Aktionsarten.\(^1\) The continuous Aktionsarten included activities and accomplishments. Activities are continuous states of affairs that, no matter at what point they cease, still constitute a case of the participant engaging in the state of affairs. Vendler’s example was that of pushing a cart. At any point at which someone stops pushing a cart, it can still be said that they pushed a cart. Accomplishments are continuous Aktionsarten that have a definite culmination point. One of Vendler’s examples was that of drawing a circle. At any point in time, when drawing a circle, a participant is in the process of drawing a circle, but if the state of affairs of drawing a circle is interrupted before the circle’s completion, then it can not be said that the person drew a circle.

The discontinuous Aktionsarten include achievements and states. Achievements are states of affairs that are punctual. Vendler’s example was that of reaching the summit, which happens in an instant. At any point before reaching the summit, the participant was doing something that led up to reaching the summit, such as climbing the mountain, but they reached the summit in an instant. States are states of affairs that happen over a span, such as being happy or knowing something. The change of state from not knowing something to knowing something is instantaneous, but the state lasts in time.

Several finer-grained distinctions have been added to the Aktionsarten of Vendler 1957[100]. Figure 5.1 summarizes the divisions presented in Boland 2006[10], Dik 1997[24], and Smith 1991[89]. The first division in Aktionsarten is between non-dynamic and dynamic uses. Non-dynamic uses are states. Carlson 1977[12] proposed two types of states.\(^2\) Individual-level states are traits that an individual possesses that generally cannot be changed, such as having blue eyes or being a certain height. Stage-level states are states that are only temporary, such as being happy or sad.

Dynamic Aktionsarten are divided into telic and non-telic (atelic). Telic means ‘goal oriented’ and is used of states of affairs that reach a point of culmination, such as the accomplishments and achievements of Vendlers 1957[100] classification. Non-telic Aktionsarten include Vendler’s activities as well as semelfactives (Smith 1991[89]). Semelfactives are punctual, atelic events such as jumping, knocking on a door, or sneezing, that can happen in a series.

\(^1\)Vendler used the terms continuous tense and discontinuous tenses, but, due to the way the word tense is being used technically in this monograph, types of Aktionsart will not be called tenses.

\(^2\)Carlson used the terms individual-level predicates and stage-level predicates.
Aktionsarten

Non-Dynamic
states
(individual level, stage level)

Dynamic

Non-Telic

Telic
Change of State/Event

Non-Punctual
Activities/Processes

Punctual
Semelfactives

Non-Punctual
Accomplishment

Punctual
Achievements
The telic Aktionsarten are also divided into punctual and non-punctual states of affairs. The non-punctual states of affairs include accomplishments, which have a duration, and the punctual include achievements, which happen in an instant.

When considering a given verb and its obligatory arguments, there is a basic Aktionsart that best describes that verb. When thinking of sneezing, there is a state of affairs in which there is a participant and an act of sneezing as in sentence (86) below:

**86. Jess sneezed.**

The Aktionsart of the verb *sneeze* is semelfactive. Sneezing is a punctual, atelic state of affairs. But the Aktionsart is not a permanent property of the predicate *sneeze* and its obligatory argument of a participant who sneezes. If, for instance, non-obligatory arguments, sometimes called adjuncts, were added, the Aktionsart would be construed as an accomplishment:

**87. Jess sneezed fifteen times in three hours.**

In sentence (87), the state of affairs of sneezing fifteen times in three hours is cumulative and durative, neither of which are properties of semelfactives, which are non-cumulative and punctual.

There are many instances in which the Aktionsart of a predicate and its obligatory arguments changes with the addition of adjuncts. Sometimes these changes are called *coercions*. Since there are relatively systematic ways in which the Aktionsart of a predicate and its obligatory arguments are altered by adjuncts or other grammatical features, it is important to characterize coercion in terms of interactions of adjuncts and grammatical features with the basic Aktionsart of a predicate.

The basic Aktionsart is a lexical property of the meaning of the predicate and its obligatory arguments. The basic Aktionsart of many predicates is likely to be the same across languages just because states of affairs like building a house are always going to be composed of accumulating steps and sneezing is always going to be perceived as being fairly punctual. However, it is known that the Aktionsart of predicates and their arguments can vary cross-linguistically (see Smith 1991[89]). Sometimes a derived Aktionsart in English is the basic Aktionsart of the predicate in another language and vice versa. Because of these known cross-linguistic variations, Aktionsart is generally held to be a lexical property of a verb and its obligatory arguments, and, consequently, something that is learned with the acquisition of the basic meaning of a predicate in a given language.
Almost every eventuality, until considering the behavior of sub-atomic particles, includes sub-eventualities that can be construed as sub-parts. Some eventualities are consciously conceived of as overlapping a number of sequential sub-eventualities of which they consist. Building a house is an eventuality that includes sub-eventualities of building a floor, building a roof, and others. In contrast, other eventualities are conceived of as having sub-eventualities that are often not salient enough to be considered. For example, the eventuality of pushing a cart includes sub-eventualities of turning a wheel on the cart, moving the cart forward, moving the pusher forward, and taking steps. Each of these sub-eventualities occur relatively simultaneously, compared to something like the sub-events of building a house. And, unless a narrative consciously slows down or speeds up the pace of eventualities, there is a way in which they are perceived relative to each other. Some eventualities are perceived as nearly instantaneous and other as quite long and drawn out, given the normal passage of time and normal way of things unfolding.

It is often the conception of an eventuality having sequential sub-eventualities or culminating sub-eventualities that lends eventualities the property of being conceived of as being durational (Dowty 1979[25], Krifka 1989[49]). The reason why building a house is considered to be durational and sneezing to be punctual is because the sequence of sub-events that are contained in a sneeze are too quick to notice compared to the process of building a house. Also, while it is possible to talk about the sub-events of building a house meaningfully, it is not the same with the sub-events of pushing a cart. Someone can be said to have built a roof or to have built many roofs, but it is less common to talk about someone having turned a wheel one-quarter turn or having turned a wheel many quarter turns.

In the classic system, there are ways of talking about what features different Aktionarten possess. Smith 1991[89] breaks Aktionarten down into the following features, which have been introduced in the previous sections, as shown in Table 5.1:

These features are used in the lexical entries of predicates in the current grammar.
Table 5.1: Features of situation types from Smith (1991:30[89]), where ‘−’ means the value is not applicable.

5.5 Perfectivity and Imperfectivity, also called Viewpoint Aspect

Perfective aspect is commonly used to convey that a state of affairs is completed. Imperfective is used for states of affairs that are seen as ongoing.

In English, perfective and imperfective are not always grammatically marked. The phrasal aspect type of progressive is often associated with imperfective grammatical marking, but the two are not the same thing in Dik’s classification. Grammatical forms like perfect aspect are called perfect in contrast to perfective, which has a general meaning of complete, bounded, non-divisible, and closed states of affairs. There are uses of simple past, for instance, which are considered to be perfective in Dik’s classification.

Dik (1997:223[24]) gives additional descriptions of what he considers to be a unified semantic meaning of imperfective aspect as non-complete, non-bounded, divisible, and open. Smith 1991[89], in contrast, describes three viewpoints: perfective, imperfective, and neutral. Perfective and imperfective have semantics similar to Dik’s: perfective viewpoints include both initial and final points. Imperfective viewpoints focus on stages that ‘are neither initial nor final, excluding endpoints’ (Smith 1991:93[89]). Neutral viewpoints include the initial point and at least one stage of a situation.

5.5.1 Progressive Aspect

The progressive aspect is marked be the use of to be as an auxiliary with an -ing ending on the main verb. Progressive aspect can occur with past, present, or future tense, and it can occur with or without perfect aspect.

88. Jess (is/was/will be) going to the store.
89. Jess (has/had) been going to the store.

The progressive is generally characterized as containing the reference time inside of the eventuality time (Dowty 1979[25]).

5.5.2 Progressive Aspect and Aktionsart

It was mentioned above that adjuncts, such as quantifying expressions like for an hour can change the natural or basic lexical Aktionsart of a predicate. Pushing a cart is atelic and has an arbitrary final point whereas pushing a cart for an hour is telic and has a final point. The presence of progressive aspect in English also changes the Aktionsart of a predicate.

Vendler’s continuous forms, the activities and accomplishments, are unchanged by the progressive aspect. In sentences (90) and (91) below, the features of the Aktionsart do not change:

90. Jess was building a house.

91. Jess was pushing a cart

The state of affairs in which Jess was building a house remains durative, cumulative, and heterogeneous. The state of affairs in which Jess was pushing a cart remains durative, atelic, and homogeneous.

The non-continuous forms are changed to have a continuous reading. The progressive only alters the nature of the state of affairs with respect to its temporal duration. It expands the state of affairs either by referring to sub-phases of a state of affairs that is generally considered to be non-durative in its basic Aktionsart, or, alternatively, expanding the reference time by means of repeating a non-durative state of affairs over an interval. Readings are ambiguous between the two. For example, the sentence Jess is sneezing can mean either that Jess is rapidly inhaling and making sounds like someone who is about to eject air in a sneeze (cumulative reading) or Jess is sneezing over and over again (iterative reading).

92. Jess was sneezing when I ran up behind him and made him scream.

On the cumulative reading, the sentence in (92) does not imply that Jess sneezed, much like other cumulative states of affairs. On the iterative reading, it could mean he did sneeze, but the iteration stopped with the interruption.
In general, achievements tend to be more easily read with the progressive as referring to a pre-state, whereas semelfactives are more commonly read as a repetition of the eventuality over a non-punctual interval.

93. *Jess was reaching the summit.*

The sentence above is easier read as Jess being involved in the culminating activities leading up to reaching the summit than as Jess reaching the summit repeatedly. With additional context, however, the repeated reading is totally non-anomalous:

94. *Jess was reaching the summit every time she climbed when she could use coca tea to help with the atmospheric sickness.*

In each case, the fact that the progressive means that the speech time is internal to the eventuality time causes the interpreter to perceive the eventuality as being over a long enough time period that the speech time fits within it. If given a made-up eventuality, an interpreter would, presumably, not know in which way the eventuality was structured. They might at first assume a naturally durational eventuality.

The progressive does not change a telic eventuality to an atelic one.

95. *Jess was building a house.*

The reading is still telic, unless an additional preposition is added:

96. *Jess was building on a house.*

97. *Jess was reaching for the summit.*

The typical distinction between *in an hour* and *for an hour* does not distinguish the cumulative reading from the iterative reading. Both take *for*:

98. *Jess sneezed in two minutes.*

99. *Jess sneezed for two minutes.*

It is only when a specific number of sneezes are mentioned that the preposition *in* is used:

100. *Jess sneezed fifteen times in two minutes.*

101. *Jess sneezed fifteen times for two minutes.*
Progressive changes states to dynamic predicates. There is a difference between sentence (102) and sentence (103) below:

102. Jess is happy.

103. Jess is being happy.

When it is said that Jess is being happy, it is as though he is displaying behaviors typical of a happy person. The use is possibly related to the agentive be discussed in Partee 1977[76]. Due to the way in which the use implies either agency or actions displaying being in a particular state, it is odd when used with i-level states as in sentence (104) below:

104. Jess is being tall.

The meaning, if it is understandable at all, is that Jess is doing something that represents the state of being tall, such as standing on stilts or a chair.

The changes that are the result of using grammatical progressive aspect marking in English are systematic and exist, as well, when perfect progressive forms are used, such as ‘has been being tall’. It will be shown, in the model, that the interactions between lexical aspect, viewpoint aspect, and tense are systematic. Furthermore, it will be shown that modals act in a predictable way with the tense and aspect of a predicate.

The progressive aspect, when used with dynamic predicates in the present tense, results in two possible readings: One in which the first participant is planning the state of affairs at the time at which the utterance occurs and one in which the first participant is participating in the state of affairs at the time at which the utterance occurs. The former reading is sometimes called a planning reading. It can be paraphrased with is going to, which is sometimes called the futurate use (Copley 2009[20]):

105. Jess is picking raspberries this afternoon.

106. Jess is going to pick raspberries this afternoon.

In the planning readings, the initial point of the state of affairs is shifted forward to the time at which the first participant had the intention. The planning phase, however, interacts differently. For instance, if Jess is interrupted before picking raspberries, that is, during the phase in which he is planning to pick them but has not begun picking them yet, it cannot be said that he picked raspberries. In contrast, if he is interrupted
in the process of picking raspberries, then it can be said that he picked raspberries. For this reason, shifting to the planning phase is considered to be a different type of state of affairs than is referred to in cases in which the state of affairs proper is being referenced.

Viewpoint aspect relies crucially on a theory of tense and a formal theory will, therefore, be presented in conjunction with a theory of tense.

In many languages, the progressive is grammatically incompatible with stative predicates. The same could be said in English, though it appears that the progressive is changing the reading of the predicate, it is more accurate to say that it is not a state being referenced by the speaker.

The progressive situates the speech time inside of the eventuality time. It does not coerce a telic eventuality into an atelic eventuality. By pragmatic inference, eventualities that are generally perceived as punctual are viewed from a perspective in which they included the speech time. There are three ways in this perception is understood. The first is through conceptually slowing the eventuality down so that the speech time fits within it. An example would be when someone talks about a video of someone sneezing and slows down the film to show the process of sneezing. This can be done narratively or conceptually as well.

The second is through iterating the eventuality, and the third is through conceptually expanding the eventuality time to include either a planning phase or the eventualities that often proceed the named eventuality in the normal way in which eventualities in the world proceed.

Slowing the eventuality down is a narrative device that involves manipulation of time. The other two readings are pervasive in any narrative. Semelfactives tend to be interpreted as iterating a semelfactive (one-time) event and achievements as including a pre-eventuality phase. The most available interpretation retains the telicity feature of the Aktionsart. It is a weaker type of pragmatic accommodation. Stative eventualities are incompatible with the progressive. When a predicate that is often stative is used with the progressive, it does not refer to a state, but to the act of doing something characteristic of a state.

5.6 Perfect Aspect

The perfect has multiple uses in English. One use is one of completion:
107. *Jess has run the algorithm successfully.*

In the sentence above, the eventuality has taken place. There is, however, an additional feeling it has, which is described as a ‘relevance to the present’. The sentence can be used to demonstrate that Jess possesses, at the time of speech, the trait of having successfully run the algorithm.

108. *Jess ran the algorithm successfully.*

Reading of the perfect also include experiential readings, resultative readings, recent news readings, and universal readings (McCawley 1971[60], Comrie 1976[17], McCoard 1978[61], Binnick 1991[9], Reed2012[81]). Experiential readings describe something that the first participant has done, as in sentence (109) below:

109. *Jess has tried pimento loaf.*

Resultative readings refer to an event that has some result state that holds at the point to which the utterance refers:

110. *Syd has painted the walls yellow.*

Recent news readings are about an event that has just occurred:

111. *Jess has just arrived.*

Universal readings are readings that describe events that hold in a time including the time of utterance.

112. *Pat has lived in Lisbon since last summer.*

Sentence (112) above does not have to mean that Pat lived in Lisbon in the past but no longer lives there, rather, it can mean that she lives there at the speech time.

5.7 Imperfect Progressive and Aktionsart

Grammatical aspect is a term that covers both the perfect in English and the imperfect progressive. Grammatical aspect involves the perspective that is taken on a given eventuality. There are at least two ways to view the interaction between Aktionsart and grammatical aspect. On one view, the grammatical aspect does not change the nature
of an eventuality in the ontology, but it can limit which part of the eventuality is being spoken about. For example, if Jess is building a house, and the house is not yet complete, talking about Jess building a house does not change the fact that eventualities of building a house are telic and culminate in the completion of the house. Rather, the progressive specifies that the temporal location of the speaker at the time of the utterance is before the culmination point. The truth value of the eventuality of the type such that there is a building event and the culmination of that building event is a house and the builder is Jess is not determined at the speech time. Ontologically, the eventuality is a cumulative and telic eventuality.

On another view, when the speaker is talking about a Jess building the house event, it is not a cumulative and telic eventuality that is evaluated. Rather, the eventuality at that time does not have a culmination point. The culmination point lies beyond what is known at the speech time.

The approach to the imperfect progressive taken by Dowty 1979[25] includes aspects of both of these approaches. It states that the eventuality under discussion is telic and culminates in a completion point, but the speech time is located such that the part beyond the speech time is merely hypothetical. It is not untrue to say Jess was building a house and continue with when she decided her time was better spent on community education and stopped the construction of the house permanently. The fact that Jess was building a house does not entail that Jess built a house.

113. Jess was building a house when she decided her time was better spent on community education and stopped the construction of the house permanently.

The discussion of entailments of the imperfect progressive have focused on the past tense of the imperfect progressive. The speaker has a perspective where they are aware that the building of the house was never completed. Narratively, the past imperfect progressive takes the interpreter back to a perspective that is situated inside the time when Jess was in the process of building a house, then transitions into another eventuality that disrupted the telic, cumulative eventuality. It is possible to do the same with the present tense:

114. Jess is building a house when she decides that her time is better spent on community education and stops the construction on the house permanently.

Sentence (114) is more strongly indicative of a narrative device than sentence (113). While sentence 113 is a fairly natural way to report an interrupted event, the present
tense version sounds like the plot summary of a script or a script for the roles people are to play in a prescribed series of events.

Generally, uses of a sentence like *Jess is building a house* take place when the speech time is included in the time of the eventuality. In order to cover the past tense, it is necessary to note that it is the reference time that is included in the eventuality. So, in order to make a more precise generalization, with the imperfect progressive, the reference time is located inside of a telic eventuality. The reference time in the present tense use is the same as the speech time (except in script-like narratives) and the reference time precedes the speech time in past tense uses. This pattern follows the common generalization that tense determines the speech time relative to the reference time, and grammatical aspect determines the relation between the reference time and the event time (Klein 1994[43]).

<table>
<thead>
<tr>
<th></th>
<th>Perfect</th>
<th>Perfective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Past</td>
<td>had gone</td>
<td>was going to</td>
</tr>
<tr>
<td>Non-Past</td>
<td>has gone</td>
<td>is going to go</td>
</tr>
</tbody>
</table>

Table 5.2: Examples of Grammatical Aspect

5.7.1 Conclusions

This chapter has discussed theories of tense and aspect, including various classifications of aspect. The next chapter will present a formal grammar in which tense and grammatical aspect are encoded.
CHAPTER 6

FORMAL GRAMMAR

6.1 Introduction

This chapter presents a Multi-Modal-Combinatory Categorial Grammar (MM-CCG) (Baldridge & Kruijff 2003[6], Steedman 2012[93]) of an English language fragment. A model-theoretic interpretation for the fragment is given in an expansion of a relativist model, as proposed in Lasersohn 2015[52]. A neo-Davidsonian event semantics (Davidson 1967[22], 1980[23], Parsons 1990[74]) is used with a feature-structure based lexicon (Pollard & Sag 1994[77], Sag, Wasow, & Bender 2003[85], Villavicenio 2002[101]). The temporal and aspectual theory is based in that of Klein 1994[43]. The temporal ontology and the event ontology follow that of Muskens 1995[68].

6.1.1 Framework for the Treatment of Tense and Aspect

Tense and grammatical aspect are represented as relations between times, using the three Reichenbachian times \textit{reference time}, \textit{utterance time}\textsuperscript{1}, and \textit{event time}. Following Klein 1994[43] and other subsequent work, tense is considered to be a relation between utterance time and reference time, and aspect is considered to be a relation between event time and reference time.\textsuperscript{2}

The focus of this work is on giving a treatment of tense and aspect which shows sensitivity to the nuances of grammatical aspect. The nuances represented reflect the observations in Reed 2012[81], in that it defines perfect and prospective grammatical

\textsuperscript{1}Reichenbach 1947[82] used \textit{speech time}.

\textsuperscript{2}Klein 1994[43] does not use this terminology. He considers the concept of reference time not to be well-defined. He calls a related notion \textit{topic time} that is, “the time for which a claim is made”(Klein 1994:3[43]) and states that it is expressed via the finite component of a sentence, where \textit{finite} generally refers to a tense-marked verbal head and non-finite to situations without a temporal location, as in \textit{Jess ride a bike}. Klein’s event time is called \textit{time of situation}. (Klein 1994:3[43]).
aspect in terms of strict temporal precedence. Perfective and imperfective aspect are handled differently.

The framework which is used borrows from several accounts and modifies them or expands them in significant ways. The combinatorial rules in the grammar are encoded using Multi-Modal Combinatory Categorial Grammar (MM-CCG) (Baldridge 2002 [5], Baldridge & Kruijff 2003 [6], Steedman 1996 [90], 2000 [92], 2012[93]).

Lexical entries are presented in terms of attribute value matrices, using syntactic features inspired by Villavicenio 2002[101], and Head-Driven Phrase Structure Grammar (HPSG) (Pollard & Sag 1994[77], Sag, Wasow & Bender 2003[85]). Linguistic expressions are interpreted in a relativist semantics based on the model presented in Lasersohn 2015[52]. The semantics encoded in the lexical entries and formal translation in this chapter differ from the semantics used in the grammars from which this work draws inspiration, but is is amenable to the grammatical framework and provides interesting insights.3

Because event time is involved in representing aspect, an event time and a general framework which can handle grammatical aspect is added to the framework of Lasersohn 2015[52]. Events are added to the types of the grammar, the categories of the logical language, and the translation in a manner inspired by Champollion 2015[13]. The approach uses neo-Davidsonian event semantics (Davidson 1967[22], 1980[23], Parsons 1990[74]) and represents events as properties of sets of events (Champollion 2015[13]).

The temporal ontology requires consideration as the treatment of aspect hinges crucially on being able to have logical relations among times. A temporal and event ontology, inspired by Muskens 1995[68] is added in order to represent tense, aspect, and modality in more detail. This ontology is used due to the emphasis it places on temporal periods, the relations defined, and its connection to an event ontology. Event variables are added to the category labels and (consequently) the types of the grammar. The treatment of events roughly follows Champollion 2015[13], treating them as sets of sets of events and then grounding them to a satisfying event with a closure operator. The closure operations in Lasersohn 2015 [52] and Champollion 2015[13] are made to serve

3There are significant differences between CCG and HPSG, especially in terms of the size of the grammar that they generate (e.g., see Kallmeyer (2010:7[39])). The use of features and values in the lexicon of CCG is not uncommon as Steedman 2000[92] uses features and Villavicencia 2002[101] is written in the framework. The particular features and values in this paper, however, have not been fully studied regarding how they affect the size of the generated grammar. It is an important question for future work.
their respective purposes and interact cohesively: The event closure operator interacts with the general existential closure used in Lasersohn 2015[52], which handles the setting of non-indexical temporal variables.

The model that Lasersohn 2015[52] presents provides a grammar and interpretation, focused especially on demonstrating the relativist treatment of predicates of personal taste. Such a model is demonstrated to provide an interpretation to other expressions that require a relativist treatment. The kind of model that Lasersohn 2015[52] presents is desirable for the interpretation of epistemic modal auxiliary verbs, which have been convincingly argued to require a relativist semantics (MacFarlane 2003[58], 2011[56]). It also provides an interpretation for expressions that do not require a relativist semantics.

6.1.2 Goals and Accomplishments of the Chapter

The purposes of the formal grammar and model are to make the details of the proposed analysis explicit and to provide a rich semantic interpretation. As it has been explained in this introductory section, the grammar that is presented in this chapter is specific to this work and uses elements of various grammars and theoretical proposals. The addition of relativist semantics for the interpretation of epistemic modals is a component that has not been discussed in broad coverage models.

The resulting type of model is used, in Chapter 7 to treat grammatical aspect. Chapters 8 and 9 show the interpretation of the various readings of modal auxiliary verbs, accounting for temporal and aspectual interactions, and providing a relativist treatment of epistemic modal auxiliaries.

6.1.3 Order of Presentation

Section 6.2 introduces the grammar and proposed category types and presents the rules of MM-CCG. The basic and phrasal expressions of the English fragment are given. Section 6.3 presents the types and the well-formed formulas of the logical language and provides a (modified version of) Lasersohn’s 2015[52] relativist model for semantic interpretation. Section 6.4.1 presents the translation of basic expressions such as noun phrases of the English fragment and their interpretation. Section 6.4 presents the lexical entries for the basic expressions and phrase features structures. It presents the use of
feature unification in the grammar. Section 6.5 presents the translations of nouns, verbs, and quantifiers.

Chapter 7 presents the semantics of tense and aspect in the grammar. Section 7.6 adds the closure operators and illustrates how various readings are treated in the grammar. Examples are given of how the theory accounts for readings English perfects and (im)perfectives in linguistic contexts without modal auxiliaries. Additions to the treatment of aspectual relations, motivated by Reed 2012[81], are incorporated. Section 7.7 is a brief conclusion recapping what has been stated and mentioning future directions of research and known issues with the current approach. The subsequent chapters, Chapters 8 and 9, show the composition of the treatment of tense and aspect with modal auxiliary verbs.

### 6.2 MM-CCG for a Fragment of English

This section presents the combinatory rules of CCG (Steedman 2000[92]) and the multi-modal operators of MM-CCG (Baldridge 2002 [5], Baldridge & Kruijff 2003 [6]). It presents the categories, basic expressions, and phrasal expressions of a fragment of English. Examples of the combinatorics of the MM-CCG are demonstrated using the basic expressions of the fragment. The syntactic rules are stated in a multi-modal version of Combinatory Categorial Grammar (MM-CCG) (Baldridge 2002 [5], Baldridge & Kruijff 2003 [6], Steedman 1996 [90], 2000 [92], 2012[93]).

Combinatory Categorial Grammar grew out of the Categorial Grammar of Ajdukiewicz 1935[2] and Bar-Hillel 1953[7]. Categorial grammars use category labels which represent functors and arguments of the functors. There is repeated application of functions through currying functions (Curry & Feys 1958[21]).

CCG has been appreciated for the correspondence it provides between the syntax and the semantics. The semantics of CCG provides a proof-theoretic interpretation of the well-formed formulas (wffs) of the language. The wffs of the translation of a fragment of a given natural language involves the composition of translations that are constrained by rules, which have a correspondence in the category rules given in the syntax.4

Each linguistic expression has a category with which it is associated. The set of cat-

4The present work departs from CCG in that it provides a model-theoretic interpretation to the semantics. The way in which a proof theory works with the translations has not been fully investigated.
Category labels are described by the following definition. Combinatory Categorial Grammar (Steedman 1996 [90], 2000 [92], 2012[93]) has the goal of using as few purely syntactic rules as necessary, encoding all language-specific variation in the lexicon.

MM-CCG is valued in that it provides a way to give language-specific combinatorial rules in a framework that works cross-linguistically. Not every combinatorial rule that CCG has applies to every combination of categories in every language. The rules that do not apply have to be given in a grammar for a particular natural language. The multimodal operators provide a formalism for giving the combinatorial restrictions directly in the categories, rather than in a list of rules or in the lexical entries, for example.

In this section, the categories and combinatorial rules for categories are presented for a fragment of English.

Definition 48. The set of categories of English syntax, Cat, is the smallest set such that:

(i) $v, e, t, i \in \text{Cat}$,

(ii) if $a \in \text{Cat}$ and $b \in \text{Cat}$, then $(a \backslash, b)$ and $(a /, b) \in \text{Cat}$,

(iii) nothing else is in Cat.

Where $v, e, i,$ and $t$ are labels for distinct sets and $\gamma$ is a member of the set of MM-CCG modal operators $M = \{\ast, \bullet, \times, \circ\}$.

The MM-CCG modal operators are a compact way of representing what basic rules of CCG can be applied to an expression. The symbols restrict associativity and permutativity. They are listed in Table 6.1 below.

<table>
<thead>
<tr>
<th></th>
<th>non-permutative</th>
<th>permutative</th>
</tr>
</thead>
<tbody>
<tr>
<td>non-associative</td>
<td>$\ast$</td>
<td>$\times$</td>
</tr>
<tr>
<td>associative</td>
<td>$\diamond$</td>
<td>$\bullet$</td>
</tr>
</tbody>
</table>

Table 6.1: Table of Symbol Meaning in Multi-Modal CCG (from Baldridge & Kruijff (2006:215))

Permutation and associativity are defined as follows:

**Generalized Weak Permutation** Where $|$ is a variable over $\backslash$ and $/$, defined only for permutable slash-types and $X$ and $Y$ are categories of expressions.

---

5The presentation of the grammar roughly follows the tradition of Montague Grammar (Montague 1973[63]). These rules, combined with the lexical entries, do much of the work that is handled in the ‘S-rules’ in Montague’s 1973[63] grammar.
\[ ((X|Y_1)\ldots|Y_n) \Rightarrow (((X|Y_n)|Y_1)\ldots). \]

**Associativity** Where \( X, Y, \) and \( Z \) are categories of expressions:

\[ i \ (Y\cap \downarrow X) / Z \Rightarrow (Y/Z)\downarrow X, \]
\[ ii \ (Y/Z)\downarrow X \Rightarrow (Y\downarrow X)/Z. \]

One motivation for MM-CCG is to have a universal set of rules that differ by language only regarding which rules apply and which modal operators are given on those that do apply.

The combinatory rules that are claimed to play a role in English (Steedman 2000, 2012[93] are given below:

Combinatory Rules in a Categorial Grammar

The foundational rule of categorial grammar is functional application.

**Forward Application** (> ) An expression of category \( X/Y \) combines with a an expression of category \( Y \) on the right, resulting in an expression of category \( X \).

**Definition 49.** \( X/Y \ Y \Rightarrow X. \)

**Backward Application** (< ) An expression of category \( Y \) combines with an expression of category \( X \cap Y \) on the right, resulting in an expression of category \( X \).

**Definition 50.** \( Y \ X \Rightarrow < X. \)

Let us assume, for now, that a noun phrase is of category \( e \) and an transitive verb is of category \( (t\cap e)/e \). If the grammar has the basic expressions \( Jess \) and \( Syd \) of category \( e \) and \( find \) of category \( (t\cap e)/e \), then the combination of them is via forward and then backward application:

\[
\frac{((t\cap e)/e) : finds \ e : Jess}{e : Syd \ (t\cap e) : Jess \ finds \ >}
\]
\[
\frac{t : Jess \ finds \ Syd \ <}{<}
\]

Figure 6.1: An example of forward and backward application rules
The use of application rules alone describe derivations equivalent to a phrase-structure grammar (Steedman 2012:80[93]). The categorial grammar of Bar-Hillel 1953 [7] explained the combinatorics and used only application, but suggested that additional rules could be necessary.

**Type Raising** $T$  an expression of category $X$ is raised to a function from a category $Y$ to a category $(Y \setminus \gamma X)$ or $(Y / \gamma X)$. Type raising is a lexical rule in Steedman 2012[93].

**Definition 51.** $X \gamma \Rightarrow_T Y / (Y / \gamma X)$. 

**Definition 52.** $X \gamma \Rightarrow_T Y \setminus (Y / \gamma X)$. 

Where $\gamma$ is a variable for MM-CCG modal operators and $\gamma$ is a possibly null set of arguments of $X$.

**Forward Composition** ($> B$)  An expression of category $X / \circ Y$ combines with an expression on its right of category $Y / \circ Z$ to result in an expression of category $X / \circ Z$.

**Definition 53.** $X / \circ Y \Rightarrow_B Y / \circ Z \Rightarrow_B X / \circ Z.$

**Backward Crossed Composition** ($< B_X$)  In backward composition, an expression of category $Y \setminus X Z$ combines with an expression of category $X \setminus Y$ to result in an expression of category $X \setminus Y Z$.

**Definition 54.** $Y \setminus X Z \Rightarrow_B X \setminus Y Z.$

**Forward Substitution** ($> S$)

**Definition 55.** $(X / \circ Y) / \circ Z \Rightarrow_S Y / \circ Z$.

**Backward Crossed Substitution** ($< S_X$)

**Definition 56.** $Y / \circ Z (X \setminus Y) / \circ Z \Rightarrow_S X / \circ Z.$

The combination of type raising and composition can lead to associativity:

The standard notion of associativity is represented in that the last two formulas combine and then combine with the first: $X(YZ)$. Through type raising, the rule is open to composition, which leads to the ordering $(XY)Z$, where $X$ is the type which is raised. Associativity can be useful in a grammar, but it can lead to over-generation of rules.
Forward and backward crossed composition rules, if unrestricted, allow permutations of expressions. Forward crossed composition has been argued to be inactive in English (Steedman 2000[92]), but both rules are useful for languages that allow scrambling.

Formal linguistics in the Montague tradition (1970[62],1973[63]) has been concerned with having a close correspondence between the categories and the types that they represent. It has also been concerned with having a close correspondence from the types that the categories represent to the translation language. The logical types are, in principle, removable, and a formal language can be written as a homomorphism from categories (assigned to linguistic expressions) to a translation language. CCG encodes this same principle in terms of a ‘Principle of Categorial Type Transparency’ and its inverse (Steedman 2000:36[92]).  

The ordinary slashes in the categorial grammar allow the morphism to have all of the properties of the rules, that is, to allow composition, associativity, and whatever properties have been defined. The modalities given on slashes are considered a type of feature that limits, in the limiting cases (that is, every modality except for “•”), the axioms which describe that category. That is, it limits the axioms in comparison to those that hold of the least limiting type.

Alternatively, all functions could be defined to be associative and compositional functions that hold of the set of linguistic expressions as a whole, with special cases being handled by limiting operators. However, because the lack of associativity, for example, holds of a set of linguistic expressions having similar combinatory properties, it is more efficient for the exposition to use modal operators limiting composition and associativity for a set of expressions than to list them in a set and state that they are not able to participate in the rules.  

---

6This rule does not appear to be mentioned in Steedman 2012[93]. The treatment of type raising in the lexicon and by a lexical rule could prevent this strict of a definition of compositionality.

7The choice between making the system overly expressive and then limiting the application on one hand and making more restrictive rules, on the other hand, is a choice that comes up often in formal grammars. For example, Montague’s 1973[63] language made all functional application introduce intensionality. In later works, authors began not adding intensionality to all functional combinations but rather added intensionality for expressions requiring it as the set of expressions that create intensional contexts.
For the sake of the exposition, the modal operators abbreviate limitations on composition and associativity that hold of a set of linguistic expressions that, in terms of simple application, resemble each other.

Following Steedman 2012[93], when the most permissive modality is assumed, there will be unmarked slashes, / and \ used rather than bulleted slashes /• and \•.

6.2.1 Categories of English

With the rules of the MM-CCG now in place, it is possible to define the categories of English that are used in the basic expressions of the language fragment. Mnemonic abbreviations are introduced in order to save space when giving categories in example derivations and in order to connect the categories with abbreviations standardly used in syntax (e.g., S for ‘sentence’, CN for common noun).

Categories of $\mathcal{F}$:

i. $Bse$ is the category construed as verbs in their base form. The category name $Bse$ abbreviates the category $((t/\cdot,(t/\cdot,v)/\cdot,i))\cdot((t/\cdot,i)/\cdot,i))\cdot((t/\cdot,i)/\cdot,i)$.

ii. $Tns$ is the category construed as tense. The category name $Tns$ represents the category $((t/\cdot,i)/\cdot,i)$.

iii. $Aspct$ is the category construed as aspect. The category name $Aspct$ represents the category $((t/\cdot,i)/\cdot,i)$

Tense and aspect are, formally, the same category as far as the MM-CCG rules are concerned. Their distributional character is handled in terms of feature matching in the lexicon, following the arguments for auxiliary ordering presented in Bach 1983[4].

Next, the categories of the functors that combine with verbal elements to satisfy the lexical argument structure of verbs are given:

v. $N$ is the category construed as nouns, including common nouns, proper nouns, and personal pronouns. The category name $N$ represents the category $e$.\footnote{are generally fewer in linguists’ models of language than those that can be interpreted extensionally (e.g., Partee & Rooth 1983[75]).}

\footnote{The category $e$ is used here following Lasersohn 2015[52], rather than the type-raised category $t/(t/e)$}
vi Det is the category construed as determiners. The category name Det stands for the category \((e/,e)\).

vii Qnt is the category construed as quantifiers. The category name Qnt stands for the category \(((t/,t)/,e)\).

viii SUBJ – NP is the category construed as the subject noun phrase. The category name SUBJ – NP abbreviates the category \(((Bse/,Bse)/,e)\).

ix OBJ – NP is the category construed as the object noun phrase. The category name OBJ – NP abbreviates the category \(((Bse/,Bse)/,e)\).

x Vcomp is the category construed as the verb phrase complement of a verb. The category name Vcomp abbreviates the category \(((Bse/,Bse)/,Bse)\).

xi S comp is the category construed as a verbal argument of another verb. The category name S comp abbreviates the category \(((Bse/,Bse)/(t/,(t/,v)))/,i)\).

Finally, there are given rules for closure of event variables and general existential closure, which provides a satisfying variable assignment.

xii ClzV is the category construed as event closure. The category name refers to the type \((t/,v)\).

xiii ClzE is the category construed as general closure. The category name refer to the type \((t/,t)\).

xiv Nothing else is in the set of Basic Expressions of English Fragment \(\mathcal{F}\).

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of Montague 1973[63]. Since Steedman 2012[93] handles type-raising as a lexical rule, it could occur in such a way if needed, however, the rules for articles and semantic role labels would need to be changed accordingly.
<table>
<thead>
<tr>
<th>Syntactic Category</th>
<th>Abbreviation</th>
<th>Common Syntactic Name</th>
<th>Basic Expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>N</td>
<td>nouns</td>
<td>Jess, Pat, badger, it, she</td>
</tr>
<tr>
<td>(e/e)</td>
<td>Det</td>
<td>determiners</td>
<td>a, the every</td>
</tr>
<tr>
<td>((t/t)/(t/e))</td>
<td>Qnt</td>
<td>quantifiers</td>
<td>every</td>
</tr>
<tr>
<td>((Bse/,Bse),e)</td>
<td>SUBJ-NP</td>
<td>subject position noun</td>
<td>SUBJECT-NP-LABEL</td>
</tr>
<tr>
<td>((Bse/,Bse)/e)</td>
<td>OBJ-NP</td>
<td>object position noun</td>
<td>OBJECT-NP-LABEL</td>
</tr>
<tr>
<td>((t,v)/(t,v))</td>
<td>Bse</td>
<td>base form of a verb</td>
<td>swim, love</td>
</tr>
<tr>
<td>((t,i)/(t,i))</td>
<td>Tns</td>
<td>past tense, non-past tense</td>
<td>PAST, NON-PAST</td>
</tr>
<tr>
<td>(t,v)</td>
<td>Aspct</td>
<td>perfects, (im)perfectives</td>
<td>PERF, PROSP</td>
</tr>
<tr>
<td>(t,i)</td>
<td>ClzV</td>
<td>event closure</td>
<td>V-CLOSURE</td>
</tr>
<tr>
<td></td>
<td>ClzE</td>
<td>general closure</td>
<td>Ξ-CLOSURE</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Syntactic Category</th>
<th>Abbreviation</th>
<th>Common Syntactic Name</th>
<th>Phrasal Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>S</td>
<td>sentences</td>
<td>Jess loves the chinchilla</td>
</tr>
<tr>
<td>(Bse,Bse)</td>
<td>NP</td>
<td>subject noun phrase</td>
<td>Jess swims.</td>
</tr>
<tr>
<td>(Bse/,Bse)</td>
<td>NP</td>
<td>object noun phrase</td>
<td>Syd loves Jess.</td>
</tr>
</tbody>
</table>

Table 6.2: Chart of selected syntactic categories, their label in more standard syntactic systems, and a natural language example of each category.
The combinatory rules of the MM-CCG presented above combine elements of the set of Basic Expressions of $\mathcal{F}$ to describe the set $\text{PHE}$ of phrasal expressions of $\mathcal{F}$.

**Definition 57.** The set $\text{PHE}$ is the smallest set such that:

i. The Basic Expressions of $\mathcal{F}$ are in $\text{PHE}$,

ii. any combination of the Basic Expressions of $\mathcal{F}$, defined by the combinatory rules of the grammar is in $\text{PHE}$, and

iii. nothing else is in $\text{PHE}$.

The set, $\text{PHE}$, describes the grammar of the fragment of English which is presented.

### 6.2.2 Sample Syntactic Derivations

The tree below shows the syntactic derivation of the noun phrase *the chinchilla* as the patient or theme of an the verb *eat*.

![Tree diagram of the phrase eat the chinchilla](image)

The semantic role label combines with the term to result in an NP node.

The semantic role label combines with the term to result in an NP node.
6.2.3 Summary of an MM-CCG for a Fragment of English

This section has presented a grammar for a fragment of English using an MM-CCG. The grammar that is given over-generates in many ways. Some examples are that there is no distinction between tense and aspect in the type system, so they can combine in any order or tense can combine twice and aspect not at all, as well as many other alternatives. It was mentioned that these issues would be constrained through features and unification. The features are encoded in the lexicon, and abbreviated on categories, as needed, following Steedman 2000[92]. Because the lexical entries involve well-formed formulas and translations of the fragment into the logical language, the lexical entries are presented in detail after the logic and interpretation, discussing how their translations are interpreted in the model.

6.3 Type Theory, Logical Language, and Interpretation

This section presents the types used in the fragment presented in this chapter. It presents the well-formed formulas of the logical language and the rules for interpretation. The next section, section 6.4 presents the translation of the English fragment into the logical language.

6.3.1 Types in the Syntax of the Logical Language

**Definition 58.** The set $Y$ of types is the smallest set such that:

(i) $e, t, v, i \in Y$.

(ii) If $a \in Y$ and $b \in Y$, then $< a, b > \in Y$.

(iii) If $a \in Y$, then $< s, a >$ and $< r, a > \in Y$.

For each type, there is a set of constants of that type and a set of numbered variables of that type. The sets of constants are labeled as $\text{Con}_a$, where $a$ is a type in $Y$. Each of the variables is labeled with a positive integer and the type to which it belongs, so $x_{i,a}$ is a variable with the numerical index $i \in I^+$ and type label $a \in Y$.

