Coordinated Science Laboratory

UNIVERSITY OF ILLINOIS – URBANA, ILLINOIS

MODIFIED UNISTOR GRAPHS AND SIGNAL FLOW GRAPHS

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Abstract

In this paper modified unistors have been defined. For example, the transconductance of a vacuum tube will be expressed by one modified unistor. A M.U. graph (modified unistor graph) is a linear graph consisting of modified unistors and can represent a linear electrical network. A node voltage equation of a M.U. graph of n edges without sources can be obtained as

$$A^{-}YA^{+'}V_{n} = 0$$

where A^{-}, A^{+}, and Y are a negative incidence matrix, a positive incidence matrix and a diagonal n by n admittance matrix.

If there exist independent current sources from the reference vertex to any vertex in a M.U. graph, then the node voltage equations can be written as

$$A^{-}YA^{+'}V_{n} = Y_{n}V_{n} = -J_{n}$$

where J_n is a column matrix representing independent current sources. It is noticed that Y_n is a connection matrix of M.U. graph.

By the Binet-Cauchy theorem, the determinant of Y_n is equal to summation of nonzero majors of $A^{-}Y$ times the corresponding majors of $A^{+'}$. Therefore, first the condition of the set of edges which forms a nonzero major should be expressed topologically. Then, it can be shown that the determinant of Y_n

is equal to the summation of $(-1)^s$ times the C.D.C. admittance products of a M.U. graph. Similarly, topological formulas of the ij cofactor Δ_{ij} and the double cofactor Δ_{iijk} of Y_n are given. These formulas are important in obtaining network functions. Finally, in Chapter 5, Mason's formula for signal flow graphs will be proved by using M.U. graphs.

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1. INTRODUCTION

Linear graphs are known to be a useful tool for analysis of electrical networks [6,9,10,11]. Here we introduce a modified unistor graph which is similar to a flow graph introduced by Coates [2]. By using a negative and a positive incidence matrix, it is easy to understand the development of topological formulas of flow graphs. Furthermore, Mason's formula [4,5] for a signal flow graph can be proved.

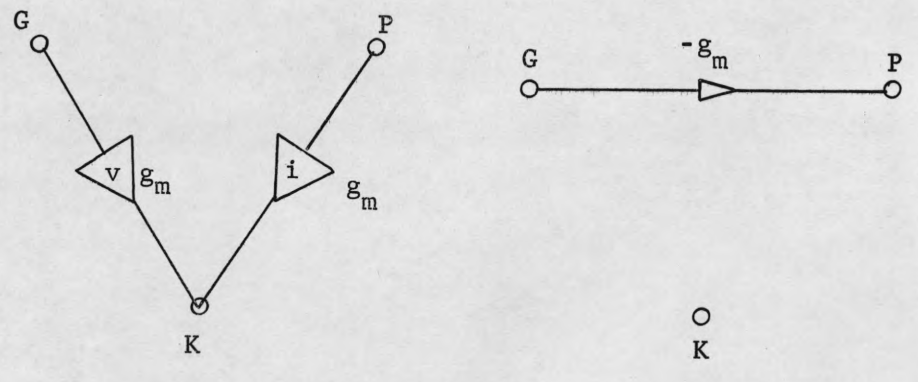
In chapter 2 and chapter 3, a modified unistor graph will be studied to obtain some fundamental properties of such graphs.

In chapter 5, Mason's formula for a signal flow graph will be proved by the use of a modified unistor graph.

2. DEFINITION AND FUNDAMENTAL PROPERTIES

2.1 Modified Unistor Graph (M.U. Graph)

It is known that the topological representation [7,8,11] of the transconductance of a vacuum tube consists of two edges, one of which is a voltage edge and the other is a current edge as shown in Figure 1a. In this paper, such an element is represented by one edge as shown in Figure 1b, and is called a "modified unistor". Also a network which consists only of modified unistors is called a "M.U. (modified unistor) graph" which is somewhat different from a unistor graph given by W. K. Chen and G. Dodd [1,3]. Since the voltage edge and the current edge of an element will be represented by only one modified unistor, the common vertex of the voltage and the current edges of every element in a network must be the reference vertex in order that the network can be represented by a M.U. graph. The formal definition of a M.U. graph is as follows:



(a) Topological representation of transconductance (b) Modified unistor of transconductance

Figure 1. Modified Unistor

Definition 1: A M.U. (modified unistor) graph is a weighted linear graph in which every edge satisfies the following: (1) each edge has an admittance as its weight, (2) let edge e_{pq} be connected from vertex p to vertex q. Also let y_{pq} be the weight of edge e_{pq}. Then the equation

$$y_{pq} \, v_{po} = i_{oq} \tag{1}$$

must be satisfied, where v_{po} is the voltage from p to the reference vertex o and i_{oq} is the corresponding current from the reference vertex o to vertex q.

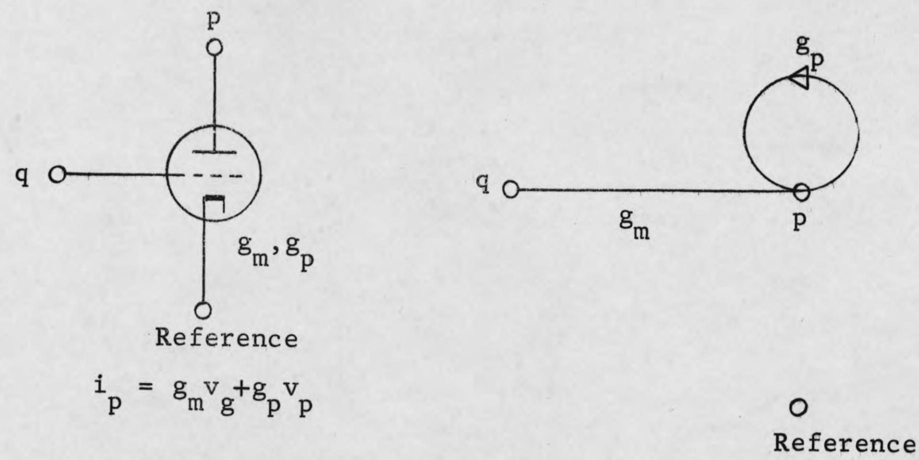
Example 1: Consider a resistor R which is located between a vertex p and the reference vertex. It can then be expressed as shown in Figure 2b.

(a) Register R

(b) Modified unistor graph of register R

Figure 2.

Example 2: The triode vacuum tube shown in Figure 3a can be described as in Figure 3b.



$$i_p = g_m v_g + g_p v_p$$

(a) Triode vacuum tube

(b) Modified unistor graph of triode vacuum tube

Figure 3.

If there exist elements which are not connected to the reference vertex in an electrical network, an equivalent network in which every element is connected to the reference vertex must be considered in order to obtain a M.U. graph of the given network.

A M.U. graph can be expressed by a matrix equation in the usual manner by using negative and positive incidence matrices [11] which are defined as follows:

Definition 2: A negative incidence matrix A^- is obtained from an incidence matrix of a M.U. graph by replacing all +1's by 0's.

Definition 3: A positive incidence matrix A^+ is obtained from an incidence matrix of a M.U. graph by replacing all -1's by 0's.

The following example will illustrate the negative and positive incidence matrices of a M.U. graph.

Example 3: An incidence matrix of the M.U. graph in Figure 4 is

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} Y_1 & Y_2 & Y_3 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \end{array} \tag{2}$$

Hence, the negative and positive incidence matrices of the M.U. graph are

$$A^- = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} Y_1 & Y_2 & Y_3 \end{array} \\ \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \end{array} \tag{3}$$

$$A^+ = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} Y_1 & Y_2 & Y_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \tag{4}$$

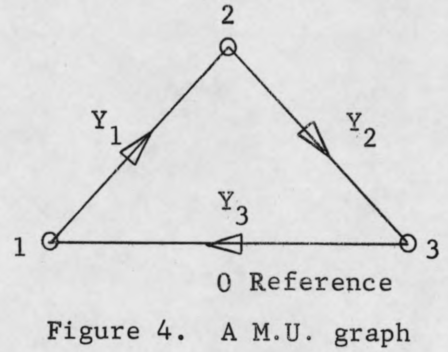


Figure 4. A M.U. graph

By Definition 1, if there exists an edge e_{pq}, which is oriented from p to q, and has edge admittance y_{pq}, the current i_{oq} from the reference vertex to the vertex q satisfies Equation (1).

The matrix expression of Equation (1) is

$$YA^{+'}V_n = I \tag{5}$$

When there are no current sources in a modified unistor graph, net current at any vertex p is zero, i.e.,

$$\sum_m i_{op_m} - \sum_n i_{p_n o} = 0 \tag{6}$$

where i_{op_m} (m = 1,2,...) and $i_{p_n o}$ (n = 1,2,...) are currents which flow into and out from vertex p.

