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J. CHANG
M. RAETHER
S. TANAKA

UNIVERSITY OF ILLINOIS - URBANA, ILLINOIS

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Experimental Observation of Wave-Wave Coupling in a Beam Plasma System*

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J. Chang†, M. Raether and S. Tanaka‡
Department of Physics and Coordinated Science Laboratory
University of Illinois, Urbana, Illinois 61801

ABSTRACT

The interaction between unstable waves has been investigated in a beam-plasma system under conditions such that nonlinear wave-particle interactions are negligible. At small wave amplitudes the results can be interpreted in terms of a simple three-wave decay process; at large amplitudes the situation is dominated by multiple wave coupling that leads to generation of satellite waves.

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†Present address: 5243 Sandia Laboratories, Albuquerque, New Mexico 87115.

‡Permanent address: Department of Physics, Faculty of Science, University of Kyoto, Kyoto, Japan.

In this report we present observations of wave-wave coupling between the linearly unstable modes of a beam plasma system in which a monoenergetic electron beam interacts with a cold plasma in the presence of collisions. This system is characterized by the dispersion relation[1]

$$\epsilon(k,\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\,\nu_c)} - \frac{\omega_b^2}{(\omega - k\,v_o)^2} = 0$$

ω_b and ω_p are the plasma frequencies of beam-and plasma electrons and ν_c is the collision frequency. For the case of spatially growing waves (ω real) the relation between the real part of k and ω is almost linear. This makes the system particularly attractive for the study of nonlinear wave interactions because the resonance conditions for three wave coupling

$$\omega_o = \omega_1 + \omega_2$$

$$k_o = k_1 + k_2$$

can be satisfied simultaneously to a good approximation for collinear waves; the mismatch in wavenumber $\Delta k = k_o - k_1 - k_2$ being typically less than 1% of k_o for the experimentally interesting case.

The experiments were carried out in a beam-plasma system that has been described in detail previously[2]: A monoenergetic electron beam of 16 kV, 60 - 100 ma and 2μ sec duration was injected into the afterglow of a pulsed discharge in Ne at a pressure of 4×10^{-2} Torr. The discharge tube was 4 cm in diameter and 50 cm long; beam and plasma were pulsed with a repetition frequency of 30 Hz. Prior to entering the plasma the beam was modulated at two frequencies f_o and f_1 (9.49 and 9.39 GHz respectively) by passing it through an X-band waveguide so that the electric field was in the direction of the beam velocity. The delay time between initiation of the discharge and injection of the beam pulse was chosen so that the plasma density corresponded to the density of maximum instability for the 9.49 GHz wave. To insure stable operation and reproducibility the neutral gas pressure variations, which were the main source of plasma density fluctuations, had to be kept below .5%. The radiation from these waves was monitored by an X-band receiver whose frequency could be swept from 8.2 to 12.4 GHz. A Ku-band receiver (13 - 18 GHz) and an if-amplifier at 100 MHz were used to monitor the sum- and difference frequency.

With the two primary waves located on the unstable, negative energy branch, there are four different ways in which the two waves can couple to a third wave: to the difference - or sum frequency on either energy branch. Coupling to the sum frequency on the positive energy branch is explosive;[3] the three other cases are of the decay type.[4] In the present experiment radiation was only detected at the difference frequency, any signal at the sum frequency, if present, was below the sensitivity of our receiver (-100 dBm). Inasmuch as the waves on the positive energy branch are damped, we may expect coupling to the difference frequency on the negative energy branch to be the most effective process. In Fig. 1 (a) the location of the three waves is indicated schematically.