The set of well-formed-formulas ($wffs$) are defined as follows by recursive definition:
1. Every variable and constant of type \( a \in Y \) is in the set of \( wffs_a \).

2. The punctuation forms ‘(’, ‘)’, ‘[’, ‘]’, and ‘.’ are in the set of \( wffs_a \).

3. If \( \alpha \in wff_{<a,b>} \) and \( \beta \in wffs_a \), then \( \alpha(\beta) \in wffs_b \).

4. If \( \alpha \in wff_a \) and \( x \) is a variable of type \( b \), then \( \lambda x \alpha \in wff_{<b,a>} \).

5. If \( \alpha, \beta \in wffs_a \), then \( \lbrack \alpha = \beta \rbrack \in wffs_t \).

6. If \( \phi, \psi \in wffs_t \) and \( x \) is a variable, then \( \neg \phi, [\phi \land \psi], [\phi \lor \psi], [\phi \rightarrow \psi], \) \( [\phi \iff \psi], \exists x \phi, \forall x \phi, \Box \phi, \Diamond \phi \in wffs_t \).

7. If \( \alpha \in wffs_a \), then \( \uparrow \alpha \rbrack \in wff_{<s,a>} \).

8. If \( \alpha \in wff_{<s,a>} \), then \( \downarrow \alpha \rbrack \in wffs_a \).

9. If \( \alpha \in wffs_a \), such that \((x_n \ldots)x_i(\ldots x_m) \) or \((x_n \ldots)x_e(\ldots x_m) \), where \( n, m \in Y \), then \( r(\alpha) \in wff_{<r,a>} \).

10. If \( \alpha, \beta \in wffs_t \), then \( \lbrack \alpha <, \beta \rbrack, [\alpha \subseteq, \beta], [\alpha \circ, \beta], [\alpha \ll, \beta] \), and \( [\alpha \sqsubseteq, \beta] \in wffs_t \), and

11. Nothing else is in the set \( wffs_a \).

The explanation of each item and important details about it are given below.

The Set of Typed \( wffs \)

A \( wffs \) of the logical language is understood as a member of \( \bigcup_{a \in Y} wffs_a \).

Equality and Predicate Logic Operators

Equality is defined for \( wffs \) of any type, resulting in an expression of type \( t \).

Logic operators, which are familiar from predicate logic, are defined for expressions of type \( t \), resulting in expressions of type \( t \).
Intensional and Extensional Operators

In contrast to Montague 1973[63], in which all functional types are intensional unless it is stated otherwise, in this grammar, functional expressions are extensional, unless there is an intensional expression present.

The ‘up’ symbol, \^, gives the intension of the expression in its scope.

The ‘down’ symbol, \_ gives the extension of an intensional expression in its scope.

Variable Assignment

The variable assignment is regarding the assignment of values to temporal variables and individual or group variables, thus, it is restricted to \( wfs \) that contain an expression of type \( i \) or type \( e \).

Any indexical expression that is not set by the referential intentions of the speaker is bound by an assignment of values to variables. There are additional stipulations in the translation regarding expressions that are always bound.

The Temporal Ordering

Temporal ordering is treated as a binary predicate that takes two expressions of type \( i \) as arguments. The nature of the temporal ordering is defined in the presentation of the model, as are the abbreviations for types of temporal relations, which can be defined in terms of \( <_t \).

Where \( a \in Y \), let \( U, T, W, V \) be disjunct sets construed as the set of individuals, temporal intervals, possible worlds, and events. Then \( D_{a,U,T,W,V} \) is the set of possible denotations of a type \( a \) corresponding to \( U, T, W, V \) and it is defined recursively below (cf. Montague 1973:24[63]):

\textbf{Definition 59. Type Domains}

\textbf{a.} \( D_e = \{ x : x \text{ is a group or individual} \} \)

\textbf{b.} \( D_t = \{ \text{truth, falsity} \} \)

\textbf{c.} \( D_i = \{ j : j \text{ is a time} \} \)

\textbf{d.} \( D_v = \{ y : y \text{ is an event} \} \)
e. If \( a \) is a type, then \( D_{<a,a>} = D_a^{\{<w,<x,t,w'>>:x \text{ is an individual, } t \text{ is a time, and } w, w' \text{ are worlds.}\}} \)

f. If \( a \) is a type, then \( D_{<r,a>} = D_a^{\{r: r \text{ is a partial function from positive integers to } D \}\}} \)

g. If \( a_1 \) is a type and \( a_2 \) is a type, then \( D_{<a_1,a_2>} = (D_{a_2})^{D_{a_1}} \).

Where, if \( X \) is a set and \( Y \) is a set, \( X^Y \) is the set of all functions with the domain \( Y \) and range in \( X \) and \( X \times Y \) is the Cartesian product of \( X \) and \( Y \), and truth values are indicated by truth and falsity.

6.3.2 Interpretation in a Relativist Model

The sort of model presented for interpretation is a relativist model in that interpretation is defined relative to various parameters. Truth not relative to the parameters, in the case of an expression with a relative reading, can be undefined. The sort of model used in this section is based closely on that of Lasersohn 2005[52]. Lasersohn’s model focuses on predicates of personal taste, but also applies to epistemic readings of modal auxiliary verbs.

Definition 60. A Relativist Model \( M = < W, I^+, T, U, V, C, Ref_a, \leq_w, <_t, \tau, Lex >, \) where:

i. \( W \) is a non-empty set construed as the set of possible worlds of \( M \),

ii. \( I^+ \) is the set of positive integers

iii. \( T \) is a non-empty set construed as the set of temporal periods of \( M \),

iv. \( U \) is a non-empty set construed as the set of individuals or groups of individuals of \( M \),

v. \( V \) is a non-empty set construed as the set of events of \( M \),

vi. \( C \) is a non-empty set construed as the set of contexts of assessment of \( M \) (Lasersohn (2015:65[52])), and, if \( c \in C \) then:

\[ \text{judge}(c) \in U \text{ is construed as the judge of } c, \]
\[ \text{speaker}(c) \in U \text{ is construed as the speaker in } c, \]
\[ \text{time}(c) \in T \text{ is construed as the time of } c, \]
\[ \text{world}(c) \in W \text{ is construed as the world of } c. \]
vii. $Ref_{ua}$ is a partial function from a finite subset of the set $I^*$ to the a subset of $U \cup T$, which is an assignment of values to variables.

viii. $\leq_w$ is a partial order on the set $W$,

ix. $<_t$ is a complete temporal precedence order on $T$,

x. $\tau$ is a function from the set $V$ into the power set of $T$, $\mathcal{P}(T)$ (construed as a function from eventualities to the temporal periods at which they occur),

xi. $Lex$ is an interpretation function defined below.

6.3.3 Temporal Ordering

Temporal periods are taken as primitive. The following abbreviations are used (inspired by Muskens 1995[68]).

**Definition 61.** Other ordering relations on $T$ are defined in terms of the complete order $<_t$:

- **temporal inclusion of $t_j$ in $t_k$, written $t_j \subseteq_t t_k$, abbreviates:**
  \[ \forall t_i ((t_k <_t t_i) \rightarrow (t_j <_t t_i)) \land \forall t_i ((t_i <_t t_k) \rightarrow (t_i <_t t_j)) \]

- **temporal overlap of $t_j$ and $t_k$, written, $t_j \circ_t t_k$, abbreviates:**
  \[ \exists t_i ((t_i \subseteq_t t_j) \land (t_i \subseteq_t t_k)) \]

When the endpoint of $t_j$ is not strictly preceded by the endpoint of $t_k$, it is written $t_j \ll_t t_k$, which abbreviates:

\[ \forall t_i (t_k <_t t_i) \rightarrow (t_j <_t t_i) \]

When a temporal period $t_j$ immediately precedes $t_k$, it is written $t_j \preceq_t t_k$, which abbreviates:

\[ (t_j <_t t_k) \land \neg \exists t_n (t_j <_t t_k <_t t_n) \]

**Definition 62.** The Ordering $<_t$

The two place relation, $<_t$, is a complete order on $T$ such that:

- **Axiom 1:** transitive: $\forall t_i \forall t_j \forall t_n ((t_i \leq_t t_j \land t_j \leq_t t_n) \rightarrow t_i \leq_t t_n)$,

- **Axiom 2:** irreflexive: $\forall t_i (\neg (t_i <_t t_i))$,
Axiom 3: comparability: $\forall t_i \forall t_j ((t_i < t_j) \lor (t_i \cap t_j) \lor (t_j < t_i))$.

Axiom 4: antisymmetric: $\forall t_i \forall t_j ((t_i \leq t_j) \land (t_j \leq t_i)) \rightarrow (t_i = t_j)$, and

Axiom 5: infinite: $(\forall t_i \exists t_j (t_i \leq t_j)) \land (\forall t_i \exists t_n (t_i \leq t_n))$

6.3.4 Mapping of Eventualities to Temporal Periods

In the temporal and event ontology given in Muskens 1995[68], which inspires this portion of the model, eventualities and temporal periods are both taken to be primitive. It is possible, though to construe temporal periods as being a series of eventualities.

The function $\tau$ (Krifka 1989[49]) maps eventualities to temporal periods. The ontology uses as close to the same relations as possible to those used for temporal periods.

Definition 63. Mapping Eventualities to Possible Worlds:
Each eventuality occurs in multiple worlds. In this ontology, following Muskens 1995[68], eventualities take place in worlds. The 2-place relation $\subset_w$ represents that an eventuality holds in a world. So $w_i \subset_w v_i$ means that the eventuality $v_i$ holds in the world $w_j$.\(^9\)

Definition 64. Comparability:
$v_i$ is comparable with $v_j$, if, and only if, $\exists w_n ((w_n \subset_w v_i) \land (w_n \subset_w v_j))$.

The relations on the domain of periods is applied to eventualities which are mapped to temporal periods:

Definition 65. Eventuality Orderings:
Comparable eventualities participate in the ordering relations which were introduced for temporal periods.

$v_i <_t v_j$ abbreviates: $(\tau(v_i) <_t \tau(v_j)) \land (\exists w_n ((w_n \subset_w v_i) \land (w_n \subset_w v_j)))$.

$v_i \leq_t v_j$ abbreviates $(\tau(v_i) \leq_t \tau(v_j)) \land (\exists w_n ((w_n \subset_w v_i) \land (w_n \subset_w v_j)))$.

$v_i \circ_t v_j$ abbreviates $(\tau(v_i) \circ_t \tau(v_j)) \land (\exists w_n ((w_n \subset_w v_i) \land (w_n \subset_w v_j)))$.

$v_i \ll_t v_j$ abbreviates $(\tau(v_i) \ll_t \tau(v_j)) \land (\exists w_n ((w_n \subset_w v_i) \land (w_n \subset_w v_j)))$.

\(^9\)Muskens (1995:24[68]) uses a functional type $\langle v, s, t \rangle$ (to use more familiar type notation) for a constant $m$, which maps an eventuality to the worlds in which it is true.
$v_i \leq_t v_j$ abbreviates $(\tau(v_i) \leq_t \tau(v_j)) \land (\exists w_n ((w_n \subset_w v_i) \land (w_n \subset_w v_j))).$

The following axioms are added in Muskens 1995[68], which will also be useful here:

**Definition 66. Eventuality Axioms**

**Axiom 6:** All eventualities take place in at least one world:

$$\forall v \exists w (w \subset_w v).$$

**Axiom 7:** Makes the past non-branching. If two eventualities, $v_i$ and $v_j$ are comparable and $v_j$ occurs in $w$, and the endpoint of $v_j$ is not to the left of the endpoint of $v_i$, then $v_i$ also occurs in $w$:

$$\forall v_i \forall v_j ((v_i \ll v_j) \rightarrow \forall w ((w \subset_w v_j) \rightarrow (w \subset_w v_i))).$$

**Axiom 8:** In each world, at each period of time, some eventuality takes place.\(^{10}\)

$$\forall t \forall w \exists v ((\tau(v) = t) \land (w \subset_w v)).$$

By axiom 7, the past is immutable “whatever has happened will always have happened”, and the following sentences are provable (Muskens 1995:24[68]):

**Definition 67. Properties of Eventuality Orderings:**

**Irreflexive:** $\forall v_i \lnot (v_i \ll v_i)$

**Transitive:** $\forall v_i \forall v_j \forall v_k (((v_i \ll v_j) \land (v_j \ll v_k)) \rightarrow (v_i \ll v_k))$

**Comparability:** $\forall v_i \forall v_j \forall v_k (((v_i \ll v_j) \land (v_j \ll v_k)) \rightarrow ((v_i \ll v_j) \lor (v_i \ll v_k) \lor (v_j \ll v_i)))$

Of the other two axioms of the temporal periods, antisymmetry is not wanted for eventualities, though it still hold of the time periods in which two eventualities occur that, if the time periods are overlapping, then the overlap is one and the same time period, but the eventualities are separate. It is important that anti-symmetry holds for the temporal ontology but not for the eventuality ontology. If each sub-event is included in an event and the event is co-existing with every sub-event, it does not make all of the eventualities exactly the same. The fact that anti-symmetry holds for periods of time and not for eventualities is solely the result of the axioms that hold of each system.

\(^{10}\)By this axiom and the first one, we get the paradox that the eventuality of no eventuality taking place both must take place in a world and must not.
Temporal periods are non-gapping, and eventualities map to temporal periods. As a result, dealing with eventualities that contain gaps requires additional ontological considerations, which will not be dealt with in detail in this model at this time.

### 6.3.5 Contexts of Assessment

The set $C$, which is construed as the set of contexts of assessment of $M$, is based on Lasersohn’s 2015\cite{52} definition. Contexts of assessment provide a *judge*, a time, and a world.

A *context*, in Lasersohn 2015\cite{52} and 2005\cite{51}, is considered to include many types of contextual information, of which the judge, time, and world are merely a few. The list is kept as small as needed for the phenomena being captured in the model, but is not explicitly limited to these three parameters.

The *judge* is the individual or group that is assessing an utterance. For many expressions, the truth conditions are such that the truth value does not change based on who is assessing the truth value of a sentence. For example, considering the sentence, *The earth is round*, the truth conditions can be stated in a non-relativist model. The truth conditions do not vary based on the person assessing the sentence, rather, it is modeled as being true if, and only if, the world is round in the actual world in which the statement was made.

Lasersohn (2005\cite{51}, 2015\cite{52}) discusses the observation that the truth conditions for predicates of personal taste are such that truth values can only be assigned relative to a judge, and that judge changes each time a different individual assesses the sentence. Sentence (115) below is an example of a use of a predicate of personal taste:

115. *Chili is tasty.*

The assertion that chili is tasty is not a matter of shared truth among individuals, rather it is the personal assessment of the speaker. If the assertion conditions, in general, depend on the personal tastes of the speaker, it is yet a different matter what the truth conditions for sentence (115) are.

It might seem that truth conditions could be assigned easily, if there were considered to be a hidden element in the semantics indicating that chili is tasty *for* the individual making the assertion. Lasersohn 2005\cite{51}, 2015\cite{52} points out problems with such an analysis in terms of *faultless disagreement*. *Faultless disagreement* refers to cases
in which people disagree but neither of them can be right or wrong in the matter. An example is given in the dialogue in sentences (116)-(117) below:

116. Jess: Chili is tasty.

117. Pat: Chili is not tasty.

If the sentence in (116) had an invisible constituent such that the truth conditions were for Chili is tasty (for Jess), then Pat’s statement in (117) would be contradicting that chili is tasty to Jess. Such a reading of the discourse does not seem intuitive. It seems, rather, that Pat is stating her own personal tastes, not commenting on Jess’s personal tastes.

The problem could be solved by claiming that the for-statement takes the nominal argument of whoever made the statement, so that sentence (116) above means, Chili is tasty for Jess. and sentence (117) means, Chili is tasty for Pat. The problem with this analysis is that it does not account for the feeling speakers have that there is a disagreement in the conversation.

Lasersohn 2005[51], 2015[52] posits that there is not only one way of interpreting predicates of personal taste, but at least two ways. These stances are called the autocentric stance and the exocentric stance. From an autocentric stance, an assessor evaluates a relative expression, such as a predicate of personal taste, from their own personal perspective. From an exocentric stance, an assessor evaluates a relative expression relative to the perspective of another.

The context of assessment is crucial in the interpretation of a relativist model as it provides the perspective from which a sentence is being considered and assigns truth values accordingly.

6.3.6 Content and Denotation

The content of an expression plays an important role in a relativist model. Contents are assigned to uses of linguistic expressions in a context. The use of an expression is defined in Definition 70 below:

Definition 68. Let \{\alpha, \beta, \gamma, \delta, \ldots\} =_{df} variables over linguistic expressions.

Definition 69. Let \{\alpha, \beta, \gamma, \delta, \ldots\} =_{df} variables over uses of linguistic expressions, with the exceptions of \tau and \lambda which are set aside as special characters for a function from events to times and the lambda operator, respectively.
Definition 70. Let $US\ E(\alpha, \alpha) =_{def} \alpha$ is a use (token) of the linguistic expression $\alpha$.

Uses are taken to be primitive (Lasersohn (2015:13[52])). A use of an expression has a particular time, place, speaker, and other contextual information associated with it.

Definition 71. Let $\{A, B, \Gamma, \Delta, \ldots\} =_{def}$ variables over contents.\(^{11}\)

Definition 72. Let $\{\}^{u} =_{def}$ a function from contents to uses given a world of use, $u \in W$.

Definition 73. Let $\left[ \right]^{w,p} =_{def}$ a function from denotations to contents given a world of evaluation $w \in W$ and a perspective $p$, where a perspective is a triple $<x,t,a>$, where $x$ is a judge, $t$ is a time, and $a$ is a perspective world.

Definition 74. $\left[\alpha\right]^{u,w,p}$ abbreviates $\left[\left\{\alpha\right\}^{u}\right]^{w,p}$ which, in turn abbreviates $\left[\left\{\alpha\right\}^{u}\right]^{w,<x,t,a>}$. Where $x$ is an individual, $t$ is a time, and $a$ is a world, construed as the ‘world of assessment’ (2015:65).

6.3.7 The Interpretation Function $Re_{f_{u,\alpha}}$

The interpretation function $Re_{f_{u,\alpha}}$ handles the assignment of values to variables.

Definition 75. Let us assume that, relative to a world of use $u$, each use $\alpha$ of an expression $\alpha$ is pragmatically associated with a set $Relevant_{u,\alpha}$ of relevant objects and with a subset $Familiar_{u,\alpha}$ of familiar objects (Lasersohn 2015:37[52]).

Let us assume that, over the course of a single use of an expression, nothing ceases to become familiar and that the things that are familiar, in the context of a complex expression, are just those things that are familiar in the context of its parts (Lasersohn 2015:38[52]).

Definition 76. Where $\alpha$ has syntactic constituents $\alpha_1, \ldots, \alpha_k$ and $1 \leq n, m \leq k$:

(a.) If $time_{u,\alpha_n}$ precedes $time_{u,\alpha_m}$, then $Relevant_{u,\alpha_n} \subseteq Relevant_{u,\alpha_m}$ and

\(^{11}\)Conventionally, characters towards the end of the Greek alphabet, particularly $\phi$, $\chi$, and $\psi$, are used as variables for sentences. So, for example, in what follows, $\phi$ is a variable over sentences, $\phi$ a variable over uses of sentences, and $\Phi$ a variable over the content of sentences.

It is important to note, however, that variables which are not at the end of the Greek alphabet do not inherently exclude sentences from the linguistic expressions, uses, or contents over which the range, unless it is explicitly stated in the introduction of the variable that the range is over a particular subset of expressions, uses, or contents.
Familiar_{u,\alpha_n} \subseteq \text{Familiar}_{u,\alpha_m}.

(b.) Relevant_{u,\alpha} = \text{Relevant}_{u,\alpha} \cup \ldots \cup \text{Relevant}_{u,\alpha_2} \text{ and } \text{Familiar}_{u,\alpha} = \text{Familiar}_{u,\alpha} \cup \ldots \cup \text{Familiar}_{u,\alpha_2} (\text{Lasersohn 2016:38[52]}).

Where time_{u,\alpha} is the time of the context for the utterance, which is usually the speech time.

Each pronoun is labeled syntactically with a referential index. In order to represent the referential intentions of the speaker, let us assume that the pragmatic context of a use \( \alpha \) (in a world \( u \)) will normally provide a function \( \text{Ref}_{u,\alpha} \) mapping positive integers from some finite subset \( I' \) of \( I^+ \) onto elements of \( \text{Familiar}_{u,\alpha} \). We regard \( \text{Ref}_{u,\alpha}(n) \) as the denotation of any constituent of \( \alpha \) bearing referential index \( n \in I' \) (Lasersohn 2015:38[52]).

**Definition 77.** Where \( \alpha \) has syntactic constituents \( \alpha_i, \ldots, \alpha_n \) (and \( 1 \leq i, j \leq n \)):

a. If \( \text{time}_{u,\alpha_n} \) precedes \( \text{time}_{u,\alpha_m} \), then \( \text{Ref}_{u,\alpha_n} \subseteq \text{Ref}_{u,\alpha_m} \)

b. \( \text{Ref}_{u,\alpha} = \text{Ref}_{u,\alpha_1} \cup \ldots \cup \text{Ref}_{u,\alpha_n} \)

Where \( n, m \in I' \), and \( \text{time}_{u,\alpha_n} \) is the time of the context in the world of use, \( u \), in most cases, ‘time’ is the speech time.

**Definition 78.** For any use \( \alpha \) and world \( u \), let us use the set of finite extensions of \( \text{Ref}_{u,\alpha} \) as assignments of values to variables, where an extension of a function \( F \) is a function which is a superset of \( F \), regarded as a set of ordered pairs (Lasersohn 2015:42[52]).

i. If a use \( \alpha \) of \( \alpha \) contains a free variable, then \( \alpha \) will denote a function whose domain is the set of all finite extensions of \( \text{Ref}_{u,\alpha} \) (Lasersohn 2016:42[52]).

ii. In cases where the referential and temporal indices (if present) on a use \( \alpha \) of \( \alpha \) are in the domain of \( \text{Ref}_{u,\alpha} \), we do not consider the extensions of \( \text{Ref}_{u,\alpha} \).

Extensions of \( \text{Ref}_{u,\alpha} \) provide assignments to elements that are in \( \text{Relevant}_{u,\alpha} \) and not to elements that are outside of the set of relevant objects.

**6.3.8 The Interpretation Function** \( \text{Lex}(\alpha, u, w, p) \)

The interpretation function \( \text{Lex}(\alpha, u, w, p) \) provides interpretations to lexical items.
Definition 79. \( \text{Lex}(\alpha, u, w, p) = \text{def} \) the “customary” definition of \( \alpha \) relative to a world of use \( u \in W \), a world of context \( w \in W \), and a perspective \( p \). Where a perspective \( p \) consists of an individual \( x \in D_x \) and time \( t \in T \), construed as the time of the context, which is often the time of use, and a world \( w' \in W \), which is construed as the “world of assessment” (Lasersohn 2015:38,65,65 footnote 59[52]).

Definition 80. Extensional and Intensional Uses of an Expression (Lasersohn 2015:67[52])
For any lexical item \( \alpha \), if US\( \text{E}(\alpha, \alpha) \), then:

a. If \( \alpha \) is extensional in \( u \), then \( \llbracket \alpha \rrbracket^{u,w,p} = \text{Lex}(\alpha, u, w, p) \);

b. If \( \alpha \) is intensional in \( u \), then \( \llbracket \alpha \rrbracket^{u,w,p} = \lambda < w', p' > \) such that \( \text{Lex}(\alpha, u, w', p') \);

The content of an expression is a function that maps any world-perspective pair to its denotation, relative to that world and perspective (Lasersohn 2015:67[52]).

Definition 81. For any use \( \alpha : \llbracket \alpha \rrbracket^{u} = \lambda < w, p > . \llbracket \alpha \rrbracket^{u,w,p} \)

6.3.9 Truth Conditions in a Relativist Model \( M \)

The truth of a sentence content \( \Phi \) is determined relative to a world \( w \) and a context of assessment \( c \):

Definition 82. \( \Phi \) is true relative to a world \( w \) and a context of assessment \( c \) iff
\( \Phi(w, < \text{judge}_c, \text{time}_c, \text{world}_c >) = \text{truth} \);
\( \Phi \) is false relative to \( w \) and \( c \) \( \Phi(w, < \text{judge}_c, \text{time}_c, \text{world}_c >) = \text{falsity} \) (Lasersohn 2015:70[52]).

Monadic truth of sentence contents is defined by universally quantifying on the perspective indices and fixing the modal evaluation world to the actual world (Lasersohn 2016:71[52]):

Definition 83. Let \( w_@ = \text{def} \) the actual world.

Where \( w_@ \in W \) and \( W \) is a non-empty set, construed as the set of possible worlds.

Definition 84. \( \Phi \) is true iff, for all individuals \( x \), times \( t \), and worlds \( w' \) : \( \Phi(w_@, < x, t, w' >) = \text{truth} \);

\( \Phi \) is false iff, for all individuals \( x \), times \( t \), and worlds \( w' \) : \( \Phi(w_@, < x, t, w' >) = \text{false} \).
The truth of a use of a sentence $\phi$ relative to contexts of assessment is defined relative to a use, a world of use, a world of modal evaluation, and a context of assessment (Lasersohn 2016:70[52]):

**Definition 85.** $\phi$ is true in a world $u$ relative to a world $w$ and a context of assessment $c$ iff $[\phi]_{u,w,<\text{judge, time, world},>} = \text{truth}$;

$\phi$ is false in a world $u$ relative to a world $w$ and a context of assessment $c$ iff $[\phi]_{u,w,<\text{judge, time, world},>} = \text{false}$.

Monadic truth is defined for sentence uses as follows (Lasersohn 2015:71[52]):

**Definition 86.** $\phi$ is true iff, for all individuals $x$, times $t$, and worlds $w'$ : $[\phi]_{w',w,<x,t,w'>} = \text{truth}$;

$\phi$ is false iff, for all individuals $x$, times $t$, and worlds $w'$ : $[\phi]_{w',w,<x,t,w'>} = \text{false}$.

The truth of a sentence $\phi$ is defined relative to quadruples of a use, a world of use, a world of modal evaluation, and a context of assessment (Lasersohn 2015:70[52]):

**Definition 87.** $\phi$ is true relative to a use $\phi$, in a world $u$, relative to world $w$ and context of assessment $c$, iff, $\text{USE}(\psi, \phi)$ and $\phi$ is true in $u$ relative to $w$ and $c$;

$\phi$ is false relative to a use $\phi$, in a world $u$, relative to world $w$ and context of assessment $c$ iff, $\text{USE}(\psi, \phi)$ and $\phi$ is false in $u$ relative to $w$ and $c$.

Monadic truth for sentences relative to uses with no relativization to contexts of assessment is defined as follows:

**Definition 88.** $\phi$ is true relative to a use $\phi$ iff, $\text{USE}(\phi, \phi)$ and $\phi$ is true;

$\phi$ is false relative to a use $\phi$ iff, $\text{USE}(\phi, \phi)$ and $\phi$ is false.

The definitions of monadic truth only apply in cases where the truth of a content, use, or sentence has a truth value that is not relative (Lasersohn 2015:71[52]).
6.4 Features Values and Unification: The Role of the Lexicon

This section presents the general rules for the translation of a fragment of English into the wffs of the logical language in Section 6.4.1 before presenting the lexicon.

Lexical entries, which contain the translations, are then presented, as relevant, throughout the rest of the chapter.

6.4.1 Translation of Fragment to wffs

There is a correspondence between the categories $e$, $t$, $v$ and $i$ of the syntax and the types $e$, $t$, $v$, and $i$ of the logical language.

**Definition 89.** The function $MAP$ is a mapping from the categories of English, $Cat$, to the set $Y$ of types, such that:

\[
MAP(e) = e,
\]
\[
MAP(t) = t,
\]
\[
MAP(v) = v,
\]
\[
MAP(i) = i,
\]
\[
MAP(A/B) = \langle< MAP(B) >, MAP(A) >, \text{ whenever } A, B \in Cat,
\]
\[
MAP(A \cap B) = \langle< MAP(A) >, MAP(B) > \text{ whenever } A, B \in Cat.
\]

The basic elements of translation are the mapping $MAP$ from categories $e$, $t$, $v$, and $i$ to types $e$, $t$, $v$, and $i$. The categories of the grammar of English translate into the corresponding types. The types, however, include two symbols, $r$ and $s$, the presence of which is encoded lexically in this presentation.\(^{12}\)

\(^{12}\)In Montague Grammar and many other grammars in the tradition of MG, the logical language is eliminable, and it is possible to translate directly from the syntax (PTQ). The logical language, then, does not express anything that cannot be expressed in the translation language. Steedman (2000:69[92]) notes that CCG takes the logical language to be the place in which proofs are constructed. Steedman 2000[92] contrasts the approaches as being proof theoretic, in the case of CCG, and model theoretic in the case of PTQ.

As a result of using the model-theoretic interpretation, there could be portions of the treatment of English that are not possible to account for in the proof theoretic tradition of CCG. The question of how the theory would be stated without a model-theoretic component is left for future work.
The symbol $s$ is familiar from Montague 1973[63] and was defined in Section 6.3:

If $a$ is a type, then $D_{<s,a>} = D_{w,<x,t,w'>}^a$ is a type, such that $x$ is an individual, $t$ is a time, and $w, w'$ are worlds.

Unlike Montague’s 1973[63] type from the domain of worlds to the domain of types, $s$ here follows Lasersohn’s 2015[52] presentation in being from the domain of the cartesian product of worlds and perspectives. Perspectives are the cartesian products of an individual, a world, and a time.

The symbol $r$, which was also defined in Section 6.3, is an assignment function:

If $a$ is a type, then $D_{<r,a>} = D_R^a$ is a type, such that $R$ is a partial function from positive integers to $D_e \cup D_i$.

The presence of $r$ depends on whether or not an expression containing a free variable has that variable satisfied indexically. In this presentation, $r$ can only be present due to something containing an expression of type $e$, that is, a (pro)nominal expression with no indexical referent, or an expression of type $i$, that is, a temporal expression with no indexically determined time.

### 6.4.2 Lexicon

Lexical entries play an important role in formal grammars. One major theoretical ideal that is foundational in CCG is that the language-specific, idiosyncratic information of a given natural language is encoded in the lexicon. The goal behind CCG is described by, "a small set of type-driven combinatory syntactic rules projects the sounds and meanings of the language specific lexicon onto the sounds and meanings of all and only the sentences of the language" (Steedman (2012:77[93])). MM-CCG construes the realization of that ideal in a way that is more practical, by allowing the slash types to express more language-specific string concatenation rules. One alternative to multi-modal operators, which has been used in implementations of CCG, involves encoding multiple lexical entries for a word, depending on the type it needed to have in a given syntactic structure (Baldridge & Kruijiff 2003 [6]). With the Multi-Modal operators, language-specific variations in the syntax are within the Universal set of combinatory rules that Steedman 2000[92] proposes. Such language-specific syntactic information is kept separate from more idiosyncratic lexical information. Examples of more idiosyncratic lexical information include, for instance, whether a verb is an equi verb or a raising verb, or whether a verb takes a verbal argument with an overt infinitival form or a base form (without to),
or whether or not a word takes irregular past tense morphology.

The lexical entries are based on those proposed in Villavicencio 2002[101] and Sag, Wasow, & Bender 2003[85], and the general demands of this research. The entries Villavicencio 2002[101] proposes are written in a unification-based generalized categorial grammar (UB-GCG). The lexical entry style will be familiar to anyone who has worked with Head Driven Phrase-Structure Grammar (HPSG) (Pollard & Sag 1994[77]). Some of the attributes and values have different names, following conventions of Combinatorial Categorial Grammar (CCG) (Steedman 2000[92]), and some are introduced in this work.

Villavicencio 2002[101] provides a concise background on inheritance-based lexicons and how they relate to categorial grammars. She provides lexical entries with categorial grammar categories, therefore, some of her decisions are incorporated into this presentation and the following summary owes much to her work. Many of the feature and value names, however, are borrowed directly from Sag, Wasow, & Bender 2003[85].

The lexicon is characterized in terms of a finite set of features and a finite set of values. Each feature takes only particular values. For example, the feature of person, PERSON, which is itself a feature of the agreement features AGR, has the possible values of 1st, 2nd, and 3rd.13

Many lexical entries share features and values of other lexical entries. For example, the basic expressions Jess and Syd are both proper names. As proper names, they both have a HEAD feature in their syntax, with a value of noun.14 They both have the agreement value of 3rd for the feature PER. An example of a features structure (also known as an Attribute Value Matrix (AVM)) for the proper name Jess, is given in Figure 6.4 below.

The lexical entry for the proper name Syd shares most of the same features and values. The only part that differs, in fact, is the orthography and fact that the value of the feature NAMED is co-indexed with the sign Sydn, rather than Jessn.15 Due to the similarities,

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13 Any features can also take the empty brackets [ ] as a value. They can be unified with any other feature (Sheiber 1986[88]).
14 Following common conventions, features are written in all capital letters and values in lower-case italics.
15 The value for gender is empty. Gender of proper names is important for pronoun agreement. The lexical entries allow unification with any gender. The gender of a given use of a proper name is considered to be pragmatically determined, set by the preferences of the individual that the proper name denotes. A simplifying assumption is made that the gender pronouns of choice remain the same in a given discourse. Uses of singular they are considered as a viable form of gender-neutral or non-binary gender pronouns in
both are considered to inherit many of their features and values from a general lexical type called a pronoun-lexeme, which Sag, Wasow, & Bender 2003[85] abbreviate pn – lxm. The version of the pn – lxm for this presentation is given in Figure 6.5 below.

Through inheritance, it is possible to abbreviate lexical entries for individual words by the values particular to those entries, noting that they are an example of a particular lexeme category. A proper name can be introduced as in Figure 6.6 below.

The major types also structure the lexicon into a lexical ontology. A full inheritance hierarchy will not be presented, but the portions will be borrowed from that presented in Sag, Wasow, & Bender 2003[85]. Inherited structures will be described in more detail as necessary for the exposition regarding the fragment of English which is the focus of this monograph.

Villavicencio 2002[101] presents a multiple-inheritance hierarchy for the lexicon. A lexical entry can inherit features and values from multiple parent-entries. Her lexicon uses default inheritance, so the inheritance is, as a result, non-monotonic: An individual lexical entry can have features which override the default inherited from a parent.

Figure 6.4: Attribute value matrix for Jess
However, portions of the lexicon have monotonic inheritance.\(^\text{16}\)

The lexical entries in this monograph encode the category and logical type information of an expression among the values for features which are not in the category types. As was mentioned in section 6.2, the category feature, along with the rules of the MM-CG over-generate. For example, common nouns, proper nouns, and pronouns share a type, \(< (r), e >\), therefore, nothing in the categorial grammar rules prevents either from combining with determiners. The difference between these categories is handled in terms of features and values in their lexical entries. The feature values, in such cases, are essentially adding granularity to the categorial rules. In much of the exposition after the presentation of the lexical entries, feature values which are relevant in a particular example will be written as a subscript. For example, if a rule involves the pronominal quality of a nominal expression, its category will be written as \(N_{\text{NAME}}\), as an abbrevia-

\(^{16}\)There will not be a need to use much non-monotonic inheritance in the fragment covered in this monograph. The only example of when it is used is when grammatical aspect changes the default lexical aspect of a predicate. However, the use of a default lexical aspect associated with a verb is really a shorthand for a more in-depth treatment of Aktionsart, which is a topic for future research. Therefore, it is not certain from the fragment used in this monograph that non-monotonic inheritance is necessary.
Features and values are handled in terms of unification (Shieber 1986[88]), which is straightforward in the cases in which defaults are not used. That is, when two signs combine, by the rules of the grammar and the translation rules, there is a sort of check on them that their features are not in contradiction. When the value for a sign’s feature does not matter, the values is written as the blank square brackets, [ ]. The blank brackets can unify with any feature value.

The argument structure list can differ in that a phrase can be considered to be complete without all of the elements on its argument structure list being present. Furthermore, the parts of qualitative aspect can change based on grammatical aspect, so predicates have some default Aktionsart features which can be changed.

The next subsections will explain what can be expected in the lexical entries for basic expressions and what the feature structures for phrasal expressions look like. In Section 6.5, the lexical entries of particular expressions will be discussed in more detail along with their interpretation.

6.4.3 Features and Values

As in HPSG (Pollard & Sag 1994[77], Pollard, Sag & Wasow 2003[85]), words are represented as signs and have three major features: PHON, SYN, and SEM. This characterization of signs is based in the work of de Saussure 1916[28]. The format used in Villavicencio 2002[101] has three major features for all signs that subsume all other features: ORTHO, CAT, SEM, for the orthographic description of the sign, the

---

17Putting relevant features as subscripts is a common technique used in Steedman 2000[92], Bach 1983[4], and in many other instances.
categorial grammar information, and the semantic information, respectively. In this presentation, a sign has four major features that subsume all others: PHON, SYN, SEM, and ARG – ST. In the current grammar, PHON only subsumes the feature ORTHO, which, as Villavicenio’s, only presents the orthographic sequence representing the sign in English. SYN subsumes several features, including CAT, which provides the syntactic category of the linguistic expression.¹⁸

The treatment of the semantics in this work differs significantly from the semantics standardly presented in HPSG and CCG. In preview, it presents an event semantics with neo-Davidsonian events and translations, which are interpreted in the relativist model. The UB-GCG uses a neo-Davidsonian based semantics implemented so that the typical argument structure of a verb is not encoded in the syntactic type, but it is in the lexical argument structure. The types in this grammar are such that there is no distinction between a transitive and intransitive verb. Each are of category Bse. The linking between the combinatorial syntactic information and the semantic role assignments is in the lexicon, using default argument structure lists (as in Villavicencio 2002[101]).

Handling argument structure in this way, from a syntactic perspective, increases the amount of combinatorial information encoded in the category of the arguments and adjuncts of a verb while decreasing that encoded in the category type of a verb.

6.4.4 Phrasal Expressions in the Lexicon

Phrasal expressions are built compositionally using syntactic categories and associated combinatorial rules, but with feature unification providing additional constraints. Villavicencio’s 2002[101] treatment puts the combinatorial rules in the lexical and phrasal entries. She uses the feature RESULT for the category that results from the combination of expressions and ACTIVE for the arguments that have not yet combined with the type.

The RESULT and ACTIVE features are useful for representing neo-Davidsonian

¹⁸ Even though they share similar-looking lexical entries, HPSG and CCG represent different approaches to grammar representation. Questions of how the features being used here relate to the type of grammar that results have not been considered in detail. Following Kallmeyer 2010[39], it is recognized that CCG is a considerably less expressive formal grammar than HPSG. Some of the greater expressiveness probably comes from the unification grammar approach and inheritance of structures (as opposed to a very small set of rules in CCG). It is important to consider how the particular features proposed and the use of unification affects the resulting grammar. Such questions, however, are left to future work.
event semantics visually in the lexicon. In the event semantics proposed in this grammar, verbs are of the category \(Bse\), and they do not differ in whether they are transitive or intransitive, for example. They do not syntactically combine with something of type \(t\) to result in something of type \((t/v)\) nor vice versa until all of the semantic roles are supplied. Rather, the linguistic expressions that combine with semantic role labels, when labeled, are of type \(Bse \circ Bse\), and they combine with the verb, the directionality depending on the location, as encoded in the type. A sample lexical entry is given in Figure 6.7 below for the predicate and argument \(Jess\) swim. The category information appears in the feature \(SYN\). The features \(RESULT\) shows the category of the phrase as it is (derived from Villavicencio 2002[101]), the active category shows the expressions with which the resulting category has already combined. In this case, the subject noun phrase, \(Jess\).

\[\begin{array}{|c|}
\hline
\text{Jess swim} \\
\hline
\text{phrase} \\
\hline
\text{PHON: ORTHO: Jess swim} \\
\hline
\text{SYN:} \\
\hline
\text{RESULT: CAT : Bse ((t\circ(t/v))\circ(Tns))\circ(Aspect)} \\
\text{FORM: finite_bse} \\
\text{ACTV: CAT: < / [SIGN: Jess], / >} \\
\text{TYPE: < (r) < i, < i, t >>> < i, < i, t >>> < i, < v, t >>>, t >>>> >} \\
\text{INDEX: v}_1 \\
\hline
\text{SEM:} \\
\hline
\text{TRANSL: } \lambda P_{<i,<i,t>,} . \lambda Q_{<i,i,t>,} . \lambda t_i . \lambda f_{<i,t>,} . \exists v \exists t_k \\
\text{RELN}(t_i, \text{time}_{a,n}) \land REL(t_i, t_k)) \land SUBJ - NP(v) = Jess_i] \\
\hline
\end{array}\]

Figure 6.7: Attribute value matrix for the phrase \(Jess\) swim

The active category list is the default argument structure of the verb. The feature \(SEM\) presents the semantic information. The type is derived from the category, as defined in section 6.4.1. The \(r\) in Figure 6.7 is an assignment of values to variables.

The index of the semantics is the same as the event to which the phrase refers (as shown in the feature \(EVENT\)). The subject noun phrase is co-indexed with the semantic
role of \textit{SWIMMER}, which is listed in the restriction.

The phrase \textit{Jess swim} denotes a set of set of events that has not yet been grounded to a particular event. The result type is \((t \land (t/v)) \land (Tns)) \land (Aspct)\).

The combination of the predicate with its arguments before the combination with aspect and tense allows the affects of arguments on the lexical aspect to be handled before combining with tense, grammatical aspect, or temporal adverbs. As stated in the introduction, a formal model of lexical aspect and the mereology of events is not presented in this paper, but, in its expansion, such semantic information is combined before grammatical elements, which, among other things, have the potential to ensure that temporal phrases align with the lexical aspect.

