A REINFORCED CONCRETE DESIGN
FOR
AN OPEN AIR AMPHITHEATRE
BY
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B. S. University of Illinois, 1903

THESIS
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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

LORING HARVEY PROVINE

ENTITLED A REINFORCED CONCRETE DESIGN FOR AN OPEN-AIR AMPHITHEATRE

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Architectural Engineer

In Charge of Major Work

Head of Department

Recommendation concurred in:

Committee

on

Final Examination
A Reinforced Concrete Design
for
An Open Air Amphitheatre

Introduction

It is the intention to make this thesis a reinforced concrete design and for that reason the subject of design and detail of the reinforcement has been described more fully in the following pages than the general plan and arrangement. While the writer has spent considerable time in planning the arrangement and has consulted several authorities on this subject, the calculations and details form the larger part of this work. A number of drawings of different forms of amphitheatre as well as arrangements have been made, but the one used here seemed the best.

Purpose

There has been a great need at the University of Illinois for an outdoor University amphitheatre, and it is the purpose of this thesis to design from a practical standpoint a structure adapted to the needs of the University, and to make it of the best construction. There has been no attempt at elaborate design or detail
but a plain substantial amphitheatre; one that will stand the wear of years and the action of the elements; substantial and useful.

There has been no place for the assembling of the student body out of doors, except on the Library steps or on bleachers on the athletic field. This was inconvenient as they were not at all adapted to the needs and entirely inadequate. The students like to assemble out of doors for "sings", open air concerts, the May Pole dance, the annual Freshman-Sophomore contest, and other like assemblages and it will be seen from the general arrangement, this building will meet this need. It is not intended for the use of baseball, or football games, requiring a large field.

General Description

The building is to be of reinforced concrete entirely. It is circular in plan, with a section on each end tangent to the curve. In plan the Amphitheatre has an open field of one hundred and five feet in diameter; around this is a concrete promenade ten feet wide; rising from this promenade are twenty rows of seats, and at the top of these is a walk, to be used as a promenade or standing room in case of large crowds. Directly in center of this amphitheatre are provided a number of boxes, which will be for the use of invited guests, or others of prominence. After careful consideration and consultations with various authorities on buildings of this kind, it was deemed advisable not to have any cross aisles between main aisles, or means of entrance except from the front. It is a well known fact that, on crowded occasions, such aisles, stairways,
etc. would be crowded by spectators standing and thus make the surrounding seats, also those back of these aisles, very undesirable.

The first row of seats is raised three feet from the promenade level, so the occupants may see over the heads of any persons who might be standing on the walk. This first row has a concrete rail eighteen inches high, and on this is a two inch gas pipe rail.

The steps are eighteen inches high and two feet and six inches wide. In the main aisles are steps one half these dimensions, making the steps easily ascended. Each step is pitched slightly to the back where there is an outlet for water. This outlet is connected to the storm water sewer.

At the rear of the upper promenade is a concrete rail ten inches thick and four feet high, with a moulded hand rail.

At first it was thought that chairs would not be necessary, but it has been decided to use chairs similar to the unupholstered opera chair. This will be much more satisfactory than benches.

As to the framing of this Amphitheatre, there are beams under each row of seats tangent to the curve at the center. These in turn are supported on girders, radiating from the center. Columns are placed under the ends of these girders and vary in length from three feet to thirty four feet. Concrete footings below frost line carry the columns.

In case there are to be class plays, or public speaking requiring a stage, this may be erected anywhere in the arena, and a free, unobstructed view for the entire audience may be had. In case of speaking, a stage can be erected at the center of the field and the voice will carry to almost the back of the amphitheatre.
Open air speaking is always an uncertainty, depending a great deal upon the atmospheric conditions; a slight breeze will make it impossible for a part of an audience to hear, no matter how close they might be to the speaker.

There are seats provided for thirty six hundred students in the main amphitheatre; there will be a row of seats at the front for one hundred more. In addition, there is room on the upper promenade for standing room for several hundred more.

**Line of Sight**

In order that the audience might all see to the best advantage, various lines of sight were considered. It is very desirable that the audience may see the entire field. In order that the sight from the first few rows of seats would be unobstructed by people standing on the promenade, the first row of seats was placed three feet above the promenade level. This allows a person sitting in the front row to see over the heads of the people standing.

The ideal arrangement for seeing is to make each seat high enough to see directly over the heads of the people directly in front. The first scheme considered was establishing a point four feet high and ten feet out from the front of the first row of seats, or a point on the ground eighteen feet out. The system used for locating the height of steps was to draw the position of the person as sitting in the front row. For the second row, a line was drawn from the assumed view point tangent to the head of the first person, which locates the eye of the second person; from this point, four
feet was measured down, giving the step line. This process was carried out for the entire twenty rows. It was found the seats assumed the line of a curve, the steps having a minimum rise at the front of about fifteen inches and increasing to about twenty seven inches at the back row. While this is the ideal condition, it is not practical, and not necessary, as other means, as explained later, solved the problem better.

The next scheme was to assume steps of a uniform height and see what point would be visible for the entire audience. This gave a point forty seven feet out from the front row of the amphitheatre. This is not at all desirable as the field nearer than this was hidden from a part of the audience.

Various other schemes were tried but they were not practical.

The scheme as finally adopted was to make the steps eighteen inches high, and stagger the seats; that is, place the seats so that those in alternate rows will be placed back of the space between the two seats in front. In other words, a person can see between the two directly in front, instead of having to look directly over their heads. This then resolved into the problem of finding the viewpoint for people looking over the heads of the people the second row in front. The scheme seemed to be the best one considered.

It gave a viewpoint at the ground for the entire audience, twenty five feet from the first row of seats, or in other words, it means the audience can see the ground twenty five feet away, over the head of a person six feet tall standing nine feet from the first row.
This last method was adopted as it gives a uniform rise to the steps and makes practically the entire field visible to the entire audience, thus all seats are desirable. The sketch, as shown, was used as the assumed position of a person sitting, and was used in the determining of seats.

Finish

The exterior, or the main elevation of the building, after the forms have been removed, is to be rubbed with a soft brick or stone, while the concrete is still damp, giving it a very desirable finish, taking off all impressions left by the form lumber, and making it a uniform finish. There are various other ways of treating this surface, such as bush hammering, after the concrete has set; by using a wire brush while the concrete is still soft, leaving the surface of the pebbles, stone and sand exposed; but the rubbed surface for this building will be more satisfactory.
For determining the working stresses to be used throughout this work, various city ordinances were consulted, and below is a table giving the various stresses allowed by the building laws of various cities, governing reinforced concrete.

