ASSESSMENT OF MOVING-MASS ACTUATORS FOR HYPERSONIC VEHICLES WITH DEPLOYABLE DECELERATORS

BY
KEVIN LOHAN

THESIS
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Adviser:
Assistant Professor Zachary R. Putnam
This study assesses internal moving-mass actuator configuration options for trajectory control in the hypersonic regime of planetary entry. Trajectory control is achieved by shifting the location of the center of gravity relative to the center of pressure to modify aerodynamic trim conditions. The vehicle is modeled as a cylinder with a deployable forebody and a moving-mass actuator that can translate along a linear track. Placing the track in the rear of the vehicle can reduce the required actuator mass fraction for a specific trim lift-to-drag ratio by up to 5%. Increasing the length of the track similarly reduces required mass fraction. Vehicle packaging density and size do not significantly influence the required actuator mass; geometric properties such as length-to-diameter ratio and the diameter of the deployable impact the required actuator mass. Using these design guidelines, and actuator mass fraction of approximately 13% is required to achieve a maximum lift-to-drag ratio similar to the Mars Science Laboratory. A range of expected hypersonic flight conditions are analyzed to determine their impact on the achievable lift-to-drag ratio. For a moving-mass actuator mass fraction of 1%, the available lift-to-drag ratio varies between 0.02 and 0.06; for a 5% mass fraction the available lift-to-drag ratio varies between 0.1 to 0.13. A study of the system response is presented across vehicle geometries, and mass motions. A preliminary closed-loop PD control is presented which reduces the 2% settling time from 9 seconds to 5 seconds and removes all oscillations. An optimal control formulation is then solved, to minimize time, which reduces the settling time from 5 seconds to 0.6 seconds. The optimal control solutions move the mass primarily in the z direction, and adding a vertical constraint to the problem only marginally increases the settling time.
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Angle of Attack [rad]</td>
</tr>
<tr>
<td>$c_A$</td>
<td>Axial Force Coefficient</td>
</tr>
<tr>
<td>$c_M$</td>
<td>Pitching Moment Coefficient</td>
</tr>
<tr>
<td>$c_N$</td>
<td>Normal Force Coefficient</td>
</tr>
<tr>
<td>CG</td>
<td>Center of Gravity</td>
</tr>
<tr>
<td>CP</td>
<td>Center of Pressure b</td>
</tr>
<tr>
<td>$d$</td>
<td>Spacecraft diameter [m]</td>
</tr>
<tr>
<td>$d_{HIAD}$</td>
<td>HIAD Diameter [m]</td>
</tr>
<tr>
<td>$H$</td>
<td>Angular Momentum [Nms]</td>
</tr>
<tr>
<td>$I_i$</td>
<td>Inertia of the $i$th element [kgm$^2$]</td>
</tr>
<tr>
<td>$I_s$</td>
<td>Inertia of the system [kgm$^2$]</td>
</tr>
<tr>
<td>$l$</td>
<td>Characteristic Length [m]</td>
</tr>
<tr>
<td>L/D</td>
<td>Lift to drag ratio</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach Number</td>
</tr>
<tr>
<td>$M_{ext}$</td>
<td>Aerodynamic moment about the CG [Nm]</td>
</tr>
<tr>
<td>$m_s$</td>
<td>Total Mass [kg]</td>
</tr>
<tr>
<td>$m$</td>
<td>Moving-Mass Actuator Mass [kg]</td>
</tr>
<tr>
<td>$m_i$</td>
<td>Mass of the $i$th element [kg]</td>
</tr>
<tr>
<td>$m_v$</td>
<td>Vehicle Mass [kg]</td>
</tr>
<tr>
<td>$m_f$</td>
<td>Mass Fraction</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Reduced Mass [kg]</td>
</tr>
<tr>
<td>$N$</td>
<td>Total number of mass elements</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of moving masses</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density [kg/m$^3$]</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Packaging density [kg/m$^3$]</td>
</tr>
</tbody>
</table>
\( q \) Dynamic Pressure [Nm²]
\( r_0 \) Distance from the CG to the moment reference center [m]
\( r_{c,i} \) Vector from the \( i \)th element to the system CG [m]
\( r_{b,i} \) Vector from the \( i \)th element to the body frame [m]
\( r_{c,b} \) Vector from the body frame to the system CG [m]
\( r_H \) HIAD Radius [m]
\( S \) Characteristic Area [m²]
\( V \) Volume [m³]
\( \omega_{c,i} \) Angular velocity of the \( i \)th element in the CG frame [kgm²]
\( x \) Unit Vector
\( x_{CG} \) x Location of the CG [m]
\( \Delta x \) Change in the x location of the CG [m]
\( z \) Unit Vector
\( z_{CG} \) y Location of the CG [m]
\( \Delta z \) Change in the y location of the CG [m]
CHAPTER 1

INTRODUCTION

Future missions to Mars require entry, descent, and landing systems to deliver larger payload masses to the surface while maintaining landing accuracy. Deceleration in the Mars atmosphere is difficult because the atmospheric density is low; decreasing the ballistic coefficient through an increase in drag area causes vehicles to decelerate higher in the atmosphere, enabling delivery of larger-mass payloads to the surface [1]. However, the size of the vehicle is limited by the fairing size of the launch vehicle. One method to bypass this restriction is to use a deployable hypersonic decelerator which can be packaged within a smaller-diameter launch vehicle fairing. The Inflatable Reentry Vehicle Experiments (IRVE) demonstrated the ability to use a hypersonic inflatable aerodynamic decelerator (HIAD) during reentry. IRVE-2 successfully demonstrated the ability to deploy an inflatable aerodynamic decelerator [2] and IRVE-3 was able to verify the results of IRVE-2 and show that a HIAD can generate lift by flying at non-zero angles of attack [3]. Aerodynamic lift can be used to control the vehicle trajectory during entry. Large lift vectors imply greater control authority. The lift-to-drag ratio is a normalization of this and is used as a control metric.

Active control of a vehicle with a deployed HIAD is difficult as complex reaction control system (RCS) jet-wake interactions in the hypersonic regime may lead to destabilization or control reversal [4]. Internal moving-mass actuator systems enable trajectory control without requiring propulsion or vehicle configuration changes. The location of the vehicle’s center of gravity (CG) is changed by shifting the location of internal moving masses. Adjusting the distance between the CG and aerodynamic center changes the aerodynamic moment, leading to a rotation in the vehicle to a new trim condition. This control strategy does not require an active RCS nor will the exterior geometry of the vehicle change during flight, avoiding complex jet-exhaust wake-flow interactions and the need to model and verify such interactions.

Several flight vehicles have been developed which implement a moving mass system for control during entry. As well as testing the performance of a HIAD, IRVE-3 demonstrated the ability to generate lift by using a moving mass system [3]. IRVE moved the back half of the vehicle to generate an offset CG. The Mars Science Laboratory was equipped with two sets of ballast masses, which were jettisoned from the vehicle to control the location of the CG during different flight phases [5]. SpaceX has also developed a movable ballast system to generate a non-zero angle of attack on the Dragon capsule during entry [6].

Previous studies of moving-mass actuation have focused on application to slender bodies and
munitions [7–12]. These moving-mass systems are used for precision targeting of both stationary and moving targets. Spinning reentry vehicles have been investigated which use a moving mass to generate a principal axis misalignment as well as an offset CG to change trim conditions [10,13]. Moving-mass actuators have also been proposed for satellite attitude stabilization for pico-class satellites [14]. Moving masses are a candidate to replace aerodynamic surfaces on conventional aircraft [15]. Recently, moving-mass systems have been applied to an entry capsule in the Mars atmosphere [16]. Example moving mass systems may be seen in Figure 1.1.

Figure 1.1: Moving Mass Candidates for (a) Slender Body [8], (b) Capsule [16], (c) HIAD [3] Entry Vehicles.

This study assesses internal moving-mass actuator configuration options for trajectory control in the hypersonic regime of planetary entry. This study will quantify how the moving-mass actuator mass impacts the trim configuration of the vehicle. Determining what parameters affects this ratio is key to developing a feasible moving mass system. The mass will be allowed to translate along a linear track. The track length and location, vehicle geometry, system size, and mass properties are also varied to determine which configuration results in the largest trim lift-to-drag ratio.

