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AUTOMATED GLOBAL OPTIMIZATION OF LOW-ENERGY TRAJECTORIES USING A HYBRID OPTIMAL CONTROL FRAMEWORK

BY

VISHWA SHAH

THESIS

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Adviser:

Dr. Victoria Coverstone
ABSTRACT

Leveraging dynamical structures found in the three-body problem provides opportunities to explore transfers that utilize considerably less fuel, thus gaining the moniker of low-energy transfers. Typically, these transfers are constructed by analyzing intersections of these dynamical structures at certain planes of interest in the form of Poincaré surfaces of section. Initial guesses gained from these maps are then used in differential correctors to obtain feasible solutions, or transcribed as non-linear problems and solved using an NLP solver to obtain locally optimal solutions at best. This process is time consuming and requires human input for seeding initial guesses. It also does not guarantee convergence or the existence of feasible or locally optimal solutions; and if successful, it generates a single trajectory of interest. A change in initial conditions or spacecraft parameters would require repeating the entire process.

Multi-phase trajectories are defined for this study as trajectories that have multiple arcs that require propulsive maneuvers to complete. As this study analyzes low-energy transfers, each of these phases incorporates the use of dynamical structures to some extent. Solving these multi-phase transfers using the same methodology described requires linking and analyzing multiple chains of Poincaré surfaces and using intuition to search the space to find a good initial guess. This becomes increasingly taxing and challenging for a mission design engineer to process and keep track of the best solutions with such a large problem space, and constantly evolving mission parameters. To add to the onus, the combinatorial space also expands dramatically as different kinds of dynamical structures are incorporated, such as patch three-body systems, resonances, and perturbed variants.

The study conducted in this thesis aims to present a framework that enables automated generation of trajectories utilizing low-energy transfers for multi-body regimes. The goal of the framework is to alleviate the effort re-
quired in creating low-energy trajectories by incorporating human intuition and numerical optimization methods in a Hybrid Optimal Control framework to rapidly produce a solution front of trajectories trading in multiple objectives that are of interest to mission design engineers. The Hybrid Optimal Control framework uses a dual-loop architecture, with an outer loop using a genetic algorithm for global search and an inner loop using a non-linear problem solver for local optimization. The outer loop uses a variable chromosome transcription to select the phase itinerary for different number of phases. The inner loop uses Monotonic Basin Hopping to seed initial guesses for the non-linear problem solver. Solutions are presented in the form of Pareto fronts trading multiple-objectives.

The work described here presents the motivation for such a tool, the mathematical models that form the foundation of the analysis, generation of relevant dynamical structures, the numerical optimization tools which formulate the search and optimization aspect of the framework, and the application of this framework to common mission concepts for impulsive and low-thrust propulsion types. Analysis of multi-phase trajectories and their impact on the quality of the solution space is conducted, and suggestions of improvements and desired features are given.
To my family, for their constant support. To my godfather, my moral compass.
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<td>Dynamical Structures</td>
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<td>LT</td>
<td>Low-thrust</td>
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<td>EMTG</td>
<td>Evolutionary Mission Trajectory Generation</td>
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<td>FBLT</td>
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<td>ML</td>
<td>Machine Learning</td>
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<tr>
<td>SRP</td>
<td>Solar Radiation Pressure</td>
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<tr>
<td>GMAT</td>
<td>General Mission Analysis Tool</td>
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<td>GUI</td>
<td>Graphical User Interface</td>
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There have been several missions in the past, as well as upcoming missions, that have leveraged dynamical structures (DS) for low-energy trajectories. The ISEE-3 mission was the first one to use a Halo orbit, about the Sun-Earth $L_1$ Lagrange point \cite{3}. The ARTEMIS-P1 was the first spacecraft to navigate to and perform stationkeeping operations around the Earth-Moon $L_1$ and $L_2$ Lagrangian points \cite{4}. The GENESIS spacecraft, launched in 2001, was sent to Sun-Earth $L_1$ \cite{5}. Future missions include the James Webb Space Telescope (JWST), that aims to place the spacecraft at the Sun-Earth $L_2$ point and the WFIRST mission, which was formally declared a mission by NASA in 2016, and aims to place a space observatory either in a Geosynchronous or $L_2$ orbit. These missions were created using the experience and intuition of mission design engineers, manually linking together trajectory phases using Poincaré maps and other visualization tools.

Therefore, there is a strong need for improved preliminary mission design tools that offer higher fidelity solutions, larger problem scope, and quicker time to solutions. Part of this demand for such improvements is driven by the larger number of possible missions per year, itself driven by new technologies that allow for smaller, lighter and more capable spacecraft. Certain dynamical regimes that are of continued interest, such as multi-body, are inherently difficult to perform trajectory optimization within; this being especially true for low-thrust (LT) trajectories. Advanced preliminary mission design tools currently exist for solving interplanetary problems, with NASA’s Evolutionary Mission Trajectory Generator (EMTG) \cite{6} arguably the best example. Equivalent tools do not exist for the LT multi-body problem; in fact existing tools for this regime are especially deficient with regards to a modern definition of an advanced preliminary mission design tool. Currently, pre-

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liminary mission design in multi-body regimes is largely limited to impulsive trajectories, where an analyst can patch together a solution. Tools such as NASA JPL’s LTOOL [7] and NASA Goddard Space Flight Center’s (GSFC) ATD [8] tool, both of which solve impulsive trajectories, require the user to ”keep track” of mission constraints and objectives, and have no automated optimization capability.

This motivated the creation of an automated global optimization framework for trajectories in multi-body regimes, with a specific focus on three-body problems. Akin to the success of hybrid optimal control (HOC) formulations for automated interplanetary global optimizers [9]–[11], an HOC framework for the CR3BP and patched-CR3BP has been developed and implemented [1], [2], [12], [13]. The goal of the framework is to enable a rapid search of the global design space and greater flexibility in specifying various mission constraints, thus allowing mission design engineers to rapidly generate and investigate complicated trade spaces. This is accomplished using a HOC framework consisting of two-loops: an outer loop for the global search which picks high-level mission parameters, and an inner loop for optimizing the local trajectory phases. Preliminary results demonstrated the success of the HOC framework at optimizing single-phase transfers to libration structures.

To solve multi-phase trajectories, a variable chromosome transcription is used, which makes it possible to handle multi-phase missions in a natural way; that is to say that the global optimizer should converge to the ”best” choice(s) of number and types of phases for a given mission problem without a priori or run-time guidance from a user. Multi-phase trajectories are more complicated to link together, especially for multi-body problem spaces which are highly sensitive. The automated generation and optimization of these trajectories is a challenging task, which enables a more comprehensive search of the problem space and the discovery of unique trajectories that may escape human intuition.
1.1 Outline

The thesis is organized in the following manner:

Chapter 2

This chapter discusses the theoretical background necessary for the formulation of the DS. The CR3BP equations of motion and equilibrium solutions are presented. A derivation of the state transition matrix and its application in a differential corrector is discussed. The generation of relevant periodic orbits and the associated invariant manifolds are presented as well.

Chapter 3

Chapter 3 explains the HOC framework in detail. The application of DS in the dual-loop architecture is presented. The choice of a genetic algorithm (GA) for the outer loop and its use in the HOC framework is detailed. Followed by the use of Monotonic Basin Hopping (MBH) and a NLP solver for the inner loop.

Chapter 4

Chapter 4 presents the results of the framework applied to the missions formulated in the previous chapter. Pareto fronts are presented and analyzed from a mission designers perspective. Interesting trajectories from these missions are discussed, with an analysis of the DS that are exploited. The effectiveness of the HOC framework at finding unique solutions is discussed, as well the limitations of the problem formulations.

Chapter 5

Future work and conclusions are given in this chapter, with a focus on the performance of the HOC framework with single phase multiphase trajectories. Issues, possible solutions and a general roadmap for the future is given as well.
CHAPTER 2

MODEL

Three-body models provide a better approximation of real world dynamics than the two-body model, while not being as complicated as a full n-body model. Increasing dynamical complexity to the three-body case unveils dynamical transport that does not exist in the simplified two-body problem. For application to trajectory optimization, we are interested in the circular restricted model of the three-body problem (CR3BP). For this study, we will be working in the planar restricted case which can easily be extended to a spatial setting.

