Granular jets

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When a solid sphere impacts on a deep layer of granular medium, it generates an ejecta sheet and a transient axisymmetric crater. The gravity-driven radial collapse of this crater generates a pressure spike, as the cavity closes up. This pressure spike drives up a narrow granular jet along the axis of symmetry. The maximum height of the jet is found to depend on the impact velocity, gravity as well as the effective viscosity of the granular medium, through a simple product of the Reynolds and Froude numbers. The presence of such granular jets, where surface tension is absent, may help pinpoint the role of surface tension for similar liquid jets.
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Worthington jets[1] are a familiar sight in light rain upon puddles and ponds. These narrow vertical jets are formed by the radial collapse of the liquid ''craters'' produced by the impacting rain drops.[2–6] Such jets can also be generated by supercritically forcing the standing Faraday waves[7,8] on a liquid surface and have recently been cast in the formalism of physical singularities[2,3] to investigate the role of the inertial focusing and the influence of surface tension on their strength.

We have *discovered* that similar narrow jets occur even for granular materials, where surface tension is absent.

The sequence in Fig. 1 shows the generation of a granular jet resulting from the impact of a solid lead sphere onto a deep flat layer of granular medium consisting of spherical glass beads. The impacting sphere produces a deep cylindrical cavity in the sand, which subsequently collapses radially under the gravity-induced ''hydrostatic'' pressure. The sand converges axisymmetrically towards the center of the cavity. Due to the relatively small compressibility of the granular medium, the radial velocity diverges as $1/r$ when the cavity closes up. This inertial focusing[9] produces a large dynamic pressure spike driving up the sand in a narrow jet along the axis of symmetry. The granular jets are quite narrow, being comparable in shape to the energetic liquid jets. The jet in Fig. 1(d) is, however, about 40 grain-diameters wide, where the grain size clearly would provide an ultraviolet cutoff in this process, thus supporting a continuum viewpoint of the flow. The porosity of the sand medium may result in an even more pronounced inertial focusing than in the liquid case, as the gas caught on the axis of symmetry can escape between the grains.

A similar setup using the impact of a solid sphere was used by Hogrefe *et al.*[4] to generate jets in a liquid.[10] The granular jets described here should clarify the relative importance of surface tension and inertial focusing in the liquid case.

The tuning of the granular jet does not depend on the layer depth[5] as in the fluids case, however, the granular jet height is strongly dependent on the grain size of the granular medium (see Fig. 2). We find that the highest jets occur by using the finest granular medium for the same impact velocity.

We should mention that vertically vibrated thin granular layers have been shown to develop ''oscillons'' that are heaps emanating from the surface after the closing of craters,[11] but those are much less energetic than the jets studied here.

Our experiments may also give insights into the constitutive properties of flowing granular media at high shear rates. The quantity we use to characterize the strength of the jetting event is the maximum height attained by the jet H_j. The absence of surface tension makes dimensional analysis more tractable here than in the liquid case. The only other physical quantities of importance are: the sphere diameter D_b, the impact velocity U, gravity g, along with material properties: the density of the sand ρ_s, the sphere material ρ_b, and finally the effective viscosity of the granular media μ_e. Here we assume that the grain size d_s only enters the problem through the effective viscosity. The dimensional analysis shows that the jet height should follow an unknown function of only three dimensionless parameters:

$$H_j/D_b = \Phi(r_\rho, \mathrm{Re}, \mathrm{Fr}),$$

i.e., a density ratio $r_\rho = \rho_b/\rho_s$, Reynolds number $\mathrm{Re} = \rho_s U D_b/\mu_e$, and a Froude number $\mathrm{Fr} = U/\sqrt{g D_b}$. We have studied this relation by keeping r_ρ constant while varying U and μ_e independently, as shown in Fig. 3. The effective viscosity of flowing granular media remains an active topic of research[12–17] and is far from fully characterized. We use the results of Savage *et al.*[13–15] based on their shear cell experimental results and the kinetic theory of granular materials,

$$\mu_e \simeq 2\rho_s d_s^2 U/D_b.$$

In the current granular flow during the impact, the granular volume fraction is not fixed and the flow may be gov-