Quantifying what vehicle parameters affect the moving-mass actuator mass required for trim is the first step to building a control algorithm for the vehicle. However since the vehicle operates based on trim control the attitude dynamics may lag behind and oscillate about the instantaneous trim condition. The vehicle open loop response must be understood prior to developing a closed loop control.

The study concludes with the development of closed loop control. A preliminary proportional derivative control law is developed to change the vehicle attitude. This control provides a baseline for vehicle performance, and may provide significant improvements from the open loop control. The problem may also be solved using optimal control. This provides a much better mass trajectory compared to a proportional derivative control.
CHAPTER 2

METHODS AND ASSUMPTIONS

2.1 Mass Properties

Figure 2.1: Assumed HIAD System Configuration with an Internal Moving Mass.

The entry vehicle is modeled as a cylinder payload with a large deployable drag area. The vehicle dimensions are governed by a length-to-vehicle diameter ratio, $l/d$, and the HIAD diameter-to-vehicle diameter ratio, $d_H/d$, (refer to Figure 2.1). The center of gravity of the spacecraft was assumed to be one third of the vehicle’s length. The moving-mass actuator mass is varied based on vehicle mass. The mass fraction of the moving-mass actuator is defined by:

$$m_f = \frac{m}{m_s} = \frac{m}{m + m_v}$$  \hspace{1cm} (2.1)

The change in CG location is described by Eq. (2.2) where $x$ is the mass location with respect to the
original CG:

\[ \Delta x_{\text{CG}} = \frac{m_x}{m_s} \]  \hspace{1cm} (2.2)

The moving masses are modeled as uniform tungsten spheres. The volume and density of the spheres are given by Equations 2.3, 2.4.

\[ V = \frac{4}{3} \pi r^3 \]  \hspace{1cm} (2.3)

\[ \rho = \frac{m}{V} \]  \hspace{1cm} (2.4)

Rearranging Equation 2.4 to solve for the Volume results in an expression for the volume as a function of mass and density. This expression may be substituted into Equation 2.3, and the radius of the moving mass is then given by Equation 2.5.

\[ r = \left( \frac{3 m}{4 \rho \pi} \right)^{\frac{1}{3}} \]  \hspace{1cm} (2.5)

2.2 Aerodynamic Properties

![Figure 2.2: Pitching moment coefficient from (a) phoenix database [17], (b) interpolated values.](image)

Data from the 2008 Mars Phoenix aeroshell were used to derive a hypersonic aerodynamic model [17]. The data included the normal, axial, and pitching moment coefficient over an angle of attack range of 0° to 16° and Mach numbers from 0 to 30 for a 70° sphere cone. These values were tabulated and a linear interpolation scheme was used to fully determine aerodynamic properties over the data range (see Figure 2.2). The aerodynamic reference area is defined by the projected frontal area of the HIAD, and the
reference length by half the HIAD diameter. The moment reference center of the data is at a location of 0.253 d from the leading edge.

2.3 Nominal Flight Conditions

Unless otherwise stated this study uses a nominal reference flight condition of an altitude of 27 km, velocity of 3977 m/s, the corresponding speed of sound is 208 m/s, and the resulting Mach number is 19 at Mars. This flight condition was generated by numerical integration of the three-degree-of-freedom equations of motion for an entry vehicle at Mars. The flight-path angle is assumed to be 0°, unless otherwise stated.

2.4 Equations of Motion

2.4.1 External Moment

Moving-mass actuators control the location of the CG. This change in CG location causes the aerodynamic forces to induce a moment about the CG, rotating the vehicle to a new trim condition. Summing the moments about the new CG results in Equation 2.6. Equation 2.6 includes four key terms: pitching moment, normal force moment, axial force moment, and dynamic moments. The normal force and axial force moments are given by their magnitude crossed with the distance from the new CG to the moment reference center, defined as \( r_{CG} \). The dynamic moment values are obtained from Atkins, and Queen [16]. To simplify the model the CG offset will be constrained to a plane resulting in Equation 2.7.
\[ M_{ext} = qS (c_m l + c_N z \times r_0 + c_A x \times r_0) + qS l \left( \frac{l}{2v} \right) \begin{bmatrix} c_{\omega(1)} \omega(2), c_{m_{\omega(2)+\hat{\omega}}} \frac{\omega(2) + \hat{\alpha}}{2}, c_{n_{\omega(3)-\hat{\beta}}} \frac{\omega(3) - \hat{\beta}}{2} \end{bmatrix} \] 

(2.6)

\[ M_{ext} = qS (c_m l + c_N \Delta x + c_A \Delta z) + qS l \left( \frac{l}{2v} \right) \left( c_{m_{\omega(2)+\hat{\alpha}}} \frac{\omega(2) + \hat{\alpha}}{2} \right) \] 

(2.7)

Under trim conditions the vehicle must satisfy:

\[ M_{ext} = 0 = c_m l + c_N \Delta x + c_A \Delta z \] 

(2.8)

### 2.4.2 Attitude Dynamics

![Vector and frame definitions](image)

The attitude dynamics may be derived through the conservation of angular momentum given by Equation 2.9. The angular momentum for a system of N particles about its center of mass is defined by Equation 2.10. The first term represents the angular momentum of the ith component about its own center of mass, and the second term is the angular momentum of the ith component about the reference point. Here \( N \) is defined as 1 + \( n \) where \( n \) is the number of moving masses. This extra component accounts for the angular momentum of the vehicle body.

\[ \dot{H} = M_{ext} \] 

(2.9)

\[ H = \sum_{i=1}^{N} I_i \cdot \omega_i + m_i \times r_{c,i} \] 

(2.10)

Taking a derivative of Equation 2.10 and equating it to Equation 2.9 will result in the dynamics for the
system. The derivative of a system of particles in an inertial frame I whose coordinates are defined by a rotating frame B is given by Equation 2.11. Using this definition for the derivative, the derivative of the angular momentum is given by Equation 2.12.

\[
\sum_{i=1}^{N} \frac{d}{dt} r_i^I = \sum_{i=1}^{N} \frac{d}{dt} r_i^B + \omega \times r_i^B
\]  

(2.11)

\[
\dot{H} = \sum_{i=1}^{N} \dot{I}_i \cdot \omega_i + I_i \cdot \dot{\omega}_i + \dot{r}_{c,i} \times m_i \dot{r}_{c,i} + r_{c,i} \times m_i \ddot{r}_{c,i} + \omega_i \times (I_i \cdot \omega_i) + \omega_i \times (r_{c,i} \times m_i \dot{r}_{c,i})
\]  

(2.12)

The term \( \dot{r}_{c,i} \times m_i \dot{r}_{c,i} \) is the cross product of velocity with itself which is always 0. Furthermore, the inertia matrix of each component may be written in terms of the inertia about the system center of mass and the summation may be written as a system inertia denoted as \( I_s \). Similar reasoning applies to the derivatives of the system inertia matrix as well. These simplification result in Equation 2.13. The simplified derivative of the angular momentum may then be seen in Equation 2.14.

\[
\dot{r}_{c,i} \times m_i \dot{r}_{c,i} = 0
\]

\[
\sum_{i=1}^{N} I_i = I_s
\]  

(2.13)

\[
\dot{H} = \dot{I}_s \cdot \omega_i + I_s \cdot \dot{\omega}_i + \omega_i \times (I_s \cdot \omega_i) + \sum_{i=1}^{N} r_{c,i} \times m_i \ddot{r}_{c,i} + \omega_i \times (r_{c,i} \times m_i \dot{r}_{c,i})
\]  

(2.14)

The above expression may be equated to Equation 2.9, resulting in Equation 2.15. The external moment, and the \( I_s \cdot \dot{\omega}_i \) term may be subtracted from both sides. By multiplying both sides of the equation, on the left, by the inverse of the system inertia an equation for the change in angular velocity may be found. This is shown in Equation 2.16.

\[
M_{ext} = \dot{I}_s \cdot \omega_i + I_s \cdot \dot{\omega}_i + \omega_i \times (I_s \cdot \omega_i) + \sum_{i=1}^{N} r_{c,i} \times m_i \ddot{r}_{c,i} + \omega_i \times (r_{c,i} \times m_i \dot{r}_{c,i})
\]  

(2.15)

\[
\dot{\omega}_i = I_s^{-1} \left\{ M_{ext} - \left[ \dot{I}_s \cdot \omega_i + \omega_i \times (I_s \cdot \omega_i) + \sum_{i=1}^{N} r_{c,i} \times m_i \ddot{r}_{c,i} + \omega_i \times (r_{c,i} \times m_i \dot{r}_{c,i}) \right] \right\}
\]  

(2.16)

This equation is valid for a system consisting of \( n \) elements. Since the dynamics are derived about the instantaneous center of mass if any element is moving the value of \( r_i \) will change for every element. Therefore it may be beneficial to write the dynamics in terms of the body frame. This is done by first
recognizing some relationships between the instantaneous center of gravity frame and the body frame.

