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BASHKOW'S A MATRIX FOR ACTIVE R,L,C, NETWORKS

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I. INTRODUCTION

In 1957 Bashkow [1] described a new method of network analysis. According to this method if voltages across capacitances and currents through inductances are used as dependent variables, a set of first order differential equations is obtained as, $\dot{y} = A\, y + F$ in which y is the column matrix of dependent variables and F represents the sources. Bryant [2] obtained an explicit form of A matrix for R, L, C networks. In this paper we shall discuss active R, L, C case such that the order of complexity of the network is equal to the number of reactive elements in the network.

II. DERIVATION OF EQUATIONS

We assume that the graph which corresponds to the network is connected and it has a tree T_p containing all capacitances and voltage sources as branches and all inductances and current sources as chords. Under these restrictions the order of A matrix is equal to the number of reactive elements in the network [4]. Our purpose is to find a way to write the A matrix by inspection of the network.

When the number of reactive elements in the network is rather small in comparison with the total number of elements, excluding independent sources, the entries of the A matrix are rather complicated. For example, when there is only one reactive element in the network the A matrix has only one entry which is equal to the natural frequency of the network and the expression of this is rather complicated if the number of elements in the network is rather high. Therefore, we shall split up the equation $\dot{y} = A\, y + F$ into two equations considering the resistive and the reactive parts of the network separately.

Let the fundamental circuit matrix B_f associated with the tree T_p be $B_f = [I\ B_{12f}]$ in which I is the unit matrix. It is well known [4] that the branch currents may be expressed in terms of chord currents as,

$$\mathbf{I}_b = \mathbf{B}'_{12f}\,\mathbf{I}_{ch} \tag{1}$$

and the chord voltages may be expressed in terms of branch voltages as,

$$\mathbf{V}_{ch} = -\,\mathbf{B}_{12f}\,\mathbf{V}_b \tag{2}$$

where $\mathbf{B}'_{12f}$ is the transpose of $\mathbf{B}_{12f}$ and $\mathbf{I}_b$, $\mathbf{I}_{ch}$, $\mathbf{V}_b$ and $\mathbf{V}_{ch}$ are the column matrices of branch currents, branch voltages and chord voltages, respectively. Let the elements in the network be partitioned into seven types c, d, e, g, l, q and r as follows:

Type	Description
c	capacitive branches
d	dependent current sources
e	dependent voltage sources
g	independent sources
l	inductive chords
q	resistive chords
r	resistive branches

Then, partitioning equations (1) and (2) conveniently and rearranging them one obtains,

$$\begin{bmatrix} \mathbf{V}_l \\ \mathbf{I}_c \end{bmatrix} = \begin{bmatrix} \mathbf{0} & -\mathbf{B}_{lc} \\ \mathbf{B}'_{lc} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{I}_l \\ \mathbf{V}_c \end{bmatrix} + \begin{bmatrix} \mathbf{0} & -\mathbf{B}_{lr} \\ \mathbf{B}'_{qc} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{I}_q \\ \mathbf{V}_r \end{bmatrix} + \begin{bmatrix} \mathbf{0} & -\mathbf{B}_{le} \\ \mathbf{B}'_{dc} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{I}_d \\ \mathbf{V}_e \end{bmatrix} + \begin{bmatrix} -\mathbf{V}_{gl} \\ \mathbf{I}_{gc} \end{bmatrix} \tag{3}$$

and

$$\begin{bmatrix} \mathbf{V}_q \\ \mathbf{I}_r \end{bmatrix} = \begin{bmatrix} \mathbf{0} & -\mathbf{B}_{qc} \\ \mathbf{B}'_{lr} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{I}_l \\ \mathbf{V}_c \end{bmatrix} + \begin{bmatrix} \mathbf{0} & -\mathbf{B}_{qr} \\ \mathbf{B}'_{qr} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{I}_q \\ \mathbf{V}_r \end{bmatrix} + \begin{bmatrix} \mathbf{0} & -\mathbf{B}_{qe} \\ \mathbf{B}'_{dr} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{I}_d \\ \mathbf{V}_e \end{bmatrix} + \begin{bmatrix} -\mathbf{V}_{gq} \\ \mathbf{I}_{gr} \end{bmatrix} \tag{4}$$