6.4.5 Summary of the Lexicon

This section explained what type of lexicon is being used in the formal grammar. It has given a general explanation of how that lexicon works with the MM-CCG that has been presented. In the next sections, specific signs and phrases are presented in the framework.

There are several approaches to what the language specific lexicon needs to contain as opposed to what is encoded by the types. Several of the features that often appear in the lexical entries, such as the person and number marking on a verb or a noun, could be handled in terms of categories. Bach 1983[4], in describing what features mean in a grammar, suggests that they abbreviate much finer-grained categorial distinctions. So, using the expression \(N_{[sg]}\), where \([sg]\) is a feature of being singular, is an abbreviation for a language fragment that contains a set of basic expressions of type \(N\) that are singular, say \(N_{[sg]}\), basic expressions of type \(N\) that are plural or mass, and so on. Alternatively, it is possible to define singular common nouns as basic expressions and derive a set of expressions that are plural, for example, via some kind of morphological change. If the morphology were handled by rules in such a way, it seems that, between basic expressions and phrasal expressions, we could define derived expressions that capture the role of agreement morphology.

The present grammar does not develop the morphological component, but rather abbreviates agreement information via lexical features.

Not all features, however, are features that can be alternatively described morphologically or as more fine-grained category labels. For example, even with a morphological
component, there needs to be exceptional rules for irregular forms. Something about the irregular basic expressions needs to indicate that the rules for plural marking, for instance, do not apply in the regular way. Doing that is not a huge deficit to a language model, but it needs to be considered that the amount of exceptionality in morphological marking for a given language creates a situation that is not as amenable to a small set of rules. Or, if it is, those rules, much as the CCG syntax, need to combine with lexical information in order to describe an appropriate output language.

6.5 Translation and Interpretation of Nouns and Non-Finite Verbs

This section provides the translation of nouns and non-finite verbs in the fragment and demonstrates how examples are interpreted, given the lexicon in Section 6.4 and the composition rules in Section 6.3.

<table>
<thead>
<tr>
<th>Variable Symbol</th>
<th>Logical Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>P, Q, R</td>
<td>(&lt; i, t &gt;, t &gt;)</td>
</tr>
<tr>
<td>T, U, V</td>
<td>Bse or (&lt; r &lt; Bse &gt;), as specified</td>
</tr>
<tr>
<td>f</td>
<td>(&lt; v, t &gt;)</td>
</tr>
<tr>
<td>g</td>
<td>(&lt; r, t &gt;)</td>
</tr>
<tr>
<td>x, y, z</td>
<td>(&lt; e &gt;) or (&lt; r, e &gt;), as specified</td>
</tr>
<tr>
<td>v</td>
<td>(&lt; v &gt;)</td>
</tr>
<tr>
<td>t</td>
<td>(&lt; i &gt;) or (&lt; r, i &gt;), as specified</td>
</tr>
<tr>
<td>i, j, k, n, m</td>
<td>numeric indices</td>
</tr>
</tbody>
</table>

Table 6.3: Variable Types

Before presenting the interpretation, a chart of variable types is given in Table 6.3 for consultation regarding translations, however, the types will often be subscripts on the variables for clarity.

6.5.1 Translation and Interpretation of Noun Phrases

As was stated in the syntax, repeated below:

**Definition 90.** Each occurrence of an expression of category N in \( \phi \) will be labeled with a positive integer, which we will call its referential index, and each occurrence of an
expression of category $B_{Tns}$ with a positive integer, which we will call its temporal index. The indices are syntactic indices used in binding and anaphoric relations (modified from Lasersohn 2015:34[52]). Where an occurrence differs from a use in that a sentence has a given occurrence of an expression, but that sentence could be used in many different ways.

Lexical entries for expressions of type $N$ and type $Bse$ will be labeled in the lexicon with an $n \in I^+$, which takes a particular numeric value in a given sentence or discourse, starting with $n$ for the first occurrence, $n + 1$ for the second occurrence, and so on.

For the sake of review, the definitions of $Lex$ and $Ref$ are repeated, briefly, below:

Let us assume that the pragmatic context of a use $\alpha$, in a world $u$, will normally provide a function $Ref_{u,\alpha}$, mapping positive integers from some finite subset $I'$ of $I^+$ onto elements of $Familiar_{u,\alpha}$. We regard $Ref_{u,\alpha}(n)$ as the denotation of any constituent of $\alpha$ bearing referential index $n \in I'$ (Lasersohn 2015:38[52]).

**Definition 91.** Where $\alpha$ has syntactic constituents $\alpha_m, \ldots, \alpha_n$ and $1 \leq i, j \leq n$:

a. If $time_{u,\alpha_m}$ precedes $time_{u,\alpha_n}$, then $Ref_{u,\alpha_n} \subseteq Ref_{u,\alpha_m}$

b. $Ref_{u,\alpha} = Ref_{u,\alpha_1} \cup \ldots \cup Ref_{u,\alpha_n}$

Where $n, m \in I'$, and $time_{u,\alpha_n}$ is the time of the context in the world of use, $u$, in most cases, ‘time’ is the speech time.

**Definition 92.** For any use $\alpha$ and world $u$, let us use the set of finite extensions of $Ref_{u,\alpha}$ as assignments of values to variables. Where an extension of a function $F$ is a function that is a superset of $F$, regarded as a set of ordered pairs (Lasersohn 2015:42[52]).

i. If a use $\alpha$ of $\alpha$ contains a free variable, then $\alpha$ will denote a function whose domain is the set of all finite extensions of $Ref_{u,\alpha}$ (Lasersohn 2015:42[52]).

ii. In cases where the referential and temporal indices (if present) on a use $\alpha$ of $\alpha$ are in the domain of $Ref_{u,\alpha}$, we do not consider the extensions of $Ref_{u,\alpha}$.

Extensions of $Ref_{u,\alpha}$ provide assignments to elements which are in $Relevant_{u,\alpha}$ and not to elements that are outside of the set of relevant objects.

**Definition 93.** $Lex(\alpha, u, w, p) =_{def}$ the “customary” definition of $\alpha$ relative to a world of use $u \in W$, a world of context $w \in W$ and a perspective $p$ which consists of an
individual $x \in D_e$ and time $t \in T$, construed as the time of the context, which is often the time of use, and a world $w' \in W$ which is construed as the “world of assessment” (Lasersohn 2015:38,65,65 footnote 59[52]).

**Definition 94.** Extensional and Intensional Uses of an Expression (Lasersohn 2015:67[52])

For any lexical item $\alpha$, if $USE(\alpha, \alpha)$, then:

a. If $\alpha$ is extensional in $u$, then $\llbracket \alpha \rrbracket^{u,w,p} = Lex(\alpha, u, w, p)$;

b. If $\alpha$ is intensional in $u$, then $\llbracket \alpha \rrbracket^{u,w,p} = \lambda < w', p' >$ such that $Lex(\alpha, u, w', p')$;

The content of an expression is a function which maps any world-perspective pair to its denotation relative to that world and perspective (Lasersohn 2015:67[52]).

**Definition 95.** For any use $\alpha : (\alpha)^u = \lambda < w, p >$ such that $\llbracket \alpha \rrbracket^{u,w,p}$

### 6.5.2 Translation and Interpretation of Proper Names

Each proper name has a lexical entry along the lines of Figure 6.8 below.

The function $Lex$ takes such a lexical entry as the $\alpha$ argument below:

a. If $USE(\alpha, Jess_{n,\text{NAME}})$ then $Lex(\alpha, u, w, p) = Ref_{u,\alpha}(n)$, provided that $Ref_{u,\alpha}(n)$ is named $Jess$ in $u$.

b. If $USE(\alpha, Pat_{n,\text{NAME}})$ then $Lex(\alpha, u, w, p) = Ref_{u,\alpha}(n)$, provided that $Ref_{u,\alpha}(n)$ is named $Pat$ in $u$.

c. If $USE(\alpha, Syd_{n,\text{NAME}})$ then $Lex(\alpha, u, w, p) = Ref_{u,\alpha}(n)$, provided that $Ref_{u,\alpha}(n)$ is named $Syd$ in $u$.

The content of a given use of a proper name in a world $u$ will denote the same individual in all possible worlds $w$, even though that individual might be named differently in different worlds (Lasersohn 2015:40,65[52]).

Recalling the definition of $USE(\alpha, \alpha)$, item (a.) above states that $\alpha$ is a token of the expression $Jess_n$. The token is given a numeric index $n$, which is its referential index (see definition above).

The AVM for $Jess$, above, is for the linguistic expression $Jess$, not for the use $\alpha$. The use $\alpha$ provides a referential index, $n$, by definition. The general AVM for $Jess$ indicates
in the MODE feature of the AVM that it is a referential expression, that is, it has the value \textit{ref}. The index of \\textit{Jess}, \textit{i}, in the AVM above, is co-indexed with the NAMED in the restriction values.

The function \( \text{Ref}_{u, \alpha} \) provides a pragmatic context, in this case, for the use of the expression \textit{Jess}. Let us assume that Jess is among the familiar in a pragmatic context, then the numeric index on \textit{Jess} is mapped to an individual. So, \textit{Jess}_n is mapped to an individual in the pragmatic context. If that name is mapped to a different individual, then a use of the name can fail to be interpretable.

The numeric index and the mapping of the numeric index to an individual are both, by definition, introduced with a use of the expression.

### 6.5.3 Translation and Interpretation of Pronouns

Each pronoun is labeled syntactically with a referential index. In order to represent the referential intentions of the speaker, let us assume that the pragmatic context of a use \( \alpha \) (in a world \( u \)) will normally provide a function \( \text{Ref}_{u, \alpha} \), mapping positive integers

---

**Figure 6.8: Attribute value matrix for \textit{Jess}**

\[
\begin{array}{c|c}
\text{pn} - \text{lxm} & \text{Jess} \\
\hline
\text{PHON: ORTHO:} & \text{noun} \\
\text{CAT : e} & \\
\text{SYN: HEAD:} & \text{PER: 3rd} \\
\text{AGR:} & \text{NUM: /sg} \\
\text{GEN: [ ]} & \\
\text{MODE: ref} & \\
\text{TYPE: e} & \\
\text{INDEX: i} & \\
\text{SEM:} & \text{TRANSL: \textit{Jess}} \\
\text{RESTR:} & \text{RELN: name} \\
\text{NAME: \textit{Jess}} & \\
\text{NAMED: i} & \\
\text{ARG-ST: / < >} & \\
\end{array}
\]
from some finite subset \( I' \) of \( I^+ \) onto elements of \( \textit{Familiar}_{u, \alpha} \). We regard \( \textit{Ref}_{u, \alpha}(n) \) as the denotation of any constituent of \( \alpha \) bearing referential index \( n \in I' \) (Lasersohn 2015:38[52]).

**Definition 96.** If \( \textit{USE}(\alpha, \alpha_i) \) and \( \alpha_i \) is a pronoun:

a. If \( \textit{Ref}_{u, \alpha}(i) \) is defined and \( \textit{Ref}_{u, \alpha} \) is a feature-respecting assignment with respect to \( u, \alpha \), then \( \textit{Lex}(\alpha, u, w, p) = \textit{Ref}_{u, \alpha}(i) \).

b. If \( \textit{Ref}_{u, \alpha}(i) \) is not defined, then \( \textit{Lex}(\alpha, u, w, p) = [\lambda r(i)] \) where \( \textit{Ref}_{u, \alpha} \triangleq r \) (Lasersohn 2015:66[52]).

Where \( u \in W \) is construed as the world of use \( i \in I \) [the set of integers] is a referential index, and \( r \) is a feature-respecting assignment of variables with respect to \( u \) and \( \alpha \).

The lexical entry for a pronoun looks as follows:

\[
\begin{align*}
\text{PHON: ORTHO:} & \quad \text{she} \\
\text{SYN: HEAD:} & \quad \text{noun} \\
& \quad \text{CAT: e} \\
& \quad \text{PER: 3}^{rd} \\
& \quad \text{AGR: NUM: /sg} \\
& \quad \text{GEN: femme} \\
& \quad \text{MODE: /ref} \\
& \quad \text{TYPE: < (r), e >} \\
& \quad \text{INDEX: n} \\
& \quad \text{RESTR: RELN: femme} \\
& \quad \text{INST: n} \\
& \quad \text{ARG-ST: / <>}
\end{align*}
\]

Figure 6.9: Attribute value matrix for the pronoun \textit{she}

The use of a pronoun differs from a proper name in several ways. First, the variable assignment must be feature-respecting, which, in this case, means that the person and number must agree between the expression and the individual to whom it refers. Second,
there is a condition for when $Ref_{u,a}$ is not defined. When it is defined, the pronoun refers to an entity which is familiar. When it is not defined, it takes extensions of $Ref_{u,a}$.

An example of a case where the function is defined would be, for instance, if the pronoun use is anaphor to a proper name used already in the discourse, provided that proper name did not fail to refer.

### 6.5.4 Translation and Interpretation of Common Nouns

Common nouns have lexical entries along the following lines:

$$\begin{array}{|c|c|}
\hline
\text{cntn} \rightarrow lxm \\
\text{PHON: ORTHO:} \quad \text{chinchilla} \\
\text{SYN: HEAD:} \quad \text{noun} \\
\text{CAT: } e \\
\text{AGR:} \quad \text{PER: 3rd} \\
\text{} \quad \text{NUM: /sg} \\
\text{} \quad \text{GEN: [ ]} \\
\text{} \quad \text{TYPE: count} \\
\text{MODE: /ref} \\
\text{TYPE: < r, e >} \\
\text{INDEX: n} \\
\text{SEM:} \quad \text{TRANSL: } \lambda r.r(n) \\
\text{} \quad \text{RESTR:}\text{ RELN: be}_a\text{.chinchilla} \\
\text{} \quad \text{CHINCHILLA: n} \\
\text{ARG-ST: } / <> \\
\hline
\end{array}$$

Figure 6.10: Attribute value matrix for chinchilla

Lasersohn (2015:44[52]) places a novelty condition on proper nouns, such that uses are not in the domain of the $Ref_{u,a}$, but rather involve extensions of $Ref_{u,a}$:

**Definition 97.** For any use $\alpha$ of a common noun $\alpha$ is felicitous in $u$ if the referential index on $\alpha$ is outside the domain of $Ref_{u,a}$ (Lasersohn 2015:44[52])

**Definition 98.** Interpretation of Common Nouns (Lasersohn 2015:66[52])
a. If US \( E(\alpha, \text{chinchilla}_{i,t_j}) \) then \( \text{Lex}(\alpha, u, w, p) = [\lambda r. r(i)] \) where \( \text{Ref}_{u,\alpha} \preceq r \) and \( r(t_j) \) is a time and \( r(i) \) is a chinchilla at \( r(t_j) \) in \( w \).

b. If US \( E(\alpha, \text{badger}_{i,t_j}) \) then \( \text{Lex}(\alpha, u, w, p) = [\lambda r. r(i)] \) where \( \text{Ref}_{u,\alpha} \preceq r \) and \( r(i) \) is a time and \( r(i) \) is a badger at \( r(t_j) \) in \( w \).

The interpretation function \( \text{Lex} \) assigns to a use \( \alpha_{i,t_j} \) of a (singular) common noun, given a world of use \( u \), a world of context \( w \), and a perspective \( p \) any individual which has a satisfying assignment in the extension of \( \text{Ref}_{u,\alpha} \) at time \( t_j \).

Unlike proper names, common nouns are given an interpretation relative to a time.

6.5.5 Translation and Interpretation of Determiners and Quantifiers

**Definition 99. Interpretation of Determiners and Quantifiers (Lasersohn 2015:66[52])**

a. For the indefinite determiner, \( a \), if US \( E(\alpha, a) \) then \( \text{Lex}(\alpha, u, w, p) = \lambda x_e. x_e \), where \( x_e \) is an individual.

b. For a singular use of the definite article, if US \( E(\alpha, \text{the}_{sg}) \) then \( \text{Lex}(\alpha, u, w, p) = \lambda x_{<r,e>}. x(r') \), where there exists some \( y_e \), such that, for all assignments \( r_r \), if \( \text{Ref}_{u,\alpha} \preceq r \) and \( x(r) \) is defined, then \( y_e = x_{<r,e>}(r) \). Where \( r' \) is any arbitrary extension of \( \text{Ref}_{u,\alpha} \), such that \( x(r') \) is defined.

c. For a plural use of the definite article, if US \( E(\alpha, \text{the}_{pl}) \) then \( \text{Lex}(\alpha, u, w, p) = [\lambda x_{<r,e>}, y] \), such that there exists some \( y \), such that, for all \( r \), if \( \text{Ref}_{u,\alpha} \preceq r \) and \( x(r) \) is defined, then \( y \) includes \( x(r) \), such that \( x(r') \) is defined.

c. For the quantifier every, if US \( E(\alpha, \text{every}) \) then \( \text{Lex}(\alpha, u, w, p) = [\lambda x_{<r,e>} \lambda g_{<r,e>}. \text{for all } r \), if \( \text{Ref}_{u,\alpha} \preceq r \) and \( x_{<r,e>}(r) \) is defined, there is at least one \( r' \), such that \( r \preceq r' \) and \( g_{<r,e>}(r') = \text{truth} \].

The indefinite determiner is of type \( < e, < r, e >> \) and, since it combines with a common noun, there is an assignment function \( r \) on that individual.

The lexical entry for the plural use of the definite article differs from the singular entry in the translation according to (c) above and in the value for \( \text{NUM} \) which is \( pl \), for plural.\(^{19}\)

\(^{19}\)This monograph does not deal in detail with plural nouns, so the use of the plural definite will not be involved in the examples given.
Thus concludes the lexical entries for the determiners and quantifiers along with the interpretation of uses of them with *Lex*.

6.5.6 Translation and Interpretation of Argument Labels

The argument labels have lexical entries. Following Sag, Wasow, & Bender 2003[85], the feature *MODE* has *none* as a possible value (Sag, Wasow, & Bender 2003:136[85]), which is applied her to the semantic role labels.

The entry for a subject noun phrase, which is also in the A0 (agent-like) role of the predicate with which it combines, appears in Figure 6.14 below.

The entries are orthographically null.20 The argument structure contains a noun, since this is a semantic role label for a noun phrase. The fact that it is in the subject position is indicated in the category information.

The entry for the semantic role label for a noun phrase argument in the object position is given in Figure 6.15 below.

---

20 In a language with more prominent case-marking, they could encode the morphological marking. The morphological marking of pronouns could be present in English in such a way, however, it is not developed in this monograph.
Figure 6.12: Attribute value matrix for *the*

The semantic information labels the semantic role as A1, which can be associated with a patient in many transitive verb places.\(^{21}\)

There are also arguments of other types besides noun phrases. In this monograph, arguments containing verbal elements are of particular interest. Figure 6.16 below presents a semantic role label for an A1 argument, which is the base form of a verb. The semantic role label combines with a verbal argument.

**Definition 100. Interpretation of Semantic Role Labels**

If \( \text{US} E(\alpha, \text{SUBJ} – \text{NP}) \) then \( \text{Lex}(\alpha, u, w, p) = \lambda x_{<r,e>} \cdot \lambda V. \lambda f. [V(\lambda v[f(v) \land A0(v) = x])] \).

If \( \text{US} E(\alpha, \text{OBJ} – \text{NP}) \), then \( \text{Lex}(\alpha, u, w, p) = \lambda x_{<r,e>} \cdot \lambda V. \lambda f. [V(\lambda v[f(v) \land A1(v) = x])] \).

If \( \text{US} E(\alpha, v – \text{Comp}) \) then \( \text{Lex}(\alpha, u, w, p) = \lambda U. \lambda V. \lambda f. [V(\lambda v[f(v) \land A1(v) = U])] \).

Before going on to the entries for verbs, the next subsection presents the rules for semantic composition, which are then demonstrated, given examples of semantic role labeled noun phrases.

\(^{21}\)More of the nature of the argument is encoded in the verb with which it composes, as will be seen in the next section.
Composition and APPLY

The composition of interpretations is handled by the function APPLY. The function differs depending on whether or not the variable assignment \( r \) is present and on whether or not the type is intensional (having an \( s \) in the type).

**Definition 101. APPLY and compositional rules (Lasersohn 2016:43,58,67)**

Where \( w \in W \) is a possible world, \( p \) is a perspective, and \( r \) is a partial function from the set \( I^+ \) of positive integers to \( D_e \cup D_t \) and where \( [\gamma]^{u,w,p} = \text{def APPLY}([\alpha]^{u,w,p}, [\beta]^{u,w,p}) \) or \( ([\beta]^{u,w,p}, [\alpha]^{u,w,p}) \), whichever is defined.

**Case A:** is a case where \([\alpha]^{u,w,p}\) is a function and \([\beta]^{u,w,p}\) is a possible argument of \([\alpha]^{u,w,p}\).
If $\alpha \in D_{<s,a2>}$ and $\beta \in D_{a2}$, then $\text{APPLY}(\alpha, \beta) = \alpha(\beta)$.

**Case B:** (i) If $\alpha \in D_{<s,a1>}$ and there is no $a2$ such that $\beta \in D_{<r,a2>}$, then $\text{APPLY}(\alpha, \beta) = [\lambda < w, p > . \text{APPLY}(\alpha(w, p), \beta)]$.

(ii) Where for every $r$ such that $\text{Re}_f u_i \preceq r$, if $[\alpha]^{u,w,p}(r)$ is defined, then $[\alpha]^{u,w,p}(r)$ is a function and $[\beta]^{u,w,p}$ is a possible argument of $[\alpha]^{u,w,p}$. If $\alpha \in D_{<s,a1>}$ and there is no $a2$ such that $\beta \in D_{<r,a2>}$, then $\text{APPLY}(\alpha, \beta) = [\lambda r. \text{APPLY}(\alpha(r), \beta)]$.

**Case C:** (i) If $\alpha \in D_{<s,a2>}$ and $\beta \in D_{<s,a3>}$, then $\text{APPLY}(\alpha, \beta) = [\lambda < w, p > . \text{APPLY}(\alpha, \beta(w, p))]$.

(ii) Where for every $r$ such that $\text{Re}_f u_i \preceq r$, if $[\beta]^{u,w,p}(r)$ is defined, then $[\alpha]^{u,w,p}$ is a function and $[\beta]^{u,w,p}(r)$ is a possible argument of $[\alpha]^{u,w,p}$. If $\alpha \in D_{<s,a2>}$ and $\beta \in D_{<s,a3>}$ then $\text{APPLY}(\alpha, \beta) = [\lambda r. \text{APPLY}(\alpha, \beta(r))]$.

**Case D:** (i) If $\alpha \in D_{<s,a1>}$ and $\beta \in D_{<s,a2>}$, then $\text{APPLY}(\alpha, \beta) = [\lambda < w, p > . \text{APPLY}(\alpha(w, p), \beta(w, p))]$.

(ii) Where for every $r$ such that $\text{Re}_f u_i \preceq r$, if $[\alpha]^{u,w,p}(r)$ and $[\beta]^{u,w,p}$ are both defined, then $[\alpha]^{u,w,p}(r)$ is a function and $[\beta]^{u,w,p}(r)$ is a possible argument of $[\alpha]^{u,w,p}(r)$. If $\alpha \in D_{<r,a1>}$ and $\beta \in D_{<r,a2>}$, then $\text{APPLY}(\alpha, \beta) = [\lambda r. \text{APPLY}(\alpha(r), \beta(r))]$. 

Figure 6.14: Attribute value matrix for the null subject noun phrase

142
Figure 6.15: Attribute value matrix for the null object noun phrase

APPLY works cyclically on a typed expression, using whatever order is defined, as stated at the beginning of the definition:

\[ \ldots \text{where } \gamma^{u,w,p} = \text{def} \, APPLY(\alpha^{u,w,p}, \beta^{u,w,p}) \text{ or } (\beta^{u,w,p}, \gamma^{u,w,p}), \]

whichever is defined.

The translation from categories to types gives a definition in the sense that types only combine in a certain way.

Case A above handles examples in which there is no variable assignment nor intensionality in either the functor or the argument.

Case B describes the composition of an intensional functor with a non-intensional argument (in B(i)), or the composition of an expression taking an assignment of values to variables (in B(ii)) with an argument that does not involve an assignment of values to variables.

The variable assignment or world-perspective pair is extracted from the composed form using lambda extraction.

Case C describes the composition of a non-intensional functor with an intensional argument (in C(i)), or the composition of a functor which does not take a variable assignment with an argument which does take a variable assignment.

As with Case B, the variable assignment or world-perspective pair is extracted from the composed form using lambda extraction.
The interpretation of each basic expression is as follows:

\[
[\delta]^{u,w,p} = \operatorname{Lex}(\delta, u, w, p) \ (A1-NP)
\]
\[
= \lambda x. A. \lambda \delta. \lambda \alpha. [V(\lambda \alpha[f(v) \land A1(v) = x])] \quad \text{(by the definition of A1-NP)}
\]

\[
[\alpha]^{u,w,p} = \operatorname{Lex}(\alpha, u, w, p) \quad \text{(indefinite article, ‘a’)}
\]
\[
= \lambda x. x(i) \quad \text{(by the definition of a)}
\]
\[ \beta^{u,w,p} = \text{Lex}(\beta, u, w, p) \] (common noun ‘chinchilla\textsubscript{1,2}’)

= \lambda r . r(1), \text{where } \text{Ref}_{u,\beta} \leq r \text{ and } r(2) \text{ is a time and } r(1) \text{ is a chinchilla at } r(2) \text{ in } w. \tag{by definition of \textit{chinchilla}}

The basic expression are combined by APPLY. Determining which is the functor and which is the argument is done with the categories of the MM-CCG.

\[ \gamma^{u,w,p} = \text{APPLY}(\alpha^{u,w,p}, \beta^{u,w,p}) \]

= \lambda r : \text{Ref}_{u,\beta} \leq r . [\lambda x . x(r(1))] \text{ where } (2) \text{ is a time and } (1) \text{ is a chinchilla at } (2) \text{ in } w. \]

= \lambda r : \text{Ref}_{u,\beta} \leq r . [r(1)] \text{ where } (2) \text{ is a time and } (1) \text{ is a chinchilla at } (2) \text{ in } w.

\[ \eta^{u,w,p} = \text{APPLY}(\delta^{u,w,p}, \gamma^{u,w,p}) \]

= \lambda r : \text{Ref}_{u,\beta} \leq r . [\lambda x_{<t,e_a} . \lambda V . \lambda f . [V(\lambda v . f(v) \land A1(v) = x)]]r(1)] \text{ where } (2) \text{ is a time and } (1) \text{ is a chinchilla at } (2) \text{ in } w.

= \lambda r : \text{Ref}_{u,\beta} \leq r . [\lambda V . \lambda f . [V(\lambda v . f(v) \land A1(v) = r(1))]] \text{ where } (2) \text{ is a time and } (1) \text{ is a chinchilla at } (2) \text{ in } w.

Any restrictions on the assignment function \( r \) are stated and then the translation of the functor and argument combine in the order defined by the MM-CCG. The \( w f s \) reduce according to the lambda calculus. The satisfying assignment of values to variables, \( r \), must meet the requirements of each basic expression.

When expressions are intensional, the intensionality is similarly extracted to the front of the combined formula, with the variables for worlds and perspectives associated with the intensional expressions. Intensional examples will be presented in the next chapter.

6.5.7 Translation and Interpretation of Base forms of Verbs

The base forms of verbs are introduced with typing that combines that of Lasersohn 2015[52] and Champollion 2015[13] and adds combinatorics for tense and aspect.

Base form verbs are of type \(< i < i, t >>, < i < i, t >>, < i << v, t >>, t >>\). The translation of a use of the verb \textit{swim} is given as an example below:

\[
\lambda P \lambda Q \lambda t_i . \lambda f_{<i,t>} . \exists v \exists t_e [\text{swim}(v) \land f(v) \land (\tau(v) = t_e) \land P(t_i, t_e) \land Q(t_i, \text{time}_{u,v})].
\]
The variables $P$ and $Q$ are variables over two place relations which relate two times to each other. Following Champollion 2015[13], verbs and other expressions are sets of sets of events, written in the translation by $f_{<i,t>}$ which abstracts over the formula in which the event to truth value variable $f$ occurs. The method can be demonstrated looking merely at a role-assigned noun phrase:

$$\lambda r.AV.\lambda f.V(\lambda v[f(v) \wedge A1(v) = r(n)])$$

Figure 6.17: Tree diagram of the noun phrase 'a chinchilla' with an A1 semantic role label

At the point in the derivation in which the noun phrase 'a chinchilla' is combined with a semantic role, it refers to the set of sets of events in which individuals that $Lex$ maps to $r(n)$ where 'chinchilla' and that are in the semantic role called A1, which is sometimes called a patient role or as the direct object argument of a verb that is generally considered transitive.22

When the semantic-role labeled noun phrase 'a chinchilla' combines with a verb phrase, then the event variable $f$ applies to the set of set of events in which that noun phrase is the object of a particular predicate. Let us say to find as an example, the set of sets is

22Recalling earlier discussions that semantic roles are a topic for a monograph on their own and not rigidly defined structurally and there is a simplifying assumption that a subject noun phrase has an $A0$ (agent-like) role and an object noun phrase has a patient-like role.
then the sets in which a chinchilla is in the direct object position of the verb *to find*.

In such a way, semantic role labeled expressions are interpreted as sets of sets of events.

The verb with which they combine has features in its lexical entry, which must unify with the semantic role labeled arguments that are present.

The lexical entry for *sleep* is given in Figure 6.18 below.

![Figure 6.18: Attribute value matrix for the word sleep](image)

In this exposition, lexical aspect is treated in terms of features on verbs. Something that has *verb* for the *HEAD* value has additional features and values for Aktionsart (based on those in Smith 1991[89] and Klein (1994:79-80[43])).

The feature *QUAL – ASP* takes *stative* or *process* as values. The features *DURA* takes *durative* or *punctual* as values. The feature *TELIC* takes the values of *telic* or *atelic*. The feature *QUANT – ASP* takes the values *semel* or *iterative.*

---

23 Associating lexical aspect with a verb is a simplifying assumption that leads to the need to override default values when arguments of a verb or aspectual marking lead to changes. In a more fully developed theory of Aktionsart, it might be possible to avoid overriding defaults.
6.6 Review and Conclusions

This chapter presented a relativist semantic model based on that in Lasersohn 2015, with several adaptations. The combinatorial rules in the syntax were encoded using MM-CCG (Baldridge 2002 [5], Baldridge & Kruijff 2003 [6], Steedman 1996 [90], 2000 [92]). Lexical entries were presented in terms of attribute value matrices, using syntactic features inspired by Villavicencio 2002[101], and HPSG (Pollard & Sag 1994[77], Sag, Wasow & Bender 2003[85]).

Events were added to the types of the grammar, the categories of the logical language, and the translation in a manner inspired by both Lasersohn 2015 [52] and Champollion 2015[13]. The event semantics used was a neo-Davidsonian event semantics (Davidson 1967[22], 1980[23], Parsons 1990[74]). A formal model description for the relativist model of Lasersohn 2015[52] was presented, including a temporal and event ontology, inspired by Muskens 2003[67]. An event time and a general framework that could handle grammatical aspect was added to the framework of Lasersohn 2015[52].

The next chapter provides a treatment of tense and aspect in the formal grammar.
CHAPTER 7

TENSE AND ASPECT IN A FORMAL GRAMMAR

7.1 Introduction

The previous chapter, Chapter 6, presented a formal grammar for a fragment of English. This chapter continues the formal grammar, showing how tense and grammatical aspect are represented. It shows how tense and grammatical aspect compose in the grammar. It shows how closure operators interact with temporal and aspectual expressions.

The temporal and aspectual theory that is presented is based on the treatment in Klein 1994[43], with the incorporation of additions to the treatment of aspectual relations, which were motivated by Reed 2012[81]. The relations presented in the temporal and event ontologies of Muskens 1995[68], 2003[67] are used as a means of expressing the semantics of temporal and aspectual expressions. Examples are given of how the theory accounts for readings of English tense and grammatical aspect.

Tense and grammatical aspect are represented as relations between times, using the three Reichenbachian 1947[82] times reference time, utterance time, and event time. Following Klein 1994[43] and other subsequent work, tense is considered to be a relation between utterance time and reference time, and aspect is considered to be a relation between event time and reference time.

The focus of this chapter is on giving a treatment of tense and grammatical aspect that shows sensitivity to the nuances of grammatical aspect, without limiting the treatment to examples with modal auxiliary verbs. The nuances represented reflect the observations in Reed 2012[81] that posit the division of aspect into two major categories of relations: Those that involve containment and those that involve precedence. The grammatical details of the treatment differ from that in Reed 2012[81], however, in that the containment relations for the perfectives and imperfectives are captured in the assignment of events

1Reichenbach 1947[82] used speech time.
to temporal periods.

7.1.1 The Tense and Aspect Puzzles Involved in Modal Auxiliary Meaning

The grammatical context in which a use of a modal auxiliary verb occurs is tied to the possible readings it can have. Sentence (118) below can be read as an epistemic statement about Jess leaving the salt out in the past or a teleological reading which places leaving the salt out as a pre-condition on some goal.

118. You must have left the salt out.

Examples construed to pre-dispose readers to each of the readings are given in sentences (119 - 120) below:

119. This bread tastes offensively bland. You must have left the salt out. (epistemic)

120. In order to produce a good loaf of bread with this much baking soda, you must have left the salt out when preparing the dry ingredients. (teleological)

Similarly, the sentence (121) below has a number of additional readings, in addition to describing a different event.

121. You must leave the salt out.

Sentence (121) can be read to be a command to leave the salt out. It can be read as part of a teleological statement. It can be read as an epistemic statement about the person frequently leaving the salt out of things.

In sentence (122) below, the leave-the-salt-out event is a telic, one time event in the future relative to the speech time. Sentence (123) below illustrates a standing request to leave the salt out whenever grandmother might be eating the bread:

122. You must leave the salt out of the bread! Grandmother has very high blood pressure. (deontic requirement)(one time)

123. You must always leave the salt out of the bread. Grandmother has very high blood pressure. (deontic requirement)(iterative)

---

2Not all speakers can interpret this as a grammatical sentence, but most, it is supposed, would allow the sentence, In order to get into the program, you must have completed five courses.
The requirement, that is, the must-event, holds of a time extending into the future of the speech time. The time of the leave-the-salt-out event and the must-event overlap.

In sentence (124) below, the leave-the-salt-out event is one that is tied to a goal. The goal is stated in terms of the dinner, indicating a specific one time instance for which the statement is relevant. The time of the leave-the-salt-out event, however, is not necessarily set to a particular time nor to a particular instance of baking bread. In sentence (125) below, the leave-the-salt-out event is tied to any instance of a want-to-produce-a-good-bread-pudding event. The must-event iterates over instances of bread making, overlapping with them.

124. In order to produce a good bread pudding for the dinner, you must leave the salt out of the bread. (teleological) (one time)

125. In order to produce a good bread pudding, you must always leave the salt out of the bread. (teleological) (iterative)

In both cases discussed so far, the deontic and teleological cases, the leave-the-salt-out event is intensional in that the truth of the entire sentence is not dependent on the leave-the-salt-out event actually taking place in the world of utterance.

The epistemic reading in sentence (126) below only has a reading in which the leave-the-salt-out event is something that is iterated in the past of the speech time.

126. Your bread pudding tastes so bland. You must leave the salt out of the bread. (epistemic) (iterative)

The must-event refers to the speaker’s thoughts about the likelihood of an iterative leave-the-salt-out event in the past, up to and including the speech time. In order to convey a one-time instance, it is necessary to use the perfect morphology:

127. Your bread pudding tastes so bland. You must have left the salt out of the bread. (epistemic) (one time)

The second verb, leave, can be interpreted either as perfective or imperfective with teleological and deontic readings but only as a generic present when with the epistemic reading.

The teleological reading is not inherently set to a particular event, although it can be, if the in-order-to clause specifies one. The teleological reading is timeless, rather. In
contrast, the deontic reading is one in which the *must*-event starts at the speech time and extends into the future.

Given an interpretation of the temporal and aspectual expressions in a sentence, independently of the interpretation of the modal auxiliary, there are some contexts, in the sense of ordering sources and conversational backgrounds, in Kratzer’s 1981[45] terms, associated with the modal auxiliary that are compatible with the temporal and aspectual interpretation of expressions in the utterance and some contexts that are not. More generally, the sentence level grammatical context has to be compatible with the context, in the sense of a pragmatic context, that includes, among other things, background knowledge and world-knowledge. Furthermore, compatibility implies that there is some relation between the grammatical expressions and the types of world-knowledge they can express.\(^3\) It is the connections between sentence-level grammatical context and pragmatic context, in the sense of background and world knowledge, that is the focus of this work, specifically, the relation between temporal and aspectual meaning on one hand, and modal readings (in the sense of Kratzer’s 1981[45] conversational background and ordering source) on the other.\(^4\)

Given the importance of tense and aspect in modal auxiliary meaning, a vital component of providing an account of modal auxiliaries in a grammar involves providing an account of temporal and aspectual meaning which not only works with the semantics of the modal auxiliaries, but also stands alone as a well-motivated theory of tense and aspect. When the temporal and aspectual portion of the grammar is independently motivated, then it is clear in the presentation that temporal and aspectual expressions have multiple readings in uses without modal auxiliaries. In the worst case scenario, composing reading-dependent modal auxiliary semantics with reading-dependent temporal and aspectual markings results in an exponential number of readings. But the worst case scenario is not what occurs. Rather, if the temporal and aspectual reading is read in a particular way, the modal auxiliary readings are limited to those which are compatible with it. It is also the case that, if the reading of a modal auxiliary is intuitively clear, then that reading carries with it only certain readings of the temporal and aspectual expressions with which it combines.

\(^3\)From the examples given, it is clear that grammatical context beyond the sentence plays a role as well. The surrounding phrases lean the reading in one way or another.

\(^4\)The modal force is considered to be part of the lexical semantics of modals, as possibility and necessity are uniformly associated with a given modal (regardless of the reading). The conversational background and ordering source, however, depend on the reading.
7.1.2 What is Included in the Formal Treatment of Tense and Aspect

In English, there are several morphological markings with temporal and aspectual meaning. For example, there is a morphological change in past versus non-past forms of finite verbs, as in the contrast between the past form *went* and the non-past form *go*. There is a morphological change in verbs when perfect aspect is present, as in the contrast between the base form *go* and the perfect form *gone*. Auxiliaries, such as *have*, and various temporal adverbs also situate an event to which a sentence is making reference.\(^5\)

These patterns use a small set of verbal endings. The perfect form of verbs, for instance, uses the endings *-ed* and *-en*, plus some irregular forms. The auxiliaries have restricted agreement in person and number, and they are a closed class of expressions.\(^6\)

In addition to the auxiliaries and verbal endings, there is lexical information that is aspectual. Most closely related to the verbal elements discussed so far is *Aktionsart* (Vendler 1957[100]), sometimes called, *lexical aspect*, which depends on the aspectual meaning of a verb in the context of its arguments. Lexical aspect deals with whether a verb is perceived as being about a durative event or a punctual one. For instance, *swim* and *solve* seem like predicates that take a span of time whereas *sneeze* and *notice* seem more instantaneous. Aktionsart is concerned with whether the internal structure of an event is something that culminates into the described event, as in *write a paper* or merely repeats without culmination, such as *ride a bike*.

Lexical aspect is distinguished from the aspectual meaning associated with auxiliaries and functional morphology. The latter can be grouped under the term, *grammatical aspect*. That is not to say, however, that the semantics expressed by lexical and grammatical aspect are separate and distinct things, rather, languages vary in what they express via lexical aspect and what they express via grammatical aspect (see Smith 1991[89], for example). But, for the sake of creating categories for a grammar and their accompanying interpretations, it is important to separate out the task of representing lexical aspect from the task of representing grammatical aspect in English. The reason is because lexical aspect is determined by a particular verbal head and its arguments, but grammatical aspect is determined by the combination of the verbal head with temporal and aspectual morphology.\(^7\)

\(^5\)Klein 1994[43] lists among temporal encodings particles, which are very common cross-linguistically. He also lists discourse organization, which plays a role in English (Klein 1994:14[43])

\(^6\)This subsection briefly reviews Aktionsart, grammatical aspect, and other concepts that were discussed more fully in Chapter 2

\(^7\)Klein 1994[43] treats Aktionsart as mainly qualitative and distinguishes it from grammatical aspect,
7.1.3 Order of Presentation

This chapter focuses on temporal and aspectual morphology as it is manifested on verbal expressions, including auxiliaries such as have and be. Section 7.2 gives an overview of how time is represented in the grammar. Section 7.3 provides the lexical entries for perfect aspect, prospective aspect, and null grammatical aspect. Section 7.4 presents the lexical entries for past and non-past tense. Section 7.5 gives examples of the composition of grammatical aspect and tense in the grammar.

The closure operators are explained in Section 7.6, with examples. The final section, Section 7.7, summaries the materials that were presented.

The account of tense, grammatical aspect, and closure is combined with the modal auxiliary system of the English fragment in Chapters 8 and 9.

7.2 Times in the Grammar

The utterance time is the time at which the utterance occurs. Each use of an expression occurs at a time. Where \( \alpha \) is a linguistic expression, the time of utterance of a use of \( \alpha \) is written \( \text{time}_{u,\alpha} \), meaning the time, usually of utterance, of \( \alpha \) in the world of utterance \( u \). The previous chapter, Chapter 6, gave the definition of the interpretation function \( \text{Lex}(\alpha, u, w, p) \), which provides interpretations to lexical items, repeated below:

**Definition 102.** \( \text{Lex}(\alpha, u, w, p) =_{\text{def}} \) the “customary” definition of \( \alpha \) relative to a world of use \( u \in W, \) a world of context \( w \in W, \) and a perspective \( p \). Where a perspective \( p \) consists of an individual \( x \in D_e \) and time \( t \in T \), construed as the time of the context, which is often the time of use, and a world \( w' \in W, \) which is construed as the world of assessment (Lasersohn 2015:38,65,65 footnote 59[52]).