The matrix expression of Equation (6)

$$A^{-}I = 0 \tag{7}$$

Equation (5) and (7) give

$$A^{-}YA^{+'}V_n = 0 . \tag{8}$$

2.2 Flow Conservation and Independent Current Sources

Suppose that there are independent current sources j_{op_v} (v = 1,2,...) in a M.U. graph which are located from the reference vertex o to the vertex p; then Equation (6) becomes

$$\sum_m i_{op_m} - \sum_n i_{p_n o} + \sum_v j_{op_v} = 0 \tag{9}$$

or

$$\sum_m i_{op_m} - \sum_n i_{p_n o} = - \sum_v j_{op_v} \tag{10}$$

The left hand side is exactly the same as that of Equation (6). Therefore Equation (7) becomes

$$A^- I = -J_n \, . \tag{11}$$

If there are independent sources which do not flow from or toward the reference vertex, then Equation (7) can be expressed as

$$A^- I = A_J J \tag{12}$$

where A_J is the incidence matrix of sources only. Then Equation (8) becomes

$$A^- Y A^{+'} V_n = A_J J \, . \tag{13}$$

Consider a source J_g which is located from vertex p to q and represented by a weighted edge g, then

$$A_J J = \begin{matrix} \\ p \\ q \end{matrix} \overset{g}{\begin{bmatrix} 0 \cdots 0\,1\,0 \cdots 0 \\ 0 \cdots 0 -1\,0 \cdots 0 \end{bmatrix}} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ J_g \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{matrix} \\ \\ \\ g \\ \\ \\ \\ \end{matrix} = - \begin{bmatrix} -J_g \\ \\ J_q \end{bmatrix} \begin{matrix} p \\ \\ q \end{matrix} = -J_n \tag{14}$$

This shows that every source from the vertex p to the vertex q can be expressed by a negative source from vertex p to the reference vertex and a positive source from the reference vertex to vertex q. Exactly the same argument can be applied by the use of superposition, when there is more than one source.

Most of the discussion in this chapter will be given in the following theorem.

Theorem 1: All the vertex potentials must satisfy the equation

$$A^{-}YA^{+'}V_n = -J_n \tag{15}$$

where J_n is the column matrix of independent current sources, each of which flows from the reference vertex.

It must be emphasized that

$$A^{-}YA^{+'} = Y_n \tag{16}$$

is the connection matrix of a M.U. graph.

3. TOPOLOGICAL FORMULAS FOR M.U. GRAPHS

3.1 Definition of C.D.C. and p-q C.D.C.

Subgraphs of a M.U. graph which are related to the determinant of Y_n of the graph are C.D.C. (covering disjoint circuits) and p-q C.D.C., which are defined as follows:

Definition 4: A C.D.C. (covering disjoint circuits) is either a directed circuit or an edge disjoint union of directed circuits of a M.U. graph which contains all vertices except the reference vertex of the M.U. graph.

Definition 5: A p-q C.D.C. is either a directed path from the vertex p to the vertex q or an edge disjoint union of a directed path from the vertex p to the vertex q and directed circuits which contains all vertices except the reference vertex of M.U. graph.

Definition 6: The number of disconnected circuits of a C.D.C. is the order of the C.D.C. Similarly, the number of disconnected circuits in a p-q C.D.C. circuit is the order of the p-q C.D.C.

Example 4: An example of Definitions 4, 5, and 6. Suppose there exists the M.U. graphs shown in Figure 5.

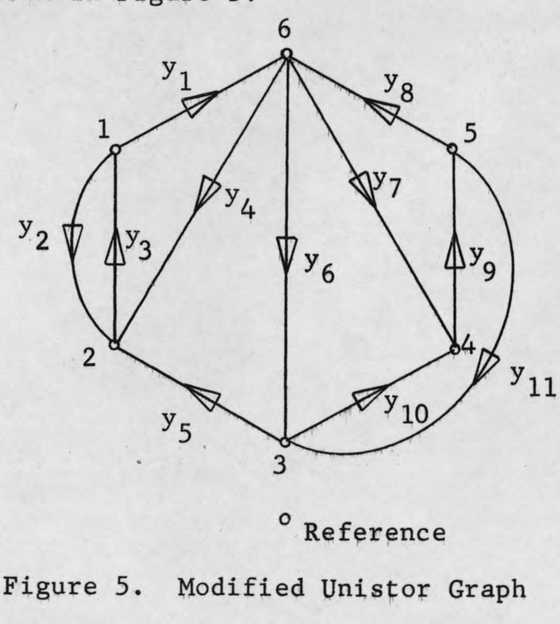


Figure 5. Modified Unistor Graph

Then one of the C.D.C. can be shown in Figure 6 and the order of this C.D.C. is two.

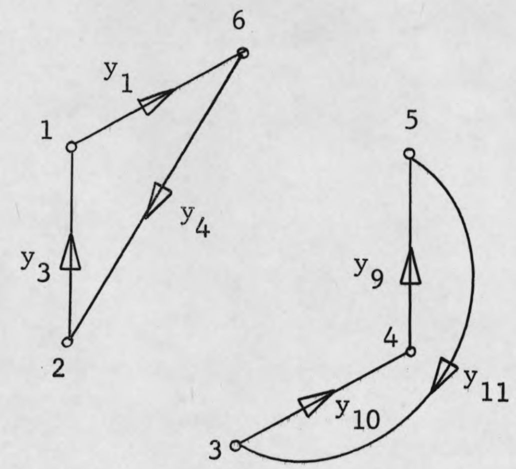


Figure 6. A C.D.C. of the M.U. graph shown in Figure 5.

Also the 5-4 C.D.C. is shown in Figure 7.

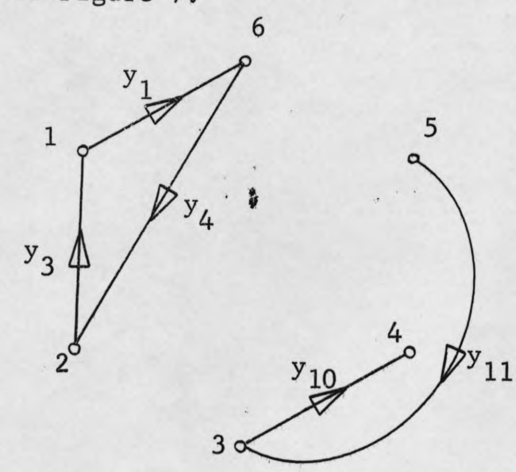


Figure 7. 5-4 C.D.C. of the M.U. graph shown in Figure 5.

The order of this 5-4 C.D.C. is one.

3.2 Evaluation of the Determinant of a M.U. Graph

By the Binet-Cauchy theorem, $|A^- Y A^{+'}|$ is the summation of all possible nonzero majors of $A^- Y$ and the corresponding majors of $A^{+'}$. Since $A^- Y$ is the matrix such that every column of A^- is multiplied by corresponding edge admittances, the next theorem can be obtained. For convenience, symbol e will be used to represent an edge in a M.U. graph as well as a row corresponding to the edge in the negative and positive incidence matrices of the M.U. graph.

Theorem 2: The determinant of $A^- Y A^{+'}$ is the summation of v-1 admittance products, where v is the number of vertices in M.U. graph.

Proof: Suppose the set of edges e_1', e_2', ..., and e_{v-1}' form a nonzero major of $A^- Y$ and the corresponding major of $A^{+'}$. Also the corresponding edge admittance are y_1', y_2', ..., and y_{v-1}'. Since each column of A^- and A^+ has only one nonzero entry, -1 and +1 respectively,

$$\text{a major of } A^- Y = \epsilon\, y_1', y_2', \ldots, y_{v-1}' \tag{17}$$

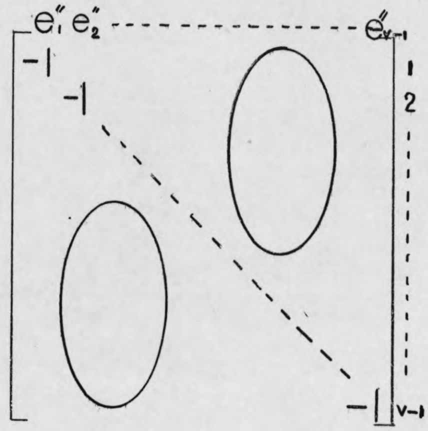
$$\text{a major of } A^{+'} = \epsilon' \tag{18}$$

where ϵ and ϵ' are $\pm$ 1. Therefore the determinant of Y_n is the summation of the v-1 admittance products.

Theorem 3: A major of A^- and the corresponding major of $A^{+'}$ are both nonzero if and only if the corresponding edges of the majors form a C.D.C.