Under this condition the dynamic wave equations[5] for the normalized amplitude $a_i(x) = \frac{1}{\sqrt{8\pi}} \left|\frac{\partial \epsilon}{\partial k_i}\right|^{\frac{1}{2}} E_i(x)$ can be solved exactly in terms of elliptic functions in two limiting cases, namely, when all linear growth rates $\varkappa_o$ are zero and when they are all equal and the mismatch $\Delta k = 0$.[6] The solutions describe a periodic energy transfer between the wave ω_o and the waves ω_1 and ω_2. The distance of maximum energy transfer will be referred to as the decay length x_o. When all linear growth rates are equal to $\varkappa$ and the initial amplitudes obey the relation $a_o^2(0) > a_1^2(0)$; $a_2^2(0) = 0$, x_o is given by

$$x_o = \frac{1}{\varkappa} \ln \left\{ 1 + \varkappa \frac{K(\gamma)}{W[a_o^2(0) + a_1^2(0)]^{\frac{1}{2}}} \right\} \tag{1}$$

where $K(\gamma)$ is the elliptic integral of the first kind, $\gamma^2 = a_o^2(0)[a_o^2(0) + a_1^2(0)]^{-1}$ and $W(\omega_o, w_1, \omega_2)$ is the coupling constant. In the present experiment, the growth rates of the high frequency waves are essentially equal and the growth rate of the wave at the difference frequency is very small. It can be shown from numerical solutions[7] that the decay length for this case is only slightly larger (10 - 15%) than that given by Eq. 1. It is convenient to express W in terms of the nonlinear growth rate $\varkappa_{NL} = W\, a_o(0)$. In the present problem this can be written

$$\varkappa_{NL} = \frac{e}{m} \frac{E_o(0)}{v_o^2} \Gamma_{NL} \tag{2}$$

Where Γ_{NL} is given by[8]

$$\Gamma_{NL} = \frac{1}{4} (\Omega_1 - K_1)^{3/2} (\Omega_2 - K_2)^{3/2} \left\{ \frac{1}{\alpha \Omega_o \Omega_1 \Omega_2} \left[\frac{K_o}{\Omega_o} + \frac{K_1}{\Omega_1} + \frac{K_2}{\Omega_2} \right] + \frac{1}{(\Omega_o - K_o)(\Omega_1 - K_1)(\Omega_2 - K_2)} \left[\frac{K_o}{\Omega_o - K_o} + \frac{K_1}{\Omega_1 - K_1} + \frac{K_2}{\Omega_2 - K_2} \right] \right\} \tag{3}$$

$$\Omega = \omega/\omega_p \quad ; \quad K = k\, v_o/\omega_p \quad ; \quad \alpha = \omega_b^{\,2}/\omega_p^{\,2}$$

A direct experimental determination of the decay length, and thereby of the coupling constant, by measuring the spatial variation of the wave amplitudes was found to be impractical due to poor spatial resolution of the microwave receiver. We therefore fixed the position of the receiver at 30 cm from the entrance aperture of the beam and varied $a_o^2(0)$ and $a_1^2(0)$ which are proportional to the modulation power $P_M(f_o)$ and $P_M(f_1)$ respectively. Fig. 1 (b) shows an example of the power in the three waves as a function of modulation power $P_M(f_o)$. In the absence of wave interactions the received power $P(f_o)$ is proportional to $P_M(f_o)$ and $P(f_1)$ should be independent of $P_M(f_o)$. The measurements show a pronounced maximum of radiated power $P(f_1)$ together with a corresponding minimum in $P(f_o)$, indicating an energy transfer from wave ω_o to wave ω_1 in qualitative agreement with the expected behavior in the decay mode. The power in the difference frequency increases with increasing power in f_o as expected, but does not exhibit a maximum. This may be due to the fact that the difference frequency is not well localized because its wavelength is comparable with the dimensions of the system. Fig. 1 (c) shows the enhancement of $P(f_1)$ for different initial modulation power of f_1. We can distinguish two distinct regimes. At low values of $P_M(f_1)$ we see the maximum in radiated power that was apparent in the previous figure. The value of $P_M(f_o)$ at which this maximum occurs corresponds to the $a_o^2(0)$ for which the decay length x_o is equal to 30 cm. For increasing modulation power $P_M(f_1)$ the maximum shifts to lower values of $P_M(f_o)$, a trend that is predicted by Eq. 1. The relation between $a_o^2(0)$ and $a_1^2(0)$ for constant decay length can be represented by a universal curve which is plotted in Fig. 2 (a) together with experimental points fitted to this curve. From the best fit the magnitude of the coupling constant can be deduced if the initial electric field is known. A direct measurement of the electric field strength is very difficult; it can, however, be calculated with reasonable accuracy from the known modulation power. From the hydrodynamic equations one can easily find the relation between the initial velocity modulation Δv and the electric field inside the plasma as

