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Effects of unmodeled high frequency dynamics on stability and performance of adaptive control schemes are analyzed. In the regulation problem global stability properties are no longer guaranteed, but a region of attraction exists for exact adaptive regulation. The dependence of the region of attraction on unmodeled parasitics is examined first. Then the general case of model reference adaptive control is considered in which parasitics can destroy stability and boundedness properties. A more robust adaptive law is proposed

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20. ABSTRACT (continued)

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# SINGULAR PERTURBATIONS AND ROBUST REDESIGN OF ADAPTIVE CONTROL*

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## Abstract

Effects of unmodeled high frequency dynamics on stability and performance of adaptive control schemes are analyzed. In the regulation problem global stability properties are no longer guaranteed, but a region of attraction exists for exact adaptive regulation. The dependence of the region of attraction on unmodeled parasitics is examined first. Then the general case of model reference adaptive control is considered in which parasitics can destroy stability and boundedness properties. A more robust adaptive law is proposed guaranteeing the existence of a region of attraction from which all signals converge to a residual set which contains the equilibrium for exact tracking. The size of this set depends on design parameters, the frequency range of parasitics and the reference input signal characteristics.

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## Introduction

Global stability of adaptive control systems, an open problem for almost two decades, was recently solved for both continuous and discrete SISO (single-input single-output) systems [1-5]. However there still remains a significant gap between the available theoretical methodologies and the potential applications of such adaptive schemes. Global stability properties are guaranteed under the "matching assumption" that the model order is not lower than the order of the unknown plant. Since this restrictive assumption is likely to be violated in applications, it is important to determine the robustness of adaptive schemes with respect to such modeling errors.

Several attempts have been made to formulate and analyze reduced order adaptive systems. Specific results such as error bounds have been obtained for adaptive observers and identifiers [6-10]. In [11] local stability has been proved for a reduced-order indirect adaptive regulator. Efforts on reduced-order direct adaptive control [12,13] have been restricted to simple first or second order examples rather than the general problem. In these examples it was shown by simulations [13] or "linearization" [12] that unmodeled parasitics can lead to an unstable closed-loop system. Analysis [6,9] of the effects of high frequency plant inputs on the performance of identifiers and adaptive observers with parasitics has determined that the inputs should be restricted to dominantly rich inputs. As a design concept the dominant richness requires that in presence of parasitics the richness condition be satisfied outside the parasitic range. It excludes wideband inputs such as noise and square waves as undesirable. The situation in adaptive control is more difficult because the plant input is generated by adaptive feedback which incorporates the unknown plant with parasitics. The schemes proposed so far do not contain a mechanism to restrict the frequency content of the plant input. The lack of this mechanism has caused the loss of robustness reported in [12,13].

The two main results of this paper are, first, an estimate of the region of attraction for adaptive regulation and second, a modification of the adaptive laws to guarantee boundedness in the case of tracking. The frequency content and magnitude of the reference input signal, the speed ratio $\mu$ of slow vs. fast phenomena, the adaptive gain and initial conditions are shown to have crucial effects on the stability of the adaptive control schemes. These results are first analytical conditions for robustness of direct adaptive control with respect to high frequency dynamics. They are obtained for a continuous-time SISO adaptive control scheme [1]. The same methodology can be extended to more complicated continuous and discrete-time adaptive control problems. The paper is organized in two main sections. The first section contains a simple motivating scalar example which illustrates the salient features of the general methodology developed in the second section.