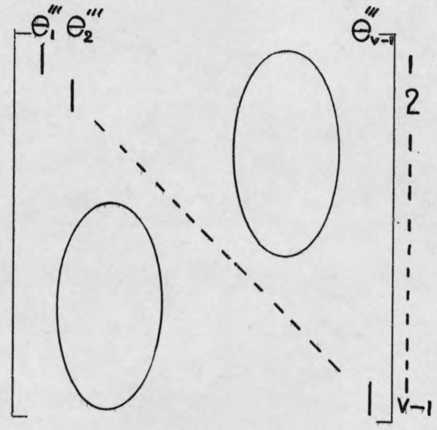
Proof: Suppose a set of edges $e_1', e_2', e_3', \ldots$, and e_{v-1}' form such a nonzero major of A^- and the corresponding major of $A^{+'}$. Then $\underset{\sim}{A}^-$ and $\underset{\sim}{A}^{+'}$, which is formed by the column corresponding edge $e_1', e_2', \ldots$, and e_{v-1}', can be changed to diagonal matrices since each column of $\underset{\sim}{A}^-$ and $\underset{\sim}{A}^{+'}$ have only one nonzero entry, -1 and +1 respectively. Therefore $\underset{\sim}{A}^{-}$ and $\underset{\sim}{A}^{+'}$ can be expressed as Equations (19) and (20) respectively by column permutations.

$\underset{\sim}{A}^{-}$ =



(19)

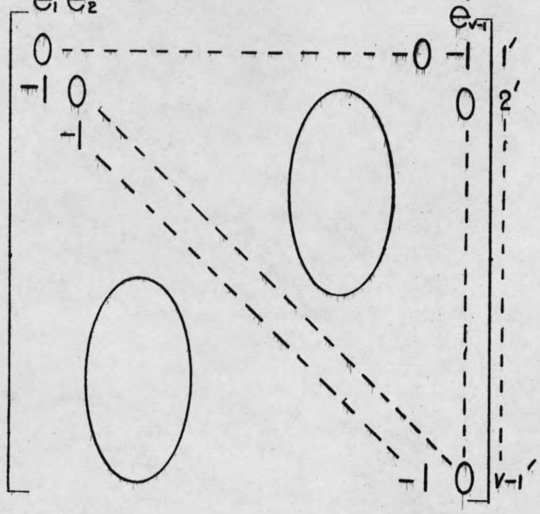
$\underset{\sim}{A}^{+}$ =



(20)

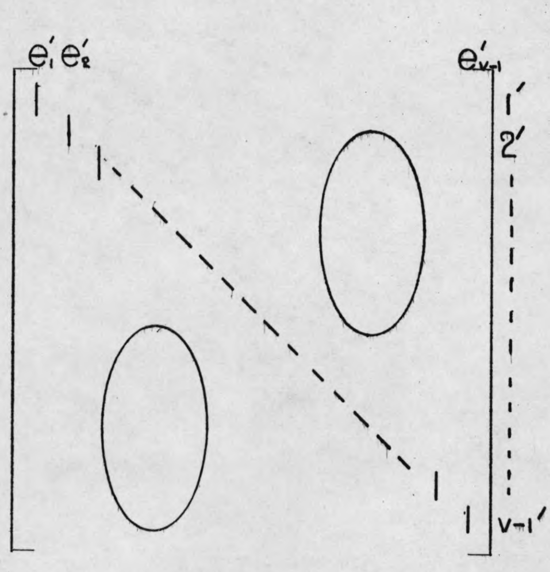
Equations (19) and (20) imply that every edge is connected to or from a different vertex and also every vertex is the initial and final vertex of exactly one edge. Therefore $e_{1'}, e_{2'}, \ldots,$ and $e_{v-1'}$ form a C.D.C. Conversely, if the set of edges $e_{1'}, e_{2'}, \ldots,$ and $e_{v-1'}$ form a C.D.C., then it is clear that $\underset{\sim}{A}^{-}$ and $\underset{\sim}{A}^{+}$ can be expressed as Equations (21) and (22).

$\underset{\sim}{A}^{-} =$



(21)

$\underset{\sim}{A}^{+} =$



(22)

Therefore $\underset{\sim}{A}^{-}$ and $\underset{\sim}{A}^{+'}$ are nonsingular, and $e_{1'}, e_{2'}, \ldots,$ and $e_{v-1'}$ form a nonzero major of A^{-} and the corresponding major of $A^{+'}$. Q.E.D.

<u>Theorem 4</u>: The determinant Δ of Y_n is equal to

$$\Delta = \Sigma(-1)^{s} \text{ C.D.C. of G admittance products} \tag{23}$$

where summation is over all possible C.D.C. and s is the order of C.D.C.

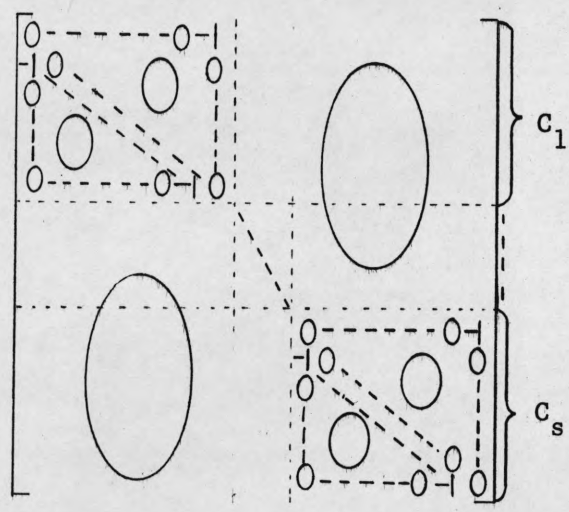
<u>Proof</u>: By the Binet-Cauchy theorem and Theorem 3, the determinant Δ of Y_n is equal to

$$\Delta = \Sigma \in \text{C.D.C. admittance products} \tag{24}$$

where $\in$ is $\pm$ 1.

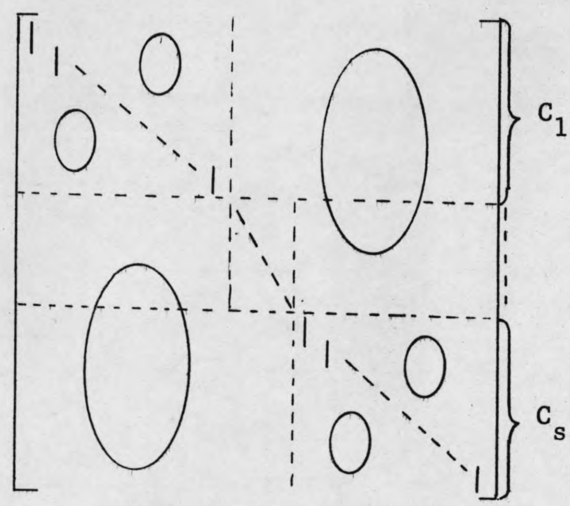
Let $\underset{\sim}{A}^-$ be the matrix whose determinant is a nonzero major of A^- and similarly $\underset{\sim}{A}^+$ be the matrix whose determinant is the corresponding nonzero major of A^+. By row permutations, $\underset{\sim}{A}^-$ and $\underset{\sim}{A}^+$ can be expressed by $\underset{\approx}{A}^-$ and $\underset{\approx}{A}^+$, respectively as

$$\underset{\sim}{A}^- = (-1)^k \underset{\approx}{A}^- = (-1)^k$$



(25)

$$\underset{\sim}{A}^+ = (-1)^{\ell} \underset{\approx}{A}^+ = (-1)^{\ell}$$



(26)

The transformations from $\underset{\sim}{A}^-$ to $\underset{\approx}{A}^-$ and $\underset{\sim}{A}^+$ to $\underset{\approx}{A}^+$ require the same number of permutations, therefore $k = \ell$. Then

$$|\underset{\sim}{A}^-||\underset{\approx}{A}^{+'}| = |\underset{\sim}{A}^-||\underset{\approx}{A}^{+'}| \tag{27}$$

Since $|\underset{\approx}{A}^{+'}| = 1$, only evaluation of $|\underset{\approx}{A}^-|$ is necessary. Suppose the set of rows C_i $(i = 1,2,\ldots,C_s)$ contains C_i rows; then to make $\underset{\approx}{A}^-$ diagonal,

$$(c_1-1) + (c_2-1) + \ldots + (c_s-1)$$

$$= (c_1+c_2 + \ldots + c_s) - s = v-1-s \tag{28}$$

permutations are necessary. Since all entries of $\underset{\approx}{A}^-$ are -1, the sign of a C.D.C. is

$$(-1)^{v-1} \cdot (-1)^{v-1-s} = (-1)^s \tag{29}$$

where s is the order of the C.D.C. Q.E.D.