The time at which an utterance of an expression occurs is customarily introduced into the interpretation of the use of that expression.\(^8\)

Reference time is represented in the present model as either being indexical, which means that it is set by the referential intentions of the speaker, or it is set by a variable

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\(^8\)There are several exceptional cases in which \( \text{time}_{u,\alpha} \) is not the utterance time. One examples is when the author is taking a perspective, as in the historical present. Another example is when a clause is embedded. But, unless it is specified otherwise, \( \text{time}_{u,\alpha} \) is customarily the time at which an utterance of \( \alpha \) occurs.

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however, the author notes that the Aktionsarten from Vendler’s 1957[100] classification do not apply neatly as a cross-linguistic descriptions of qualitative aspect.
assignment $r$ (Lasersohn 2015[52]). The variable assignment $r$ is an assignment that satisfies the pragmatic constraint of assigning values to the set of relevant individuals, groups, and times (Lasersohn 2015[52]), as given in Definition 76 in Chapter 6.

Tense marking on verbs in English is one kind of linguistic expression that determines whether the reference time is past or non-past relative to $time_{u,a}$. If the speaker is referring to a specific, known time, then the interpretation is indexical, that is to say, it is provided with a numeric value that is a mapping from a finite subset of the positive integers to a subset of the relevant and familiar times, $Familiar_{u,a}$. If the time is not a specific time that is set pragmatically by the referential intentions of the speaker, then a linguistic expression is provided with an assignment of values to variables $r$. The assignments are limited to those that extend $Ref_{u,a}$ and are constrained to be limited to times in the set $Relevant_{u,a}$.

Events are mapped to times by the function $\tau$ (Krifka 1989[49]). The events mapped to times are built into a model in which interpretation takes place. Given a world, an event either did or did not occur at a time in that world, given what is modeled in the ordered domains of entities $D_e$ and times $D_t$. The event time is that time that $\tau$ provides to the event. For an event $v$, $\tau(v) = t \in D_t$, that is, the temporal period $t$ is the time at which the event $v$ occurs. The ontology of time and events follows Muskens 1995[68], 2003[67], as presented in Chapter 6. The Aktionsart of an expression and the mereology of events is also a significant factor in meaning, however, it is not represented in detail in this presentation.

### 7.3 Grammatical Aspect

Grammatical aspect is of category $(i/,t/,i)$ and type $< i < i, t >$. It denotes a function from temporal periods to temporal periods to truth values. It can be construed as a relation between two times.

Grammatical aspect is either a default of placing the reference time equal to the event time (as placed through perspectival aspect), being perfect, or being prospective.

The base form of the verb combines with an aspect node, which can have some aspectual morphology associated with it, or can be an aspect associated with the base form. For example, `sleep`, by itself, if only marked for tense, has simple present reading, which is generally interpreted as having imperfective aspect, as shown in Figure 7.1.
The treatment of grammatical aspect follows the observations of Reed 2012[81], who claims that perfect aspects, which include perfects and prospectives, signify a precedence relation between the event and reference time.

With perfect aspect, the event time strictly precedes the reference time.

**Definition 103.** *PERF* is construed as perfect aspect and translates to $\lambda t_i \lambda t_j . (t_i < t_j)$, where $t_i$ and $t_j$ are variables of type $i$.

With prospective aspect, the reference time strictly precedes the event time.

**Definition 104.** *PROSP* is construed as prospective aspect and translates to $\lambda t_i \lambda t_j . (t_j < t_i)$, where $t_i$ and $t_j$ are variables of type $i$.

Due to the antisymmetry of the temporal model, the definitions in 103 and 104 above allow times to overlap, with co-extensive times being, formally (and intuitively), the same time.

When there is no overt marking of grammatical aspect as prospective or perfect, the aspectuality is taken to be neutral and the reference and event time claimed to be overlapping:

**Definition 105.** *NULL* is construed as the default when no grammatical aspectual marking is present. It translates to $\lambda t_i \lambda t_j . (t_i \cap t_j)$, where $t_i$ and $t_j$ are variables of type $i$.

The *NULL* grammatical aspect has a relation between the reference time and the event time that is not one of precedence, but of containment, indicating that the lack of perfect or prospective marking results in a perfective or imperfective reading, depending on other factors, such as the Aktionsart of the expressions and temporal adverbs.

Due to the interaction of translations of aspect-taking predicates with the lambda calculus, the event time assigned by $\tau$ is the time that goes in for $t_i$ and the reference time that goes in for $t_j$. Therefore, the perfect aspect describes a relation such that the
event time precedes the reference time and the prospective aspect describes a relation such that the reference time precedes the event time.\(^9\)

The lexical entries for perfect and prospective aspect are given in Figure 7.2 and Figure 7.3, respectively.

![Figure 7.2: Attribute value matrix for perfect aspect](image)

The lexical entries are for two orthographic ways in which the perfect and prospective can be expressed in English. The orthographic entry for the perfect aspect is a use of \textit{to have} with perfect morphology on the verb with which it combines. The orthographic entry for the prospective aspect is a use of \textit{to be going} with an infinitival form of the verb with which it combines.

The perfect has numerous readings that are encompassed by the lexical entry, all of which involve the precedence relation. In order to discuss how the account of the perfect aspect accounts for the readings, the implications of claiming strict precedence of the event time before the reference time in an antisymmetric temporal model will be discussed.

Figure 7.4 give the lexical entry for the default when no grammatical aspect marking is present.

Reed 2012\cite{ref81} describes an interval called the Event Reference Interval (ERI). The ERI is a corollary of the precedence relation, < in her temporal model. It is the interval

\(^9\)There are many other issues with the prospective aspect, including whether or not it introduces intensionality, that are not undertaken here.
that represents the time between the end point of the event time and the beginning point of the reference time, in the case of perfects. In the case of prospectives, the ERI is between the end point of the reference time and the beginning point of the event time.

Given the temporal and event ontologies used in this monograph, the ERI can is represented a temporal period between the end time of the event and the start time of reference.

In order to give a definition of the ERI in this model, the notion of initial points and final points needs to be defined.

**Definition 106.** The initial point of an event $v$ is that $i \in \tau(v)$ such that there does not exist an $i' \in \tau(v)$, distinct from $i$, such that $t' \prec t$.  

**Definition 107.** The final point of an event $v$ is that $i \in \tau(v)$ such that there does not exist an $i' \in \tau(v)$, distinct from $i$, such that $t < t'$

**Definition 108.** The initial point of a reference time $t$ is that point $i \in t$ such that there does not exist an $i' \in t$, distinct from $i$, such that $t' \prec t$.  

**Definition 109.** The final point of a reference time $t$ is that $i \in t$ such that there does not exist an $i' \in t$, distinct from $i$, such that $t < t'$

With the definitions for the initial and final points in place, it is possible to give a formal definition of the ERI in the current model:
$t_i <_t ERI <_t t_j$

Where $t_i$ is the final point of the event and $t_j$ is the initial point of the reference time, for the perfect.

For the prospective, $t_i$ is read as the final point of the reference time and $t_j$ is read as the initial point of the event time.

There is a special case that Reed 2012[81] discusses in which the final point of the event is co-extensive with the initial point of the reference time. The special case is evidenced in universal perfect readings. An example of a sentence favorable to a universal perfect reading is given in sentence (128) below, on the reading in which Syd still lives in Greece at the time of speech:

128. *Syd has lived in Greece (since 1980).*

In the current model, the co-extensive nature of $t_i$ and $t_j$ is captured through the asymmetry of the model. Axiom 4 of the model, repeated below, states that if two point precede each other, they are the same point:

Axiom 4: antisymmetric: $\forall t_k \forall t_j ((t_k \subseteq_t t_j) \land (t_j \subseteq_t t_k)) \rightarrow (t_k = t_j)$, where temporal inclusion of $t_j$ in $t_k$, written $t_j \subseteq_t t_k$, abbreviates:

$\forall t_i ((t_k <_t t_i) \rightarrow (t_j <_t t_i)) \land \forall t_i ((t_i <_t t_k) \rightarrow (t_i <_t t_j))$
Axiom 4 can be restated as:

$$\forall i, j, k \in T(((t_k < t_i) \rightarrow (t_j < t_i)) \land ((t_i < t_k) \rightarrow (t_i < t_j)))$$

$$\land (((t_j < t_i) \rightarrow (t_k < t_i)) \land ((t_i < t_j) \rightarrow (t_i < t_k))) \rightarrow (t_k = t_j)$$

The universal perfect reading is a special case in which the end point of an event is co-extensive with the initial point of the reference time.

A corollary of this model is that, in universal perfect readings, the ERI is equal to the initial point of the reference time, which is itself equal to the final point of the event time.\(^{10}\)

The universal perfect reading is a special case. In most readings of the perfect, the end point of the event and the initial point of the reference time are not co-extensive, and the ERI occupies a place in between them.

Sentence (129) below illustrates a reading of the *experiential* perfect:

129. *Jess has picked raspberries (before).*

On these readings, the perfect has to do with having had the experience of picking raspberries. The ERI can be understood to strictly precede the initial point of the reference time in these cases.

Perfектs are also used for recently occurring events, as in sentence (130) below and for resultatives as in sentence (131) below:

130. *Pat has arrived!*

131. *Syd has put on her costume for the show.*

Both of these readings have an ERI of some duration that precedes the reference time.

As Reed 2012[81] notes, the ERI does not need to be quantified over or explicitly specified as it falls out of the immediate precedence relations. Reed 2012[81] demonstrates the importance of this interval cross-linguistically in the description of perfects in Scottish Gaelic, which has morphological distinctions among readings involving the length of the ERI.

\(^{10}\)It might be the case that the event time extends further into the reference time than the initial point (p.c. Sylvia Shreiner). It is not clear, if this is the intuition, how to strictly maintain the theoretical claim that precedence defines the perfect and prospective as something distinct from (im)perfectives. It seems that overlap would be necessary in the temporal definition of the universal perfect. It might be the case that a pragmatic account of the universal perfect is the preferred account (see Reed 2012[81] for a discussion). These questions are left for future work.
The lexical aspect of a verb provides additional constraints that are especially relevant for non-perfect forms, but this information is provided by the lexical entry and not in the combinatorial node.

7.4 Tense

The past and non-past tense morphemes set the relation between the speech time and the reference time.

The type of tense is \(<i, <i, t>>\) and translates as follows:

**Definition 110.** *PAST* tense translates to \(\lambda t_j \lambda t_k. (t_j < t_k)\), where \(t_j\) and \(t_k\) are variables of type \(i\).

**Definition 111.** *NON-PAST* tense translates to \(\lambda t_j \lambda t_k. (t_j \subseteq t_k)\), where \(t_j\) and \(t_k\) are variables of type \(i\).

In the combinatorics of the translations, the reference time variable goes in for \(t_j\) and \(time_{\alpha,\alpha}\), where \(\alpha\) is the containing linguistic expressions goes in for \(t_k\).\(^{11}\) The lexical entries for tense are given in Figure 7.5 and Figure 7.6 below.

```
[PAST
 PHON: ORTHO: V-en
 SYN: HEAD:
   past
   CAT : (i/\(\alpha(t_i, t_j)\))
 MODE: temporal
 TYPE: \(<i < i, t>>\)
 INDEX: none
 SEM: TRANSL: \(\lambda t_i \lambda t_j. (t_i < t_j)\)
       RELN: Precede
       PRECEDER: \(n\)
       PRECEDED: \(m\)
 ARG-ST: \(<[t_i]_n, [t_j]_m>\)
```

Figure 7.5: Attribute value matrix for past tense

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\(^{11}\)The verb *will* is treated among the modality readings.
Figure 7.6: Attribute value matrix for non-past

7.5 Summary and Examples

There are several points regarding this analysis of tense and aspect that are important to note.

7.5.1 The Perfective and Imperfective Distinction

Perfective and imperfective aspect are characterized in terms of the mapping of $\tau$ (Krifka [49]) of events to times and in the lexical Aktionsart features of a given use of a verbal element. The event being referred to, given the use of a verbal element with progressive marking, is an event that is associated with a typical type of that event. The typical event may have a distinguished endpoint, as in the case of verb phrases that are often understood to be telic (e.g., a build-a-house event), but the use of the progressive in referring to such a typical does not necessarily include the endpoint. The event being referred to by means of the progressive may have different features in regard to Aktionsart than that of the typical event with which it is associated, in particular, the progressive use may have the feature of being atelic, even though a use of a non-progressive verb form to refer to the same kind of event would be telic. This approach most closely follows Parsons 1990[74]. Distinctions among perfective and imperfective verbal elements are
much more complicated than this brief discussion suggest, however, they are not a major foci of the present work. The treatment sketched above is one that would benefit from development and additional literature review in future work.

7.5.2 Perfect and Non-Perfect Aspect

Grammatical aspect is translated as a relation among two temporal points. These temporal points are the time to which $\tau$ ([49]) maps an event and the time that is the reference time. The reference time can be set to a particular time per the referential intensions of the speaker or it can quantify over a (pragmatically restricted) set of times (following Lasersohn 2015[52]). There are three types of grammatical aspect. The types of grammatical aspect are perfect aspect, prospective aspect, and neutral aspect. Perfect and prospective aspect are treated as precedence relations on the event and reference times. The interval induced by strict precedence, the ERI, is associated with the state of having done the event described by the main verb (following Reed 2012[81]). The particular setting of the reference time is what distinguishes the difference between universal readings and resultative readings, in which the state continues into the reference time, on one hand and experiential and recent past readings on the other hand, in which the ERI strictly precedes the reference time. Prospective aspect refers to cases in which the reference time strictly precedes the event time.

Neutral grammatical aspect refers to cases in which there is no morphological marking indicating a precedence relation between the event and reference times. In such cases, the event and reference time are considered to overlap with stricter relations, such as equivalence, being dictated by a combination of factors including the Aktionssart of the verb, the mapping $\tau$ from events to times, and the referential intensions of the speaker regarding the reference time. In comparison to Reed 2012[81], the aspectual class of perfects and the aspectual class of perfectives are still distinct classes with variations of readings within them. However, in this monograph, the perfect is treated as a grammatical aspectual element and perfectives are determined between the lexicon and the event mapping. The neutral aspect can still encode information such as the event time being included in the reference time (similar to grammatical treatments of perfective) and the reference time being included in the event time (similar to grammatical treatments of imperfective). Perfects, as treated in this grammar, situate event times relative to reference times with precedence orderings and can combine with verbal
expressions that have progressive marking, for example.

There is a prevalent ambiguity regarding how mappings of \( \tau \) are construed with respect to iterative or semelfactive (one-time) readings. The difference is encoded in lexical items in the \( QUANT - ASP \) features. Klein 1994[43] discusses the complications determining iterative versus one-time readings in detail and concludes that, while there are some influencing grammatical factors, the difference comes down to how events are typically construed in the linguistic context of the use. In the examples given in section 7.1.1, it was apparent that both iterative and one-time readings were available in sentences with modal auxiliary verbs. When discussing the semantics of modal auxiliaries, it is important to keep in mind that some such differences in event construal are not particular to sentences with modal auxiliary verbs, but rather are a pervasive source of multiple readings of temporal and aspectual properties more generally.

An example of the base form of a verb combining with aspectual and temporal information is illustrated in the Figure 7.7 below showing a non-perfect and non-past use of the sentence \textit{Jess swims}.$^{12}$

\textsuperscript{12}The details of closure have not been presented yet, so the tree goes up to the first closure operator, the semantics of which are presented in the next section.
\begin{align*}
\eta : & \text{Jess}_1 \text{swims}_2, \text{VB} \\
& \lambda r \lambda f \exists v \exists z [\text{swim}(v) \land f(v) \land (\tau(v) = z) \land (z \circ r(t_i)) \land (\neg (r(t_i) \subseteq \text{time}_{u,v}))]
\end{align*}

\begin{align*}
\delta : & \text{NON - PAST, Tns} \\
& \lambda x \lambda y (\neg (x \subseteq_t y))
\end{align*}

\begin{align*}
\gamma : & \text{Jess}_1 \text{swim}_2, \text{Bse} \\
& \lambda Q \lambda r \lambda f \exists v \exists z [\text{swim}(v) \land f(v) \land (\tau(v) = z) \land (z \circ r(t_i)) \land Q(t_i, \text{time}_{u,v})]
\end{align*}

\begin{align*}
\beta : & \text{NEUTRAL, Aspect} \\
& \lambda x \lambda y (x \circ_t y)
\end{align*}

\begin{align*}
\alpha : & \text{Jess}_1 \text{swim}_2, \text{Bse} \\
& \lambda P \lambda Q \lambda t_i \lambda f \exists v \exists z [\text{swim}(v) \land f(v) \land (\tau(v) = z) \land P(t_i, z) \land Q(t_i, \text{time}_{u,v}) \land A0(v) = \text{Jess}_1]
\end{align*}

Figure 7.7: Tree diagram for Jess swims
In the tree above for *Jess swims*, the event and reference time are in an inclusion relation due to the neutral grammatical aspect. The speech time is included in the reference time because the event is durative and the speech time comparatively punctual. The simple present is more easily read as iterative than as a one-time event, but either reading is possible.

Sentence (132) below is an example in which the context gives the simple present a one-time swim-event reading:

**132. Jess swims the lap!**

The style is that of a sports announcer or the kind of artificial use of the simple present found in children’s books. Sentence (133) below is an example in which the context leads the progressive to have an iterative reading:

**133. Jess is swimming more these days.**

The different readings available with respect to the QUANT—ASP feature appear to be available even when the main arguments of a verb are present and the tense and aspect are determined.

For the present progressive sentence, *Jess is swimming*, the translation would not differ as the aspect would remain neutral and the tense remain non-past, however, the Aktionsart features would differ in that the qualitative aspect would be *stative* (*QUAL—ASP : /stative*) and the telicity *atelic* (*TELIC : atelic*). The most likely reading of the present progressive is a one-time swimming event that includes the speech time.

The fact that the translations are identical, outside of Aktionsart features, captures the similarities between the present tense and the present progressive.

The translation of the perfect aspect in the sentence, *Jess has eaten the chinchilla* appears below:

\[
A_0(v) = \text{Jess} \land A_1(v) = \text{the chinchilla}
\]

The perfect aspect leads to a translation in which the event time strictly precedes the reference time. The use of the definite article leads to a resultative reading of the perfect in which the state of Jess eating the chinchilla holds into the speech time.\(^{13}\) The speech time overlaps the with reference time.

\(^{13}\)In contrast, the use of *a chinchilla* in the sentence *Jess has eaten a chinchilla* causes the reading to be more likely an experiential reading.
The past perfect sentence, *Jess had eaten the chinchilla*, differs in that the reference time strictly precedes the speech time:

\[ \lambda r. \lambda f. \exists v. \exists z [eat(v) \land f(v) \land (\tau(v) = z) \land (z <_{r} r(t)) \land (r(t) <_{r} time_{u,\eta})] \land A0(v) = Jess \land A1(v) = the\text{chinchilla} ] \]

The meaning is such that the event time of the Jess-eat-the-chinchilla event precedes the reference time and the reference time precedes the speech time.

It is interesting to note that the arguments that Reed 2012[81] makes about the ERI being a result of strict precedence are predicted to hold for past tense in this model as well. There is time between the reference time and the speech time, which can overlap with the speech time. Unlike past tense, perfect aspect is recognized to be sensitive to a *state of having done* the event described by the lexical verb and its arguments. The use of the recent past as opposed to the distant past is one way in which the possible interval between reference time and speech time could be considered to be linguistically relevant.

### 7.6 Translation and Interpretation of Closure Operators

In this grammar, the closure of the set of sets of events to a satisfying event and the closure of variables by an assignment given by *Ref* interact.

In order to find a satisfying event, it must be considered how the reference time relates to the event time, given the linguistic expressions used. In sentence (134), there is an event described that precedes the reference time by the definition of the perfect aspect and the default lexical telicity of a purchasing situation:

134. *Jess had purchased the party supplies.*

The temporal marking on *had* as past places the reference time before the speech time.

In order to consider the relation between the types of closure, let us assume that the event time is not set solely by the referential intentions of the speaker. That is to say, the speaker is asserting that the purchasing event happened and there is some reference time that the speaker is aware of that follows the purchasing event.
In order to find a satisfying event, it is necessary to know to which relevant span of
time the person is referring and to find an event that $\tau$ maps to a time that starts before
the reference time. Since the reference time is already set indexically, it merely needs to
be the case that there is a satisfying event that precedes that time. Once that satisfying
event is found, it has a time that $\tau$ maps it to, that time precedes the indexical reference
time, and there is no need for additional temporal closure.

That is to say, in the case in which the reference time is indexical and the events are
quantified over, there is no need of a temporal closure operation because the satisfying
event has an event time.

Let us now consider the case in which the event and its time are both known to the
speaker but the reference time is unknown. In order for this scenario to make sense, it
must be the case that the event time does not equal the reference time. This scenario
seems to be implausible, since the reference time is determined by linguistic expressions
and since it stands in a relation to the event time.

Let us finally consider the case in which both the reference time and the event time
are not known exactly to the speaker. It is possible to imagine a case in which the event
time is contained in a pragmatically loose reference span in the past, but the speaker
does not know at what time that was.

135. **Syd got a kitten from the shelter.**

Let us assume that, in the present, the speaker knows that Syd has a cat that was
obtained from the shelter, but she does not know the age of the cat nor the date at which
Syd obtained it. Given that the sentence is not one with an intensional context, the set of
sets of events under consideration are those in the actual world (or world of evaluation)
in which Syd got a cat from the shelter at a time before the speech time. The truth
conditions are satisfied if Syd did this once, and they are satisfied, intuitively, even if
there were more than one time and the one satisfying the event closure were a different
one than that about which the speaker is talking.

The example motivates having the event closure satisfied by any event prior to the
speech time and setting the value of the reference time to any that contains that event
time (per the definition of tense constraining the reference time to precede the speech
time).

These data motivate having the event closure occur in the derivation before the gen-
eral closure operation and containing a variable $i$ for the event time that $\tau$ assigns, in
cases where there is a potential for multiple events (therefore multiple values for $\tau(v)$) to satisfy the truth conditions of the sentence.

In Champollion’s analysis, there is a closure operation that states that there is a satisfying event, changing the type from sets of sets of event to a truth value. A simplified version, ignoring tense and aspect, appears in Figure 7.8.

Ignoring tense, aspect, and variable assignments and looking just at the way in which events are characterized, it can be seen that the closure operator reduces the set of sets of events to a particular event satisfying the description.

In the present account, event closure it ordered before the general operation of existential closure, so event closure can be of type $<< v, t >$, $t >$ or of type $< r, << v, t >$, $t >>$, depending on whether or not there are variables that have their value set by a variable assignment.

Champollion’s closure is of type $< v, t >>$:

**Definition 112.** $\lambda v. true$

Lasersohn’s general existential closure is of type $<< r, t >$, $t >$ when there is a variable assignment and unnecessary when there is not one.

**Definition 113.** $\lambda P. \exists r [Ref_{u,o} \leq r \wedge P(r) = truth]$

An illustration of the closure of a saturated verb is given in the (partial) tree in Figure 7.9
Figure 7.9: Partial tree diagram of closure of a saturated verb
The interaction of the closure operators will be discussed in more detail in the account of modal auxiliaries.
Table 7.1: Some Expressions with their Syntactic Categories and Logical Types. There is generally a potential for expressions to have a type $r$ in front when they contain an individual, group, or time that requires a variable assignment. There is also the potential for any expression to have an $s$ in front, if it is in an intensional context. Some of these details have not been pictured in the chart as it would require listing multiple forms.
7.7 Review and Conclusions

This chapter presented a treatment of tense and grammatical aspect in the formal grammar that was given in Chapter 6. The temporal relations introduced were used in the lexical entries for tense and aspect.

The closure operations in Lasersohn 2015 [52] and Champollion 2015[13] were made to serve their respective purposes and will be shown to interact in interesting ways in the account of modality.

The formal treatment of modal auxiliary verbs in the model can now be presented.
CHAPTER 8
EXTENDING THE GRAMMAR TO MODALITY

In this chapter, several hypotheses will be presented, defended, and incorporated into the grammar of the previous chapter. The hypotheses presented in this chapter deal with the grammar and the composition of modal auxiliary meaning with other linguistic expressions.

There are two major paradigms of modal auxiliary verbs: Those that take temporal verbal morphology directly on the modal auxiliary verb and those that have unchanging forms. The two paradigms behave differently in a variety of environments and are given different grammatical characterizations.

It is demonstrated that modal auxiliaries have a distinction between control verbs and raising verbs, similar to the distinction among lexical verbs that take verbal arguments.

Sentences that contain modal auxiliary verbs describe two events. One is the event that the modal auxiliary verb and co-occurring expressions describe and the other is the event that the second verb and co-occurring expressions describe.

Each of the two events referred to in a sentence with a modal auxiliary verb has a verbal head with certain properties. Each of these verbal heads combine with aspectual and temporal meaning. As a result, the time of each event stands in a relation to a reference time. The reference times are placed by tense relative to the same utterance time (since the expressions occur together in the sentence), but the pragmatic time, time_{u_o} is not always the utterance time, so it can differ in the relationship it is in with each reference time via tense.

The event and reference times interact systematically depending on two major factors: (1) the lexical meaning of the verbs and the events they help characterize and (2) the morphological temporal and aspectual marking on the verbal elements. There are other features that contribute to the location of the times including the Aktionsart and temporal adverbs, but these are not discussed in as much detail.

Readings that demonstrate assessment sensitivity are only compatible with modal
auxiliaries that do not display tense marking (i.e., Paradigm B modal auxiliaries). Assessment sensitivity is also only compatible with modal auxiliaries that are semantically raising verbs. The difference between modal auxiliary readings that are assessment sensitive and those that are not is handled in terms of temporal and event closure of the event associated with the modal auxiliary verb.

8.1 Motivating Two Grammatical Paradigms

This section presents the case that some modal auxiliary verbs encode tense marking on the modal morphologically and others do not have tense marking on the modal auxiliary verb.

The general research question under consideration is why some modal auxiliary verbs appear to express a past and non-past tense in the morphology of the modal auxiliary verb and other ones do not express temporal morphology on the modal auxiliary, but appear to form the past tense by means of combing with the perfect form of the main verb.

Section 8.1.1 discusses the ways in which modal auxiliary verbs, in general, appear to differ from other verbs, and, more specifically, to differ from other auxiliaries with respect to tense marking. Section 8.1.2 provides evidence that some modal auxiliary verbs have tense marking and some do not and motivates two paradigms of modal auxiliary verbs called Paradigm A and Paradigm B. Section 8.1.5 discusses the paradigmatic distinctions in terms of their behaviour in Sequence of Tense (SoT) contexts. Section 8.1.8 discusses the presence of perfect verb forms with non-past uses of Paradigm A modal auxiliaries.

8.1.1 Modal Auxiliary Verbs in Contrast to Other Auxiliary Verbs

Modal auxiliary verbs, compared to other auxiliaries, display unique patterns with respect to tense marking. Generally, a declarative sentence in English has agreement marking with the person and number of the subject noun phrase on the first verb, as shown in sentences (136-138) below:

136. Jess goes to school.
137. Jess is going to school.
138. Jess has gone to school.

In contrast, with modal auxiliary verbs, there is no person and number agreement with the subject noun phrase, as indicated by the grammatical sentence in (139) below, in contrast to the ungrammatical example in sentence (140) below:

139. Jess can go to school.

140. *Jess cans go to school.

The modal auxiliary cannot appear with person and number agreement with the subject noun phrase.

Modal auxiliary verbs also differ from other auxiliary verbs in that they do not have in infinitival form with to. In general, auxiliaries appear in the typical infinitive form if they are selected by an embedding verb that takes an infinitival complement. They do not differ from lexical verbs in this sense, as shown in sentences (141)-(143).

141. Jess likes to go to school.

142. Jess likes to be going to school.

143. Jess likes to have been chosen for the part.

Modal auxiliary verbs do not follow the same pattern. They cannot occur as complements and they do not take the infinitive, as shown in sentences (144)-(145) below:

144. *Jess likes to can go to school.

145. *Jess likes can go to school.

Compared to other auxiliaries, modal auxiliary verbs, in English, display less morphology with regard to person and number agreement with the subject noun phrase. They do not have infinitive forms nor progressive forms.

Tense and aspectual patterns with modal auxiliary verbs are also different. The first verb in a sequence usually has tense and agreement marking, with the following verbs displaying some morphological agreement variation.

The difference between go and progressive, be going involves the auxiliary use of be, which can be marked for person and number agreement with the subject noun phrase and marked for tense with the non-auxiliary verb taking a standard -ing ending (e.g., is going to).
The difference between *go* and the perfect form *to have gone* similarly can involves agreement with the subject and standard morphology on the next verb in the sequence (e.g., *has gone*). Usually, it takes a form ending in -*ed*, -*en*, or -*t* (e.g., *billed*, *eaten*, or *slept*).

With a modal auxiliary verb, the modal does not show a change. The progressive form of *go* in *might go* is *be going* and the perfect form of *go* in *might have gone* is *have gone*.

These data that have been presented so far indicate that modal auxiliaries have immutable forms with respect to tense, aspect, and agreement marking. There are, however, other data that challenge this assumption, as, in these data, modal auxiliaries appear to have tense marking, but only certain modal auxiliary verbs and somewhat irregularly, or so it appears at first.

8.1.2 Tense Marking and Modal Auxiliary Verbs

One type of modal reading that has a past and non-past form are examples of ability uses of *can* and *could*:

146. *Jess can swim a mile.*
147. *Jess could swim a mile when she was 45.*

One type of modal reading that does not have a past and non-past form on the modal auxiliary verb are examples of epistemic uses of *might*:

148. *Jess might come to the party tomorrow.*
149. *Jess might have come to the party last night.*

The modal auxiliary *might* does not change to reflect whether the tense is past or non-past. It appears that the past is represented by the use of the perfect form *have come* in contrast to *come*.

The hypotheses that are presented in response to these data begin with the proposal of two paradigms of modal auxiliary verbs. Those modal auxiliary verbs that have morphological change directly on the modal auxiliary indicating a difference in temporal meaning will be called Paradigm A modal auxiliaries. Those modal auxiliaries that do not exhibit a morphological change directly on the modal auxiliary verb and describe past scenarios via the use of perfect on the main verb will be called Paradigm B modal auxiliaries.
8.1.3 Paradigm A Past and Non-Past

The strongest evidence for Paradigm A modal auxiliary verbs having past and non-past tense morphology comes from the fact that, intuitively, if the speech time is moved from concurrent with a state of affairs being described to following the state of affairs being described, the only linguistic change is the form of the modal auxiliary and the only semantic change is that of the location of the reference time to a location that strictly precedes the speech time.

Let us assume a context such that in sentence (150) below the reading of the modal auxiliary can is an ability reading:

150. Syd can run a mile.

It is possible to say sentence (150) at a time when Syd’s capacity to run a mile holds. Sentence (151) describes the same scenario, but when Syd’s capacity to run a mile held in the past of the speech time:

151. Syd could run a mile.

Paradigm A uses of will as a volitional marker behave similarly across temporal instantiations. The use of volitional will describes an inclination of the first participant in a span including the speech time. When used in the past, it describes a habit or tendency across a past span.¹

152. Jess will run all day without getting tired, that is how she is.

153. Jess would run all day without getting tired, that is how she was.

A chart of non-past and past Paradigm A modal auxiliaries appears in table 8.1 below.

<table>
<thead>
<tr>
<th>Non-Past</th>
<th>Past</th>
</tr>
</thead>
<tbody>
<tr>
<td>will</td>
<td>would</td>
</tr>
<tr>
<td>can</td>
<td>could</td>
</tr>
<tr>
<td>may</td>
<td>–</td>
</tr>
<tr>
<td>(shall)</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 8.1: Table of Paradigm A Non-Past and Past Forms

¹The use of will that is often considered to be future is discussed later, after the formal theories of tense and aspect are presented.
8.1.4 Paradigm B Modals in Past and Non-Past Uses

Paradigm B modal auxiliaries, when used with the base form of the main verb (or auxiliary in the case of progressive), are used for describing a relation to an event that is non-past relative to the speech time. It can include the speech time or be in the future relative to the speech time, largely depending on the Aktionsart and grammatical aspect of the main verb.

For example, the epistemic use of the modal *might* is about the possibility of something in the present or future of the speech time, depending on the Aktionsart of the main predication:

154. *Jess might come to the party.*

155. *Jess might be sick.*

In sentence (154), the event of Jess coming to the party is in the future of the speech time. In sentence (155), the state of being sick is most felicitously read as including the speech time. There is a less-prominent reading of sentence (155) in which it means something like *Jess might become sick.* Crucially, neither sentence licenses a reading in which the main event strictly precedes the speech time.

If the epistemic reading is stipulated to be constant, but the perfect is used rather than the base form of the verb, and it appears that the only matter that changes is the time of the event about which an epistemic claim is made. The epistemic knowledge state is that state that holds at the speech time (when the speaker is the assessor), just as in non-past uses:

156. *Jess might have come to the party.*

157. *Jess might have been sick.*

A contrast with Paradigm A forms is noticeable. In Paradigm A, the tense of the modal auxiliary determines, in the case of ability uses, whether the capacity of the first participant to participate in the event described by the main verb is a capacity that includes the speech time or strictly precedes it. In Paradigm B forms, the form of the main verb, in the cases of epistemic uses, determines whether the event about which the author is speculating is situated strictly before the speech time or not.
8.1.5 Sequence of Tense Contexts

The difference between the Paradigm A and Paradigm B forms will be demonstrated with reference to speaker intuitions in SoT contexts.

SoT contexts are known to result in changes of non-past tense main verbs to a grammatical past tense (Abusch 1997[1]):

158. Jess: Syd is sick.

159. Pat: Jess said that Syd was sick.

At the time when Jess said sentence (158), assuming she spoke truthfully, the state of Syd being sick held. At the time when Pat reports what she said in (159), it is possible that Syd is still sick, but it is not necessary that she be. Sentence (159) can be used perfectly felicitously even if Syd is sick at the time when Pat says the sentence. In such cases, the past tense form of the main verb, *was*, shows grammatical tense agreement with the past tense embedding verb, *said*.

8.1.6 Paradigm A Modal Auxiliaries in Sequence of Tense Contexts

A similar scenario occurs when *can* or *will* appears in such a context:

160. Jess: Syd can come to the party.

161. Pat: Jess said that Syd could come to the party.

Let us assume an ability reading of *can* in sentence (160) and let us assume that the reading does not change in the report of the sentence in sentence (161).

The party could be an event that is in the future of both sentences and the use of the past form *could* is entirely felicitous and expected. The grammatical past tense is present for agreement with the past tense embedding verb *said*.

The same pattern takes place with volitional *will*:

162. Jess: Syd will wear heels to walk his dog, he always does.

163. Pat: Jess said that Syd would wear heels to walk his dog.

A base form of *WOLL* is standard in treatments of *would* as the past tense form of *will* (e.g., Abusch 1997[1], Iatridou 2000[38]). That convention will be followed and
an analogous form COLL is used for a base form with which non-past can and past could are associated. The details of the morphological process are abbreviated and the features [PAST] and [NON-PAST] are features of the respective forms. The SoT effects on Paradigm A modal auxiliaries have been shown to be the same as those that occur with non-modal tensed verbs. In Abusch 1997[1], uses of would as the past tense of WOLL are described in terms of grammatical tense change, but there is not a similar discussion of can and could, though they work in parallel ways with respect to tense marking and SoT contexts.

Both Paradigm A modals, COLL and WOLL display SoT effect analogous to those displayed by main verbs. The reading does not change with the tense change.

Paradigm A Modals That Do Not Participate in Sequence of Tense Alternations

The modal auxiliary shall, however, does not participate in the SoT pattern, since the use of should is not productive as a past tense form of shall in dominant dialects of modern dominant varieties of English in the Midwestern United States. In fact, uses of shall are so rare, that it takes an amusingly archaic-sounding context for me to even come up with an example I might say:

164. Jess: Syd shall meet her doom.

In reported speech, it is only possible, according to my intuitions, to report sentence (164) by using shall in the SoT context:

165. Pat: Jess said that Syd shall meet her doom.

If the form is changed to should, the only possible reading for me changes to one of a priority reading:

166. Pat: Jess said that Syd should meet her doom.

These intuitions are common among native speakers of dominant varieties of English in the Midwestern United States, but probably less common among other varieties of English. The theory presented here is for dominant varieties of English in the Midwestern United States, the idealized and generalized variety that overlaps with my idiolect.

According to the same intuitions, there is a Paradigm A use of may, but it does not take a past form might. In my dialect, the two modals are nearly indistinguishable
semantically, with a few exceptional contexts. One of those exceptional contexts is the use of *may* in a request for permission to do something or as a formal means of granting permission. A similar use of *might* would not occur in dominant varieties of English in the Midwestern United States, to my knowledge:

167. *Jess*: Syd may have another cookie.

168. *Pat*: Jess said that Syd might have another cookie.

While sentence (168) is an acceptable sentence, it is not an SoT form of (167), and it does not have a permission reading, but one of the possibility that Syd will have another cookie.

8.1.7 Sequence of Tense and Paradigm B Modal Auxiliaries

It was shown in the section above that SoT effects occur on Paradigm A modal auxiliaries, changing the present form to the morphological past form for grammatical tense agreement with the main verb. If the perfect in Paradigm B modal auxiliary uses is the same as other past tense morphemes, than it is predicted to show similar patterns, but it does not.

169. *Pat*: Syd might be coming to the party.

170. *Jess*: Pat said Syd might be coming to the party.

It is not possible to report Pat’s statement as in sentence (171):

171. *Jess*: Pat said Syd might have been coming to the party.

Sentences (170) and (171) have different meanings, and only the former can be used to report what was said in sentence (169).

In sentence (171), the reading is one in which, if it is kept epistemic, the actual utterance that Pat made was *Syd might have been coming to the party*.

There is a meaning change when the perfect is used with Paradigm B modal auxiliaries, but the meaning change is primarily temporal in nature.

For the present, the following assumptions are made: (1) Paradigm B modal auxiliaries do not have temporal morphology (2) When they occur with the perfect, the perfect appears to act as a temporal marker of the past tense (3) The way in which past
tense is expressed in Paradigm B modal auxiliary uses (by combining with the perfect) is less productive in the grammar than past tense morphological markers that appear directly on the verb, which is evidenced in their contrasting behaviour in SoT contexts. A chart of Paradigm B modal auxiliaries appears in Table 8.2.

<table>
<thead>
<tr>
<th>Non-Past</th>
<th>Past</th>
<th>SoT</th>
<th>Non-Past Modal Perf</th>
<th>Past Modal Perf</th>
</tr>
</thead>
<tbody>
<tr>
<td>would</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>would have</td>
</tr>
<tr>
<td>could</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>could have</td>
</tr>
<tr>
<td>may</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>may have</td>
</tr>
<tr>
<td>might</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>might have</td>
</tr>
<tr>
<td>should</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>should have</td>
</tr>
<tr>
<td>must</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>must have</td>
</tr>
</tbody>
</table>

Table 8.2: Table of Paradigm B Forms

The perfect uses are recorded in the table as past modal perfect forms. The past tense is left empty in the table because the column is reserved for past tense marking on the modal auxiliary verb, which does not occur in Paradigm B forms.

8.1.8 Paradigm A Modal Auxiliaries and Non-Past perfect

Additional evidence for two paradigms comes the distribution of modal auxiliaries with respect to the perfect. It is often only possible to use the perfect with the non-past tense form. When it is used with the past tense form, the reading changes to a counterfactual or epistemic reading.

The use of the perfect is most productive with will, and it results in past in the future readings:

172. (By that time,) Jess will have turned sixty.

The presence of the perfect does not change the meaning of future uses of will from being a future tense marker in that it still places the event in the future of the speech time. The use of the perfect shifts the event to being past of reference time.

Uses of can are only possible with the perfect, in my dialect, in very restricted environments when they occur with negation:

173. Jess can’t have turned sixty!
This is a means of expressing surprise when the event is taken to be new information. It can be paraphrased as:

**174. It cannot be the case (now) that you have already turned sixty.**

The uses of *will have* have future in the past readings and the uses of *can’t have* have a use of the perfect that is better characterized as past with relevance to the present (where the present is the speech time in this case).

The use stands in contrast to the Paradigm B epistemic use of *could*:

**175. Jess couldn’t have turned sixty!**

**176. In order to pass the drug test, you can’t have ingested drugs in the last two months.**

This is a priority reading that can be paraphrased as:

**177. At the time of the drug test, it cannot be the case that you had ingested drugs in the two months prior to that time.**

There is no use of *shall have* in my dialect. Uses of *may have* have only Paradigm B epistemic readings, except, perhaps very marginally, as a polite priority paraphrase of sentence (176):

**178. In order to pass the drug test, you may not have ingested drugs in the last two months.**

Uses of the perfect are rare and not fully productive in modern dominant varieties of English in the Midwestern United States, but contrast with Paradigm B modal auxiliaries.

This chapter, so far, has proposed a tense-bearing paradigm of modal auxiliary verbs that is called *Paradigm A*. The paradigm has historical origins and is not fully productive in modern dominant varieties of English in the Midwestern United States. A chart summarizing the uses and distribution of Paradigm A modal auxiliaries appears in Table (8.3).

### 8.1.9 Summary and Conclusions Regarding Paradigm A and Paradigm B Modal Auxiliaries

The modal auxiliaries *could* and *would* occur with the base form of the main verb but can either be Paradigm A or Paradigm B uses. When the use is a Paradigm A use, the
state of affairs being described is before the speech time. When the use is a Paradigm B use, the state of affairs is non-past with respect to the speech time.