2.1 Planar Circular Restricted 3-Body Problem

The PCR3BP describes the motion of a massless body under the influence of two main bodies, the primary and the secondary, with their relative motion being circular. In the case of the work presented here, the Earth is the primary and the Moon is the secondary. The system is described in a rotating coordinate frame. The mass is normalized with the mass parameter

\[ \mu = \frac{M_2}{M_1 + M_2} \]

where \( M_1 > M_2 \), \( M_1 \) being the mass of the Earth, and \( M_2 \) being the mass of the Moon. The normalized masses are \( m_1 = 1 - \mu \) for the primary, and \( m_2 = \mu \) for the secondary. The third body, the spacecraft, is considered massless. The positions of the bodies are also normalized. This puts the primary body, \( m_1 \) at \((-\mu,0)\) and the secondary body, \( m_2 \) at \((1-\mu,0)\). The

\footnotetext{This chapter contains previously published material from [1] and [2]. The copyright owner provides permission to reprint.}
equations of motion for the PCR3BP are described as

\[ \ddot{x} - 2\dot{y} = \frac{\partial \bar{U}}{\partial x} \]  
\[ \dot{y} + 2\dot{x} = \frac{\partial \bar{U}}{\partial y} \]  

(2.2)  
(2.3)

where

\[ \bar{U} = \frac{x^2 + y^2}{2} + \frac{1 - \mu}{r_1} + \frac{\mu}{r_2} \]  
(2.4)

where \( r_1 \) and \( r_2 \) are equal to the distance from the spacecraft to the primary and secondary, respectively

\[ r_1^2 = (x + \mu)^2 + y^2, \quad r_2^2 = (x - 1 + \mu)^2 + y^2 \]  
(2.5)

The dynamics of the PCR3BP permit an integral of motion to exist in the synodic reference frame, known as the Energy integral. The Energy integral of the system is given by

\[ E = \frac{V^2}{2} - \bar{U} \]  
(2.6)

\[ V^2 = \dot{x}^2 + \dot{y}^2 \]  
(2.7)

where \( \bar{U} \) is given in Eq. 2.4. The coordinates of the equations in the rotating frame use the following conventions: the sum of the masses, \( m_1 + m_2 = 1 \). The distance between \( m_1 \) and \( m_2 \) is normalized to 1. The angular velocity of \( m_2, \omega, \) around \( m_1 \) is normalized to 1. Therefore, \( m_2 \) is moving around \( m_1 \) in a circular orbit with period \( 2\pi \). The origin of the system is set at the barycenter of the \( m_1 \) and \( m_2 \). The x-axis is defined by the line connecting \( m_1 \) and \( m_2 \), with \( m_2 \) on the positive x-axis. For the Earth-Moon system, the
relevant parameters are

\[ \mu = 0.012154 \]
\[ m_1 = 0.987845 \]
\[ m_2 = 0.012154 \]
\[ m_1(x,y) = (-0.012154, 0) \]
\[ m_2(x,y) = (0.987845, 0) \]
\[ \omega = 1 \]
\[ t^* = 384400 \]
\[ t^* = 375201.53 \]

2.2 Euler-Lagrange Points

Due to the dynamics of the CR3BP, there exist five stationary points known as Euler-Lagrange points, designated as \( L_j \) with \( j \in \{1,2,3,4,5\} \) (Figure 2.1. The points associated with \( j \in \{1,2,3\} \) are the collinear points and are unstable. The \( j \in \{4,5\} \) points are the equilateral points and are stable. For large values of \( \mu \), the \( L_4 \) and \( L_5 \) undergo a bifurcation and also become unstable. For trajectory optimization, the most attention is given to \( L_1 \) and \( L_2 \).
2.3 Periodic Orbits near $\mathcal{L}_1$ and $\mathcal{L}_2$

The PCR3BP permits the existence of several families of periodic and quasiperiodic orbits around the Euler-Lagrange points. These orbits are usually approximated analytically by linearizing the equations of motion about the Euler-Lagrange points.

\[
\begin{align*}
x' &= -k A_y \cos(\lambda t + \phi) \quad (2.8) \\
y' &= A_y \sin(\lambda t + \phi) \quad (2.9) \\
z' &= A_z \sin(\nu t + \psi) \quad (2.10)
\end{align*}
\]

where the $'$ indicates the coordinates relative to the Euler-Lagrange point. $A_y$ and $A_z$ are the amplitudes, $\lambda$ and $\nu$ are the frequencies, and $\phi$ and $\psi$ are the phase angles of the in-plane and out-of-plane motion respectively. As we are working in the PCR3BP, we are interested in the planar periodic orbits obtained by setting $A_z$ to zero, known as Lyapunov orbits.

2.3.1 Differential Correction

The analytical approximations in Eq. 2.8 are used to generate initial guesses for the desired symmetric Lyapunov orbits. However, these initial guesses are not accurate enough for mission design. Therefore, a numerical scheme needs to be implemented to improve accuracy and guarantee that periodic orbits are generated within the desired tolerance. The most popular method of doing so, is the single shooting differential correction scheme. The single shooting differential corrector tweaks the initial values to minimize some error in the final values. [14] This iterative process continues until a periodic orbit is guaranteed. The Lyapunov orbits are symmetric about the $y = 0$ plane, which means they pass through the $y = 0$ plane twice, and they pierce this plane orthogonally each time. Define the initial state of a simple periodic symmetric orbit as $X(t_0)$. This orbit starts at the $y = 0$ plane with a positive $\dot{y}$. Define $X(t_{T/2})$ as the state of the orbit at half of its orbital period at the $y = 0$ plane with negative $\dot{y}$. The states must have the following form to satisfy the symmetry and orthogonality conditions at $t_0$ and $T/2$ respectively.

\[
X(t_0) = [x_0 \ 0 \ 0 \ \dot{y}_0]^T \quad (2.11)
\]
Assume we have an initial guess from our analytic approximation, \( \hat{X}(t_0) \), that is close to an initial state of a desirable periodic orbit. We integrate this state forward until it crosses the \( y = 0 \) plane. At this point of crossing, we have a new state \( \hat{X}(t_{T/2}) \).

\[
\hat{X}(t_{T/2}) = [x_{T/2} \ 0 \ 0 \ y_{T/2}]^T
\]  

(2.12)

This deviates from the form expressed in Eq. 2.12. The initial state of the trajectory must be adjusted so as to drive \( \hat{x}_{T/2} \) to zero. We have two initial conditions that can be varied to achieve the desired final state, \( x \) or \( \dot{y} \). The relationship between the final state and the initial state is given by the following linearized equations

\[
\delta X(t_{T/2}) = \Phi(t_{T/2}, t_0)\delta X(t_0) + \frac{\partial X}{\partial t}(T/2)
\]  

(2.14)

where \( \Phi(t_{T/2}, t_0) \) is the state transition matrix, flowing from \( t_0 \) to \( t_{T/2} \). The state transition is numerically obtained by integrating the following equations simultaneously

\[
\dot{\bar{x}} = f(\bar{x})
\]  

(2.15)

\[
\dot{\Phi}(t, t_0) = Df(\bar{x}(t))\Phi(t, t_0)
\]  

(2.16)

with the initial conditions

\[
\bar{x}(t_0) = \bar{x}_0
\]  

(2.17)

\[
\Phi(t_0, t_0) = I_{4 \times 4}
\]  

(2.18)

where \( \bar{x}_0 \) is the initial guess for the periodic orbit, and \( I_{4 \times 4} \) is the identity matrix of size \( 4 \times 4 \). The Jacobian matrix \( Df(\bar{x}(t)) \) is

\[
Df(\bar{x}(t)) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -U_{xx} & -U_{xy} & 0 & -2 \\ -U_{yx} & -U_{yy} & -2 & 0 \end{bmatrix}
\]
Keeping $x_0$ fixed, we can obtain the required correction to $\dot{y}_0$ from expanding Eq. 2.14 as

$$\delta \dot{x}_{T/2} = \Phi_{34} \delta \dot{y}_0 + \ddot{x}_{T/2} \delta \dot{\hat{T}}/2$$

(2.19)

$$0 = \delta y_{T/2} = \Phi_{24} \delta \dot{y}_0 + \dot{y}_{T/2} \delta \dot{\hat{T}}/2$$

(2.20)

where $\Phi_{ij}$ is an element of the state transition matrix. $\dot{x}_{T/2}$ is obtained from the equations of motion evaluated at crossing the $y = 0$ plane. Therefore, the desired change in $\dot{y}_0$ is given by

$$\delta \dot{y}_0 \approx (\Phi_{34} - \frac{\dot{x}_{T/2}}{\dot{y}_{T/2}} \Phi_{24})^{-1} \dot{x}_{T/2}$$

(2.21)

Upon achieving tolerance, a periodic orbit like Figure (2.2) is obtained.

### 2.3.2 Continuation Method

Once we have successfully generated a periodic orbit around the Euler-Lagrange point, a continuation method can be used to traverse through the family of periodic orbits to obtain an orbit of desired energy $E^*$. This is possible due to the existence of the Energy integral, the constant of motion for the PCR3BP and the fact that the Lyapunov orbits vary smoothly when parametrized by energy.