3.3 Evaluation of Cofactors of a M.U. Graph

Theorem 5: The cofactor Δ_{ii} of Y_n is equal to

$$\Delta_{ii} = \Sigma(-1)^{s'} \text{ C.D.C. of } G_{-i} \text{ admittance products} \tag{30}$$

where G_i is a subgraph of G which doesn't contain vertex i, s' is the order of a C.D.C. of G_{-i} and summation is over all possible C.D.C. of G_{-i}.

Proof: The cofactor Δ_{ii} is the determinant $[A^-Y]_{-i}A^{+'}_{-i}$, where $[A^-Y]_{-i}$ and $A^{+'}_{-i}$ are obtained by deleting row i and column i from A^-Y and $A^{+'}$ respectively. Notice that $[A^-Y]_{-i}$ is equal to $A^-_{-i}Y$, where A^-_{-i} is obtained by deleting row i from A^-. Also A^-_{-i} and A^+_{-i} are the negative and positive incidence matrices of graph G_{-i} which is obtained from a given M.U. graph by elimination of all edges which are connected to and from the vertex i.

The remaining part of the proof is exactly the same as that of Theorem 4. Q.E.D.

<u>Theorem 6</u>: The cofactor Δ_{ij} of Y_n is equal to

$$\Delta_{ij} = \Sigma(-1)^{s'} \text{i-j C.D.C. of G admittance products} \tag{31}$$

where G is a given M.U. graph, s' is the order of i-j C.D.C. of G, and summation is over all possible i-j C.D.C. of G.

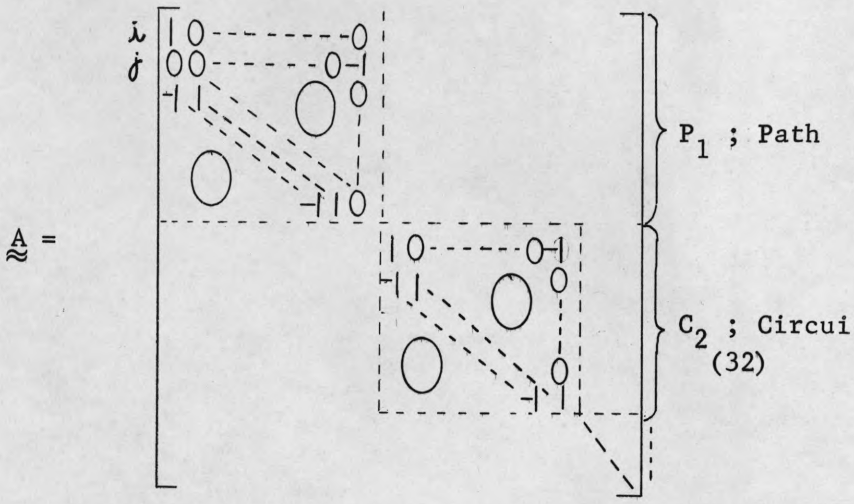
<u>Proof</u>: It is clear that Δ_{ij} is equal to $(-1)^{i+j}$ times the determinant of $A^-_{-i}YA^{+'}_{-j}$ where A^-_{-i} and $A^{+'}_{-j}$ are obtained by deleting row i and column j from A^- and $A^{+'}$ respectively. Suppose $e_{1'}, e_{2'}, \ldots,$ and $e_{v-2'}$ form a nonzero major of $A^-_{-i}Y$. Since there is only one nonzero element in each column of A^-_{-i}, the edges $e_{1'}, e_{2'}, \ldots,$ and $e_{v-2'}$ are the edges which are connected to all vertices except the vertex i and the reference vertex, and the converse is also true.

Since it is only necessary to consider the case when a major of $A^-_{-i}Y$ and the corresponding major of $A^{+'}_{-j}$ are both nonzero, suppose $e_{1'}, e_{2'},$ and $e_{v-2'}$ form a nonzero major of $A^{+'}_{-j}$. Then the edges $e_{1'}, e_{2'}, \ldots,$ and $e_{v-2'}$ should be incident at all the vertices except j, and the converse is also true. Hence edges $e_{1'}, e_{2'}, \ldots,$ and $e_{v-2'}$ should be incident at all vertices except vertices i, j and of course the reference vertex. Adding edge e_{ji} from j to i makes edges $e_{1'}, e_{2'}, \ldots, e_{v-2'}$ and e_{ji} incident at all vertices except the reference vertex. Since there are v-1 vertices except the reference vertex and there are v-1 edges, these edges form a C.D.C.

Conversely, suppose there exists an i-j C.D.C. which consists of edges $e_{1'}, e_{2'}, \ldots,$ and $e_{v-2'}$. Let $G(e_{ji})$ be a graph obtained by adding edge e_{ji} from vertex j to vertex i in G. Also let $A^-(e_{ji})$, $A^+(e_{ji})$ and $Y(e_{ji})$ be a negative, a positive and the corresponding diagonal admittance matrices of $G(e_{ji})$ such that removal of the column corresponding to e_{ji} from $A^-(e_{ji})$ and $A^+(e_{ji})$ produces A^- and A^+ of G, and removal of the row and the column corresponding to e_{ji} in $Y(e_{ji})$ produces Y of G. Since edges $e_{1'}, \ldots, e_{v-2'}$ and edge e_{ji} form a C.D.C., by the definition of an i-j C.D.C., the determinants of square submatrices $\tilde{A}^-(e_{ji})$ and $\tilde{A}^+(e_{ji})$ whose columns are $e_{1'}, e_{2'}, \ldots, e_{v-2'}$ and e_{ji} are both nonzero. Since $\tilde{A}^-(e_{ji})$ is nonsingular and e_{ji} is connected from j to i, there exists only one nonzero element in row i at the intersection of column e_{ji}. Hence removal of row i and column e_{ji} makes the determinant of the resultant submatrix nonzero. However this resultant submatrix is $\tilde{A}^-_{-i}$ whose determinant is the major of A^-_{-i} which corresponds to the i-j C.D.C. Similarly, the only nonzero entry in row j of $\tilde{A}^+(e_{ji})$ is at the intersection of column e_{ji}. Therefore, by the property of a C.D.C., the removal of row j and column e_{ji} of $\tilde{A}^+(e_{ji})$ makes a non-singular matrix. However, this resultant matrix is $\tilde{A}^+_{-j}$ whose determinant is a major of A^+_{-j}. Thus the major of A_{-i} and the corresponding major of A^+_{-j} corresponding to an i-j C.D.C. are both nonzero. Therefore cofactor Δ_{ij} is the summation of ϵ times (v-2) admittance products of all possible i-j C.D.C.'s of G, where ϵ is either +1 or -1.

The sign ϵ of each i-j C.D.C. will be considered next. Let $\tilde{A}$ be the matrix of order ((v-1) by (v-2)) obtained from an incidence matrix of a M.U. graph by taking only the columns corresponding to the edges in an i-j C.D.C.

By the definition of an i-j C.D.C., $\underset{\sim}{A}$ can be transformed by permutation of rows and columns to $\underset{\approx}{A}$ as in Equation (31).



(32)

Let $\underset{\approx}{A}^{-}_{-i}$ be the matrix obtained from $\underset{\approx}{A}$ by removing row i and replacing all 1's by 0's and $\underset{\approx}{A}^{+}_{-j}$ be the matrix obtained from $\underset{\approx}{A}$ by removing row j and replacing all -1's by 0's. Also, without loss of generality, let $i < j$. Then the transformation $\underset{\sim}{A}^{-}_{-i}$ to $\underset{\approx}{A}^{-}_{-i}$ requires $j-2 + k$ permutations, where $j-2$ is the number of permutations necessary to place jth row at the top of the matrix and k is the permutations of other rows. Similarly the transformation $\underset{\sim}{A}^{+}_{-j}$ to $\underset{\approx}{A}^{+}_{-j}$ requires $i-1 + \ell$ permutations where $i-1$ is the required number of permutations for ith row and ℓ is for the permutation of other rows. It is clear that ℓ can be equal to k. After this transformation $\underset{\approx}{A}^{+}_{-j}$ is diagonal. Hence, only $\underset{\approx}{A}^{-}_{-i}$ must be considered in order to determine the sign of an i-j C.D.C. Let a path P_i contain p_1 vertices, and a circuit C_i contain c_i vertices ($i = 2,3,\ldots,s$). Then the following permutations make $\underset{\approx}{A}^{-}_{-i}$ diagonal.

$$(p_1-1) + (c_2-1) + \dots + (c_s-1)$$

$$= (p_1+c_2+c_3 + \dots + c_s) - s = v-2-s \tag{33}$$

Therefore the sign ϵ of a i-j C.D.C. will be

$$(-1)^{i+j} \cdot (-1)^{i+j-3+2k} \cdot (-1)^{v-2} \cdot (-1)^{v-2-s} = (-1)^{s-1} \tag{34}$$

where $(-1)^{v-2}$ is from the determinant of the diagonalized $\underset{\approx}{\bar{A}}_{-i}$ because all nonzero entires of $\underset{\approx}{\bar{A}}_{-i}$ are -1. Therefore the sign of cofactor is $(-1)^{s-1}$ which is equal to $(-1)^{s'}$. Q.E.D.