<table>
<thead>
<tr>
<th>Building Laws Governing Reinforced Concrete</th>
<th>Cleveland</th>
<th>New York</th>
<th>San Francisco</th>
<th>Buffalo</th>
<th>Toronto</th>
<th>Boston</th>
<th>St. Louis</th>
<th>Philadelphia</th>
<th>Prussian Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of Modulus of Elasticity of Steel to Concrete</td>
<td>15</td>
<td>12</td>
<td>15</td>
<td>12</td>
<td>15</td>
<td>15</td>
<td>12</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>Tensile Stress in Steel</td>
<td>16000</td>
<td>1600</td>
<td>12 E.L.</td>
<td>16000</td>
<td>16000</td>
<td>16000</td>
<td>14000</td>
<td>7000</td>
<td>16000</td>
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<tr>
<td>Compressive Stress in Steel</td>
<td>10000</td>
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<tr>
<td>Shearing Stress in Steel</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
<td></td>
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<tr>
<td>Extreme Fibre Stress on Concrete in Compr.</td>
<td>*700</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>800</td>
<td>600</td>
<td>1/8 U</td>
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<tr>
<td>Concrete in Direct Compression</td>
<td>*500</td>
<td>350</td>
<td>350</td>
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<td>350</td>
<td>350</td>
<td>500</td>
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</tr>
<tr>
<td>Shearing Stress in Concrete</td>
<td>50</td>
<td>50</td>
<td>75</td>
<td>50</td>
<td>50</td>
<td>60</td>
<td>25</td>
<td>75</td>
<td>64</td>
</tr>
<tr>
<td>Bending Moment in Beams Continuous</td>
<td>1/16 w</td>
<td>1/8 w</td>
<td>1/8 w</td>
<td>1/8 w</td>
<td>1/8 w</td>
<td>1/8 w</td>
<td>1/8 w</td>
<td>1/8 w</td>
<td>1/10 w</td>
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<tr>
<td>Bending Moment in Slabs Continuous</td>
<td>1/10 w</td>
<td>1/10 w</td>
<td>1/12 w</td>
<td>1/10 w</td>
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<tr>
<td>Bending Moment in Square Floor Panels</td>
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<td>1/20 w</td>
<td>1/20 w</td>
<td>1/20 w</td>
<td>1/20 w</td>
<td></td>
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<tr>
<td>Method of Calculation</td>
<td>S.L.</td>
<td>5 L.</td>
<td>S.L.</td>
<td>5 L.</td>
<td>5 L.</td>
<td>5 L.</td>
<td>5 L.</td>
<td>5 L.</td>
<td></td>
</tr>
<tr>
<td>T Section Amount Allowed as Part of Beam</td>
<td>6 b</td>
<td>10 b</td>
<td>5 t</td>
<td>10 b</td>
<td>5 b</td>
<td>1/2 Span</td>
<td>1/2 Span</td>
<td>20 + 1/5 l</td>
<td></td>
</tr>
<tr>
<td>Columns- Ratio of Maximum Ratio M to W</td>
<td>16</td>
<td>12</td>
<td>15</td>
<td>16</td>
<td>12</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Requirements of Tests</td>
<td>3 L.</td>
<td>3 L.</td>
<td>2 L.</td>
<td>3 L.</td>
<td>3 L.</td>
<td>2 L.</td>
<td>2 L.</td>
<td>2 L+D.</td>
<td></td>
</tr>
</tbody>
</table>

Note: S.L. = Straight Line Formula.  
E. L. = Elastic Limit.  
T = Hooped Columns.  
*600# If test shows F.S. of 3 in 90 days.
It will be seen from this table that the Cleveland laws have been just recently revised and conform to the best practice. The stresses as specified will generally be used in this work. For live loads for buildings used for public assemblies:

<table>
<thead>
<tr>
<th>City</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York</td>
<td>90 # per sq ft</td>
</tr>
<tr>
<td>Chicago</td>
<td>100 #</td>
</tr>
<tr>
<td>Philadelphia</td>
<td>120 #</td>
</tr>
<tr>
<td>Boston</td>
<td>150 #</td>
</tr>
<tr>
<td>St. Paul</td>
<td>125 #</td>
</tr>
</tbody>
</table>

In these calculations we will use the Chicago requirements of 100 # per square foot, in addition to the weight of the floor construction.

Straight Line Law of Stress Deformation

Working loads and stresses will be used throughout these calculations. Should the stresses and loads be the ultimate, the parabolic relation would have to be used, since there would be considerable deformation in the concrete.

Since working loads and stresses are to be used, the straight line law of stress variation is sufficiently accurate. The comparison of experimental results with theoretical analysis of authorities, show the simple beam theory as generally employed, neglecting tension in the concrete, can be used with confidence. The results appear to show that, calculated on the basis of such theory, the yield point of the steel may safely be taken as the ultimate
9.

strength of the reinforced beams; that the crushing strength of concrete, as determined by tests on cubes seasoned under similar conditions as the beams, will be fully developed in the beam.

The usual assumptions will be made,- that the loads are applied at right angles to the length of the beam; that a plane section before bending, remains a plane section after bending; that the metal and surrounding concrete stretch together; the tensile strength of the concrete is entirely neglected, and the modulus of elasticity of concrete in compression, is constant.

Calculations

Formulas - Straight line assumption and neglecting tension in concrete. Using stresses as specified by Cleveland building laws.

Determining percentage of reinforcement.

\[ p = \frac{n}{b d} \quad (1) \]

where \( p \) = steel ratio \( \frac{A}{b d} \)

\( n = \) ratio \( \frac{E_s}{E_c} \)

\( f_s \) = unit fiber stress in steel

\( f_c \) = unit fiber stress in concrete at its compressive face

In the Cleveland ordinance-

\( n = 15 \)

\( f_s = 16000 \) # per square inch

\( f_c = 700 \) " " " " 
Substituting in the formula-

\[
p = \frac{\frac{1}{6000}}{\frac{16000}{700} \left( \frac{16000}{15 \times 700} + 1 \right)}
\]

\[
p = .0087\%
\]

This percentage of reinforcement for the given concrete and ratio of working stresses, has the same value for all sizes of beams.

If a percentage of reinforcement as given above, is adopted, then, theoretically, the concrete will fail by compression at the same time the steel will fail by tension, which is the ideal condition. If a percentage less than that given above is used, then the steel will fail and the strength of the steel will govern the strength of the beam. If a greater percentage is chosen, the steel will be in excess, and the strength of the concrete will determine the strength of the beam.

Neutral Axis and Arm of Resisting Couple.

Since we are using the straight line formula -

\[
k = \sqrt{2 \, pn + (pn)^2} - pn \quad (2)
\]

Where the values are the same as given for equation (1)

Substituting in equation (2) values from (1)-
\[ k = \sqrt{2 \times 0.0087 \times 15 + (0.0087 \times 15)^2} - 0.0087 \times 15 \]

\[ k = 0.397 \]

This formula shows that the neutral axes of all beams of a given concrete and of a given percentage of reinforcement, are at the same proportionate depth. As "k" increases, "j" decreases, but not in the same ratio. For common values, \((n = 15, \text{ and } p = 0.0075 \text{ to } 0.011)\)- the value of "j" does not vary much with "p", the average value of "j" is about 7/8.

\[ j = 1 - \frac{1}{3} k \]

(3)

\[ j = 0.868 \]

Formulas for Slabs, Beams, and Girders.

For the strength of the steel:

\[ M = f_s \ p_j \ bd^2 \quad \text{bd}^2 = \frac{M}{f_s \ p_j} \]

(4)

For the strength of the concrete:

\[ M = \frac{1}{2} f_c \ k_j \ bd^2 \quad \text{bd}^2 = \frac{M}{\frac{1}{2} f_c \ k_j} \]

(5)

Since we use a value of "p" that balances \(f_s\) and \(f_c\), it makes no difference which formula we use:

Reinforcing against Shrinkage and Temperature Stresses.