First, a single parameter of the known periodic orbit is perturbed and differentially corrected to a new member of that periodic family. The original periodic orbit has an energy $E_1$, the new periodic orbit has an energy $E_2$. The difference in energy is

$$E_2 - E_1 = \frac{1}{2} \int_0^T (\dot{x}^2 + \dot{y}^2) dt$$

(2.22)

where $T$ is the period of the orbit. In practice, this can be achieved by solving the equations of motion with a third-order Runge-Kutta method.

Figure 2.2: Differentially corrected Lyapunov orbit around the Earth-Moon $L_2$ Euler-Lagrange point
$E_2$. Thus, $E^*$ either lies between $E_1$ and $E_2$, or on either side of those bounds. This information allows us to linearly perturb the initial conditions appropriately until we differentially correct to an orbit of desired energy $E^*$. Though linear schemes are sufficient for orbits within the same family, pseudo-arclength schemes might be necessary for orbits of different families as state space curves are not always well-modeled by polynomials. [15] For more details on the derivation of the equations of motion, differential correctors, continuation methods and dynamical systems theory, please refer to [14], [16], [17].

2.4 Invariant Manifold Theory

Having generated periodic orbits of the desired energy, we can exploit the instability in these orbits to generate the associated invariant manifolds. These manifolds are a union of arcs that are created from perturbations on the periodic orbits along directions of the stable/unstable eigenvectors associated the monodromy matrix. The collinear Euler-Lagrange points are unstable and hence the periodic orbits around those points are unstable. To exploit these instabilities, we need to analyze the local stability characteristics. This is done efficiently by analyzing the eigenvalues\(^1\) of the monodromy matrix [16]. The monodromy matrix is obtained by propagating the state transition matrix for one whole orbital period. The state transition matrix $\Phi(t)$ can be computed as stated in Eqs. 2.15-2.18. The eigenvalues of the monodromy matrix for Lyapunov orbits are of the following form

$$\lambda_1 > 1, \quad \lambda_2 = \frac{1}{\lambda_1} < 1, \quad \lambda_3 = \lambda_4 = 1 \quad (2.22)$$

where $\lambda_1$ and $\lambda_2$ are real, and $\lambda_3$ and $\lambda_4$ are equal to 1. $\lambda_1$ is the eigenvalue associated with the unstable invariant manifold since $\lambda_1 > 1$ implies exponential growth. Similarly, $\lambda_2$ is the eigenvalue associated with the stable invariant manifold. The eigenvectors associated with the stable and unstable eigenvalues are defined as $v^S$ and $v^U$ respectively. The stable and unstable eigenvectors at time $t_i$ can be obtained using the monodromy matrix and the

\(^1\)formally the Floquet exponents
stable/unstable eigenvectors of the monodromy matrix, \( v^S \) and \( v^U \).

\[
v_i^S = \Phi(t_i, t_0)v_0^S \quad (2.23)
\]
\[
v_i^U = \Phi(t_i, t_0)v_0^U \quad (2.24)
\]

A small perturbation, \( \epsilon \), is then applied to the state of the orbit at that time, \( \bar{X}_i \), along the respective eigenvector. However, the magnitude of the eigenvectors grows exponentially along an unstable orbit and therefore must be normalized. Thus, the final state equations are given by

\[
\bar{X}_i^S = \bar{X}_i \pm \epsilon \frac{v_i^S}{|v_i^S|} \quad (2.25)
\]
\[
\bar{X}_i^U = \bar{X}_i \pm \epsilon \frac{v_i^U}{|v_i^U|} \quad (2.26)
\]

where in our implementation, \( \epsilon \) is set to 1E-10. The sign of perturbation dictates if it is an interior or exterior manifold. The eigenvector dictates if it is a stable or unstable manifold. Therefore there are four types of manifold arcs, \( \mathcal{W}_i^S \) = Stable primary, \( \mathcal{W}_i^S \) = Stable secondary, \( \mathcal{W}_i^U \) = Unstable primary, \( \mathcal{W}_i^U \) = Unstable secondary. The union of the evolution of all periodic orbit states that are perturbed in the unstable directions, constitute the unstable invariant manifolds. These perturb states will flow away from the periodic orbit forward in time. Similarly, perturbations of the periodic orbit in the stable directions will result in perturb states that flow away from the periodic orbit backwards in time. These invariant manifolds approximate global transport structures of the solar system. Koon et al. refer to it as a network of dynamic “super highways” and rightfully so, utilization of invariant manifolds of three-body systems can result in low-energy transfers between systems [14]. With the generation of these structures, it is possible to find low-energy transfers which can be optimized, which is the primary objective of the following section.
Figure 2.3: Stable exterior (red) and Unstable exterior (green) invariant manifold arcs emerging from a Lyapunov orbit around the $L_2$ Euler-Lagrange point.
CHAPTER 3
HYBRID OPTIMAL CONTROL FRAMEWORK

The HOC framework used in this study consists of a dual-loop architecture to enable automated trajectory optimization. An overview of this architecture is shown in Figure 3.1. The outer loop uses the non-dominated sorting genetic algorithm II (NSGA-II); for brevity it will be referred to as the GA. The GA selects and improves on categorical and real-valued parameters, which allow for the formulation of the inner loop optimal control problems. The inner loop problems are converted to nonlinear programs (NLP) and solved using the NLP solver SNOPT. Control parameters that have not been selected by the GA are randomly chosen at solution time by a heuristic search algorithm; monotonic basin hopping (MBH). The solution of these NLPs then provide the inputs for the fitness evaluation of the GA population. This framework allows for a global search of the nonlinear problem space with careful exploration of local minima for each candidate trajectory. Further, the NSGA-II enables the solution of multi-objective optimization problems, which enables the production of Pareto fronts; providing an engineer with knowledge of how to trade various mission components, e.g. mass, time of flight, science objectives, etc...

3.1 Outer Loop

The purpose of the outer loop is to pick categorical and real valued parameters that optimize the transfer. This is done using an evolutionary algorithm (EA). There are a few different multi-objective evolutionary algorithms (MOEA) that have been applied to trajectory optimization in the past. Genetic Algorithms have strong precedent in spacecraft trajectory op-

\footnote{This chapter contains previously published material from [1] and [2]. The copyright owner provides permission to reprint.}
Figure 3.1: The Hybrid Optimal Control framework.

Optimization with favorable results. [10], [11], [18], [19] For this application, the Non-dominated Sorting Genetic Algorithm (NSGA-II) was chosen due to its improved runtime (O(MN^2)) over its competitors and excellent results in other trajectory generators such as NASA Goddard’s Evolutionary Mission Trajectory Generator (EMTG). [11], [20] NSGA-II solves a multi-objective optimization problem, where the goal is to choose design variables $\vec{u}$, such that the components of a vector-valued function are minimized. A multi-objective problem is defined as:

$$\text{Minimize } \vec{J}(\vec{u}) = [J_1(\vec{u}), J_2(\vec{u}), ..., J_{n_{obj}}(\vec{u})]^T$$

subject to:

$$c(\vec{x}, \vec{u}) \leq 0$$

(3.1)

c(\vec{x}, \vec{u}) is a vector of constraint functions, often nonlinear, that must be satisfied for a solution to be feasible or optimal.
A GA mimics natural selection and reproduction of a population of possible solutions to evolve the selected parameters to an optimized design space. A population in this work, is a set of potentially feasible trajectory solutions, each characterized by their design variables. Each trajectory and its design variables are then referred to as an individual of the population. An initial population is created by setting the parameters in $\vec{X}$ to random values within defined bounds for each individual. Each individual is then evaluated based on the cost function and assigned a fitness value. The fitness value is the rank of the individual in comparison to the rest of the population. The individuals are ranked on their merit based on the objective functions. This forms the parent pool. Two parents are selected at random and are “mated”, to create two new children. The design parameters of the child population are a combination of their parents’ parameters. This process is known as a “crossover”. Mutations are then applied to the child generation to further randomize the design space. This is done by selecting random individuals and parameters from the child population and setting new random values for them. A mutated child will have some of its design parameters randomized, adding to the diversity of the population. This child population forms the parent population of the next generation. This process is then repeated until the maximum number of generation has been crossed.