Thus far we have not used the relation between dependent sources and element currents and voltages. We assume that this relation is as follows:

$$\begin{bmatrix} \underset{\sim}{I}_d \\ \underset{\sim}{V}_e \end{bmatrix} = \underset{\sim}{D} \begin{bmatrix} \underset{\sim}{I}_l \\ \underset{\sim}{V}_c \\ \underset{\sim}{I}_q \\ \underset{\sim}{V}_r \end{bmatrix} \tag{5}$$

where $\underset{\sim}{D}$ is a real matrix. Thus, it is seen that controlling currents are the currents of passive chords and controlling voltages are the voltages of passive branches of T_p. Furthermore, we assume that at each row of $\underset{\sim}{D}$ there is exactly one non-zero entry and it is a real constant. On the other hand, from the third postulate of network analysis we have $\underset{\sim}{V}_l = \underset{\sim}{L}\,\dot{\underset{\sim}{I}}_l$, $\underset{\sim}{I}_c = \underset{\sim}{C}\,\dot{\underset{\sim}{V}}_c$, $\underset{\sim}{I}_q = \underset{\sim}{G}_q\,\underset{\sim}{V}_q$ and $\underset{\sim}{V}_r = \underset{\sim}{R}_r\,\underset{\sim}{I}_r$ where $\underset{\sim}{L}$, $\underset{\sim}{C}$, $\underset{\sim}{G}_q$ and $\underset{\sim}{R}_r$ are the diagonal matrices of inductances, capacitances, resistive chord conductances and resistive branch resistances, respectively and $\dot{\underset{\sim}{I}}_l$ and $\dot{\underset{\sim}{V}}_c$ are the time derivatives of $\underset{\sim}{I}_l$ and $\underset{\sim}{V}_c$. By making use of these relations and the quation (5) the equations (3) and (4) take the following forms:

$$\begin{bmatrix} \dot{\underset{\sim}{I}}_l \\ \dot{\underset{\sim}{V}}_c \end{bmatrix} = \begin{bmatrix} \underset{\sim}{P}_1 \\ \underset{\sim}{P}_2 \end{bmatrix} \begin{bmatrix} \underset{\sim}{I}_l \\ \underset{\sim}{V}_c \end{bmatrix} + \begin{bmatrix} \underset{\sim}{Q}_1 \\ \underset{\sim}{Q}_2 \end{bmatrix} \begin{bmatrix} \underset{\sim}{I}_q \\ \underset{\sim}{V}_r \end{bmatrix} + \begin{bmatrix} \underset{\sim}{V}_{gtl} \\ \underset{\sim}{I}_{gtc} \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} \underset{\sim}{M}_1 \\ \underset{\sim}{M}_2 \end{bmatrix} \begin{bmatrix} \underset{\sim}{I}_q \\ \underset{\sim}{V}_r \end{bmatrix} = \begin{bmatrix} \underset{\sim}{N}_1 \\ \underset{\sim}{N}_2 \end{bmatrix} \begin{bmatrix} \underset{\sim}{I}_l \\ \underset{\sim}{V}_c \end{bmatrix} + \begin{bmatrix} \underset{\sim}{V}_{gtq} \\ \underset{\sim}{I}_{gtr} \end{bmatrix} \tag{7}$$

Thus, dependent sources have been eliminated. It will later be shown that the matrix $\underset{\sim}{M} = \begin{bmatrix} \underset{\sim}{M}_1 \\ \underset{\sim}{M}_2 \end{bmatrix}$ is non-singular. Therefore, by solving the equation (7) for $\begin{bmatrix} \underset{\sim}{I}_q \\ \underset{\sim}{V}_r \end{bmatrix}$ and substituting it into equation (6) the $\underset{\sim}{A}$ matrix is obtained and solving the resulting differential equation system $\begin{bmatrix} \underset{\sim}{I}_l \\ \underset{\sim}{V}_c \end{bmatrix}$ is obtained. Then using equation (7) $\begin{bmatrix} \underset{\sim}{I}_q \\ \underset{\sim}{V}_r \end{bmatrix}$ is calculated. Thus, it is seen that when the solution of the system of equations (6) and (7) is known all element currents and voltages in the network may be calculated easily.

