TWO ESSAYS IN ASSET PRICING

BY

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DISSERTATION

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in Finance
in the Graduate College of the
University of Illinois at Urbana-Champaign, 2017

Urbana, Illinois

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Abstract

The first essay, *Knowledge Capital and Innovation Efficiency Effects on Stock Returns*, provides a novel framework for understanding innovation in the asset pricing literature. Prior research shows that stock returns are increasing in firms' innovative efficiency. In a dynamic model of investment in physical and knowledge capital, this effect can arise rationally as innovative efficiency amplifies risks associated with investment and investors require compensation for these risks. I identify operating leverage and expansion option channels as the main drivers of the risk premium. Simulations of panels of firms with heterogeneous technology can reproduce the economic magnitude of the empirical return effect. The model further implies that the effect should be stronger for firms with high operating leverage and low book-to-market ratios. These predictions are supported by the data.

The second essay, *Operating Leverage, R&D Intensity, and Stock Returns*, studies interaction effects of operating leverage and R&D intensity on stock returns. A production-based asset pricing model with knowledge capital has an implication that R&D intensive firms earn higher expected stock returns among high fixed costs firms. An investment strategy that bought R&D intensive firms and sold R&D weak firms earn 0.67% to 1.26% per month in high fixed cost portfolios, while the strategy is not profitable in low fixed costs portfolios. In regression analysis, one standard deviation increase in R&D expenditures is associated with 1.85% to 2.12% increase in yearly stock returns for above median fixed costs firms. By the recursive nature of knowledge capital accumulation, the value of knowledge capital itself is sensitive to the economic situation. R&D intensive firms' values aggravate faster with fixed costs in bad times and investors require compensation for the risk. In short, the value of knowledge capital itself is risky, R&D intensive firms are more exposed to the risky nature of knowledge capital, and fixed costs amplify the risk.

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Chapter 1

Knowledge Capital and Innovation Efficiency Effects on Stock Returns

1.1 Introduction

In recent years, neo-classical investment-based models have been successful in explaining the relation between firm characteristics and expected stock returns through the firms optimal choice of the physical capital investment. Even though the physical capital is one of the most important components of firm operation, the standard investment-based models lack many other meaningful aspects of firms decision process.[1] Research and Development (R&D) is one of the essential activity for firms to increase their productivity by improving product quality, reducing production costs, or expanding the market (Hall, Mairesse, and Mohen, 2009). And it involves optimal choices on how much firms should allocate their resources. R&D is of great significance in business both at the national level and at the firm level. USA spends 432 billion dollars on R&D in 2014, which is 2.74% of the GDP[2] and firms that perform R&D spend 3.5% of their sales on R&D on average. Especially, computer and electronic products industry or pharmaceuticals and medicines industry spend up to 10.2% - 13.4% of sales in 2014.[3]

This paper investigates R&D expenditures, physical capital investment and the expected stock returns under the neo-classical framework. I propose a new framework to consider the R&D decisions and knowledge capital and deliver the recently reported empirical regularity about R&D and stock returns. By the sensitivity analysis of the model, I isolate the mechanisms that incur the effect. I also draw the implications for stock returns from the model and present evidence in the data.

[1] There had been models employing other components of production. I will discuss later.

[2] http://www.oecd.org/sti/inno/researchanddevelopmentstatisticsrds.htm

[3] https://www.nsf.gov/statistics/2016/nsf16315/nsf16315.pdf

R&D has been a subject of an empirical asset pricing literature since Chan, Lakonishok, and Sougiannis (2001) find that firms with high R&D expenditures earn higher stock returns. Li (2011) reports that the positive relationship between R&D and stock returns are stronger for firms with financial constraints. Gu (2015) shows that market competition has the interaction effect with R&D. Many studies focus on the relationship between quantities of R&D and stock returns. Instead, this paper pays attention to the quality of R&D; that is, how well firms operate R&D with the same dollar amount spent.

Hirshleifer, Hsu, and Li (2013) show that innovative efficiency, measured by patent citations over R&D expenditures or by the number of patents over the cumulative sum of R&D, predicts higher stock returns. Table 1 shows primary results of the Hirshleifer, Hsu, and Li (2013). It reports the Fama-Macbeth (1973) return regression on innovative efficiency. As innovative efficiency (IE) rises by one standard deviation, stock returns increase by 0.08% to 0.11% per month after controlling for other firm characteristics.[4] They document that mispricing plays a role in the positive relationship between innovation efficiency and stock returns. Investors do not fully understand the complex technologies of the firm. As a result, Innovation efficient firms draw limited attention, and the stock price is undervalued.

This paper constructs the model that can explain innovative efficiency effects on stock returns in the neoclassical framework. It needs not imply investor's limited attention. Agents' rational decisions on R&D and investments can drive such effect.

I extend an investment-based model by incorporating both endogenous physical capital and knowledge capital.[5] In most investment-based models, capital (physical, labor, knowledge etc.) evolves and depreciate linearly. It increases exactly as much as invested and depreciates proportionally. However, in this paper, knowledge capital has an alternative law of accumulation from the standard accumulation. I find the micro-foundation for knowledge capital accumulation function from applied microeconomics and management field. A large

[4] Hirshleifer, Hsu, and Li (2013) also perform portfolio analysis. The 66% high IE minus 33% low IE portfolio yields about 35-46 basis points of Carhart-4 factor alpha per month.

[5] There have been extensions of investment-based models by employing an additional component of the firm operation. Belo, Lin, Bazdresch (2014) consider labor and physical capital in the production function to account for the relation between hiring rate and stock returns. Lin (2012) adopts physical capital and intangible capital to explain the positive relation between stock returns and R&D expenditures. The model in this paper is different from the previous models in that it considers the quality of R&D and knowledge capital do not accumulate in the standard linear form.

body of research on R&D points out that the standard physical capital accumulation formula is not suitable for the knowledge capital build-up. It is because a standard accumulation of knowledge capital implies the negative relation between past R&D and current R&D, which is not consistent with the data. They suggest an alternative form that gets around the problem.[6] Following those critiques and suggestions, I adopt the knowledge capital law of motion as follows.

$$z_{t+1} = \mathrm{e}^{\nu_{t+1}} z_t^{\zeta_z} (1 + h_t)^{\zeta_h} \tag{1.1}$$

The next period knowledge capital z_{t+1} is determined by the current knowledge capital z_t and R&D expenditures $h_t \geq 0$, where ν_{t+1} is an i.i.d. random shock on individual firm technology and ζ_z denotes depreciation rate of knowledge capital. The key parameter in this equation is innovation efficiency ζ_h. It is the knowledge capital elasticity of R&D expenditures. High ζ_h firms can achieve more in knowledge capital in the next period with the same R&D spending than low ζ_h firms. This formulation can embody variations across firms in the quality of R&D in addition to the quantity of R&D.

The existing knowledge and current efforts on increasing knowledge are complementary in generating additional knowledge. The knowledge capital that has been already acquired helps in producing new knowledge capital. This feature differentiates the knowledge capital from the standard setup of physical capital accumulation.

The first contribution of this paper is to explore what difference the equation (2.1) makes on firms' policy decision and risk exposure. To understand the impact of the knowledge capital accumulation, I construct two simplified firms that only differ in the capital accumulation. One follows the standard linear capital accumulation and the other follows the knowledge capital accumulation form in equation (2.1). By comparing them, I find the optimal R&D expenditures increases more sharply with aggregate productivity than the physical capital investment. It is because R&D has an additional marginal benefit of enhancing the future operating environment. R&D expenditures become pro-cyclical, which is a stylized fact in the data. This characteristic makes knowledge intensive firms more sensitive to the

[6] Hall and Hayashi(1989), Klette (1996), and Hall, Mairesse, and Mohnen (2009)

economic states.

Incorporating the feature of the knowledge capital, the model can quantitatively deliver the innovation efficiency effects under the realistic model setup. Firms choose both physical capital and knowledge capital simultaneously and the model includes many aspects of operating environment such as quasi-fixed costs, time-varying risk premium[7], which are standard in investment-based asset pricing literature. The model shows that the risk loading β rises with innovation efficiency.

The paper runs simulations to investigate if the model can reproduce the economic magnitude of empirical analysis. I run Fama-Macbeth regressions of stock returns on innovation efficiency with the simulated data. The result shows economically strong enough relationship between them after controlling for firm characteristics such as size, book-to-market ratio, and profitability. It is also consistent with the existing empirical regularities, such as book-to-market effects, and size effects.

To isolate the mechanism behind the result, I simulate the model with alternative parameters. The main implication is that innovation efficiency intensifies the existing risk sources associated with the investment. The iterature has explored the risk sources of physical capital investment. Carlson, Fisher, and Giammarino (2004) suggest operating leverage (OL) effects as one of the risk determinants. Operating leverage effect is that fixed(or quasi-fixed) costs make a firm value more sensitive to the economic states since the firm still have to pay the costs even in bad times. Knowledge capital provides investment opportunities and makes the firm more prosperous in good times, but in bad times, it does not provide as many opportunities as in good times. Thus, innovation efficient firms become more vulnerable to the fixed costs.

Hackbarth and Johnson (2015) document an expansion option effect. Investment opportunities are like having real call options, firms expands (or exercise the options) when the prospect of the marginal increase in firm value is maximized over the adjustment cost. It means that firms are more likely to make an investment when firms can generate more cash

[7]Pricing kernel is given exogenously in this paper. The aggregate productivity fully determines the dynamics of the pricing kernel. Agents do not care long-run consumption growth or stochastic volatility. I do not attempt to match aggregate moments or explain the predictability of market returns in this paper.

flow with a dollar spent on the capital. Those firms should exchange riskless cash to risk asset, and their firm values become more sensitive to the economic states. Innovation efficient firms have more invest opportunities from knowledge capital, and it intensifies the expansion option effect.

I examine empirical evidence by testing the model predictions. The model suggests that innovation efficiency amplifies the operating leverage effect and expansion option effects. If the effect only comes from the investors' limited attention, one should not be able to find any variation in the IE effect depending on operating leverage or the book-to-market ratio. When portfolios are formed by double sorting on OL and IE, the IE effects are stronger in high OL firms. Likewise, double sorts on book-to-market ratio and IE shows strong IE effect in low book-to-market firms. These results corroborate the channels that the model suggests.

My paper is based on the prior investment-based asset pricing model. Cochrane (1991) uses investment returns to understand stock returns. Zhang (2005) introduces the impacts of inflexibility on stock returns. Gomes and Schumid (2010) show how simultaneous decisions on leverage and investment can affect the systematic risk of the firm. Belo, Lin, and Bazdresch (2014) investigate the impact of labor market frictions on stock returns. Hackbarth and Johnson (2015) analyze the positive relationship between profitability and systematic risk by the expansion and contraction option effects of investment.

Research on R&D is also closely related to my work as well. Griliches (1979), Hall, Mairess, and Mohnen (2009) survey huge literature on R&D and knowledge capital in economics. Hall, Griliches, and Hausman (1986), Klette (1996) attempt to measure the effects of R&D with different specifications from the cobb-douglas production function.

The outline is as follows. Section 2 analyze the effect of the alternative capital accumulation function. Section 3 discusses the full model that includes realistic aspects of firm operation. Section 4 calibrates and analyzes the solution of the model. Section 5 simulates the model and tests the simulated data in the same way as empirical papers would do. Section 6 investigates the economic intuition of the model solution and simulation. Section 7 analyzes the testable implications from the model with data. The final section concludes.

1.2 Analysis of knowledge capital

This section investigates the impact of the distinctive knowledge capital accumulation on firms' policy and risk. I construct two versions of simple models that only differ in capital accumulation function. The first model accumulates capital linearly while the second model accumulates the capital as in equation (1). They represent the physical capital firms and knowledge capital firms, respectively. The model setup for both firms is as simple as possible to contrast the effect of the accumulation function clearly. By comparing two firms, I can show the difference in policy decisions between the impact of the knowledge capital accumulation and the standard physical capital accumulation. The purpose of this analysis is simply to contrast two firms. The models lack many aspects of the realistic operating environment such as capital adjustment cost, the time-varying price of risk, external financing costs, etc. Thus, the results are qualitative.

Table 2.1 panel (A) summarizes the model setup. Aggregate productivity follows mean reverting AR (1) process. The stochastic discount factor is the simplified version of full model SDF. The SDF has the constant Sharp ratio which does not embed the time-varying risk premium. The only difference between two firms is capital accumulation function and the profit scaling constant parameter κ in the knowledge capital firm. To enable comparing two different kinds of firms, I simulate both firms and set κ to match the long-run mean market value of two firms. Thus, they have the same market value at the steady state, and this makes the policies comparable in that the difference is not originated from the firm value.

The aggregate economy calibration is set as similar as possible to the full model, which mainly follows the calibration in Li, Lividan, and Zhang (2009). Table 2.1 panel (B) shows the calibration of the simple models. As the full model solution, the problems are solved by value iteration. The aggregate productivity X_t are discretized by Rouwenhourst (1995) procedure.

1.2.1 Policy comparison

The differential accumulation of physical capital and knowledge capital results in the difference in policy decisions. Those policies have implications for firms risk dynamics. This section investigates the policy difference between physical capital firms and knowledge capital firms and shows how knowledge capital firms can be more sensitive to the economics situation.

The figure 1.1 shows the investment and R&D policies of the two firms. Panel (A) and (B) represents the investment policy of physical capital firm and R&D expenditures policy of knowledge capital firm, respectively. In a good state of an economy, both firms increase their spending on the resources. It is notable that knowledge capital firm increases its expenditures much faster than the physical capital firm. Especially, when firms are in a small area, the increasing rate difference is huge. Knowledge capital firm raises R&D expenditures explosively when the economic situation reaches a certain level.

One can see the difference more clearly in panel (C) and (D). They are two-dimensional figures that plot the spending on capital when the firms are small (both k and z are 0.38) and large (both k and z are 8.30), respectively. Knowledge-capital firm increase R&D expenditures faster than investment of physical capital firm in panel (C). In the form of knowledge accumulation in equation (1), there is an extra marginal benefit of R&D expenditures in addition to the immediate increase in operating profit. It enhances the future operating environment. The knowledge capital that has been already acquired helps to generate new knowledge capital. Thus, R&D expenditure rises sharply with aggregate productivity. Knowledge capital has a feature that could be more sensitive to the economic state.

Panel (D) shows not much difference in R&D expenditures and investment of large firms. R&D expenditures and investment are similar in panel (D). It is because the marginal benefit of existing knowledge capital decreases in size. As in equation (1), the next period knowledge capital is increasing and concave in the existing knowledge capital. Although existing knowledge capital helps in generating new knowledge, the marginal impact of knowledge capital diminishes.

The simple model analysis implies that R&D expenditures are pro-cyclical since R&D

expenditures increase sharply as economic situation gets better. It is a stylized fact in the literature that R&D expenditures are pro-cyclical. Figure 2.1 plots the weighted average of R%D expenditures growth rate in Compustat universe and real GDP growth rate. The figure updates the survey in Barlevy (2007). The figure shows that the R&D growth rate is positively covarying with the real GDP growth. R&D growth rate tends to drop in NBER recession years. The correlation coefficient between R&D growth rate and the real GDP growth rate is 0.38 while the correlation between weighted average of Compustat capital expenditures growth rate and the real GDP growth rate is 0.3.

1.2.2 Expected excess returns

In this section, I show how expected excess returns vary depending on innovation efficiency. Figure 1.3 panel (A) plots the expected excess returns of firms that differ in innovation efficiency ζ_h. It shows that expected excess returns rise with the innovation efficiency. As innovation efficiency increases, the increments in marginal benefit of R&D depending on economic states also rises. Thus, they spend more on R&D expenditures. Figure 1.3 panel (B) depicts that R&D-to-knowledge capital ratio increases with innovation efficiency ζ_h. Innovation efficient firms make riskier choices on R&D than inefficient firms. Consequently, the values of those firms become more sensitive to the economic states.

The model in this section is too simplified to explain innovation efficiency effect enough. However, this model can illustrate how innovation efficiency that is embedded in knowledge capital accumulation can affect the firms' risk and expected excess returns. The effect in this section is one source of the innovation efficiency effect, which amplifies the mechanisms that are suggested in the later section.

1.3 Model

1.3.1 Model

Following Li, Lividan, and Zhang (2009), the stochastic discount factor m_{t+1} is defined by

$$\log m_{t+1} = \log\eta + \gamma_t(x_t - x_{t+1})$$

$$\gamma_t = \gamma_0 + \gamma_1(x_t - \bar{x})$$

$0 < \eta < 1$ is time discount parameter. Parameter $\gamma_0 > 0$, and $\gamma_1 < 0$ are constants which characterize the maximum sharp ratio. It increases with γ_0. If $\gamma_1 = 0$, the economy has the constant sharp ratio. With $\gamma_1 < 0$, the maximum sharp ratio varies over time counter-cyclically. x_t is aggregate productivity process in the economy. the pricing kernel solely depend on x_t, which is exogenously given. It follows AR(1) process.

$$x_{t+1} = \bar{x} + \rho_x\ (x_t - \bar{x}_t) + \sigma_x \epsilon^x_{t+1}$$

$\bar{x}$ is long run mean of aggregate productivity. ρ_x is persistence of the aggregate productivity. σ_x is volatility of the aggregate productivity. ϵ^x_{t+1} is the aggregate productivity shock, which follows standard normal distribution.