Use 0.0066 of the cross section.
12.

Slab Design

Live load = 100 # per square foot

Chairs or seats = 10 # " " "

Assume slab = 40 # " " "

Total load = 150 # " " "

Span = 2' - 6"

In the slab calculations, no advantage can be taken of the continuous action of the slab, since it is supported on each beam and is a separate slab; therefore, we will use \( M = \frac{Wl^2}{8} \)

\[ M = \frac{150 \times 2.5^2}{8} = 117 \text{ ' # or 1404 " #} \]

For thickness, assume a breadth of 12"

For steel \( bd^2 = \frac{M}{f_s p_j} = \frac{1404}{16000 \times .0037 \times .868} = 11.62 \)

For concrete \( bd^2 = \frac{M}{\frac{1}{2} f_c k_j} = \frac{2 \times 1404}{700 \times .397 \times .868} = 11.64 \)

This shows that with this percentage of steel, the values for thickness are the same.

Assume \( b = 12" \) \( d^2 = \frac{11.62}{12} = .98" \)
We will make the slab 3" thick as a minimum thickness; for practical reasons it should not be made thinner.

Assume ½" of fireproofing on the underside of the slab -

\[ d = 2\frac{1}{4}" \]

Area of steel = $0.0087 \times 12" \times 2\frac{1}{4}" = 0.2349 sq."

Use a wire fabric for reinforcement.

Triangular reinforcement 2" mesh, with 2 wires # 4 in each longitudinal, and # 14 cross wires. This gives a total area per foot width of 0.2349 sq. " of metal.

This form of reinforcement will also provide against any temperature or shrinkage cracks.

For the finish use a wearing coat ¼" thick, composed of Portland cement, sand and marble, or granite dust. This finish is to be applied before the concrete base has had a chance to set and is to be a part of the step. The step is to pitch backward one quarter of an inch, and from there pitched to the drain. All steps are to be finished thus.

For Slab of Upper Promenade.

Span = 12' - 0"
Live load = 100 # per square foot
Assume slab = 75 # " "
Total load = 175 # " "

\[ M = \frac{175 \times 12^2}{8} = 3150 \ ' # \ or \ 37800 " \ # \]

\[ bd^2 = \frac{37800}{120.8} = 313 \]
14.

Assume \( b = 12'' \)

\[
d^2 = \frac{313}{12} = 26.1''
\]

\[
d = 5.1'' \quad \text{Add} \quad 0.9'' \text{ for fireproofing.}
\]

Total thickness 6''

Area of steel required -

\[
A = p b d = 0.0087 \times 12 \times 5.1 = .53'' \quad \text{of steel per foot width}
\]

Use \( \frac{1}{2}'' \) twisted square bars \( 5\frac{1}{2}'' \) centers.

For temperature and shrinkage stress \( \frac{1}{2}'' \) twisted

\( 2' - 0'' \) centers.

Use the same finish and same method of applying as called
for on steps.

Beam Design

The Cleveland ordinance allows the use of the formula

\[
\frac{WL}{M=10}
\]

for continuous beams, but owing to the beams not being in
true alignment, we will not use this formula for the beams, but al­
low the continuity over the support as an additional factor of safety.

The beams for supporting are all tangent to the curve of
the seats and therefore there will be a slightly heavier section of
beam at the support. The forms will be made perfectly straight
for the back of the beam and curved to the proper line for the
front.

This difference between the curve and the
beam line has been computed and is found to
equal .6 of a foot at the extreme back step
15.

and .3 of a foot at the inner step. The maximum additional weight at the back step, at each support, due to this curve, amounts to 500 pounds, gradually decreasing to 250 # at the lowest step.

It would not be practical to vary each and every step from the front to the back step, therefore, the area between two rows of girders has been divided into portions about 10 feet in width, as shown in the drawing to the left.

Beams will be computed for these varying lengths as noted.

The depths are practically fixed by the height of step and thickness of slab.

**Top Girder of Seats - Girder marked (A)**

Span = 22' - 0" - Load from seats = 150 # x 1½' x 22' = 4125 #

" " walk = 175 # x 6' x 24' = 25200 #

Wt. of beam 15" x 29" - 22' x 150 # = 9900 #

Total load on girder (A) = 39225 #

\[
M = \frac{39225 \times 22}{8} = 107870 \text{ '# or 1,294,440 " #}
\]

\[
b d^2 = \frac{1294440}{120.8} = 10715
\]

Assume b = 15" d² = \frac{10715}{15} = 714

\[
d = 26.8" \text{ Add 2.2" for fireproofing.}
\]
16.

Steel A = \( p \times b \times d = 0.0087 \times 15 \times 26.8 = 3.5 \) in

Use 5 - 7/8" twisted square rods = 3.8 in

Size and spacing of shear loops as shown on the details.

Investigate beam (A) for internal stresses.

Bond or Resistance to Slipping of Reinforcing Bars.

In order to have beam action there must be a proper web connection between the tension and the compressive portions of the beam. Where there is no metallic web reinforcement, the concrete of the beam acts as this web. In transmitting the increment of tension from the reinforcing rods to the surrounding concrete, there is developed a tendency of the rods to slip in the concrete, and the amount of resistance to slip thus developed, is called bond, and is measured in terms of the area of the surface in contact with the concrete. It will be seen that the total bond developed on the surface of the bars in one inch of length, is equal to the total change in total tensile stress in the bar for the same inch of length.

Without going into the derivation of the formula:

Let \( m \) = number of bars

\( o \) = the circumference of one bar

\( u \) = bond developed per unit of area of surface of bar

\( V \) = the total vertical shear at the given section

\( d' \) = the effective depth or \( = j \cdot d \)

Then \( u = \frac{V}{mod'} \) or \( u = \frac{V}{moj \cdot d} \)
This equation is true for bars horizontal to the support; if a part of the rods are inclined upwards, \( d' \) will be variable and the formula will need to be modified.

Then for girder (A)-

\[
V = \text{maximum at the support} = \frac{\text{load}}{2}
\]

\[
V = \frac{39300}{2} = 19650 \, \#
\]

The reinforcement was 5 - 7/8" square rods.

\[
m_o = 5 \times 4 \times \frac{7}{8} = 17\frac{1}{2} \, \frac{\#}{"}
\]

\[
u = \frac{19650}{17.5 \times .868 \times 26.8} = \frac{19650}{407} = 48.3 \, \# \text{ per square inch.}
\]

With a factor of safety of 5, a working stress of from 50 to 75 \# per square inch is allowable for smooth rods; for deformed bars, about 30% more. Therefore, we see the bond stress is sufficient at the location of maximum stress.

Since it would be necessary to embed the bar a length of from 60 to 75 diameters, we will use a twisted bar and bend a hook on each end, thereby adding to the strength of the bar concerning slipping.

Vertical and Horizontal Shearing Stresses.