3.4 Example of Evaluation of the Determinant and Cofactor

Consider a M.U. graph as in Figure 8 whose Y_n is given in Equation (35).

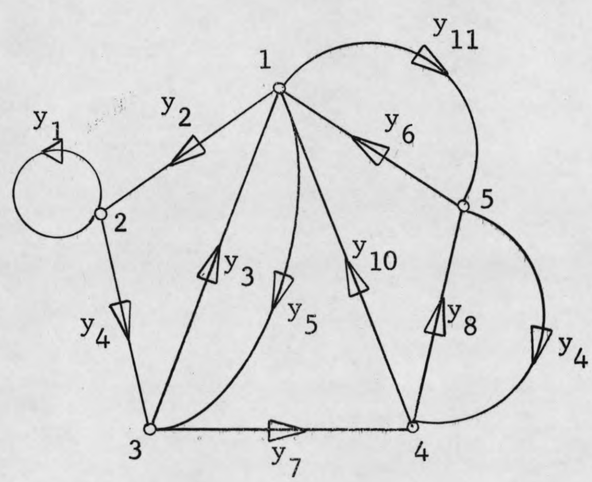


Figure 8. M.U. Graph

$$Y_n = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & -y_3 & -y_{10} & -y_6 \\ -y_2 & -y_1 & 0 & 0 & 0 \\ -y_5 & -y_4 & 0 & 0 & 0 \\ 0 & 0 & -y_7 & 0 & -y_9 \\ -y_{11} & 0 & 0 & -y_8 & 0 \end{bmatrix} \tag{35}$$

The determinant can be obtained directly from the M.U. graph by Theorem 4.

$$\Delta = y_2y_3y_4y_8y_9 - y_2y_4y_6y_7y_8 - y_1y_3y_5y_8y_9 + y_1y_5y_6y_7y_8 \tag{36}$$

where each C.D.C. is shown in Figure 9a, 9b, 9c and 9d.

Similarly, cofactor Δ_{54} can be obtained by Theorem 6 as

$$\Delta_{54} = -y_2y_3y_4y_9 + y_2y_4y_6y_7 + y_1y_3y_5y_9 - y_1y_5y_6y_7 \tag{37}$$

where each 5-4 C.D.C. is shown in Figure 10a, 10b, 10c and 10d.

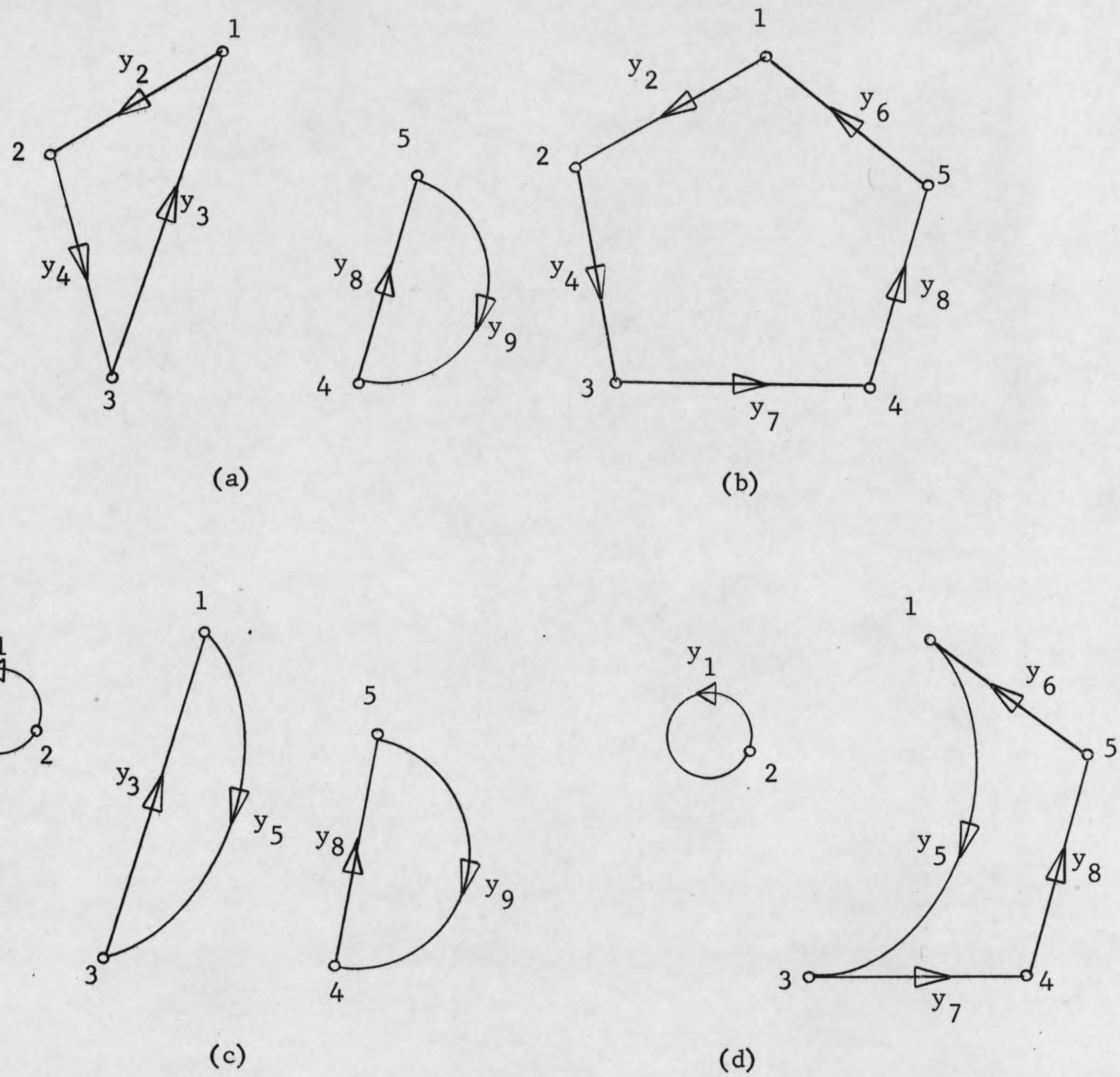


Figure 9. A C.D.C. of M.U. graph shown in Figure 8.