Unlike gradient-based optimization algorithms, a GA does not require any knowledge of the derivative information. As the first generation is generated randomly, no initial guesses are necessary to start the optimization process. This also makes the GA a heuristic search algorithm which, unlike deterministic algorithms, is not confined to local search spaces. NSGA-II has also been applied to other trajectory optimization problems like interplanetary transfer to great success. [11], [19], [21], [22]

There are two types of transfers studied, impulsive and LT. Both of these transfers consider time-of-flight, $\Delta T$, as an objective. The impulsive case also optimizes for minimum $\Delta V$; that is magnitude of velocity change. The LT case optimizes for minimum fuel usage $\Delta M$. The total fuel expenditure in the impulsive case is defined as

$$\Delta V \equiv J_{\Delta V}(\vec{u}) = \sum_{i=1}^{i=n} \sqrt{\Delta V_{i,x}^2 + \Delta V_{i,y}^2}$$

(3.2)
In Eq. 3.2, the individual $\Delta V^2_{i,x}$ is the square of the change in the x-velocity at an impulse. $\Delta V^2_{i,y}$ is defined similarly. The subindex $i$ indicates the impulse at the $i$-th maneuver. The second objective function is the total time-of-flight $\Delta T$, defined as

$$\Delta T \equiv J_{\Delta T}(\vec{u}) = T_{parking} + T_{transfer} + T_{manifold}$$

(3.3)

In Eq. 3.3, $T_{parking}$ is the sum of time on parking orbits, $T_{transfer}$ is sum of transfer time for each phase, $T_{manifold}$ is the sum of time spent on manifolds and Lyapunov orbits. When considering LT missions, the goal is to maximize final mass. However, as the GA is minimizing all objectives, the negative of the objective value is considered. The objective function in this case is the negative of the initial mass minus the total mass consumed:

$$\Delta M \equiv J_{\Delta M}(\vec{u}) = -m_0 + \sum_{s \in S} b_s \Delta t_s u_s$$

(3.4)

where $m_0$ is the initial mass, $S$ is an index set for the LT burn segments of our transcription, $b_s$ is the constant mass flow rate for the $s$ segment, $\Delta t_s$ is the time interval of the $s$ segment, and $u_s \in [0, 1]$ is the duty cycle for the $s$ segment.

### 3.1.1 NSGA-II parameters

NSGA-II utilizes a fast non-dominated sorting approach that sorts individuals based on their domination rank. An individual $p_1$ dominates $p_2$ if, for all objectives, $p_1$ is better than or equal to $p_2$ and $p_1$ is strictly better than $p_2$ for at least one objective. The parent and child populations are both evaluated and individuals with the best (lowest) rank are selected for the new generation. These features allow the GA to solve multi-objective problems and create Pareto fronts, while maintaining diversity. The NSGA-II can be tuned by tweaking the mutation and crossover probabilities. A mutation probability, $P_{mut}$, and a crossover probability, $P_{cr}$, are set for both variants of NSGA-II and shown in Table (3.1). These probabilities dictate when a mutation and crossover occurs through the evolution process. Results can be improved by resizing the workspace. This is done by changing the bounds for the design parameters mentioned in Table (3.1). Single-phase transfers
<table>
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<th>Parameter</th>
<th>Description</th>
<th>value</th>
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<tr>
<td>$P_{mut}$</td>
<td>Probability of mutation</td>
<td>0.15</td>
</tr>
<tr>
<td>$P_{cr}$</td>
<td>Probability of crossover</td>
<td>1.0</td>
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</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{mut}$</td>
<td>Probability of mutation</td>
<td>0.05</td>
</tr>
<tr>
<td>$P_{cr}$</td>
<td>Probability of crossover</td>
<td>0.5</td>
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{gen}$</td>
<td>Maximum number of generations</td>
<td>100/200</td>
</tr>
<tr>
<td>$pop_size$</td>
<td>Size of population</td>
<td>48/96</td>
</tr>
</tbody>
</table>

Table 3.1: NSGA-II GA parameters

have a population size of 48 with $N_{gen}$ of 100. Multi-phase trajectories are more complex, thus requiring a larger $pop\_size$ of 96 and longer $N_{gen}$ of 200.

3.2 Variable length Chromosome transcription

Global optimization only considering transfers with a fixed number of phases, will have a fixed number of design parameters. This is convenient as standard genetic crossover operations can be applied to such populations. However, in general a mission designer will not know a priori the number of phases to be used for a given mission. It is thus advantageous to have a search method that autonomously uncovers solutions that may vary in the number of phases. But carrying out standard GA operations for varying length chromosomes, which results in our case for a population having different numbers of phases, is a difficult task. For multi-phase trajectories, the number of design parameters depends on the number of phases. Standard crossover operators can no longer be applied to these as the chromosomes might not be of the same length. Similar challenges are encountered in interplanetary trajectory optimization for multiple gravity assists. One way of solving this problem is the use of a fixed size chromosome with a hidden gene [23], or a null gene [11]. Another approach is segregating the population into subpopulations, with each subpopulation having a different chromosome length. This allows the use of standard crossover techniques [24], as each subpopulation only mates and evolves within itself.

The approach implemented in this study builds on the null gene and hidden
gene approaches. Each individual has a fixed chromosome length dictated
by the maximum number of phases, \( n_{\text{phasemax}} \), set for that problem. Each
individual is then given a \( n_{\text{phase}} \) value, which decides the number of phases
for that transfer trajectory. \( n_{\text{phase}} \) also dictates the number of active genes in
the chromosome. This transcription segregates genes as active and inactive,
where only the active genes are selected for crossover, mutation and fitness
evaluation.

A mission starts at a parking orbit, executes \( n_{\text{phase}} \) transfers and injects
to the target orbit. The continuous design parameters are broken into two
categories, parking orbits and manifold arcs. Parking orbit information is
represented by 2 genes, and manifold parameter information is represented
by 6 genes as described in Table 3.2. The discrete design parameters are
segregated into two categories as well, phase information is provided by 1 +
\( (n_{\text{phase}} - 1) \) genes, and manifold arc type information is represented by 2
genes as described in Table 3.3. Figure 3.2 illustrates the three possible
chromosomes available for an Earth-to-Moon transfer with a phase-max of
3. The active genes are shown in green, and the inactive genes in red. The
first row of genes are the "real" or continuous parameters chosen by the GA.
The second row of genes are the "binary" or discrete. The first chromosome
is for a three-phase transfer. Most of the genes are active, except some of
the \( t_{\text{transfer}} \) genes which are redundant. The second chromosome is for a
two-phase transfer, in which the second manifold arc is inactive, as well as
the second \( p_{\text{type}} \) as there are only two phases. The third chromosome is
for a single-phase or direct-to-goal transfer, the goal here being a parking
orbit around the Moon. Thus, both the manifold phases are inactive, as the
spacecraft will do a direct transfer. Both the phase types are inactive, as
there is no phase option other than to go to the goal.

3.3 Inner Loop

A transfer phase is defined as a two-point boundary value problem (TPBVP).
The TPBVP is then formulated as a non-linear problem (NLP) which can
be solved using any NLP solver. The solutions presented here have been
solved using the Sparse Nonlinear OPTimizer (SNOPT) [25]. Astrodynamics
problems tend to be sparse in nature due to formulation of the objective,
Figure 3.2: An example of a three-phase, two-phase and single-phase chromosome for an Earth to Moon transfer with a phase-max of 3. Active genes (green), inactive genes (red)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>Parking Orbit Parameters</td>
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</tr>
<tr>
<td>$t_{parking}$</td>
<td>Time of flight on parking orbit</td>
</tr>
<tr>
<td>$t_{transfer}$</td>
<td>Time of flight on transfer arc</td>
</tr>
<tr>
<td>Manifold Arc Parameters</td>
<td></td>
</tr>
<tr>
<td>$t_{transfer}$</td>
<td>Time of flight on transfer arc</td>
</tr>
<tr>
<td>$\tau_2^S$</td>
<td>Time of flight on stable manifold arc</td>
</tr>
<tr>
<td>$\tau_1^S$</td>
<td>Time of flight on Lyapunov orbit</td>
</tr>
<tr>
<td>$E^*$</td>
<td>Energy of Lyapunov orbit for Earth</td>
</tr>
<tr>
<td>$\tau_1^U$</td>
<td>Time of flight on Lyapunov orbit</td>
</tr>
<tr>
<td>$\tau_2^U$</td>
<td>Time of flight on unstable manifold arc</td>
</tr>
</tbody>
</table>

Table 3.2: NSGA-II Parameters - Continuous

<table>
<thead>
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<th>Parameter</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>Mission Discrete Parameters</td>
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<tr>
<td>$n_{phase}$</td>
<td>Number of phases</td>
</tr>
<tr>
<td>$p_{type}$</td>
<td>Type of phase</td>
</tr>
<tr>
<td>Manifold Arc Discrete Parameters</td>
<td></td>
</tr>
<tr>
<td>$W_1^S$</td>
<td>Type of stable manifold (interior or exterior)</td>
</tr>
<tr>
<td>$W_1^U$</td>
<td>Type of unstable manifold (interior or exterior)</td>
</tr>
</tbody>
</table>

Table 3.3: NSGA-II Parameters - Discrete

control vectors and constraints, and thus sparse solvers like SNOPT provide significant computational advantages. SNOPT solves non-linear problems of
the form:

\[
\begin{align*}
\text{Minimize } f(\vec{x}) \text{ subject to: } \\
\vec{x}_{lb} \leq \vec{x} \leq \vec{x}_{ub} \\
c(\vec{x}) \leq 0 \\
A\vec{x} \leq 0
\end{align*}
\] (3.5)

where \(\vec{x}_{lb}\) and \(\vec{x}_{ub}\) are the lower and upper bounds on the decision vector \(\vec{x}\), \(c(\vec{x})\) is a vector of nonlinear constraints, and \(A\) is a matrix for linear constraints. The initial guesses for the NLP control parameters are seeded by a stochastic search algorithm, Monotonic Basin Hopping (MBH), which is discussed later.