Equations (6) and (7) may be written by the inspection of the network as follows:

Let the general entry of the matrix $\underset{\approx}{P}_1$ be denoted by $p_{ij}^{(1)}$. In general $p_{ij}^{(1)}$ may be expressed as $p_{ij}^{(1)} = p_{ija}^{(1)} + p_{ijp}^{(1)}$ in which $p_{ija}^{(1)}$ comes from the dependent sources in the fundamental circuit i and $p_{ijp}^{(1)}$ comes from the element j of the column matrix $\begin{bmatrix} \underset{\sim}{I}_l \\ \underset{\sim}{V}_c \end{bmatrix}$. We assume that the orientation of a fundamental circuit is chosen to agree with that of the defining chord; also the orientation of a fundamental cut-set is chosen to agree with that of the defining branch. Evidently, the problem does not lose anything of its generality by these conventions.

$p_{ij}^{(1)}$ is written by the inspection of the fundamental circuit i defined by the inductive chord of inductance L_i. Let us first consider passive part of $p_{ij}^{(1)}$.

$p_{ijp}^{(1)} = 0$ if $i = j$. Otherwise,

$p_{ijp}^{(1)} = \frac{1}{L_i}$ if element j of the column matrix $\begin{bmatrix} \underset{\sim}{I}_l \\ \underset{\sim}{V}_c \end{bmatrix}$ is in the circuit i and the orientations of the circuit i and element j do not coincide.

$p_{ijp}^{(1)} = \frac{-1}{L_i}$ if element j is in the circuit i and the orientations coincide.

$p_{ijp}^{(1)} = 0$ if element j is not in the circuit i.

The general expression of active part is $p_{ija}^{(1)} = \frac{1}{L_i} \sum_k a_{ijk} d_{jk}$.

$a_{ijk} = 1$ if the circuit i contains a dependent voltage source controlled by the element j with a coefficient d_{jk} and the orientations of the circuit and dependent voltage source do not coincide. $a_{ijk} = -1$ if the orientations coincide. The above rule applies to the general entry $q_{ij}^{(1)} = q_{ija}^{(1)} + q_{ijp}^{(1)}$ of $\underset{\approx}{Q}_1$ considering column matrix $\begin{bmatrix} \underset{\sim}{I}_q \\ \underset{\sim}{V}_r \end{bmatrix}$ instead of $\begin{bmatrix} \underset{\sim}{I}_l \\ \underset{\sim}{V}_c \end{bmatrix}$ with the exception that $q_{ijp}^{(1)}$ not necessarily be zero for $i = j$.

The entry i of the column matrix $\underset{\sim}{V}_{gtl}$ has the form $v_{gi} = \frac{1}{L_i} \sum_k a_{ik} v_{ik}$ in which v_{ik} is the voltage of the independent voltage source k in the circuit i. $a_{ik} = 1$ if the orientations of the circuit and the voltage source do not coincide; $a_{ik} = -1$ if the orientations coincide.

The general entries $p^{(2)}_{ij}$, $q^{(2)}_{ij}$ and i_{gi} of the matrices $\underset{\sim}{P}_2$, $\underset{\sim}{Q}_2$ and $\underset{\sim}{I}_{gtc}$ are written in exactly the same way as $p^{(1)}_{ij}$, $q^{(1)}_{ij}$ and v_{gi}, respectively, considering the fundamental cut-set i defined by capacitive branch of capacitance C_i instead of the fundamental circuit i defined by L_i and considering current sources instead of voltage sources.