First the position of the center of mass may be written as its sum of the position of each component in the body from multiplied by its respective mass divided by the total mass. A similar expression may be written for velocity and acceleration, as shown in Equation 2.17. Note that the summation is over \( n \), since the vehicle body is at the origin its contribution will be 0.

\[
\begin{align*}
\mathbf{r}_{c,b} &= -\frac{1}{m_s} \sum_{i=1}^{n} m_i \mathbf{r}_{b,i} \\
\dot{\mathbf{r}}_{c,b} &= -\frac{1}{m_s} \sum_{i=1}^{n} m_i \dot{\mathbf{r}}_{b,i} \\
\ddot{\mathbf{r}}_{c,b} &= -\frac{1}{m_s} \sum_{i=1}^{n} m_i \ddot{\mathbf{r}}_{b,i}
\end{align*}
\] (2.17)

Writing a vector in the center of mass frame given the body frame may be produced in similar fashion.

\[
\mathbf{r}_c = \sum_{i=1}^{n} m_i \mathbf{r}_i + m_v \mathbf{r}_b
\] (2.18)

Writing the position, and acceleration of the center of mass in the center of mass frame will always be 0, however this expression may be written by applying the above equation.

\[
\begin{align*}
\mathbf{r}_{c,c} &= \frac{1}{m_s} \sum_{i=1}^{n} m_i \mathbf{r}_{c,i} + m_v \mathbf{r}_{c,b} = 0 \\
\dot{\mathbf{r}}_{c,c} &= \frac{1}{m_s} \sum_{i=1}^{n} m_i \dot{\mathbf{r}}_{c,i} + m_v \dot{\mathbf{r}}_{c,b} = 0 \\
\ddot{\mathbf{r}}_{c,c} &= \frac{1}{m_s} \sum_{i=1}^{n} m_i \ddot{\mathbf{r}}_{c,i} + m_v \ddot{\mathbf{r}}_{c,b} = 0
\end{align*}
\] (2.19)

These equations may then be rearranged to solve for the position of the center of mass in terms of the body frame resulting in the following set of equations

\[
\begin{align*}
\mathbf{r}_{c,b} &= -\frac{1}{m_i} \sum_{i=1}^{n} m_i \mathbf{r}_{c,i} \\
\dot{\mathbf{r}}_{c,b} &= -\frac{1}{m_i} \sum_{i=1}^{n} m_i \dot{\mathbf{r}}_{c,i} \\
\ddot{\mathbf{r}}_{c,b} &= -\frac{1}{m_i} \sum_{i=1}^{n} m_i \ddot{\mathbf{r}}_{c,i}
\end{align*}
\] (2.20)

There are two expressions describing the position, velocity, and acceleration of the center of mass in the body frame given by Equations 2.17 and 2.20. These expressions may be equated as shown in Equation 2.21, and the position, velocity, and acceleration of each component in the center of mass frame may be solved for as shown in Equation 2.22. The relationship between the center of mass frame and body
frame may now be written in terms of the body frame using Equations 2.20 and 2.22 as shown in Equation 2.23

\[
\frac{1}{m_s} \sum_{i=1}^{n} m_i r_{b,i} = \frac{1}{m_v} \sum_{i=1}^{n} m_i r_{c,i} \\
\frac{1}{m_s} \sum_{i=1}^{n} m_i \dot{r}_{b,i} = \frac{1}{m_v} \sum_{i=1}^{n} m_i \dot{r}_{c,i} \\
\frac{1}{m_s} \sum_{i=1}^{n} m_i \ddot{r}_{b,i} = \frac{1}{m_v} \sum_{i=1}^{n} m_i \ddot{r}_{c,i} 
\]

(2.21)

\[
\sum_{i=1}^{n} r_{c,i} = \frac{m_v}{m_s} \sum_{i=1}^{n} r_{b,i} \\
\sum_{i=1}^{n} \dot{r}_{c,i} = \frac{m_v}{m_s} \sum_{i=1}^{n} \dot{r}_{b,i} \\
\sum_{i=1}^{n} \ddot{r}_{c,i} = \frac{m_v}{m_s} \sum_{i=1}^{n} \ddot{r}_{b,i} 
\]

(2.22)

\[
r_{c,b} = -\frac{1}{m_s} \sum_{i=1}^{n} m_i r_{b,i} \\
\dot{r}_{c,b} = -\frac{1}{m_s} \sum_{i=1}^{n} m_i \dot{r}_{b,i} \\
\ddot{r}_{c,b} = -\frac{1}{m_s} \sum_{i=1}^{n} m_i \ddot{r}_{b,i} 
\]

(2.23)

However these equations may not be substituted into Equation 2.19 yet since these equations are summed over \( n \), and don’t include the contribution from the vehicle body. The dynamics may then be re-written to sum over \( n \) given by Equation 2.24. Then substituting Equations 2.20 and 2.22 into Equation 2.24 results in Equation 2.25. A reduced mass may be introduced by Equation 2.26. By using the reduced mass, and rearranging the summations in Equation 2.25 results in Equation 2.27.

\[
\dot{\omega}_i = I_s^{-1} \left\{ M_{ext} - \left[ \dot{J}_s \cdot \omega + \omega \times (J_s \cdot \omega) \right] + r_{c,b} \times m_v \dot{r}_{c,b} + \omega_i \times (r_{c,b} \times m_v \dot{r}_{c,b}) \right.
\]
\[
+ \left. \sum_{i=1}^{n} r_{c,i} \times m_i \dot{r}_{c,i} + \omega_i \times (r_{c,i} \times m_i \dot{r}_{c,i}) \right\} 
\]

(2.24)
\[ \dot{\omega}_i = I_s^{-1} \left\{ M_{ext} - \left[ I_s \cdot \omega_i + \omega_i \times (I_s \cdot \omega_i) - \sum_{i=1}^{n} \mu_i \dot{r}_{c,i} - \sum_{i=1}^{n} \mu_i \dot{r}_{c,i} + \omega_i \times \left( \sum_{i=1}^{n} \mu_i \dot{r}_{c,i} \right) \right] \right\} \] (2.25)

\[ \mu_i = \frac{m_i \mu_v}{m_s} \] (2.26)

\[ \dot{\omega}_i = I_s^{-1} \left\{ M_{ext} - \left[ \dot{I}_s \cdot \omega_i + \omega_i \times \left( \dot{I}_s \cdot \omega_i \right) + \sum_{i=1}^{n} \mu_i \dot{r}_{c,i} \times \dot{r}_{b,i} + \omega_i \times \left( \mu_i \dot{r}_{b,i} \times \left( \sum_{i=1}^{n} \mu_i \dot{r}_{c,i} \right) \right) \right] \right\} \] (2.27)

The order of the cross products in Equation 2.27 may be switched, and the acceleration and velocity terms may be factored resulting in Equation 2.28.

\[ \dot{\omega}_i = I_s^{-1} \left\{ M_{ext} - \left[ \dot{I}_s \cdot \omega_i + \omega_i \times \left( \dot{I}_s \cdot \omega_i \right) + \sum_{i=1}^{n} \mu_i \dot{r}_{c,i} \times \dot{r}_{b,i} + \omega_i \times \left( \mu_i \dot{r}_{b,i} \times \left( \sum_{i=1}^{n} \mu_i \dot{r}_{c,i} \right) \right) \right] \right\} \] (2.28)

A final relation between the center of mass frame and body frame may be derived by subtracting the first line from Equation 2.23 and 2.22 to give Equation 2.29. This relation may then be simplified by recognizing that the sum of all of the masses and the vehicle mass is equivalent to the system mass resulting in Equation 2.30

\[ r_{c,b} - \sum_{i=1}^{n} r_{c,i} = -\frac{1}{m_s} \sum_{i=1}^{n} m_i r_{b,i} - \frac{m_v}{m_s} \sum_{i=1}^{n} r_{b,i} \] (2.29)

\[ r_{c,b} - \sum_{i=1}^{n} r_{c,i} = -\sum_{i=1}^{n} r_{b,i} \] (2.30)