In mechanics of beams it has been shown that there exist throughout a beam, vertical and horizontal shearing stresses of varying intensities, and that at any point in a beam, the vertical
shearing unit stress is equal to the horizontal shearing unit stress there developed. The total tension in all beams varies along the length of the beam, as does also the total compression. The horizontal shearing stress may be considered to transmit the increase of the total tensile stresses in the reinforcing bars, (which is transmitted to the surrounding concrete by the bond stress) to the corresponding compression in the compression area of the concrete.

Let \( v \) = horizontal unit shearing stress
\( b \) = the breadth of beam

Then \( v = \frac{V}{bd'} = \frac{V}{bjd} \)

This equation is based on the use of straight rods, and will have to be modified if rods are bent up, since \( d' \) will vary then. This equation gives the horizontal shearing unit stress, and also the vertical shearing unit stress, at a point just above the level of the bars. This stress is true for all points between the neutral axis and the steel; above the neutral axis, the shear follows the law, as in homogeneous beams.

\[
v = \frac{19650}{15 \times 0.868 \times 26.3} = \frac{19650}{350} = 56.3 \text{ # per square inch.}
\]

This stress is a little in excess of the working stress, but since we are to use twisted bars, bent up, also shear loops, this stress will be safe. By making the beam 17 inches thick, we can re-
duce this stress to the allowable 50 #, but for the reasons given above, this is not necessary.

Diagonal Tension in Concrete

It is shown in mechanics of beams that whenever vertical and horizontal shearing stresses are set up in the web of the beam, tensile, or compressive and shearing stresses exist in every diagonal direction. Only the horizontal components of these stresses enter into the determination of the bending moment. When there is no metallic web reinforcement, all the diagonal stresses are taken by the concrete. The angle of the diagonal direction at which the maximum stress exists, depends upon the relative values of the shearing stress and the horizontal tension existing in the concrete. Since we assume there is to be no tension in the concrete considered —

then \( t = v \)

and the maximum diagonal tension makes an angle of 45 degrees with the horizontal, and is equal in intensity to the vertical and horizontal shearing stress.

The best method of computing this stress seems to be, to compute the horizontal and vertical shearing unit stress, and make all comparisons on the basis of this stress.

Then if \( t = v \) for beam (A) \( t = 56 # \) per square inch.
Vertical and Diagonal Reinforcement.

Since the diagonal tension may be resolved into horizontal and vertical components, the web stresses, or at least a part of them, may be resisted by use of stirrups, to take the vertical component of the diagonal tension, and by bending up the reinforcing rods into a diagonal position.

For designing the stirrup, it is very seldom that a beam need carry more than 100 lb per square inch average shearing stress, we will assume the concrete to carry 30 lb per square inch shearing stress, and the steel must carry the balance, at a working stress of 16000 lb per square inch. If the area of the cross section =bd, then the shear to be carried is 70 bd; and as the tendency to rupture is on a line inclined at 45 degrees, this shear may be considered as the load to be carried by the web reinforcement in a length equal to the depth of the beam.

The necessary steel area is then -

$$A = \frac{70 \times bd}{16000} = 0.0043 \times bd$$

Beam 15" x 26" \(bd = .390\) \(\frac{\text{"}}{\text{"}}\) \(A = 0.0043 \times 390 = 1.67\) \(\frac{\text{"}}{\text{"}}\) for a length equal to \(d\) or 26".

If the stirrups are spaced 6" centers, or about \(\frac{d}{4}\), then the steel area = \(\frac{1.67}{4} = .42\) \(\frac{\text{"}}{\text{"}}\). This would require a 3/8" rod in a double loop.
According to Talbot: \( \frac{V}{b} = \frac{v}{d'} \) or \( \frac{V}{b} = \frac{v}{j'd} \)

Beam 15" x 26"  \( V = \frac{19650}{15 \times .868 \times 26.8} = 56.3 \) #

If stirrups are 6" centers, then the stress in the two prongs of the stirrup = 6 \( b \) \( v = 6 \times 15 \times 56 = 5040 \) #; if we use a double loop then we have four prongs and the stress in each prong equals \( \frac{5040}{4} = 1260 \) # each. \( A = \frac{1260}{16000} = .07866 \) ". This value agrees very closely with the value determined above.

Another authority says \( Y = \frac{j'd}{P} \)

Where \( Y \) = spacing, center to center of stirrups required at any section

\( j'd \) = the effective depth of beam

\( V \) = external vertical shear at any section

\( V_c \) = total vertical shearing stress assumed to be carried by the concrete = \( bd' \times V_c \)

\( P \) = total stress in one stirrup = area metal of stirrup x allowable unit stress.

Substituting the stirrup as determined by our first formulae:

\[ Y = \frac{.868 \times 26 \times (\text{area of 4-3/8" rods})}{19650 - (15 \times 26 \times 30 \#)} \]

\[ Y = 20" \text{ o.c.} \]

Thus we see there are various formula for determining, and therefore only so called estimates can be made.

Close spacing is of more importance even than size. For this
work we will use the size of stirrup and the spacing as were determined by the first equation. This spacing agrees fairly well with Mr. E. L. Ransome's empirical rule for spacing stirrups, which is, "the first stirrup is placed at one fourth the depth of the beam from the end; the second stirrup is placed at one half the depth of the beam from the first, and third stirrup is placed at three fourths the depth of the beam from the second," and continuing.

It is always best to place a few stirrups in the center of all beams, although theoretically they may not be necessary. Under concentrated loads it will be necessary to space the stirrups closely together.

Girder at back of Upper Promenade - A'

Span = 24'-0" Load from walk 175 # x 6' x 24' = 25200 #
" " Parapet wall 3' x 1' x 24' x 150 # = 10800 #
Weight of beam = 9900 #

\[ M = \frac{45900 \times 24}{8} = 137700' \# \] or \( 1,652,400'' \# \)

\[ bd^2 = \frac{1,652,400}{120.8} = 13650 \]

Assume \( b = 18'' \)
\[ d^2 = \frac{13650}{18} = 760 \]
\[ d = 27.6'' \]

Make beam 18" x 30"
23.

Area steel \[ A = pbd = 0.0087 \times 18 \times 27.6 = 4.33 \text{" metal} \]

6 - 7/8 " twisted square bars

With shear loops of size and spacing as shown.

Bond \[ u = \frac{V}{mojd} = \frac{22950}{6 \times 3 \frac{1}{2} \times 0.868 \times 27} = 47 \text{ # per square inch.} \]

(which is allowable)

Vertical and horizontal shear -

\[ v = \frac{V}{bjd} = \frac{22950}{18 \times 0.868 \times 27} = 54.5 \text{ # per square inch.} \]

(which is allowable on account of the stirrups and rods bent up)

We will use 3/8 " round double stirrups.
Design of Beam (B) - Typical of Panel (5)

Total load on beam next to back beam -

- Live load = 100 # per square foot
- Slab 3 " = 40 #  "  "  "
- Chairs = 10 #  "  "  "
- Assume beam = 80 #  "  "  "
- Add for extra wt. beam = 10 #  "  "  "
- Total load = 240 #  "  "  "

Total load on beam 240 # $\times 2\frac{1}{2} ' \times 22 ' \text{ (max. span) } = 13200 #

\[
\text{Span } = 22' - 0'' \quad M = \frac{13200 \times 22}{8} = 36300 ' \text{ # } = 435600 '' \text{ #}
\]

\[
bd^2 = \frac{M}{f_s \text{ psj}} = \frac{435600}{120.8} = 3620
\]

Assume \( b = 8'' \quad d^2 = \frac{3620}{8} = 452 \)

\( d = 21.3 '' \)

Add 2.7 " for fireproofing.