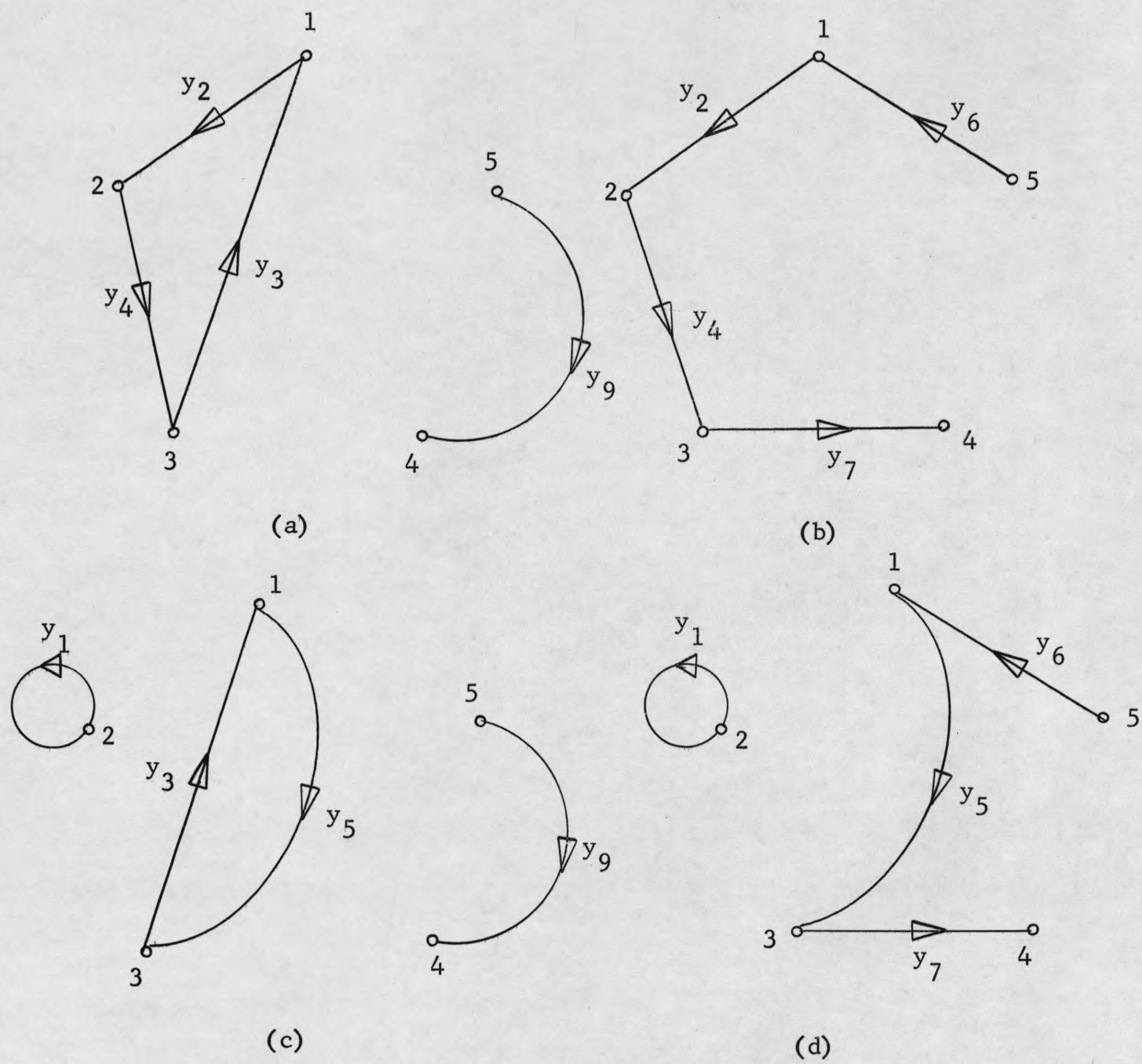


Figure 10. 5-4 C.D.C.'s of M.U. graph shown in Figure 8.

4. TOPOLOGICAL FORMULAS FOR SHORT CIRCUIT FUNCTION

Let N be a four terminal network shown in Figure 11.

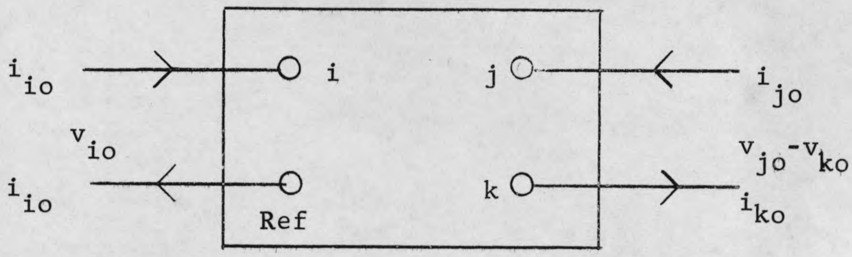


Figure 11. Four terminal network

$$\begin{bmatrix} 0 \\ \vdots \\ 0 \\ v_{io} \\ 0 \\ \vdots \\ 0 \\ v_{jo} \\ 0 \\ \vdots \\ 0 \\ v_{ko} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \Delta_{11} & \Delta_{21} & \Delta_{31} & \cdots \\ \Delta_{12} & \Delta_{22} & \cdots & \\ \Delta_{13} & \cdots & & \\ \vdots & \vdots & & \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ i_{io} \\ 0 \\ \vdots \\ 0 \\ i_{jo} \\ 0 \\ \vdots \\ 0 \\ i_{ko} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (38)$$

Equation (38) can be written as

$$\begin{bmatrix} v_{io} \\ v_{jo} \\ v_{ro} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \Delta_{ii} & \Delta_{ji} & \Delta_{ki} \\ \Delta_{ij} & \Delta_{jj} & \Delta_{kj} \\ \Delta_{ir} & \Delta_{jr} & \Delta_{rr} \end{bmatrix} \begin{bmatrix} i_{io} \\ i_{jo} \\ i_{ro} \end{bmatrix} \tag{39}$$

By letting $v_{jk} = v_{jo} - v_{ko}$ and $i_{jk} = i_{jo} = -i_{ko}$, Equation (39) becomes

$$\begin{bmatrix} \Delta_{io} \\ v_{jk} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \Delta_{ii} & \Delta_{ji}-\Delta_{ki} \\ \Delta_{ij}-\Delta_{ik} & \Delta_{jj}-\Delta_{jk}-\Delta_{kj}-\Delta_{rr} \end{bmatrix} \begin{bmatrix} i_{io} \\ i_{jk} \end{bmatrix} \tag{40}$$

Let us take the inverse of Equation (40).

$$\begin{bmatrix} i_{io} \\ i_{jk} \end{bmatrix} = \frac{1}{\frac{\Delta_{ii}(\Delta_{jj}-\Delta_{jk}-\Delta_{kj}+\Delta_{kk})}{\Delta^2} - \frac{\Delta_{ji}-\Delta_{ki}}{\Delta}\,\frac{\Delta_{ij}-\Delta_{ik}}{\Delta}}$$

$$\begin{bmatrix} \frac{\Delta_{ii}}{\Delta} & -\frac{\Delta_{ji}-\Delta_{ki}}{\Delta} \\ \frac{\Delta_{ij}-\Delta_{ik}}{\Delta} & \frac{\Delta_{jj}-\Delta_{jr}-\Delta_{kj}+\Delta_{kk}}{\Delta} \end{bmatrix} \cdot \begin{bmatrix} V_{io} \\ V_{jk} \end{bmatrix} \tag{41}$$

However,

$$\frac{1}{\frac{\Delta_{ii}(\Delta_{jj}-\Delta_{jk}-\Delta_{kj}+\Delta_{kk})}{\Delta^2} - \frac{\Delta_{ji}-\Delta_{ki}}{\Delta} \cdot \frac{\Delta_{ij}-\Delta_{ik}}{\Delta}}$$

$$= \frac{1}{\frac{1}{\Delta^2}(\Delta_{ii}\Delta_{jj}-\Delta_{ii}\Delta_{jk}-\Delta_{ii}\Delta_{kk}-\Delta_{ji}\Delta_{ij}+\Delta_{ji}\Delta_{ik}+\Delta_{ki}\Delta_{ij}-\Delta_{ki}\Delta_{ik})}$$

$$= \frac{\Delta^2}{\Delta(\Delta_{iijj}+\Delta_{iikk}-\Delta_{iijk}-\Delta_{iikj})} \tag{42}$$

Hence,

$$\begin{bmatrix} i_{io} \\ i_{jk} \end{bmatrix} = \frac{1}{\Delta_{iijj}+\Delta_{iikk}-\Delta_{iijk}-\Delta_{iikj}} \begin{bmatrix} \Delta_{ii} & -\Delta_{ij}+\Delta_{ik} \\ -\Delta_{ij}+\Delta_{ik} & \Delta_{jj}-\Delta_{jk}-\Delta_{kj}+\Delta_{kk} \end{bmatrix} \begin{bmatrix} V_{io} \\ V_{jk} \end{bmatrix} \tag{43}$$

This equation shows that the topological formula for double cofactors is important in analysis of networks.

<u>Theorem 7</u>: The double cofactor Δ_{iijk} of Y_n is equal to

$$\Delta_{iijk} = \Sigma(-1)^{s'} \text{ j-k C.D.C. of } G_{-i} \text{ admittance products} \tag{44}$$

where G_{-i} can be obtained from the vertex i from the M.U. graph G, s' is the order of j-k C.D.C. of G_{-i} and summation is over all possible j-k C.D.C. of G_{-i}.

<u>Proof</u>: It is clear that Δ_{iijk} is equal to the determinant of the matrix which is obtained by deleting the i and jth rows and i and kth columns. However, deleting the ith row and column gives the cofactor Δ_{ii} of Y_n of G,

which means that Δ_{iijk} is the j-k cofactor of the admittance matrix of G_{-i}. Thus by Theorem 6,

$$\Delta_{iijk} = \Sigma(-1)^{s'} \text{ j-k C.D.C. of } G_{-i} \text{ admittance products} . \quad (45)$$

Q.E.D.

5. PROOF OF MASON'S FORMULA FOR SIGNAL FLOW GRAPHS BY M.U. GRAPHS

5.1 Introduction

Mason's signal flow graph [4,5] is one of the graphical representations of linear equations. If by some modification of a signal flow graph, the resultant graph represents a system of linear equations similar to that stated in chapter 2, then the modified graph is a M.U. graph, and it is possible to prove Mason's formula by the method of chapter 3.