The left hand side of Equation 2.30 appears twice in Equation 2.28, and may be substituted. The order of the cross products may then be switched again to change the sign resulting in the final form of the system dynamics in the body frame.

\[ \dot{\omega}_i = I_s^{-1} \left\{ M_{ext} - \left[ \dot{I}_s \cdot \omega_i + \omega_i \times \left( \dot{I}_s \cdot \omega_i \right) + \sum_{i=1}^{n} \mu_i r_{b,i} \times \dot{r}_{b,i} + \omega_i \times \left( r_{b,i} \times \dot{r}_{b,i} \right) \right] \right\} \] (2.31)
CHAPTER 3

EVALUATION OF CONTROL AUTHORITY

A preliminary analysis is conducted to determine the largest change in trim lift-to-drag ratio, a measure of control authority during atmospheric flight. Determining what parameters affects this ratio is key to developing a feasible moving mass system. The vehicle is modeled with a moving mass constrained to a linear track. The track length and location, vehicle geometry, system size, and mass properties are also varied to determine which configuration results in the largest trim lift-to-drag ratio. Vehicle configuration is then changed to allow for the mass to move over a 2 dimensional region, and the optimal mass location is then found for this new configuration. The trim analysis then concluded with an analysis of the Mach dependency of the aerodynamics.

For the first set of analyses the vehicle is modeled as a uniform cylinder with a large HIAD. A single mass is modeled, and its motion is constrained to a linear track. The location, and length of the track is

Figure 3.1: Assumed HIAD System Configuration with an Internal Moving Mass.
allowed to vary, along with the size of the mass. A representation of the vehicle may be seen in Figure 3.1. In this example the mass is constrained to a linear track along the back edge of the vehicle. The vehicle length is twice its diameter, and the HIAD diameter is 5 times the vehicle diameter.

The analysis is conducted by determining the moving mass location which results in no moment on the vehicle. This is done by discretizing the track into points where the moment on the vehicle will be evaluated. Under trim conditions the vehicle must satisfy Equation 2.8, which is re-written below for convenience. The right hand side of Equation 2.8 is solved for each point created an array of moment values. The values are then interpolated to find the location which satisfied Equation 2.8.

\[ 0 = c_M l + c_N \Delta x + c_A \Delta z \]

3.1 Moving Mass Actuator with a Linear Track

To determine the effect of mass placement on the requirement for trim, a preliminary study was conducted with forward and aft track placement for the moving-mass actuator. This would test the case with a maximum and minimum distance between the CG and center of pressure. The location of the center of pressure varies with angle of attack, but remains behind the CG for all cases. Tracks of the following lengths were considered to determine their contribution to the achievable L/D.

1. A track length of \( d \)
2. A track length of 0.9 \( d \)
3. A track length of 1.1 \( d \)
4. A track whose length spans the vehicle diameter but is shortened based on the radius of the moving mass. This case is labeled as \( d_m \) in subsequent figures

The first 3 cases provide a baseline performance as well as general trends on how the track length affects the required mass for trim. For the final case, the mass is assumed to be a uniform tungsten sphere whose diameter is found by solving Equation 2.5 leading to diameters between 0 and 0.53 m. The vehicle is modeled as a 50 t vehicle with a 5 m diameter main body and a 25 m diameter HIAD.

The resulting mass fractions required for trim with a forward track can be seen in Figure 3.2a and an aft track in Figure 3.2b. For the forward track the mass fraction required for trim conditions at an L/D of 0.24 is 20%, for an aft track the same L/D requires a mass fraction of 15%. Placing the moving mass further aft results in a reduction in the mass required for trim by 5%. For the 50 t test vehicle a mass fraction reduction of 5% would correspond to a weight reduction of 2500 kg. For both a forward and aft track increasing the track length provides some reduction in required mass fraction. This reduction is
negligible at small L/D. However at an L/D of 0.24, the difference in mass fraction can be between 6% and 3% depending on the length of the track. For a capsule shaped vehicle this may be an important trade off.

In order to determine how well the moving mass system translates to various spacecraft, a study is conducted where the system is modeled as a 50 t vehicle with a 5 m diameter cylinder. The packaging density of this cylinder is calculated to be 763.94 kg/m$^3$, and is assumed constant. Using this packaging density, appropriate geometries can be constructed for 10 t, and 1 t vehicles. The resulting mass fraction required for trim can be seen in Figure 3.3a. For each vehicle the mass fraction required for trim is the same causing all curves to overlap. To achieve a trim condition of an L/D of 0.24 all vehicles require a mass fraction near 13%. Although mass fraction remains the same the total mass required for each case varies based on Equation 2.1. For a 50 t vehicle this corresponds to a mass of 6500 kg, which is far greater than the 130 kg required for the 1 t vehicle.

Prior crewed spacecraft packaging densities have followed a power law with respect to the total volume [18] given by

$$\rho_p = 454.5V^{-0.24}$$

where $\rho_p$ is in kg/m$^3$, and $V$ is in m$^3$ Given a desire vehicle mass Equation 3.1 can be used to find the volume of each vehicle, then vehicle dimensions can be obtained based on the proportions mentioned above. Again the mass fraction required for trim is the same for all vehicle sizes (see Figure 3.3b). In this case with varying packaging density and size the mass fraction required for trim is the same as that of the case with constant packaging density. Varying size, or packaging density of the vehicle has a negligible
contribution to the mass fraction required for trim.

Figure 3.3: Mass Fraction Required for Trim Conditions for Various Vehicle Sizes Following (a) Constant 
(b) Varying Packaging Density.

The mass fraction being the same for all cases of vehicle size and packaging density come as a result of 
all the aerodynamic moments being change proportionally. The vehicle geometry is governed by the 
volume which is proportional to $d_H^3$. Assuming the same flow conditions both $N$ and $A$ are proportional to 
d_H^2. They are multiplied by a moment arm which is also proportional to $d_H$, making the their moment 
proportional to $d_H^3$. The pitching moment on the spacecraft is proportional to $d_H^3$. The moment from the 
moving mass is at least two orders of magnitude smaller than the aerodynamic moments so it’s neglected. 
The moments on the spacecraft can be considered to be only the aerodynamic moments which are all 
scaled by the same factor of $d_H$. Then in order to trim at the same conditions the vehicle must have a 
proportional change in mass thus making the mass fraction for all three cases the same.

The test vehicle was defined as a cylinder with a length to diameter ratio of 2, changing this ratio may 
effect the ability to control the vehicle. To test this the length to diameter ratio, l/d is varied between 0.5, 
1, and 2. As l/d approaches 0 the vehicle becomes shorter and wider, as l/d approaches infinity the vehicle 
becomes longer and more slender. The resulting mass fraction required for trim conditions can be seen in 
Figure 3.4(a). The geometry plays a role in determining the required mass to achieve trim conditions. This 
is largely due to the difference in moment arm for each configuration. The l/d = 2 vehicle has a smaller 
HIAD diameter bringing the center of pressure closer to the CG changing the stability of the vehicle. This 
less stable vehicle requires a smaller moment in order to produce similar trim conditions. This change is 
relatively insignificant when L/D is small, yet as L/D increases this effect becomes more prominent. At an 
L/D of 0.24, the mass fraction required for trim is 2% less for a l/d = 2 vehicle compared to a 0.5 vehicle.
The longer and more slender the vehicle the more efficient the moving-mass system will be.

The prior simulation assumed the HIAD-diameter-to-vehicle-diameter ratio remained at 5 to 1. If the HIAD diameter is instead fixed to a constant value and the vehicle geometry is changed independently there are different requirements for trim conditions. Under this assumption the HIAD diameter was chosen to be 33 m to be consistent with the l/d = 1 vehicle from the prior simulation. The required mass fraction for trim can be seen in Figure 3.4(b). In this case the short and wider vehicle now required far less mass compared to the long and slender vehicle. Fixing the HIAD diameter will fix the location of the center of pressure making the moment arm in the x direction nearly the same for all the test cases. Now the track length becomes the deciding factor in determining the required mass fraction. In this case there is a 4% change in required mass fraction making the shorter and wider more favorable compared to the long and slender vehicle.

Figure 3.4: Mass Fraction Required for Trim Conditions for Various Geometries with (a) 5:1 HIAD Diameter:Vehicle Diameter Ratio (b) 33 m HIAD Diameter.