Area of steel \( A = \text{pbd} = .0087 \times 8 \times 21.3 = 1.48 '' \)

Use 3 - 3/4" twisted square bars = 1.68 "

Beam 8" $\times 24''$

Rods 3 - 3/4" square twisted

Shear loops - 3/8" and spaced as shown on drawing.
Investigate the internal stresses for Beam (B).

**Bond**

Total load from floor = 13200 #

Beam 8" x (21.3" + 2.7")

Rods 3 - 3/4" twisted square rods

\[ V = \frac{13200 \#}{2} = 6600 \# \]

\[ u = \frac{V}{mod'} = \frac{V}{moj'd} \]

\[ u = \frac{6600}{3 \times 3 \times 868 \times 21} = 40 \# \text{ per square inch.} \]

This stress is well within the safe working stress of 60 #.

**Vertical and Horizontal Shearing Stresses.**

\[ v = \frac{V}{b \cdot j \cdot d} \]

\[ v = \frac{6600}{8 \times 368 \times 21} = 46 \# \text{ per square inch.} \]

This stress is allowable since it is within the safe working stress.

Both the bond stress and shearing stress will be greatly reinforced as an additional factor of safety by the stirrups and the rods bent up.
Diagonal Tension.

Since \( t = v \) \( t = 46 \) # per square inch

\( 8'' \times 21'' \times .0043 = .72 \) " necessary in a length equal to the depth.

If stirrups are spaced 7" O.C., or \( \frac{1}{3} d \), then \( \frac{.72}{3} = .24 \) " required for each stirrup which requires a double loop stirrup 5/16" in diameter. Since the other stirrups in Beam (A) etc. are 3/8", we will make these stirrups 3/8" in diameter.

Beams for Section # 4

Total load per square foot the same on these beams as those in Section (5).

Total load on beam 240 # \( \times 2\frac{1}{2}' \times 20' \) (max. span) = 12000 #

\[
M = \frac{12000 \times 20}{8} = 30000 \text{ #}, \text{ or } 360000 \text{ " #}
\]

\[
bd^2 = \frac{360000}{120.8} = 2975
\]

Assume \( b = 7" \) \( d^2 = \frac{2975}{7} = 425 \)

\( d = 20.7" \) Add 3.3 " for fire proofing.

Area of steel \( A = p \times b \times d = .0087 \times 7 \times 20.7 = 1.26 \) " of metal.

Use 3 - 3/4" twisted square bars.

With shear loops spaced as shown.

This beam is so near the size of beams for Panel (5), it is doubtful whether there is any economy in reducing the size.
Beams for Section # 3

Maximum Span = 18'-0"
Total load 240# x 2 1/2' x 18' = 10800 #

\[
M = \frac{10800 \times 18}{8} = 24300\#'
\]

Thus we see the bending moment is gradually decreasing. It is necessary to keep all the beams of certain depths owing to the uniform rise of steps, consequently the thickness must be reduced. For the depth of beam necessary for the steps, the minimum thickness should be not less than 7 inches, therefore it will not be economical to reduce the beams any smaller.

The reinforcing might be reduced some in size, but 3/4" bars are the "base" or the basis for costs, therefore it will be cheaper to use 3/4 inch bars and not get into bars requiring an extra on price. The handling and piling bars all one size is cheaper than having various sized rods to sort, pile and handle.

The question of forms is also another consideration. By making the beams all the same size, the forms as laid out for one beam will do exactly for any other by changing the length.

For the above reasons we will make the beams under the steps, except the one at the top, a uniform size of 8" x 24", with 3 - 3/4" twisted square bars. The loops will be of the size and spaced as shown on the plans and details.
Girders.

In the design of the girders we have considered the seating area divided into four equal divisions as shown in the accompanying drawing. The girders and columns hereafter designed will be lettered to correspond to the letters in the sketch.

Girder L

See above drawing for length of divisions.

Average length of area supported by girder = \( \frac{22.0 + 19.57}{2} = 20.78' \)

Live load = 100 # per square foot
Slab = 40 # " " "
Beam = 90, # " " "
Chairs = 10 # " " "
Total load = 240 # " " "

The average load on each beam = 240 \( \times 2.5' \times 20.8' = 12480 \) #

Weight of assumed girder = 15" \( \times 26" - 11'-0" = 4460 \) #
Reaction at right support, hereafter known as \( R_e = 28400 \) #

\[ R_L = (12480 + 4460) - R_e \]

Maximum moment occurs at point 5'-0" from left support.

\[ M_{\text{max}} = 994,056" \] #

\[ \frac{bd^2}{f_s p_j} = \frac{994000}{120.8} = 8230 \]

Assume \( b = 15" \) then \( d = \frac{8230}{15} = 550 \)

\[ d = 23.5" - \text{(call it 24" + 2")} \]

\[ A = pbd = .0087 \times 15" \times 24" = 3.13" \]

Use 6 - 3/4" twisted square rods = 3.36" "

Since we are using square twisted rods, or rods with a mechanical bond, there will be no danger from slipping. Also we are using stirrups and also a portion of the rods bent up at the supports the safe horizontal and vertical shearing stresses will not be in danger of being exceeded.
Diagonal Tension

Area of metal required in stirrup $s = 0.0043$ bd

$= 0.0043 \times 15 \times 24 = 1.55''$

for a length of beam equal to the depth.

For 6'' spacing $\frac{1.55}{4} = 0.39''$ required for each stirrup

$\frac{4}{4} = 1''$ " " " " loop

Use a 3/8" rod with a double loop.

Girder K

See drawing for length of divisions.

Average length of area supported by girder $\frac{19.57 + 17.13}{2} = 18.35$

Average load on each beam $= 240 \times 2.5 \times 18.35 = 11010 \#$

Weight of assumed girder $= 15'' \times 26'' - 12' = 4860 \#$
Max. Moment occurs at center of span.

\[
M_{\text{max}} = 90200'' \# \text{ or } 1,082,400''\#
\]

Then

\[
bd^2 = \frac{M}{f_b P_j} = \frac{1082400}{120.8} = 8400
\]

Assume \(b = 15''\) then

\[
d^2 = \frac{8400}{15} = 560
\]

\(d = 23.7''\) call it 24'' (+2'')

\[
A = \pi b d = 0.0087 \times 15 \times 24 = 3.13''
\]

Use 6 - 3/4'' twisted square rods = 3.36''

Use 3/8'' rods, with double loop for stirrup.

Girder J

See drawing for length of divisions.

Average length of area supported by girder \(= \frac{17.13 + 14.69}{2} = 15.91''\)

The average load on each beam \(= 240'' \times 2.5'' \times 15.9'' = 9540''\)

Weight of assumed girder \(15'' \times 26'' - 12' - 0'' = 4860''\)

We see from these reactions that the load is slightly decreasing; therefore theoretically we can reduce the size of the beam and the area of reinforcement, but for practical reasons, it is much better to leave them the same size as Girder K.
Girder H.