Operating profit $\pi_{i,t}$ is

$$\pi_{i,t} = \mathrm{e}^{x_t} z_{i,t}^{\alpha_z} k_{i,t}^{\alpha_k} - f k_{i,t}$$

$k_{i,t}$ is physical capital and $z_{i,t}$ is knowledge capital. f is proportional costs to the physical capital(quasi-fixed costs), $0 < \alpha_k < 1$, and $0 < \alpha_z < 1$ are output elasticity of physical capital and knowledge capital, respectively. They govern how much output can be produced with the given amount of physical capital and knowledge capital. Physical capital $k_{i,t}$ follows

$$k_{i,t+1} = i_{i,t} + (1-\delta)k_{i,t} \tag{1.2}$$

δ is depreciation rate for physical capital. $i_{i,t}$ is investment on physical capital. Firms cannot divest their physical capital in this model. Investment, $i_{i,t}$, should be greater than zero, $i_{i,t} > 0$. There exists a physical capital adjustment cost, $c_{i,t}$. It is as follows

$$c_{i,t} = \frac{a}{2}\left(\frac{i_{i,t}}{k_{i,t}}\right)^2 k_{i,t}$$

The adjustment cost is convex on investment and decreasing in physical capital. a is constant. This is standard set-up for the adjustment cost in the literature.

This paper differs from existing investment-based models in that knowledge capital, $z_{i,t}$ is chosen endogenously. The knowledge capital determines the profitability of the firm. Firms should spend their resources on R&D to manage their profitability for every period. Recently, Kung and Schmid (2015) suggests the model that endogenizes R&D and innovation to explain aggregate returns. They adopt innovation sector that only innovates while final good sector buys the technologies from the innovation good sector. Their set up is for explaining aggregate level innovation and market returns. This paper, on the other hand, intends to describe a firm-level decision regarding innovation. I focus on how firms allocate resources on R&D, how much they get out of it, and its implication on the cross-sections of expected returns. The law of motion of knowledge capital is defined by

$$z_{i,t+1} = \mathrm{e}^{\nu_{i,t+1}} z_{i,t}^{\zeta_z} (1 + h_{i,t})^{\zeta_h} \tag{1.3}$$

$$\nu_{i,t+1} \sim \mathbf{N}(0, \sigma_\nu)$$

$\nu_{i,t+1}$ is an i.i.d. random shock on the outcome of R&D. $h_{i,t} > 0$ denotes R&D expenditures. Divestment on knowledge capital is not allowed in this model. [8] ζ_z is future knowledge capital elasticity of current knowledge capital. ζ_z can capture industry competitiveness or depreciation rate of knowledge capital. ζ_h is the knowledge capital elasticity of R&D expenditures, which is innovation efficiency in this model. This knowledge capital accumulation distinguishes the model from other multi-factor models with another component of production. The distinctive features of knowledge are detailed in the discussion section.

When firms cannot finance investment and R&D with internal capital, external financing costs are incurred. An equity issuance cost, $\lambda(e_{i,t})$ is

$$\lambda(e_{i,t}) = (\lambda_0 + \lambda_1 |e_{i,t}|) I_{\{e_{i,t} < 0\}}$$

λ_0 is fixed issuance cost,and λ_1 is variable issuance cost. $I_{\{e_{i,t}<0\}}$ is an indicator function

[8] In practice, disinvestment in knowledge capital is not impossible in some sense, for example, sales of patent or goodwill. This model does not contain those features for simplicity

that gives value one when $e_{i,t} < 0$, and gives zero otherwise.

Equity issuance, $e_{i,t}$ is

$$e_{i,t} = \pi_{i,t} - i_{i,t} - h_{i,t} - c_{i,t}$$

Distribution to shareholders, $d_{i,t}$ is

$$d_{i,t} = e_{i,t} - \lambda(e_{i,t})$$

For each period, a manger maximizes firm value $v(k_{i,t}, z_{i,t}, x_t)$ by choosing investment $(i_{i,t})$ and R&D $(h_{i,t})$.

$$v(k_{i,t}, z_{i,t}, x_t) = \max_{\{i_{i,t}, h_{i,t}\}} \{d_{i,t} + E[m_{t+1} v(k_{i,t+1}, z_{i,t+1}, x_{t+1})]\} \tag{1.4}$$

1.3.2 Discussion

This section discusses the setup of the model further in detail. This model assumes the production to be the Cobb-Douglas function. The motivation is endogenizing the profitability which is the residual value in the output functions of neo-classical models. Having Cobb-Douglas production function with knowledge capital is a natural extension of the neo-classical models.

Extending neo-classical models is not the only reason the model is set up the way it is. I find the empirical evidence about the relationship between knowledge capital and physical capital. The model implies that knowledge and physical capital are somewhat complementary. The more knowledge capital the firm has, the more investment opportunities available for firms. It naturally comes from the assumption of Cobb-Douglass function. Figure 1.5 shows that physical capital investment is high for innovation efficient firms. However, one of conventional thought is that IE efficient firms invest less in physical capital and, instead, spend much more on developing knowledge capital because their business is more focused on knowledge. One might think that knowledge capital and physical capital are kinds of substitutable. Many start-up firms in silicon valley might play a role to generate such stereotypes.

If knowledge and physical capital are complementary, the coefficients of IE should be positive. Table 2.2 presents regressions of investment on IE. One standard deviation increase in log(1+IE) is associated with about 0.15 % increase in log(1+Capx/at) with t-statistics 3.72 to 3.97. This implies that innovative efficient firms do invest more than inefficient firms, which is consistent with the model predictions.

Equation (1.3) is the crucial assumption that differentiates the model from other investment-based models. The knowledge capital accumulation is inspired by Hall and Hayashi (1989) and Klette (1996). Klette (1996) points out that it might not be realistic to apply the conventional capital accumulation (equation (1.2)) to the knowledge capital. With the standard specification, existing capital do not play a role in generating additional capital. However, the empirical analysis points out that existing knowledge gives the firm more incentives to spend on R&D (Hall, Griliches, and Hausman, 1986). On the other hand, multiplicative form as in equation (1.3) is different from the linear accumulation.

I assume $0 < \zeta_z < 1$ and $0 < \zeta_h < 1$. Under these assumptions, the knowledge capital has the following characteristics.

$$z'_z > 0, z'_h > 0, z'_{zz} < 0, z'_{hh} < 0, z'_{hz} > 0$$

z' is the next period knowledge capital and subscripts are partial derivatives. Current knowledge makes easier for firms to advance their knowledge capital ($z'_z > 0$). R&D expenditures increase future knowledge capital ($z'_h > 0$). Current knowledge capital and R&D expenditure is concave on future knowledge capital ($z'_{zz} < 0, z'_{hh} < 0$). Not like physical capital accumulation, knowledge capital accumulation shows that the former successful R&D gives firms more incentives to invest on R&D ($z'_{hz} > 0$). These characteristics are consistent with the literature in management and applied microeconomics. Recently, macroeconomics and finance areas adopt these features of knowledge and share the similar spirit. Comin and Gertler (2006) and Croce et al. (2016) shows R&D behavior that differs in output elasticity of R&D at macro-level and at firm-level.

1.4 Model solution

1.4.1 Calibration

Output elasticity of physical capital, α_k and the elasticity of knowledge capital, α_z are 0.6 and 0.3 respectively. α_k is similar to Li, Lividan, and Zhang (2009). Output elasticity of "knowledge capital" (α_z) is set to make output elasticity of "R&D" be 0.045 for the innovative inefficient firm and be 0.15 for the efficient firm. Prior research has tried to measure output elasticity of R&D at firm level or industry level. The calibration in this paper utilizes the previous empirical analysis. The elasticities vary mostly from 0.02 to 0.2 (Hall, 2009). Output elasticity of "R&D" is just output elasticity of knowledge capital times knowledge capital elasticity of R&D ($\alpha_z \times \zeta_h$). In the baseline calibration, α_z is set to 0.3. Thus, I set ζ_h 0.15 for innovation inefficient firms so that output elasticity of "R&D" becomes 0.045, and set ζ_h 0.45 for innovation efficient firms so that output elasticity of "R&D" is 0.15.

The long run mean of aggregate productivity $\overline{x}$ is set to -3.2 to have the book-to-market ratio from simulated data to match the ratio from the data. Many of parameters follow calibration in Li, Lividan, Zhang (2009), the persistence of aggregate productivity ρ_x is 0.983. The standard deviation of aggregate productivity σ_x is 0.0023. Quasi-fixed cost, f is 0.01. Constant price of risk parameter γ_0 is 50 and Time-varying price of risk parameter γ_1 is -1000. This stochastic discount factor yields countercyclical time-varying risk premium. The physical capital depreciation rate δ is 0.01. Physical capital adjustment cost, a is 15. Time discount parameter η is 0.994. Fixed component of external financing cost, λ_0 is 0.08. Proportional financing cost, λ_1 is 0.025. Unfortunately, to my knowledge, no prior research estimates the parameters for knowledge capital. I set future knowledge capital elasticity of existing knowledge capital, ζ_z is 0.993. It is because knowledge capital depreciates slower than physical capital does. I set standard deviation of knowledge capital accumulation σ_ν is 0.01, which is larger than the volatility of aggregate productivity. It is reasonable to assume that building knowledge capital entails more uncertainty than the uncertainty of the entire economic productivity.

1.4.2 Value and policy functions

Figure 1.4 depicts the value function $v(k, z, x)$, defined in equation (1.4). The shape of value is consistent with the existing literature and adds the another dimension, knowledge capital. Firm value increases with physical capital, knowledge capital, and aggregate productivity. The value functions are monotonically increasing and concave in both k and z. Many investment-based models show that firm values are increasing and concave in k. The firm value in is also increasing and concave in knowledge capital, z. Knowledge capital exhibits decreasing return to scale as well as physical capital does. Marginal contribution of knowledge capital to the firm value decreases as a firm accumulates knowledge capital; meanwhile, it gets easier for firms to innovate as in equation (1.3).

Firm values for good time (dash-dot line) in figure 1.4 are the values when the aggregate productivity is above the one unconditional standard deviation away from the mean. Firm values in bad time (dotted line) have the aggregate productivity the one unconditional standard deviation below from the average. When all other things are equal, innovation efficient firms have higher values than the inefficient firms.

Figure 1.5 shows investment and R&D policies as a function of physical capital. In all panels, both investment and R&D increases in good times and decreases in bad times. For IE firms (panel (B) and (D)), the impact of the economic situation is significant. Innovation efficient firms spend on R&D much larger in a good state while they cut down drastically on R&D in a bad time. IE firms have more investment opportunities and more substantial marginal benefit R&D in economic boom, but knowledge capital does not provide investment opportunities enough in bad times. In panel (C), economic states do not have effects very much on R&D policies for innovation inefficient firms. They barely spend on R&D regardless of the aggregate productivity. It is because innovation inefficient firms are better to spend on physical capital or dividends if internal capital is available. It is similar to the fact that many firms do not spend on R&D at all.

1.4.3 Beta

Following Cochrane (2001,p.19), I define $\beta_{i,t}$ as

$$\beta_{i,t} = \frac{-Cov[r_{i,t+1}, m_{t+1}]}{Var[m_{t+1}]}$$

Following Li, Lividan, Zhang (2009), I can compute β by backing it out from the following one factor beta-pricing equation.

$$E[r_{i,t}] = r_t^f + \beta_{i,t}\lambda_t^m$$

where $r_t^f \equiv 1/E[m_t]$ is the risk free rate, and $\lambda_t^m \equiv Var[m_{t+1}]/E[m_t]$ is the price of risk.

Figure 1.6 panel A shows β as a function of physical capital capital. I set the aggregate productivity at mean level and knowledge capital at the midpoint of the range. I depicted βs when R&D efficiency, ζ_h are 0.15, 0.3, and 0.45. Solid line is the case of $\zeta_h = 0.15$. dotted line is the β when $\zeta_h = 0.3$. Dash-dot line is the case of $\zeta_h = 0.45$. When all other things are equal, β increases with innovation efficiency. This figure suggests that innovation efficient firms load more on risk, and consequently, they have higher stock returns

Figure 1.6 panel B shows β as a function of knowledge capital in case of $\zeta_h = 0.15$, 0.3, and 0.45. As in the panel (A), aggregate productivity is set to mean value and the physical capital is at the midpoint of the grid range. β increases with ζ_h in this direction as well. The shape of β is a bit humped in the figure 1.6. In some state, the risk gets higher for large firms than the small firms. It might be related to the external financing costs. It is not reported in the paper but if firms do not incur external financing costs, humps in the beta figures disappear. Financing costs hinder firms' investment and these bindings make the firms' risk non-monotonic.

1.5 Simulation and results

I simulated 1000 panels for the benchmark model. Each panel has 5000 firms and 720 months. First 240 months are dropped to mitigate the effect of the initial conditions. I run well-known empirical analysis (Fama-Macbeth regressions) for each panel and calculate cross-panel averages of empirical results.

1.5.1 Unconditional moments

I report the unconditional moments of the model simulation in the Table 2.4. The overall fit seems acceptable. Moments that are related to the overall economy and risk premium, the annual average risk-free rate, and the volatility of annual risk-free rate are close to the data.

For the firm level moments, the annual investment-to-book-asset ratio is close to the data for both innovation efficient and inefficient firms. The efficient firm has higher annual R&D to book asset ratio than data, while innovation inefficient firm does not. However, when scaling down with the market value of asset, then the ratios for both firms are a little lower than the data. The book-to-market ratios higher than the average of data for inefficient firms and lower than the average for IE firms. Simulated data does not represent the entire firms. Instead, I compare the two groups (innovation efficient firms and inefficient firms). The moments need not have to be the same as the quantities or the entire market. The moments from the data lies in between the moments of two groups, and this supports that each group represents innovation efficiency well.

The average return on equity for both groups is higher than the data. It is because ROE from the model in the table 2.4 is actually defined as a profitability, $(k_{i,t+1} - k_{i,t} + d_{i,t})/k_{i,t}$. It can be slightly different from the accounting measure, ROE. There might be many another type of costs for the net income in the accounting ROE so that it might lower the value. Another potential issue is that innovation inefficient firm has higher ROE than ROE of innovation efficient firm. It is because of decreasing-return-to-scale of physical capital. As R&D efficiency increases, more investment opportunities are open to the firm. The efficient firm grows faster and has more physical capital. The profitability measure is scaled by physical capital. Thus, it lowers the unconditional profitability of the IE firm. Size effect dominates the increase in knowledge capital effect.

The volatility of the investment and R&D to the market value ratio for both firms are lower than the data. It is because the simulated data does not represent entire economy. The simulated investment-book asset volatility is within-firm volatility. If there are many kinds of firms in the economy, then the unconditional moment will rise.

1.5.2 Innovation efficiency and expected stock returns

This section demonstrates with the model simulation that the IE effect rises rationally. I examine whether simulated data from the calibrated model can deliver the IE effect.

Table 2.5 presents the results of Fama-Macbeth regression (1973) with the simulated data from the baseline model. In column (1), the innovation efficiency effect is positive but not statistically significant. The investment-based model has a feature generating strong size effect because of the operating leverage and decreasing returns to scale. If it is not controlled, size effect dominates the innovation efficiency effect. The efficient firm tends to be bigger firm since they have more investment and R&D opportunities. When size is controlled in column (2)-(4), the efficiency effect becomes positive and significant. IE firms earn 2.31% per month more than inefficient firms when size is controlled. When both book-to-market ratio and size are controlled, the IE firms earn 1.65% per month.

In column (4), I verify that IE effect is not an artifact of failure to control for various firm characteristics. IE effect is still positive and significant after controlling for R&D expenditures, book-to-market ratio, profitability, investment, and size, which are well known as predictors of stock returns. The innovation efficient firm still earn 1.32% more than the inefficient firm per month. The effect of control variables is consistent with the literature. The magnitude of the book-to-market effect reduces when profitability and investment are controlled. It is consistent with the recent findings in Fama and French (2014) that the book-to-market effect is mitigated when investment and profitability factors are considered. The positive and significant coefficient of R&D expenditures are in line with the empirical findings that Chan, Lakonishok, Sougiannis (2001) document. Size and value effects are still high in the regression.

Investment-based model with exogenous profitability (e.g.,. Li, Lividan, Zhang (2009)) can produce the negative relationship between investment and stock returns. High profitability firms invest more, but high profitability means low risk in their model. Thus, investment and expected returns have negatively related. However, since the profitability is somewhat endogenously chosen in this paper, the relationship between investment and profitability is not that simple. High investment does not necessarily mean low profitability and high profitability does not necessarily mean the low returns in this model. It could higher the expected returns. This is probably the reason for the low the significance level of the investment coefficient in table 2.5 column (4).