The main contribution to the moment on the vehicle is the moment induced by the aerodynamic forces. These forces are magnified by a large moment arm which depends on the distance between the CG and center of pressure. This distance is manipulated by the moving mass yet it’s also possible to design a vehicle which has an offset CG. In doing so the nominal L/D will change and the required mass fraction may be different. The CG position is changed to be 1/10 of the radius both above and below the center line resulting in a shift in the required mass fraction as seen in Figure 3.5. The 0 case shows the required mass fraction curve with no CG offset for comparison. This case is symmetric about 0 L/D which comes as a result of the vehicle being symmetric. The other two cases are very similar however the only symmetry is in their nominal L/D. These small differences are a result of the slight change in moment arm which result
from flying at non-zero angles of attack.

For all cases to increase L/D beyond the nominal L/D the moving mass needs to be placed in the lower corner of the vehicle and to decrease the L/D the mass should move to the upper corner. This relationship can be thought of as having a desired CG location for a given L/D. For positive L/D this desired CG location will be in the lower half of the vehicle. If the CG is shifted downwards then these two points will be closer together making the required mass fraction smaller. If the CG is shifted upwards then this distance is increased and a larger mass fraction is needed.

For a given mass fraction the control authority is different for each case. If a mass fraction of 2% is given, a symmetric vehicle can achieve an L/D between -0.069 and 0.069 which corresponds to a change in L/D of 0.138. If the CG is shifted down then the achievable L/D is between 0.165 and 0.217 which gives a change of 0.052, and if the CG is shifted up the L/D is between -0.222 and -0.16 which gives a difference of 0.062. Keeping the CG on the vehicle center line will more than double the range of obtainable L/D. However if a specific L/D is required then it may be more beneficial to offset the CG and allow the moving mass to provide control around that point. The nominal L/D for a CG which is offset by 1/10 of the radius downwards is 0.188. To bring a symmetric vehicle to this L/D a mass fraction near 9% would be required, and to generate similar control authority as a vehicle with an offset CG an increase in mass fraction larger than 2% is needed.

![Figure 3.5: Mass Fraction Required for Trim Conditions for an Offset CG.](image)

Prior studies have tested the mass fraction required at specific points in order to achieve a certain trim condition. A more realistic study is conducted where there is a given mass of mass fraction 10%, 15%, and 20% which is then moved along a track. The track is placed in back, and the length is limited by the size of the moving mass. Since the vehicle size does not matter so a 50 t vehicle was used, and the initial length
Figure 3.6: Mass Location Required for Trim Conditions for Various Mass Fractions.

to diameter ratio of 2 was used. The resulting mass location can be seen in Figure 3.6 where the y location is normalized by the vehicle radius. As expected the curve moved towards the lower edge of the vehicle as the L/D requirements increase. Since a 10% mass fraction is below the requirement for an L/D of 0.24 the maximum achievable L/D is 0.16 whereas the 20% mass fraction case is able to achieve all angles while having some unused length of track.

3.2 External Moving Mass System

To allow for a wider range of vehicle maneuvers the moving mass will be allowed to travel in both x and z. Rather than having a track at some fixed location within the vehicle, a track will be attached from the HAID to a tether point on the vehicle. By allowing this tether point to vary along the edge of the vehicle the location through which a mass may move is constrained to a triangle. For a two mass system the candidate location for a mass may then be seen in Figure 3.7 a. For these studies the diameter of the HAID is shrunk to only be twice the vehicle diameter.

An exhaustive Search over this domain is very computationally and time intensive, as a result an optimization scheme was used to determine the optimal location more rapidly. The optimization problem was formulated as a two stage optimization problem. For the outer loop the design vector is the location of both of the masses, and the objective function is the angle of attack. The inner loop optimization takes the angle of attack as the design vector, and the moment about the CG is the objective function. The algorithm starts by assuming given mass location which is passed into the inner loop. The moment on the vehicle is then calculated for angle of attack values between 0 and 16° for the given location. The moment
is then squared which makes the data convex, in other words as the distance from the optimal location increases the moment increases. Convex problems are more easily solved by gradient based optimization schemes. To further speed up the algorithm a surrogate model was fitted to the mesh. In this case a radial basis function was used to fit a surface through the points. On this surface any gradient based optimization scheme will, such as Newton’s method, will converge to the optimal location very quickly. This optimum will be global due to the convexity of the problem. The angle of attack which results in 0 moment is then passed out, and taken as the current trim angle of attack. The outer phase then uses this angle of attack value as its objective function, and varies the mass location to find what mass location maximizes this angle of attack. Another Newton’s method is used to solve this optimization problem for the mass location.

The mass location which yields the largest lift-to-drag ratio can be seen in Figure 3.7 b. This result moves the CG as far off the center line and aft as possible. Taking another look at Equation 2.8 the CG offset only effects the last two terms. By moving the CG off the center line the term multiplying the axial force coefficient increases, whereas moving the CG further aft increases the term multiplying the normal force coefficient. This result holds for all flight conditions and vehicle geometries.

The aerodynamic properties vary as a function of Mach number which changes the max achievable lift-to-drag ratio as the vehicle decelerates. Figure 2.2 shows the aerodynamic pitching moment coefficient as a function of Mach number. The coefficient is nonlinear and its magnitude decreases toward zero for a Mach number of 15 at low angles of attack. To quantify this effect a trim analysis was done using Equation 2.8 for Mach numbers between 2 and 25 to determine the lift-to-drag ratio for moving mass mass fractions below 6%. The resulting lift-to-drag ratio can be viewed in Figure 3.8. Here it can be observed that for a given mass fraction the lift-to-drag ratio varies significantly with both Mach number and moving-mass mass fraction. For a mass fraction of 1% the maximum achievable lift-to-drag ratio is 0.06 near a Mach number of 17. However a lift-to-drag ratio of 0.06 cannot be sustained through entry. For a
Figure 3.8: Achievable lift-to-drag ratio for a given Mach number and moving mass actuator mass fraction.

The majority of the Mach numbers tested the lift-to-drag ratio is between 0.02 and 0.03 with the average being 0.04. To maintain a lift-to-drag ratio of 0.06 a mass fraction of 3% is required. The spike in achievable lift-to-drag is only present when the pitching moment coefficient drops near 0. Since this drop in pitching moment coefficient is more prominent for low angles of attack, as angle of attack increases the Mach number dependence in the available lift-to-drag ratio decreases. For a moving mass fraction of 5%, its maximum available lift-to-drag ratio is 0.14 which occurs near a Mach number of 12, and the smallest lift-to-drag ratio is 0.11.

Figure 3.9: Contour plots of the achievable lift-to-drag ratio for a given Mach number and moving mass actuator mass for a vehicle which is (a) twice as long (10.6 m), (b) HIAD diameter twice as large (10.6 m)

To further evaluate the Mach dependent aerodynamics on other vehicle types the above study was
repeated for two more vehicle configurations, one where the vehicle is twice as long (10.6 m), and another where the HIAD diameter is twice as large (10.6 m). The resulting lift-to-drag ratios can be seen in Figure 3.9. The case where the payload is longer has a significant change in the maximum achievable lift-to-drag ratio. The maximum lift-to-drag ratio jumps from 0.16 to 0.27, and the range of lift-to-drag ratios for a given Mach number has also changed. For a mass fraction of 1% the maximum lift-to-drag ratio varies between 0.04 and 0.1, nearly double the 5.3 m case. A 5% mass fraction for the original case would correspond to a 2.5% mass fraction for a vehicle which is twice as long. This huge change comes as a result of the vehicle CG being shifted very far back from the moment reference center which results in a very large moment arm on the normal force. However this configuration may not be stable since the CG is shifted so far back, and flyable angle of attacks are limited due to aero thermal considerations.

When the HIAD diameter is doubled the maximum lift-to-drag ratio increases 0.016 to 0.017. Again there is a trend to have certain Mach numbers resulting in an increase to the available lift-to-drag ratio, however the shape of the contours changed for this case. For a moving mass fraction of 5% the range of available lift-to-drag ratios changed from between 0.11 to 0.14, to 0.13 to 0.15. A 5% mass fraction for the original case would correspond to roughly a 4.5% mass fraction for the case where the HIAD is twice as large.