### 3.3.1 Low-Thrust Transcription

The inner loop solves optimal control problems; both impulsive and LT. In the LT case, the finite-burn low-thrust (FBLT) transcription is used. Figure 3.3 offers a pictorial representation of a single FBLT phase.

![Figure 3.3: A single FBLT phase with unsatisfied match-point constraints.](image)
The outer loop solver specifies boundary conditions for each FBLT phase; in the three-body problem these are states on manifold arcs, libration point orbits as well as various parking orbits. The phase is broken into several segments; the examples in this study used 10 segments total for both forward and backward propagation half-phases of each FBLT phase. SNOPT has the ability to choose $3 + 2n$ control parameters for each phase, where $n$ is the number of segments. The first 3 control parameters are coast times at the start and end of the boundary conditions and the total time-of-flight for the phase. The $2n$ parameters consist of a duty cycle $u_i$ and thrust angle $\phi_i$ for each segment. In the current framework, engine models with constant mass flow rate and $I_{sp}$ are used, giving a maximum thrust of 1N. The thrust angle and thrust level are held constant over segments. Figure 3.3 indicates that varying the thrust level on each segment is possible and will be done in future work; this is shown by the red arrows varying in length along some segments. See Ellison [26] for more information regarding engine modeling and analytic derivatives that can be used with this transcription.

The $3 + 2n$ initial control parameters for the FBLT phase are chosen by MBH using a Pareto distribution. SNOPT then solves an NLP based on these initial guesses. The constraints included in the NLP include bounds on the control variables, which are provided by a user. The coast times in non-dimensional time units must be $[0, 5]$, and time-of-flight for each phase $[0, 30]$; in Earth days this corresponds to maximums of 21.7 and 130.2 days respectively. The controls are chosen such that $u_i \in [0, 1]$, and $\phi_i \in [0, 2\pi)$. Additional constraints are given by the match-point constraints: $|x_F^\dagger - x_B^\dagger|$; these states are shown in Figure 3.3. The match-point constraints ensure continuity of the position, velocity and also the mass (although not shown). Thus, SNOPT has 23 control parameters available to optimize a trajectory. These parameters are summarized in Table (3.4).

3.3.2 Low Thrust Spiral Approximation

The current framework makes use of LT spiral approximations. In this case, the only control parameter is the final energy at which the spiral should terminate. The $L_1$ point is used as a reference, allowing the energy control parameter to be choosen within 250E-4 of the $L_1$ energy. The LT spiral is
<table>
<thead>
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<th>Parameter</th>
<th>Description</th>
<th>Lower bound</th>
<th>Upper bound</th>
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</thead>
<tbody>
<tr>
<td>$dt_{1/2}$</td>
<td>Half transfer time</td>
<td>0.0</td>
<td>15.0 (65.139 days)</td>
</tr>
<tr>
<td>$dt_c^f$</td>
<td>Forward coast time</td>
<td>0.0</td>
<td>3.0 (13.029 days)</td>
</tr>
<tr>
<td>$dt_c^b$</td>
<td>Backward coast time</td>
<td>0.0</td>
<td>3.0 (13.029 days)</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>Forward arc thrust angle</td>
<td>0.0</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>$u_1$</td>
<td>Forward arc thrust percent</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\theta_{10}$</td>
<td>Forward arc thrust angle</td>
<td>0.0</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>$u_{10}$</td>
<td>Forward arc thrust percent</td>
<td>0.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 3.4: SNOPT control parameters

simply a thrust in the anti-velocity direction with full thrust approximation; this is 1N in the examples to be shown. The spiral maximizes the change in energy for a fixed time.

### 3.3.3 Impulsive Transcription

Impulsive missions are solved with phases similar to that shown in Figure 3.3, but with far less complexity. For the problems shown here, there are $3+6$ control parameters to be seeded with MBH and solved with SNOPT for each impulsive phase. Again the boundary conditions of the impulsive phase are set by the outer loop. The 3 time parameters are the same as that of FBLT; consisting of coasts and time-of-flight with the same bounds. The 6 control parameters to be chosen are $\Delta v_1, \phi_1, \Delta v_m, \phi_m, \Delta v_2, \phi_2$ where the subscript 1 indicates the thrust parameters after the initial coast time, $m$ is a thrust at the match-point, and 2 is a thrust before commencing a final coast. Again $\phi$ is the thrust angle in $[0, 2\pi)$ and $\Delta v$ is a magnitude change of velocity that is in the range $[0, 5]$, corresponding to a max 5.1 km/s maneuver. Thus, SNOPT has 9 control parameters available. These parameters are summarized in Table (3.5).

All NLP solvers, including SNOPT, require an initial guess for the solution. The solutions then obtained are within some neighbourhood of that initial guess. Therefore each solution will end up at a local minimum of the neighbourhood. Initial guesses can be generated in many different ways. For a simple TPBVP, a two-body Lambert arc can be used as an initial guess.
Another approach is using approximations generated by shape-based methods. [27], [28] One of the more recent and successful methodologies utilized in the field of spacecraft trajectory design is Monotonic Basin Hopping (MBH), which is robust, generalized and can autonomously create initial guesses for a wide range of problems. [11]

### 3.3.4 Monotonic Basin Hopping

MBH is a multistart algorithm that stochastically searches a solution space and utilizes an NLP solver for local optimization [29]. It traverses through the search space via random walks to find globally optimal solutions. In problems such as this, there are usually several local minima, often in clusters where one local minima is better than others. Thus, the goal of MBH is to explore and exploit the local minima in the cluster. First, MBH picks a random initial guess \( \vec{x} \). SNOPT is run using \( \vec{x} \) till a feasible solution \( \vec{x}^* \) is found. If no solution is found, then that point is discarded and a new random point is provided. Having found a feasible solution, MBH applies a random perturbation vector to \( \vec{x}^* \). This is known as a “hop”. SNOPT is then run with the new perturbed vector \( \vec{x}_p^* \). If the new solution is feasible and better than the previous solution, \( \vec{x}_p^* \), then it replaces \( \vec{x}^* \) as the new best solution and the MBH process is applied again. If the new solution is not feasible or inferior, MBH performs a new hop from the current \( \vec{x}^* \) and a \( MBH_{not\_improve} \) counter is incremented. If \( MBH_{not\_improve} \) exceeds a predefined threshold value, MBH resets and generates a new random guess \( \vec{x} \). All feasible solutions are stored. The “reset” operator allows MBH to explore the global search.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Lower bound</th>
<th>Upper bound</th>
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</thead>
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<td>( dt_{1/2} )</td>
<td>Half transfer time</td>
<td>0.0</td>
<td>15.0 (65.139 days)</td>
</tr>
<tr>
<td>( dt^f_c )</td>
<td>Forward coast time</td>
<td>0.0</td>
<td>3.0 (13.029 days)</td>
</tr>
<tr>
<td>( dt^b_c )</td>
<td>Backward coast time</td>
<td>0.0</td>
<td>3.0 (13.029 days)</td>
</tr>
<tr>
<td>( \theta^f )</td>
<td>Forward arc velocity angle</td>
<td>0.0</td>
<td>( 2\pi )</td>
</tr>
<tr>
<td>(</td>
<td>v</td>
<td>^f )</td>
<td>Forward arc velocity magnitude</td>
</tr>
<tr>
<td>( \theta^m )</td>
<td>Mid-course velocity angle</td>
<td>0.0</td>
<td>( 2\pi )</td>
</tr>
<tr>
<td>(</td>
<td>v</td>
<td>^m )</td>
<td>Mid-course velocity magnitude</td>
</tr>
<tr>
<td>( \theta^b )</td>
<td>Backward arc velocity angle</td>
<td>0.0</td>
<td>( 2\pi )</td>
</tr>
<tr>
<td>(</td>
<td>v</td>
<td>^b )</td>
<td>Backward arc velocity magnitude</td>
</tr>
</tbody>
</table>

Table 3.5: SNOPT control parameters
space effectively while the “hop” operator exploits the clusters to find the best local minima. MBH is run until the stopping conditions are met, which can be either a certain number of iterations or some maximum time limit. At the end of the process, the best solution is returned.