$$E_o(0) = \frac{m}{e} \Delta v(\omega - k\, v_o) = \frac{3^{1/4}}{2} \frac{m}{e} \Delta v \omega_p (\alpha \omega_p / 2 \nu_c)^{1/2} \tag{4}$$

for modulation at the frequency of maximum instability. A modulation power of 100 mW corresponds to an electric field $E_o(0)$ of 0.42 V/cm. The so determined value of Γ_{NL} is indicated in Fig. 2 (b) together with the theoretical curve in the vicinity of the most unstable mode. Although there is considerable uncertainty in the absolute experimental determination of Γ_{NL}, it appears that there is a disagreement between the experimental and theoretical value. Whether the source of this discrepancy can be traced to the inadequacy of the theory is not easy to ascertain, however, in view of the fact that the wavelength of the low frequency mode is comparable to the decay length we cannot expect the theoretical model to provide more than a rough approximation to the experimental situation.

At larger modulation power $P_M(f_1)$ the points of maximum amplitude deviate substantially from the theoretical curve (Fig. 2 (a)) and the interaction changes in character. The frequency spectrum of the received radiation reveals that apart from the two primary waves numerous satellites appear at frequencies $f_o \pm n f_2$ and $f_1 \pm n f_2$, where $f_2 = f_o - f_1$. An example is shown in Fig. 3. This indicates a high degree of remixing of the primary waves with the difference frequency. The physical mechanism may be very similar to that described by Goldman et.al.[9] where they consider the mixing of a high frequency wave with a low frequency wave to generate first order Stokes and anti-Stokes modes. This aspect of the experiment will be dealt with in detail in a forth coming publication.

Summarizing, we can say that the interaction between unstable waves in a beam plasma system leads to phenomena that are in general agreement with the theoretical ideas of three wave coupling in a decay mode. Since the system is nonresonant as far as wave-particle interactions are concerned, the only other nonlinear process that could interfere is particle trapping.[10] The critical trapping field

$$E_{cr} = \frac{1}{4} k \frac{m}{e} (v_{ph} - v_o)^2 = \frac{\sqrt{3}}{32} k \frac{m v_o^2}{e} \left(\frac{\alpha}{\nu_c} \omega_p\right) \qquad (5)$$

is of the order of 10 V/cm and well below the fields that may be encountered in the experiment. Although dynamically possible, trapping can nevertheless be ruled out because the transit time of the electrons through the system is smaller than, or at most of the order of the trapping time $\tau_{TR} = 2\pi\left(\frac{m}{ekE}\right)$. An electron can therefore not even execute one oscillation in the potential trough and the concept of trapping becomes meaningless.