The effect of profitability on stock returns is one thing to note in this model. It is well documented that high profitability firms earn higher stock returns. (Robert Novy-Marx, 2013). However, existing investment based models often fail to incorporate this empirical regularity. For example, when the profitability is exogenous as in Zhang (2005), profitability and the risk premium has negatively related. It is because profitable firms are less sensitive to the fixed costs. In other words, profitable firms would be more robust to the economic shocks compared to the less-profitable firms when they have the same fixed costs. However, in this model with endogenous knowledge capital, the negative relationship between profitability and stock returns are weakened away. IE firms tend to have high profitability, but it does not mean the firms is less risky in this setup. IE firms become more sensitive to the fixed costs, and they might have more growth options. I investigate the mechanisms about why they are not less risky in detail in the next chapter. In sum, the model with endogenous knowledge capital can produce innovation efficiency effects and offer richer interpretations in firm's characteristics and stock returns while maintaining other empirical regularities in asset pricing literature.

1.5.3 Robustness

In the previous section, two kinds (innovation efficient and inefficient firms) of the firms are simulated and analyzed. This section investigates if the model still holds other cross-sectional patterns of returns within the same group. To verify the validity of the model, I check whether homogeneous simulated firms (on innovation efficiency) are still consistent with the present empirical regularities.

Table 2.6 panel (A) reports the Fama-Macbeth regression results of the inefficient firms. Panel (B) presents the regression coefficients for the IE firms. Overall the results seem to be similar with the previous literature. In both panels, the more firms spend on R&D, the higher stock returns they earn, which is consistent with Chan, Lakonishock, and Sougianni

(2001). High book-to-market firms earn 1.3% to 1.5% per month within the inefficient firms while the value effect among the efficient firms is not as strong as in the inefficient firms. Size effects are significant in both panels. Profitability is negatively correlated with stock returns for the inefficient firms. It is consistent with the invest based asset pricing models with exogenous profitability. Controlling for the innovation efficiency constant generates similar results to the model with exogenous profitability. Introducing endogenous knowledge capital can mitigate this relationship.

1.6 Isolating mechanism

The literature has shown some rational explanations about the IE effect. For example, Hirshleifer, Hsu, and Li (2013) denote that firms are innovative efficient because they have purchased innovative efficient and, at the same time, risky technology. Thus, they have higher expected returns. Another argument is that innovative activity itself is associated with the economic uncertainty. Berk, Green, and Naik (2004) points out that even though R&D technology itself can be diversified out, the continuing or abandoning R&D decision could be related to the economic situation.

However, interpretations above focus only on the technology itself, they do not consider the innovation efficiency together with physical capital. What I try to concentrate on is that knowledge capital and physical capital are interdependent, since firm manager make choices for both simultaneously. Innovation efficiency intensifies the risks that are already nested in investment. This section investigates how innovation efficiency increases the risk of the firm through the interaction between knowledge capital and physical capital.

1.6.1 Intensifying risks associated with investment

Operating leverage is one of the well-known source of the risk premium on investment. Innovation efficiency intensifies the risk premium mechanisms of operating leverage. It is because the operating profit for IE firms could be more sensitive to economic shocks. Relative operating profit to its fixed cost[9] decrease faster with negative shocks for IE firms while the ratio rises faster in good times. Knowledge capital provides abundant investment opportunities in good times but it cannot in bad times. so firms with high fixed cost are more sensitive to the economic situation. It is different from the existing investment-based models, which the firm profitability is exogenous. In those models, high profitability firms have robust operating-profit-to-fixed-cost ratios, since the individual firm productivity is orthogonal to the economic situation. High profitability just higher the relative operating-profit-to-fixed-cost ratio.

However, when knowledge capital is endogenized and the firm should choose the how much profitable they will be, then they choose the more profitable state (high in knowledge capital) when the economic outlook is prosperous, but they reduce putting in R&D expenditures in an economic downturn. As a result, the ratio of operating profit to fixed cost becomes more volatile, and risk premium gets higher for IE firms. It is consistent with the empirical evidence that R&D expenditures are procyclical.[10]

An expansion option effect is another source of risk on investment. Hackbarth and Johnson (2015) presents how this mechanism affects equity risk dynamics. While firms are investing, stockholders expose themselves to more risk instead of holding riskless cash. The risk is associated with the quantity of investment. The value of stock becomes more sensitive to the productivity shocks as firms are investing more since they have to spend more riskless cash to acquire the risky asset. Hackbarth and Johnson call this expansion option effect. HJ model has exogenous productivity and firms chooses the timing of the expansion, and investment. On the other hand, the model in this paper does not determine the timing of the expansion. Instead, firms choose profitability level and investment. IE firms get more knowledge capital with less R&D expenditure and have more investment opportunities, and they are willing to take the opportunities to maximize the firm values. In the sense that IE firms spend more on riskless cash for expansion, it could be interpreted as they are exposed to the expansion option effect.

[9] It is the proportional cost in this model, but the economic intuition is the same.

[10] There might be a concern that price of R&D is fluctuating, so it might distort the real R&D activities. However, the alternative measure of R&D still shows the procyclical patterns. See Barlevy (2007).

Inflexibility is another well-explored source of risk associated with an investment. Zhang (2005) suggests the asymmetric adjustment cost of investment yields the value premium since firms with more assets in place, which are more costly to liquidate, deteriorate faster in bad times. In this model, it seems that the innovation efficiency does not intensify the inflexibility effect. It is because a manager can transfer the sources from one to the other when the inflexibility of physical capital increases. In other words, the existence of the different type of asset could mitigate the inflexibility effect in the model. For example, when firms have a huge amount of the asset-in-place that is not reversible, it increases risk when profitability is exogenous. However, since profitability is the function of resources, the marginal benefit of increasing profitability rises with the amount of asset in places. They put more resources on knowledge capital. It keeps firms from losing their values by the inflexibility of asset-in-place.

It does not mean the inflexibility does not play a role in the risk premium. Knowledge capital itself is more difficult to disinvest by its nature, and together with physical capital, operating profit cannot be freely adjusted depending on the economic situation. This irreversibility on total assets(knowledge capital and physical capital) still can affect the risk premium. The model in this paper points to shortcomings on incorporating this aspect of risk.

Simulation results from alternative parametrization support the arguments above. Table 2.7 shows Fama-Macbeth regression coefficients from the simulated returns with alternative parameters. Column (1) is the baseline result. Column (2) is the case when there is no proportional cost, $f = 0$. Since there are no fixed costs in this calibration, there is no operating leverage effect. IE firms earn 0.23% per month more than the inefficient firms in this case. Innovation efficiency effect is still statistically significant, but the magnitude of the effect is reduced more than 1%, compared to the benchmark case. This result indicates that innovation efficiency plays a role in stock returns through the operating leverage channel.

Column (3) represent low output elasticity of physical capital ($\alpha_k = 0.55$), and Column (4) presents the case with high elasticity of physical capital ($\alpha_k = 0.65$). The output elasticity of physical capital is related to the investment opportunities. How much profit percentage increases when a firm invests extra one unit. Considering increasing property of production function, firms with high elasticity would invest more than firms with the low elasticity when other things are equal. Of course, the α_k does not only captures the expansion options, since the parameter affects the output from the existing physical capital as well, however, in some sense, but it can also be interpreted as firms have more expansion options when α_k is high. They expose themselves more to the productivity shocks by acquiring the risky asset in return for giving up holding riskless cash. Thus, if innovation efficiency intensifies the expansion option effect, then the effect should increase with the elasticity. Simulated data shows the innovation efficiency effect increases with high α_k and decreases with low α_k . In column (3) IE firms earn 0.25% more per month, which is smaller than the baseline case. In column (4) ($\alpha_k = 0.65$), innovation efficient firms earn 1.93% more per month, which is larger than the baseline case.

Column (5) and (6) are about the price of risk parameters. Column (5) is when the price of risk is constant, and Column (6) is when the price of risk is lower than the baseline case. Constant price of risk does not make much difference in innovation efficiency effect. With the low price of risk, innovation efficiency effect is mitigated. Column (7) has no capital adjustment cost. It affects the inflexibility of the capital. In standard investment based model. Low capital adjustment cost lowers the systematic risk. However, inflexibility channel is reduced in this model. It does not make any big difference from the baseline case.

In sum, innovation efficiency increases the risks associated with the investment. Since innovation efficient firms hold more investment opportunities, and consequently, firms are more exposed to the risk associated with the investment. Operating leverage effect and expansion option effect are intensified.

1.6.2 Time-varying risk loadings

One might raise a concern if innovation efficiency effect can be explained by the CAPM, then why existing empirical risk measures can't capture the effects. For example, Fama-French factors should be able to explain the risk loadings regarding R&D. Hirshleifer, Hsu, and Li (2013) and Cohen, Diether, and Malloy (2013) find their results are robust to the Fama-French factors. It is not inconsistent with the model. The model implies the risk loading varies over time. Aggregate productivity x_t and a random R&D outcome variable ν_t moving around over time. These variables allow the risk loadings not stable over time since risk loadings depend on those state variables. Existing empirical measures are limited to capture the time-series variant component of the risk loadings. Figure 1.7 presents the example of the beta path from the benchmark simulation for an innovation efficient firm and an innovation inefficient firm. Both firms have volatile risk loadings. Those volatile features of the real beta might generate the failure of empirical risk measures.

1.7 Testable Implications

In this section, I explore the empirical evidence that could support the theoretical work documented in the previous sections. The model suggests some testable implications. This section analyzes whether data corroborates the proposed mechanisms of the innovation efficiency effect, operating leverage channel and output elasticity of physical capital channel (α_k).

Data consists of the intersection of COMPUSTAT, CRSP, and NBER patent data. Investment analysis is based on yearly frequency data, and portfolio analysis uses monthly frequency data. Missing values on accounting data are converted to zero.

1.7.1 Operating leverage

In the previous section, simulation results imply that innovation efficiency intensifies the risk associated with operating leverage. Table 2.7 shows that innovation efficiency effect on stock returns is larger for high operating leverage firms. This section tests whether it is consistent with data. If the mechanism is valid, one should be able to find stronger innovation efficiency effect in high operating leverage firms.

The measure for analysis follows Gu, Hackbarth, and Johnson (2015). I run five-year rolling window regressions of operating costs on lagged operating costs, the current sale and lagged sale with quarterly data. By this rolling window regression, I can estimate how operating costs are associated with the current sale. The annual firm-level operating leverage is the predicted operating costs when current sale is zero,[11] scaled by current sale. As a predictor, I use annual mean values of lagged operating costs and lagged sale. Operating cost is the sum of costs of good sold and selling, general, and administrative expenses.[12]

Table 2.8 presents the results of 3-by-3 double sort portfolio analysis. Firms are sorted independently by operating leverage measure and innovation efficiency. The intersection of the two sorts forms the portfolio returns. Consistent with the model prediction, alpha and returns spread between high IE firms and low IE firms are large in high operating leverage portfolios. The spreads are from 0.44 to 0.85% per month when innovation efficiency is measured by citation scaled by R&D. The spread is the largest and significant in Fama-French 5 factor model. Within the same operating leverage portfolio, operating leverage does not vary much depending on innovation efficiency. When patent/R&D capital is used for innovation efficiency, the alpha or return spreads are large in high operating leverage portfolios as well.

1.7.2 Output elasticity of physical capital

Table 2.6 shows that innovation efficiency effect on stock returns is larger for high physical capital share to the profit (α_k). If the mechanism is valid, then the innovation efficiency effect should be larger in high capital share to profit firms. One problem is that it is unobservable variable. It might be associated with expansions option in a sense that firm can earn more profit with a dollar spent on physical capital. However, it is not solely about expansion option.

Even though an empirical measure equivalent to physical capital share in the model is not available, the model presents the consequences of high physical capital share. Figure 1.8 shows how firm values varies depending on α_k. The slope of the line from the origin represents the book-to-market ratio. As α_k increases, the book- to-market ratio decreases. It is natural that firms making more sales with less capital should be valued more. Thus, they have the low book-to-market ratio. I assume that the book to market ratio captures α_k to some degree

[11]The predicted operating costs when the current sale is zero captures the intuition of "fixed costs".
[12]The portfolio analysis with an alternative measure, operating cost scaled by sale, gives the similar results

and sort firms with the book-to-market ratio. Of course, it would be the perfect measure for capital share to sales, since the book-to-market ratio could capture many other things. However, it could be one of the necessary conditions for the validity of the mechanism that innovation efficiency effect is stronger in low book-to-market firms.

Table 1.10 presents the results of double sort portfolio analysis by the book-to-market ratio and innovation efficiency. Alpha and return spread between high IE firms and low IE firms are from 0.26 to 0.39 for low book-to-market firms while they are negligible for the high book to market firms. Average book to market ratios does not vary much in IE as long as firms fall in the same group of the book-to-market ratio. This means that book-to-market ratio does not play a role much for innovation efficiency effects. These results suggest that innovation efficiency effects are more associated with the low book-to-market firms and this might be related to the mechanisms of high capital share to profit implied in the model.

1.8 Conclusion

Following the framework of the investment-based model, I study the mechanism that generates the innovation efficiency effect on stock returns; IE firms earn high subsequent stock returns. I analyze how innovation efficiency interacts with well-explored risk sources regarding investment. IE firms get riskier when they suffer high fixed costs. When they have more investment opportunities, IE firms take more risky projects. I found some empirical evidence that is consistent with the proposed mechanisms. The data shows that IE firms tend to invest more, which is implied by the model as well. Double sort portfolio analysis presents that innovation efficiency effect is stronger in high operating leverage firms and low book-to-market ratio firms.

This paper could be a good start of understanding how to value intangible knowledge capital, and how the knowledge capital intensive firms' cost of equity should be measured. This paper stands in line with recent findings of the relationship between the operating environment and firm's risk; features that increase firm value are not always decreasing equity risk loadings.

The model has a limitation that it does not specify how variation in innovation efficiency across firms exists in the economy. It just assumes that two different efficiencies of the firms exist and I analyze how the efficiency works for stock returns. Future work will need to incorporate how the efficiency heterogeneity are generated in the economy and how they evolve over time. Future work will need to address these points. With the augmented model, my model could be one of the useful tools to understand innovation activities under the neoclassical framework on the asset pricing side.

Besides, not only for asset pricing implication, my model might shed light on understanding the simultaneous dynamics of corporate investment policies and R&D policies. Model predictions of interactions between physical and knowledge capital can provide some testable implications. These predictions would stimulate future empirical research.

1.9 Figures and Tables

Figure 1.1: Investment and R&D policy
This figure represents the optimal policies of the physical capital-only firms and knowledge capital-only firms. k is physical capital, z is knowledge capital and x is aggregate productivity. I/K is investment over physical capital and R/Z is R&D expenditures over knowledge capital. Panel (A) shows the optimal investment for the physical capital firms. Panel (B) depicts the R&D expenditures of the knowledge capital firms. Panel (C) compares the investment and R&D expenditures when both firms are small (k and z are both 0.38) and panel (D) shows the comparison when they are large firms (k and z are 8.30).

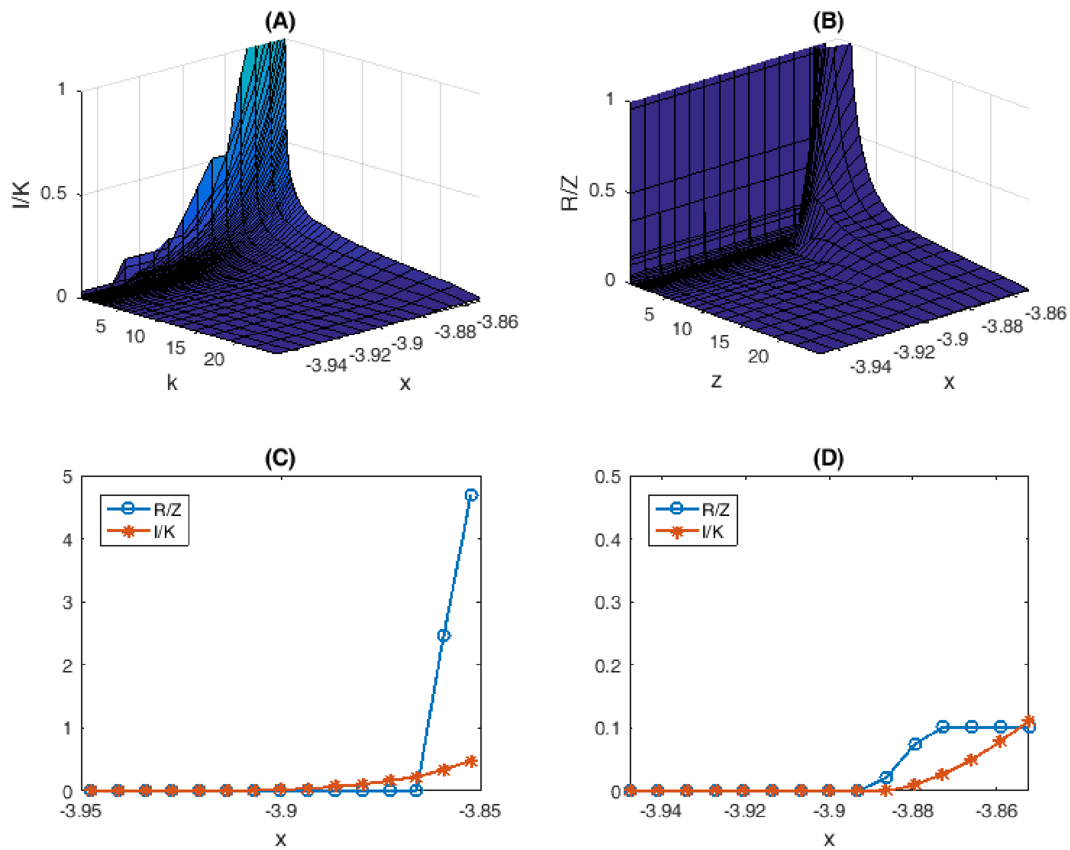

Figure 1.2: R&D growth rate over the business cycle
This figure shows the COMPUSTAT weighted average R&D expenditures growth rate and real GDP growth rate. The sample for the R&D consists of all domestic observations in COMPUSTAT each year that reported positive R&D in both that year and the previous year. The weights are the lagged R&D expenditures and the sample is winsorized at 1% and 99% level. The grey area is recession.