To summarize, MBH explores the search space until a feasible solution is found. Then, the cluster is exploited to find better local minimums. The exploitation process continues until the $MBH_{not\_improve}$ threshold is crossed, at which MBH does a global reset and returns to exploration of the search space. The algorithm is explained in Algorithm 1 given below.

**Algorithm 1 MBH + NLP algorithm**

```
generate random point $x$
run SNOPT to obtain $x^*$ using $x$ as initial guess
if no feasible solution then
    generate new random point $x$
elseredef $x = x^*$
    store $x^*$
end if
while stopping condition not met do
    perturb $x$ to generate $x'$
    run SNOPT to obtain $x^*$ using $x'$ as initial guess
    if $x^*$ is feasible and $f(x^*) \leq f(x)$ then
        $x = x^*$
        store $x^*$
    else if $x^*$ is infeasible and $MBH_{not\_improve}$ exceeded then
        reset
    end if
end while
return best $x^*$
```

MBH has two stopping conditions, $MBH_{max\_runtime}$ and $MBH_{max\_trials}$ as described in Table (3.6) whereas SNOPT has a single stopping condition $SNOPT_{max\_runtime}$. The values of these variables had to be tweaked over a few test runs to provide a good trade off between exploration and optimization. Due to the complexity of the problem, SNOPT is generously capped at a 5 second limit to optimize a “good” initial guess. Thus, beyond 5 seconds the initial guess is considered “bad” and is reset. $MBH_{max\_runtime}$ is set to 120 seconds so as to guarantee enough exploration of the search space and to obtain atleast one feasible solution. This provides atleast 20-24 MBH
iterations, which is sufficient.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$MBH_{not_improve}$</td>
<td>Max # of iterations without improving</td>
<td>100</td>
</tr>
<tr>
<td>$MBH_{max_stepsize}$</td>
<td>Max stepsize of “hop”</td>
<td>1.0</td>
</tr>
<tr>
<td>$MBH_{max_runtime}$</td>
<td>Max CPU runtime</td>
<td>120 (seconds)</td>
</tr>
<tr>
<td>$MBH_{max_trials}$</td>
<td>Max # of iterations</td>
<td>10,000</td>
</tr>
<tr>
<td>$SNOPT_{f_tol}$</td>
<td>SNOPT tolerance for feasible solutions</td>
<td>1E-2</td>
</tr>
<tr>
<td>$SNOPT_{o_tol}$</td>
<td>SNOPT tolerance for optimal solutions</td>
<td>1E-6</td>
</tr>
<tr>
<td>$SNOPT_{max_runtime}$</td>
<td>SNOPT CPU runtime</td>
<td>5 (seconds)</td>
</tr>
</tbody>
</table>

Table 3.6: Inner Loop parameters

Each decision variable in the vector $\vec{x}$ must have an upper and lower bound. These bounds are predefined as seen in Table (3.5) and (3.4). Other researchers have investigated the effects of different distributions on the efficiency of MBH. The use of long tailed distributions such as Cauchy or Pareto distributions has been noted to improve MBH efficiency in certain classes of problem [30]. Some quick test runs indicated that picking random values from a uniform distribution provide satisfactory results. In the future, the effect of distributions, in particular long tailed distributions will be investigated further.
The HOC framework is now applied to missions in Cis-Lunar space. Single phase trajectories from Earth-to-$EML_2$, and $EML_1$ to $EML_2$, and multi-phase trajectories from Earth-to-$EML_2$ and Earth-to-Moon are investigated. Solutions are illustrated as a family of Pareto fronts trading $\Delta V$ vs. Time-of-Flight for the impulsive missions, and Final Mass vs. Time-of-Flight for the low-thrust missions. The mathematical definitions of these objects are expressed in Equations 3.2, 3.3, 3.4. For the remainder of the section, the following nomenclature will be used to highlight multi-phase solutions in Pareto fronts using color. Single phase transfers will be marked in black, 2-phase transfers in red, and 3-phase transfers in blue. For trajectories, stable manifolds will be depicted in green, unstable manifolds in red, parking orbits and low-thrust spirals in blue, transfer arcs in black, and coast arcs in light gray. The coast arcs here represent coasting periods along the boundary conditions selected by inner loop.

4.1 Impulsive transfer: single-phase

The parameters for the mission are described in Table (4.1). Missions starting at Earth begin in a circular parking orbit with an altitude of 800 km. The spacecraft then executes impulsive maneuvers to connect to manifold arcs (if necessary) and inject into the goal orbit at $EM-L_2$. $EM-L_1$ to $EM-L_2$ transfers are as expected, starting at a $L_1$ orbit, coasting along the unstable manifold arc, and then transferring to the $EM-L_2$ stable manifold arc.

\footnote{This chapter contains previously published material from [1] and [2]. The copyright owner provides permission to reprint.}
Table 4.1: Parking orbit parameters for two example missions using impulsive propulsion; Earth to $EM-L_2$ transfer and Earth to Lunar transfer

<table>
<thead>
<tr>
<th>Mission Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parking orbit altitude</td>
<td>800</td>
<td>km</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.0</td>
<td>-</td>
</tr>
</tbody>
</table>

4.1.1 Earth-to-$EM-L_2$ mission (Single-phase)

Figure 4.1 illustrates a single smooth pareto front for a single-phase transfer from LEO to $EM-L_2$. There are two clusters that can be seen for the $\sim$ direct transfers (left) and $\sim$ low-energy transfers that connect to the stable manifolds using a small $\Delta V$ (right). The minimum-fuel trajectory seen in Figure 4.2 takes about 30 days and utilizes a low lunar insertion point to minimize the $\Delta V$ required for the transfer.

Figure 4.1: Family of pareto fronts for a LEO to $EM-L_2$ single-phase impulsive transfer
4.1.2 $EM-L_1$-to-$EM-L_2$ mission (Single-phase)

This mission begins at an $L_1$ orbit of an arbitrary energy, and transfers to an $L_2$ orbit of an arbitrary energy. Both energies are selected by the outer loop. The pareto front in Figure 4.3 is a single smooth pareto showing low-energy transfers ranging from $\sim 23$ to $\sim 25$ days in time of flight. The minimum-fuel transfer shown in Figure 4.4 requires $\sim 0.5 \text{ cm/s}$ of $\Delta V$, indicating that the natural transfer connecting the two libration point orbits was found. This mission was not studied using LT propulsion as the HOC framework was able to find the natural transfers for this mission, indicating that the same could be done for LT mission.

4.2 Low-thrust transfer: single-phase

The parameters for the parking orbit, spacecraft and mission are described in Table (4.2). Missions starting at Earth begin in an eccentric parking orbit at GTO with a semi-major axis of 24000 km. The spacecraft then executes
low-thrust manuevers to connect to manifold arcs (if necessary) and inject into the goal orbit at $L_2$.

Table 4.2: Parking orbit and spacecraft parameters for two example missions using low-thrust propulsion; Earth to $EM-L_2$ transfer and Earth to Lunar transfer.

<table>
<thead>
<tr>
<th>Mission Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parking orbit altitude</td>
<td>24000</td>
<td>km</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.7263</td>
<td>-</td>
</tr>
<tr>
<td>Initial mass</td>
<td>1000.0</td>
<td>kg</td>
</tr>
<tr>
<td>Isp</td>
<td>1000.0</td>
<td>s</td>
</tr>
<tr>
<td>$b$</td>
<td>1.0196E-4</td>
<td>kg/s</td>
</tr>
</tbody>
</table>

4.2.1 Earth-to-$EM-L_2$ mission (single-phase)

The pareto in figure 4.5 shows a cascading structure, indicating lackluster gains in mass fraction for trajectories with time of flights ranging from $\sim 25$ to $\sim 43$ days. Beyond that, $\sim 2\%$ mass fraction improvements can be gained
Figure 4.4: Minimum-fuel $EM-L_1$ to $EM-L_2$ transfer. $\Delta V = 5E-6$ km/s, $\Delta T = 48.95$ days at most, as demonstrated by the maximum delivered mass solution in figure 4.7. On the other side of the front, the minimum-time solution shown in figure 4.6 takes only $\sim$ 20 days with a mass fraction of $\sim$ 64 %. However the trade-off trajectory here at the center of the front provides a mass fraction of $\sim$ 68 % for only $\sim$ 3 more days in time of flight.