5.2 Proof of Mason's Formula

A linear equation which can be represented by Mason's signal flow graph has the form

$$\begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{array}\begin{array}{c} \begin{array}{cc} y_o & x_1\ x_2 \cdots\cdots x_n \end{array} \\ \left[\begin{array}{c|c} & \\ C_1 & C_2 \\ & \end{array}\right] \end{array} \begin{bmatrix} y_o \\ x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad (46)$$

This can be changed to

$$\begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{array}\begin{array}{c} \begin{array}{cc} y_o & x_1\ x_2 \cdots\cdots x_n \end{array} \\ \left[\begin{array}{c|c} & \\ C_1 & C_2\text{-}U \\ & \end{array}\right] \end{array} \begin{bmatrix} y_o \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (47)$$

or by adding

$$y = \alpha x_1 \,. \tag{48}$$

Equation (47) becomes

$$\begin{array}{c} \\ y_o \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{array} \begin{array}{c} \begin{array}{cccc} y_o & x_1 & x_2 \cdots\cdots & x_n \end{array} \\ \left[\begin{array}{c|c} -1 & \alpha \quad 0 \cdots\cdots 0 \\ \hline & \\ C_1 & C_2 - U \\ & \end{array}\right] \end{array} \begin{bmatrix} y_o \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \tag{49}$$

Equation (49) can be expressed as M.U. graph (see chapter 2). By Equations (46) and (49), it can be seen that a M.U. graph can be obtained from a signal flow graph by the following steps.

(1) Addition of -1 circuits to all the vertex in the graph.

(2) Addition of an edge, whose weight is α, from the output vertex (in this case vertex x_1) to the input vertex (in this case vertex y_o).

(3) Addition of a reference vertex.

If Equation (46) is consistent, then the determinant of Equation (49) is zero. From Equation (48) we have

$$\frac{1}{\alpha} = \frac{x_1}{y_o} \tag{50}$$

and $\frac{1}{\alpha}$ is the quantity which should be evaluated.

Consider a Mason's signal flow graph G^m having v vertices. Let G be a M.U. graph obtained by using steps (1) and (2), then by Theorem 4 the determinant of Y_n of G is

$$\Delta = \Sigma(-1)^s \text{ C.D.C. of G admittance products .} \tag{51}$$

For convenience, symbols U_i, L_i^q, P^q and $L_{\sim i}^r$ are defined as follows:

<u>Definition 7</u>: U_i is the ith set of negative unit circuits which is added in the step (1) to obtain a M.U. graph from a signal flow graph.

<u>Definition 8</u>: L_i^q is a set of an edge and vertex disjoint union of i directed circuits consisting of q edges of a signal flow graph G^m.

<u>Definition 9</u>: P^q is an oriented path from the input vertex to the output vertex consisting of q edges.

<u>Definition 10</u>: For a given oriented path P^q, $L_{\sim i}^r$ is a set of either an edge or a vertex disjoint union of i directed circuits consisting of r edges of G^m such that the set and P^q are also disjoint each other.

By Equation (51) and the above four definitions, determinant Δ of G can be expressed as

$$\begin{aligned}\Delta = {} & (-1)^v U_v + \Sigma(-1)^{v-p-1} L_i^p U_{v-p} \text{ adm. prod.} + \Sigma(-1)^{v-p+2} L_2^p U_{v-p} \text{ adm. prod.} \\ & + \ldots + \Sigma(-1)^{v-p+k} L_k^p U_{v-p} \text{ adm. prod.} + \ldots + \Sigma(-1)^v \, L_v^v \text{ adm. prod.} \\ & + \alpha \, \Sigma \, P^r \, \{(-1)^{v-r} U_{v-r-1} + \Sigma(-1)^{v-q-r+1} L_{\sim 1}^q \, U_{v-q-r-1} \text{ adm. prod.} + \ldots \\ & + \Sigma(-1)^{v-q-r+k} \, L_{\sim k}^q \, U_{v-q-r-1} \text{ adm. prod.} + \ldots + \Sigma(-1)^{v-r} \, L_{\sim v-r-1}^{v-r-1} \text{ adm. prod.}\}\end{aligned} \tag{52}$$

where if L_i^p is empty, then $(-1)^{v-p-i}$ $L_i U_{v-p}$ adm. prod. is zero, and if either P^r or $\underset{\sim}{L}_i^q$ is empty, then $\alpha P^r (-1)^{v-q-r+i}$ $\underset{\sim}{L}_i^q$ $U_{v-q-r-1}$ adm. prod. is zero. The above result is true, because of the following two reasons: (1) Suppose an edge and vertex disjoint union of k directed circuits is obtained from G^m. Also suppose the circuits together contain k edges. By Definition 8, the collection of these circuits is symbolized by L_k^p. It is obvious that L_k^p consists of p vertices. In order to form a C.D.C. v-p negative unit circuits must be picked, therefore L_k^p U_{v-p} is a C.D.C., and the sign of this C.D.C. is $(-1)^{v-p+k}$ which is in Equation (52). (2) Suppose that a path from the output vertex to the input vertex is obtained from G^m which consists of r edges, which is symbolized by P^r. This path with edge α forms a directed circuit which contains r+1 vertices. Also suppose an edge and vertex disjoint union of k directed circuits such that these circuits and the circuit formed by α and P^r are disjoint. By Definition 10, these k circuits are symbolized by $\underset{\sim}{L}_k^q$, under the assumption that these circuits consist of q edges. It is clear that $\underset{\sim}{L}_k^q$ consists of q vertices.

In order that $\underset{\sim}{L}_k^q$ and the circuit formed by α and P^r are in a C.D.C., v-r-1 negative unit circuits must be chosen. Hence every term which contains α must be of the form $\alpha P^r (-1)^{v-q-r+k}$ $\underset{\sim}{L}_k^q$ $U_{v-q-r-1}$ adm. prods.

Because U_i adm. prods. is equal to $(-1)^i$, Equation (52) can be written as

$$\begin{aligned}\Delta = {} & 1 - \Sigma L_1^p \text{ adm. prod.} + \Sigma L_2^p \text{ adm. prod.} + \ldots \\ & + \Sigma(-1)^k L_k^p \text{ adm. prod.} + \ldots \\ & - \alpha \Sigma P^r \{1 - \Sigma \underset{\sim}{L}_1^q \text{ adm. prod.} + \Sigma \underset{\sim}{L}_2^q \text{ adm. prod.} \\ & + \Sigma(-1)^k \underset{\sim}{L}_k^q \text{ adm. prod.} + \ldots \} \qquad (53)\end{aligned}$$

By setting determinant Δ being zero, we will obtain Mason's formula as

$$\frac{1}{\alpha} = \frac{\Sigma\, P^r \{1 - \Sigma\, \underset{\sim}{L}_2^q \text{ adm. prod.} + \Sigma\, \underset{\sim}{L}_2^q \text{ adm. prod.} - \dots \}}{1 - \Sigma\, L_1^p \text{ adm. prod.} + \Sigma\, L_2^p \text{ adm. prod.} - \dots} \tag{54}$$

Q.E.D.

6. AN EXAMPLE OF EVALUATION OF VERTEX POTENTIAL BY TOPOLOGICAL FORMULAS

Let the given system be the network of an amplifier shown in Figure 12.

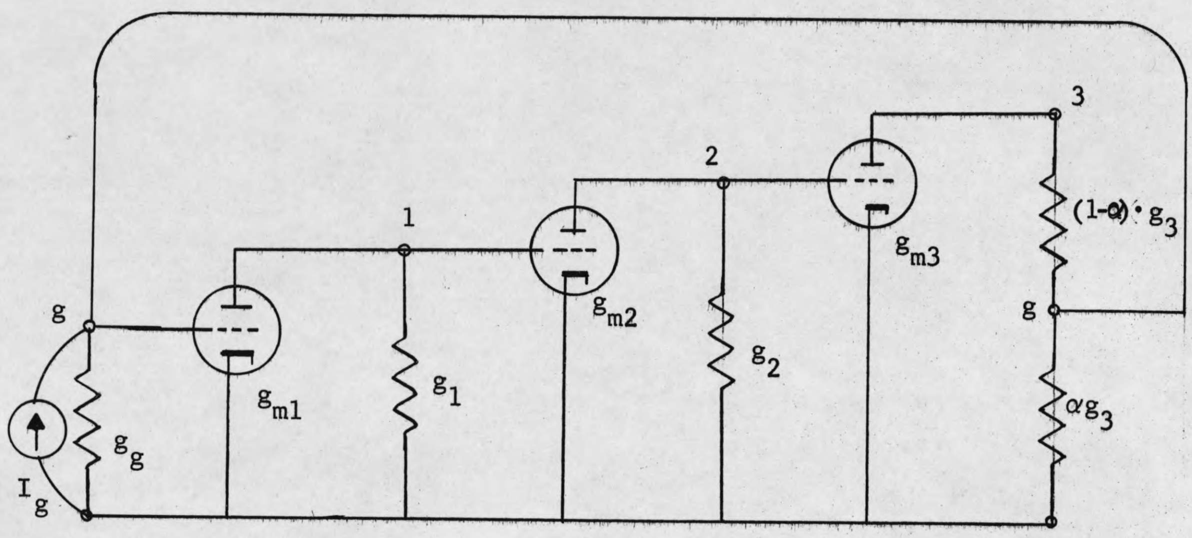


Figure 12. The given network N

The corresponding M.U. graph can be expressed in Figure 13, and the matrix expression of linear equations is written as Equation (55).