The requirements for trim conditions have been determined in terms of the moving-mass mass fraction. The size and packaging density of the vehicle have little effect on the mass fraction required for trim conditions, as the vehicle proportions remain the same. Slender vehicles require less mass in order to achieve the same trim conditions compared to a shorter vehicle. Placing the track in back of the vehicle rather than the front can provide a significant reduction in actuator mass fraction. To maximize lift-to-drag ratio the masses should be placed as far aft and off the center line as possible resulting in the largest possible moment arm to the aerodynamic forces. There is a strong dependency on both moving-mass mass fraction and Mach number on the available lift-to-drag ratio. The available lift-to-drag varies by as much as 0.03 over Mach numbers from 2 to 25, and the Mach number at which it peaks is between 10 and 17 depending on mass fraction.
CHAPTER 4

OPEN LOOP SYSTEM RESPONSE

A change in the location of the center of gravity will modify the trim condition. This trim condition is what changes the vehicle attitude. However the attitude will not response instantaneously to a change in trim condition. The vehicle attitude may lag behind and oscillate about the trim condition. In order to create a closed loop control for this system an understand of how it responds to a command developed.

4.1 System Response

In order to characterize the feasibility of moving-mass trim control an analysis of the system response was conducted. For this study a numerical simulation of the vehicle dynamics was developed for a vehicle with two moving masses whose combined mass fraction is 3.2%. Both masses are placed at the rear of the vehicle, and the lower mass is moved towards the lower edge of the HIAD (see Figure 8 a). As the mass translates, the vehicle CG location changes causing the vehicle to maneuver to a new trim condition. For a preliminary analysis the lower mass was translated at a velocity of 1 m/s. The resulting system dynamics is plotted in Figure 8 b. The mass takes 5 seconds to reach its final position, and just over 9 seconds for the system to reach its 2% settling time. Throughout the transient phase the system has dampening oscillations about its instantaneous trim condition.

Figure 8 b shows that as the mass begins to translate along the track the angle of attack decreases rather than increasing. This effect occurs due to the change in momentum of each mass with respect to the system CG. As the mass begins to translate the aerodynamic forces are small, and the vehicle rotation is small. Equation 2.16 shows that initially the only term which contributes to the angular acceleration would be the fourth term. The fourth term represents the change in the linear momentum’s contribution to the angular momentum of the system. Since the lower mass has moved the CG has shifted which makes the distance from each mass element to the CG different. For the upper mass and vehicle body this term is small. However for the lower mass this term results in a moment on the order of 10 N·m. Since this term is negative the vehicle counter rotates causing the dip in angle of attack. As the vehicle begins to rotate the other terms in Equation 2.16 begin to dominate the dynamic response and the vehicle begins to rotate towards its instantaneous trim configuration.

This motion is similar to that of an inverted pendulum on a cart. In order to balance the pendulum
Moving Mass
Moving Mass
(a)

Figure 4.1: (a) Path of the moving mass (b) System response for a mass with a velocity of 1 m/s

and move the cart, it must first be swayed backwards to adjust the angle of the pendulum, then the system may translate forwards. With the current system configuration the masses are both placed at end the rear edge of the vehicle, and may not sway back. The initial counter-rotation may not be avoided with the current system configuration.

To ensure that both representations of the dynamics given by Equations 2.16 and 2.31 a quick simulation of the dynamics in both frames was conducted. The above system was redone for both cases and the results were plotted in Figure 4.2. The dynamics appear on top of one another meaning that the systems are equivalent. There are small errors, however this is attributed to numeric instability, which decreases as the time step decreases.

Figure 4.2: Coplot of the system response for dynamics written in the center of mass frame, and the body frame
Other system configurations were assessed to determine their contribution to the system response. First, two additional vehicle geometries were examined as before, one with a vehicle which is twice as long, and one with a HIAD diameter which is twice as large. The system was examined as before with a mass translating at 1 m/s. These results are plotted in Figure 4.3.

Figure 4.3 shows the jump in maximum achievable angle of attack, corresponding to higher L/D as shown in Figure 3.9, for the case which is twice as long. The trim angle of attack has jumped from 6.5° to 11°. However this case also has much larger oscillations, as well as a longer settling time. For the case with a HIAD which is twice as large there are significantly smaller oscillations than before, and these oscillations dampen out before the system reaches steady state. For both these cases the same initial drop in angle of attack is observed as before.

In an attempt to remove the initial drop in angle of attack a case where the mass translated based on an acceleration law rather than constant velocity was conducted. The mass accelerates with an acceleration of 1 m/s² until it reaches the center, then accelerates by 1 m/s² in the opposite direction. This case is plotted in Figure 4.4. Again the vehicle drops in angle of attack prior to increasing however the drop is much smaller than the prior analyses. Furthermore the magnitude of the oscillations is smaller than before, with a smaller transient phase.

4.2 Stability

With the current vehicle configuration as well as other configurations examined there may be some static stability issues. Figures 8 and 4.3 show an initial jump in the instantaneous trim angle of attack from 0 to
Figure 4.4: System response for the case of a mass which translates based on an acceleration law.

Figure 4.5: Bounds for the center of pressure location for angles of attack between 0 and 16°. This comes as a result of the center of gravity being behind the center of pressure for the initial configuration. The location of the center of pressure is dependent on the HIAD diameter and varies between 0.28 and 0.9 $d_H$ from the nose of the vehicle. Since the CG is located at 0.33 $d_H$ from the leading edge for some angles of attack the CG will be ahead of the center of pressure making the vehicle statically unstable. Figure 4.5 shows the bounds of the center of pressure location on the vehicle along with the center of gravity. For the test vehicle the center of gravity was behind the center of pressure until the angle of attack became larger than 2°. As the moving masses continue to translate the vehicle regains static stability.

This stability issue is more obvious when looking at the cases where the vehicle is twice as long or the case where the HIAD is twice as large (see Figure 4.3). For the case where the HIAD is twice as large, the center of pressure is behind the CG for all angles of attack, so there is no initial jump in the trim condition. For the case where the HIAD is twice as large the CG is even further behind the center of pressure, so the trim angle of attack jumps to 5° rather than 2°.
Across all mass trajectories there is an initial counter rotation resulting from the mass motions contribution to the angular momentum. Vehicle geometries with longer payloads have larger oscillations and longer settling times but also higher achievable angles of attack, vehicles with a larger HIAD have smaller oscillations which dampen very quickly. The more stable a vehicle is the smaller the achievable angle of attack becomes.
CHAPTER 5

CLOSED LOOP SYSTEM RESPONSE

A preliminary proportional derivative control law is developed to change the vehicle attitude. This control provides a baseline for vehicle performance, and may provide significant improvements from the open loop control. The problem may also be solved using optimal control. This may provide a much better mass trajectory compared to a proportional derivative control.

5.1 Proportional Derivative Control

A closed-loop controller was developed to determine how much the performance of the system could be improved relative to the open-loop case. In order to ensure that the vehicle is stable for all angles of attack the vehicle was given a larger HIAD putting the CG ahead of the center of pressure. A PD control algorithm determined the desired moment on the vehicle. The integral term was not used since the vehicle is stable and the oscillations will naturally dampen out. To make the initial configuration more stable the masses are both started at the front edge of the search domain rather than the back. A Newton’s method optimization scheme was then used to search the domain for the location which results in the desired moment on the vehicle. To make the problem convex the optimization is attempting to minimize the error in the moment. A first case study was conducted to determine the system response for a case where the acceleration, and velocity of the masses are unbounded. The system was commanded to reach an angle of attack of 1° using constant proportional and derivative gains. The resulting dynamics can be seen in Figure 5.1.

The black circles mark the second time step and every tenth time step afterwards. Both masses start at the furthest forward point, on the edge of the HIAD. After the first step there is a large jump on both masses. The masses are both required to move in order to negate the initial dip in the system response. Afterwards the lower mass and the upper mass move in opposite directions. This keeps the center of mass in nearly the same place, but the higher order terms which depend on the velocity and acceleration of the masses are being used to control the vehicle dynamics.