4.3 Single-phase summary

Overall, the single-phase transfers converge smoothly to a single pareto front for the impulsive and LT cases. The HOC framework is successful at obtaining a trade of solutions for the dual objectives while also finding the minimum objective solutions. Figures 4.1, 4.3, 4.5 show the growth of pareto fronts from generations 20, 50, and 100, with the final generation marked in solid black.

For the Earth-to-$EM-L_2$ case, the search space for time of flight is about 5 to 33 days. It is clear that single-phase solutions form the short time of flight
section of the solution space. In contrast, multi-phase solutions will show how that trade space can be extended and searched for more complex chains that provide greater ΔV savings. This is shown in the following section where the variable length chromosome transcription is utilized to solve for multi-phase solutions.

4.4 Impulsive transfer: multi-phase

The parameters for the parking orbit, final orbit, spacecraft and mission are described in Tables (4.3), (4.4). Missions start at Earth in a circular parking orbit with an altitude of 800 km. The spacecraft then executes impulsive maneuvers to connect to manifold arcs (if necessary) and inject into the goal orbit at \( L_2 \) or the Moon depending on the mission.
Figure 4.6: Minimum-time single-phase low-thrust transfer from GTO to $EM-\mathcal{L}_2$, $\frac{M_f}{M_o} = 64.3\%$, $\Delta T = 20.34$ days.

<table>
<thead>
<tr>
<th>Mission Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parking orbit altitude</td>
<td>800</td>
<td>km</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Lunar orbit altitude</td>
<td>5000</td>
<td>km</td>
</tr>
</tbody>
</table>

Table 4.3: Parking orbit parameters for two example missions using impulsive propulsion; Earth to $EM-\mathcal{L}_2$ transfer and Earth to Lunar transfer

<table>
<thead>
<tr>
<th>Mission Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_2$ Orbit</td>
<td></td>
</tr>
<tr>
<td>Max # of phases</td>
<td>2</td>
</tr>
<tr>
<td>Phase types</td>
<td>Earth-Moon $L_1$</td>
</tr>
<tr>
<td>Lunar Orbit</td>
<td></td>
</tr>
<tr>
<td>Max # of phases</td>
<td>3</td>
</tr>
<tr>
<td>Phase types</td>
<td>Earth-Moon $L_1$, Earth-Moon $L_2$</td>
</tr>
</tbody>
</table>

Table 4.4: Mission parameters for two example missions using impulsive propulsion; Earth to $EM-\mathcal{L}_2$ transfer and Earth to Lunar transfer

4.4.1 Earth-to-$EM-\mathcal{L}_2$ mission (multi-phase)

Figure 4.8 illustrates candidate solutions for an Earth-to-$L_2$ mission. As expected, the single phase trajectories have a shorter time-of-flight. The
minimum-fuel trajectory is a two-phase transfer shown in Figure 4.9. The trajectory passes near the secondary and onto the stable $W_i^S$ manifold of an $L_1$ Lyapunov orbit. It then coasts along the Lyapunov orbit and perturbs off the unstable $W_e^U$ manifold arc and uses an impulse to coast to the $W_i^S$ manifold arc of the $L_2$ Lyapunov orbit where a final insertion burn is executed. The total time-of-flight of the transfer is 187 days, with a total $\Delta V$ of 3.15 km/s. Though this solution uses the least fuel, the time-of-flight is quite long. Analyzing the Pareto front reveals solutions that trade fuel savings with better time-of-flight.

Figures 4.10 and 4.11 shows two characteristic transfers from Earth to $L_2$. These kinds of transfers are often found as the minimum-fuel transfers for a single-phase Earth to $L_2$ transfer [1]. The short and long transfers possess similar fuel savings but significantly different time-of-flights, with the long transfer taking almost twice as long. The short transfer takes 2.22% more fuel than the two-phase minimum fuel transfer while requiring 138 fewer days.
4.4.2 Earth-to-Moon mission (multi-phase)

The family of Pareto fronts seen in Figure 4.12 illustrate a segregation in the front between single-phase and multi-phase trajectories. The lighter colored points are from earlier generations of the front, providing information on the evolution of trajectories with respect to the number of phases. The single-phase trajectories are few initially but converge more quickly to their final values. Comparatively, the three-phase trajectories are spread out all over the search space and take longer to converge, as indicated by the slower movement of the clusters. All single-phase trajectories possess $\Delta V$s greater than 4.2 km/s, and a $\Delta T$ of less than 20.0 days. There are a few two-phase solutions that are of interest, as seen in Figure 4.15. The majority of three-phase solutions require less than 4.0 km/s and take longer than 100 days to complete.

A simple direct transfer with a time-of-flight of 3.09 days to the Moon is shown in Figure 4.13. The minimum-fuel three-phase transfer is seen in Figure 4.15. The spacecraft links from a parking orbit to an $L_1$ Lyapunov
Figure 4.9: Minimum-fuel two-phase LEO to $EM-L_2$ transfer. $\Delta V = 3.15$ km/s, $\Delta T = 187.00$ days, marked with red diamond on Figure 4.8.

orbit and then to an $L_2$ Lyapunov orbit, and finally to a parking orbit around the Moon; a long transfer of $\sim 153$ days, but requiring only 3.80 km/s. In comparison, the two-phase transfer from Figure 4.15 uses only 1.84% more fuel while saving $\sim 87$ days in transfer time.

4.5 Low-thrust transfers: multi-phase

The parameters for the parking orbit, final orbit, spacecraft and mission are described in Tables (4.5), (4.6). Missions starting at Earth begin in an eccentric parking orbit at GTO with a semi-major axis of 24000 km. The spacecraft then executes low-thrust maneuvers to connect to manifold arcs (if necessary) and inject into the goal orbit at $L_2$. 
4.5.1 Earth-to-$L_2$ mission (multi-phase)

The family of Pareto fronts in Figure 4.16, show a clear distinction between the single-phase and two-phase trajectories. The two-phase transfer offers superior payload capacity, however the trade-off with time-of-flight is not as impressive as the impulsive cases. The maximum delivered mass transfer shown in Figure 4.17 delivers 11.03% more payload for $\sim$3 times the transfer time compared to the minimum-time solution. Comparitively, the trajectory
Figure 4.11: Long ($\Delta V = 3.24$ km/s, $\Delta T = 95.34$ days) single-phase transfers from LEO to $EM-L_2$. Marked with black diamonds on Figure 4.8.

in Figure 4.19 trades 9.93\% more payload for 10 more days.

4.5.2 Earth-to-Moon (multi-phase)

The Earth-to-Moon Pareto front seen in Figure 4.20 is filled with single-phase trajectories for all except one lone two-phase trajectory. The two-phase trajectory from Figure 4.23 is the maximum delivered mass solution in the search, providing 10.62\% more payload capacity compared to the minimum-

<table>
<thead>
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<th>Mission Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_2$ Orbit</td>
<td></td>
</tr>
<tr>
<td>Max # of phases</td>
<td>2</td>
</tr>
<tr>
<td>Phase types</td>
<td>Earth-Moon $L_1$</td>
</tr>
<tr>
<td>Lunar Orbit</td>
<td></td>
</tr>
<tr>
<td>Max # of phases</td>
<td>3</td>
</tr>
<tr>
<td>Phase types</td>
<td>Earth-Moon $L_1$, Earth-Moon $L_2$</td>
</tr>
</tbody>
</table>

Table 4.6: Mission parameters for two example missions using low-thrust propulsion; Earth to $EM-L_2$ transfer and Earth to Lunar transfer.
time transfer shown in Figure 4.19 on the left, while taking $\sim 72$ days longer. However, similar to the Earth-to-$L_2$ mission, it offers a small increase compared to other solutions with a significantly shorter transfer time. Figure 4.19 (left) represents a more favorable trade-off, providing 8.31% more payload compared to the minimum-time transfer, for only an additional $\sim 12$ days in transfer time.

4.6 Multi-phase summary

The four example missions demonstrate the trade-offs available to a mission design engineer when investigating multi-objective problem spaces as compared to solving multiple single-objective problems. The minimum/maximum objective solutions are interesting to study, but from a mission design perspective, the solutions of interest are in the middle of the Pareto space trading both objectives as observed from the examples provided above. These solutions tend to be harder to find and require further exploration of the solution
Figure 4.13: Minimum-time single-phase LEO to Lunar transfer trajectory. $\Delta V = 4.68$ km/s, $\Delta T = 3.09$ days. Marked with a black diamond on Figure 4.12.

space. The Pareto fronts also provide valuable information on the evolution of the multi-phase trajectories. Figure 4.20 shows a surprising lack of two-phase trajectories. This is also seen in Figure 4.12 where single-phase and three-phase trajectories are seen in abundance, but two-phase trajectories which could offer superior trade offs are fewer. This can be attributed to the increased difficulty of solving problems in multi-body systems. It can also be due to the architecture implemented in our framework. If an individual phase from a multi-phase trajectory is unable to converge to a feasible solution, then the entire chromosome is marked as poor and penalized heavily in its objective values to deter other individuals from mating with that individual. This can be mitigated by keeping track of which phase fails to converge, marking those genes as inactive, and only penalizing those particular phases.