## I. The Scalar Reduced-Order Adaptive Control Problem

We start with a simple example of reduced-order adaptive control in which the output $y_p$ of a second order plant

$$\dot{y}_p = a_p y_p + 2z - u, \qquad a_p > 0 \tag{1.1}$$

$$\mu\dot{z} = -z + u \tag{1.2}$$

with unknown constant parameters $a_p$ and $\mu$, is required to track the state $y_m$ of a first order model

$$\dot{y}_m = -a_m y_m + r(t) \qquad a_m > 0 \tag{1.3}$$

where u is the control input and $r = r(t)$ is a reference input, a uniformly bounded function of time. This example serves as a motivation for and an introduction to the general methodology to be developed in the next section. As in our earlier work [6], the model-plant mismatch is due to some "parasitic" time constants which appear as multiples of a singular perturbation parameter $\mu$ and introduce the "parasitic" state $\eta$. In (1.1), (1.2) the parasitic state is defined as $\eta = z - u$ resulting into the following representation

$$\dot{y}_p = a_p y_p + 2\eta + u \tag{1.4}$$

$$\mu\dot{\eta} = -\eta - \mu\dot{u} \tag{1.5}$$

where the "dominant" part (1.4) and "parasitic" part (1.5) of the plant appear explicitly.

If we apply to the plant with parasitics (1.4), (1.5) the same adaptive law which we would have applied to the plant without parasitics, that is if we use the control

$$u = -K(t)y_p + r(t) \tag{1.6}$$

and the adaptive law

$$\dot{K} = \gamma e y_p \qquad \gamma > 0 \tag{1.7}$$

we obtain

$$\dot{e} = -a_m e - (K(t)-K^*)(e+y_m) + 2\eta \tag{1.8}$$

$$\mu\dot{\eta} = -\eta + \mu[\gamma e(e+y_m)^2 - K(K-a_p)(e+y_m) + 2K\eta + Kr - \dot{r}] \tag{1.9}$$

$$\dot{K} = \gamma e(e+y_m) \tag{1.10}$$

where

$$e \triangleq y_p - y_m, \qquad K^* \triangleq a_m + a_p. \tag{1.11}$$

The existing theory of adaptive control [14,15] guarantees stability properties for the case without parasitics, $\mu=0$, when (1.8), (1.9), and (1.10) reduce to

$$\dot{\bar{e}} = -a_m\bar{e} - (\bar{K}(t)-K^*)(\bar{e}+y_m) \tag{1.12}$$

$$\dot{\bar{K}} = \gamma\bar{e}(\bar{e}+y_m). \tag{1.13}$$

<u>Lemma 1</u>: For any bounded initial conditions $\bar{e}(0)$, $\bar{K}(0)$ the solution $\bar{e}(t)$, $\bar{K}(t)$ of (1.12), (1.13) is uniformly bounded and $\lim_{t\to\infty} \bar{e}(t) = 0$, $\lim_{t\to\infty} \bar{K}(t) = K_s$ where constant $K_s$ is in general a function of $\bar{e}(0)$, $\bar{K}(0)$. Furthermore if $r(t)$ is sufficiently rich then $\lim_{t\to\infty} K(t) = K^*$, independent of $\bar{e}(0)$, $\bar{K}(0)$.

The above example illustrates some of the robustness questions to be answered in this paper. Given that the adaptive system without parasitics, in this case (1.12), (1.13), possesses properties such as in Lemma 1, how will these properties be altered by the parasitics that is, what are the stability properties of (1.8) to (1.10)? Which modification of the adaptive law would help to preserve some of the desirable properties? The perturbation parameter $\mu$ provides us with a means to answer such questions in a semi-quantitative way using the orders of magnitude $O(\mu^\nu)$, noting that for $\mu$ small, the quantity $O(\mu^\nu)$

is small when $\nu > 0$ and large when $\nu < 0$. The smallness of $\mu$ implies that the parasitics are fast and that neglecting them, $\mu = 0$, we concentrate on the slow, that is the "dominant", part of the plant.