Equation (7) is written by the inspection of fundamental circuits and cut-sets defined by resistive chords and branches. Let the general entry of the matrix $\underset{\sim}{M}_1$ be denoted by $m^{(1)}_{ij} = m^{(1)}_{ija} + m^{(1)}_{ijp}$. Then, $m^{(1)}_{ijp} = 1$ if $i = j$. Otherwise, $m^{(1)}_{ijp} = G_i$ if the element j of the column matrix $\begin{bmatrix} \underset{\sim}{I}_q \\ \underset{\sim}{V}_r \end{bmatrix}$ is in the circuit i defined by the resistive chord i of conductance G_i and the orientations of the circuit i and the element j coincide. $m^{(1)}_{ijp} = -G_i$ if the element j is in the circuit i but the orientations do not coincide and $m^{(1)}_{ijp} = 0$ if element j is not in circuit i. The active part of $m^{(1)}_{ij}$ is, $m^{(1)}_{ija} = G_i \sum_k a_{ijk} d_{jk}$.

$a_{ijk} = 1$ if the circuit i contains the dependent voltage source k controlled by the element j with a coefficient d_{jk} and the orientations coincide. $a_{ijk} = -1$ if the orientations do not coincide. Similar operation is repeated for the general entry $n^{(1)}_{ij} = n^{(1)}_{ija} + n^{(1)}_{ijp}$ of the matrix $\underset{\sim}{N}_1$ but the signs are reversed: $n^{(1)}_{ijp} = G_i$ if the element j of the column matrix $\begin{bmatrix} \underset{\sim}{I}_l \\ \underset{\sim}{V}_c \end{bmatrix}$ is in the circuit i defined by G_i and the orientations do not coincide. $n^{(1)}_{ijp} = -G_i$ if the orientations coincide. $n^{(1)}_{ijp} = 0$ if the element j is not in the circuit i. The active part has the form $n^{(1)}_{ija} = G_i \sum_k a_{ijk} d_{jk}$ and $a_{ijk} = 1$ if the circuit i contains a dependent voltage source controlled by the element j with a coefficient d_{jk} and the orientations do not coincide. $a_{ijk} = -1$ if the orientations coincide. The general entry v_{gi} of the column matrix $\underset{\sim}{V}_{gtq}$ has the form $v_{gi} = G_i \sum_k a_{ik} v_{ik}$ and is written in exactly the same way as the general entry of the column matrix $\underset{\sim}{V}_{gt1}$. Finally, the general entries $m_{ij}^{(2)}$, $n_{ij}^{(2)}$ and i_{gi} of the matrices $\underset{\sim}{M}_2$, $\underset{\sim}{N}_2$ and $\underset{\sim}{I}_{gtr}$ are written in exactly the same way as $m_{ij}^{(1)}$, $n_{ij}^{(1)}$ and v_{gi}, respectively, considering the fundamental cut-set i defined by resistive branch of resistance R_i.

After having the equations (6) and (7) written in order to calculate the $\underset{\sim}{A}$ matrix first of all the inverse of the matrix $\underset{\sim}{M}$ is to be calculated and this is the most involved part of the calculation. Therefore, we shall discuss the properties of the matrix $\underset{\sim}{M}$.

III. THE PROPERTIES OF THE MATRIX $\underset{\sim}{M}$

The $\underset{\sim}{M}$ matrix may be written by the inspection of a subnetwork N_m of the given network N. Let a dependent source be called "$\underset{\sim}{M}$-controlling" dependent source if it is in a fundamental cut-set defined by a resistive branch and controlled by a resistive element or it is in a fundamental circuit defined by a resistive chord and controlled by a resistive element. The subnetwork N_m consists of all resistive elements and $\underset{\sim}{M}$-controlling dependent sources in the network N and is formed as follows:

Let all elements but resistive branches and $\underset{\sim}{M}$-controlling dependent sources of the tree T_p be short-circuited and then removed from the graph. The resulting graph T_{pm} is connected and has no circuit. Hence it has a tree character and consists of all resistive branches and $\underset{\sim}{M}$-controlling dependent voltage sources of the tree T_p. Therefore, if we connect a resistive chord to this T_{pm}, between its original vertices, the obtained fundamental circuit consists of resistive branches, $\underset{\sim}{M}$-controlling dependent voltage sources and the defining resistive chord of original one in the network N. Let us connect all resistive chords and $\underset{\sim}{M}$-controlling dependent current sources between their original vertices. Then there is a one-to-one correspondence between the fundamental circuits defined by resistive chords and $\underset{\sim}{M}$-controlling dependent current sources in the new network N_m and the original network N. Therefore, the voltage equations for all fundamental circuits defined by resistive chords and current equations for all fundamental cut-sets defined by resistive branches in the subnetwork N_m give a set of equations which are equivalent to the set,

$$\begin{bmatrix} \underset{\sim}{M}_1 \\ \underset{\sim}{M}_2 \end{bmatrix} \begin{bmatrix} \underset{\sim}{I}_q \\ \underset{\sim}{V}_r \end{bmatrix} = \underset{\sim}{0}$$

Hence the matrix $\underset{\sim}{M}$ is non-singular.

Let us assume that the given network N has no $\underset{\sim}{M}$-controlling dependent source i.e., the subnetwork N_m has no source. In this case, we shall show that the determinant Δ of the matrix $\underset{\sim}{M}$ and the cofactors Δ_{ii} on the main diagonal may be written by inspection of the subnetwork N_m.

Let the fundamental circuit matrix of N_m be $\underset{\sim}{B}_{fm} = [\underset{\sim}{I}\ \underset{\sim}{B}_m]$. Since $\underset{\sim}{M}$ is written by the inspection of fundamental cut-sets and circuits of N_m it is easily seen that,

$$\underset{\sim}{M} = \begin{bmatrix} \underset{\sim}{I} & \underset{\sim}{G}_q \underset{\sim}{B}_m \\ -\underset{\sim}{R}_r\ \underset{\sim}{B}'_m & \underset{\sim}{I} \end{bmatrix}$$

and using Laplace's expansion we obtain det $\underset{\sim}{M} = \Delta =$ det $[\underset{\sim}{I} + \underset{\sim}{G}_q\ \underset{\sim}{R}_m]$ in which $\underset{\sim}{R}_m = \underset{\sim}{B}_m\ \underset{\sim}{R}_r\ \underset{\sim}{B}'_m$. Let $\underset{\sim}{M}_g = [\underset{\sim}{I} + \underset{\sim}{G}_q\ \underset{\sim}{R}_m]$ and $\underset{\sim}{R}_m = [r_{ij}]_{nxn}$ then,

$$\underset{\sim}{M}_g = \begin{bmatrix} 1 + G_1 r_{11} & G_1 r_{12} & \cdots\cdots\cdots & G_1 r_{1n} \\ G_2 r_{21} & 1 + G_2 r_{22} & \cdots\cdots\cdots & G_2 r_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ G_n r_{n1} & G_n r_{n2} & \cdots\cdots\cdots & 1 + G_n r_{nn} \end{bmatrix}$$

where n is the number of fundamental circuits in N_m. It is seen that each element g_{ij} of the j^{th} column of $\underset{\sim}{M}_g$ mah be expressed as $\alpha_{ij} + \beta_{ij}$ in which $\alpha_{ij} = 0$ if $i \neq j$, $\alpha_{ij} = 1$ if $i = j$ and $\beta_{ij} = G_i\ r_{ij}$. Therefore, the determinant Δ of $\underset{\sim}{M}_g$ may be expressed as the sum of determinants of two matrices the elements of whose j^{th} columns being respectively α_{ij} and β_{ij} and the other columns unchanged. Applying this operation to each column of M_g repeatedly and simplifying the resulting determinants we obtain,