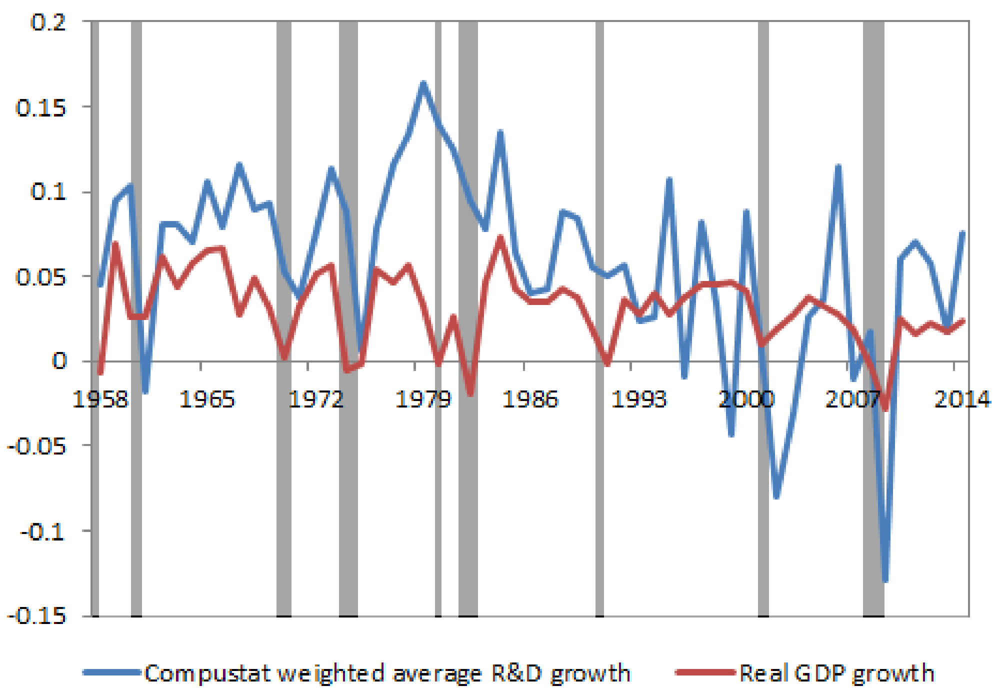

Figure 1.3: Expected excess returns
Panel (A) shows the expected excess returns for the knowledge capital firms. After solving problems, conditional expectation can be implemented via matrix multiplication. Expected return is defined as $E_t[r_{t+1}] \equiv E_t[v_{t+1}]/(v_t - div.)$. 'div' is dividend. Expected excess return is the expected returns minus risk free rate, $r_f \equiv 1/E_t[m_{t+1}]$. Panel (B) plots the R&D expenditures-to-knowledge capital ratio as functions of knowledge capital z. I fix the aggregate productivity to long run mean. The arrows indicate the direction of the increase in innovation efficiency ζ_h. Firms with $\zeta_h = 0.23$ are plotted as crosses. Firms with $\zeta_h = 0.25$ and $\zeta_h = 0.27$ are plotted as asterisks and circles, respectively.

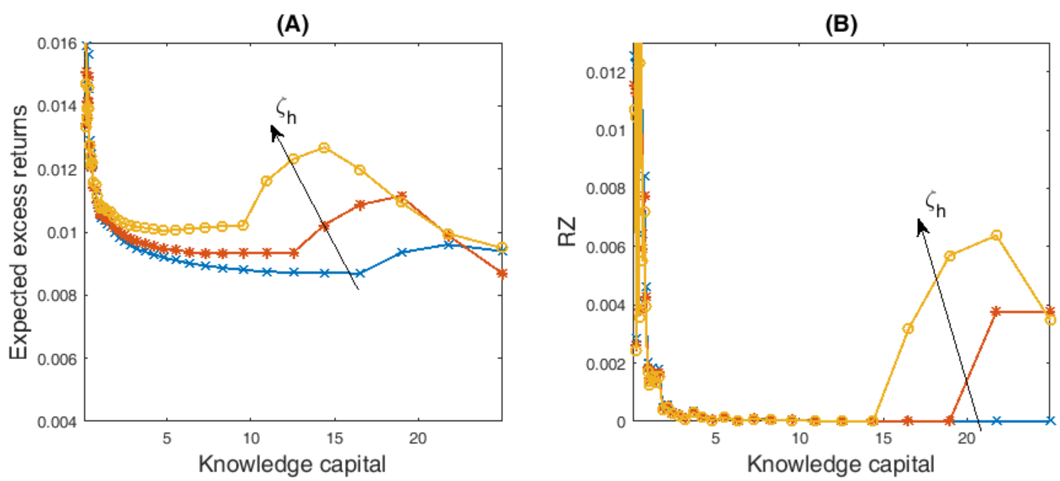

Figure 1.4: Firm values.
This figure depicts the value function(v(k,z,x)). Panel (A) and (C) shows how firm value increases as physical and knowledge capital increases for innovation inefficient firm($\zeta_h = 0.15$), respectively. Panel (B) and (D) shows how firm value increases as physical and knowledge capital increases for innovation efficient firm($\zeta_h = 0.45$), respectively.

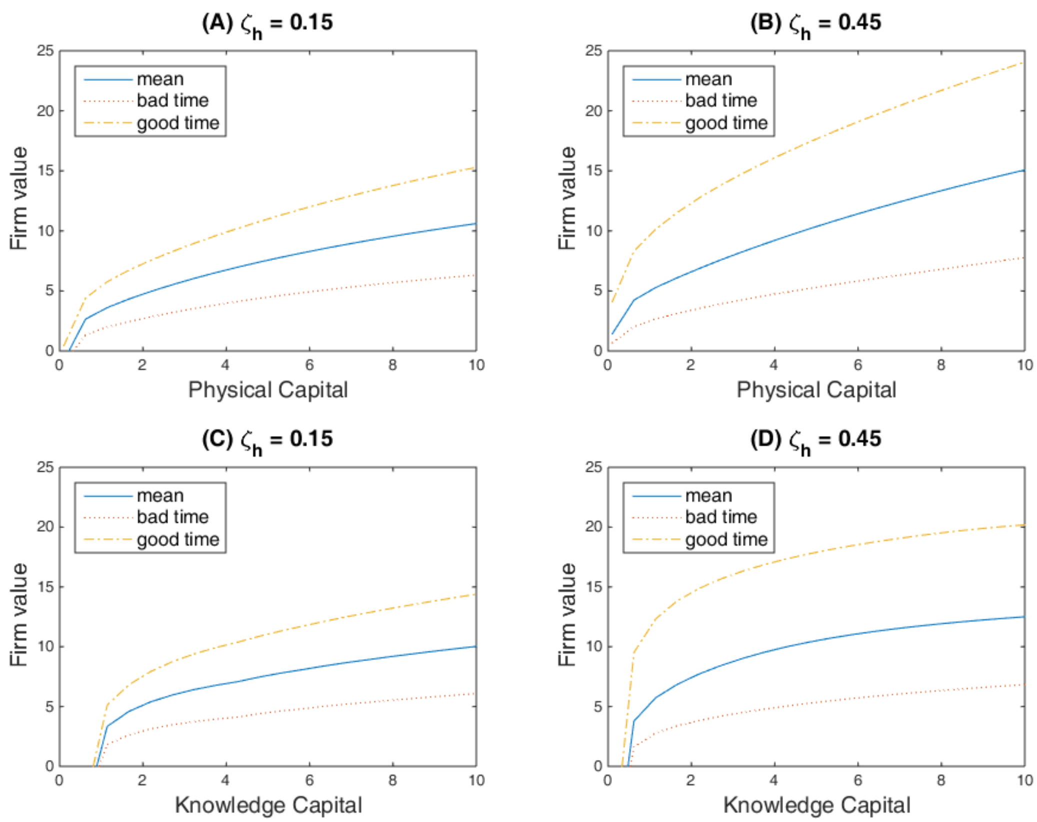

Figure 1.5: Optimal Policy functions.
This figure plots the optimal investment and R&D policies. IK is investment over physical capital and RK is R&D expenditure scaled by physical capital. Panel (A) and (C) are policy functions of innovation inefficient firms($\zeta_h = 0.15$). Panel (B) and (D)are policy functions of innovation efficient firms($\zeta_h = 0.45$).

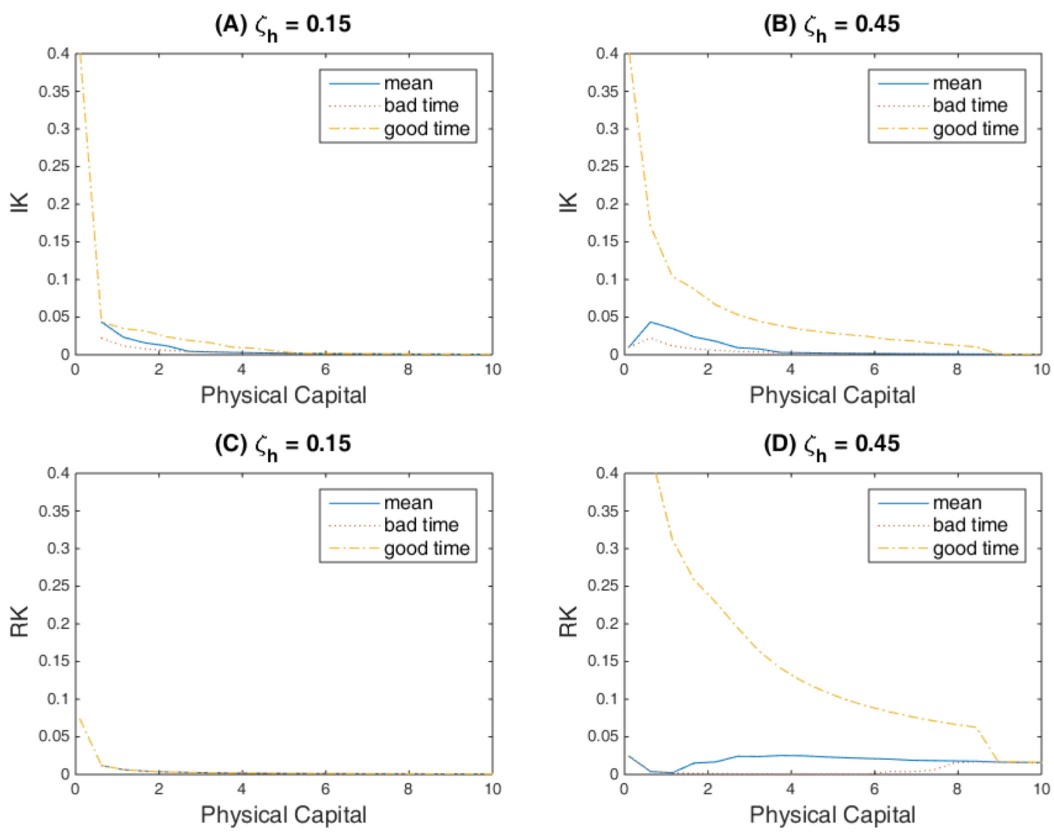

Figure 1.6: Innovation efficiency and Beta.
This figure shows the beta in equation (16). Panel (A) shows how beta as a function of physical capital changes when knowledge capital is fixed at the midpoint of its range. Panel (B) shows beta as a function of knowledge capital.

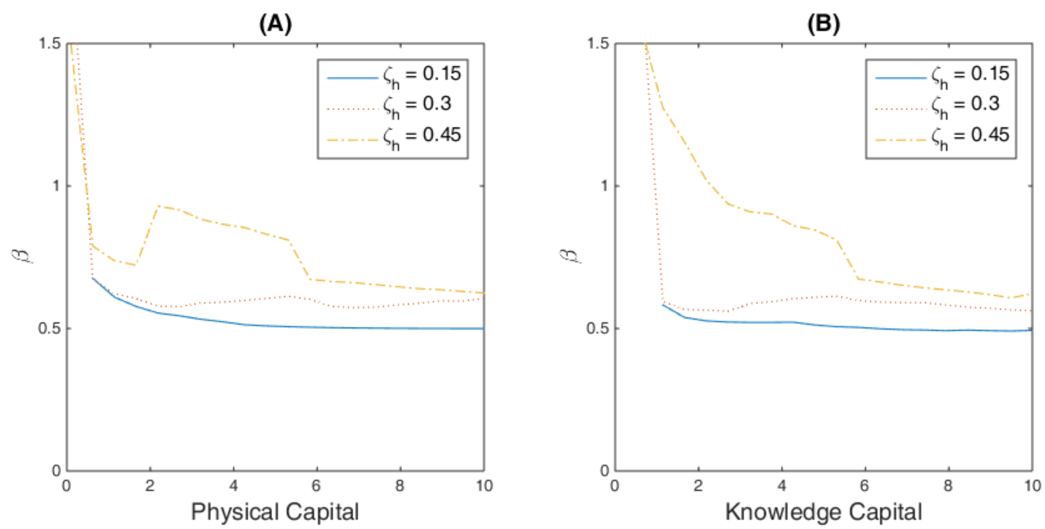

Figure 1.7: An example beta path
This figure shows the beta in equation (16). Panel (A) shows how beta as a function of physical capital changes when knowledge capital is fixed at the midpoint of its range. Panel (B) shows beta as a function of knowledge capital.

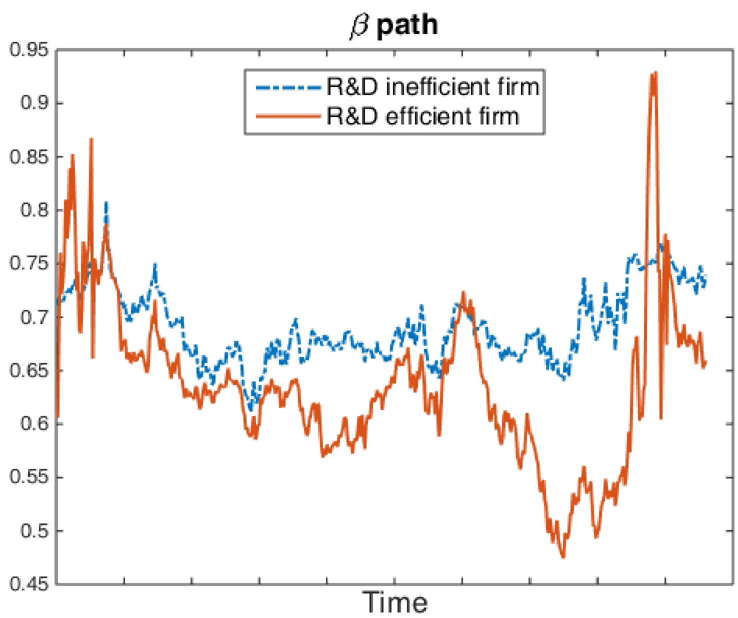

Figure 1.8: The book-to-market ratio and output elasticity of physical capital
Panel (A) plots values of the innovation inefficient firm. Panel (B) plots values of the innovation efficient firm depending on α_k. The slope from the origin represents the inverse of book-to-market ratio.