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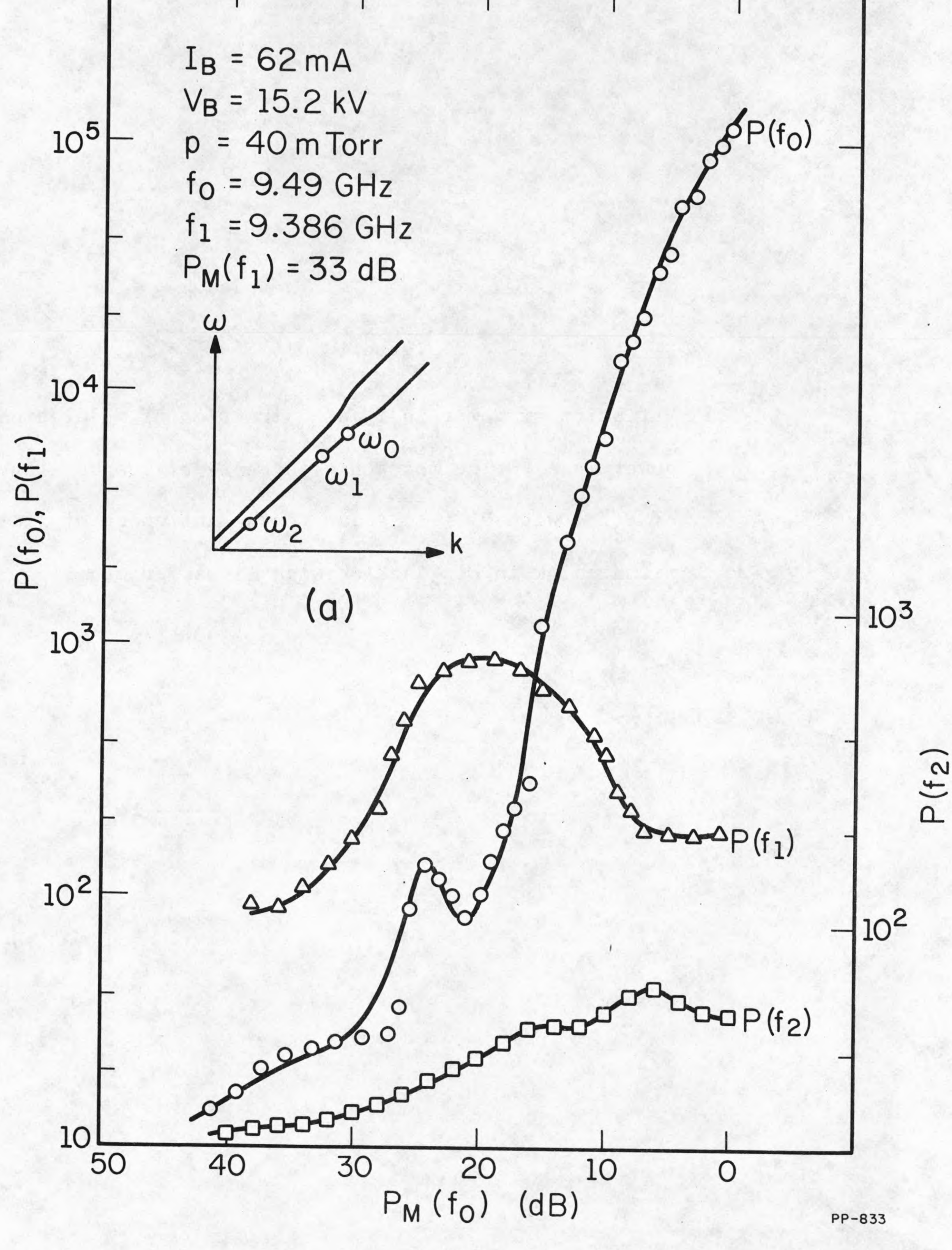
Figure 1.

(a) Location of the three waves on the dispersion diagram. The lower curve corresponds to the negative energy branch.

(b) Power dependence of the three waves as a function of modulation power of the wave with frequency f_o. 0 dB corresponds to a modulation power of 100 mW and the indicated dB values refer to attenuation with respect to this power level.