As we shall see a first property to be lost due to parasitics is global stability. In the case of regulation, that is when $y_m = 0$, $r(t) = 0$, the boundedness of the solutions $e(t)$, $K(t)$ and the convergence of $e(t)$ to zero as $t \to \infty$ is preserved, but is not global. It possesses a domain of attraction whose size we describe by estimating the orders of magnitudes of the axes of an elipsoid $\mathcal{D}(\mu)$. In the tracking problem, when $r(t) \neq 0$ the adaptive system with parasitics such as (1.8) to (1.10) may not converge to or may not even possess an equilibrium. A practical goal is then to guarantee some boundedness properties. We show that a redesign, which may sacrifice some properties of the ideal system without parasitics, results in the convergence from any point in $\mathcal{D}(\mu)$ to a disk $\mathcal{B}(\mu)$ around the origin in the e, $\eta$-plane. The design objective is then to make $\mathcal{D}(\mu)$ as large as possible and $\mathcal{B}(\mu)$ as small as possible. Let us illustrate this discussion by analyzing the regulation problem and the tracking problem for the example (1.1) to (1.3).

a. <u>Regulation</u>: In the regulation problem expressions (1.8) to (1.10) become

$$r(t) = 0,\ y_m(t) = 0,\ \ e(t) = y_p(t) \tag{1.14}$$

$$\dot{y}_p = a_p y_p + u + 2\eta \tag{1.15}$$

$$\mu\dot{\eta} = -\eta - \mu\dot{u} \tag{1.16}$$

$$u = -K(t)y_p \tag{1.17}$$

$$\dot{K} = \gamma y_p^2 \tag{1.18}$$

and the objective is to drive $y_p$ to zero despite the presence of parasitics while assuring that all the signals in the closed loop system (1.15) to (1.18) remain bounded. It is important to note that the open loop system (1.15), (1.16) might not be stabilizable by constant gain output feedback for a given value of $\mu$. If this is the case then there is no hope that the adaptive controller (1.17), (1.18) will stabilize the equilibrium of (1.15), (1.16). The following lemma characterizes parasitics for which a linear output stabilizing feedback law exists.

Lemma 2: There exists a $\mu_1 > 0$ and a constant $K_o$ such that for all $\mu \in (0,\mu_1]$ the system (1.15), (1.16) with the feedback law

$$u = -K_o y_p \tag{1.19}$$

is an asymptotically stable closed-loop system. Furthermore,

$$\mu_1 < \frac{1}{2a_p} \tag{1.20}$$

and

$$\frac{1}{\mu} - a_p > K_o > a_p \tag{1.21}$$

We now establish the stability properties of the adaptive control system (1.15) to (1.18) for $\mu < \mu_1$.

Theorem 1. There exists $\mu^* < \mu_1$ and positive numbers $\alpha < 1/2$, $c_1$, $c_2$ such that for $\mu \in (0,\mu^*]$ any solution $y_p(t)$, $\eta(t)$, $K(t)$ of (1.15) to (1.18) starting from the set

$$\mathcal{D}(\mu) = \{y_p, \eta, K: |y_p| + |K| < c_1\mu^{-\alpha}, |\eta| < c_2\mu^{-\alpha-1/2}\} \tag{1.22}$$

is bounded and $y_p \to 0$, $\eta \to 0$, $K(t) \to$ constant as $t \to \infty$.

Proof: Let $K_1 > a_p$ be a constant and consider the function

$$V(y_p, \eta, K) = \frac{y_p^2}{2} + \frac{(K - K_1)^2}{2\gamma} + \frac{\mu}{2}(\eta + 2y_p)^2 \tag{1.23}$$

Observe that for each $\mu > 0$, $c > 0$, $\alpha > 0$ the equality

$$V(y_p, \eta, K) = c\mu^{-2\alpha} \tag{1.24}$$

defines a closed surface $S(\mu, \alpha, c)$ in $R^3$. The derivative of V along the solution of (1.5) to (1.18) is

$$\dot{V} = -(K_1 - a_p)y_p^2 - \eta^2 + \mu(\eta + 2y_p)(\gamma y_p^3 + Ka_p y_p + 2a_p y_p - 2Ky_p - K^2 y_p + 2K\eta) \tag{1.25}$$