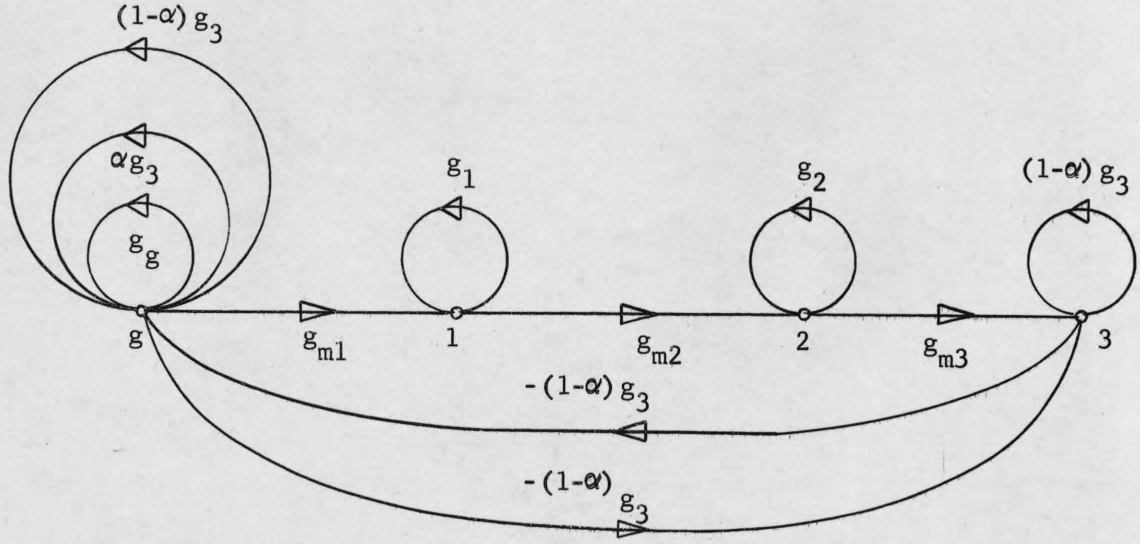


Figure 13. M.U. graph of Network N

$$
\begin{array}{c} \\ g \\ 1 \\ 2 \\ 3 \end{array}
\begin{array}{c} \begin{array}{cccc} g & 1 & 2 & 3 \end{array} \\
\begin{bmatrix} g_g+\alpha g_3+(1-\alpha) g_3 & 0 & 0 & -(\alpha-1) g_3 \\ g_{m1} & g_1 & 0 & 0 \\ 0 & g_{m2} & g_2 & 0 \\ -(\alpha-1) g_3 & 0 & g_{m3} & (1-\alpha) g_3 \end{bmatrix} \end{array}
\begin{bmatrix} V_g \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -I_g \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (55)
$$

Then the determinant of Y_n of G can be obtained by the topological formulas which are in chapter 3.

$$\Delta = g_{m1} \cdot g_{m2} \cdot g_{m3} (1-\alpha) \cdot g_3 - (1-\alpha)^2 \cdot g_3^2 \cdot g_1 \cdot g_2 + g_1 \cdot g_2 \cdot (1-\alpha) \cdot g_3$$

$$\{g_g + \alpha g_3 + (1-\alpha) \cdot g_3\}$$

$$= (1-\alpha) \cdot g_{m1} \cdot g_{m2} \cdot g_{m3} \cdot g_3 - (1-\alpha)^2 \cdot g_3^2 \cdot g_1 \cdot g_2 + (1-\alpha) \cdot g_1 g_2 g_3 g_g$$

$$+ \alpha (1-\alpha) \cdot g_1 \cdot g_2 \cdot g_3^2 + (1-\alpha)^2 \cdot g_1 \cdot g_2 \cdot g_3^2 \qquad (56)$$

Then the cofactor Δ_{gg} of Y_n should be obtained. According to chapter 3, G_{-i}, which is obtained by deleting the vertex i of G, will be necessary and shown in Figure 14.

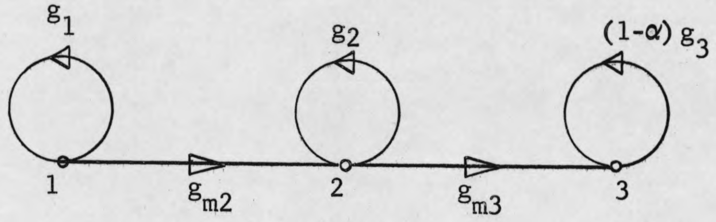


Figure 14. G_{-i} of G

By Figure 14, the cofactor Δ_{gg} is as follows:

$$\Delta_{gg} = -(1-\alpha) \cdot g_1 g_2 \cdot g_3 \qquad (57)$$

Therefore the vertex potentials V_g of N is

$$V_g = \frac{(1-\alpha)\cdot g_1 g_2 g_3 \ I_g}{(1-\alpha)\, g_{m1}\cdot g_{m2}\cdot g_{m3}\cdot g_3 - (1-\alpha)^2\cdot g_3^2\cdot g_1 g_2 + (1-\alpha)\cdot g_1 g_2 g_3\cdot g_g + \alpha(1-\alpha)\cdot g_1 g_2 g_3^2 + (1-\alpha)^2\cdot g_1 g_2 g_3^2}$$

$$= \frac{g_1 g_2 \ I_g}{g_{m1} g_{m2} g_{m3} + g_1 g_2 g_3 + \alpha g_1 g_2 \cdot g_3} \qquad (58)$$

BIBLIOGRAPHY

1. Chen, W. K., "General Topological Analysis of Linear Systems", Ph.D. Thesis, University of Illinois, Urbana, Illinois; 1964.

2. Coates, C. L., "General Topological Formulas for Linear Network Functions", Trans. Inst. Radio Engrs., CT-5, 30-42 (March 1958).

3. Dodd, G. G., "Principles and Applications of Unistor Graphs", Ph.D. Thesis, University of Illinois, Urbana, Illinois; 1964.

4. Mason, S. J., "Feedback Theory - Some Properties of Signal Flow Graphs", Proc. Inst. Radio Engrs., 41, 1144-1156 (Sept. 1953).

5. Mason, S. J., "Topological Analysis of Linear Non-Reciprocal Networks", Proc. Inst. Radio Engrs., 45, 829-838 (June 1957).

6. Mayeda, W., and Seshu, S., "Topological Formulas for Network Functions", Bull. No. 446, University of Illinois Engineering Experiment Station (1957).

7. Mayeda, W., "Topological Formulas for Active Networks", Report No. 8, U.S. Army Contract DA-11-022-ORD-1963, University of Illinois (January 1958).

8. Mayeda, W., "Topological Formulas for Nonreciprocal Networks and Networks with Transformers", Proc. of NEC, Vol. XIV (October 1958), 631-643.

9. Okada, S., Onodera, R., and Orui, H., "Topological Treatments of Four-Terminal Networks", PAAG Memories of the Unifying Study of Basic Problems in Engineering and Physical Science by Means of Geometry, Vol. 2, div. A(1958), 5-31.

10. Percival, W. S., "Solution of Passive Electrical Networks by Means of Mathematical Trees", J. Inst. Elect. Engr., 100, Part III, (London 1953), 143-150.

11. Seshu, S. and Reed, M. B., Linear Graphs and Electrical Networks, (Addison-Wesley, 1961).

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13. ABSTRACT

In this paper modified unistors have been defined. For example, the transconductance of a vacuum tube will be expressed by one modified unistor. A M.U. graph (modified unitsor graph) is a linear graph consisting of modified unistors and can represent a linear electrical network. A node voltage equation of a M.U. graph of n edges without sources can be obtained as

$$A^{-}YA^{+\prime}V_n = 0$$

where A^{-}, A^{+}, and Y are a negative incidence matrix, a positive incidence matrix, and a diagonal n by n admittance matrix.

If there exist independent current sources from the reference vertex to any vertex in a M.U. graph, then the node voltage equations can be written as

$$A^{-}YA^{+\prime}V_n = Y_nV_n = -J_n$$

where J_n is a column matrix representing independent current sources. It is noticed that Y_n is a connection matrix of M.U. graph.

By the Binet-Cauchy theorem, the determinant of Y_n is equal to summation of nonzero majors of $A^{-}Y$ times the corresponding majors of $A^{+\prime}$. Therefore, first the condition of the set of edges which forms a nonzero

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13. ABSTRACT (Continued)

major should be expressed topologically. Then, it can be shown that the determinant of Y_n is equal to the summation of $(-1)^s$ times the C.D.C. admittance products of a M.U. graph. Similarly, topological formulas of the ij cofactor Δ_{ij} and the double cofactor Δ_{iijk} of Y_n are given. These formulas are important in obtaining network functions. Finally, in Chapter 5, Mason's formula for signal flow graphs will be proved by using M.U. graphs.