See drawing for length of divisions.

Average length of area supported by girders = \( \frac{14.69 + 12.25}{2} = 13.47 \) average length of area supported "by girders = \( \frac{14.69 + 12.25}{2} = 13.47 \)

The average load on each beam \( 240 \times 2.5 \times 13.47 = 8081 \) #

The weight of the assumed girder \( 15'' \times 26'' = 12' = 4860 \) #

Maximum moment occurs at a point 5 feet from the left support.

\[ M_{\text{max}} = 64300 \text{ #} \text{ or } 771600 \text{ " #} \]

\[ \text{then } bd^2 \cdot \frac{M}{f_5jd} = \frac{771600}{120.8} = 6030 \]

Assume \( b = 15'' \) - then \( d^2 = \frac{6030}{15} = 400 \)

\[ d = 20'' \text{ total depth } = 20'' + 2'' = 24'' \]

\[ A = p \cdot b \cdot d = .0087 \times 15 \times 20 = 2.61'' \]

Use 5- 3/4 twisted square rods -

Use 3/8" round rods, with double loop for stirrups.

From this we see the beam is 2 inches shallower than the other girders and we are using one rod less for reinforcing.
Columns

There is a scarcity of comparable data on the strength of reinforced concrete columns from which reliable formulae might be derived. Experiments to determine the compressive strength of concrete, even when made on carefully prepared test specimen, give a great range of results and therefore the allowable stresses used in designing should be conservative values.

There are three methods of increasing the allowable compressive stress in a column:

1st - The use of a very rich mixture for the columns only.

2nd - The introduction of longitudinal reinforcement, which is usually tied together by horizontal bands or loops.

3rd - Hooping the columns to prevent lateral expansion when under the load.

It has been proven by numerous tests, the results of a rich mixture of concrete; this needs no further remarks - It is desirable and necessary that there be some longitudinal reinforcement, since, owing to the monolithic character, more or less bending moment will be put into the columns by various conditions of loading. This reinforcement also carries a part of the direct compressive stress. The ratio of the moduli of elasticity of the two materials may be used with a certain degree of accuracy for working stresses but as higher stresses are used, this module will vary. In connection with these longitudinal rods, horizontal loops or binders must be used at intervals apart, not exceeding the side of the core; their
purpose being to hold the longitudinal rods in place and prevent buckling.

With the hooped column, greater stresses may be used on the core of the column. The hooping prevents lateral movement of the contained concrete when it has about reached its ultimate value. This hooping usually consists of wire spirals with a pitch varying from $1\frac{1}{2}''$ to $4''$. Hooping undoubtedly prevents sudden failure by preventing failure through shear on an oblique plane. All hooped columns should have sufficient longitudinal reinforcement to take care of bending stresses.

In the design herein submitted, hooped concrete columns were designed for the loads to be carried but in order to use the high stresses permissible with this style of column, the column was entirely too small. There is nothing to be gained in the use of hooped concrete columns unless high stresses may be used. For this reason the hooped concrete was replaced by a plain concrete column with longitudinal reinforcement.

It will be noted hereafter that this Amphitheatre presents rather an unusual condition of rather light loads with long columns. Therefore the columns are all designed for a greater load than the superimposed load because of the length of column and the size of beam resting on the column.
Columns and Footings

Column G

Load on Column G equals the load from the beam at the back of the upper promenade and equals 45900 #.

Length of column is considered from the center line of the girder to three feet below grade line.

Length 34'-6"

According to the Cleveland ordinance, the ratio of the length of the column to the width is 16, therefore requiring a column with a core 26" in diameter. If this were used then the stress on the concrete would be too low and not at all practical. We will therefore use a smaller column and reduce the stress proportionately.

Marsh gives the following formula for long columns:

\[
P_i = \frac{P}{1 + \frac{K^2 C I^2}{\pi^2 E_c (r_c^2 + r_s^2)}}
\]

Where

- \(P_i\) = Safe load on long column in pounds
- \(P\) = Safe load calculated on a short column
- \(K\) = A coefficient depending on method of support at ends
- \(C\) = Ultimate resistance of concrete in pounds
- \(I\) = Length of column in inches
- \(E_c\) = Modulus of elasticity of the concrete
- \(r_c\) = Radius of gyration of the reinforcement
- \(r_s\) = Radius of gyration of concrete assumed being of total sectional area of column
From recent tests on plain concrete we can assume -

\[ C = 2000 \# \quad \text{and} \quad E_c = 2,000,000 \# \]

The formula then reduces to -

\[
P_1 = \frac{P}{1 + \frac{K^2 l^2}{9870 \left( r_c^2 + r_s^2 \right)}}
\]

Since our beam is 15 inches wide, it would therefore not be advisable to make the core of the column less than 15 inches. We will try a column 15 inches x 15 inches, with one 7/8 inch square rod in each corner. If this column were of a height not exceeding 16 times the side, then the load which this column would support is

\[
P = C(bd + 14 A)
\]

Then

\[
P = 500 \left( 15 \times 15 + 14 \times 4 \times .77 \right)
\]

\[ P = 134000 \# \]

For a long column we will substitute this value in our formula given above -

\[
P = \frac{134000}{1 + \frac{1190 \times 144}{2 \times 9870 \left( \frac{15 \times 15 + 7 \times 7.4}{12} \right)}}
\]

\[ P = 119200 \# \]

This load is in excess of the given load but when a smaller section was considered, we found the calculated load to correspond too closely with the actual conditions. With this column the load may be applied directly to the core or effective area of the column. The column that will be used will be a 15" x 15" core, with 4 - 7/8" square rods, and one and one half inches of fireproofing outside.
Horizontal loops to be 3/8" round bars, spaced 12" centers.

The Cleveland building law gives a rule to use for long columns, which is -

\[
\text{Stress per square inch} = \frac{500 \times \text{least side in inches}}{\text{Unsupported length in feet}}
\]

Therefore, \( S = \frac{500 \times 15}{34} = 220 \) # per square inch

Total load the column will carry equals 15" x 15" x 220# = 49500#

therefore the column is safe, exclusive of the longitudinal steel.

Footing for Column G

Load from beams = 45900 #

Weight of column 21" x 21" - 34'-6" = 15800 #

Load on footing = 61700 #

Assumed weight of footing = 3000 #

Total load on ground = 64700 #

Assume pressure on soil at 3000 # per square foot -

Then, \( \frac{64700}{3000} = 21.6 \) square feet required.

Make footing 4.65 feet on each side or 4'-8" square.
The allowable vertical shear in the concrete is given in the building laws as 50 pounds per square inch.

Required depth of footing then, \[ \frac{61700}{4 \times 21 \times 50} \] = 14.4" Thus to keep the column from shearing through the footing, the footing should be 14.4" thick. It should be noticed however that in this shearing no account was taken of the rods to resist the shear.

As the shear decreases toward the edge of the footing and this depth of footing is not required to resist the bending moment developed, the footing can be stepped off or battered.