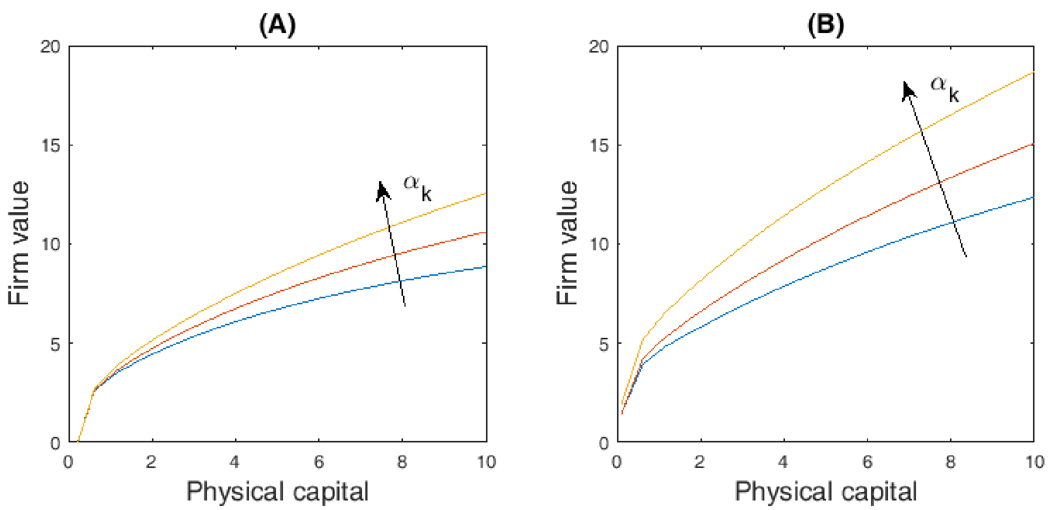

Table 1.1: Fama-Macbeth (1973) regression of stock returns on innovative efficiency (in percent) from Hirshleifer, Hsu, and Li (2013).
This table presents the result of Fama-Macbeth(1973) regressions from Hirshleifer, Hsu, and Li (2013). IE stands for innovation efficiency. RDC is the 5-year cumulative R&D expenditures with a depreciation rate of 20%. RD is R&D expenditures. BTM is book to market ratio. CapEX is capital expenditures. ME is market equity. The sample period is from 1976 -2006. The sample consists of the intersection of CRSP, Compustat, IBES, and NBER patent database. Finance, insurance,and real estate sectors are dropped in the sample.

	IE = citations/RD		IE = patents/RDC	
ln(1+IE)	0.11	0.07	0.08	0.04
	(5.13)	(2.92)	(4.44)	(1.81)
ln(1+RD/ME)		0.13		0.13
		(3.22)		(3.28)
ln(Size)	-0.30	-0.30	-0.30	-0.30
	(-3.13)	(-3.14)	(-3.06)	(-3.13)
ln(BTM)	0.39	0.37	0.39	0.37
	(6.73)	(6.34)	(6.76)	(6.32)
ln(1+CapEx/ME)	-0.11	-0.12	-0.11	-0.12
	(-3.09)	(-3.25)	(-3.09)	(-3.25)
ROA	0.15	0.15	0.15	0.16
	(2.93)	(3.16)	(2.96)	(3.20)
Controls	YES	YES	YES	YES

Table 1.2: Simple models and calibration
This table presents the model setup of physical capital firms and knowledge capital firms.

Panel A. Model setup

	Physical capital firm	Knowledge capital firm
Aggregate productivity	$X_{t+1} = \bar{X} + \rho_x(X_t - \bar{X}_t) + \sigma_X \epsilon^X_{t+1}$	
SDF	$\log M_{t+1} = \log\eta + \gamma_0(X_t - X_{t+1})$	
Capital accumulation	$K_{t+1} = I_t + (1-\delta)K_t$	$Z_{t+1} = Z_t^{\zeta_z}(1 + H_{t+1})^{\zeta_h}$
Production function	$\Pi^k_t \equiv \mathrm{e}^{X_t} K^\alpha_t$	$\Pi^z_t \equiv \kappa \mathrm{e}^{X_t} Z^\alpha_t$
Dividends	$D^k_t \equiv \Pi^k_t - I_t$	$D^z_t \equiv \Pi^z_t - H_t$
Firm values	$V^k_t \equiv \max_{I_t}\{D_t + E_0[M_{t+1}V^k_{t+1}]\}$	$V^z_t \equiv \max_{H_t}\{D_t + E_0[M_{t+1}V^z_{t+1}]\}$

Panel B. Calibration

Description	**Parameter**	**Value**
Output elasticity of capital	α	0.65
Mean aggregate productivity	$\overline{X}$	-3.9
Persistency of aggregate productivity	ρ_x	0.983
Volatility of aggregate shock	σ_x	0.0023
Time discount parameter	η	0.994
Constant price of risk parameter	γ_0	50
Capital depreciation rate	δ	0.01
Knowledge capital retainment rate	ζ_z	0.993
Innovation efficiency	ζ_h	0.25

Table 1.3: Innovation efficiency and investment.
This table presents the panel regression coefficients(in percent) and their t-statistics of physical capital investment on innovation efficiency. Sample period is from 1977 to 2006, and consists of COMPUSTAT, CRSP, and NBER patent data. IE stands for innovation efficiency, measured by patents number granted divided by R&D capital and by number of citations scaled by past cumulative R&D expenditures. CAPX is capital expenditures. AT is total asset. Size is market equity on December in year t-1. ROA is income before extraordinary items plus interest and related expense divided by lagged total asset, $(ib_t + xint_t)/at_{t-1}$. AG is asset growth rate, $(at_t - at_{t-1})/at_t$. NS is net stock issues, $log(shout_t/shout_{t-1})$. AD is advertising expense. shout is split adjusted share outstanding. expense(compustat item xad). Fama-French 48 industry code is used for controlling for industry fixed effect. All variables are winsorized at the 1% and 99% level and normalized to mean 0 and standard deviation 1.

	log(1+CAPX/AT)			
	patents/RDC		citation/RD	
log(1+IE)	0.1490	0.1511	0.1572	0.1580
	(3.91)	(3.97)	(3.72)	(3.72)
log(BE/ME)	-0.4927	-0.5048	-0.4684	-0.4889
	(-10.23)	(-10.43)	(-9.40)	(-9.85)
Size	0.6433	0.6004	0.6847	0.6158
	(11.55)	(10.53)	(12.05)	(10.59)
log(1+RD/ME)	0.1349	0.1803	0.1159	0.1917
	(4.18)	(5.25)	(3.52)	(5.48)
ROA		0.0876		0.1547
		(2.13)		(3.27)
AG		0.1222		0.2611
		(2.98)		(4.92)
NS		-0.1026		-0.1594
		(-2.74)		(-3.41)
log(1+AD/ME)		-0.1519		-0.1888
		(-3.32)		(-4.15)
year FE	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes

Table 1.4: Calibration.
This table summarizes the calibration for the benchmark model. innovation efficiency in parenthesis is for the innovation efficient firm.

Description	**Parameter**	**Value**
Physical capital share	α_k	0.6
Knowledge capital share	α_z	0.3
Mean aggregate productivity	$\overline{x}$	-3.2
Persistency of aggregate productivity	ρ_x	0.983
Volatility of aggregate shock	σ_x	0.0023
Volatility of R&D	σ_ν	0.01
Time discount parameter	η	0.994
Proportional cost	f	0.01
Constant price of risk parameter	γ_0	50
Time-varying price of risk parameter	γ_1	-1000
Capital depreciation rate	δ	0.01
Capital adjustment cost	a	15
Fixed equity issuance cost	λ_0	0.08
Proportional equity issuance cost	λ_1	0.025
Knowledge capital retainment rate	ζ_z	0.993
Innovation efficiency	ζ_h	0.15 (0.45)

Table 1.5: Unconditional moments of the simulated panel.
Risk free rate and standard deviation of risk free rate are from Campbell, Lo, and Mackinlay, 1997. The average investment to book asset is from Abel and Eberly (2001). Volatility of Investment to book asset is from Hennessy, and Whited, 2005. The average book to market ratio is from Pontiff, and Schall 1998. The rest are from Chan, Lakonishock, and Sougianni, 2001. innovation inefficient firms has the $\zeta_h = 0.15$ and innovation efficient firms has $\zeta_h = 0.45$. Return on equity in simulated data is calculated as $(k_{i,t+1} - k_{i,t} + d_{i,t})/k_{i,t}$.

	Data	Inef. firms	Ef. firms
The annual average risk free rate	0.018	0.019	0.019
The volatility of annual risk free rate	0.030	0.023	0.023
The average annual investment to book assets ratio	0.150	0.119	0.125
The volatility of annual investment to book assets ratio	0.077	0.006	0.025
The average annual R&D to book assets ratio	0.076	0.039	0.115
The average annual R&D to market value ratio	0.058	0.046	0.041
The average book to market ratio	0.668	1.118	0.439
The average return on equity(profitability)	0.088	0.184	0.137

Table 1.6: Fama-Macbeth regression results from the simulated data

This table presents Fama-Macbeth(1973) regression results with the simulated data. The dependent variable is excess return. 1000 panels are simulated. Each panel has 5000 firms and 720 months. First 240 months are dropped. For each panel Fama-Macbeth regression is implemented. The coefficients and t-statistics are cross panel averages. EFF is a dummy variable that gives one when $\zeta_h = 0.45$ and gives zero otherwise. RK is R&D expenditures scaled by physical capital. BEME is book to market ratio. Profitability is defined as$(k_{i,t+1} - k_{i,t} + d_{i,t})/k_{i,t}$. IK is investment to book value ratio. Size is log of market value. All independent variables except dummy variables are normalized and winsorized at 1% and 99% level.

	(1)	(2)	(3)	(4)
EFF	0.0008	0.0231	0.0165	0.0132
	(1.20)	(9.13)	(7.71)	(5.43)
RK				0.0164
				(2.38)
BEME	0.0134		0.0091	0.0034
	(12.73)		(5.81)	(3.10)
Prof.				0.0020
				(-0.33)
IK				-0.0006
				(0.38)
Size		-0.0194	-0.0124	-0.0114
		(-12.39)	(-9.72)	(-7.76)
Const.	0.0132	-0.0021	0.0039	0.0101
	(4.24)	(-0.77)	(1.35)	(2.12)

Table 1.7: Monthly Fama-Macbeth regression: homogenous panel.
This table presents Fama-Macbeth(1973) regression results with the simulated data. Panel A is simulated with innovation inefficient firms (ζ_h =0.15)and Panel B is simulated with the innovation efficient firms (ζ_h =0.45) The dependent variable is excess return. 1000 panels are simulated. Each panel has 5000 firms and 720 months. First 240 months are dropped. For each panel Fama-Macbeth regression is implemented. All independent variables are defined as in table 4.

Panel A: ζ_h =0.15

	(1)	(2)	(3)	(4)	(5)	(6)
RK	0.0032	0.0023	0.0025	0.0029	0.0006	0.0007
	(12.36)	(11.09)	(11.32)	(12.69)	(2.86)	(2.29)
BEME		0.0138				0.0158
		(6.90)				(4.60)
Prof.			-0.0009			
			(-7.96)			
IK				-0.0007		
				(-2.37)		
Size					-0.0066	-0.0051
					(-11.03)	(-10.36)
cons.	0.0186	0.0192	0.0187	0.0191	0.0187	0.0195
	(5.27)	(5.42)	(5.26)	(5.27)	(5.02)	(5.25)

Panel B: ζ_h =0.45

	(1)	(2)	(3)	(4)	(5)	(6)
RK	0.0012	0.0014	0.0023	0.0006	0.0006	0.0004
	(6.72)	(7.53)	(2.73)	(2.51)	(7.54)	(7.09)
BEME		0.0003				0.0019
		(0.10)				(1.70)
Profit.			0.0004			
			(0.44)			
IK				-0.0007		
				(0.07)		
Size					-0.0016	-0.0035
					(-1.50)	(-2.40)
cons.	0.0085	0.0080	0.0084	0.0079	0.0075	0.0102
	(2.54)	(2.31)	(2.52)	(2.15)	(2.07)	(2.53)

Table 1.8: FM regression results from the alternative parametrization.
This table presents the Fama-Macbeth (1973) regression results with alternative parametrezation. 1000 panels are simulated. Each panel has 2500 innovation efficient firms and 2500 innovation inefficient firms are simulated for 720 months and firs 240 months are dropped. Independent variables are defined the same as in the table 3. Column (1) is benchmark calibration. I set alternative parameters such as zero proportional cost $f = 0$ in column (2), low capital share $\alpha_k = 0.55$ in column (3). high capital share $\alpha_k = 0.65$ column (4), constant sharp ratio $\gamma_1 = 0$ in column (5), low sharp ratio $\gamma_0 = 30$ in column(6), and low capital adjustment cost $a = 0$ in column (7).

	BM	$f = 0$	$\alpha_k = 0.55$	$\alpha_k = 0.65$	$\gamma_1 = 0$	$\gamma_0 = 30$	$a = 0$
EFF	0.0132	0.0023	0.0025	0.0193	0.0157	0.0054	0.0150
	(5.43)	(2.74)	(3.44)	(7.25)	(5.54)	(8.77)	(4.44)
RK	0.0164	0.0003	0.0083	0.0029	0.0031	-0.0004	0.0067
	(2.38)	(1.17)	(1.48)	(1.34)	(2.24)	(-2.84)	(3.47)
BEME	0.0034	0.0020	0.0474	0.0011	0.0014	0.0018	0.0021
	(3.10)	(4.45)	(4.73)	(2.52)	(2.34)	(5.27)	(4.23)
Prof.	0.0020	0.0001	0.0040	0.0001	0.0008	0.0004	-0.0002
	-(0.33)	(-0.35)	(2.15)	(-1.19)	(0.10)	(1.89)	(-0.40)
IK	-0.0006	0.0014	-0.0025	-0.0002	-0.0012	-0.0002	-0.0064
	(0.38)	(1.90)	(-0.24)	(0.41)	(-0.20)	(-0.64)	(-0.95)
Size	-0.0114	-0.0031	-0.0087	-0.0148	-0.0146	-0.0063	-0.0119
	(-7.76)	(-3.53)	(-7.79)	(-8.11)	(-6.63)	(-12.01)	(-6.55)
Const.	0.0101	0.0092	0.0141	0.0029	0.0066	0.0086	0.0062
	(2.12)	(1.71)	(3.34)	(0.97)	(1.12)	(3.99)	(1.40)

Table 1.9: Double sort portfolio analysis. (operating leverage and innovation efficiency). This table presents the double sort portfolio analysis with innovation efficiency and operating leverage. Sample period is from 1982 to 2007, and the sample data consists of COMPUSTAT, CRSP, and NBER patent data. At the end of June each year t, portfolios are independently sorted into 3 groups by innovation efficiency and operating leverage. Innovation efficiency measures are the same in the table (1.1). The operating leverage measure follows Gu, Hackbarth, and Johnson (2015). I run five year rolling window regressions of operating costs on sale and lagged sale with quarterly data. The annual firm-level operating leverage measure is the sum of intercept of the regression and predicted operating cost scaled by sale. Operating cost is the sum of costs of good sold and selling, general, and administrative expenses.

Panel A. IE = citation/RD

	4 factor model alphas				5 factor model alphas				returns
	IE low	IE mid	IE high	H-L	IE low	IE mid	IE high	H-L	H-L
low	0.01	0.11	0.20	0.18	-0.07	-0.16	0.11	0.18	-0.01
	(0.08)	(0.69)	(2.11)	(1.00)	(-0.38)	(-1.08)	(1.21)	(0.99)	(-0.05)
mid	-0.10	0.14	0.13	0.24	-0.24	0.02	0.06	0.30	-0.13
	(-0.63)	(0.97)	(1.04)	(1.20)	(-1.53)	(0.12)	(0.46)	(1.56)	(-0.67)
high	-0.10	0.21	0.47	0.58	-0.38	0.44	0.47	0.85	0.44
	(-0.48)	(1.01)	(1.96)	(1.80)	(-1.73)	(2.24)	(1.93)	(2.70)	(1.41)

	Median OL		
	IE low	IE mid	IE high
low	0.003	0.001	0.006
mid	0.131	0.133	0.128
high	0.450	0.448	0.418

Panel B. IE = patent/RDC

	4 factor model alphas				5 factor model alphas				returns
	IE low	IE mid	IE high	H-L	IE low	IE mid	IE high	H-L	H-L
low	0.07	0.04	0.22	0.15	0.01	-0.16	0.13	0.12	0.04
	(0.42)	(0.31)	(2.33)	(0.87)	(0.07)	(-1.10)	(1.40)	(0.68)	(0.22)
mid	-0.06	0.08	0.21	0.26	-0.22	0.03	0.07	0.29	0.00
	(-0.35)	(0.54)	(1.63)	(1.41)	(-1.45)	(0.17)	(0.56)	(1.56)	(-0.01)
high	0.06	0.21	0.45	0.39	-0.12	0.40	0.47	0.59	0.20
	(0.32)	(1.04)	(1.89)	(1.24)	(-0.60)	(2.12)	(1.93)	(1.91)	(0.65)

	Median OL		
	IE low	IE mid	IE high
low	0.004	-0.001	0.005
mid	0.133	0.131	0.127
high	0.452	0.458	0.413

Table 1.10: Double sort portfolio analysis (book to market ratio and innovation efficiency). This table presents the double sort portfolio analysis with book to market ratio and innovation efficiency. Sample period is from 1982 to 2007, and the sample data consists of COMPUSTAT, CRSP, and NBER patent data. At the end of June each year t, portfolios are independently sorted into 3 groups by innovation efficiency and book-to-market ratio. Innovation efficiency measures are the same as in table 1.