179. *Syd could crawl in the cupboards (when he was a child).*

180. *Syd could crawl in the cupboards (in the event of an earthquake).*

181. *Syd would crawl in the cupboards (for fun when he was a child).*

182. *Syd would crawl in the cupboards (if there were an earthquake).*

Surrounding clauses help disambiguate the readings. Even when there is not a clause like the ones in parenthesis above, there are often adjacent sentences that differ depending on the reading of *could* or *would*.

The readings can be entirely obscured in SoT contexts, which leads to even more confusion when considering that Paradigm A non-past tense forms *can* and *will* also occur as *could* and *would*, respectively in SoT contexts.

183. *Pat was surprised that Jess could fry the fish.*

The full set of readings and the forms the exhibit are displayed in Table 8.4.

Table 8.4 puts entries on the same row if the Paradigm A forms and Paradigm B forms have forms that look identical. It does not imply any relations between the forms. Each Paradigm has a separate semantics from the other.

<table>
<thead>
<tr>
<th>Non-Past</th>
<th>Past</th>
<th>SoT</th>
<th>Non-Past Perf</th>
<th>Past Perf</th>
</tr>
</thead>
<tbody>
<tr>
<td>will</td>
<td>would</td>
<td>would</td>
<td>will have</td>
<td>–</td>
</tr>
<tr>
<td>can</td>
<td>could</td>
<td>could</td>
<td>(can’t have)</td>
<td>–</td>
</tr>
<tr>
<td>may</td>
<td>–</td>
<td>–</td>
<td>(may not have)</td>
<td>–</td>
</tr>
<tr>
<td>(shall)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 8.3: Table of Paradigm A Forms

8.1.10 Recap of Points Made Thus Far

There is evidence for two paradigms of modal auxiliaries in the grammar. At first glance, it seems that Paradigm A and Paradigm B merely represent two ways to express the past tense in a two-verb sequence featuring a modal auxiliary as the first verb in the
Table 8.4: Morphological Forms of Paradigm A and Paradigm B Modal Auxiliaries in Various Contexts

sequence. Paradigm A expresses the past tense through morphological marking on the modal auxiliary and Paradigm B by the use of perfect on the second verb.

The idea of two ways of expressing tense runs into problems, however, when looking further into the distribution of these verbs. In sequence of tense contexts, where grammatical tense agreement is attested to occur with non-modal verbs, Paradigm A modal auxiliaries change their tense as other tense-bearing verbs. If the perfect form of the second verb were to be merely another means of expressing the past tense, then we might expect to see non-past uses change grammatically to past uses in SoT contexts. As Abusch 1997[1] noted, and as is obvious to our intuitions, they do not change form in SoT contexts. Changing the form changes the meaning and cannot be merely a form of grammatical agreement.

We see additional distributional curiosities in the ways in which they combine with perfect aspect. While Paradigm B modal auxiliaries occur with perfect and appear to merely change their temporal semantics, Paradigm A modal auxiliaries do not use perfect to express the past tense as it is expressed on the modal auxiliary. Given these patterns, there is no obvious reason why Paradigm A modal auxiliaries should not freely combine with perfect marking, resulting in some change in meaning related to the completed state of something or to ‘past with relevance to the present’. It does not appear to be possible, however, to use a Paradigm A modal auxiliary in the past tense with perfect marking. Rather, it is only non-past Paradigm A modal auxiliary verbs that combine
with the perfect aspect.

These data show patterns that appear irregular but that make sense if they are considered in terms of two paradigms, one that displays temporal morphology and one that does not.

8.2 Control and Raising in Modal Auxiliary Verbs

Section 8.1 proposed two paradigms of modal auxiliary verbs. Paradigm A modal auxiliaries have morphological tense marking on the modal auxiliary verb and Paradigm B modal auxiliaries do not.

This section describes another important contrast among modal auxiliary verbs, which is, the contrast between modal auxiliaries that are control verbs and modal auxiliaries that are raising verbs. Paradigm A modal auxiliaries are more frequently control verbs. Paradigm B epistemic readings are raising verbs, and Paradigm B priority readings are raising verbs.

In order to present the difference between control and raising verbs, an example of a Paradigm A reading is treated along side an example of a Paradigm B reading. The two modal auxiliaries and readings used, that is, ability readings of COLL and epistemic readings of might, show contrasts between control and raising as well as between readings that are not assessment sensitive and readings that are assessment sensitive.

8.2.1 Control and Raising in Non-Modal Verbs

There are many verbs that take verbal or sentential complements. The focus of this chapter is modal auxiliary verbs, but it is helpful to consider the classes of verb-complement taking verbs. One class is that of equi or control verbs and the other is that of raising verbs.

An example of a control verb is tries and an example of a raising verb is continue. The primary notion behind the labeling is that, with control verbs, the subject noun phrase is a semantic argument of both the embedding verb and the complement verb.

For example, in sentence (184) below, Jess is the individual who tries and the individual standing in a relation to the picking event:

184. Jess tries to pick strawberries on Saturdays.
In contrast, with raising verbs, the subject noun phrase is the semantic argument of the inner verb, but not of the outer verb.

In sentence (185) below, Jess stands in a relation to the picking event, but it is the Jess-pick-strawberries event that continues:

**185. Jess continues to pick strawberries on Saturdays.**

The contrast between control and raising verbs is sometimes made more clear if the subject noun phrase is a semantically empty use of *it* or *there*. In sentence (186) below, the semantically empty use of *there* does not work as a subject:

**186. *There try to be problems with the release.***

In contrast, with raising verbs, the use of a semantically empty subject noun phrase sounds natural:

**187. There continue to be problems with the release.**

The differences between control and raising verbs are also apparent when an active sentence is compared with a passive one. Raising verbs do not change the roles of the semantic arguments, but control verbs do.

**188. Jess tried to hire Syd.**

**189. Syd tried to be hired by Jess**

In sentence (188) above, it is Jess who is trying and in sentence (189) above, it is Syd who is trying.

In contrast, with raising verbs, whether passive or active, the same event continues:

**190. Jess continued to eat chinchillas.**

**191. Chinchillas continued to be eaten by Jess.**

The difference between control and raising verbs semantic in that it is about semantic role assignment, but it interacts with the syntax in that the subject noun phrase position is structurally filled.
8.2.2 Modal Auxiliaries

Modal auxiliary verbs have been treated as a type of raising verb in some analyses (e.g., Steedman (2000:68)[92]). Epistemic readings of *might* follow the expected patterns for raising verbs. In sentence (192) below, *Jess* has an agentive role in the *hire-Syd* event, but *Jess* does not have an agentive role in a *might*-event.

192. *Jess* might hire *Syd*.

On epistemic readings of *might*, the modal auxiliary does not express something that *Jess* plays a role in, rather it expresses a lack of certainty regarding whether *Jess* will hire *Syd* or not.

The epistemic reading has similar patterns to raising verbs in terms of the possibility of semantically empty subjects, as in sentences (193 - 194) below:

193. *There* might be a storm coming this afternoon.

194. *It* might be difficult to use the back exist during construction.

There does not seem to be an agentive semantic role assigned to the subject noun phrase by the modal auxiliary *might*.

In the case of passive sentences, the semantic roles do not reverse (in comparison to an active sentence with the same actors), which is what is expected for raising verbs. An active sentence and a passive sentence are given below:

195. *Jess* might hire *Syd*.

196. *Syd* might be hired by *Jess*.

Classifying epistemic readings of *might* as a type of raising verb seems to be well supported by the data, but it is not as easy to extend the raising verb analysis to all readings of modal auxiliary verbs.

The similarity between raising verbs and epistemic modal auxiliary verbs on one hand and between control verbs and dynamic modal auxiliary verbs on the other, has been recognized (Sag et al. 2003[85]). The ability reading of *can*, which is a type of dynamic reading, behaves more like a control verb.

In sentence (197) below, *Jess* is in an agentive semantic role with respect to the *run-event*, and she is in also in an agentive role with respect to the *can-event*.

197. *Jess* can run a five minute mile.
Uses of ability \textit{COLL} are more difficult to construct with a semantically empty subject. Attempts to do so, as in sentence (198) below, appear to shift the reading from an ability reading to a circumstantial reading, if it is considered to be semantically acceptable at all:

\textbf{198.} \textit{?There can be storms this afternoon.}

In passive and active sentences, there is a contrast in the roles, given the position of each argument. Sentences (199-200) provide an example:

\textbf{199.} Jess can beat Syd in the wrestling ring.

\textbf{200.} Syd can be beaten by Jess in the wrestling ring.

Although it is more subtle to discern than with \textit{try}, it is yet apparent that it is Jess’s capacities that are under discussion in sentence (199) above, and Syd’s capacities that are under consideration in sentence (200).

Readings of epistemic \textit{might} and (dynamic) ability \textit{COLL} provide evidence that some modal auxiliaries belong to the class of raising verbs and some belong to the class of control verbs.

\subsection*{8.2.3 Ability Readings of \textit{COLL}}

Raising and control are represented in CCG with a variable in the place where a subject noun phrase would be relative to the main verb (Steedman 2000:67-68[92]):

\textbf{201.} \textit{Jess\textsubscript{i} tries anaphor\textsubscript{i} to leave.} (equi/control verb)
\texttt{try'((leave' ( ana' Jess\textsubscript{i})), Jess\textsubscript{i})} (Steedman 2000:67[92])

\textbf{202.} \textit{Jess\textsubscript{i} continues anaphor\textsubscript{i} to leave.} (raising verb)
\texttt{might'(leave' (Jess\textsubscript{i}))} (Steedman 2000:68[92])

The difference in the types of sentence are encoded in terms of how the interpretation is assigned to the anaphor with respect to the rest of the grammar. In the control case, the verb \textit{tries} selects the subject noun phrase as does the verb \textit{leave}. In the raising case, only the verb \textit{leave} selects the subject noun phrase as an argument. The phrase \textit{Jess to leave} is what the verb \textit{continue} selects.
The analysis in HPSG is similar. The argument structure of the embedding verb selects the subject noun phrase and the verb phrase in both cases. However, in the case of control, semantic roles are given to each individually, whereas, with raising, a semantic role is given to a ‘Jess-leave’ situation. The subject noun phrase is only structurally related to the first verb and that verb does not assign it any semantic roles.

Although the difference between control verbs and raising verbs is encoded in the semantics, it has some sympathetic vibrations in the syntax. The typical tests for whether a verb is a control or raising verb highlight the syntactic consequences. The most common consequence is that raising verbs are predicted, at least in most theories, to be able to take semantically null subjects, such as pleonastic there and it, but control verbs are not.

8.2.4 Traditional Semantics of Modal Auxiliaries with Control and Raising

The classification of modal auxiliaries as raising verbs is congruent with the common formal semantic interpretation in which a modal operator takes scope over a proposition.

Let us assume, along with most of the literature, that modal auxiliary verbs are expressions that create intensional environments. The expressions in their scope are interpreted intensionally, but their truth conditions are given according to a semantic translation involving logical necessity or possibility. In traditional accounts, the modal auxiliary itself represents a logical relation, that of logical necessity or possibility (represented by $\Box$ or $\Diamond$, respectively). Readings are determined by constraints on which worlds are accessible, orderings on the worlds (Lewis 1973[54], Kratzer 1981[45]), and the premises from which an agent is reasoning (Kratzer 1981[45]).

Chapter 4 presented Lewis’s 1973[54] analysis of counterfactual conditionals, in which modal auxiliaries were treated as modal logic operators ranging over worlds. In Lewis’s truth conditions, repeated in Definition 114 below, the propositions $\phi$ and $\psi$ are what are under consideration across worlds. The modal operator, $\Box \rightarrow$ is a logical operator.

Definition 114. $\phi \Box \rightarrow \psi$ is true at a world $w$, according to a system of spheres $S_w$, if, and only if, either

(i) no $\phi$-world belongs to any sphere $S$ in $S_w$, or
(ii) some sphere \( S \) in \( S_w \) does contain at least one \( \phi \)-world, and \( \phi \rightarrow \psi \) holds at every world in \( S \).

In the statement, *If kangaroos had no tails, they would topple over*, that Lewis represents with the formula: \( \phi \Box \rightarrow \psi \), \( \phi \) is the proposition that kangaroos have no tails and \( \psi \) is the proposition that kangaroos topple over. The details of the grammatical marking are omitted.

Similarly, in Kratzer’s 1981[46] definitions, it is a proposition \( \phi \) that is situated by the modal logic notions of (graded) possibility and necessity.

The control reading of ability *can*, however, suggests that there is a subject noun phrase that is not in the scope of the modal operator, but, rather, stands in an agentive (first participant) semantic relation to it. The referent of that subject noun phrase is the same as the referent of the agent of the second verb.

If such readings exist, which appears to be the case, then there must be an individual standing in a relation to the modal logic operator, outside of its scope.\(^2\) If the traditional logical form is represented by a modal operator and a proposition, \( \Box \phi \), the control readings argue for a semantics in which there are elements outside of the scope of the modal operator. However, making such a semantic change cannot involve treating the logical symbol characterizing the modal auxiliary as a two place predicate, as \( \Box(\alpha)(\phi) \), where \( \alpha \) is the subject marked noun phrase of category (Bse/Bse) and \( \phi \) is the proposition headed by the verb occurring after the modal auxiliary. The formula \( \Box(\alpha)(\phi) \) places \( \alpha \) in the scope of the modal, in the traditional notion of being in the scope of a modal, but, in terms of the meaning of the modal auxiliary verb, it is not in the scope of the modal. It is extensionally, in the actual world, that the person holds a capacity to do whatever is described by the proposition in the scope of the modal. The desired formula for control modal auxiliaries would place only the event that the second verb is the head of in the scope of the modal operator:

\[
\text{COLL}(Jess_i)(\Box_{\text{ability}}(\text{swim}'(\text{anaphor}_i)))
\]

This sub-section has provided a background on control and raising verbs and demonstrated that the categories apply to readings of modal auxiliaries. The next sub-section will provide the lexical entry and semantic treatment of ability readings of *can*, given

\(^2\)The readings exist in cases other than ability uses of *can*. The agentive nature of the subject noun phrase is even more prominent in volitional uses of *will*, for example, as in the sentence, *I WILL win the race today*, stated by a runner as a statement of personal commitment to winning.
these data and the Paradigmatic distinctions discussed in Section 8.1 above.

The lexical entry for ability COLL appears in Figure 8.1 below.

ability COLL

word

PHON: ORTHO: [COLL]

SYN: HEAD:

\[
\begin{bmatrix}
\text{verb} \\
\text{CAT} : Bse \\
\text{FORM} : bse
\end{bmatrix}
\]

MODE: ref

TYPE: \(< i < i, t >>, < i < i, t >>, < i << v, t >, t >>\)

INDEX: \(v_1\)

SEM:

TRANSL: \(\lambda P \lambda Q \lambda t_1. \lambda f_{<v,t>>}. \exists v \exists t_2. [COLL'(v) \land f(v) \land (\tau(v) = t_2) \land P(t_1, t_2) \land Q(t_1, t_2)]\)

RESTR: \(< i < i, t >>, < i < i, t >>, < i << v, t >, t >>\)

CARG: \(< v, t >\) and \(< v, t >\)

Figure 8.1: Attribute value matrix for ability \textit{COLL}

The category of modal auxiliary verbs is \textit{Bse}, the same as other verbs, and the type is \(< i < i, t >>, < i < i, t >>, < i << v, t >, t >>\), as with other verbs.

The semantic entry notes that the index of the event indicated by an ability use of \textit{COLL} is \(v_1\), and the mode is that of a referential expression. The restriction states that the type of relation it encodes is a capacity-relation and it indicates an event \(v_1\), a first participant who has the indicated capacity, indexed with \(i\), and a event \(v_2\) in the scope of an intensional operator. The intensional event is indexed to the complement verb and its arguments. For example, in the sentence, \textit{Jess can run.}, the event \(v_2\) is the intension
of a Jess-run event.\footnote{For verbs that are not modal auxiliaries, the semantics of the verb are often given in terms of its arguments structure and semantic role assignment, with the translation merely being an instance of the verb with some special marking, such as an apostrophe, distinguishing it from the lexical item:}

Control is handled semantically in that the person of whom the modal predicates a capacity and the event describing the capacity are both semantic arguments in the restriction of the lexical entry.

In the case of ability readings of \textit{COLL}, the description of the accessible worlds are those worlds in which the individual or group that the subject noun phrase denotes has relevant capacities that are as they are in the world of utterance, which can often be assumed to be the actual world.

With ability uses of \textit{COLL}, the information in the scope of the modal is a proposition, as discussed regarding sentence (204), Jess running a mile is interpreted intensionally. The notion of being ‘in the scope of the modal auxiliary’ has been used based on previous literature but has not been defined in this grammar. What is meant is that the event that is interpreted intensionally due to the semantic selection properties of the modal auxiliary verb. The proposition is the \textit{CAPACITY} argument in its closed form. In the compositional semantics used in this grammar, the intensional operator applies to a set of sets of events rather than to a proposition (i.e., an expression of type \( t \)). After the event closure \( ClzV \), however, the set of sets of events has a satisfying event relative to an accessible world. Sometimes it is more clear in the exposition to discuss the proposition rather than the sets of sets of events even though, technically, the intensional operator applies to the latter.

The modal itself also describes an event, specifically an event relating the first participant to the capacity. Its reading as a verb is extensional and does not involve consideration across possible worlds. This idea can be seen as similar to use of the verb \textit{think} that

\[ \lambda y. ea'f(y) \]

It is common to treat the verb as having a denotation that can be written as the set of ‘eaters’ and ‘eaten things’.

In this grammar, the entry (of just a verb that has not combined with any arguments) is:

\[ \lambda P. \lambda Q. \lambda t. \lambda f. \exists x. [ea'(y) \land f(v) \land P(x, z) \land Q(x, y)] \]

Entries for modal auxiliaries do not need to have any more complicated lexical semantics than what is already given in their syntactic and semantic entries. However, it would be an interesting idea to develop the notion of a capacity in such a way that it related to properties that the individual or group denotes by the subject noun phrase had and, thereby, place more explicit conditions on the accessibility relations. These developments are left for future work.
contributes intensionality to the thing thought, but does not get interpreted intensionally itself. That is to say, the thinking event is something occurring in the actual world, but the truth conditions do not depend on the thing being thought being true in the world in which it is thought.

8.2.5 Epistemic Readings of *might*

Epistemic *might* differs from ability uses of *COLL* in several ways.

The subject noun phrase of the sentence below, *the badger*, is the agent in an event regarding eating a chinchilla:

203. *The badger might eat a chinchilla.*

On the epistemic reading, there is no paraphrase of the sentence such that the badger stands in a *might* relation to the badger eating the chinchilla. The *might* relation is that of the knowledge state of the speaker to the event of the badger eating a chinchilla.

The lexical entry appears in Figure 8.2 below.

The verb category and type are the same as that of ability *COLL*, but the subcategorization information differs. The subject noun phrase is labeled to be token-identical to the noun phrase for which the second verb subcategorizes.

The set of set of events that the verb describes, $v_2$ is the argument of the epistemic *might* event. The *might* event itself is the other event, $v_1$.

8.2.6 Conclusions Regarding Ability *COLL* and Epistemic *might* as Control and Raising Verbs

This section has discussed the notions of control verbs and raising verbs and shown that there are modal auxiliary verbs that fall into each class. Ability readings of *COLL* behave as control verbs and epistemic readings of *might* behave as raising verbs. A lexical entry was given for each verb showing how the difference between control and raising are portrayed in the semantics.

---

*There is a reading where the badger stands in such a position. That is, where the badger is thinking to himself, ‘I might like to eat a chinchilla just now.’ and the sentence is repeating that thought. The reading could be paraphrased as ‘The badger is considering eating a chinchilla.’.*
might

word

PHON: ORTHO: might

SYN: HEAD: CAT: Bse

FORM: bse

MODE: ref

TYPE: <i < i, t >>, < i < i, t >, < i << v, t >, t >

INDEX: v₁

TRANSL: λPλQ.λt₁.λf<v,t>s.∃v∃t₂

[might(v) ∧ f(v) ∧ (τ(v) = t₂) ∧ P(t₁, t₂) ∧ Q(t₁, time_{α,α})]

RELN: epistemic

EVENT: v₁

ARG: PLEON ∨ STRUCTᵢ

ARG: ◊v₂m where v₂ is of type << v, t >, t >,

which contains an anaphoric element anaᵢ, and

where the accessibility relation, Rep_{epi}t is such that

Rep_{epi}(w, w') iff w' is a world that

is epistemically accessible to the judge.

Figure 8.2: Attribute value matrix for epistemic might
The next section presents the ways in which each verb composes with its arguments, how the modal auxiliaries compose with tense and aspect, and how event closure and general closure interact with their semantics.

8.3 Tense and Aspect with Ability *can*

8.3.1 Two Events in Control Readings

The definition of a model $M$ in the last chapter included an ontology of events such that events are things that exist in worlds and that map to times via a function $\tau$.

Uses of the Paradigm A modal auxiliary $\textit{COLL}$ express an ability or capacity of the individual or group to which the linguistic expression in the subject noun phrase position refers, (henceforth called the first participant). There are two events: One event is the capacity of the first participant to engage in an event. The event in which the first participant engages is the second event.

Although the two events share the same first participant, they can have different event times and different Aktionsart properties. In sentence (204) below, the capacity that Jess has is a state that holds over a span that includes the speech time.

204. *Jess can run a mile.*

The second event, $v_2$, which is the set of sets of events in the scope of the intensional operator, involves Jess’s running of a mile, occurs within the state of Jess’s capacity. The topic of the sentence is about a particular time span in which she holds the capacity. Sentence (204) is still true if there is a Jess running a mile event in a world besides the actual world, provided that things about Jess in that world are similar enough to the actual world that they support her capacity to run a mile in the world of utterance. That is to say, the capacity does not entail that an instance of the $v_2$ event actually occurred.

8.3.2 Syntactic Representation and Translation of Ability $\textit{COLL}$

The typed syntactic tree for the sentence *Jess can swim* is given in an MM-CCG below:
Figure 8.3: Tree of \textit{Jess can swim}
Figure 8.4: Tree of *Jess can swim*
The interpretation tree for the syntactic tree above is presented in portions, explaining each part in a bottom-up way. The first semantic sub-tree is for the base form of swim and the anaphoric element in the subject position.

Node $\delta\delta$ above introduces the anaphoric expression of type $e$, which is co-indexed with a referential expression of the same type. The use of $ana_0$ follows that of Steedman (2000:67[92]) and it encodes the notion of control in that it is co-indexed with the subject noun phrase of the modal auxiliary and the entity to which it refers a semantic argument of both the verb swim and the modal auxiliary. The sub-tree above shows its role as the element in the subject noun phrase position of the verb swim. Node $\gamma\gamma$ above is the argument role label for expressions that have an agentive flavor, which is called A0 in the translation. Node $\beta\beta$ gives the composition of the anaphor and the argument label according to APPLY.

Node $\epsilon\epsilon$ above shows the translation of the verb swim in its base form without aspect or tense marking present. The base form composes with its semantic role labeled arguments, which are constrained by the semantics of the lexical entry in the RELN feature. The translation present above has replaced $\lambda t_i$, where the bound $t_i$ occurs as an argument of $P$ and $Q$ with a variable assignment $r$ to the temporal index (2) that is on the verb. If the reference time of the expression were set indexically to a specific time, that time would go in for $t_i$, but, in this case, it is assumed that $t_i$ is determined by a variable assignment.

The indexical time, $time_{\epsilon\epsilon}$, is generally the time of utterance, however, there are well
known cases in which the reference time is not placed relative to the time of utterance, but rather to some contextually determined time. Because the verb *swim* does not occur in a tense-taking position, *time* will percolate up the tree until the verb *swim* and its arguments are given the semantic role label of being an argument of another verb, at which point, the time is indexed to that node of the tree and given a variable assignment.

What this treatment means is that, in such non-tense taking positions, the role that an indexical utterance time usually plays is replaced by a time borrowed from the tense-taking verb, which is partly lexically determined and partly determined by the type of embedding.

For example, some sequences of auxiliaries and verbs involve overlap of the events to which they refer and some involve containment. In the sequence *Jess wants to get a car* the desire is a state that extends into the point of the event of getting a car. It could be the case that Jess wants to get a car and then gets a car after the desire is over, but that does not, intuitively, seem to be a satisfying ‘Jess-get-a-car’ event for the utterance *Jess wants to get a car* if it happens after the desire passes. Ability *COLL*, in contrast, involves a more substantial overlap of the capacity and the swimming event. The difference in temporal relations is generally described by the overlap operator *τ*, however, a more...
specific relation can be defined based on the Aktionsart of the two verbs.

In the case of a sentential argument for ability COLL, the time that it borrows is the reference time of the tense-taking verb, COLL that is determined by a variable assignment and the reference time of the ‘swim’-event contains or equals (i.e., \( \subseteq_t \)) the reference time of the ability. The intuition is that, if someone says, Jess can swim, the capacity referenced holds at the same time as the potential ‘Jess-swim’-event, but Jess may have also swum before the reference time of the capacity or after. More will be said about this relation after the semantics of the modal auxiliary are presented.

Node \( a \alpha \) above shows the composition of the base form of the verb with its semantic role labeled argument. The result is the base form of swim with a swimmer, \( (A0) \), who is co-referenced with a referential expression in a higher node. The variable assignment \( r \) handles both the assignment of a referent to the anaphor and an assignment of a reference time.

The next sub-tree, below, continues the tree above, and shows the verb swim, along with its semantic role labeled argument, combining with aspect and tense nodes. Although swim is not in a tense-taking position, it still has a tense node that, rather than relating reference time to utterance time, relates reference time to a time determined by the lexical nature of the embedding verb.

There could, alternatively, be another name for this node such as TENSELESS or EMBEDDED – TENSE, however, because NON – PAST is the unmarked form and because the same temporal relation of overlap is useful, the difference in labels seems merely mnemonic. The present approach also has the advantage of not assuming the absence of tense means grammatically tense-less at a point in the composition in which the embedding element is not yet present.

Node \( \omega \) above provides the translation of non-perfect aspect, which is overlap, in contrast to perfect aspect, which involves precedence (Reed 2012[81]). Node \( \sigma \) states that the tense defaults to non-past, which is an inclusion relation.

Node \( \rho \) shows the composition of swim with its A0 argument and aspect and tense.

The next sub-tree, below, shows the composition of COLL with its sentential complement.

The node \( \pi \) shows an intensional \( S – COMP \), since the lexical entry for the verb COLL with which it combines is stipulated to contain an intensional complement. The restrictions on the accessible worlds are lexically defined by COLL and define the abstraction, \( \lambda < w, p > \) in the intensional \( S – COMP \) to be restricted to those worlds in
Figure 8.7: Tree of Jess can swim

(continued from ρ: ana'₄ swim₂)

| ρ: ana'₄ swim₂
| λr.λf(∃v₂ ∃z₂[swim(v₂) ∧ f(v₂)]
| A0(v₂) = r(4) ∧ (τ(v₂) = z₂) ∧ (z₂ ⊈ r(2)) ∧ (time₉ₓₓ ⊈ r(2)))
Where Ref₉ₓₓₓₓ, and (4) is co-indexed
to a referring expression with the same denotation as ana'₄, and (2) is a time.

σ: NON-PAST
λx.λy(y o₁ x)

υ: ana'₄ swim₂
λx.λQf(∃v₂ ∃z₂[swim(v₂) ∧ f(v₂)]
A0(v₂) = r(4) ∧ (τ(v₂) = z₂) ∧ (z₂ ⊈ r(2)) ∧ Q(time₉ₓₓₓₓ, r(2)))
Where Ref₉ₓₓₓₓ, and (4) is co-indexed
to a referring expression with the same denotation as ana'₄, and (2) is a time.

ω: NON-PERF
α: ana'₄ swim₂
λx.λy(x o₁ y)

Figure 8.8: Tree of Jess can swim

(continued)

κ: COLL₁ ana'₄ swim₂
λr.λ< w, p > λPλQλf(∃v₁ ∃z₁[COLL(v₁) ∧ f(v₁)]
A1(v₁) = [∃v₂ ∃z₂[swim(v₂) ∧ f(v₂)]
A0(v₂) = r(4) ∧ (τ(v₂) = z₂) ∧ (z₂ ⊈ r(2)) ∧ (r(1) ⊈ r(2))](w)(p)]
(τ(v₁) = z₁) ∧ P(r(1), z₁) ∧ Q(time₉ₓₓₓₓ, r(1)))]
Where Ref₉ₓₓₓₓ, and (4) is co-indexed
to a referring expression with the same denotation as ana'₄, and (1) and (2) are times and
where Re₉ₓₓₓₓₓ, u holds of any w ∈ W such that the capacities of r(4)
are as they are in the world of use, u.

ϕ: COLL₁
λr.λPλQλf(∃v₁ ∃z₁[COLL(v₁) ∧ f(v₁)]
(τ(v₁) = z₁) ∧ P(r(1), z₁) ∧ Q(time₉ₓₓₓₓ, r(1)))]

ξ: ana'₄ swim₂, ^ S-COMP
λr.λ< w, p > λYλVλf[V(∃v₂ ∃z₂[swim(v₂) ∧ f(v₂)]
A0(v₂) = r(4) ∧ (τ(v₂) = z₂) ∧
(z₂ ⊈ r(2)) ∧ (time₉ₓₓₓₓ ⊈ r(2))](w)(p)]
Where Ref₉ₓₓₓₓₓ, and (4) is co-indexed
to a referring expression with the same denotation as ana'₄, and (2) is a time.

π: ^ S-COMP
λ < w, p > λYλVλf
[V(λv[f(v) ∧ A1(v) = Y(w)(p)])]

ρ: ana'₄ swim₂ (continued above)

Figure 8.8: Tree of Jess can swim
which the capacities of the referent \( r \) assigns to (4) are as they are in the world \( u \) of utterance.

The lexical semantics of \textit{COLL} also constrains the indexical time of the sentential compliment to be whatever \( r \) assigns to the reference time of \textit{COLL}. The overlap relation \( o_t \) is made more specific due to the Aktionsart of each verb. The reference time of the state of capacity is thus contained in the reference time of the second event.

It is this containment that accounts for the iterative interpretation of the internal event. The modal auxiliary stretches the time such that an event that is intuitively shorter than a stative capacity is understood to have the potential to iterate (with gaps) over the span of the state of capacity.

The forcing of a state of repetition or iteration on the second verb provides an explanation for the paucity of progressive forms of the internal verb in uses of ability \textit{COLL}. The progressive is odd with ability \textit{COLL} readings:

205. *Jess can be swimming.

206. *Back in the day, Jess could be swimming.

In contrast, the Paradigm B modal works naturally with the progressive:

207. Jess might be swimming.

208. Jess might have been swimming.

Progressive aspect similarly stretches an event into a stative interval and is therefore redundant in uses of ability \textit{COLL}.

The next sub-tree below shows the combination of the modal auxiliary with its other semantic role labeled argument, the subject noun phrase:

The sub-tree below shows the combination of the modal auxiliary with tense and aspect:

Viewing the sub-tree above, it is also possible to consider how the past tense of \textit{COLL}, \textit{could}, is translated. As with \textit{can}, the reference time of the internal event contains the reference time of the capacity, but the reference time for the capacity precedes the speech time:

\[
\delta': \ \text{Jess}_{4} \text{can}_{4} \text{ana}_{4} \text{swim}_{2}
\]
\[
\lambda r. l < w, p > \lambda f(\exists v_1 \exists z_1 [COLL(v_1) \land f(v_1) \land A0(v_1) = r(4)] \land \ A1(v_1) = [\exists v_2 \exists z_2 [swim(v_2) \land f(v_2)]]
\]
are stated for the top node \( \phi \):

\[
\lbrack \phi \rbrack^{u,w} = \text{truth} \quad \text{if for at least one } w' \in W, \text{ such that } R_{\text{capac}u,w'}
\]

where \( R_{\text{capac}u,w'} \) holds of any \( w \in W \) such that the capacities of \( r(4) \)
are as they are in the world of use, \( u \).

The final sub-tree, given below, shows the closure operations of event closure and
variable assignment closure.

The truth conditions for the ability reading of \( \text{COLL} \) in the sentence, \( \text{Jess can swim} \),
are stated for the top node \( \phi \):

\[
\lbrack \phi \rbrack^{u,w} = \text{truth} \quad \text{if for at least one } w' \in W, \text{ such that } R_{\text{capac}u,w'}
\]

where \( R_{\text{capac}u,w'} \) holds of any \( w \in W \) such that the capacities of \( r(4) \)
are as they are in the world of use, \( u \).

The final sub-tree, given below, shows the closure operations of event closure and
variable assignment closure.

The truth conditions for the ability reading of \( \text{COLL} \) in the sentence, \( \text{Jess can swim} \),
are stated for the top node \( \phi \):

\[
\lbrack \phi \rbrack^{u,w} = \text{truth} \quad \text{if for at least one } w' \in W, \text{ such that } R_{\text{capac}u,w'}
\]
(continued) δ: Jess can ana1 swim2
\[
\exists l < w, p > \exists f(\exists v_1 \exists z_1 [COLL(v_1) \land f(v_1) \land A0(v_1) = r(4) \land \\
A1(v_1) = [\exists v_2 \exists z_2 [swim(v_2) \land f(v_2) \land \\
A0(v_2) = r(4) \land (r(v_2) = z_2) \land (z_2 \circ r(2)) \land (r(1) \subseteq r(2)])(w)(p)] \\
(\tau(v_1) = z_1) \land (r(1) \circ z_1) \land (time_{u,q} \subseteq r(1)_{\text{static}})] \\
\text{Where } Ref_{u,\text{non-perf}}, \text{ and} \\
\text{Ref}_{u,\text{non-perf}}(4) \text{ is named } Jess \text{ in } u \text{ and } (1) \text{ and } (2) \text{ are times, and} \\
\text{where } R_{\text{capa}w}, u \text{ holds of any } w \in W \text{ such that the capacities of } r(4) \\
\text{are as they are in the world of use, } u.
\]

ε: NON-PAST
\[
\lambda x. \lambda y(y \circ x) \\
\zeta: Jess COLL1 ana1 swim2
\[
\exists l < w, p > \lambda Qf(\exists v_1 \exists z_1 [COLL(v_1) \land f(v_1) \land A0(v_1) = r(4) \land \\
A1(v_1) = [\exists v_2 \exists z_2 [swim(v_2) \land f(v_2) \land \\
A0(v_2) = r(4) \land (r(v_2) = z_2) \land (z_2 \circ r(2)) \land (r(1) \subseteq r(2)])(w)(p)] \\
(\tau(v_1) = z_1) \land (r(1) \circ z_1) \land Q(time_{u,q}, r(1)))] \\
\text{Where } Ref_{u,\text{non-perf}}, \text{ and} \\
\text{Ref}_{u,\text{non-perf}}(4) \text{ is named } Jess \text{ in } u \text{ and } (1) \text{ and } (2) \text{ are times, and} \\
\text{where } r(time_{u,q}) \text{ is equal to the reference time of the tensed (higher) verb, and} \\
\text{where } R_{\text{capa}w}, u \text{ holds of any } w \in W \text{ such that the capacities of } r(4) \\
\text{are as they are in the world of use, } u.
\]

η: NON-PERF
θ: Jess COLL1 ana1 swim2
\[
\lambda x. \lambda y(x \circ y) \\
\text{(continued above)}
\]

Figure 8.10: Tree of Jess can swim
\( \phi: E \text{ Jess} \,\!_{4} \text{ can swim}_2 \)

\( \lambda < w, p > \exists r(\exists v_1 \exists z_1 \{COLL(v_1) \land A0(v_1) = r(4) \land \\
A1(v_1) = [\exists v_2 \exists z_2 \{swim(v_2) \land \\
A0(v_2) = r(4) \land (r(v_2) = z_2) \land (z_2 \subseteq r(2)) \land (r(1) \subseteq r(2))] \}(w)(p)] \land \\
(\tau(v_1) = z_1) \land (r(1) \subseteq z_1) \land (time_{u,\beta} \subseteq r(1))) \}) \]

Where \( \text{Ref}_{u,\beta} \) and

\( \text{Ref}_{u,\beta}(4) \) is named \textit{Jess} in \( u \) and (1) and (2) are times, and

where \( R_{\text{capa}W}, u \) holds of any \( w \in W \) such that the capacities of \( r(4) \)

are as they are in the world of use, \( u \).

\( \alpha: ClzE \)

\( \forall r A(\exists r(\exists v_1 \exists z_1 \{COLL(v_1) \land A0(v_1) = r(4) \land \\
A1(v_1) = [\exists v_2 \exists z_2 \{swim(v_2) \land \\
A0(v_2) = r(4) \land (r(v_2) = z_2) \land (z_2 \subseteq r(2)) \land (r(1) \subseteq r(2))] \}(w)(p)] \land \\
(\tau(v_1) = z_1) \land (r(1) \subseteq z_1) \land (time_{u,\beta} \subseteq r(1))) \}) \]

Where \( \text{Ref}_{u,\beta} \) and

\( \text{Ref}_{u,\beta}(4) \) is named \textit{Jess} in \( u \) and (1) and (2) are times, and

where \( R_{\text{capa}W}, u \) holds of any \( w \in W \) such that the capacities of \( r(4) \)

are as they are in the world of use, \( u \).

\( \gamma: ClzV \)

\( \delta: \text{ Jess} \,\!_{4} \text{ can} \_\!_{1} \text{ and} \_\!_{1} \text{ swim}_2 \)

\( \lambda f. \text{true} \) \hspace{1cm} (continued above)

Figure 8.11: Tree of \textit{Jess can swim}
Where $Ref_{u,\phi,r}$, and $Ref_{u,\phi}(4)$ is named $Jess$ in $u$ and (1) and (2) are times, and where $R_{\text{capa}u,w}$ holds of any $w \in W$ such that the capacities of $r(4)$ are as they are in the world of use, $u$.

The truth conditions do not involve relativization or assessment sensitivity. They involve existential quantification over accessible worlds for the $A_1$ argument of the modal auxiliary $COLL$.

### 8.3.3 Truth in a Relativist Model for Ability $COLL$

According to the definitions for monadic truth of an utterance, the modal evaluation world is set to the actual world and the utterance world is also set to the actual world:

- $\alpha$ is true in a world $u$ relative to a world $w$ and a context of assessment $c$ if, and only if, 
  
  $[\alpha]_{u,w,<\text{judge},\text{time},\text{world}>} = \text{truth}$;

- $\alpha$ is false in a world $u$ relative to a world $w$ and a context of assessment $c$ if, and only if, 
  
  $[\alpha]_{u,w,<\text{judge},\text{time},\text{world}>} = \text{false}$.

Because the perspective is irrelevant for the truth evaluation, the definition can be written in terms of monadic truth with no relativization to contexts:

- $\alpha$ is true relative to a use $\alpha$ if, and only if, $USE(\alpha, \alpha)$ and $\alpha$ is true;

- $\alpha$ is false relative to a use $\alpha$ if, and only if, $USE(\alpha, \alpha)$ and $\alpha$ is false.

The intensional nature of the sentence is closed at the level of $\alpha$.

Some background will be given on why $p$ is not relevant for $COLL$, that is, on why $COLL$ does not need a relativist interpretation in order to set the stage for the discussion of epistemic might in the next sections, which does require a relativist semantics.

Saying that it has a truth value that does not consider perspectives means that there is some fact of the matter whether Jess has the capacity in the actual world to swim. Two people could disagree on what her capacities are in the actual world and on whether or not those capacities merit the claim that she can swim. But there is a fact of the matter such that Jess either can or cannot swim in the world of the utterance. Any declarative sentence can be hedged with epistemic expressions, such as ‘I think that Jess can swim.’, but these expressions relate the person’s belief state to Jess’s capacity to swim and do not cause the truth value of the capacity to depend on their knowledge state any more than in a sentence like, ‘I think that the earth has doubled in size in the last 200 years.’ changes the truth value of whether or not the earth has doubled in size in the last 200
years. The question with \textit{COLL} in comparison to epistemic modals is whether or not the modal auxiliary itself is epistemic, and it seems it is not.

\textbf{209.} \textit{Jess can swim across the lake.}

\textbf{210.} \textit{No, he can’t, he tried this morning and failed.}

Assume two perspectives \( p \) and \( p' \) such that the \( x \) of \( p \) is not the same \( x \) of \( p' \), let us call that \( x \) ‘\( y \)’. Let us assume that \( y \) knows more about Jess’s capacities than \( x \) does. Saying that it matters what their knowledge state is is equivalent to saying that determining the capacities is partly epistemic in that it relates to knowledge states. In contrast, saying that the accessibility relation is determined by what Jess’s capacities are in the world of the utterance means that the truth value is determined at the time of utterance, given her capacities.

\subsection*{8.3.4 Actuality Inferences with Ability \textit{COLL}}

The actuality inference associated with ability readings comes when the world of utterance itself is the world in which the capacity is satisfied by the modal operators. The repetition of the event of the second verb is due to the assignment of a habitual interpretation of the second verb, unless the particular lexical nature of the second event make habitual readings difficult to sustain.

Sentences expressing sets of events that are easy to be realized in the world of utterance lend themselves more easily to actuality inferences:

\textbf{211.} \textit{Jess can see the mountains from her window.}

\textbf{212.} \textit{Syd can jump on a trampoline.}

In contrast to events that, though one might try repeatedly, have the most salient pragmatic reading in which they have been accomplished only once in the world of utterance or maybe not ever:

\textbf{213.} \textit{Pat can win a triathlon.}

\textbf{214.} \textit{Jess can climb Mt. Everest.}

The role that the semantics of the second verb plays and its likelihood of ease of occurrence and repetition is further exemplified by cases that lend themselves to a one-time-only reading as in:
215. John can circumcise (or decapitate) himself.