The upward pressure on the triangle A C D and the downward pressure on the triangle D C E are each equal to one fourth the column load.

The maximum bending moment is in the plane X X and may be obtained by taking moments about this line. We first find the center of gravity of the triangle A C B which equals nineteen inches from X X, and the center of gravity of the triangle D C E which equals
seven inches from X X.

\[
\text{Then } M = \frac{61700}{4} \times (19 - 7) = 185100 \text{ " #}
\]

This moment is resisted by a section in the plane X X. The width considered available may be taken equal to the width of the column plus \( 1\frac{1}{2} \) times the depth of the footing.

This width \( 21" + (1\frac{1}{2} \times 8") = 33" \)

The moment per foot width of section \( \frac{185100}{33 \times 12} = 67500 \text{ " #} \)

Since the concrete in the footing must resist shear as well as the compression stress due to bending, we will use -

\[ p = .007 \text{ reinforcement} \]

\[
\frac{bd^2}{f_y} = \frac{M}{16000 \times .007 \times .868} = \frac{67500}{97.2} = 691
\]

But we assumed \( b=12" \) then \( d^2 = \frac{691}{12} = 57.6" \)

\( d = 7.59" \) - make it 8"

\( A = p \times b \times d = .007 \times 12 \times 8 = .672 \text{ " of metal per foot width.} \)

Use 5/8" twisted square bars 7" centers.

Part of these rods will be bent up and all will have hooks on the ends. Being a deformed bar and having the ends bent into hooks, there will be no danger of the rods slipping in the concrete.

There is another method of figuring this footing in which the over hanging portion of the footing is considered as a cantilever
fixed on the line of the column base. The first method will be used throughout these calculations.

Column F

Load from Girder L = 23830 # eccentric.

" " Girder A = 39225 # concentric.

Although this column has an eccentric load, it will not be considered as such, since there is always more or less uncertainty of the actual location of the center of pressure; and also the girder rods run through the column almost to the opposite edge.

This eccentricity when a column supports a crane girder, or other similar construction where the eccentricity is definitely known, should be considered and great care exercised in the design. Also this column will be made larger than theoretically required, so there will be no danger from this condition of loading.

Length of column from center line of girder to top of footing equals 32'-6".

Since the girders are 15" wide, we will assume a column with a core 15"x15", with 4 - 7/8" square rods, and a concrete protection 1½" thick around the outside.

By referring to the column just designed we find the same size column, 34 feet long, will be good for 119200 #; this same column will be used here. Therefore we see that the column will be strong enough to carry the direct load and also the stresses due to the eccentric loading.

Column is to be 21"x 21", with 4 - 7/8" square rods, with horizontal loops.
41.

Footing of Column F

Load = 63055 #
Column 21"x21" - 32'-6" = 15000 #
Load on footing = 78055 #
Assume footing = 5600 #
Total load on ground = 83655 #

Assuming the same soil for all columns -

Then \( \frac{83655}{3000} = 27.88 \) " required

Make footing 5.28 ' on each side, or 5'-4" square.

For shearing, the depth = \( \frac{78055}{4 \times 21 \times 50} \) = 18.6"

\[ M = \frac{78055}{4} (21 - 7) = 273000 " # \]

Available width through center 21" + (1\( \frac{1}{2} \) x 11) = 37" = 3.08'
M per foot = \( \frac{273000}{3.08} = 90000 \) #

\[ bd^2 = \frac{90000}{97.2} = 926 \]

Assume \( b = 12" \)

\[ d^2 = \frac{926}{12} = 77.2 \]

\[ d = 8.79" \]

Since 11" is needed for shear, we will make the footing 11" to the reinforcing rods and 3" below the rods.

\[ A = p b d = 0.007 \times 12 \times 11 = 0.924" \] of metal per foot width.

Use 5/8" twisted square rods 5" centers.

Column E

Load from Girder K = 29955 #

" " Girder L = 30630 #

Load on column = 60585 #

Length of column from center line of girder to top of footing equals 23'-6".

Since there is only 3000 # difference in load between Column E and Column F, we will use the same size column.

Column 21" x 21", with 4 - 7/8" square bars and horizontal loops.
Footings.

Load = 60585 #
Column 21" x 21" - 23'-6" = 10800 #
Load on footing = 71385 #
Assume footing = 5600 #
Total load on ground = 76985 #

Assuming the same soil -

Then the bearing area = \[
\frac{76985}{3000} = 25.66 \text{ sq ft}
\]

Make footing 5.08' on each side or 5'-1" square.

Since the footing for Column F is 5'-4" square, we will make this footing the same thickness and with the same reinforcement.

Column D

Load from Girder J = 22305 #
" " Girder K = 29995 #
Total load on col. = 52300 #

Length of column from center line of girder to top of footing equals 20'-0".

Assume a column 15" x 15" with a core 13" x 13", with 4 - 7/8" square rods.

This column is within the requirements of the building laws,
which require the side of a column to be 1/16 the length; therefore, at 500 # per square pressure on the concrete, the load that can be supported by this column $13'' \times 13'' \times 500 = 84500 \#$.

The column would carry the load without any steel, but it is advisable to use the steel.

The column will be $15'' \times 15''$ with 4 - $7/8''$ square bars, and horizontal loops.

**Footing for Column D**

<table>
<thead>
<tr>
<th>Description</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load on column</td>
<td>$52300 #$</td>
</tr>
<tr>
<td>Column $15'' \times 15''$ - 20'</td>
<td>$4700 #$</td>
</tr>
<tr>
<td>Total load on footing</td>
<td>$57000 #$</td>
</tr>
<tr>
<td>Assume footing</td>
<td>$4500 #$</td>
</tr>
<tr>
<td>Total load on ground</td>
<td>$61500 #$</td>
</tr>
</tbody>
</table>

Pressure on soil at 3000 \#:

Then area required $= \frac{61500}{3000} = 20.5 \text{ sq ft}$.

Make footing 4.5' on each side, or 4'-6'' square.
45.

For shearing, the depth = \( \frac{57000}{4 \times 15 \times 50} = 19" \)

\[ M = \frac{57000}{4} (18 - 5) = 185250 " \] #

Available width through center \( 15" + (1\frac{1}{2} \times 11) = 31" = 2.6' \)

\[ M \text{ per foot} = \frac{185250}{2.6} = 71250 " \] #

\[ bd^2 = \frac{71250}{97.2} = 733 \]

Assume \( b = 12" \)

\[ d^2 = \frac{733}{12} = 61 \]

\[ d = 7.82" \] - call it 8"

Since 11" is needed for shear, we will make the footing as shown with \( d = 11" \). This also keeps the depth of footings uniform.

\[ A = \pi b d = .007 \times 12 \times 11 = .924 " \] of metal per foot width

Use 5/8" square twisted bars, 5" centers.
Column C

Load from Girder H = 17915 #
" Girder J = 20715 #
" beam directly over column = 8820 #
Total load on column = 47450 #

Length of column from center line of girder to top of footing equals 10'-6".

We will make this column the same as Column D.

Column 15" x 15", with 4 - 7/8" square rods, with horizontal loops.

Footing.