The stronger conclusion from these results is that multi-phase solutions take longer to converge to optimized solutions, as evident from the Pareto fronts. Therefore it may be best to give an opportunity to evolve before being put in the same pool as single-phase solutions. This provides motivation to investi-
Figure 4.14: Minimum-fuel three-phase LEO to Lunar transfer trajectory. $\Delta V = 3.80$ km/s, $\Delta T = 153.20$ days. Marked with a blue diamond on Figure 4.12.

gate the impact of dynamic population sizing [24] for global optimization in multi-body regimes. Having specific bins for single-phase, two-phase, three-phase etc. for the first quarter generations would allow ample exploration of the multi-phase solution space and possibly create a more diverse Pareto front. The null gene approach has been shown to provide superior flexibility in the evolution of multi-phase trajectories and is the natural next step from the current hidden gene formulation [11]. An important feature observed in the trajectories for all the missions, both impulsive and low-thrust cases, is the presence of coast arcs (show in light gray) picked by the NLP solver. The GA picks parameters to generate the optimal control boundary conditions, but it is the NLP that chooses the coasting time away from these boundary conditions. This allows the trajectories to evolve much quicker and brings to light an interesting trade study of separating and mixing the parameter space between the outer loop and inner loop. This gets more challenging once you involve external automation and optimization capabilities, like automated Poincaré intersection detection [12], [31].
Figure 4.15: Two-phase LEO to Lunar transfer trajectory. $\Delta V = 3.87$ km/s, $\Delta T = 65.98$ days. Marked with a red diamond on Figure 4.12.

Figure 4.16: Family of Pareto fronts for a GTO to $EM-L_2$ low-thrust transfer.
Figure 4.17: maximum delivered mass two-phase low-thrust transfer from GTO to \( EM-L_2 \). \( \frac{M_f}{M_o} = 74.545 \% \), \( \Delta T = 148.63 \) days. Marked with a red diamond on Figure 4.16.

Figure 4.18: Multi-objective trade-off \( \frac{M_f}{M_o} = 73.764 \% \), \( \Delta T = 54.95 \) days)
Figure 4.19: Minimum-time ($\frac{M_f}{M_o}=67.097\%$, $\Delta T=44.99$ days) single-phase low-thrust transfers from GTO to $EM-L_2$. Marked with black diamonds on Figure 4.16.

Figure 4.20: Family of Pareto fronts for a GTO to Lunar low-thrust transfer.
Figure 4.21: Minimum-time single-phase GTO to Lunar low-thrust transfer. \(\frac{M_f}{M_o} = 65.053\%\), \(\Delta T = 35.00\) days. Marked with a black diamond on Figure 4.20.

Figure 4.22: Multi-objective trade-off \(\frac{M_f}{M_o} = 70.464\%\), \(\Delta T = 47.27\) days) single-phase GTO to Lunar low-thrust transfers. Marked with black diamond on Figure 4.20.
Figure 4.23: maximum delivered mass two-phase GTO to Lunar low-thrust transfer. $\frac{M_f}{M_o} = 71.967 \%$, $\Delta T = 108.11$ days. Marked with a red diamond on Figure 4.20.
CHAPTER 5
CONCLUSION AND FUTURE WORK

5.1 Conclusion

The multi-phase extension of the HOC framework is successful at producing families of Pareto fronts for a variety of missions; using both impulsive and low-thrust propulsion. The Earth-to-EM-$L_2$ and Earth-to-Moon impulsive missions are strong candidates for using multi-phase trajectories, as demonstrated by the merits of Figure 4.19 and Figure 4.15. The Pareto fronts in Figure 4.12 and Figure 4.16, illustrate a large portion of the problem space that is solved by multi-phase trajectories, which would otherwise be unexplored. Generating solutions without the automated HOC framework would require manually selecting the phase types and manifold parameters, generating Poincaré maps, identifying intersections of interest, and then seeding a differential corrector with the hand-selected patch-points to obtain a feasible solution or a control transcription with non-linear programming to hopefully yield a locally optimal solution. For multi-phase trajectories, this time-consuming process would need to be repeated multiple times. The framework presented here eliminates that required human effort and allows for rapid prototyping of missions while exploring a wide variety of solutions that may not be intuitive for human engineers. The low-thrust missions solved here are for a specific propulsion system and initial mass. To replicate the effort for another propulsion system or initial mass would require a considerable effort without the use of this tool. The entire process of generating Poincaré maps and intelligently choosing patch-points would have to be repeated for every change in mission constraints, parameters or goals. Doing so with the automated HOC framework requires tweaking only a few values in the problem.

\footnote{This chapter contains previously published material from [1] and [2]. The copyright owner provides permission to reprint.}
setup script, without the need to even recompile the code.

5.2 Future work

The goal of this project was to lay foundation for mission design tool that automates rapid prototyping of preliminary trajectories for a wide variety of missions. To enable this, the HOC framework presented in this document was developed. To expand the capabilities of this mission design tool, several features and improvements need to be made. This is reflected in the works of [31],[13],[12],[32].

5.2.1 Multi-phase transcription improvements

Moving forward, the multi-phase transcription must be improved to be more robust and flexible. For starters, information from individual parts of a multi-phase trajectory must be exploited in crossover operations. A null gene formulation will be implemented due to its improved flexibility and better search of the combinatorial space. This is crucial for the convergence of the GA, especially when the choice of the number of DS increases. Dynamic population sizing will be investigated to gauge in merits for evolution of multi-phase trajectories in separated bins before mixing in the population.

5.2.2 Inner loop problem formulation

The inner loop problem formulation can be improved by incorporating the outer loop parameters. Having the outer loop search the global space, and the inner loop tweak the local space has been shown to improve results as supported by preliminary tests. Including the generation of the DS in the inner loop widens the local search space, thus reducing the number of generations needed to converge to better solutions. The trade-off here is the significant increase in runtime for generating and integrating the DS.
5.2.3 Rapid generation of DS

To enable the generation of DS in the inner loop without significantly increasing the runtime, approximation methods are a likely candidate. Initial studies in cubic convolution and other methods have yielded favorable results. Taking lessons learnt from that study and incorporating them into the HOC framework is necessary before improving the inner loop formulation. Another possible approximation scheme is Machine Learning (ML) approaches that use regression to create approximate models.

5.2.4 Resonance orbits and manifolds

Incorporation of resonance orbits and manifolds in the HOC framework is an important milestone to endgame and tour design. Initial applications of the HOC framework to missions involving transfer to resonance orbits have proven very successful. The next step starts with more robust generation of various families of resonance orbits in different systems. This is necessary to properly investigate the options available for such missions, like the Europa Clipper/Lander.

5.2.5 Automated manifold intersection Detection

Automated Poincaré surface intersection is necessary to exploit advanced dynamical structures and patched three body transfers between different systems. Preliminary results of such efforts have been implemented by Aurich et al. to great success. In the future, the computational runtime and parallelization of that tool must be expanded and incorporated fully into the HOC framework to improve runtime.

5.2.6 Four-body models

Certain missions may benefit from exploiting the gravitational perturbations of the Sun, and while patch three body solutions are good approximations for such transfers, full optimization in a four-body model would provide a much better approximation of the trajectory and benefit from not being constrained by the patch three body solution space.
5.2.7 Validation of trajectories

While the three-body problem is a good model for rapid prototyping trajectories and building intuition, higher fidelity models are necessary to evaluate the feasibility of these trajectories in real world scenarios. Firstly, these trajectories must be optimized in a full ephemeris model with additional perturbations such as solar radiation pressure (SRP) and J2 perturbations for the Earth, Jupiter etc. Secondly, these trajectories must be evaluated in a flight ready mission analysis tool like NASA GSFC’s General Mission Analysis Tool (GMAT) [33]. GMAT is a space mission design software system that includes high-fidelity space system models, local optimization and targeting capabilities, as well as user features like the fully-featured interactive Graphical User Interface (GUI) with customizable plots, reports and data products. Having the ability to port trajectories to GMAT is an important feature that will validate the trajectories generated in simpler models.
BIBLIOGRAPHY