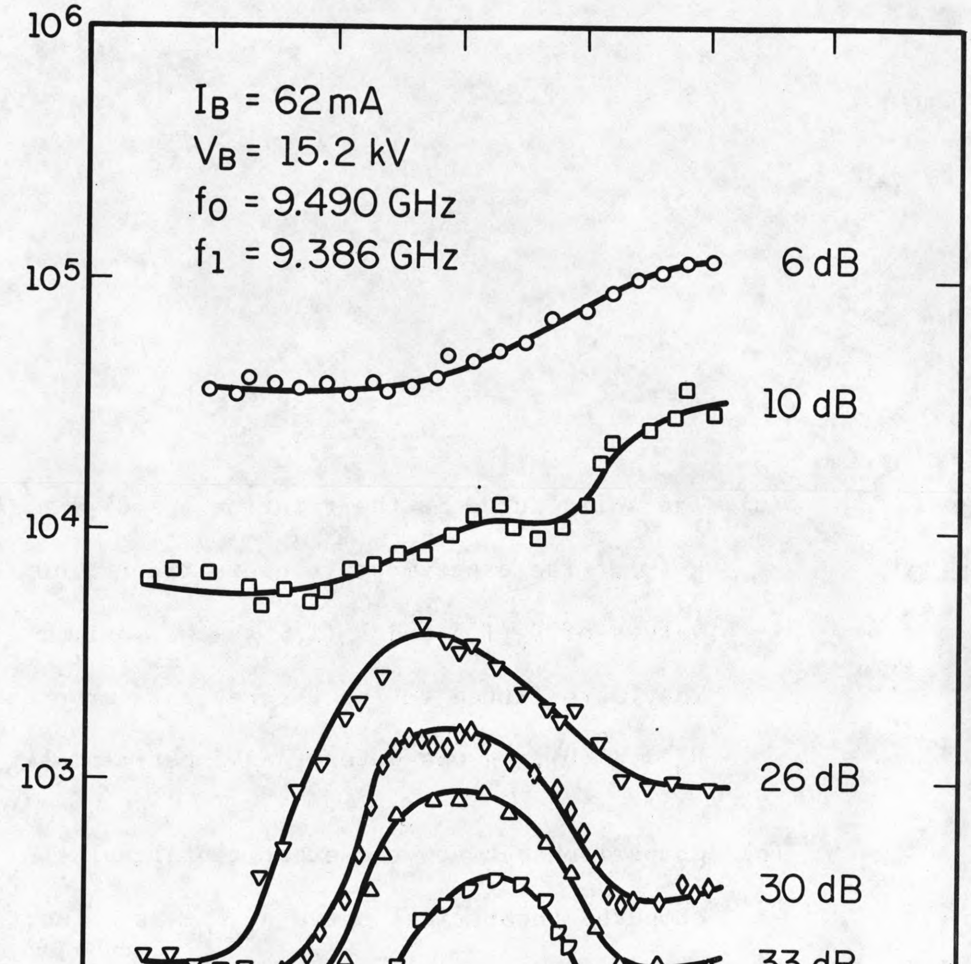


I_B = 62 mA
V_B = 15.2 kV
p = 40 m Torr
f_0 = 9.49 GHz
f_1 = 9.386 GHz
$P_M(f_1)$ = 33 dB
ω
k
ω_0
ω_1
ω_2
(a)
10^5
10^4
10^3
10^2
10
$P(f_0), P(f_1)$
$P(f_0)$
$P(f_1)$
$P(f_2)$
10^3
10^2
50
40
30
20
10
0
$P_M(f_0)$ (dB)
PP-833

(b)

Figure 1.

(c) Variation of power in f_1 as a function of modulation power in f_o. The monotonic increase of the baselevel of $P(f_1)$ with $P_M(f_o)$ is due to the influence of finite mismatch Δk in conjunction with linear growth.