Because John has but one shot at self-decapitation, probably even in the accessible worlds, it is hard to even read the uses as capacity reading as the one-time nature of them seems more compatible with a permission reading. At best, the capacity reading is easier if John has a history of circumcising or beheading others or some obsession with cutting off more and more of himself.

Two pragmatic generalizations can be drawn. One is that the second event is interpreted in the closest accessible world and that factors such as how easily someone does a thing affect how close that world is. The other is that the internal event tends to have an iterative or habitual reading, unless it refers to an event that is pragmatically unlikely to be possible more than once.

8.3.5 Conclusions Regarding Tense and Aspect with Ability COLL

An example was given of the sentence Jess could swim. in order to illustrate the interaction of aspect and tense in the sequence of a modal auxiliary and another (non-auxiliary) verb. Special attention was given to the formula for tense in the second verb. It was posited that tense in non-finite clauses that verbs select is filled by a relation of the clause to the selecting clause. The selecting clause set the relation of its reference time to time_{u,a} in the normal way in which past and non-past tense operate. The dependent clause, however, had a reference time that was related to the reference time of the selecting verb. As a result, that event inherits the relation of strictly preceding time_{u,a} or not strictly preceding it via the temporal properties of the modal auxiliary verb.

At this point, it should be clear that two events are needed to establish the proper relations among event times and reference times in addition to being intuitively the case that the statement of one’s capacity in the actual world is a different event than the event that the person has the capacity to do.

The modal auxiliary is represented as something quite like other verbs that select for arguments that have non-finite verbs and as having a lexical entry that has Aktionsart properties, argument selection properties, and a specific temporal behaviour.

A pragmatically determined accessibility relation is being used, as is traditionally the case, to determine in what worlds the intensional argument is evaluated. The modal force of possibility is present in the sense of the truth conditions existentially quantifying over accessible worlds.
There are, however, many more components to the meaning of the modal auxiliary than the event in its scope (Kratzer’s proposition) and a context (Kratzer’s set of accessible worlds against which the modal is evaluated). The notion of modal force is present, Kratzer’s exact notion of a conversational background could be incorporated as well as an ordering source.

If Kratzer-style modal auxiliary analysis is seen as the part of modal meaning that is unified, what is presented here is the part that is lexical and grammatical in the sense of being reading specific in its grammatical behaviour and translation.

There will, however, be generalizations across readings by paradigm.

8.4 Tense, Aspect, and Epistemic might

The syntactic tree for the raising verb might is the same as for the control tree for ability COLL:

This syntactic treatment contrasts with that given in Steedman 2000[92] in which the subject noun phrase combines with the modal auxiliary, any other arguments of the second verb combine, and then the modal auxiliary and subject noun phrase combine with the un-tensed verb phrase. The semantic account, however, is similar to Steedman’s account, which also uses the anaphoric element.

Babby 2009[3] argues explicitly that the second verb in a sequence with modal aux-
(continued from $\theta$: Jess$_4$ might$_1$ $ana'_4$ swim$_2$ $Bse$)

$v$: Jess$_4$, SUBJ-NP ($Bse\backslash,Bse$)  $\kappa$: Jess$_4$ might$_1$ $ana'_4$ swim$_2$ $Bse$

$\lambda$: SUBJ-NP ($($Bse\backslash,Bse$/,.N)$) $\mu$: Jess$_4$, $N$

$\nu$: might$_1$ $Bse$  $\xi$: $ana'_4$ swim$_2$, S-COMP ($Bse\backslash,Bse$)

$\pi$: S-COMP  $\omega$: $ana'_4$ swim$_2(t/,(t/,.v))$

$((Bse\backslash,Bse),(t/,(t/,.v)))$  $\uparrow^{\uparrow}$

Figure 8.13: Tree of Jess might swim

(continued from $\omega$: $ana'_4$ swim$_2$)

$\sigma$: NON-PAST ($((t/,.i)/,.i)$)  $\nu$: $ana'_4$ swim$_2$ ($((t/,.i)/,.i)(t/,(t/,.v))$)

$\omega$: NON-PERF ($((t/,.i)/,.i)$)  $\alpha$: $ana'_4$ swim$_2$ $Bse$

$\beta\beta$: $ana'_4$ SUBJ-NP ($Bse\backslash,Bse$)  $\epsilon\epsilon$: swim$_2$ $Bse$

$\gamma\gamma$: SUBJ-NP ($($Bse\backslash,Bse$/,.N)$)  $\delta\delta$: $ana'_4$ $N$

Figure 8.14: Tree of Jess might swim
iliaries has its arguments taken over by the auxiliary and cannot be understood to be control verbs because they ‘lack an $i$ node’.

The semantics of the second verb remain the same up until the $S − COMP$ node, at which point the lexical semantics of the modal auxiliary reading result in a different translation.

Specifically, a semantic role label is not given to the referring expression with which the anaphor is co-indexed, the modal auxiliary has different temporal properties, and the tense of the second verb does not have the same constraints imposed.

In the case of epistemic might, the time of utterance, $time_{u}$, is linked to the second verb as an artifact of the utterance. This step is crucial in representing the difference between uses with the perfect aspect and uses without the perfect aspect.
(continued) κ: might₁ ana₂′ swim₂

\[ \lambda r \lambda w . p > \lambda \forall \lambda \forall \lambda f [ (\exists v_1 \exists z_1 \text{might}(v_1) \land f(v_1)) \land \\
\quad A_1(v_1) = [\exists v_2 \exists z_2 \text{swim}(v_2) \land f(v_2)] \land \\
\quad A_0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circlearrowleft r(2)) \land (\text{time}_w \sqsubseteq r(2))](w)(p) \]

(τ(v₁) = z₁) ∧ P(r(1), z₁) ∧ Q(timeₜₖ, r(1))]

Where Refₜₖ,ₜₚ,₀, and (4) is co-indexed
to a referring expression with the same denotation as ana₄′, and (1) and (2) are times and
where Refₑₜₖ,₀, hold of any w ∈ W such that w is compatible with the knowledge the judge x has in world a at time the time of assessment t.

Figure 8.15: Tree of Jess might swim
The accessibility relation for epistemic readings of *might* is defined such that the accessible worlds are those worlds that are compatible with the knowledge that the judge $j$ has in the world of assessment $a$ at the time of assessment $t$.

The proposition that is judged relative to those parameters is $Jess_4 \swim_2$:

$$
\exists v_2 \exists z_2 [\swim(v_2) \land f(v_2) \land A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ r(2)) \land (time_{\alpha} \subseteq r(2))]
$$

Where a satisfying variable assignment for $r$ is given on closure. The proposition that the judge assesses is relative to the time of utterance of the sentence. A judge can assess a sentence the day after it is said, and the time at which it was said is what determines the temporal location of the event that the second verb describes. In the example, above, the verb the second verb describes is the $Jess - \swim$-event. Assessment sensitivity does not relocate the event that the original sentence described, it rather states that the truth value is relative to the time and the person assessing the sentence.

The accessibility relation is defined for the assessor, since it is the assessor’s knowledge state that is relevant for assessment.

When the subject noun phrase is combined it occurs as a syntactic element, but the lexical semantics do not list it as an $A0$ of the modal auxiliary, rather, it is listed as not having a semantic role assigned by the modal auxiliary, but rather being semantically empty. It’s only semantic role assignment comes from its co-indexation with the $A0$ of $v_2$: 
continued from θ: Jess might swim

\[ \lambda r \lambda t < w, p > \lambda P \lambda f (\exists v_1 \exists v_2 \text{might}(v_1) \land f(v_1) \land \text{PLEON}(v_1) = r(4) \land A1(v_1) = [\exists v_2 \exists w \text{swim}(v_2) \land f(v_2)] \land A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \subseteq r(2)) \land ((\text{time}_{a,t} \subseteq r(2)))(w)(p) \land (\tau(v_1) = z_1) \land P(r(1), z_1) \land Q(\text{time}_{a,t}, r(1)))] \]

Where \( R_{\text{epistem}}(4) \) is named Jess in \( u \) and (1) and (2) are times and

where \( R_{\text{epistem}} w', a \) holds of any \( w' \in W \) such that \( w' \) is compatible with the knowledge that the judge \( x \) has in world \( a \) at time of assessment \( t \).

\[ \mu \alpha \lambda V \lambda f \quad \text{Where } R_{\text{epistem}}(4), \text{provided that } R_{\text{epistem}}(4) \text{ is named Jess in } u. \]

\[ \lambda \mu \alpha \lambda V \lambda f \quad \text{Where } R_{\text{epistem}}(4), \text{provided that } R_{\text{epistem}}(4) \text{ is named Jess in } u. \]

Figure 8.16: Tree of Jess might swim
The sub-tree below shows the combination of the modal auxiliary with tense and aspect. Epistemic readings of might, like other Paradigm B modal auxiliaries, does not take tense, although it is in the tense-taking position, nor does it take aspectual morphology. The default for each of these is overlap:

Considering what epistemic might means, the reference time of might is the time of the knowledge state that meets the assertion conditions, in the case of the speaker. The event it represents is an event of standing in a speculative relation to a proposition. The event of speculation is always at the same time as the knowledge state that satisfies the assertion conditions.

The internal event, however, can be either overlapping the reference time of that event or strictly preceding it. When the aspectual marking is non-perfect, there is overlap. When the aspectual marking is perfect (as in the sentence Jess might have slept.), the event time strictly precedes the reference time.

For an assessor, the time $t$ of assessment can be such that the time of utterance is in the past. However, unless there is some situation temporal expression such as, ‘Well, at that time, I thought that Jess might swim’, the assessment involves the knowledge state of the assessor at the time of assessment. The accessibility relation, likewise, is defined as being based on compatibility with the world from which the assessor is assessing the proposition.

By means of these temporal interactions, the event marked $v_2$ is associated with a particular event time and the sentence refers, at the time of utterance, to that event. In contrast, the event marked $v_1$ is situated relative not to the world of use $u$, but to the world of assessment $a$.

The discussion thus far assumes an autocentric stance rather than an exocentric or acentric stance. Lasersohn (2015:99-105)[52] defines an autocentric stance to be the stance taken relative to ones own current perspective. Any other perspective that can be taken represents an exocentric stance. An acentric stance is considered to be a ‘birdseye-view’ on an event that does not take any perspective.

The use of the perfect with epistemic might has a similar translation, but with the event time of the swim event strictly preceding the reference time of the swim event, in the sentence Jess might have swum:

$$\phi: E \text{Jess}_4 \text{might}_1 \text{swim}_2$$
$$\lambda w, p > \exists r(\exists v_1 \exists z_1[might(v_1)] \land STRUCT(v_1) = r(4)) \land$$
$$A1(v_1) = [\exists v_2 \exists z_2[swim(v_2)] \land$$
(continued) δ: Jess might\textsubscript{1} ana\textsubscript{2} swim\textsubscript{2}
\[
\lambda r. t < w, p > A(f(\exists v_1 [\text{might}(v_1) \land f(v_1) \land \text{PLEON}(v_1) = r(4) \land \\
A(\exists v_2 [\text{swim}(v_2) \land f(v_2)])
\]
\[
A_0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ t, r(2)) \land (\text{time}_u \subseteq r(2)])(w)(p)
\]
\[
(\tau(v_1) = z_1) \land (r(1) \circ z_1) \land (\text{time}_u \subseteq r(1))
\]
Where \(\text{Ref}_{u,\delta}x\), and
\[
\text{Ref}_{u,\delta}(4) \text{ is named } J_{\text{ess}} \text{ in } u \text{ and } (1) \text{ and (2) are times, and}
\]
where \(\text{Re}_{\text{epistem}} w', a \) holds of any \( w' \in W \) such that \( w' \) is compatible with the knowledge \( j \) has in world \( a \) at time \( t \).

\begin{figure}[h]
\centering
\begin{align*}
\theta: \text{NON-PERF} & \quad \text{Jess might}\textsubscript{2} ana\textsubscript{3} swim\textsubscript{4} \\
\lambda x. y. (x \circ y) \\
\end{align*}
\caption{Tree of Jess might swim}
\end{figure}
\[ \phi: E \text{ Jess}_4 \text{ might}_1 \text{ swim}_2 \]
\[ \lambda < w, p > \exists r \exists v_1 \exists z_1 [\text{might}(v_1) \land \text{STRUCT}(v_1) = r(4) \land \\
A1(v_1) = [\exists v_2 \exists z_2 \exists \text{swim}(v_2) \land \\
A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_1 r(2)) \land (\text{time}_n \subseteq r(2)))(w)(p)] \\
(\tau(v_1) = z_1) \land (r(1) \circ_1 z_1) \land (\text{time}_n \subseteq r(1)))] \\
\text{Where } \text{Ref}_u, \text{true}, \text{and} \]
\[ \text{Ref}_u(4) \text{ is named } \text{Jess} \text{ in } u \text{ and } (1) \text{ and } (2) \text{ are times, and} \]
where \( \text{Ref}_u w', a \) holds of any \( w' \in W \) such that \( w' \) is compatible with the knowledge \( j \) has in world \( a \) at time \( t \).

\[ \alpha: \text{ClzE} \]
\[ \lambda t \exists r [\text{Ref}_u, r \leq r \land t(r) = \text{true}] \]
\[ \beta: \text{Jess}_4 \text{ might}_1 \text{ ana}_4' \text{ swim}_2 \]
\[ \lambda t \exists r [\text{Ref}_u, r \leq r \land t(r) = \text{true}] \]
\[ \lambda r < w, p > \exists r \exists v_1 \exists z_1 [\text{might}(v_1) \land \text{STRUCT}(v_1) = r(4) \land \\
A1(v_1) = [\exists v_2 \exists z_2 \exists \text{swim}(v_2) \land \\
A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_1 r(2)) \land (\text{time}_n \subseteq r(2)))(w)(p)] \\
(\tau(v_1) = z_1) \land (r(1) \circ_1 z_1) \land (\text{time}_n \subseteq r(1)))] \\
\text{Where } \text{Ref}_u, \text{true}, \text{and} \]
\[ \text{Ref}_u(4) \text{ is named } \text{Jess} \text{ in } u \text{ and } (1) \text{ and } (2) \text{ are times, and} \]
where \( r(\text{time}_n) \) is the time of utterance, and
where \( \text{Ref}_u w', a \) holds of any \( w' \in W \)
such that \( w' \) is compatible with the knowledge \( j \) has
in world \( a \) at time \( t \).

\[ \gamma: \text{ClzV} \]
\[ \delta: \text{Jess}_4 \text{ might}_1 \text{ ana}_4' \text{ swim}_2 \]
\[ \lambda f: \text{true} \text{ (continued above)} \]

Figure 8.18: Tree of \text{Jess might swim}
\[
A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 <_t r(2)) \land \left(\text{time}_{u,\phi} \subseteq_t r(2)\right))(w)(p) \\
(\tau(v_1) = z_1) \land (r(1) \circ_t z_1) \land \left(\text{time}_{u,\phi} \subseteq_t r(1)\right))
\]

Where \(\text{Ref}_{u,\phi}\), and
\(\text{Ref}_{u,\phi}(4)\) is named \textit{Jess} in \(u\) and (1) and (2) are times, and
where \(\text{Rep}_{\text{epist}}w', a\) holds of any \(w' \in W\) such that \(w'\) is compatible with the knowledge \(j\) has in world \(a\) at time \(t\).

The perfect works in such a way that the event that is being considered, that event labeled \(v_2\), above, is in the past relative to the speech time. However, the event signified by the modal auxiliary, labeled \(v_1\), above, has a time that varies according to the context of assessment.

### 8.4.1 Truth Conditions for Epistemic \textit{might}

Epistemic \textit{might} has a relativist interpretation, so the truth conditions of a a use of \(\alpha\) where \(\alpha\) is the \textit{The badger might have eaten a chinchilla} are given below:

**Definition 115.** \(\alpha\) is true relative to a use \(\alpha\), in a world \(u\), relative to world \(w\) and context of assessment \(c\) if \(\text{iff US } E(\alpha, \alpha)\) and \(\alpha\) is true in \(u\) relative to \(w\) and \(c\);

\(\alpha\) is false relative to a use \(\alpha\), in a world \(u\), relative to world \(w\) and context of assessment \(c\) if \(\text{iff US } E(\alpha, \alpha)\) and \(\alpha\) is false in \(u\) relative to \(w\) and \(c\) (Lasersohn 2015:70).

The truth conditions for the epistemic reading of \textit{might} in the sentence, \textit{Jess might swim.}, are stated for the top node \(\phi\):

\[
\left[\phi\right]_{u,w,p(<x,t,a>)} = \text{truth} \text{ if for at least one } w' \in W, \text{ such that } \text{Rep}_{\text{epist}}u, w', \\\n[\exists v_2 \exists z_2 [\text{swim}(v_2) \land \left(\text{time}_{u,\phi} \subseteq_t r(2)\right))(w)(p)] \text{ in } w' \text{ given } p, \\\n\text{and } \exists r(\exists v_1 \exists z_1 [\text{might}(v_1) \land \text{STRUCT}(v_1) = r(4)\land A0(v_1) = [\exists v_2 \exists z_2 [\text{swim}(v_2) \land A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 <_t r(2)) \land \left(\text{time}_{u,\phi} \subseteq_t r(2)\right))(w)(p)] \land (\tau(v_1) = z_1) \land (r(1) \circ_t z_1) \land \left(\text{time}_{u,\phi} \subseteq_t r(1)\right))]
\]

Where \(\text{Ref}_{u,\phi}\), and
\(\text{Ref}_{u,\phi}(4)\) is named \textit{Jess} in \(u\) and (1) and (2) are times, and
where \( r(time_{u,c}) \) is the time of utterance, and

where \( R_{epistem}w', a \) holds of any \( w' \in W \)

such that \( w' \) is compatible with the knowledge \( j \) has

in world \( a \) at time \( t \).

The truth conditions involve relativization to the perspective of the judge in the context, \( x \) at the time of assessment \( t \) in the world of assessment \( a \). They involve existential quantification over accessible worlds for the A1 argument of the modal auxiliary \( might \).

8.4.2 Perfect as a Modal Event Argument

In the previous discourse, lack of tense marking was associated with defaulting to a general overlap between the indexical \( time_{u,c} \) and the reference time. The indexical time, furthermore, was shown to be a time other than speech time when verbs occur in a position that does not allow them to take tense.

If epistemic \( might \) had tense marking, then it would be possible for the reference time to precede the assessment time. Such an example would be one in which the judge considers the intensional proposition relative to a prior knowledge state.

It has been argued that such readings only happen when there are extra elements outside of the modal auxiliary conveying that it is a past knowledge state against which the intensional proposition is expressed:

216. At the time, I thought Jess might swim.

Sentences such as (216) are no longer assessment sensitive, much like Lasersohn’s 2015[52] examples of predicates of personal taste that have a judge mentioned in the for-phrase:

217. Roller coasters are fun for Jess.

It has been argued in von Fintel and Gillies 2008[102] that uses of epistemic \( might \) can be used to refer to a past knowledge state via the use of perfect aspect on the verb in the intensional proposition. The authors use sentence (218) below as an illustration.

The authors comment, “Sophie is looking for some ice cream in the freezer. There is none in there. Asked why she opened the freezer, she replies:

218. There might have been ice cream in the freezer.”
At first, the reading seems metaphysical, however, there is no if-clause that provides an obvious context for a counterfactual.

I would say that this example relies on the discourse placing the opening event in the past much like the embedding sentence creates a ‘relic’ assessment.

In the debate about assessment sensitivity, it is important to note that sentence (218) is not assessment sensitive from any given time of assessment, but only in taking the same past perspective time \( t \) to which the speaker indexically refers. From the perspective of that fixed time, however, two judges can demonstrate the same kind of faultless disagreement that happens in other assessment-sensitive uses.

In this sense, it is a valid argument for a reading in which the perfect aspect appears to be combining with the modal event and synchronizing it with the internal event time.

It is interesting to note that the quality of the example does not depend on the stative verb and pleonastic there:

**219.** Q: Why did you open the freezer?

**220.** A’: Ice cream might have been in the freezer.

**221.** A”: Maintenance might have thawed the freezer.

To my ear, these dialogs feel like they are missing an I thought for context.

The semantic translation for such sentences would involve fixing the assessment time to be before the utterance time and ignoring certain intervening information that prevent the assertion conditions. Following the proposal by Schmerling 1983[86], this type of use of Paradigm B modal auxiliaries provides evidence for a complex perfect modal form, the similarity that she notes to the reduced form, might / / .

The reading that is claimed to exist does not differ with respect to assessment sensitivity, it differs in that the context of assessment is situated in the past relative to the speech time and the time \( t \) of assessment is fixed to that time. The knowledge state of one judge \( a \), still differs from that of another judge, \( a’ \), and there can be faultless disagreement about the possibility of the past event.

This use of the perfect would be represented semantically as follows:

---

5Schmerling 1983[86] does not make a claim that these two readings have a phonological difference. Rather, she proposed only the modal perfect form. Whether or not uses that associate the perfect with the modal event are more reduced phonetically than forms in which the second event is in the perfect is a topic for empirical research.
\[ \phi: E \text{Jess}_{4} \text{ might}\] have swum_{2} \\
\lambda < w, p > \exists r(\exists v_{1} \exists z_{1}[\text{might}(v_{1}) \land STRUCT(v_{1}) = r(4)] \land \\
A1(v_{1}) = [\exists v_{2} \exists z_{2}[\text{swim}(v_{2}) \land \\
A0(v_{2}) = r(4) \land (\tau(v_{2}) = z_{2}) \land (z_{2} <_{T} r(2)) \land (time_{a,\phi} \subseteq_{T} r(2)))](w)(p)] \\
(\tau(v_{1}) = z_{1}) \land (r(1) <_{T} z_{1}) \land (time_{a,\phi} \subseteq_{T} r(1)))] \\
Where \text{Ref}_{u,\phi \neq r}, and \\
\text{Ref}_{u,\phi}(4) \text{ is named } \text{Jess} \text{ in } u \text{ and (1) and (2) are times, and} \\
\text{where } \text{Rep}_{w} \text{ holds of any } w' \in W \text{ such that } w' \text{ is compatible with the} \\
\text{knowledge } j \text{ has in world } a \text{ at time } t.

The reference time of the \text{might have} event precedes that of the assessment time and it set to the same time as the speaker intended. The reading can be thought of as having the perfect marking on the main verb lead to a perfect reading on it and the auxiliary \text{have} being perfect marking on the modal auxiliary itself.

### 8.5 Conclusions

The following claims have been supported so far:

1. All verbs, including modal auxiliary verbs and non-finite verbs combine with an aspect relation and a tense relation.

2. The only grammatical aspect marking is perfect, which places the event time strictly before the reference time of the event.

3. If a verbal expression does not have perfect aspect marking, then it is considered to be non-perfect, which means that the event time does not strictly precede the reference time. That is to say, for example, they can be overlapping, or the event can be in the future of the event time.

4. The lexical aspect of a verb contributes nuances to the way in which it situates the event and reference times, with stative events having a longer duration, for example.

5. Modal auxiliary verbs have stative Aktionsart.
6. If a verb is in a tense-taking position, then the past tense morphology places the reference time strictly before the speech time and any non-past morphology is interpreted as meaning that the reference time does not strictly precede the speech time.

7. If a verb is in a position where it does not take tense, its temporal relation is one relating its reference time to the reference time of an tensed verb. The cases we looked at involved a tensed embedding verb, and they were shown to have a relation of the second reference time being contained in the first reference time.

The next chapter discusses how ability *COLL* and epistemic *might* relate to other modal auxiliaries.
CHAPTER 9

THE LANDSCAPE OF MODAL AUXILIARY VERBS

9.1 Introduction

The last chapter, Chapter 8, introduced a theory of how modal auxiliaries combine with tense and aspect. Two paradigms were presented; They were called Paradigm A and Paradigm B. It was posited that Paradigm A modal auxiliaries take tense morphologically on the modal auxiliary, and Paradigm B modal auxiliaries refer to an event in the past through the use of perfect morphology on the following verb.

It was shown that, among modal auxiliaries, there are those that behave as control verbs and those that behave as raising verbs. A grammar was given for two exemplary cases; Ability COLL is a Paradigm A control verb, and epistemic might is a Paradigm B raising verb. Based on the exemplary cases in Chapter 8, there are several hypotheses available regarding the relationship among tense, grammatical aspect, semantic role labels, modal scope, and assessment-sensitivity.

<table>
<thead>
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<th></th>
<th>Tense</th>
<th>SUBJ SRL</th>
<th>SUBJ in scope</th>
<th>GramAspt</th>
<th>Asses-Sensitive</th>
</tr>
</thead>
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<tr>
<td>COLL:</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>'might':</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 9.1: Patterns with Ability COLL and epistemic might

Table 9.1 presents the two readings, ability COLL and epistemic might, on the horizontal axis and the various features under consideration on the vertical axis. The columns are labeled as follows, where each label has a binary value of yes or no. The first column is labeled Tense. It takes the value yes if the modal auxiliary verb takes temporal morphology and no otherwise. The second column is labeled SRL to SUBJ-NP. This column takes the value of yes if the modal auxiliary verb assigns a semantic role label to the subject noun phrase and no if it does not. The third column is labeled SUBJ-NP in scope. It takes the value yes if the subject noun phrase is in the scope of
the intensional operator that is introduced by the modal auxiliary verb, and takes the value *no* otherwise. The fourth column is labeled *GramAspt*. It takes the value *yes* if the form takes grammatical aspect on the verb following the modal auxiliary and *no* otherwise. The fifth column is labeled *Asses-Sensitive*. It takes the value *yes* if the reading is assessment-sensitive and *no* otherwise.

Based on Table 9.1, hypotheses that suggest themselves for testing include, among others, the following:

- Modal auxiliaries that take temporal morphology also assign a semantic role label to the subject noun phrase.
- Modal auxiliaries that assign a semantic role label to the subject noun phrase do not interpret that subject noun phrase in the scope of the intensionality of the modal auxiliary.
- Assessment sensitivity occurs with grammatical aspect.
- Assessment sensitivity does not occur with tense.
- Assessment sensitivity does not occur when the subject noun phrase is interpreted extensionally.

When checked against Paradigm A and Paradigm B more broadly, however, the main hypothesis that is supported is that tense-bearing modal auxiliaries do not co-occur with assessment sensitive readings, as shown in Table 9.2 below, where the labels are exactly as in Table 9.1, except that there are three values, and the value ‘–’ means that some modal auxiliaries in the paradigm have the trait listed and some do not:

<table>
<thead>
<tr>
<th></th>
<th>Tense</th>
<th>SRL SUBJ</th>
<th>SUBJ in scope</th>
<th>GramAspt</th>
<th>Assess-Sensitive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paradigm A:</td>
<td>Yes</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>No</td>
</tr>
<tr>
<td>Paradigm B:</td>
<td>No</td>
<td>–</td>
<td>–</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 9.2: Patterns with Paradigm A and Paradigm B

What happens in the actual distribution of modal auxiliary readings is that control and raising are represented in both paradigms, but assessment sensitivity is restricted to Paradigm B.

The next sections will provide evidence for the proposed grouping and give examples of how each reading is translated. The patterns of SoT effects are described in terms of
These data and the use of perfect aspect with tensed (Paradigm A) modal auxiliaries is translated. The table of forms presented earlier in this chapter is repeated here, in Table 9.4, with relevant updates. The traditional taxonomic categories of hypothetical conditionals, counterfactual conditionals, and teleological uses are treated as grammatical forms that can have various readings present.

### 9.2 Dynamic Modal Auxiliaries Readings

The dynamic modal auxiliaries include ability *COLL*, dispositional *WOLL*, circumstantial *COLL*, and quantifications *COLL* and *WOLL*. All of the dynamic modal auxiliaries are Paradigm A modal auxiliaries. All of them are control verbs except for circumstantial *COLL*.

Dynamic modal auxiliaries are unique among the taxonomic categories in that they are associated with actuality inferences. The only other modal auxiliary that has actuality inferences are ‘future in the past’ uses of *WOLL*, and the uses might be argued to be actuality entailments, if the event precedes the speech time.

As has been stated, the category of dynamic modal auxiliaries is a heterogeneous category of expressions. Future uses of *WOLL* lie outside of the three major categories in

<table>
<thead>
<tr>
<th>Control:</th>
<th>Paradigm A (tensed)</th>
<th>Paradigm B (un-tensed &amp; assessment-sensitive)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ability</td>
<td>COLL</td>
<td>priority should</td>
</tr>
<tr>
<td>volitional</td>
<td>WOLL</td>
<td>priority must</td>
</tr>
<tr>
<td>quantificational</td>
<td>COLL</td>
<td></td>
</tr>
<tr>
<td>quantificational</td>
<td>WOLL</td>
<td></td>
</tr>
<tr>
<td>permission</td>
<td>COLL</td>
<td></td>
</tr>
<tr>
<td>permission</td>
<td>WOLL</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Raising:</th>
<th>Paradigm A (tensed)</th>
<th>Paradigm B (un-tensed &amp; assessment-sensitive)</th>
</tr>
</thead>
<tbody>
<tr>
<td>circumstantial</td>
<td>COLL</td>
<td>epistemic should</td>
</tr>
<tr>
<td>future</td>
<td>WOLL</td>
<td>epistemic might</td>
</tr>
<tr>
<td></td>
<td></td>
<td>epistemic may</td>
</tr>
<tr>
<td></td>
<td></td>
<td>epistemic must</td>
</tr>
<tr>
<td></td>
<td></td>
<td>epistemic could</td>
</tr>
<tr>
<td></td>
<td></td>
<td>speculative could</td>
</tr>
<tr>
<td></td>
<td></td>
<td>speculative might</td>
</tr>
<tr>
<td></td>
<td></td>
<td>speculative may</td>
</tr>
<tr>
<td></td>
<td></td>
<td>speculative should</td>
</tr>
</tbody>
</table>

Table 9.3: Patterns with Paradigm A and Paradigm B
Table 9.4: Morphological Forms of Paradigm A and Paradigm B Modal Auxiliaries in Various Contexts

<table>
<thead>
<tr>
<th></th>
<th>Non-Past</th>
<th>Past</th>
<th>SoT</th>
<th>Tensed-Perf</th>
<th>Perf</th>
<th>Subj</th>
<th>Subj-Perf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Para A</td>
<td>will</td>
<td>would</td>
<td>would</td>
<td>will have</td>
<td>–</td>
<td>would</td>
<td>would have</td>
</tr>
<tr>
<td></td>
<td>can</td>
<td>could</td>
<td>could</td>
<td>(can’t have)</td>
<td>–</td>
<td>could</td>
<td>could have</td>
</tr>
<tr>
<td></td>
<td>may</td>
<td>–</td>
<td>–</td>
<td>(may not have)</td>
<td>–</td>
<td>might</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>(shall)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Para B</td>
<td>could</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>could have</td>
<td>could</td>
<td>could have</td>
</tr>
<tr>
<td></td>
<td>may</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>may have</td>
<td>may</td>
<td>may have</td>
</tr>
<tr>
<td></td>
<td>might</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>might have</td>
<td>might</td>
<td>might have</td>
</tr>
<tr>
<td></td>
<td>should</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>should have</td>
<td>should</td>
<td>should have</td>
</tr>
<tr>
<td></td>
<td>must</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>must have</td>
<td>must</td>
<td>must have</td>
</tr>
</tbody>
</table>

many taxonomies, as discussed in Chapter 2. It is reasonable to consider future \textit{WOLL} among the dynamic uses, in which case, it could be said more definitely that, if a reading has an actuality inference, than it is a dynamic modal use. The inverse of this statement holds that, if something is not in the else set of modal auxiliaries, known as dynamic modals, it does not lend itself to actuality inferences. In other words, priority modal auxiliaries and epistemic modal auxiliaries do not have actuality inferences, whether or not dynamic modal auxiliaries and future \textit{WOLL} together form a conceptually cohesive category.

Ability uses of \textit{COLL} were treated in the last chapter, Chapter 8. They can have past or non-past tense. The second verb describes an event, \( v_2 \), which has a reference time contained in the reference time of the capacity event \( v_1 \). If the capacity ends before speech time, so do the events that satisfy the truth conditions for the reading. This section presents the semantics of the other dynamic modal auxiliaries and discusses their temporal properties, their modal properties, and their association with actuality inferences.

9.2.1 Dispositional \textit{WOLL}

Dispositional uses of \textit{WOLL} involve the repeated habits or compulsions of the first participant or a telic event that the first participant is surmised to be involved in based on their habits or compulsions.
dispositional WOLL

**word**

**PHON: ORTHO:** [WOLL]

**SYN:**

<table>
<thead>
<tr>
<th>HEAD: CAT : Bse</th>
</tr>
</thead>
<tbody>
<tr>
<td>FORM: bse</td>
</tr>
</tbody>
</table>

**MODE:** ref

**TYPE:** \(< i < i, t >>, < i < i, t >>, < i < v, t >, t >>\)

**INDEX:** \(v_1\)

**TRANSL:** \(\lambda P \lambda Q \lambda t_i \lambda f_{<v,t>} . \exists v \exists t_z \)

\[ WOLL'(v) \land f(v) \land (\tau(v) = t_z) \land P(t_i, t_z) \land Q(t_i, t_j) \]

**RELN:** volition

**EVENT:** \(v_1\)

**DISPOSITION-OF:** \(i\), where \(i\) is the index on an expression of type \(< i < i, t >>, < i < i, t >>, < i < v, t >, t >>\),

**RESTR:** \(< i < i, t >>, < i < i, t >>, < i < v, t >, t >>\)

**DISPOSED-TO:** \(\square v_2\) where \(v_2\) is the index of an expression of type \(< v, t > t >\), which contains an anaphoric expression *ana*; and , where the accessibility relations \(R_{dispo}\) is such that \(R_{dispo}(w, w')\) \(w'\) is a world where the habit and disposition of the individual or group referred to by \(i\) are as they are in \(w\).

Figure 9.1: Attribute value matrix for dispositional WOLL

### 222. Jess will take the morning shift.

The truth conditions for the sentence *Jess*\(_4\) *will*\(_1\) *eat*\(_2\) *a chinchilla*\(_5,2\), taken as a disposition of Jess to be the kind of person who eats a chinchilla, are given below:

\[
\text{[[}\phi^{u,w}\text{]} = \text{truth} \text{ if for all } w' \in W, \text{ such that } R_{dispo}u, w',
\]

\[
[\exists v_2 \exists z_2[\text{bite}(v_2) \land A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_1 r(2)) \land (r(1) \subseteq r(2))]\text{ in } w'
\]

and \(\exists r(\exists v_1 \exists z_1[WOLL(v_1) \land A0(v_1) = r(4) \land A1(v_1) = [\exists v_2 \exists z_2[\text{eat}(v_2) \land A0(v_2) = r(4) \land A1(v_2) = r(5) \land (\tau(v_2) = z_2) \land (z_2 \circ_1 r(2)) \land (r(1) \subseteq r(2))](w')]]\)

(\(\tau(v_1) = z_1\) \land (r(1) \subseteq z_1) \land (\text{time}_{u,\phi} \subseteq r(1))])

Where \(Ref_{u,\phi}r\), and \(Ref_{u,\phi}(5)\) is a chinchilla at time \(r(2)\) in \(u\) and \((1)\) and

229
(2) are times, and where $R_{\text{tra}u}, w'$ holds of any $w \in W$ such that the desires and compulsions of $r(4)$ at time $r(2)$ are as they are in the world of use, $u$.

Past habits are sometimes described by dispositional WOLL, where the form of the verb is past would, and the events being described occur before the speech time.

**223. Back in the day, I would always take the early morning shifts.**

The past form behaves predictably in the semantics placing the event time being referenced before the speech time. Even if the speaker still takes the early morning shifts, it is the past span to which the speaker is referring in the sentence. The past span is made explicit by the phrase back in the day, but it can be supplied contextually. While the past tense of the modal COLL indicates that the referenced temporal span is in the past relative to speech time, when providing a single sentence out of context, the use is easily confusable with a hypothetical use of would as in sentence (224) below:

**224. I would always take the early morning shifts, if I were you.**

### 9.2.2 Quantificational (Dynamic) COLL and WOLL

The dynamic reading of COLL that is called quantificational is a control reading and its semantics are similar to the semantics of ability COLL.

Quantificational readings of COLL express a capacity of a generic subject:

**225. A chinchilla can bite.**

The meaning of sentence (225) is that something that is a chinchilla is associated with biting. An analysis on par with ability seems adequate to deal with such examples, where the accessibility relation is defined to include worlds in which chinchillas are as they are in the world of utterance.

Quantificational readings do not allow empty subjects and passivization changes the meaning:

**226. A chinchilla can bite a badger.**

**227. A badger can be bitten by a chinchilla.**
Figure 9.2: Attribute value matrix for quantificational COLL
The semantic arguments of quantificational COLL are the individuals who have a trait, stated in the subject noun phrase, and the trait that they have, stated in the second verb and the arguments associated with it.

The truth conditions for the sentence \textit{A chinchilla$_4$ can$_1$ bite$_2$.}, where \( \phi \) represents the closed sentence, are given below:

\[
\begin{align*}
[\phi]^{u,w} &= \text{truth} \text{ if for at least one } w' \in W, \text{ such that } R_{\text{trait } u, w'}, \\
&[\exists v_2 \exists z_2 [\text{bite}(v_2) \land A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_t r(2)) \land (r(1) \subseteq_t r(2))]} \\
&\text{in } w' \\
&\text{and } \exists r(\exists v_1 \exists z_1 [\text{COLL}(v_1) \land A0(v_1) = r(4) \land A1(v_1) = [\exists v_2 \exists z_2 [\text{bite}(v_2) \land A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_t r(2)) \land (r(1) \subseteq_t r(2))] (w')]} \\
&(\tau(v_1) = z_1) \land (r(1) \circ_t z_1) \land (\text{time}_{u,\phi} \subseteq_t r(1)))
\end{align*}
\]

Where \( R_{\text{trait } u, \phi} \), and \( R_{\text{trait } u, \phi} (4) \) is a chinchilla at time \( r(2) \) in \( u \) and (1) and (2) are times, and where \( R_{\text{trait } u, w'} \) holds of any \( w \in W \) such that the traits of \( r(4) \) at time \( r(2) \) are as they are in the world of use, \( u \).

The semantic similarity between ability COLL and quantificational COLL is also intuitive as the capacity of a generic or plural subject can be used to express the traits of a species or the functionality of a type of manufactured product (e.g., as in, ‘\textit{An SUV gets good mileage.}’).

Quantificational readings of WOLL do not differ much from readings of COLL in their lexical entries and translations. They differ only in their accessibility relations and in their modal force.

228. \textit{A badger will bite.}

The semantic arguments of quantificational WOLL are the individuals who have a trait, stated in the subject noun phrase, and the trait that they have, stated in the second verb and the arguments associated with it.

The truth conditions for the sentence \textit{A badger$_4$ will$_1$ bite$_2$.}, where \( \phi \) represents the closed sentence, are given below:

\[
\begin{align*}
[\phi]^{u,w} &= \text{truth} \text{ if for all } w' \in W, \text{ such that } R_{\text{trait } u, w'}, \\
&[\exists v_2 \exists z_2 [\text{bite}(v_2) \land A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_t r(2)) \land (r(1) \subseteq_t r(2))]} \\
&\text{in } w' \\
&\text{and } \exists r(\exists v_1 \exists z_1 [\text{WOLL}(v_1) \land A0(v_1) = r(4) \land A1(v_1) = [\exists v_2 \exists z_2 [\text{bite}(v_2) \land}
\end{align*}
\]
quantificational WOLL

**word**

**PHON:** ORTHO: [WOLL]

**SYN:**

- **HEAD:** CAT: Bse
- **FORM:** bse

**MODE:** ref

**TYPE:** $<i < i, t >, < i < i, t >, < i << v, t >, t >$

**INDEX:** $v_1$

**SEM:**

- **TRANSL:** $\lambda \mathcal{P}_\lambda Q \lambda t, \lambda f_{<v,t>}, \exists v \exists t_{v'}$
  - $[WOLL'(v) \land f(v) \land (\tau(v) = t_v) \land P(t_i, t_z) \land Q(t_i, t_j)]$

- **RELN:** trait

- **EVENT:** $v_1$

- **TRAIT-OF:** $i$, where $i$ is the index on an expression of type $<i < i, t >, < i < i, t >, < i << v, t >, t >$,

- **RESTR:** $<< i < i, t >, < i < i, t >, < i << v, t >, t >>$

- **TRAIT:** $\Box v_2$ where $v_2$ is the index of an expression of type $<< v, t > t >$, which contains an anaphoric expression $ana_i$ and, where the accessibility relations $R_{trait}$ is such that $R_{trait}(w, w')$ iff $w'$ is a world where the traits of the individual or group referred to by $i$ are as they are in $w$.

Figure 9.3: Attribute value matrix for quantificational WOLL
A0(v2) = r(4) \land (\tau(v2) = z2) \land (z2 \circ_1 r(2)) \land (r(1) \subseteq_1 r(2)))(w')]

(\tau(v1) = z1) \land (r(1) \circ_1 z1) \land (\text{time}_{u,.} \subseteq_1 r(1)))]

Where \(Ref_{u,.}\) and \(Ref_{u,.}(4)\) is a badger at time \(r(2)\) in \(u\) and \(1\) and \(2\) are times, and where \(R_{\text{trait}u,.}w_0\) holds of any \(w_2\) \(W\) such that the traits of \(r(4)\) at time \(r(2)\) are as they are in the world of use, \(u\).