$$\Delta = 1 + \sum_{i=1}^{n} G_i\ r_{ii} + G_1 G_2 \begin{vmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{vmatrix} + \ldots + G_{n-1} G_n \begin{vmatrix} r_{n-1,n-1} & r_{n-1,n} \\ r_{n,n-1} & r_{nn} \end{vmatrix} + \ldots +$$

$$+ G_1 G_2 \ldots G_n \begin{vmatrix} r_{11} & r_{12} & \ldots & r_{1n} \\ r_{21} & r_{22} & \ldots & r_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ r_{n1} & r_{n2} & \ldots & r_{nn} \end{vmatrix} \qquad (8)$$

The last determinant in equation (8) is the determinant of $\underset{\sim}{R}_m$ and others are the determinants of submatrices of $\underset{\sim}{R}_m$. $\underset{\sim}{R}_m = \underset{\sim}{B}_m\ \underset{\sim}{B}'_m$, therefore, writing the loop-resistance matrix of N_m and setting all chord resistances to zero $\underset{\sim}{R}_m$ is obtained. In fact $\underset{\sim}{R}_m$ is the loop-resistance matrix of the network N_{ms} obtained from N_m by short-circuiting and then removing the chords. Therefore, the value of the determinant $|\underset{\sim}{R}_m|$ of the matrix $\underset{\sim}{R}_m$ is equal to the summation of chord resistance product for all chord sets of the network N_{ms}[4]. Using this fact we may write $|\underset{\sim}{R}_m|$ by the inspection of N_m as follows. Take one branch from each fundamental circuit of N_m; multiply the resistances of these branches; do this operation for all combinations and add these products together. Then the result is $|\underset{\sim}{R}_m|$ and multiplying $|\underset{\sim}{R}_m|$ by $G_1 G_2 \ldots G_n$ we obtain the last term of equation (8). On the other hand the other determinants in equation (8) are the determinants of the submatrices of $\underset{\sim}{R}_m$ and a term with a determinant of order n-i multiplied by $G_1 G_2 \ldots G_{n-i}$ is written in exactly the same way as

$G_1 G_2 \ldots G_n \left|\underset{\sim}{R}_m\right|$ considering the network N_{n-i} obtained from N_m by open-circuiting the chords $G_{n-i+1}, G_{n-i+2}, \ldots, G_n$ i.e., consider n-i fundamental circuits at a time instead of n at a time. The number of determinants of order n-i in equation (8) is $\binom{n}{n-i}$ which is the number of combinations of n different objects taken n-i at a time. Thus, each term of equation (8) is written by inspection of N_m and summing them Δ is obtained. Δ_{ii}'s are obtained in a similar way. Suppose row i of the matrix $\underset{\sim}{M}$ represents the fundamental circuit i. If row i and column i are deleted from $\underset{\sim}{M}$ the resulting matrix $\underset{\sim}{M}_{-i}$ is the $\underset{\sim}{M}$ matrix of the network N_{-i} obtained from N_m by open-circuiting the chord which defines the fundamental circuit i. Therefore Δ_{ii} is the determinant Δ of the network N_{-i}. On the other hand if row i represents the cut-set i then $\underset{\sim}{M}_{-i}$ is the $\underset{\sim}{M}$ matrix of the network N_{-i} obtained from N_m by short-circuiting the branch which defines the fundamental cut-set i and consequently Δ_{ii} is the determinant Δ of the network N_{-i}.

IV. CONCLUSIONS

In this paper a way of writing the $\underset{\sim}{A}$ matrix of Bashkow by inspection of active R,L,C network is given. It has been assumed that the network has a tree which contains all capacitances and voltage sources as branches and all current sources and inductances as chords. In this case two matrix equations may be written by inspection and solving these equations the $\underset{\sim}{A}$ matrix, currents and voltages of passive elements may be calculated. One of these matrix equations represents the resistive part of the network and if the network does not contain resistive elements then there is only one matrix equation and the $\underset{\sim}{A}$ matrix may be written directly.

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