To further analyze the system response, another case was examined where the target angle of attack was raised to 5° rather than 1°. The new system response is plotted in Figure 5.2. Here the system dynamics looks very similar to the prior case. Both have similar transient phase, and similar settling time,
Figure 5.1: (a) Response for a controlled mass with a $1^\circ$ target (b) mass locations for the system response despite one case commanding a larger angle of attack. If the position of the masses are examined there are is a large difference in the mass response. In the $1^\circ$ case the masses have an initial jump then stay very near the new location, but in the $5^\circ$ case the lower mass has a far larger range of motion. The upper mass follows the same trend of moving slightly forwards, however the lower mass jumps, the proceeds back towards its starting position. Based on prior analyses the maximum angle of attack is obtained when the lower mass is moved as far forward and off center line as possible, and the upper mass is as far back and near the center line as possible. Since this case is near the maximum achievable angle of attack the final position of the lower mass will be far from the center line.

Figure 5.2: (a) Response for a controlled mass with a $5^\circ$ target (b) mass locations for the system response
To constrain the motion of the mass to something more realistic a bound on the velocity is added. The magnitude of the velocity is required to be less than 1m/s, limiting the range the mass may translate. The new system dynamics can be seen in Figures 5.3 and 5.4 for a 1° and 5° targets.

The system response is slightly different than before. The rise time, and settling time are both larger, but the structure of the transient phase is now different. In the case where there is instantaneous control the system dynamics initially jumps up very fast, but with the velocity constrain the system dynamics initially rise much slower. This change only results in an increase in settling time from from 3 to 4 seconds. The motion of the masses is now completely different for both the two velocity constrained cases and the
two unconstrained cases. For the $1^\circ$ target the masses both move along the diagonal boundary. The upper and lower mass motions are very similar with the upper mass moving more towards the center line than the lower mass. For the $5^\circ$ case the lower mass doesn’t move much at all, rather the upper mass translates almost all the way along the vertical boundary.

A final study is conducted by adding a constraint on the acceleration. In this case each component of the acceleration is constrained to be less than $1 \text{m/s}^2$. The prior velocity constraint is still enforced for this case. As a result the dynamics are simulated for a $1^\circ$ and $2^\circ$ targets which can be seen in Figures 5.5 and 5.6.

![Figure 5.5](image1.png)  
**Figure 5.5:** (a) Response for a controlled mass with a $1^\circ$ target (b) mass locations for the system response

![Figure 5.6](image2.png)  
**Figure 5.6:** (a) Response for a controlled mass with a $1^\circ$ target (b) mass locations for the system response
For these two cases the system responses are nearly identical to the case where only the velocity is constrained. Again despite similar system responses there are different mass locations. Both cases have a similar structure where the upper mass moving along the vertical boundary, and the lower mass moves diagonally inwards. The $2^\circ$ case moves further from the initial condition than the $1^\circ$ case, and the lower mass moves at a different angle.

To better understand the motion of the masses the velocity profile of the masses is observed. The upper mass is labeled as $m_1$ and the lower mass is $m_2$. Looking at the system response form Figures 5.5 and 5.6 the settling time of the system is near 6 seconds. looking at the velocity profile for both cases there is a non-zero velocity for both masses at 6 seconds. Even at the final time both masses are still moving. This implies that while the system rotation is at steady state the masses within the vehicle may still be translating.

\begin{figure}[h]
\centering
\begin{subfigure}{0.45\textwidth}
\includegraphics[width=\textwidth]{fig5a.png}
\caption{Velocity of the moving masses for (a) a $1^\circ$ target}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\includegraphics[width=\textwidth]{fig5b.png}
\caption{Velocity of the moving masses for (b) $1^\circ$ target}
\end{subfigure}
\caption{Velocity of the moving masses for (a) a $1^\circ$ target (b) $1^\circ$ target}
\end{figure}

5.2 Optimal Control

While observing the closed loop control it was found that infinitely many mass trajectories may result in the same system performance. Consider the case where the control is attempting to reach a desired moment on the vehicle as above. If the location of the top mass is fixed there is a function of equations which result in the same moment (See Figure 5.8). If the top mass is then moved a new function of lower mass positions will result in the same moment as before. As a result many trajectories may satisfy all constraints on the vehicle while reaching the same conditions. This implies that the control may accomplish some objective function.

The optimal control problem is first solved for a system where a single moving mass is allowed to
translate anywhere within the vehicle body. The velocity and acceleration of the mass is again constrained to be less than 1 m/s and 1 m/s$^2$ receptively. The vehicle angle of attack is bounded to remain within the aerodynamic database ($\pm 16^\circ$). The termination criteria are that the angle of attack reaches $1^\circ$ and the angular velocity is 0 rad/s. The state vector is a vector which contains the angle of attack, angular velocity, mass location, and mass velocity. The mass acceleration is the control vector. Since the control is the mass acceleration, and the derivative is a state, the problem is solved more easily in the body frame. By using the body frame there is no need to calculate the velocity and acceleration of the system center of mass. The formation may be seen in Equation 5.1. The problem dynamics are the same as for the PD control and are re-written in Equation 5.3.

\[
\min_x J = t_f \\
\text{s.t } h_1(x) = u - a_{\text{nom}} \leq 0 \\
h_2(x) = \|\dot{x}\|_2 - v_{\text{nom}} \leq 0 \\
h_3(x) = t_f - t_{\text{nom}} \leq 0 \\
h_4(x) = \alpha(t) - 16 \leq 0 \\
h_5(x) = x_{\text{lb}} \leq x \leq x_{\text{ub}} \\
g_1(x) = \alpha(t_f) - \alpha_{\text{nom}} = 0 \\
g_2(x) = \omega(t_f) - \omega_{\text{nom}} = 0
\] (5.1)

\[
x = [\alpha, \omega, x, z, \dot{x}, \dot{z}] (5.2)
\]
\[
\dot{\omega} = I_s^{-1} \left\{ M_{\text{ext}} - \left[ \dot{I}_s \cdot \omega + \omega \times (I_s \cdot \dot{\omega}) + \sum_{i=1}^{n} r_i \times m_i \ddot{r}_i + \omega \times (r_i \times m_i \dot{r}_i) \right] \right\}
\]

\[
M_{\text{ext}} = qS \left( c_M l + c_N z \times r_{CG} + c_A x \times r_{CG} \right)
\]

\[
\Delta x_{CG} = \frac{m \dot{x}}{m_s}
\]

An initial study was conducted with a single 10 kg moving mass whose initial condition is at the vehicle center of gravity. Figure 5.9 shows the vehicle angle of attack and angular velocity, Figure 5.10 shows the mass location in the body frame, Figure 5.11 is corresponding mass velocity as a function of time, and Figure 5.12 is the mass acceleration as a function of time. The control was able to find a solution which reached the 1° target in less than 0.6 seconds. The PD control took nearly 6 seconds to reach 1° in angle of attack for the same mass constraints. The mass moves nearly vertically for a majority of its trajectory, with a small translation in the x direction near the center of the trajectory. The acceleration constraint is active throughout the entire trajectory, and a majority of the acceleration acts in the z direction. The z acceleration is nearly a bang-bang control. While the acceleration in the z direction switches from negative to positive the x component of acceleration has a positive impulse. A seconds impulse in the x direction is required to slow the mass’s translation in the x direction. At the time which this occurs there is a dip in the acceleration in the z direction. The velocity constrain is never active throughout the trajectory due to the short duration of the trajectory. Since the acceleration is limited to 1 m/s² and the problem is solved in 0.6 seconds there isn’t enough time for the mass to reach 1 m/s.

![Figure 5.9: (a) Angle of attack and (b) angular velocity](image-url)
Figure 5.10: Mass location

Figure 5.11: (a) x velocity of the mass (b) z velocity of the mass as a function of time

Figure 5.12: (a) x acceleration of the mass (b) z acceleration of the mass as a function of time
Another case was examined where there are two moving mass elements which again may translate anywhere within the vehicle. For this case the masses no longer start at the origin but are offset by \( \pm 0.5 \text{ m} \) in the \( z \) direction, and they are allowed to move independently. The optimal control problem was again solved for the same terminal conditions and the objective function remains the final time. The system dynamics may be seen in Figures 5.13-5.16. The vehicle angle of attack and angular velocity are nearly identical to the prior case. The system takes slightly longer to reach its termination criteria, but the mass trajectories are different. Again there is a focus to drive the mass as far down as possible, but the \( x \) component of the trajectory is mirrored for the two masses. The upper mass moves down and forward, while the lower mass moves down and aft. Furthermore the masses move much farther in the \( x \) direction than the prior case. The \( x \) acceleration again begins with an impulse to start the mass motion, but the acceleration to bring the mass to rest occurs over a longer period of time. The \( z \) component of the acceleration is again nearly bang-bang and as large as it can be without bringing the magnitude of the acceleration above \( 1 \text{ m/s}^2 \). The acceleration of the two masses in the \( z \) direction lie on top of one another, while in the \( x \) direction they are opposite. Velocity lie on top of one another and the \( x \) components of velocity are opposite. Here again the velocity constraint is not active since the problem is resolved in a such a short time scale.