Panel A. IE = citation/RD

	4 factor model alpha spread				5 factor model alpha spread				returns
	IE low	IE mid	IE high	H-L	IE low	IE mid	IE high	H-L	H-L
low	-0.16	0.15	0.23	0.39	-0.12	-0.02	0.14	0.26	0.38
	(-0.93)	(1.13)	(2.39)	(1.97)	(-0.70)	(-0.16)	(1.43)	(1.32)	(1.81)
mid	0.14	-0.05	0.06	-0.08	0.14	-0.24	0.00	-0.14	-0.26
	(0.89)	(-0.36)	(0.48)	(-0.45)	(0.92)	(-1.63)	(-0.02)	(-0.83)	(-1.54)
high	0.00	0.10	0.00	0.00	0.02	-0.13	0.02	0.00	0.13
	(-0.01)	(0.41)	(0.01)	(0.01)	(0.10)	(-0.53)	(0.09)	(0.01)	(0.62)

	Average book to market ratio		
	IE low	IE mid	IE high
low	0.235	0.234	0.255
mid	0.633	0.589	0.615
high	1.554	1.426	1.408

Panel B. IE = patent/RDC

	4 factor model alpha spread				5 factor model alpha spread				returns
	IE low	IE mid	IE high	H-L	IE low	IE mid	IE high	H-L	H-L
low	-0.15	0.13	0.25	0.40	-0.02	-0.06	0.19	0.21	0.43
	(-0.95)	(0.93)	(2.72)	(2.20)	(-0.10)	(-0.42)	(2.06)	(1.14)	(2.18)
mid	0.21	-0.04	0.03	-0.18	0.28	-0.15	-0.08	-0.36	-0.25
	(1.29)	(-0.29)	(0.22)	(-1.11)	(1.80)	(-0.92)	(-0.64)	(-2.25)	(-1.63)
high	0.13	-0.10	0.16	0.03	0.08	-0.14	0.10	0.02	0.22
	(0.79)	(-0.42)	(0.82)	(0.13)	(0.49)	(-0.57)	(0.51)	(0.08)	(1.02)

	Average book to market ratio		
	IE low	IE mid	IE high
low	0.231	0.234	0.251
mid	0.623	0.584	0.614
high	1.550	1.481	1.402

Chapter 2

Operating Leverage, R&D Intensity, and Stock Returns

2.1 Introduction

Does a firm accumulate knowledge as much as the firm wants in exchange for the same price of knowledge all the time? The same one dollar spent on research and development (R&D) across firms or over time does not result in the same amount increase in knowledge capital. The management and mircoeconomics literature has extensively explored this point. It finds that knowledge capital accumulation depends on many aspects of a firm. However, this intuition has received little attention in the asset pricing literature. There are production-based models that incorporate knowledge capital as an additional factor of production, but the evolution of the knowledge capital is not fundamentally different from the standard physical capital accumulation in those models.

The accumulation form of capital has implications for a firm's risk. Lee (2017) introduces the distinctive knowledge capital accumulation function that is different from the conventional linear accumulation in the investment-based model framework and finds implications for empirical regularities on innovation and stock returns. Knowledge capital accumulation is characterized by its salient feature, which is that existing knowledge makes it easier for firms to produce new knowledge. R&D expenditures (behavior of accumulating knowledge capital) have an additional marginal benefit that enhances future operating environment. This recursive additional value of knowledge capital varies depending on economic prospect. Thus, knowledge capital itself has risky nature, compared to other factors of production which does not have such accumulation feature.

This paper starts from introducing the production based asset pricing models with a factor of production that well represents the recursive characteristics of R&D. And the paper investigates how the quantity of the R&D is related to the firms operating leverage with respect to firms' risk. I solve the model for two firms that are only different in fixed costs (high and low fixed costs) and analyze how the R&D intensity - stock returns relation differs across two firms. I find that R&D intensive firms earn high stock returns among high fixed costs firms. The simulation analysis supports the finding. It shows that the model can produce the sizable economic magnitude of the interaction effect between R&D intensity and fixed costs.

As I mentioned earlier, knowledge capital has risky nature by the recursive future benefit of R&D. R&D intensive firms are more exposed to this risky nature of knowledge capital. It is because their firm values consist of the more future values of knowledge capital, which are exposed to productivity shocks. Fixed costs reinforce the sensitivity of R&D intensive firms. It widens the firm value variation depending on economic states. R&D intensive firm values decrease fast in bad times because of the costs. This operating leverage mechanism amplifies the covariance between R&D intensive firm values and economic states.

Based on this intuition, I run empirical analysis on R&D and stock returns. Using double-sorted portfolio analysis and Fama-Macbeth regression analysis, I find R&D intensity effects on stock returns are associated with the operating leverage. In portfolio analysis, I find the interaction effect between R&D expenditures and fixed costs on stock returns. The positive R&D and stock relation is much stronger in high fixed cost portfolios. Excess returns and risk adjusted alphas are monotonically increasing R&D in high fixed cost portfolios while not in low fixed costs portfolios. Excess returns of bottom 30%, mid 40%, and top 30% of R&D intensity value-weighed portfolios in high fixed costs are 0.46%, 0.82%, and 1.13% per month respectively. Carhart four factor alpha and Fama-French five factor alphas have the same patterns. The Fama-French 5 factor alpha spreads between top 30% R&D intensity firms and bottom 30% R&D intensity firms have 1.26% per month with t-statistics of 3.94. Meanwhile, in low fixed costs portfolios, excess returns of bottom 30%, mid 40%, and top 30% of R&D intensity value-weighed portfolios are 0.74%, 0.79%, and 0.95% per month and its spread between high low are not statistically significant.

The positive association between R&D intensity effects and fixed costs on stock returns are found in yearly Fama-Macbeth regression analysis as well. In firms with low fixed costs, one standard deviation increase in R&D is associated with about 1% to 1.4% increase in yearly stock returns in Fama-Macbeth regressions with weak statistical significance. However, in firms with high fixed costs, the coefficients of R&D on stock returns increase by 1.85% - 2.12% with t-statistics of 2.21-2.54.

These return patterns are robust to the inclusion of variables that are known to have interaction effect with R&D intensity on stock returns. Firms financial constraint (Li, 2009) and product market competition (Gu, 2016) are closely related to R&D intensity effects on stock returns, and innovation efficiency (Hirshleifer, Hsu, and Li, 2013) is positively associated with R&D intensity and stock returns. In subsample portfolio analysis, R&D intensity effects on stock returns still come from high fixed costs portfolios. In Fama-Macbeth regressions, R&D and fixed cost relation is robust to those variables as well. One thing to note is that Interaction effect between low fixed costs firms and R&D intensity is robust after controlling for innovation efficiency. This is consistent with the intuition that quantity of R&D has independent dimension on top of the quality of R&D in explaining stock returns.

To summarize, this paper provides better understanding of knowledge capital. This paper adds an additional layer of firm operation to the production-based asset pricing literature. Empirical facts about R&D activity does not fit the physical capital investment framework well. Lee (2017) provides the new framework for understanding knowledge capital by extending production based asset pricing model. This paper documents supporting evidence of the framework by finding empirical results based on the model predictions.

This paper is related to the growing literature on production-based asset pricing. Zhang (2005) investigates the effects of asymmetric adjustment costs on value premium. Belo, Bazdresch, and Lin (2014) incorporate labor adjust costs in explaining cross-section of stock reuturns. Gu, Hackbarth, and Jonhson (2016) investigates the relation between inflexibility and operating leverage. This paper is also related to the literature on R&D expenditures. Hall, Griliches, and Hausman (1986), Klette (1996) suggest the new knowledge capital accumulation function alternative to the standard linear form. Li (2011) finds the R&D intensive firms earn high stock returns when firms are financially constrained. Gu (2016) finds that the strong positive R&D-stock return relation exists among high product market competition.

The paper proceeds as follows. Section 2 introduces a model and develop hypothesis.

Section 3 shows empirical results. Section 4 concludes.

2.2 Model

I employ the model developed by Lee (2017) of knowledge capital to study the risk premium implications for fixed costs and R&D intensity. The model incorporates the realistic knowledge capital accumulation in the production based asset pricing model framework. The model shows innovation efficiency can increase firms risk exposure to the aggregate productivity shock by optimal choice of investment and knowledge capital policy. Innovation efficiency is captured by a single parameter that governs the degree of knowledge capital accumulation with a dollar spent on R&D. This paper focuses on the quantity of R&D implications for stock returns. I review the model and study model implications for the interaction effects between fixed costs, and R&D intensity on stock returns.

2.2.1 Overview of the model

The framework of the model mostly follows Lee (2017). The stochastic discount factor m_t is governed by aggregate productivity shock x_t.

$$\log\, m_{t+1} = \log\eta + \gamma_t(x_t - x_{t+1})$$

$$\gamma_t = \gamma_0 + \gamma_1(x_t - \bar{x})$$

Where η is time discount parameter and Sharpe ratio is determined by parameter γ_0 and γ_1. The aggregate productivity x_t follows the AR(1) mean-reverting process with long-run mean $\bar{x}$ and volatility σ_x.

$$x_{t+1} = \bar{x} + \rho_x\ (x_t - \bar{x}) + \sigma_x \epsilon^x_{t+1}$$

Where ϵ^x_{t+1} is standard normal i.i.d. shock. The production function $\pi_{i,t}$ is given by

$$\pi_{i,t} = \mathrm{e}^{x_t} z_{i,t}^{\alpha_z} k_{i,t}^{\alpha_k} - f k_{i,t}$$

in which α_z and α_k are output elasticity of knowledge capital and physical capital, respectively. Let f denote the quasi fixed costs. Let $k_{i,t}$ and $z_{i,t}$ be physical capital and knowledge capital, respectively. The phsyical capital $k_{i,t}$ evolves as follows.

$$k_{i,t+1} = i_{i,t} + (1-\delta)k_{i,t}$$

Where $i_{i,t}$ is physical capital investment and δ is depreciation rate. The physical capital investment entails the capital adjustment costs $c_{i,t}$. It is defined by

$$c_{i,t} = \frac{a}{2}\left(\frac{i_{i,t}}{k_{i,t}}\right)^2 k_{i,t}$$

where $a > 0$ is a constant parameter. Knowledge capital $z_{i,t}$ accumulation function is given by

$$z_{i,t+1} = \mathrm{e}^{\nu_{i,t+1}} z_{i,t}^{\zeta_z}(1 + h_{i,t})^{\zeta_h}$$

$$\nu_{i,t+1} \sim \mathbf{N}(0, \sigma_\nu)$$

This form of accumulation function distinguishes this model from other production based asset pricing models. A firm spends on R&D $h_{i,t}$ to increase their knowledge capital. The current knowledge capital $z_{i,t}$ makes it easier for firms to produce the next period knowledge capital $z_{i,t+1}$. Let ζ_z and ζ_h denote knowledge retention rate and innovation efficiency. Let $\nu_{i,t}$ be the normal random R&D shocks with standard deviation σ_ν.

Distribution to shareholder, $d_{i,t}$ is

$$d_{i,t} = \pi_{i,t} - i_{i,t} - h_{i,t} - c_{i,t}$$

For each period, a manger maximizes firm value $v(k_{i,t}, z_{i,t}, x_t)$ by choosing investment $(i_{i,t})$ and R&D $(h_{i,t})$.

$$v(k_{i,t}, z_{i,t}, x_t) = \max_{\{i_{i,t}, h_{i,t}\}} \{d_{i,t} + E[m_{t+1}v(k_{i,t+1}, z_{i,t+1}, x_{t+1})]\} \tag{2.1}$$

I calibrate 14 parameters $(\alpha_k, \alpha_z, \bar{x}, \rho_x, \sigma_x, \sigma_\nu, \eta, f, \gamma_0, \gamma_1, \delta, a, \zeta_h, \zeta_z)$ in monthly basis. The parameter values that govern entire economy follows the previous studies, especially Li, Lividan, and Zhang (2009). Time discount parameter η is 0.994, sharp ratio parameters γ_0 and γ_1 are 50 and -1000, respectively. The persistence of the aggregate productivity ρ_x is $0.95^{1/3}$ and its conditional volatility σ_x is 0.007/3. I set long-run mean aggregate productivity $\bar{x}$ is -3.25 to match the book-to-market ratio of the simulated data to the real data. The output elasticity of knowledge capital α_z and physical capital α_k are 0.3 and 0.6, respectively. Depreciation rate of the physical capital δ is 0.01. The physical capital adjustment cost parameter a is 15. Knowledge capital retaining parameter ζ_z is 0.993 and innovation efficiency ζ_h is 0.25. The quasi-fixed cost parameter f is set to be 0.008 for high fixed costs firms and 0.004 for low fixed costs firms. Lastly, the standard deviation of R&D random shock σ_ν is 0.01.

2.2.2 Hypothesis development

To analyze the cross-sectional effects of fixed costs on the relation between R&D intensity and stock returns, I use the model to solve for the risk premium and R&D expenditures for firms of differing degrees of fixed costs. In the model, fixed costs have nothing to do with the R&D policy decisions. It is intuitive since fixed costs are by definition costs that occur regardless of the decisions on a firm's resource allocation. It does not appear in the first order condition of optimal R&D expenditures. Thus, variation in fixed costs does not have direct effects on how much resources would be allocated to R&D policy[1]. However, it does have impacts on firm value sensitivity to productivity shocks. The intuition behind the effects of fixed costs on firms risk via knowledge capital is the fundamentally similar to the physical capital case. I start to explain the mechanism from the physical capital case and get back to knowledge capital.

The relation between financial leverage and equity risk is not clear because it depends on investment opportunities available to the firm[2]. Likewise, the operating leverage does not necessarily increase the risk of the firm. Gu, Hackbarth and Jonhson (2016) show under

[1]Quasi-fixed costs can have impact on R&D decisions through the effect on physical capital decisions. In the model fixed costs increase proportional to physical capital

[2]See Gomes and Schumid (2010)

the real option framework that operating leverage effects on stock returns depend on the inflexibility of capital.

In physical capital investment framework such as in Zhang (2005) or Li, Lividan, and Zhang (2009), fixed costs increase the risk of the firms when the physical capital is inflexible to adjust. Fixed costs and inflexibility of capital have an interaction effect on risk. If firms can freely invest and disinvest their capital with no costs, they can always maintain an optimal level of capital in response to the productivity shocks. Even if they are subject to high fixed costs, firm values do not deviate much from the optimal level. Thus, fixed costs alone do not have the substantial impact on the firm value sensitivity to productivity shocks. It affects the risk premium together with the inflexibility of a firm.

Inflexibility is the characteristic of the capital that makes it difficult to sell the assets in bad times and prevent firms from placing the assets enough in good times. It is about off-optimality of capital placement. Due to this feature, inflexibility is the seed of firm value variation in economic states. However, inflexibility itself, without fixed costs, does not have much impact on a firm's risk. Even though the capital level deviates from the optimal point, if firms are not obliged to any costs, the impact on firm values is not substantial. Together with fixed costs, inflexibility matters for a firm's risk. Firms with fixed costs suffer a much faster decrease in values in bad times when their capital is costly to adjust. In other words, the degree of the inflexibility widens the variation in values of physical capital depending on economic states when firms are facing fixed costs.

Knowledge capital inherits the similar mechanism into it. Knowledge capital has, by its nature, sensitive variation in values depending on economic situation. Since the existing knowledge helps in generating new knowledge, a dollar spent on R&D has an additional value that enhances future R&D conditions. In good times, the marginal benefit of knowledge capital increases fast because the increased knowledge will facilitate the more valuable future R&D while in bad times, the additional future value shrinks. Because of this recursive characteristic of knowledge capital, its value becomes sensitive to economic situation. Future productivity affects how much a dollar spent on R&D will be better off in the future. Consequently, R&D expenditures policy becomes pro-cyclical. These R&D policy decisions and the recursive nature of knowledge capital spread out the variation of firm values in economic states.

Like physical capital with inflexibility case, fixed costs reinforce the risk of R&D intensive firms. R&D intensive firms are sensitive to the recursive future value of knowledge capital since their firm value come from the future value of knowledge capital more than R&D weak firms. When those R&D intensive firms are subject to fixed costs, the costs make firms more vulnerable to productivity shocks. Firm values decline fast in bad times because of them. The fixed costs strengthen the correlation between R&D intensive firm values and economic situation.

Figure 2.1 depicts the risk loadings and R&D policies of two firms of ex-ante differing in fixed costs. The left panel shows betas and right panel depicts R&D expenditure scaled by knowledge capital. Risk loadings are obtained as follows.