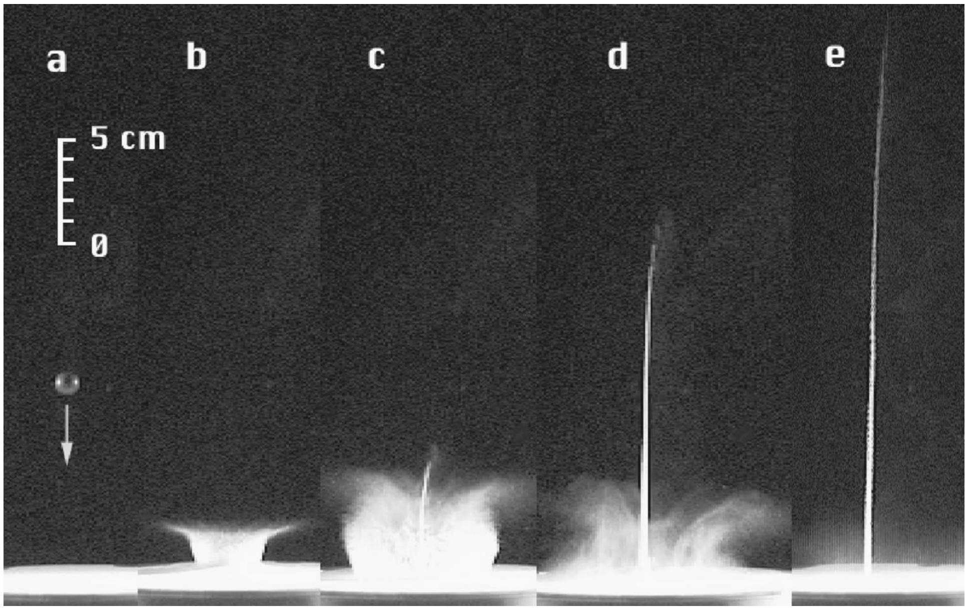

FIG. 1. Granular jet generated by the impact of a lead sphere ($U=5.5$ m/s, $D_b=1.34$ cm, $\rho_b=11.5$ g/cm^3) onto a deep layer of granular medium, consisting of spherical glass beads 0.080 mm in diameter. The medium sits in a circular glass container 12.2 cm deep and 18.8 cm in diameter. (a) $t=-18$ ms; (b) $t=25$ ms; (c) $t=92$ ms; (d) $t=160$ ms; (e) $t=260$ ms. The upward speed of the granular jet is about one-third of the sphere impact velocity.

erned more by compressive rather than shear stresses, therefore, the above model is only a rough approximation.

Furthermore, note that with the above definition of the effective granular viscosity, the Reynolds number simply reduces to a geometric factor $\mathrm{Re}=\frac{1}{2}(D_b/d_s)^2$, which is a constant for each grain size. The effective dynamic viscosity from the above formulas varies here in a range between 4 and 200 times that of water. The corresponding Reynolds numbers are between 1200 and 14 000.

Dimensional analysis does not provide the form of the function Φ. However, using the above definition we find that the normalized jet height collapses using a simple product of the Froude and Reynolds numbers, as shown in Fig. 3.

Instead of using the Reynolds number one could simply replace it by the dimensionless ratio D_b/d_s in the above dimensional analysis. This would, however, obscure the effects of frictional dissipation, which clearly plays a crucial role in this process. Ignoring friction, one can construct a simple energy balance between the kinetic energy of the impacting sphere $\propto \rho_b D_b^3 U^2$ and the potential energy of the jet column $\propto \rho_s g d_s^2 H_j^2$, where we assume the jet thickness is proportional to d_s. This produces the following scaling: $H_j/D_b \propto r_\rho \mathrm{Fr}\sqrt{\mathrm{Re}}$, which does not collapse the data in Fig. 2. In other words, were one to increase the grain diameter d_s by a factor of 10 (i.e., reduce Re by 100) for the same impact conditions, then the frictionless flow would predict a 10-fold reduction in H_j, whereas the experiments suggest a 100-fold reduction. This shows clearly that viscous forces play a role in the granular jetting, as well as gravity and inertia.

High-speed video clips showing the granular jet can be viewed at our web site *http://www.tam.uiuc.edu/Faculty/Thoroddsen/GranularJet.html*

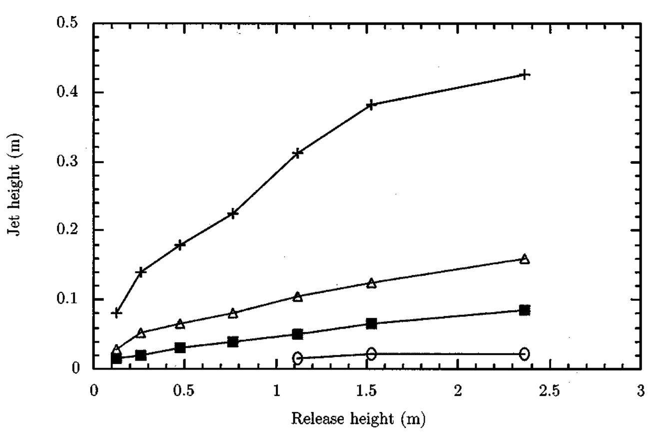

FIG. 2. The maximum height attained by the granular jet versus the release height of the lead sphere. Granular media consist of spherical glass beads with four different mean diameters: $d_s=0.080$ mm (+), 0.118 mm (△), 0.176 mm (■), and 0.275 mm (○). The glass has a density of 2.48 ± 0.05 g/cm^3. The material was poured slowly between two identical glass containers and leveled using a metallic ruler dragged over the surface. This was done before every impact, to avoid any effects of compaction, which can greatly alter the results for the finest grains.

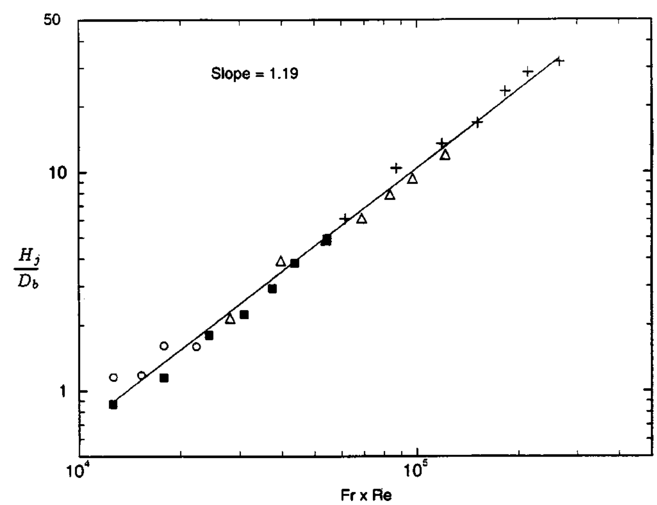

FIG. 3. The dimensionless plot of the maximum height attained by the granular jet, versus the jetting parameter Fr×Re. Granular media consist of spherical glass beads with four different mean diameters: $d_s=0.080$ mm (+), 0.118 mm (△), 0.176 mm (■), and 0.275 mm (○).

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