The difference between quantificational COLL and quantificational WOLL has to do with the strength of the trait that the first participant possesses. Because the traits are usually stated about a generic first participant, it is hard to think of past readings, but they work with extinct species:

229. A pterodactyl could peck you.

230. A pleiosaur would attack you in the water.

9.2.3 Circumstantial COLL

Circumstantial uses of COLL are understood to be uses in which there is a set of circumstances allowing something to occur. An example that is discussed in Kratzer 1981[45] is that of having the physical capacity to play an instrument, but being stranded on an island without that instrument. In such a case, a person has the capacity to play the instrument, but the circumstances disallow it. The main difference between circumstantial and capacity readings is the type of accessibility relation that the second event has placed on it by the modal. The accessible worlds are those in which circumstances are considered.

Uses of Paradigm A past could are common in narratives. The example from earlier is repeated below:

231. In the 1930s, my grandfather could cultivate a hectare of maize for $50.

The reading means that the author’s grandfather lived in a time when circumstances around him allowed the cultivation of a hectare for $50. It is true of a case when the circumstances were that way whether or not he ever did cultivate a hectare for $50.

The past tense places the time of the intension of the grandfather-cultivate-hectare event inside of the reference time of COLL, which is strictly before the speech time, given the meaning of the past tense.
Figure 9.4: Attribute value matrix for circumstantial COLL

\[
\begin{array}{l}
\text{word} \\
\text{PHON: ORTHO: [COLL]} \\
| \text{verb} \\
\text{SYN: HEAD: CAT : Bse} \\
| \text{FORM: bse} \\
\text{MODE: ref} \\
\text{TYPE: } < i < i, t >, < i < i, t >, < i << v, t >, t > \\
\text{INDEX: } v_1 \\
\text{SEM:} \\
\text{TRANSL: } \lambda P \lambda Q \lambda t, \lambda f_{< v, t >}, \exists v \exists t_x \\
[\text{might}(v) \land f(v) \land (\tau(v) = t_x) \land P(t_i, t_x) \land Q(t_i, time_{n,o})] \\
\text{RELN: circumstantial} \\
\text{EVENT: } v_1 \\
\text{ARG: } PLEON \lor STRUCT_i \\
\text{CIRCUMSTANCES: } v_2 m \text{ where } v_2 \text{ is of type } < v, t >, t >, \\
\text{which contains an anaphoric element } ana_i, \text{ and} \\
\text{where the accessibility relation, } R_{circumst} \text{ is such that} \\
R_{circumst}(w, w') \text{ iff } w' \text{ is a world where the circumstances are} \\
as they are in the actual world.
\end{array}
\]
9.2.4 Actuality Inferences in Sentences with Dynamic Paradigm A Modal Auxiliary Verbs

Actuality inferences are associated with dynamic Paradigm A modal auxiliary verbs. The section on ability *COLL* discussed that actuality inferences take place when the world satisfying the existential quantifier is the actual world. Actuality inferences are easier to draw in cases in which the leap from the capacity to do the event is very close to actuality doing the event. Therefore uses like in sentence (232) have strong actuality inferences because the leap from having the capacity to see the mountains from one’s window to actually seeing the mountains from one’s window, barring inability to see or a very dirty window, is a small conceptual leap.

232. *Jess can see the mountains from her window.*

Examples of sensory verbs are used to illustrate actuality inference primarily because isolated sentences, not knowing anything about the fictional character *Jess* lead readers to assume she has more common visual capacities. However, in uses, it might not be the case that actuality inferences are stronger in sensory verbs than in other cases.

Given a public figure, whose capacities are at least approximately known by many readers, the actuality inference in sentence (233) is at least as strong as that of a sensory verb:

233. *Michael Jordan can jump 48 inches.*

It would be odd to say sentence (233) if Michael Jordan had never been known to jump so high in reality.\(^1\)

The question has arisen (e.g., Hacquard 2006[34]) what function the modal auxiliary has if it is merely stating something that did happen. In the present account, the role of the modal auxiliary in such cases has two functions. First, the modal auxiliary describes an event of a capacity and, in doing so, is not semantically empty outside of quantification over worlds. Second, the aspectual properties of the modal auxiliary add meaning that cannot be expressed as well with other present tense uses. The modal auxiliary is stative and stretches the jumping event, in sentence (233) more than a single satisfying jump-48-inches event is relevant, it also implies that several such events happened over the interval that is the reference time of the modal auxiliary.

\(^1\)It is not likely that Michael Jordan can still jump 48 inches at the time at which this is being written, however, the use of *could* has stronger actuality inferences and is also confusable with a hypothetical use of *could*. 

236
The options for stretching a telic event to iterate over time without a modal auxiliary present include the progressive and the simple present, neither of which make the same statement:

234. *Michael Jordan is jumping 48 inches.*


The progressive can be read to mean jumping 48 inches again and again in an interval including the speech time or it can be read to mean he is squatting down ready to jump (pre-state reading). The ambiguity is undesirable and the gapping in the iterative reading seems less strongly implied.

The simple present conveys the iteration and the span including speech time, however, it appears imply that every time he jumps he jumps 48 inches.

It is only the use with *can jump 48 inches* that least ambiguously conveys iterative jumping 48 inches while leaving it open that jumping less than 48 inches is also a possibility.²

The other dynamic modal auxiliaries are similar to ability *COLL* in that the accessibility relations that satisfy them are related to the way that the first participant is in the actual world. They involves the traits and dispositions of the first participant.

With circumstantial *COLL*, there are still actuality inferences with pleonastic subject noun phrases:

236. *There can be issues with the software.*

In such uses, there are probably known issues with the software and the phrasing is obfuscating the fact.

9.2.5 Summary of Dynamic Paradigm A Modal Auxiliary Verbs

Dynamic Paradigm A modal auxiliary verbs are all control verbs except for cases of circumstantial *COLL*, which behaves more as a raising verb.

²Professor Tania Ionin points out that the notion that the 48-inch jump is his maximum is probably due to the general way in which numerical modifiers are interpreted more than the modal auxiliary meaning. It can be seen, however, that the inference that it is his infrequent maximum is only expressed with ability *can*. The simple present infers frequent jumping at that height while also having the inference that it is his maximum.
Actuality inferences are strong in dynamic modal auxiliaries because they involve accessibility relations that operate on the capacities and traits of individuals or things being as they are in the actual world. They contribute a unique iterativity of the event in their scope such that it is seen as possibly repeated and as the defining or crowning case of the capacity.

Dynamic Paradigm A modals account for uses of *could* and *would* to describe iterated events in the past. The actuality inferences with *would* as a past habitual use of *WOLL* pass many of the tests for actuality entailments in that they cannot be denied:

237. *When I was a child, I would catch chinchillas for dinner with my bare hands, *(but I never actually caught one.)*

The inferences seem equally strong with *could* as a past ability:

238. *When I was a child, I could eat chinchillas raw, *(but I never actually did.)*

The past tense appears to strengthen the actuality inferences to the point that they are entailments.

Dynamic modal auxiliaries have been recognized as a distinct category from priority modal auxiliaries and epistemic modal auxiliaries (Portner 2009[80]). Lassiter 2011[53] presents a novel approach to the semantics of epistemic and deontic modal auxiliaries and notes that the approach does not extend to dynamic modal auxiliaries. Nuyts 2005[71] claims that dynamic modal auxiliaries have the meaning of *quantificational aspect*. Although a formal definition of quantificational aspect is not given, the author is referring to the fact that the readings contain a capacity over a span that differs aspectually from both the simple present and the present progressive form. Bhatt 1999[8] and Hacquard 2006[34] both discuss dynamic modals in terms of their actuality inferences. Hacquard 2006[34] defends the use of quantificational possible world semantics even in uses in French that have actuality entailments.

In this work, the data in English are treated as ranging over possible worlds, but constraining the accessibility relation based on the capacities, traits, or dispositions of the first participant. Because all readings but the circumstantial readings involve control, the first participant is interpreted extensionally with respect to the modal auxiliary. The first participant of the modal auxiliary is co-indexed with the anaphoric element marking the first participant of the verb that marks the second event. The anaphoric element and the rest of that verbs arguments are interpreted intensionally. The strength of actuality
inferences are due to the intuitive similarity between the actual world, in which the first participant has a capacity and the closest world in which the first participant’s capacities are as they are in the actual world. Often, the actual world is the world in which the proposition in the scope of the modal auxiliary holds.

The quantificational aspect to which Nuyts 2005[71] refers is represented by the interaction between the modal auxiliary and the event in its scope. The temporal and aspectual interactions between modal auxiliaries and the events in their scope are determined lexically by the modal auxiliary. With dynamic modal auxiliaries, the modal auxiliary expresses a state that holds in the actual world. The state is durative. The reference time of the event in the scope of the modal contains the state that is referenced. Being in a position in which tense is not present, it has its reference time set relative to the reference time of the modal auxiliary. Generally, the containment of a capacity state within the event stativizes the containing event by stretching it. The progressive has three prominent interpretations. On one interpretation, the progressive refers to a portion of a typical (telic) event that does not include a culmination point. One another interpretation, a typical event is iterated over a state. On yet another interpretation, the pre-state of a typical event is what is referenced. The use of dynamic modal auxiliaries favors the interpretation in which an event is iterated over a time span.

9.3 Priority Readings

Priority Readings involve the goals and priorities of an individual or community. Paradigm A has priority readings in the form of volitional readings of WOLL permission readings of COLL and may. Paradigm B has priority readings in the form of should and must.

All of the priority readings involve control, but only the Paradigm B readings allow readings that involve assessment sensitivity.

9.3.1 Volitional WOLL

The volitional uses describe an intent or forced action of the first participant. The uses are hard to distinguish from uses of will that are considered to be future, but they often come across most clearly in cases in which there is a first person subject and where the statement is read as if it is a promise or commitment. In sentence (239) below, the
volitional reading is one in which the speaker is stating a self-imposed commitment:

239. *I will eat a chinchilla.*

There are also volitional uses in which the speaker is placing an obligation on the addressee:

240. *You will eat that chinchilla I cooked for you.*

In sentence (240) above, the volition is that of the speaker and it is placed on the addressee, having overtones of forcing the addressee participate in the second event.

The coercive reading could have a label other than volitional, but it is included here as both place an obligation on the first participant, which is in the SUBJ-NP position, to be the first participant of the second event.

The reading is one in which the first participant is an argument of the modal and the event is situated by the tense of the modal auxiliary:

The truth conditions for the sentence *Jess will eat a chinchilla* are given below

\[ \phi_{u,w}^w = \text{truth} \text{ if for all } w' \in W, \text{ such that } R_{\text{volite}}u, w',
\[
[ \exists v_2 \exists z_2 [\text{bite}(v_2) \wedge A0(v_2) = r(4) \wedge (\tau(v_2) = z_2) \wedge (z_2 \circ_i r(2)) \wedge (r(1) \subseteq_i r(2))] ]
\]

in \( w' \)

and \( \exists r(\exists v_1 \exists z_1 [\text{WOLL}(v_1) \wedge A0(v_1) = r(4) \wedge A1(v_1) = [\exists v_2 \exists z_2 [\text{eat}(v_2) \wedge A0(v_2) = r(4) \wedge A1(v_2) = r(5) \wedge (\tau(v_2) = z_2) \wedge (z_2 \circ_i r(2)) \wedge (r(1) \subseteq_i r(2))] ](w')] ]
\]

(\( \tau(v_1) = z_1 \) \wedge (r(1) \circ_i z_1) \wedge (\text{time}_{u,\phi} \subseteq_i r(1))))]

Where \( \text{Ref}_{u,\phi} \), and \( \text{Ref}_{u,\phi}(5) \) is a chinchilla at time \( r(2) \) in \( u \) and (1) and (2) are times, and where \( R_{\text{volite}}u, w' \) holds of any \( w \in W \) such that the desires and compulsions of \( r(4) \) at time \( r(2) \) are as they are in the world of use, \( u \).

9.3.2 Permission *COLL* and *may*

Permission uses of *COLL* are a special case in that they involve granting of permission at the speech time. The reading of *Jess can swim* as a permission, means that someone is allowing Jess to swim.

241. *Jess can swim.*
volitional WOLL

<table>
<thead>
<tr>
<th>word</th>
</tr>
</thead>
<tbody>
<tr>
<td>PHON: ORTHO: [WOLL]</td>
</tr>
<tr>
<td>SYN:</td>
</tr>
<tr>
<td>HEAD: CAT: Bse</td>
</tr>
<tr>
<td>FORM: bse</td>
</tr>
<tr>
<td>MODE: ref</td>
</tr>
<tr>
<td>TYPE: (&lt; i &lt; i, t &gt;&gt;, &lt; i &lt; i, t &gt;&gt;, &lt; i &lt;&lt; v, t &gt;, t &gt;&gt;)</td>
</tr>
<tr>
<td>INDEX: (v_1)</td>
</tr>
<tr>
<td>SEM:</td>
</tr>
<tr>
<td>TRANSL: (\lambda P \lambda Q \lambda t_1 \lambda f_{sv,t} \exists v \exists t_z)</td>
</tr>
<tr>
<td>([WOLL'(v) \land f(v) \land (\tau(v) = t_z) \land P(t_1, t_z) \land Q(t_1, t_j)])</td>
</tr>
<tr>
<td>RELN: volition</td>
</tr>
<tr>
<td>EVENT: (v_1)</td>
</tr>
<tr>
<td>VOLITION-OF: (i), where (i) is the index on an expression of type (&lt; i &lt; i, t &gt;&gt;, &lt; i &lt; i, t &gt;&gt;, &lt; i &lt;&lt; v, t &gt;, t &gt;&gt;),</td>
</tr>
<tr>
<td>RESTR: (&lt; i &lt; i, t &gt;&gt;, &lt; i &lt; i, t &gt;&gt;, &lt; i &lt;&lt; v, t &gt;, t &gt;&gt;)</td>
</tr>
<tr>
<td>VOLITED: (\Box v_2) where (v_2) is the index of an expression of type (&lt; v, t &gt; t &gt;), which contains an anaphoric expression (ana); and, where the accessibility relations (R_{volite}) is such that (R_{volite}(w, w')) iff (w') is a world where the desires and compulsions of the individual or group referred to by (i) are as they are in (w).</td>
</tr>
</tbody>
</table>

Figure 9.5: Attribute value matrix for volitional WOLL
permission COLL

word

PHON: ORTHO: [COLL]

SYN: HEAD: CAT: Bse

FORM: bse

[verb]

MODE: ref

TYPE: \(<i < i, t >>, <i < i, t >>, <i << v, t >, t >>\)

INDEX: \(v_1\)

TRANSL: \(\lambda P \lambda Q \lambda t_i \lambda f_{<v,t>} .\exists v \exists t_c\)

\([COLL'(v) \land f(v) \land (\tau(v) = t_c) \land P(t_i, t_c) \land Q(t_i, t_j)]\)

RELN: permission

EVENT: \(v_1\)

PERMITTED-FOR: \(i\), where \(i\) is the index on an expression of type \(<i < i, t >>, <i < i, t >>, <i << v, t >, t >>,\)

RESTR: \(<i < i, t >>, <i < i, t >>, <i << v, t >, t >>,\)

PERMITTED: \(\diamond v_2\) where \(v_2\) is the index on an expression of type \(<< v, t > t >\), which contains an anaphoric expression \(ana\); and \(,\) where the accessibility relations \(R_{permission}\) is such that \(R_{permission}(w, w')\) iff \(w'\) is a world where the individual or group referred to by \(i\) has permission to \(v_2\).

Figure 9.6: Attribute value matrix for permission COLL

Following the format of other Paradigm A forms, it is the case that Jess is allowed to swim at a time not strictly preceding the speech time. The reference time of the intension of the ‘Jess-swim’ event is situated within the reference time of the permission.

The results are intuitive and its representation in the grammar involves a straightforward extension of the other forms of COLL with an accessibility relation that limits the interpretation to those worlds in which Jess has permission to swim.

242. Jess can swim.

The truth conditions for the sentence Jess\(_4\) can\(_1\) swim\(_2\), where \(\phi\) represents the closed sentence, are given below:

\[ [\phi]^{u,w} = \text{truth} \text{ if for all } w' \in W, \text{ such that } R_{\text{trail}} t_u, w', \]
\[ \exists v_2 \exists z_2 [swim(v_2) \wedge A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ r(2)) \land (r(1) \subseteq r(2))] \]
in \( w' \)
and \( \exists r(\exists v_1 \exists z_1 [COLL(v_1) \land A0(v_1) = r(4) \land A1(v_1) = [\exists v_2 \exists z_2 [swim(v_2) \wedge A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ r(2)) \land (r(1) \subseteq r(2)))](w')) \]
\( (\tau(v_1) = z_1) \land (r(1) \circ z_1) \land (time_{u,\phi} \subseteq r(1))) \]
Where \( Ref_{u,\phi}, \) and \( Ref_{u,\phi}(4) \) is named Jess in \( u \) and (1) and (2) are times, and where \( R_{\text{permission}}u, w' \) holds in the closest \( w \in W \) such that \( r(4) \) has permission at \( time_{u,\phi} \) to swim.

The granter of permission is often the speaker, but permission can be reported as well without specifying who originally granted the permission.

Permission readings of \( COLL \) have past forms, and, by the grammar written, those past forms are predicted to mean that the state of having permission ended before the speech time.

**243. Back in the day, you could cross the border with no hassle.**

The semantics constrain the intension of the satisfying event to be ones that occur in that span. There could still be permission in the present, but that is not the span of time being referred to by the use with \( could. \)

Permission uses of \( may \) are not, in my dialect, distinguishable from those with \( COLL. \)

Permission uses of \( COLL \) and dynamic uses differ in their aspectual interpretation. If the sentence, ‘Jess can swim in the competition lanes.’ is understood as a permission reading, then the most likely reading is one in which there is a particular swimming event scenario under consideration in which Jess is allowed to swim in the competition lanes. Permission readings can mean that, from the time of permission forward, the event repeats. For example, that Jess is permitted to swim in the competition lanes any time she uses the pool or that Jess is permitted to swim in the competition lanes every time she comes to class.

In contrast, on the ability reading, swimming in the competition lanes is something that Jess is able to do, regardless of a particular event.

When something is read as telic and one-time, the permission reading is more accessible than the ability reading.

---

3It does not even occur in my dialect since elementary school forced use of it. It is included here because in the belief that some people actually use it.
It has been noted that, regardless of one’s capacities to do something, if, due to permission or circumstances they cannot do them, there is an inference that they are also not able, as our capacities are not solely independent of external factors (Kratzer 1981[45]).

9.3.3 Tense and Assessment Sensitivity in Priority Modal Auxiliaries

The data with epistemic might and ability COLL invite a hypothesis that lack of tense on a modal leads to assessment sensitivity.

It can be said that the presence of tense on a modal auxiliary verb is incompatible with assessment sensitivity, but, so far, it is only the epistemic reading that has been shown to be assessment sensitive. Paradigm B also includes priority modals:

244. Jess shouldn’t have gone there.

MacFarlane 2013[57] argued for assessment sensitivity of certain goal-oriented modal auxiliaries such as ought to in a controlled context.

The main point MacFarlane 2013[57] is using to test assessment-sensitivity is the ability of a person to change the truth value they assess a proposition to have, given information involved or additional perspectives. However, the notion of assessment-sensitivity defined in the relativist model only requires that the truth value be possible to change based on parameter settings.

The examples that MacFarlane[57] gave of assessment sensitive priority modals were cases in which people have a shared goal and various perspectives on the facts of the world. Given the goal and the facts of the world, there is a change in the truth value given to a priority statement.

The scenario is one in which Jess is driving to work and Jess, Pat, and Syd all want Jess to arrive at work as quickly as possible. Syd is riding with Jess. At an intersection, Jess and Syd have the first-person perspective of the driver and passenger of the car and Pat has a perspective from a surveillance camera above the intersection. The question arises whether Pat should run a red light or not. From Syd’s perspective, she should not run the light because she does not have full visibility of the risks, and a crash would make her later for work than stopping for the light would. Syd might think:

245. Jess should not run the red light.

However, from Pat’s perspective on the surveillance camera, it is clear that there is no threatening oncoming traffic for a long distance. Pat might think:
Both perspectives are valid, given the ordering of priorities and the epistemic knowledge available to the person who could assert them.

The question regarding assessment sensitivity is whether or not, upon arrival at work, when Syd communicates with Pat and Pat provides the information she had, would Syd’s assessment of his earlier statement result in a different truth value?

The problems presented do not seem to differ significantly from those posed by epistemic modals, they merely involve a goal that forms a priority ordering.

It is useful to draw a distinction between priority modal uses that involve shared goals and those that involve laws, the goals of which might not be shared and that might not change due to information growth.

Discussing shared ethics and laws can easily get into non-linguistic questions regarding whether or not there is some fact of the matter regarding what is ethical. Questions such as these are far beyond the scope of this treatment. It will be assumed that, whether or not there is some objective fact of the matter regarding the ethicality of a decision, the truth value of sentences that rely on a shared goal or shared ethics can vary based on the knowledge state of the assessor.

There is another way in which disagreement can occur, and that is disagreement with the ethical standard that is presupposed. In a case where an office requires a dress code, an e-mail comes around saying:

247. From now on, employees should wear business casual clothing to work every day.

An office member who disagrees with the idea can respond:

248. No we should not.

To mean that she disagrees with the rule and could argue that point and convince the group as a whole to change the policy. This type of disagreement constitutes a disagreement regarding what the priority ordering is and amounts to a personal perspective on what is and is not a priority.

It stands in contrast to a case such as the traffic light scenario discussed above in which the goal is agreed upon but the means of achieving it differ.

The kind of relativity discussed by MacFarlane 2013[57] was one in which the goals or ethics are not being debated, but the assessors can assign different truth values based
to a sentence depending on what they know about how to reach the goal or follow the ethics.

The only priority modals that disallow an assessment-sensitive readings are permission \textit{COLL} and volitional \textit{WOLL}:

\textbf{249. You can’t kill an albino skunk in the USA.}

This is the only phrasing that means that the laws prevent it in an objective way that does not depend on the assessor. The ethics behind the laws can be argued, but, given a law against killing albino skunks, the sentence is true, regardless who the assessor is.

Uses of Paradigm B modal auxiliaries \textit{should} and \textit{must} present a recommendation or obligation that invites some individual assessment:

\textbf{250. You shouldn’t kill an albino skunk in the USA.}

\textbf{251. You mustn’t kill an albino skunk in the USA.}

They are recommendations based on the consequences and shared acceptance of the consequences as un-desirable rather than merely repeating a law. They are also based on knowledge update.

Take for example the use of radon to kill ringworm. In the 1930’s in the USA, some doctors considered using radon to treat the skin fungus ringworm in children was a good idea. They later realized that the benefits were not worth the longterm consequences. Given the knowledge of the consequences of radon exposure, other methods are used.

Knowing what is known in the present, the truth value of sentence (252) would remain true, for example, if medical boards still allowed the use of radon to kill ringworm:

\textbf{252. You can still treat ringworm with radon.}

In contrast, many people would have different truth assessments for sentences (253) and (254) below:

\textbf{253. You should still treat ringworm with radon.}

\textbf{254. You must still treat ringworm with radon.}

Furthermore, even at the time of use, there might have been multiple opinions in the medical community about sentences (253) and (254) above, regardless of how doctors felt about its use, there was permission to do it, so sentence (252) above was true, regardless of the assessor.
Permission readings are Paradigm A readings that also change their truth value in the sense that permission has a pragmatically restricted endpoint. On a permission reading, sentence (252) above is false. But this does not make it assessment sensitive. It only changes the span of the permission. It is possible to say sentence (255) below:

255. *Doctors could treat ringworm with radon.* *(permission reading, e.g., ‘were allowed to’)*

In contrast, in the present, very few people would agree with the past tense forms of sentences (253) and (254) above:

256. *Doctors should have treated ringworm with radon.*

257. *Doctors must have treated ringworm with radon.* *(teleological reading, e.g., ‘In order to treat patients ethically’)*

Rather than knowledge states alone, deontic Paradigm B readings involve an additional parameter of goals and priorities of agents. The goals and priorities change over time, leading to assessment sensitivity. For a doctor who believes that ringworm in the present is more problematic than causing a child to lose all of her hair and get Parkinsonism in the present and get bone cancer 50 years in the future, apparently, the truth value is different. Arguments to the contrary involves discussions of ethics regarding whether or not community ethics override that of an individual doctor, that, although they are important questions, involve a much more complex definition of *truth* and *falsity* than the semantics of other natural language expressions are being required to fulfill.

These cases indicate that Paradigm B priority modal auxiliaries *should* and *must* can be either assessment sensitive or receive a non-relativist treatment. In contrast, Paradigm A priority modal auxiliaries only have a non-relativist reading.

### 9.3.4 Control Paradigm B Modal Auxiliaries

Priority readings of *must* and *should* are the only Paradigm B modal auxiliaries that are control verbs.

They have a semantics like that of control verbs, but the first participant enters into the requirement placed on them.

*Jess should swim:*
should
word

PHON: ORTHO: should

SYN:

HEAD: CAT: Bse

FORM: bse

MODE: ref

TYPE: \(<i < i,i,t >>, <i << i,i,t >>, <i << i,v,t >>, t >>\)

INDEX: \(v_1\)

TRANSL: \(\lambda P \lambda Q \lambda t_1 \lambda f_{<v,t>} \exists v \exists t_2 (should'(v) \land f(v) \land (\tau(v) = t_2) \land P(t_1, t_2) \land Q(t_1, time_{u, \alpha}))\)

RELN: deontic

EVENT: \(v_1\)

ARG: \(A0_i\)

ARG: \(\circ v_2 m\) where \(v_2\) is of type \(<v, t >, t >,\)

which contains an anaphoric element \(ana_i,\) and

where the accessibility relation, \(R_{\text{priority}}\) is such that

\(R_{\text{priority}}(w, w')\) iff \(w'\) is a world that

is deontically accessible to \(w.\)

Figure 9.7: Attribute value matrix for should
The truth conditions for the deontic reading of *should* in the sentence, ‘*Jess should swim.*’, are stated for the top node $\phi$:

\[
\left[\phi\right]_{u, w, p(\langle x, t, a \rangle)} = \text{truth} \quad \text{if for at least one } w' \in W, \text{ such that } R_{\text{epistem}} u, w', \\
[\exists v_2 \exists z_2 [\text{swim}(v_2) \land \\
A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_t r(2)) \land (\text{time}_{u, t} \subseteq_t r(2))] (w(p)) \text{ in } w' \text{ given } p,
\]
and $\exists r(\exists v_1 \exists z_1 [\text{should}(v_1) \land A0(v_1) = r(4) \land \\
A1(v_1) = [\exists v_2 \exists z_2 [\text{swim}(v_2) \land \\
A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_t r(2)) \land (\text{time}_{u, t} \subseteq_t r(2))] (w(p))]) (w(p)) \text{ in } w' \text{ given } p$

*Jess must swim:*

The truth conditions for the priority reading of *must* in the sentence, ‘*Jess must swim.*’, are stated for the top node $\phi$:

\[
\left[\phi\right]_{u, w, p(\langle x, t, a \rangle)} = \text{truth} \quad \text{if for at least one } w' \in W, \text{ such that } R_{\text{priority}} u, w', \\
[\exists v_2 \exists z_2 [\text{swim}(v_2) \land \\
A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_t r(2)) \land (\text{time}_{u, t} \subseteq_t r(2))] (w(p)) \text{ in } w' \text{ given } p,
\]
and $\exists r(\exists v_1 \exists z_1 [\text{must}(v_1) \land A0(v_1) = r(4) \land \\
A1(v_1) = [\exists v_2 \exists z_2 [\text{swim}(v_2) \land \\
A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_t r(2)) \land (\text{time}_{u, t} \subseteq_t r(2))] (w(p))]) (w(p)) \text{ in } w' \text{ given } p$

Where $Ref_{u, \phi, t}$, and
$Ref_{u, \phi, 4}$ is named *Jess in* $u$ and (1) and (2) are times, and
where $r(\text{time}_{u, t})$ is the time of utterance, and
where $R_{\text{priority}} w', a$ holds of any $w' \in W$
such that $w'$ is a deontically accessible world, and there is no $w''$ such that
$w'' < w'$ and $[\exists v_2 \exists z_2 [\text{swim}(v_2) \land \\
A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_t r(2)) \land (\text{time}_{u, t} \subseteq_t r(2))] (w(p)) \text{ in } w' \text{ given } p$.

249
must
word

PHON: ORTHO: must

SYN:           [verb
               HEAD: CAT: Bse
               FORM: bse]

MODE: ref

TYPE: < i < i, t >, < i < i, t >, < i << v, t >, t >>

INDEX: v₁

SEM: TRANSL: λP.λQλt₁.λf<ι,τ> .∃v∀t₂
 [must(v) ∧ f(v) ∧ (τ(v) = t₂) ∧ P(t₁,t₂) ∧ Q(t₁, time_uₐ)]

EVENT: v₁

RESTR: ARG: A₀₈

ARG: ∃v₂m where v₂ is of type << v, t >, t >,
which contains an anaphoric element anaᵢ, and
where the accessibility relation, R_priority is such that
R_priority(w, w’) iff w’ is a deontically accessible world.

Figure 9.8: Attribute value matrix for must
such that \( w' \) is deontically accessible, and there is no \( w'' \) such that \( w'' < w' \) and 
\[
\exists v_2 \exists z_2 \left( \text{swim}(v_2) \land A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ \tau r(2)) \land (\text{time}_{u,\xi} \subseteq r(2))) \right)(w)(p) \] in \( w'' \) given \( p \).

In the cases in which the reading is not relativist, the parameter \( p \) is insignificant and can be left out of the truth values.

Events in the Past with Paradigm B Priority Modals

Priority readings with \textit{must} have interesting results when they are about an event that precedes the reference time. When used with the perfect form:

\textbf{258. Jess must have run a mile at least once before she can enter the program.}

The reading that results is one in which there is a dependency between the clause with the modal auxiliary and another piece of information that relate to each other in a timeless way.

There is not a specific reference time relative to the speech time at which \textit{must} run a mile, rather, running a mile is a stipulation on entering the program. The reference time, therefore, is tied to the capacity to enter the program event and not the speech time.

\subsection{9.3.5 Summary and Conclusions Regarding Priority Modal Auxiliaries}

Priority modal auxiliaries are represented both in Paradigm A and in Paradigm B. While Paradigm A readings are not assessment sensitive, Paradigm B readings can have assessment sensitive readings.

Paradigm A modal auxiliaries take tense on the modal auxiliary directly and they do not have assessment sensitive readings. Paradigm B modal auxiliaries, in contrast, do not have tense on the modal auxiliary and allow assessment sensitive readings.

Paradigm A has priority readings including permission and volition. Paradigm B has readings of obligations and requirements.
9.4 Paradigm B Raising Modal Auxiliary Verbs: Speculative and Epistemic Readings

The epistemic readings beyond might differ very little in their representations. Uses of must have more modal force. Those with may are, in my dialect, indistinguishable from those with might in many uses.

Speculative readings are like epistemic readings, but either the Aktionsart of the second event or another temporal expression situates them in such a way that they strictly follow the time of utterance.

Jess may swim:

The truth conditions for the epistemic reading of may in the sentence, ‘Jess may swim’, are stated for the top node φ:

\[ [φ]_{w, w'. \qa{\langle k, t, a \rangle}} = \text{truth} \text{ if for at least one } w' \in W, \text{ such that } R_{\text{epistem}}u, w', \]
\[\exists v_2 \exists z_2 [\text{swim}(v_2) \land \\
A_0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_i r(2)) \land (\text{time}_{u,k} \subseteq_i r(2))](w)(p)\] in \(w'\)
given \(p\),
and \(\exists r(\exists v_1 \exists z_1 [\text{may}(v_1) \land \text{STRUCT}(v_1) = r(4) \land \\
A_1(v_1) = [\exists v_2 \exists z_2 [\text{swim}(v_2) \land \\
A_0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_i r(2)) \land (\text{time}_{u,k} \subseteq_i r(2))](w)(p)\] \(\tau(v_1) = z_1) \land (r(1) \circ_i z_1) \land (\text{time}_{u,k} \subseteq_i r(1)))\]
Where \(\text{Ref}_{u,\phi \land r}\), and
\(\text{Ref}_{u,\phi}(4)\) is named \(\text{Jess}\) in \(u\) and (1) and (2) are times, and
where \(r(\text{time}_{u,k})\) is the time of utterance, and
where \(\text{R}_{\text{epistem}}w', a\) holds of any \(w' \in W\)
such that \(w'\) is compatible with the knowledge \(j\) has
in world \(a\) at time \(t\).

Where \(< w', p' >\) is a world-perspective pair such that \(w' \in W\) is accessible from \(u\),
with the accessibility relation being constrained in such a way that only those worlds
that are compatible with the judge’s knowledge state are accessible and the perspective
is set to that of the author at the time of speech in the world of speech for assertion
conditions and to whoever is the judge at whatever time they are judging where truth
conditions are concerned.

The truth conditions for the epistemic reading of \textit{must} in the sentence, ‘\textit{Jess must swim.’}, are stated for the top node \(\phi\):

\[
\left[\phi\right]_{u,w,p(<x,t,a>)} = \text{truth} \text{ if for all } w' \in W, \text{ such that } \text{R}_{\text{epistem}}u,w', \]
\[\exists v_2 \exists z_2 [\text{swim}(v_2) \land \\
A_0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_i r(2)) \land (\text{time}_{u,k} \subseteq_i r(2))](w)(p)\] in \(w'\)
given \(p\),
and \(\exists r(\exists v_1 \exists z_1 [\text{must}(v_1) \land \text{STRUCT}(v_1) = r(4) \land \\
A_1(v_1) = [\exists v_2 \exists z_2 [\text{swim}(v_2) \land \\
A_0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_i r(2)) \land (\text{time}_{u,k} \subseteq_i r(2))](w)(p)\] \(\tau(v_1) = z_1) \land (r(1) \circ_i z_1) \land (\text{time}_{u,k} \subseteq_i r(1)))\]
Where \(\text{Ref}_{u,\phi \land r}\), and
\(\text{Ref}_{u,\phi}(4)\) is named \(\text{Jess}\) in \(u\) and (1) and (2) are times, and
where \(r(\text{time}_{u,k})\) is the time of utterance, and
where \(\text{R}_{\text{epistem}}w', a\) holds of any \(w' \in W\)
must

word

PHON: ORTHO: must

SYN:

[HEAD: CAT : Bse ]

FORM: bse

MODE: ref

TYPE: < i < i, t >, < i < i, t >, < i < < v, t >, t >

INDEX: v_1

TRANSL: \( \lambda P \lambda Q \lambda t_1 . \lambda f_{<v,t>} . \exists v \exists t_\epsilon \)

[must'(v) \land f(v) \land (\tau(v) = t_\epsilon) \land P(t_1, t_\epsilon) \land Q(t_1, time_{\alpha, \alpha})]

RELN: epistemic

EVENT: v_1

ARG: PLEON \lor STRUCT_i

ARG: \( ^{\alpha}v_2m \) where \( v_2 \) is of type \( << v, t >, t >, \) which contains an anaphoric element \( ana_i \), and where the accessibility relation, \( R_{epistem} \) is such that \( R_{epistem}(w, w') \) iff \( w' \) is a world that is epistemically accessible to the judge.

Figure 9.10: Attribute value matrix for must
such that \( w' \) is compatible with the knowledge \( j \) has in world \( a \) at time \( t \).

Where \( < w', p' > \) is a world-perspective pair such that \( w' \in W \) is accessible from \( u \), with the accessibility relation being constrained in such a way that only those worlds that are compatible with the judge’s knowledge state are accessible and the perspective is set to that of the author at the time of speech in the world of speech for assertion conditions and to whoever is the judge at whatever time they are judging where truth conditions are concerned.

The differences between forms are minor and all variations in are the accessibility relations.

*Jess could swim:*

The truth conditions for the epistemic reading of *could* in the sentence, ‘*Jess could swim,*’ are stated for the top node \( \phi \):
[\phi]_{u,w,p(\langle x,t,a,r \rangle)} = \text{truth} \text{ if for at least one } w' \in W, \text{ such that } R_{\text{epistem}}u,w',

[\exists v_2 \exists z_2[\text{swim}(v_2) \land

A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_t r(2)) \land (time_{u,\phi} \subseteq_t r(2))(w)(p)] \text{ in } w'
given p,

and } \exists r(\exists v_1 \exists z_1[\text{might}(v_1) \land \text{STRUCT}(v_1) = r(4) \land

A1(v_1) = [\exists v_2 \exists z_2[\text{swim}(v_2) \land

A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_t r(2)) \land (time_{u,\phi} \subseteq_t r(2))(w)(p)]

(\tau(v_1) = z_1) \land (r(1) \circ_t z_1) \land (time_{u,\phi} \subseteq_t r(1))]

\text{Where } Ref_{u,\phi|a,r}, \text{ and }

Ref_{u,\phi}(4) \text{ is named } Jess \text{ in } u \text{ and (1) and (2) are times, and where } r(time_{u,\phi}) \text{ is the time of utterance, and }

\text{where } R_{\text{epistem}}w', a \text{ holds of any } w' \in W

\text{such that } w' \text{ is compatible with the knowledge } j \text{ has}
in world } a \text{ at time } t.

\text{Jess should swim:}

The truth conditions for the epistemic reading of should in the sentence, ‘Jess should swim.’, are stated for the top node } \phi:

[\phi]_{u,w,p(\langle x,t,a,r \rangle)} = \text{truth} \text{ if for at least one } w' \in W, \text{ such that } R_{\text{epistem}}u,w',

[\exists v_2 \exists z_2[\text{swim}(v_2) \land

A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_t r(2)) \land (time_{u,\phi} \subseteq_t r(2))(w)(p)] \text{ in } w'
given p,

and } \exists r(\exists v_1 \exists z_1[\text{should}(v_1) \land \text{STRUCT}(v_1) = r(4) \land

A1(v_1) = [\exists v_2 \exists z_2[\text{swim}(v_2) \land

A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_t r(2)) \land (time_{u,\phi} \subseteq_t r(2))(w)(p)]

(\tau(v_1) = z_1) \land (r(1) \circ_t z_1) \land (time_{u,\phi} \subseteq_t r(1))]

\text{Where } Ref_{u,\phi|a,r}, \text{ and }

Ref_{u,\phi}(4) \text{ is named } Jess \text{ in } u \text{ and (1) and (2) are times, and where } r(time_{u,\phi}) \text{ is the time of utterance, and }

\text{where } R_{\text{epistem}}w', a \text{ holds of any } w' \in W

\text{such that } w' \text{ is compatible with the knowledge } j \text{ has}
in world } a \text{ at time } t.

The difference in meaning among epistemic modal auxiliary uses is very slight. Epistemic might and may have the same semantics. Epistemic must involves a stronger
should
word
PHON: ORTHO: should

SYN:  
  HEAD:  
  CAT:  
  FORM: 

MODE: ref
TYPE: <i < i, t >>, < i < i, t >>, < i << i, t >>, t >>
INDEX: v

SEM:
  EVENT: v
  ARG: PLEON \lor STRUCT
  ARG: \diamond v_2 m where v_2 is of type << v, t >>, t >, which contains an anaphoric element ana, and where the accessibility relation, R_{epistem} is such that R_{epistem}(w, w') iff w' is a world that is epistemically accessible to the judge.

Figure 9.12: Attribute value matrix for should
modality than *might* and *may*. Epistemic uses of *should* and *could* have lexical connotations of obligation and possibility, respectively. The connotations, however, are not part of the truth conditions.

Epistemic modal auxiliaries have been shown to have assessment sensitive readings. There are cases, however, in which the situating context prevents assessment sensitivity. One that was previously discussed was the use of a statement such as *‘At the time, I thought that Jess might have swum.’* The use of *I thought* sets the judge to the speaker and time to a particular setting.

These readings differ from other uses in that the perfect aspect situates the modal event, \( v_1 \) to precede the speech time.

9.5 ‘Future’ WOLL

Future uses of *WOLL* are generally considered either to be tense marking or to be a modal use that displays tense.

The decision in this work is to treat future *WOLL* as a modal auxiliary that places particular constraints on the location of the reference time of the embedded event within the modal auxiliary’s reference span.

The meaning of the auxiliary *WOLL* is one of anticipating a particular change of state or continuation of a state. That state is described by the second event.

259. *The event will begin.*

Future *WOLL* has an additional property that other forms do not. That is, it passes the tests to be a subject raising verb rather than a control verb.

Future *WOLL* allows semantically empty subject noun phrases:

260. *There will be rain tomorrow.*

When passivized, future *WOLL*, conveys the same relation between arguments:

261. *Jess will find Syd.*

262. *Syd will be found by Jess.*

These data suggest that the first participant in a sentence with future *WOLL* does not have an agent role with respect to the modal auxiliary.
Future WOLL presents a unique case. It has the subject raising properties that, so far, have been associated with Paradigm B modal auxiliaries. However, it has tense marking on the modal auxiliary as it can describe a future event with the non-past marking and a future in the past event with the past marking.

The truth conditions for the future reading of WOLL in the sentence, ‘Jess will swim.‘, are stated for the top node φ:

\[
[φ]_{u,w,p(\langle x,t,\rangle)} = \text{truth} \quad \text{if for all } w' \in W, \text{ such that } R_{future}u, w', \\
\exists v_2 \exists z_2[\text{swim}(v_2) \land \\
A0(v_2) = r(4) \land (τ(v_2) = z_2) \land (z_2 \circ_t r(2)) \land (time_{u,ξ} \subseteq_t r(2)))](w)(p)] \text{ in } w' \\
given p, \\
\text{and } \exists r(\exists v_1 \exists z_1[WOLL(v_1) \land STRUCT(v_1) = r(4) \land \\
A1(v_1) = [\exists v_2 \exists z_2[\text{swim}(v_2) \land \\
A0(v_2) = r(4) \land (τ(v_2) = z_2) \land (z_2 \circ_t r(2)) \land (time_{u,ξ} \subseteq_t r(2)))](w)(p)]]
\]
(τ(v₁) = z₁) ∧ (r(1) ⊔₁ z₁) ∧ (timeₜₕ ⊓₀₁ r(1)))]

Where Refₜₕᵢᵣ, and

Refₜₕᵢᵣ(4) is named Jess in u and (1) and (2) are times, and

where r(timeₜₕᵢᵣ) is the time of utterance, and

where Rfuture(w, w') iff w' is a world that

w' is a world such that what is the case in w is the case in w'

This monograph takes the view that will is not epistemic in this use, but indicates
that the speaker is certain of which world he is in. Since we assume a model with
truth values to all propositions, epistemic modals are considered in terms of confusion
regarding which world we are in. Uses of will are therefore either true or false at the
time of utterance in the world of evaluation, but they can involve a world-of-evaluation
error.