Figure 2.1 reveals the joint relation between fixed costs, R&D intensity, and risk premium. Since fixed cost has no impact on first order condition on R&D expenditures, the quantity of R&D has no difference between two firms. However, firm value sensitivity to stochastic discount factor (beta), which is subject to productivity shocks, shows differences between two firms. Beta rises sharply as optimal R&D intensity level increases for high fixed costs firms. Meanwhile, risk loadings of low fixed costs firms are not increasing as sharply as in high fixed costs firms. This result implies that R&D intensity and stock return relation is amplified via fixed costs.

2.2.3 Simulation results

Table 2.1 reports the results for the simulation analysis with the model of two ex-ante different firms in fixed costs. I simulate 200 panels of 5000 firms for 40 years. Panel A reports moments of real data and simulated data. I compute the real data moments using the sample which is used in the portfolio analysis. The reported moments are book-to-market ratio, R&D expenditures scaled by market equity, capital expenditures scaled by market equity, excess stock returns, and volatility of excess stock returns. Overall, simulation moments seem not to be substantially different from the moments in the data. Simulated volatility is smaller than the volatility in the real data. It is because the heterogeneity in the cross-section of firms only comes from the random R&D shock in the model simulation and this is not enough to capture entire sample variation. Considering that there exist only two kinds of firms in the simulation, the average volatility 0.259 is quite reasonable.

Panel B reports the long-run averages of simulated operating leverage to show that difference in parameter (f) results in the actual variation in operating leverage. It shows that high f_h firms (f=0.008) have indeed higher operating leverage (quasi-fixed costs (fk) divided by sales ($\mathrm{e}^{x_t} z_{i,t}^{\alpha_z} k_{i,t}^{\alpha_k}$) than low f_h firms (f=0.004).

Panel C reports the average Fama-Macbeth regressions (1974) regression results. The results show that stock returns are increasing in R&D more sharply when firms have high fixed costs. Column (1) and (2) present that the model is consistent with the existing literature. Stock returns are increasing in R&D intensity without controls and after controlling for book-to-market ration and size. Column (3) and (4) illustrate that the positive relation between R&D intensity and stock returns get stronger in high fixed costs firms. The reason that column (3) shows positive coefficient is that in simulation high fixed costs firms moves around in small physical capital and knowledge capital states while low fixed costs firms stays in large physical and knowledge capital states. Because of this size effects, the fixed costs dummy variable has positive value. One standard deviation increase in R&D intensity is associated with 0.1% to 0.12% increase in monthly stock returns in high fixed costs firms.

The quantity of effects is also substantial considering two firms are only different in fixed costs. In terms of their R&D capability (or innovation efficiency), both firms are identical in the simulation, which are known to be important variable for R&D intensity. These results show models implication that fixed costs plays important role in determining the relation between R&D intensity and stock returns.

2.3 Empirical results

2.3.1 Data and measures

I obtain stock returns, share outstanding from Center for Research in Security Prices (CRSP) and draw accounting variables such as research and development expenditures, total assets, and sales from Compustat. I also extract the patent data such as the number of patents issued and the number of citations from the NBER U.S. Patent Citations Data File. I use intersection of CRSP and Compustat for a main empirical analysis. For any analysis that requires firm-level patent data, I use intersection of three dataset, CRSP, Compustat, and NBER patent data.

The sample includes NYSE, AMEX, and NASDAQ common stocks. I exclude the financial firms, which have four-digit SIC codes between 6000 and 6999. I use sample from 1981 to 2013. Even though Compustat started to record R&D expenditures from 1975, the sample starts from 1981. This is because there were not enough observations in 70s to construct 9 portfolios. The portfolios contain stocks with non-missing quasi-fixed costs (QFC) and R&D expenditures. QFC is operating leverage measure that I construct following Gu, Hackbarth, and Johnson (2016). It is rolling window regression estimates of fixed costs divided by sales. The intuition of the measure is that fixed costs can be captured by the operating costs estimates when sales are zero. Operating costs are the sum of Compustat variable Selling, General and Administrative Expense (XSGA) and Cost of Goods Sold (COGS). The detail of construction equation is as follows. Using Compustat quarterly data, I run 60 months rolling window regressions of operating costs on lagged operating costs, current sale, and lagged sale. The regression equation is as follows

$$cost_{i,q} = \beta_{i,0} + \beta_{i,1} cost_{i,q-1} + \beta_{i,2} sale_{i,q} + \beta_{i,3} sale_{i,q-1} \epsilon_{i,q}$$

Where $cost_{i,q}$ is operating costs at quarter q. Quasi-fixed costs (QFC) is defined as follows using the average values of quarterly operating costs and quarterly sales over the year ($costsmean_{i,t-1}$ and $salemean_{i,t-1}$).

$$QFC_{i,t} = \frac{\beta_{i,0} + \beta_{i,1} costmean_{i,t-1} + \beta_{i,3} salemean_{i,t-1}}{salemean_{i,t-1}}$$

2.3.2 Portfolio analysis

This section examines the interaction effects on stock returns between R&D intensity and operating leverage by double-sorting portfolio analysis. Table 2.2 reports summary statistics of portfolios double sorted by QFC and R&D intensity. At the end of the year t, I form portfolios based on the bottom 30%, middle 40%, and top 30% of QFC and R&D intensity measured independently in year t-1. After assignment, I hold portfolios one year until the end of June in year t+1. All portfolios are value-weighted portfolios.

Variables in table 2.2 are mean values except for QFC. QFCs are median values in table 2.2. Book-to-market ratios are different across R&D portfolios but there is no trend in variation. QFC are similar across R&D portfolios. Size has variation across R&D portfolios, In low QFC group, average market equity of low R&D, mid R&D, and high R&D firms are 6744, 5722, and 2060 million dollars respectively. However, this relation is mechanical since R&D expenditures scaled by market equity. High QFC portfolios has lower return on asset (ROA) than low QFC portfolios, it is natural because high fixed costs firms might have low profitability and consequently show low ROA. Overall, differences between portfolios appear to be economically small except the mechanical features driven by sorting.

Table 2.3 shows a interaction effects between operating leverage and R&D intensity on stock return. It reports the results of double sorted portfolio analysis by R&D intensity and QFC. Panel A documents value-weighted excess returns and factor model alphas for high QFC portfolios (top 30%) and low QFC portfolios (bottom 30%). Excess returns are monotonically increasing in R&D intensity only in high fixed costs portfolios. In high QFC portfolios, return spreads (including abnormal return spreads) between high R&D and low R&D portfolios are 0.67% to 1.26% per month with t-statistics of 1.81 to 3.94. Meanwhile, there is no upward trend in returns by R&D intensity in low fixed costs portfolios and return spreads are not economically and statistically significant. In low QFC firms, high minus low R&D portfolio return spreads are -0.03% to 0.21% per month with t-statistics of -0.11 to 0.86. Panel B reports equal-weighted excess returns and alphas. In equal-weighted portfolio analysis, the R&D intensity effects are statistically significant in both low QFC firms and high QFC firms. However the economic significance are different between low QFC firms and high QFC firms. In low QFC portfolios, return spreads between high R&D firms and low R&d firms are 0.46% to 0.62% per month, while the spreads are 0.98% to 1.32% per month in high QFC portfolios. These results are consistent with the model prediction that the positive relation between R&D intensity and stock returns are much stronger in high fixed costs firms.

I also run subsample analysis to investigate whether other R&D related variables affect the interaction effects on stock returns between R&D intensity and fixed costs. I test whether fixed costs just capture the effects of any other R&D related variables that are known to affect stock returns. Li (2011) documents that financial constraints plays a role in R&D intensity effects on stock returns. It shows that the financial constraints makes R&D intensive firms risky because they have to abandon on going R&D projects. One might raise a concern that fixed costs happen to capture the financial constraints and the financial constraints truly drive the interaction effects on stock returns with R&D intensity. Subsample tests show that the effects do not come from the financial constraints. I divide the sample into two groups by financial constraints measure developed by Whited and Wu (2006) and run the same portfolio analysis.

Table 2.4 documents the results of subsample analysis with respect to financial constraints. Panel A reports the value-weighted returns and panel B reports equal-weighted returns. It reveals that financial constraints are not driving the interaction effects between fixed costs and stock returns. In both financially constrained firms and unconstrained firms, return spreads between R&D intensive firms and R&D weak firms are economically and statistically significant in high fixed costs firms. In financial unconstrained firms, value-weighted excess return and alpha spreads between R&D intensive firms and R&D weak firms in high fixed cost portfolios are 0.7% to 1.25% per month with t-statistics of 1.71 to 3.24. These results support that fixed costs contain independent effects on stock returns from financial constraints. Equal weighted excess return and alpha spreads are similar to the value-weighted return spreads. In high QFC firms, R&D intensity effects on stock returns are stronger in both financial constraints groups.

Gu (2016) documents that the R&D intensity effect on stock returns are stronger within firms in competitive product market. It shows that R&D projects are more likely to fail under the severe competition and this competition increases the risk of the firm. To show that operating leverage effects are not driven by market competition, I sort stocks into two groups additionally by the market competition. The Herfindahl-Hirschman Index is used for product market competition. It is defined as sum of squared market shares. Industries are classified with three-digit standard industrial classification codes from CRSP. Every year t-1 HHI index are calculated with sales data in Compustat. Two groups are formed by HHI index and portfolios are constructed by operating leverage and R&D intensity within each groups of HHI index.

Table 2.5 results are supportive. In low competition sample the returns spreads between R&D intensive firms and R&D weak firms are stronger in high fixed costs firms. In value-weighted portfolio returns, R&D intensive firms earn 0.74% to 1% per month than R&D weak firms in high QFC firms. Meanwhile, return spreads in low QFC firms are not statistically significant. In equal-weighted portfolio returns, the economic significance of R&D intensity effects are much stronger in high QFC portfolios regardless of product market competition. Overall, this results presents that the product market competition do not drive the fixed costs effects

2.3.3 Cross-sectional regressions

The next tests employ yearly Fama-Macbeth (1973) cross-sectional regressions each year to investigate the predictive power of interaction effects between fixed costs and R&D intensity on stock returns. I control variables that are known to have an effect on stock returns. Size, book-to-market ratio, momentum, capital expenditures, profitability, and industry fixed effects are considered as independent variables. I introduce dummy variable QFCHigh which is one if quasi-fixed costs is above median and zero otherwise. The key variable in the regression is the interaction variable between R&D expenditures and QFChigh dummy variable. It captures how much the coefficients of R&D expenditures increase when QFC is above median. All variable definition and sample constructions are the same as in the portfolio analysis. Yearly excess returns are constructed by compounding monthly access returns from July in year t to June in year t+1.

Table 2.6 demonstrates that the interaction effects between fixed costs and R&D intensity are robust to controlling for many variables. Column (1) shows that R&D intensity has positive effects on stock returns in entire sample. One standard deviation increases in R&D intensity is associated with 2.3% per year increase in stock returns. This result is consistent with the finding in Chan, Lakonishok, and Sougiannis (2001). Column (2) (4) include QFCHigh dummy variables and interaction term (log(1+R&D/ME)*QFCHigh) to check if R&D intensity effects are stronger with operating leverage. The results are consistent with portfolio analysis. The interaction terms are statistically significant while the statistical significance of R&D intensity coefficients decrease. Ones standard deviation increase in R&D intensity increase stock returns by 1.85% - 2.12% per year with t-statistics of 2.21 to 2.54 when quasi-fixed costs are above median. These results imply that the positive relation between R&D intensity and stock returns mainly come from high fixed costs firms.

I examine whether my findings are robust to the other R&D related variables in the regression framework. As in the portfolio analysis, financial constraints and product market competitions are controlled in the regression analysis. I adopt the same WW index and HHI index for the control variables. Table 2.7 documents the Fama-Macbeth regression coefficients after controlling for financial constraints and market competition. The results are consistent with portfolio analysis. For above median QFC firms, one standard deviation increase in R&D intensity is associated with 2.36% - 2.37% increase in stock returns after controlling for financial constraints. The coefficients of interaction term decrease to 1.59%-1.86% after controlling for product market competition and its interaction with R&D intensity. This might be because the fixed costs and product market competition share time varying common features by the characteristics of the industry. However, the interaction effects between R&D intensity and operating leverage is still economically and statistically significant.

Innovation efficiency is also crucial in the relation between R&D and stock returns. It is how much a firm can accumulate knowledge capital with a dollar spend on R&D. It is usually measured by number of patents granted (PRDC) or citations (CRD) in a year scaled by past cumulative R&D expenditures. Hirshleifer, Hsu, and Li (2013) document the empirical evidence that innovation efficiency is positively related to the subsequent stock returns because of investors inability to understand firm technology. Lee (2017) shows that innovation efficiency effects on stock returns can arise rationally as innovation efficiency amplifies the risk associated with operating leverage and expansion options. Lee (2017) also shows empirical evidence that innovation efficiency effects are stronger in high fixed costs firms. Innovation efficient firms would spend more on R&D expenditures because their marginal benefit on R&D is large. Innovation efficiency might be the driver of the interaction effects between R&D intensity and fixed costs.

I employ Fama-Macbeth regressions to access whether R&D intensity itself has independent effects on stock returns or not. Innovation efficiency measures are included as independent variables in regressions analysis and interaction terms between R&D intensity and innovation efficiency measures are included. Since patent data only exist up to 2006, the number of observations in the sample is decreased in this analysis.

Table 2.8 reports the results. Column (1) shows the baseline results still well hold in the decreased number of observations. Stock returns are rising in 1.79% per year with one standard deviation increase in R&D intensity. Column (2) to (5) reports the regression results when innovation efficiency is controlled. Column (2) and (3) adopt PRDC as an innovation efficiency measure and Column (4) and (5) adopts CRD as a control variable. Controlling for innovation efficiency do not have impact on my baseline results. Coefficients on interaction term between R&D intensity and fixed costs dummy variables are 1.91% to 2% with t-statistics of 2.11-2.19. R&D intensity through PRDC and CRD (interaction terms between R&D intensity and innovation efficiency) do not have significant impact on stock returns. In sum, joint effects of R&D intensity and fixed costs have independent information about stock returns on top of innovation efficiency.

2.4 Conclusion

Investment-based asset pricing has focused on how operating environment are linked to a firm's risk relying on physical capital framework. Naturally, most papers implicitly assume that other factors of production would inherit the same features of the physical capital. Even in multi-factor models, the structures of different factors are mostly similar to each other except for adjustment costs or parameter calibration. Consequently, in the simplified framework leaning toward physical capital, the analysis misses the implications of fundamental differences across production factors for a firms risk.

I employ the distinctive accumulation function for knowledge capital and provide understanding on R&D activity and a firms risk. The model predicts that a firm's fixed costs amplify the risk incurred by the nature of knowledge capital and R&D intensive firms are subject to the risk mechanism. Empirically, as model predicts, this paper documents fixed costs are associated with the relation between R&D intensity and stock returns. I find that robust R&D intensity-return relation exists among high fixed costs firms. Overall, this paper demonstrates theoretically and empirically that a realistic consideration of different types of production factors enables us to explain diverse dimensions of firms risk dynamics, which would enrich our understanding of risk.

2.5 Figures and tables

Figure 2.1: Effect of fixed costs on beta and R&D policy.
This figure represents risk loadings (beta) and optimal R&D policy for firms with fixed costs of $f_h = 0.008$ (plotted as triangles) and $f_l = 0.004$ (circles). The horizontal axis is the knowledge capital in both panels. Physical capital is set to be one standard deviation less than the simulated long-run average value for high fixed costs firms. I fix the aggregate productivity at the long-run average level.

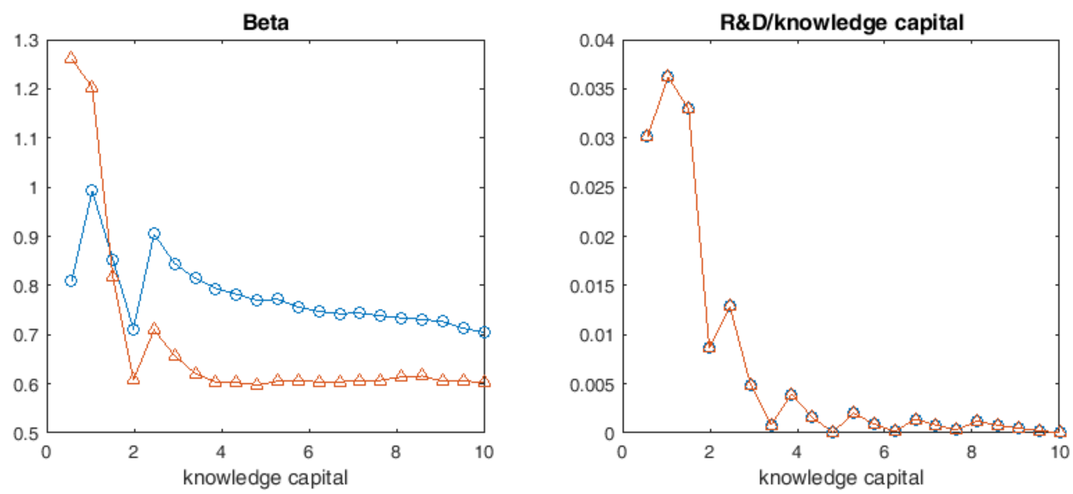

Table 2.1: Simulation results.