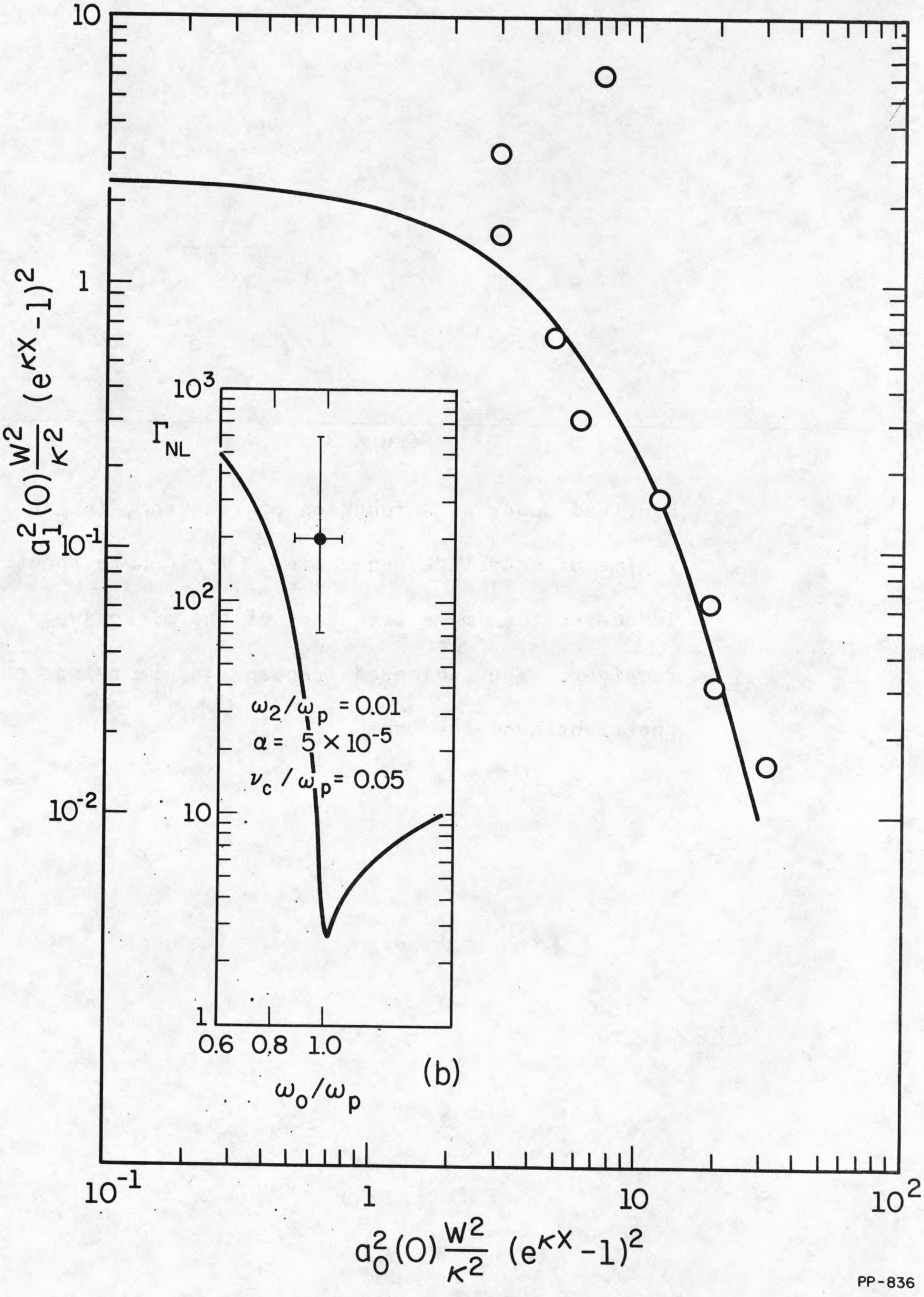
I_B = 62 mA
V_B = 15.2 kV
f_0 = 9.490 GHz
f_1 = 9.386 GHz
6 dB
10 dB
26 dB
30 dB
33 dB
36 dB
40 dB
43 dB
46 dB
10^6
10^5
10^4
10^3
10^2
10
1
$P(f_1)$
50
40
30
20
10
0
$P_M(f_0)$ (dB)
PP-786

(c)

Figure 2.

(a) The solid curve is the relation $(a_o^2(0) + a_1^2(0))\frac{1}{\varkappa^2}(e^{\varkappa x_o}-1)^2 W^2 = K^2(\gamma)$. The experimental points are the corresponding values of $P_M(f_o)$ and $P_M(f_1)$ at the maximum of $P(f_1)$. They were fitted to the theoretical curve assuming that $P_M \propto a^2(0)$. $\varkappa$ was determined experimentally as 0.25 cm^{-1}.

(b) Comparison between the experimental value of Γ_{NL} and the computed theoretical curve of Γ_{NL} as a function of frequency ω_o for fixed difference frequency. The vertical error bars correspond to the uncertainty in x due to the finite aperture of the receiving horn; the horizontal error bars indicate the uncertainty of position of the frequency f_o with respect to the plasma of frequency.