263. Jess will win first place in the race tomorrow. (future)

Future WOLL is not assessment-sensitive in this treatment, however, it requires a
larger model than ‘up to speech time’ for truth evaluation.

Future in the past uses, as in sentence (264) below involve a setting of the reference
time before the speech time through the past tense on the modal auxiliary WOLL:

264. Jess would win the first place in the race. (future in the past)

The event, v₂ is placed in the future relative to the reference time of the modal auxili-
ary via the lexical properties of how the second verb relates to the modal auxiliary:

The truth conditions for the future reading of WOLL in the sentence, ‘Jess will swim.’,
are stated for the top node φ:

\[ [φ]_{u,w,p(\langle,x,t,x\rangle)} = \text{truth} \text{ if for all } w' \in W, \text{ such that } Rfuture u, w', \]

\[ [∃v₂∃z₂[swim(v₂)∧
A0(v₂) = r(4) ∧ (τ(v₂) = z₂) ∧ (z₂ ⊔₁ r(2)) ∧ (timeₜₕᵢᵣ ⊓₀₁ r(2)))](w)(p)] \text{ in } w' \]
given p,

and \[∃r(∃v₁∃z₁[WOLL(v₁) ∧ STRUCT(v₁) = r(4)∧
A1(v₁) = [∃v₂∃z₂[swim(v₂)∧
A0(v₂) = r(4) ∧ (τ(v₂) = z₂) ∧ (z₂ ⊔₁ r(2)) ∧ (timeₜₕᵢᵣ ⊓₀₁ r(2)))](w)(p)] \]
(τ(v₁) = z₁) ∧ (r(1) ⊔₁ z₁) ∧ (timeₜₕᵢᵣ ⊓₀₁ r(1)))\]
Where \( \text{Ref}_{u\beta|t} \), and
\( \text{Ref}_{u\beta}(4) \) is named \textit{Jess} in \( u \) and (1) and (2) are times, and
where \( r(\text{time}_{u,c}) \) is the time of utterance, and
where \( \text{Ref}_{\text{future}}(w, w') \) iff \( w' \) is a world that
in which the proposition with which \( v_1 \) is associated is supported at time \( t_2 \).

The treatment of future in the past does not require that the event, \( v_2 \) has taken place already at speech time. However, if it has, then there are actuality entailments.

9.5.1 Perfect Aspect with \textit{will}

Generally, Paradigm A modal auxiliaries have tense on the modal auxiliary and no grammatical aspect on the second main verb. The major exception is the use of future perfect as in sentence (265) below:

\textbf{265.} The badger will have bitten the chinchilla (by the time we get back home).

The predicted meaning is that there is an expectation at the speech time that there is a change of state from the badger not being bitten by the chinchilla to being bit by the chinchilla. The reference time of the state of expectation contains the reference time of the chinchilla being bitten event.

According to the theory in place, the perfect aspect merely places the event of the chinchilla being bit before the reference time of the chinchilla being bit event. As the reference time of the expectation is a durative state, there is no complication in it containing both.

The truth conditions for the future reading of \( \textit{WOLL} \) in the sentence, ‘\textit{Jess will have swum.}’ are stated for the top node \( \phi \):

\[
[\phi]_{u,w,p(<x,t,w>)}^{u,w,p(<x,t,w>)} = \text{truth} \text{ if for all } w' \in W, \text{ such that } \text{Ref}_{\text{future}}u, w', \\
[\exists v_2 \exists z_2[\text{swim}(v_2) \land \\
A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_t r(2)) \land \text{time}_{u,\xi} \subseteq_t r(2)])(w)(p)] \text{ in } w' \text{ given } p,
\]

and \( \exists r(\exists v_1 \exists z_1[\text{WOLL}(v_1) \land \text{STRUCT}(v_1) = r(4) \land \\
A1(v_1) = [\exists v_2 \exists z_2[\text{swim}(v_2) \land \\
A0(v_2) = r(4) \land (\tau(v_2) = z_2) \land (z_2 \circ_t r(2)) \land \text{time}_{u,\xi} \subseteq_t r(2)])(w)(p)] \land \\
(\tau(v_1) = z_1) \land (r(1) \circ_t z_1) \land \text{time}_{u,\xi} \subseteq_t r(1)))] \]
Where \( \text{Ref}_{u \beta \epsilon t} \), and
\( \text{Ref}_{u \beta}(4) \) is named \( 	ext{Jess} \) in \( u \) and (1) and (2) are times, and
where \( r(\text{time}_{u,d}) \) is the time of utterance, and
where \( R_{\text{future}}(w, w') \) iff \( w' \) is a world that
in which the proposition with which \( v_1 \) is associated is supported at time \( t_2 \).

The lexical meaning of the future places the reference time of the \( v_2 \) event strictly after the reference time of the modal auxiliary event \( v_1 \). An additional interval is created between the event \( v_2 \) and the reference time of the event ensuring that the endpoint of \( v_2 \) is before the reference time of the modal auxiliary, as predicted by the perfect aspect.

9.5.2 Summary and Conclusions Regarding Future \( 	ext{WOLL} \)

Future uses of \( \text{WOLL} \) are treated as a modal auxiliary that has lexically specified a relation between the modal auxiliary event and the event \( v_2 \) such that \( v_2 \) is strictly preceded by an indexical time that can be the speech time (in simple future) or can be a contextually specified time, as in future in the past uses. The use of future perfect creates an additional interval between the speech time and the reference time of the modal auxiliary by the strict precedence relation that the perfect aspect contributes to the verb in \( v_2 \).

9.6 Perfect Aspect and Modal Auxiliary Verbs

It has been claimed that perfect aspect combines with modal auxiliary verbs in a compositional way. If the combination is compositional, then perfect aspect is predicted to behave in basically the same way, whether it occurs with or without a modal auxiliary verb. It should still situate the event time before the reference time. It should allow all possible readings: Universal readings, resultative readings, experiential readings, and hot-news readings. Of course, not every sentence allows every reading. Even when modal auxiliaries are not present, there are sentential factors and world knowledge that influence the reading (see Reed 2012[81], for example). What needs to be shown is that there exist linguistic contexts that support the uses supporting each reading when modal auxiliary verbs are present.
Data can be found to show that the readings of perfects are all attested with modal auxiliaries (the following examples are based on those in Reed 2012[81]):

266. Jess might have met Cher. (might have had the experience)
267. Jess might have spilled her coffee. (surmised from a stain)
268. Jess might have lived in Arizona ever since 2005.

If the use of might is read as epistemic, it yields each of the readings with the addition of uncertainty.

Teleological uses also display various perfect readings:

269. In order to apply to the program, you must have met with the department chair.
270. In order to apply to the program, you must have submitted your documentation (by February 1st).
271. In order to apply to the program, you must have been living in the state since 2005.

The perfect should also have current relevance, and they seem indistinguishable from sentences with the perfect that do not contain modal auxiliaries, in this respect.

The present perfect should be incompatible with past tense adverbials. This is complicated in the case of the modalized examples because the difference between past and present perfect is not encoded in the perfect form of to have that occurs with modals. It is always a base form have. The tests allow the perfect forms to take past adverbials:

272. Jess could have eaten yesterday.

What is expected with respect to tense is that the perfect behaves differently with present tense modal auxiliaries. In Paradigm A, it should not take the past adverbials because the modal is non-past:

273. *Jess will have eaten yesterday.
274. Jess will not have eaten yesterday.
275. (?) Jess can’t have eaten yesterday.
276. *Yesterday, Jess can’t have eaten something.
277. Yesterday, Jess might have eaten.
This bears out, to the extent that there are examples. As noted, the perfect does not occur with past tense Paradigm A modal auxiliaries. The reason is semantic in that it would mean that instances of an ability held in the past of the time span of the ability, or that the event enabled by circumstances held before the span in which the circumstances held.

Paradigm B modal auxiliaries are in a tense-taking position, but they do not take tense marking.

The event time of an assessment-sensitive Paradigm B modal, I have argued, is the current time for the judge. The uncertainty (in epistemic modals), is the uncertainty that the judge holds at the time of assessment. The reference time of the modal, then, is always current.

Within that currency, the second event can be either strictly before that currency, or not strictly before it. Strictly before is expressed by the perfect. Therefore, when someone says on Monday, ‘Jess could be in France.’ and learns on Thursday that she is in town, they might still be thinking the same thought (the same proposition), but they would express it as ‘Jess could have been in France.’

The modal event of my uncertainty continues, but the event of the second verb, that is, the event of which the speaker was uncertain, must be appropriately situated before assessment or utterance. The reference time of the internal event, however, is situated to strictly follow that event when the perfect marking is present.

Paradigm B modal auxiliaries complicate the scenario because another event and reference time are layered into the sentence. In epistemic readings, the modal event time is equal to the modal reference time. The times are set to the assessment time and always include the other event in the time. In fact, it is this inclusion that is often characterized as the ‘scope of the modal’.

When the second event is in the perfect, the event time still precedes the reference time, but both spans are in the scope of the modal event (of uncertainty, in epistemic modals). The meaning of the modal perfect is present perfect. It places the event being referenced strictly before the reference time.

The present tense however, is not set by tense because there is not tense marking available to the Paradigm B modal perfect at all. The reference time of the second event is set either indexically (by the intentions of the speaker) or it quantifies over a pragmatically constrained span. The utterance time is an artifact of the utterance.

Something resembling a tense relation, then, is derivable through the pragmatic refer-
ence time and the act of utterance. The perfect indicates that the event under discussion is before the reference time, which is itself anchored to speech.

The event time is always specific, however, the communication of that time is determined by the reference time. If the reference time of the second event is indexical, the past adverbial is felicitous.

Consider a case such that it is under consideration whether or not Jess rode a horse to a particular demonstration at a particular time:

278. Jess might have ridden a horse to the protest (yesterday, on September 18th).

In contrast, the case in which the experiential reading is considered (whether Jess has ever had the experience of riding a horse or not) is not so:

279. Jess might have ridden a horse (in the past, before, *yesterday, *on September 18th)

When the second event is non-perfect, the utterance time is construed to be in the reference time, and the reference time is considered to contain the event time. This is encoded as default present-tense-like relation and default non-perfect-like relation.

The modal event must, however, contain the second event. Because the time of assessment is open-ended into the future, the assessor’s perspective on the second event changes. The utterance time is an artifact that allows the assessor to situate the event.

When the second event is non-perfect, the assessor forms a tense relation via the artifact of the utterance time and the time of their assessment. If the assessor were to restate the utterance at assessment time, they would situate the second event strictly before the reference time (using perfect). However, this involves a content versus character distinction as the utterance time is indexical.

The distinction has led to debate regarding the truth conditions of epistemic modal auxiliaries and whether or not someone can use the perfect to situate their epistemic perspective in the past relative to the repetition of the statement.

With a situating temporal context, such as ‘at the time, I thought that Jess might have ridden a horse.’, the epistemic perspective is situated in the past, however, it is done via the embedding clause. Judgments vary on whether merely saying, ‘Jess MIGHT have ridden a horse.’ is possible to read as a statement of the speaker’s past epistemic perspective. I believe it is not. I believe it is read as metaphysical (not epistemic) in that context.
When the original utterance was in the perfect, the assessment and the speech time are both in the result state of the (hypothetical) event. Therefore, there is no difference specified grammatically between the original epistemic utterance and the epistemic utterance characterizing the assessor’s perspective. But one can be very far in the future.

9.7 Subjunctive Modal Auxiliaries

There are other multi-clausal expressions that are common with modal auxiliaries. The most discussed one is that of counterfactual conditionals. It was mentioned in the background chapters that Lewis 1973[54] presented an account of counterfactual conditionals that represented important parts of their semantic interpretation while choosing to simplify their compositional semantics to a single operator that included the subjunctive mood of the antecedent clause, the modal auxiliary, and perfect or non-perfect verbs occurring with the modal auxiliary.

Given what has been presented so far, the present theory makes some predictions about the compositional semantics of counterfactuals.

There has not been an account proposed so far for anything like a conditional in general nor for the subjunctive mood.

Let us start with some relatively un-controversial assumptions. Let us assume that if introduces an intensional context such that the clause it occurs in receives truth conditions relative to worlds.

The nature of the subjunctive mood in the antecedent clause is part of what will be discussed. It is sometimes characterized as taking a past and not a present form, even if the thing it hypothetically alters is something that, if altered, would hold at the speech time and not strictly before it. In this work, the morphology is not considered to be past but to be subjunctive.

280. *If I were a man, I might get into a study group.*

That has a different meaning than the sentence with the present tense in the antecedent:

281. *If I am a man, I might get into a study group.*

The modal auxiliary that occurs in the consequent clause has two possible forms: One in which the verb that follows it is non-perfect and one in which the verb that follows it is in the perfect.
The options in terms of temporal and aspectual morphology for the Paradigm B modal auxiliaries that occur in hypothetical and counterfactual sentences is no different than in other clauses.

They may or may not have the semantic interpretation that they have in other places. If they do have the same semantic interpretation, then the non-perfect form describes an event that contains another event. The reference time of the second event is contained in the reference time of the first one. That reference time does not strictly precede the speech time.

The interpretation yields the correct results with respect to hypothetical conditionals:

282. If I were a man, then I might get into a study group.

The reading of *might* is generally one that is epistemic, but, in this case, it is not clear that an epistemic reading is what is desired. Whatever *might* means here, let us assume it is a state of *might* that is not strictly before $time_{ua}$, which is set to the speech time. The event of getting into a study group is contained in the span of the reference time of *might* and, consequently, is also not strictly before the speech time.

The reading of *might* is better read as a potential. This is a reading that has not been discussed so far but that some authors, such as Coates 1983 [16] have recognized as distinct from epistemic readings.

The boundaries between potential readings and epistemic readings is hard to define, but that difficulty is not surprising because it reduces to a difference in what kinds of scenarios are pragmatically accessible.

If future uses of *will* are considered at one end of a spectrum of knowledge-based speculation and and past epistemic uses of *might* at the other, potential readings can be seen as someplace in between these two readings.

Potential readings speculate about something that is not known to be the case. Sometimes they could be described as epistemic modals about the future. They make a statement relative to the speaker’s knowledge state about something that is not known because it is situated in the future of the time of speech.

Uses of *could* give some of the clearest examples:

283. The civil unrest could lead to increased violence by the state.

The use of *could* does not necessarily state a limitation of the speaker’s knowledge state, but rather, reports a scenario of which the speaker has knowledge but of which the future outcome involves randomness.
The reading could be viewed as circumstantial, but circumstantial alone does not fully characterize it either because the circumstance of civil unrest do not allow violence by the state. It could be argued that civil unrest produces an environment that is conducive to state violence, but there is an element of the unknown conveyed that is not as present in circumstantial readings.

In the case of would, the reading is hypothetical (when read as non-past):

284. *Civil unrest would lead to increased violence by the state.*

It is demonstrated that potential readings are one reading that occurs in hypothetical and counterfactual statements.

Potential readings encapsulate an event that is non-past from the perspective of the information supporting the assertion.

<table>
<thead>
<tr>
<th></th>
<th>Non-Past</th>
<th>Past</th>
<th>Non-PastPlup</th>
<th>Non-PastSubjn</th>
<th>Non-PastPerfSubjn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pdgm A</td>
<td>will</td>
<td>would</td>
<td>will have</td>
<td>would</td>
<td>would have</td>
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<tr>
<td></td>
<td>can</td>
<td>could</td>
<td>(can’t have)</td>
<td>could</td>
<td>could have</td>
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<tr>
<td></td>
<td>may</td>
<td>–</td>
<td>(may not have)</td>
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<td>–</td>
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<tr>
<td></td>
<td>(shall)</td>
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<td>–</td>
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</tr>
<tr>
<td>Pdgm B</td>
<td>could</td>
<td>–</td>
<td>could have</td>
<td>could</td>
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<td>may</td>
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<td>should</td>
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<td>should have</td>
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<td></td>
<td>must</td>
<td>–</td>
<td>must have</td>
<td>must</td>
<td>must have</td>
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</tbody>
</table>

Table 9.5: Morphological Forms of Paradigm A and Paradigm B Modal Auxiliaries in Various Contexts

### 9.8 Grammar of Conditional Clauses with Modal Auxiliary Verbs

Counterfactual conditionals are a structure in which a modal auxiliary verb occurs that is often differentiated grammatically from other forms in English.

The antecedent clause contains the past perfect form of the main verb and the consequent clause contains a modal auxiliary verb followed by the past participle form of the main verb.
If kangaroos had no tails, they would tip over. (from Lewis 1973[54])

The use of the past perfect in the antecedent clause is generally considered to be a subjunctive use of the past perfect. It has interesting behavior when the copula is used:

If kangaroos were to have no tails, they would tip over. (from Lewis 1973[54])

Counterfactual conditionals are quite easy to identify in English due to their grammatical components.

Not every language has the same type of grammatical form for counterfactual conditionals. Some languages mark more information grammatically in counterfactuals than English and some mark less. This is not a survey of how counterfactuals are encoded in languages of the world, but it is worth noting that their grammatical components, beyond the basic conditional form, differ cross-linguistically so, when constructing a compositional semantic analysis, much of the analysis will be specific to English. Iatridou 2000[38], discussed below, provides some cross-linguistic evidence for the generalizations she characterizes and also shows what appear to be consistent patterns in how past tense and subjunctive mood interact in several related languages and posits that it might be a universal. Such research is an ongoing area of study.

9.8.1 Lewis’ Counterfactuals

Lewis 1973[54] mentions the grammatical components of counterfactuals briefly, but then notes that he is setting the grammatical issues aside. From that point on, he standardly changes the antecedent clause from one in the subjunctive to simple tense and the consequent clause from one with a modal auxiliary verb and a past participle form to one in simple tense. Lewis works entirely with past counterfactuals.

\[ \phi \Box \rightarrow \psi \]

Removing the modal auxiliary verb from the proposition representing the consequent in the semantic representation is common. The modal is treated as a modal logic operator that quantifies over accessible worlds, checking the truth value of the consequent according to whatever semantics an author has proposed. Much like semantic treatments of determiners and quantifiers, modal auxiliary verbs have a meaning interpretation that has a strong logical component. Lewis demonstrates that representing modal auxiliary verbs as modal logic operators that take a proposition in their scope leads to interesting results in modeling truth conditions.
The grammatical details of the propositions posited to be in the scope of the modal operator are not fully developed. For the sake of the semantic analysis, Lewis 1973[54] ignores the temporal and aspectual components, associating them with the $\square\rightarrow$ operator.

Many questions remain about the form of the antecedent and consequent, if a theory such as Lewis’s 1973 theory is to be added to a grammar. Given that the past perfect is in the antecedent of a counterfactual conditional, what role does temporal placement of the event being described play in determining which worlds are $\phi$-worlds? Presumably, there is a salient time-frame in which the event in the antecedent proposition is hypothesized to take place. If someone says, *If Jess had been rich, she would not have lost her house*, where the interlocutors know that she lost her house in 1985, the truth value is not determined by how rich Jess is at the speech time or how rich she was in 1963. So, the temporal placement of the antecedent plays a role in the evaluation of a counterfactual conditional. The temporal and aspectual morphology are part of the meaning of the proposition that has to be modeled in a grammar of counterfactuals.

Lewis’s 1973[54] theory states the truth values as being such that the counterfactual conditional is true if, and only if, all the $\phi$-worlds in the system of spheres most similar to the world of evaluation are also $\psi$-worlds. The temporal relation of the antecedent to the consequent matters in this case as well. When talking about $\phi$-worlds and $\psi$-worlds, in the context of counterfactual meaning, it is the case that the antecedent occurs temporally before the consequent. Counterfactual conditionals differ from epistemic conditionals with the same temporal and aspectual morphology in this way. In epistemic conditionals, the consequent can temporally precede the antecedent (but it does not have to).

287. *If Jess had been rich, she would not have lost her house.* (counterfactual)

288. *If Jess had been rich, she would have to have won the lottery.* (epistemic)

These are some questions that are dealt with in the subsequent literature reviewed in the next subsections.

It is not being suggested that Lewis 1973[54] intended truth values to be determined without temporal properties, rather, it is being demonstrated that, in order to have a compositional analysis of counterfactual conditionals in a grammar, it is highly desirable to systematically encode the temporal contributions of the temporal and aspectual morphology of the sentences that occur as the antecedent and consequent.
9.8.2 Iatridou on Counterfactuals

Iatridou 2000[38] presents a grammatical account of counterfactuals. Much of her data is in Greek, which is different than English, but it provides some insights due to the presence of more grammatical morphology. Iatridou proposes answers to some of the questions that Lewis’s 1973[54] proposal did not seek to explain. Her proposed solution involves treating the past tense as contributing an Exclusion Feature. The exclusion feature can range over times or worlds and has a different interpretation in each case. When ranging over times, the Exclusion Feature excludes the speech time from the reference time. When ranging over worlds, the Exclusion Features excludes the actual world of the speaker from the set of worlds under consideration.

The details of Iatridou’s 2000[38] analysis that are relevant to this study are presented below.

Iatridou’s work contains hypothetical conditionals and counterfactual conditionals, as shown below:

289. If he were smart, he would be rich. (Iatridou 2000:232[38])

290. If he had been smart, he would have been rich. (Iatridou 2000:232[38])

Iatridou calls counterfactual conditionals past counterfactuals and hypothetical conditionals present counterfactuals when the Aktionsart of the antecedent event is stative or future less vivid when the Aktionsart of the antecedent is telic. Iatridou also accounts for embeddings under the verb wish.

Iatridou claims that counterfactuality of the sentences is a conversational implicature (2000:232[38]) due to previous literature showing that the antecedent can be denied without resulting in a contradiction and the fact that the statement of the falsehood of the same proposition does not result in redundancy.

She also claims that the conversational implicature extends to the consequent being taken as false, but I do not share this intuition, primarily because it seems possible to take the consequent of a hypothetical to be in the future relative to the speech time.

291. If Jess were smart, she would be rich.

It seems more accurate to say we do not know if she is rich or if she will be in the future. The consequent holds in worlds in which the antecedent is true, but it may hold in other worlds besides. Intuitively, if the person had won the lottery and was rich, it
should not affect the truth conditions of the sentence. In Lewis’ 1973[54] model it does not. The truth conditions are such that, in the world most similar to the actual world in which she is smart, she is also rich. The truth conditions do not restrict the possibility that is more like the actual world (or even is the actual world) in which she is not smart, but she is rich.

There do seem to be assertion conditions in English such that, if someone knows that Jess is already rich, sentence (291) is odd. According to Iatridou (2000:233[38]), Greek has a morphological distinction present in the antecedent that distinguishes these two scenarios. In the case that implies that the speaker knows the state of Jess’s finances, it is not possible to deny the antecedent.

Iatridou 2000[38] refers to the past perfect form as pluperfect. Either name is used with frequency for the form.

Iatridou corroborates, using English and Greek data that “a pluperfect, which is commonly assumed to contain two levels of past uses one of its ‘past’ layers for [counterfactual] purposes and the other for temporal purposes.” (2000:240[38])

Iatridou’s 2000[38] Theory of Past Tense

Iatridou presents an analysis of the past perfect as having two past tenses, following Steedman 1997[91] and others. She presents a ‘skeletal’ meaning of past tense that can apply to times or worlds, depending on the context of use. The skeletal semantics of the past tense that she presents is given below:

**Definition 116.** *ExclusionFeature is that feature that, when it ranges over times, we call 'past tense'.*

\[ T(x) \text{excludes} C(x) \]

Where \( T(x) \) is *Topic(x)*, which is described as ‘the topic we are talking about.’

\( C(x) \) is described as ‘the x that, for all we know, is the x of the speaker.’

**Definition 117.** *When the Exclusion Feature ranges over times:*

\( T(t) \) is the time interval that we are talking about.

\( C(t) \) is the time interval that, for all we know, is the time of the speaker, that is, the utterance time.
The meaning is that the topic time excludes the utterance time.

It is assumed under Iatridou’s analysis that the there is not a future tense but that will is a modal auxiliary verb. Therefore, the exclusion of the speech time results in a past reference (for the speaker, apparently).

**Definition 118.** *When the Exclusion Feature ranges over worlds:*

\[ T(w) \text{ represents the set of worlds that we are talking about.} \]
\[ C(w) \text{ represents the set of worlds that, for all we know, are the worlds of the speaker, which Iatridou notes to be the actual world.} \]

The meaning is that the actual world is excluded from the topic worlds.

In a simple clause, the past tense ranges over times not including the speech time. This description resembles the typical characterization of past tense as placing the reference time before the speech time. The reference time is generally considered to be equal to the speech time.

**292. Jess was rich.**

The past tense, in Iatridou’s 2000[38] theory, is an exclusion feature that excludes the speech time from being contained in the reference time.

In contrast, the present perfect in a non-modal sentence moves the reference time to the past of the speech time while still computing truth conditions relative to the actual world.

Iatridou’s analysis is particularly intended to handle the fact that the present perfect form is used in the antecedent to mark hypothetical and does not place the event time before the speech time but rather has a meaning of being about non-actual worlds. Thus, the present perfect, in hypothetical uses, indicates exclusion of the actual world from the worlds of evaluation, without moving the reference time to the past.

The Greek future less vivid form is a hypothetical with a telic antecedent event. Typical hypotheticals, which Iatridou calls Present Counterfactuals, have stative antecedent events.

The past perfect, or pluperfect, has two instances of the exclusion features. One that places the event in the past relative to speech time and one that excludes the actual world from being the world of evaluation.
Epistemic uses have a temporal use of past tense in the antecedent because they do not exclude the actual world. They can be epistemic uncertainty regarding which world we are in.

Condoravdi 2002[19] does not describe her work as dealing with counterfactuals, but the metaphysical readings she describes include them as a strong component. Also, the pragmatic rules she states have a lot to do with whether facts can be determined at the time of utterance.

Lewis’ analysis does not attempt to treat counterfactuals in terms of a formal grammar.

Iatridou 2000[38] considers what the individual morphemes in counterfactual conditionals, hypothetical conditionals, and wish-statements contribute to the semantics. She proposes an analysis in which the pluperfect contributes two past tenses and provides two analysis of past tense: One that ranges over times and one that ranges over worlds.

The nature of an Exclusion Feature in temporal and irrealis uses has some important differences. The temporal ontology presented has an ordering of intervals, so it is helpful to state Iatridou’s feature in more standard terms of strict temporal precedence.

\[ \text{Ref}(i) < \text{Speech}(i) \]

The temporal version of the exclusion feature is merely simple past tense, since Iatridou is considering only past and present where present is equivalent to the speech time.

Iatridou is careful to say that what is labeled here as the reference time is ‘the time interval that we are talking about’ which allows for the speech time to be included in an event time, so long as the reference time strictly precedes the speech time.

293. Jess had been sick

The analysis seems to be more straightforward using standard accounts of the past tense as locating the reference time before the speech time and the perfect aspect locating the event time before the reference time.

With worlds, there is an exclusion of the actual world from the worlds being considered in the evaluation of the proposition.

First, this approach goes against the arguments made in Lewis 1973[54] stating that the actual world is not excluded from the worlds of evaluation for counterfactual conditionals, but, rather, when it does not contain the antecedent, the assessor must look
to the nearest worlds that do. When it does, the counterfactual conditional has truth conditions like a material conditional.

Iatridou supports her decision with data from Modern Greek, showing that epistemic uncertainty about the truth value of the antecedent is an assertion condition for the Modern Greek form that parallels the \textit{if} -clause of English counterfactual conditionals. When the speaker is certain of the truth value of the antecedent, a different morpheme is used and, in such cases, the use of the antecedent clause is veridical, thus, denying it results in a contradiction.

The speaker’s epistemic uncertainty regarding the truth value of the antecedent clause could be described as a mild felicity condition on asserting a counterfactual in English, although it is totally fine to use a counterfactual statement to corroborate the truth of the corresponding material conditional.

The encoding of epistemic uncertainty in the antecedent clause in Modern Greek requires its own semantic representation that interacts with the counterfactual conditional. A solution to this issue will not be presented here.

In the counterfactual conditionals, which are similar in English and Modern Greek, the actual world would be excluded from the worlds being talked about, that, formally, seems to mean that it is either inaccessible when considering truth conditions or pragmatically excluded in some rather unclearly stated way.

Second, the exclusion of the actual world, even if considered to be pragmatic, differs significantly from the exclusion of the speech time in past tense. The temporal case is not at all pragmatic but firmly holds to be the case in the meaning of the past tense morphology.

Third, there is nothing in Iatridou’s theory preventing the Exclusion Feature from twice applying to the exclusion of the actual world, and it is not clear what that would mean.

Iatridou calls the ‘fake’ past tense a use of the pluperfect morphology to indicate a pragmatic constraint that the proposition is about an irrealis scenario.

The attempt to present a unified skeleton of meaning for the past tense glosses over several things that are essential even in the skeletal meaning. First, it does not handle the fact that the exclusion of the actual world is about pragmatic assertion conditions (at best) while the temporal exclusion is about the semantics of the temporal morphology. Second, it avoids referencing the ordered structure of intervals and excludes the future from consideration so that exclusion of one thing, the speech time, can be used as an
identifying part of the semantics. Third, it is not clear what advantage the skeletal meaning has over having multiple ways of using the past and the perfect, considering the parts of its skeletal semantics that are not well fleshed-out.

Iatridou 2000[38] does, however, present an important point that is that the use of the pluperfect for irrealis meaning and for past tense is common cross-linguistically, indicating that there is some unified meaning of the morpheme.

Such reasoning has a long tradition in linguistics. Often, underlying such patterns there is a history of changes in use resulting in polysemy of the forms. How the semantics and pragmatics of morphemes and other linguistic expressions diverge and converge is a vast area of study in its own right. Too much attention to these developments in a working model of a grammar, however, can result in compromised semantic analysis for the sake of unification. For this reason, the approach taken here is to give the best working semantics possible to the component parts and then discuss what their commonalities are, as it is relevant.

In one sense, a given language user does still confuse the meanings of the pluperfect morphology, because sentences that allow epistemic and metaphysical readings are common. Going back to the data that led to Kratzer’s analysis of modal auxiliary verbs as having a context as part of their meaning, it is sometimes non-linguistic features, or at least non-sentence-level linguistic features, that determine the difference among readings.

Given these two perspectives, it is the reading in the context that determines semantic content pluperfect morphology has. The assertion conditions, assessment conditions, and truth conditions for a given reading relate to the pluperfect morphology in different ways. The skeletal structure in Iatridou 2000[38] does not significantly reduce that problem, especially if times and worlds have the expected structures defined elsewhere.

The goal has been to find grammatical components of conditionals that determine the reading of an modal auxiliary verb. The grammatical differences looked at in Iatridou 2000[38] are differences between verbal morphology in hypothetical conditionals and counterfactual conditionals. Features of the Aktionsart of the antecedent event influence the temporal properties of hypothetical conditionals. Specifically, stative antecedents are contemporary with the consequent whereas telic antecedents precede the consequent clause.

Other sentence-level linguistic features that distinguish among epistemic and counterfactual meanings, not included in Iatridou 2000[38] include the use of the modal
expression *have to have* in the consequent, which are generally associated only with epistemic readings, and the use of *if not for* (Henderson 2011[36]), which is associated only with counterfactual readings of the conditional.

The distributional properties of auxiliaries have been the topic of much research. Limiting the discussion to modal auxiliary verb, and considering the two proposed paradigms, there are several options.

Paradigm A modal auxiliaries, which have a non-past form, a past form, a semi-productive non-past perfect form and a dubious (for now) past perfect form allow the verb phrase occurring with the modal to be in a progressive form as well. Paradigm B modal auxiliaries, which have non-past forms and non-past perfect forms, can also have progressive forms. Table 9.6 shows the complete list of forms under consideration.
<table>
<thead>
<tr>
<th>Paradigm A</th>
<th>Non-Past</th>
<th>Past</th>
<th>Non-PastPerf</th>
<th>PastPerf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Progressive</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>will find</td>
<td>would find</td>
<td>will have found</td>
<td>would have found</td>
<td></td>
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<tr>
<td>can find</td>
<td>could find</td>
<td>can’t have found</td>
<td>could have found</td>
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<tr>
<td>may find</td>
<td>–</td>
<td>may not have found</td>
<td>might have found or may have found</td>
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<tr>
<td>shall find</td>
<td>–</td>
<td>–</td>
<td>should have found</td>
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<td>Progressive</td>
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<tr>
<td>will be finding</td>
<td>would be finding</td>
<td>will have been finding</td>
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<td>can be finding</td>
<td>could be finding</td>
<td>can’t have been finding</td>
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<td>may not be finding</td>
<td>might have been finding or may have been finding</td>
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<td>shall be finding</td>
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<td>should have been finding</td>
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<td>Paradigm B</td>
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<td>Non-Progressive</td>
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<td>must be finding</td>
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<td>must have been finding</td>
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Table 9.6: Combinatorics of the verbal elements of modal auxiliary clauses
9.9 Summary and Conclusions

This chapter discussed the full landscape of modal auxiliary readings in terms of the distinctions between Paradigm A and Paradigm B, between raising verbs and control verbs, and between readings that permit assessment sensitivity and readings that do not.

It was shown that many temporal and aspectual properties are accounted for in terms of the theory that was in place for treating ability COLL and epistemic might.

It was argued that assessment-sensitivity is only present with modal auxiliary verbs that do not display tense directly on the modal auxiliary. Assessment-sensitive readings are characterized by the relativization of the truth value to a perspective, which is not fixed but changes depending on who the judge is and the time of assessment.

There were shown to be several contexts that serve to situate an utterance relative to a particular perspective. Much like the use of for Jess in a sentence like ‘Peanut butter is tasty for Jess.’, in Lasersohn’s 2016[52] account of predicates of personal taste, the use of additional contextualizing elements can make a given utterance have a non-relativist reading.

The question of why the Paradigm B forms are ‘defunct’, as Abusch 1997[1] states, is accounted for structurally by the fact that Paradigm B modal auxiliary verbs do not mark past tense grammatically on the verb and, therefore, cannot be in grammatical agreement with the past tense embedding verb.

Subjunctive uses of modal auxiliaries were treated as a separate declension and can apply to most readings. Contra to Iatridou 2000[38], the past tense and subjunctive mood in English are treated as having separate meanings.
It was presented at the beginning of this monograph that modal auxiliary verbs have readings that interact with temporal and aspectual properties of the linguistic expressions in the sentences in which they occur.

A background on taxonomies of modal auxiliaries was given, followed by a review of Montague grammar and the standard modal logic accounts of modal auxiliary verb semantics.

Next the background of tense and aspect was presented before giving a formal grammar for tense, aspect, and modality.

10.1 Major Findings

The major contributions of this work include three primary areas. The first area is that of providing a particular characterization of modal auxiliary paradigms that solves a number of problems in taxonomic treatments of modal auxiliary verbs. The second area is that of providing a formal grammar and interpretation that allows the composition of tense, grammatical aspect, and modal auxiliary verbs. The third major area is the finding that assessment sensitivity only occurs in one of the paradigms, Paradigm B, and, as a corollary, assessment sensitivity and tense do not co-occur with modal auxiliary verbs in English.

10.1.1 A Unified Theory of Modal Auxiliary Meaning

It was brought up that this theory could be viewed as having a weakness over other theories of modal auxiliary verbs because it appears to make their semantics less unified. The basis for this claim comes from a comparison with Kratzer’s 1981[45] theory.
As covered in Chapter 4, Kratzer’s 1981[45] theory claimed that all readings of modal auxiliary verbs could be characterized by a conversational background, an ordering source, and the modal force that they have. In the grammar in Chapters 8 and 9, modal auxiliaries are presented with lexical entries in the grammar. The conversational background, ordering source, and modal force are part of the semantics of the lexical entry, however, they are not the entirety of the entry. The difference is especially apparent with control verbs. In the lexical entry for control verbs, the modal auxiliary selects a proposition as well as another semantic argument, which is in the syntactic location of the subject noun phrase.

The claims made in Kratzer 1977[44] and 1981[45] are indeed phrased in an attention-getting manner. Looking back on the claims, more than thirty years after they have become the standard theory of modal auxiliary semantics, it is not immediately apparent what the context was in which they were made. Based on the texts, however, it is clear that Kratzer did not at the time disbelieve that there was more to modal auxiliary meaning than what she made the focus of her work. At the time of writing the 1981 paper, the author had already been thinking intently on the topic for at least four years. Interestingly, the 1981 paper begins by setting the stage for a unified theory with a statement to the effect that there are many nuances of modal auxiliary meaning that the author herself felt to be distracting.

Nothing at all is said about the syntax, beyond stating that modal expressions come in various forms. The forms are presented in order to say that she is limiting the scope of the study to modal auxiliary verbs and modal adverbs. Although not working on the syntax, it is improbable that the author considered the idea that the notion of unification that she was proposing encompassed every component of modal auxiliaries and modal adverbs in a grammar. Such a strict interpretation of what Kratzer meant by unified entails that modal auxiliaries and modal adverbs do not even encode categorial distinctions in how they form constituents and combine in a grammar.

A slightly less strict interpretation of what Kratzer 1981[45] meant by saying the account was unified could lead to expecting that at least the semantic component of the lexical entries for modal auxiliary verbs should include nothing beyond a conversational background, an ordering source, and a modal force. On this interpretation of unified, the account presented in this monograph does indeed stand outside of such a unified theory because it involves the semantic argument of the subject noun phrase in control readings. Going back to the claims made in Kratzer 1977[44], a modal auxiliary or
modal adverb takes two arguments, a proposition and a context. The control readings take three arguments, a proposition, a context, and an individual.

Is the account in this monograph less unified because some readings take an extra argument that others do not? It depends on what the goals of unification are meant to accomplish. Kratzer’s foundational works were not dealing with syntax, they were dealing with semantics. In the semantic sense, what would disprove her notion of unification is if you could find a modal expression the semantics of which could not be characterized in terms of a conversational background, an ordering source, and modal force. The theories have undergone a number of changes and innovations, such as lumps of thought (Kratzer 1989[47]), which changed the structure of the propositions in order to solve problems with interpretation. The addition of a semantic subject outside the scope of the modal operator does not challenge the notion of unification in terms of the three central axes that Kratzer 1981[45] proposed. The information in the scope of the modal logic operator is related to the natural language semantics in this monograph in the same manner that it was in Kratzer’s 1981[45].

What could be seen to present a more serious problem is the treatment of some modals in terms of their temporal properties because temporal and event information enters into the portion of the semantic representation that is in the scope of the modal logic operator. These portions of natural language semantics, however, are treated in terms of the temporal and event ontologies and not as part of the modal logic. Kratzer 1981[45] does not present a temporal logic, so there is no prediction in the work regarding what lies or does not lie in the temporal logic surrounding uses of modal expressions.

In summary, the perspective taken in this work is that Kratzer 1981[45] was providing a unified account of how the information in the scope of the modal logic operator is to be handled in modal logic as it applies to natural language. The use of a conversational background, ordering source, and modal force for that portion of the semantics is woven into the proposed grammar. This work can be seen as building on the unified account of Kratzer 1981[45], adding a formal grammar and temporal logic, and providing a compositional treatment of their interactions. The proposal in this monograph does not undo previous work in unifying the semantics of modals, rather it integrates those discoveries into a more fine-grained notion of context.
10.2 Future Work

This work mentioned that Aktionsart and temporal adverbs play a role in situating the events in an utterance with a modal auxiliary verb. The formal treatment of these components of the meaning were not developed in the present work. This work only looked at the sentence in which the modal auxiliary verb occurs, however, the surrounding context is another factor that could work with the semantics to further limit the availability of multiple readings.

Modal adverbs are often covered in treatments of modality. In this work, they were set aside because the focus was heavily on the combinatorics of tense and aspect with modal auxiliary verbs. Modal adverbs have been used as evidence for treating modal auxiliaries on par with gradable adjectives (Lassiter 2011[53]). The semantics presented here has interesting interactions since only a few of the readings have a modal adverb counterpart. Questions that this analysis bring up include whether or not there are two events with modal adverbs and whether or not assessment sensitivity works as expected in the theory.

Experimentally, this work has stated hypotheses that open up a number of questions regarding how a reading is determined and what features influence the strength of actuality inferences.

There are several direction in SLA studies. At what point in SLA does assessment sensitivity become apparent? Does it function in the same way as with the majority of first language speakers? What are the affects of L1 on the acquisition of assessment-sensitivity? In a more directly applied sense, how can tense, aspect, and modality be taught in a more helpful way, given that the accurate acquisition is very difficult?

10.2.1 Logical Models and Probabilistic Information

The original goal of this project involved a computational model of the proposed theory, implemented in a formal grammar. Distributional information, obtained by my concurrent studies in corpus linguistics and natural language processing, was going to be used as a feature indicating the probability of a particular reading. For the sake of an academic dissertation, I was directed to propose some reconciliation of what the distributional information meant in terms of a formal model of language. After several discussions with advanced scholars, it became clear that no one has a satisfactory
answer to the question of how to use distributional information in traditional formal semantic models and that it is, rather, a very active area of research that would require a dissertation in itself.
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