This table shows simulated results of the model in section 2. I simulate 200 panels of 5000 firms for 40 years. All simulation results are cross-panel averages. Panel A reports the average ratios from the simulated and real data. Panel B shows the average of operating leverage for high fixed costs firms (f_h =0.008) and low fixed costs firms ($f_l = 0.004$), respectively. The simulated operating leverage is defined as quasi-fixed costs (fk) divided by sales ($e^{x_t} z_{i,t}^{\alpha_z} k_{i,t}^{\alpha_k}$). Panel C reports average values of Fama-Macbeth (1974) monthly regression results. Firms are simulated by two firms of differing fixed costs parameters f . In panel A ME is market equity. Inv is capital expenditures. In panel C, HighFC is a dummy variable which gives one if fixed cost parameter f is 0.008 (high), otherwise is zero. R&D is R&D expenditures scaled by market equity. Size is natural log of market equity. BM is physical capital to market equity ratio. The significance levels 1%, 5%, and 10% are indicated by ***, **, and *, respectively.

Panel A: Target moments

	Data	Simulation
Boot to Market ratio	0.770	0.728
R&D/ME	0.063	0.098
Inv/ME	0.090	0.077
Average excess stock returns	0.107	0.103
Volatility of excess stock returns	0.699	0.259

Panel B: Moments of simulated operating leverage

	f_l	f_h
Average OL	0.12	0.21
Standard deviation of OL	0.005	0.011

Panel C: Return regressions with simulated data (in percent)

	(1)	(2)	(3)	(4)	(5)
R&D*HighFC				0.1200**	0.0988**
				(2.11)	(2.09)
HighFC			0.0732*	0.0805	0.1649**
			(1.69)	(1.29)	(2.29)
R&D	0.0844***	0.0767***		0.0715***	0.0527***
	(2.47)	(3.31)		(5.38)	(5.14)
BM		0.1500			0.0913
		(1.12)			(0.79)
Size		-0.0171			-0.0479
		(-0.17)			(-0.11)

Table 2.2: Summary statistics.

This table shows the summary statistics of double sorted portfolios by quasi-fixed costs (QFC) and R&D intensity. Size is market equity in millions at the end of June at year t. QFC is operating leverage measure that I construct following Gu, Hackbarth, and Johnson (2016). It is rolling window regression estimates of fixed costs divided by sales. ROA is the sum of Compustat variable *ib* (income before extraordinary items) and *xint* (interest and related expense - total) scaled by asset. Asset growth is the growth rate of Compustat variable *at*

QFC	1			2			3		
log(1+R&D/ME)	1	2	3	1	2	3	1	2	3
Book to Market ratio	0.781	0.647	1.173	0.812	0.631	1.093	0.783	0.513	0.847
Size	6744.762	5722.800	2060.348	2948.636	3837.916	1262.932	1088.266	1868.613	517.514
QFC	0.005	0.000	-0.007	0.130	0.135	0.143	0.394	0.448	0.538
log(1+R&D/ME)	0.003	0.031	0.143	0.003	0.032	0.146	0.003	0.036	0.179
log(1+capx/ME)	0.062	0.052	0.054	0.063	0.051	0.048	0.063	0.045	0.040
ROA	0.080	0.082	0.032	0.084	0.081	0.015	0.023	-0.022	-0.117
Asset growth	0.140	0.132	0.065	0.149	0.136	0.059	0.242	0.241	0.049

Table 2.3: Portfolio analysis

This table shows the portfolio excess returns and factor model alphas (Carhart four factor model and Fama-French five factor model) sorted by quasi-fixed costs and R&D intensity. At the end of the year t, I form portfolios based on the bottom 30%, middle 40%, and top 30% of QFC and R&D intensity measured independently in year t-1. After assignment, I hold portfolios one year until the end of June in year t+1. Panel A reports value-weighted portfolio returns and panel B reports equal-weighted portfolio returns excluding 20% smallest size firms in the sample. All returns and alphas are in percent.

Panel A: Value weighted portfolio returns (in percent)

	Low R&D	Mid R&D	High R&D	High - Low	t-stat
	Low QFC				
Excess returns	0.74	0.79	0.95	0.21	0.86
Carhart four factor alpha	0.19	0.22	0.16	-0.03	-0.11
Fama-French five factor alpha	0.06	-0.04	0.11	0.05	0.22
	High QFC				
Excess returns	0.46	0.82	1.13	0.67*	1.81
Carhart four factor alpha	-0.27	0.23	0.46	0.73**	2.17
Fama-French five factor alpha	-0.49	0.31	0.76	1.26***	3.94

Panel B: Equal weighted portfolio returns (in percent, excluding 20% smallest size firms)

	Low QFC				
Excess returns	0.75	0.94	1.37	0.62***	3.72
Carhart four factor alpha	0.07	0.23	0.49	0.42***	2.70
Fama-French five factor alpha	-0.22	0.01	0.24	0.46***	2.93
	High QFC				
Excess returns	0.41	0.76	1.39	0.98***	3.58
Carhart four factor alpha	-0.26	0.23	0.74	0.99***	4.44
Fama-French five factor alpha	-0.40	0.27	0.91	1.32***	6.11

Table 2.4: Subsample double sorts

This table reports monthly portfolio excess returns and abnormal returns (alphas) of subsamples, which are divided into two groups by financial constraints. Portfolios are constructed in the same way in table 3 within subsample of financial constraints. Financial constraints are measure by Whited-Wu index (2006).

Panel A: Value weighted portfolio returns (in percent)

	WW below median					WW above median				
	Low R&D	Mid R&D	High R&D	High - low	t-stat	Low R&D	Mid R&D	High R&D	High - low	t-stat
	Low QFC					Low QFC				
Excess returns	0.71	0.82	0.94	0.22	0.88	1.05	0.31	1.30	0.26	0.52
Carhart four factor α	0.19	0.26	0.17	-0.03	-0.11	0.15	-0.45	0.44	0.29	0.59
Fama-French five factor α	-0.01	0.00	0.11	0.12	0.48	0.39	-0.29	0.46	0.07	0.14
	High QFC					High QFC				
Excess returns	0.54	0.79	1.24	0.70*	1.71	-0.19	0.95	1.18	1.37***	3.76
Carhart four factor α	-0.20	0.24	0.65	0.85**	2.18	-0.92	0.21	0.41	1.33***	3.64
Fama-French five factor α	-0.45	0.31	0.80	1.26***	3.24	-0.74	0.42	0.70	1.44***	3.93

Panel B: Equal weighted portfolio returns (in percent, excluding 20% smallest size firms)

	Low QFC					Low QFC				
Excess returns	0.77	1.00	1.35	0.58***	3.63	0.70	0.70	1.29	0.60*	1.92
Carhart four factor α	0.08	0.29	0.49	0.41***	2.63	0.08	0.04	0.41	0.33	1.03
Fama-French five factor α	-0.28	0.01	0.12	0.40***	2.50	0.06	0.12	0.33	0.28	0.85
	High QFC					High QFC				
Excess returns	0.78	0.99	1.47	0.69**	2.29	0.03	0.59	1.37	1.34***	4.66
Carhart four factor α	0.07	0.43	0.79	0.72***	2.60	-0.56	0.06	0.74	1.29***	4.77
Fama-French five factor α	-0.25	0.27	0.70	0.95***	3.46	-0.44	0.24	0.97	1.41***	5.07

Table 2.5: Subsample double sorts

This table reports monthly portfolio excess returns and abnormal returns (alphas) of subsamples, which are divided into two groups by product market competition. Portfolios are constructed in the same way in table 3 within subsample of product market competition, respectively. Product market competition is measured by Herfindahl-Hirschman Index (HHI).

Panel A: Value weighted portfolio returns (in percent)

	HHI below median					HHI above median				
	Low R&D	Mid R&D	High R&D	High - low	t-stat	Low R&D	Mid R&D	High R&D	High - low	t-stat
	Low QFC					Low QFC				
Excess returns	0.79	0.72	1.21	0.43	1.43	0.83	0.97	0.77	-0.07	-0.21
Carhart four factor α	0.25	0.24	0.64	0.39	1.28	0.29	0.27	-0.24	-0.53	-2.00
Fama-French five factor α	0.36	-0.07	0.58	0.22	0.70	0.02	0.06	-0.38	-0.41	-1.49
	High QFC					High QFC				
Excess returns	0.35	0.84	1.09	0.74*	1.82	0.77	0.88	1.23	0.46	0.98
Carhart four factor α	-0.31	0.31	0.43	0.74**	1.98	-0.03	0.18	0.54	0.56	1.29
Fama-French five factor α	-0.38	0.44	0.63	1.00***	2.63	-0.34	0.11	0.96	1.30***	3.26

Panel B: Equal weighted portfolio returns (in percent, excluding 20% smallest size firms)

	Low QFC					Low QFC				
Excess returns	0.76	0.89	1.43	0.67***	2.95	0.77	0.99	1.35	0.59***	2.82
Carhart four factor α	0.10	0.22	0.65	0.55***	2.49	0.08	0.26	0.44	0.36*	1.77
Fama-French five factor α	-0.08	0.09	0.47	0.55**	2.37	-0.30	-0.01	0.15	0.44**	2.15
	High QFC					High QFC				
Excess returns	0.22	0.72	1.45	1.23***	4.19	0.52	0.82	1.18	0.66**	2.14
Carhart four factor α	-0.37	0.24	0.81	1.17***	4.62	-0.19	0.16	0.51	0.70**	2.41
Fama-French five factor α	-0.47	0.37	1.01	1.48***	5.86	-0.36	0.08	0.59	0.95***	3.29

Table 2.6: Yearly Fama-Macbeth Regressions
This table shows results from yearly Fama-Macbeth regressions of excess returns on R&D intensity, dummy variable (QFCHigh) which is one when quasi-fixed costs (QFC) is above median, and their product. I obtain yearly excess returns by compounding monthly excess returns. Mom is the prior 12 months returns with one month gap between the holding period and current month. I use Fama-French 48 industry codes for industry dummy variables. All independent variables are winsorized at 1% level and normalized. The significance levels 1%, 5%, and 10% are denoted by ***, **, and *, respectively.

	(1)	(2)	(3)	(4)
log(1+R&D/ME)*QFCHigh		0.0185**	0.0189**	0.0212**
		(2.21)	(2.25)	(2.54)
QFCHigh		0.00763	0.00791	0.0118
		(0.58)	(0.60)	(0.95)
log(1+R&D/ME)	0.0225**	0.00955	0.0108	0.0141
	(2.27)	(1.07)	(1.19)	(1.63)
logBM	0.0246**	0.0250***	0.0274***	0.0269***
	(2.57)	(2.76)	(2.88)	(3.03)
logSIze	-0.0224	-0.0216	-0.0212	-0.0256*
	(-1.62)	(-1.60)	(-1.58)	(-1.95)
MOM	-0.00298	-0.00306	-0.00380	-0.00505
	(-0.34)	(-0.35)	(-0.44)	(-0.60)
log(1+capx/ME)			-0.00363	-0.00329
			(-0.74)	(-0.67)
ROA				0.0201**
				(2.46)
Industry dummy	Yes	Yes	Yes	Yes

Table 2.7: Yearly Fama-Macbeth Regressions with R&D related control variables
This table shows results from yearly Fama-Macbeth regressions of excess returns on R&D intensity, dummy variable (QFCHigh) and their product, as well as R&D related control variables. The control variables WW and HHI are Whited Wu (2006) index and Herfindahl-Hirschman Index (HHI), respectively. I use Fama-French 48 industry codes for industry dummy variables. All independent variables are winsorized at 1% level and normalized. The significance levels 1%, 5%, and 10% are denoted by ***, **, and *, respectively.

	(1)	(2)	(3)	(4)
log(1+R&D/ME)*QFCHigh	0.0236**	0.0237**	0.0159*	0.0186**
	(2.45)	(2.47)	(1.87)	(2.18)
QFCHigh	0.0171	0.0170	0.00721	0.0114
	(1.53)	(1.54)	(0.55)	(0.92)
log(1+R&D/ME)	0.0152	0.0173*	0.00907	0.0135
	(1.69)	(1.86)	(1.06)	(1.60)
logBM	0.0198**	0.0257***	0.0253***	0.0271***
	(2.41)	(3.07)	(2.82)	(3.11)
logSIze	-0.0430***	-0.0408***	-0.0215	-0.0256*
	(-3.38)	(-2.99)	(-1.59)	(-1.95)
MOM	-0.00297	-0.00340	-0.00304	-0.00495
	(-0.35)	(-0.42)	(-0.35)	(-0.60)
WW	-0.0302**	-0.0263		
	(-2.47)	(-1.51)		
WW*log(1+R&D/ME)	0.000870	0.000276		
	(0.17)	(0.05)		
HHI			-0.00642	-0.00672
			(-0.84)	(-0.89)
HHI*log(1+R&D/ME)			-0.0126*	-0.0130*
			(-1.85)	(-1.98)
ROA		0.00941		0.0205**
		(0.75)		(2.52)
INV		-0.00632		-0.00311
		(-1.33)		(-0.64)
Industry dummy	Yes	Yes	Yes	Yes

Table 2.8: Yearly Fama-Macbeth Regressions with innovation efficiency measures
This table shows results from yearly Fama-Macbeth regressions of excess returns on R&D intensity, dummy variable (QFCHigh) and their product, as well as innovation efficiency measures. Following Hirshleifer, Hsu, and Li (2013) I use PRDC (number of patents granted scaled by past 5 year cumulative R&D expenditures with depreciation of 20% per year) and CRD (number of citation scaled by past 5 year R&D expenditures) as innovation efficiency measures. All independent variables are winsorized at 1% level and normalized. Fama-French 48 industry codes are used for industry dummy variables. The significance levels 1%, 5%, and 10% are denoted by ***, **, and *, respectively.

	(1)	(2)	(3)	(4)	(5)
log(1+R&D/ME)*QFCHigh	0.0200**	0.0197**	0.0198**	0.0197**	0.0191**
	(2.19)	(2.16)	(2.17)	(2.19)	(2.11)
QFCHigh	0.0120	0.0116	0.0120	0.0112	0.0108
	(0.95)	(0.92)	(0.95)	(0.89)	(0.86)
log(1+R&D/ME)	0.0175*	0.0186*	0.0159	0.0174*	0.0149
	(1.74)	(1.87)	(1.62)	(1.75)	(1.53)
logBM	0.0189*	0.0191*	0.0189*	0.0199*	0.0200*
	(1.83)	(1.84)	(1.81)	(1.91)	(1.91)
logSIze	-0.0300**	-0.0297**	-0.0296**	-0.0284**	-0.0283**
	(-2.19)	(-2.16)	(-2.16)	(-2.10)	(-2.10)
MOM	-0.00783	-0.00810	-0.00823	-0.00799	-0.00795
	(-0.82)	(-0.84)	(-0.86)	(-0.84)	(-0.84)
ROA	0.0158	0.0156	0.0149	0.0154	0.0151
	(1.45)	(1.41)	(1.33)	(1.44)	(1.40)
INV	-0.00343	-0.00415	-0.00414	-0.00462	-0.00423
	(-0.56)	(-0.67)	(-0.66)	(-0.73)	(-0.67)
PRDC		0.00696*	0.00492		
		(1.83)	(0.98)		
PRDC*log(1+R&D/ME)			-0.00409		
			(-0.49)		
CRD				0.0130	0.0102
				(1.68)	(1.61)
CRD*log(1+R&D/ME)					-0.00829
					(-0.87)
Industry dummy	Yes	Yes	Yes	Yes	Yes

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Appendix A

A.1 Numerical methods for chapter 1

To solve the model, I use the value function iteration method. $k_{i,t}$ and $z_{i,t}$ are endogenous state variable. x_t is exogenous state variable. I transform continuous state variables into discrete variables by creating the grids for each variables. I discretize the aggregate productivity shock by using the Rouwenhourst (1995) procedure. Physical capital and knowledge capital have 20 grid points, and aggregate productivity shock has 9 grid points. Policy vectors, physical capital and knowledge capital have 750 grid points each. Since $\nu_{i,t}$ is realized after decisions are made and follows *i.i.d.* standard normal distribution, it does not add extra dimension in computation. Linear interpolation is used to approximate firm values while finding policies to satisfy the equation (1.4).

Appendix B

B.1 Numerical methods for chapter 2

To solve the model numerically, I use the value function iteration procedure. $k_{i,t}$ and $z_{i,t}$ are choice variables and x_t is an exogenous state variable. Continuous state variables are converted into discrete grid points for all state variables. I use the Rouwenhourst (1995) procedure to generate discrete aggregate productivity states. Physical capital and knowledge capital have 20 grid points, and aggregate productivity state has 5 grid points. Policy vectors, physical capital and knowledge capital have 500 grid points each. I use linear interpolation to approximate firm values while finding policies to satisfy the equation (2.1).