![Figure 5.13: (a) Angle of attack and (b) angular velocity](image-url)
Figure 5.14: Mass location

Figure 5.15: (a) x velocity of the mass (b) z velocity of the mass as a function of time

Figure 5.16: (a) x acceleration of the mass (b) z acceleration of the mass as a function of time
The optimal control problem is reformulated for a mass within the area behind the HIAD as in the PD control case. The problem must include new constraints on the mass location, and the new formulation may be seen in Equation 5.4 with the new design vector in Equation 5.5. There are two new constraint equations which depend on a variable, m, which is the slope of the line connecting the tip of the HIAD to the rear of the vehicle. The equation for this line is presented in Equation 5.6. Additionally the bounds on the x, z are no longer the maximum and minimum, length and diameter, but depend on the size of the HIAD. The relationship for the bounds on the moving mass x and z location is written in Equation 5.7, where the lower bound on x is the x value where HIAD ends. The dynamics equations are similar to before, but the aerodynamic moment includes a dynamic pitching moment term.

\[
\min_x J = \int_0^t d\tau
\]

\[s.t\]

\[
h_1(x) = \|u\|_2 - a_{nom} \leq 0
\]

\[
h_2(x) = \|\dot{x}_1\|_2 - v_{nom} \leq 0
\]

\[
h_3(x) = z - (m \ast (x - (l + le)) + d/2) \leq 0
\]

\[
h_4(x) = -(m \ast (x - (l + le)) - d/2) - z \leq 0
\]

\[
h_5(x) = t_f - t_{nom} \leq 0
\]

\[
h_6(x) = \alpha(t) - 16 \leq 0
\]

\[
h_7(x) = x_l \leq x \leq x_u
\]

\[
g_1(x) = \alpha(t_f) - \alpha_{nom} = 0
\]

\[
g_2(x) = \omega(t_f) - \omega_{nom} = 0
\]

\[x = [\alpha, \omega, x_1, z_1, x_2, z_2, \dot{x}_1, \dot{z}_1, \dot{x}_2, \dot{z}_2]\]  

\[m = \frac{(d_h - d)/2}{((d_h - d)/(2 * \text{tand}(70)) - l)}\] (5.6)
\[ x_{ub}(\text{upper mass}) = \begin{bmatrix} \frac{(d_h - d)}{(2 \cdot \tan(70))} \\ d \end{bmatrix} \]
\[ x_{lb}(\text{lower mass}) = \begin{bmatrix} \frac{(d_h - d)}{(2 \cdot \tan(70))} \\ -d_h \end{bmatrix} \]
\[ x_{ub}(\text{upper mass}) = \begin{bmatrix} l \\ dh \end{bmatrix} \]
\[ x_{ub}(\text{lower mass}) = \begin{bmatrix} l \\ -d \end{bmatrix} \] (5.7)

w.r.t.

\[ \dot{\omega} = I_s^{-1} \left\{ M_{ext} - \left[ \dot{I}_s \cdot \omega + \omega \times (I_s \cdot \dot{\omega}) + \sum_{i=1}^{n} r_i \times m_i \dot{r}_i + \omega \times (r_i \times m_i \dot{r}_i) \right] \right\} \]
\[ M_{ext} = qS (c_M l + c_N z \times r_{CG} + c_A x \times r_{CG}) \]
\[ \Delta x_{CG} = \frac{mx}{m_s} \] (5.8)

Since the control is attempting to reach a 1° angle of attack target the solution is sensitive to initial conditions. The first initial condition tested is when the masses are placed at the furthest position from the centerline on the HIAD interface, the same location as the PD control. The optimal control problem is solved and the resulting system performance is shown in Figures 5.17-5.22. The problem is solved in just over 0.5 seconds which is an improvement from the prior case of the mass inside the vehicle, however the system response is different. The angle of attack has very little change for the first 0.1 seconds, but the masses are moving during this time. The upper mass has a similar motion to the prior cases, where it is nearly bang-bang control in the z direction with a small shift in the x direction. The lower mass motion is far more erratic. The mass initially moves inward counteracting the motion of the top mass, however it then loops back to its initial condition after 0.075 seconds. Then it begins to move along the diagonal boundary for the remainder of the trajectory. The acceleration of this mass is more chaotic than the upper mass, with far more switching between positive and negative acceleration.
Figure 5.17: (a) Angle of attack and (b) angular velocity

Figure 5.18: Mass location for (a) upper mass (b) lower mass

Figure 5.19: Mass location in the x direction as a function of time
Figure 5.20: Mass location in the z direction as a function of time for the (a) upper mass (b) lower mass.

Figure 5.21: (a) x velocity of the mass (b) z velocity of the mass as a function of time.

Figure 5.22: (a) x acceleration of the mass (b) z acceleration of the mass as a function of time.
Another mass initial condition was considered where the masses are started on the vehicle body at the HIAD interface, in other words the masses are shifted inwards compared to the previous case. The resulting system dynamics may be seen in Figure 5.23-5.28. The time to reach the terminal condition is larger than the previous case with different dynamics. Here the time until the vehicle starts rotating is smaller than the previous case, but the angular velocity has a region where the angular velocity is relatively unchanged. The lower mass moves as expected with most of its motion in the z direction and a small shift in the x direction while the z acceleration switches sign. Comparing this case with the previous case you can see that the acceleration of mass 1 in Figure 5.22 is very similar to the acceleration of mass 2 in Figure 5.28. The upper mass motion is different from all other cases in that it moves the same order of magnitude in both the x and z directions.

Figure 5.23: (a) Angle of attack and (b) angular velocity

Figure 5.24: Mass location for (a) upper mass (b) lower mass
A preliminary PD control is developed which is able to improve the system performance. A control
where the mass is allowed to move instantaneously within the search domain only has a 1-2 second faster response than if the mass is constrained by velocity or acceleration. Despite the similarities in the system response he mass locations for all of these cases is very different. Optimal control is able to significantly improve the response time of the system beyond that of the PD control. The response time between the optimal control cases are all less than 0.6 s which is a almost an order of magnitude improvement.
CHAPTER 6

CONCLUSIONS

This thesis presents internal moving-mass actuator performance for various system configurations. The study presented the requirements for trim conditions in terms of the moving-mass mass fraction. The size and packaging density of the vehicle have little effect on the mass fraction required for trim conditions, as the vehicle proportions remain the same. Slender vehicles require less mass in order to achieve the same trim conditions as a shorter vehicles. Placing the track in back of the vehicle rather than the front can provide a significant reduction in actuator mass fraction. To maximize lift-to-drag ratio the masses should be placed as far aft and off the center line as possible resulting in the largest possible moment arm to the aerodynamic forces. There is a strong dependency on both moving-mass mass fraction and Mach number on the available lift to drag ratio. The available lift-to-drag varies by as much as 0.03 over Mach numbers from 2 to 25, and the Mach number at which it peaks is between 10 and 17 depending on mass fraction.

Furthermore the dynamics of using a moving mass system was analyzed. General system response is presented for various geometries and mass translations. Across all cases there is an initial counter rotation resulting from the mass motions contribution to the angular momentum. Vehicle geometries with longer payloads have larger oscillations and longer settling times but also higher achievable angles of attack, vehicles with a larger HIAD have smaller oscillations which dampen very quickly.

Closed loop control for the system is then developed. A preliminary PD control is developed which is able to improve the system performance. The control is able to remove the initial counter-rotation as well as reduce rise time of the system. A control where the mass is allowed to move instantaneously within the search domain only has a 1-2 second faster response than if the mass is constrained by velocity or acceleration. The difference between a system which is constrained based on only velocity vs that of a system with an additional acceleration constraint is very small. Despite the similarities in the system response he mass locations for all of these cases is very different. Optimal control is able to significantly improve the response time of the system beyond that of the PD control. The response time between the optimal control cases are all less than 0.6 s which is a almost an order of magnitude improvement. For these cases only the acceleration constraint is active since the response time is so short that the masses never reach their maximum velocity.
REFERENCES