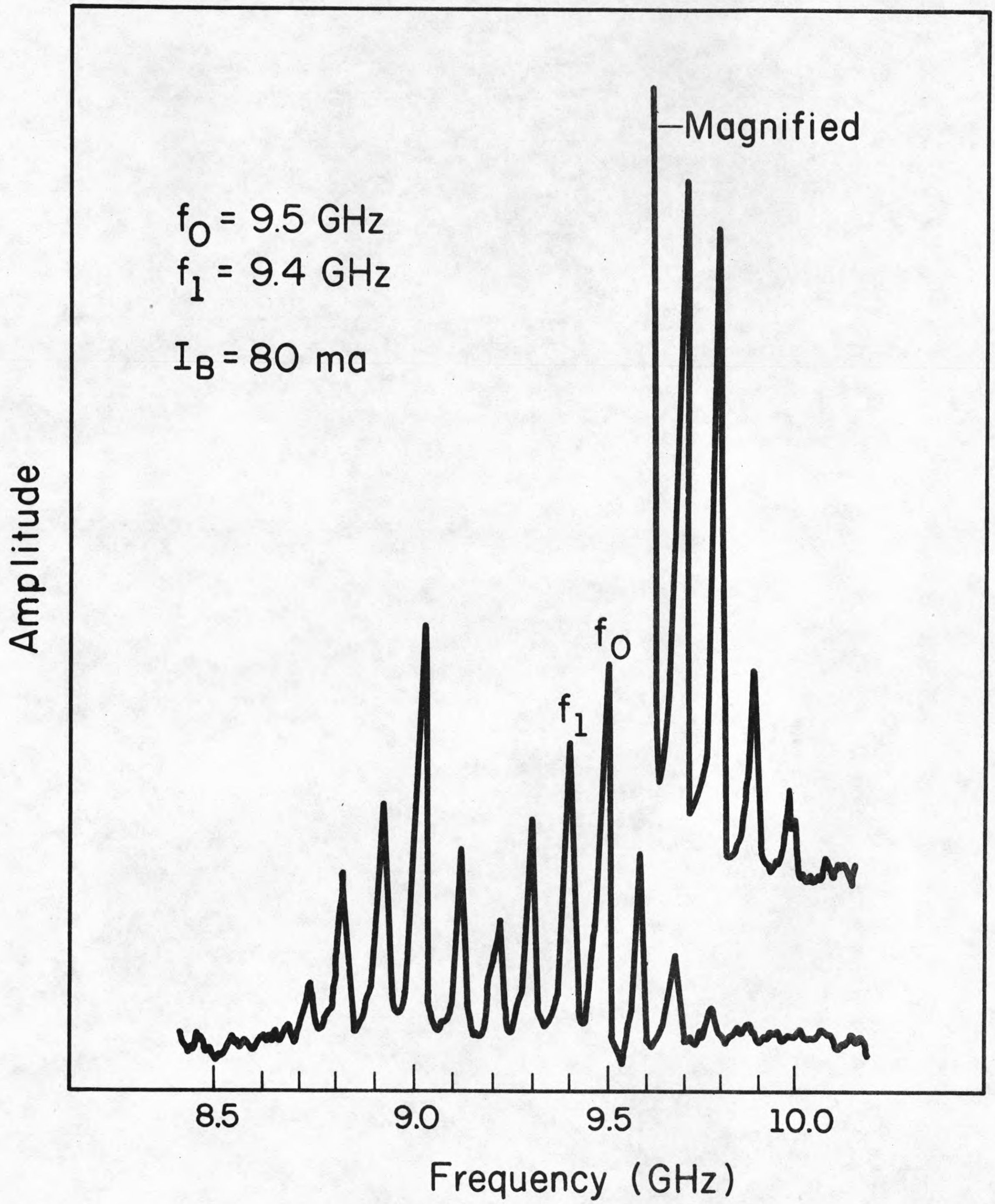
10
1
10^{-1}
10^{-2}
$a_1^2(0)\frac{W^2}{\kappa^2}(e^{\kappa X}-1)^2$
10^{-1}
1
10
10^2
$a_0^2(0)\frac{W^2}{\kappa^2}(e^{\kappa X}-1)^2$
10^3
Γ_{NL}
10^2
10
1
0.6
0.8
1.0
ω_0/ω_p
$\omega_2/\omega_p = 0.01$
$\alpha = 5\times10^{-5}$
$\nu_c/\omega_p = 0.05$
(b)

PP-836

(a)

Figure 3.

Received power as a function of frequency in the regime of satellite generation. The double spectrum is due to the image frequency of the microwave receiver. The indicated frequency scale refers to the right hand spectrum.



—Magnified
f_0 = 9.5 GHz
f_1 = 9.4 GHz
I_B = 80 ma
Amplitude
f_0
f_1
8.5
9.0
9.5
10.0
Frequency (GHz)

PS-791

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13. ABSTRACT

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