Load on column = 47450 #
Weight of column 15" x 15" - 10'-6" = 2350 #
Load on footing = 49800 #
Assume weight of footing = 4800 #
Total load on ground = 54600 #

Assuming 3000 # per square foot pressure on the soil -

\[
\frac{54600 \text{ #}}{3000} = 18.2 \text{ ft}^2
\]

The area required = 18.2 ft^2

Make footing 4.25 on each side, or 4'-3" square
For shearing, the depth \( \frac{49800}{2 \times 15 \times 50} = 16.6" \)

\[ M = \frac{49800}{4} (17 - 5) = 149400 " \]

Available width \( 15 + (1\frac{1}{2} \times 11) = 31\frac{1}{2}" = 2.625' \)

M per foot \( = \frac{149400}{2.625} = 57000 " \)

\[ bd^2 = \frac{57000}{97.2} = 585 \]

Assume \( b = 12" \)

\[ d = \sqrt{\frac{585}{12}} = 48.75 \]

\( d = 6.98" \) necessary.

We will make this footing \( d = 11" \) to correspond to the other footings - also for shear -

\[ A = p b d = 0.007 \times 12 \times 11 = 0.924 " \] of metal requires.

Use 5/8" rods, 5" centers.
Footing for Wall at Lower Promenade

Load from Girder H = 19265 #
Weight of concrete under end of H 15" x 18" x 3'-0" = 840 #
Weight of wall 10" x 6'-0" x 12'-3" = 9200 #
Load from lower step 150# x 1'-3" x 12'-3" = 2300 #
Weight of concrete below ground line to top of footing 3' x 13" x 3' = 1690 #
Load on footing = 33295 #
Assume weight of footing = 3000 #
Total load on ground = 36295 #

Pressure soil 3000 # per square foot -

\[ \frac{36295}{3000} = 12.09 \text{ sq ft} \]

In order to make the center of pressure coincide with the center of the footing, we have in the figure the condition of loading.

and by moments find the center line of the loads comes to a point 1.05' from girder load; therefore about this center line, lay off
the footing as shown in the drawing. There will be no reinforcement necessary in this footing.

Materials

Perhaps a word might be said about the quality of the materials to be used. The steel, as figured, is known as medium steel, having an ultimate strength of 60000 to 70000 # per square inch, and shall conform to the Manufacturers' Standard Specifications as revised in 1903.

The concrete shall be machine mixed and for columns, in the proportion of 1 part of Portland cement, 2 parts of clean sand, and 4 parts of stone. For slabs, beams and girders, the proportions may be 1 of cement, 3 of sand, and 5 of stone. The usual care to be exercised in the mixing and placing of the concrete.

The cement to be of an approved brand of Portland cement, and shall conform to the Specifications of the American Society for Testing Materials. The requirements for the tensile strength for briquettes, one inch square in section-

For neat cement, 24 hours in moist air 200 #, 7 days (one day in air and six in water) 500 #, 28 days (one day in air and twenty seven in water) 600 #.

For one part cement, to three parts standard sand, 7 days (one day in air and six days in water) 175 #, 28 days (one day in air and twenty seven in water) 250 #.

The sand shall be clean, sharp and graded as to size.
The stone shall be broken stone, free from dust, and small enough to pass through a three quarter inch ring.

The forms should demand care in construction and method of support. Either wood or sheet iron may be used. If wood is used, only planks 2 inches thick shall be used and the inner face dressed. All joints between plank to be made with beveled edges so the swelling due to absorbing the water will not cause them to twist or bulge the faces. If sheet iron, or steel is used, they must be securely backed and braced, so when the concrete is placed they will hold their shape. All forms to be securely held in place by supports at intervals strong enough to carry the dead weight of the concrete while it is setting; also any live load that may be imposed upon them.

The forms not to be removed within ten days after the last concrete has been placed and at that time only with the consent of the Inspector.

After the forms have been removed, and while the concrete is still more or less damp, go all over the faces of the concrete with a soft brick or stone, giving a rough effect and removing all imperfections left by the forms. This rubbed finish is to apply to columns, beams, girders and walls exposed to view. The seats, both the riser and tread, the steps and promenade are to have a smooth wearing surface.

After the erection of the building various tests may be required within a reasonable time after completion. The tests must sustain twice the live load for which the section was designed without any sign of failure.
General

For this design, square twisted, medium steel rods are to be used, except for beam stirrups and column loops. That the twisting of the bar gives a mechanical bond, and greatly reduced the tendency to slip, has been proven by tests in laboratories, and also in the actual construction. A square bar, twisted cold, has a higher ultimate strength than the same bar untwisted. This twisting also will show up any defective bar, as the twisting tends to compact the fibres all the closer and a bar that is not uniform or homogeneous will be evident from the fracture caused.

At each support certain bars will be bent up and pass over the support into the adjoining beam, while the remaining bars in the beam will be straight and run from the center line of one support to the center line of the other. Thus we are providing for a negative bending moment over the support, making an additional factor of safety, which was not considered in the calculations. All bars are to have a hook six inches long and bent at right angles on each end, thereby adding another factor of safety against slipping.

In each beam and girder there will be placed vertical stirrups, varying from a close spacing at the ends, to a greater distance at the center. These stirrups will help resist any diagonal tension. These stirrups, in connection with a part of the reinforcing rods bent up and having hooks on the ends, will provide amply for the internal stresses developed in the beams.
In these calculations all sections of beams were considered rectangular; there was no place where the tee section could be considered.

Some city building laws will allow a reduction of the live load on girders, columns and footings. In this work the full live load was considered as carried clear through the construction and on to the ground. Since there will be no live load at all unless it is loaded to the full capacity of the amphitheatre.

For the purpose of protection to the steel, the steel in the columns will be spaced two inches away from the surface of the concrete, in beams, the lower layer of rods shall be kept one and one half inches from the bottom, for spacing the rods apart in the bottom, a minimum of one and one half diameters will be maintained both between the rods and from the outside rod to the edge of the concrete. For footings, the rods will be kept three inches above the bottom face; for the slabs, the protection will vary from one half inch to three quarters of an inch.

It will be noticed certain parts of the construction have been increased in size, or made to correspond in size to certain other parts. This is advisable within limits, for the reasons as given. A greater uniformity in the sections will minimize the cost of detailing parts, the cost of labor in assembling and not having so many different sizes as to make it confusing. By using certain sizes of reinforcement, without designing down to the last fraction of area, the ordering, sorting, piling, handling and placing of rods will be made much easier and more economical. Since the 3/4" rod is taken as a base price, it is always advisable if possible,
to keep the rods required above the "base". Therefore, for practical reasons we have preserved a certain uniformity of sections.

In some cases wire netting has been placed in the forms before the reinforcement is placed. This holds the concrete firmly in place at the bottom of the beam and helps unite the beam more firmly.

On account of the length of columns F and G, lateral bracing has been provided. The horizontal struts have been made 12 inches by 16 inches in section, and having 4 - 7/8" twisted square rods, with 3/8" round loop for each. The diagonal bracing has been made the same.
PART PLAN - FOOTINGS

HALF PLAN - AMPHITHEATRE

FRONT ELEVATION

REAR ELEVATION

AN OPEN AIR AMPHITHEATRE

Scale 1:16" = 1' 0"