A detailed analysis of (1.25) shows that there exist constants $\alpha < 1/2$, c, $\mu^*$ such that $\dot{V} \leq 0$ for each $\mu \in (0, \mu^*]$ and all $y_p$, $\eta$, K enclosed in $S(\mu, \alpha, c)$. Moreover $\dot{V} = 0$ only at the equilibrium $y_p = 0$, $\eta = 0$, K = constant. The same analysis shows that there exist positive constants $c_1$, $c_2$ such that the set

$$\mathcal{D}(\mu) = \{y_p, \eta, K: |y_p| + |K| < c_1\mu^{-\alpha},\ |\eta| < c_2\mu^{-\alpha-1/2}\} \tag{1.26}$$

is enclosed by the surface $S(\mu, \alpha, c)$ and any solution of (1.15) to (1.18) starting from $\mathcal{D}(\mu)$ remains inside $S(\mu, \alpha, c)$. Furthermore inside $S(\mu, \alpha, c)$ V is a non-increasing function of time which is bounded from below and hence converges to a finite value $V_\infty$. Since $\ddot{V}$ is bounded, $\dot{V}$ is uniformly continuous for all $y_p$, $\eta$, K enclosed in $S(\mu, \alpha, c)$ and therefore $\lim_{t \to \infty} \dot{V} = 0$ i.e. $y_p \to 0, \eta \to 0$ and $K \to$ constant as $t \to \infty$.

Remark 1. It can also be shown that increasing adaptive gain $\gamma$ for a fixed $\mu$ reduces the size of the domain $\mathcal{D}(\mu)$ and the stability properties of Theorem 1 can no longer be guaranteed if $\gamma \geq O(1/\mu)$.

Remark 2. As $\mu \to 0$, domain $\mathcal{D}(\mu)$ becomes the whole space $R^3$, that is the adaptive regulation problem (1.15) to (1.18) is well posed with respect to parasitics.

Remark 3. Theorem 1 is more than a local result because it shows that given any bounded initial condition $y_p(0)$, $\eta(0)$, $K(0)$, there always exists $\mu^*$ such that for each $\mu \in (0,\mu^*]$ the solution of (1.15) to (1.18) is bounded and $y_p \to 0$, $\eta \to 0$, $K \to$ constant as $t \to \infty$.

Remark 4. Since Theorem 1 is only a sufficient condition it is of interest to examine whether the stability properties of Lemma 1 are indeed lost for initial conditions outside the set (1.22). From Lemma 2 and the fact that $K(t)$ is non-decreasing it can be seen that instability occurs if $K(t_o) > \frac{1}{\mu} - a_p$.

As an illustration of the stability properties established by Theorem 1 simulation results for (1.15) to (1.18) with $a_p = 4$ and different values of $\mu$, $\gamma$ and initial conditions are plotted in Figures 1 to 4.* In addition to $y_p(t)$ also V with $K_1 = 7$ is plotted against time to show whether all the signals in the closed loop remain bounded. In Fig. 1a,b where $\mu = 0.05$, $\gamma = 5$, $y_p(0) = 1.0$, $\eta(0) = 1.0$, $K(0) = 3$, the objective of the regulator is achieved since $y_p \to 0$ and V is bounded. Increasing $y_p(0)$ from 1 to 2.4 and keeping all the other conditions the same as in Fig. 1, the regulator fails its objective and $y_p \to \infty$ as shown in Fig. 2. With the same initial conditions as in Fig. 1a,b but with $\mu = 0.07$ instead of $\mu = 0.05$, $y_p \to \infty$ as indicated in Fig. 3. Figure 4 shows the effect of increasing the adaptive gain $\gamma$. With the same initial conditions as in Fig. 1a,b but with $\gamma = 30$ instead of $\gamma = 5$ regulation fails and $y_p \to \infty$.

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*All figures appear at the end of the paper.

b. Tracking: Returning now to the tracking problem we note that for a general $r(t) \neq 0$ the system (1.8) to (1.10) need not possess an equilibrium. The best we can expect to achieve in this case is to guarantee that the solutions starting in $\mathcal{D}(\mu)$ remain bounded and converge to a disk $\mathcal{B}(\mu)$ around the origin in the e, η-plane.

To prove such a result we modify the adaptive law (1.10) as

$$\dot{K} = -\sigma K + \gamma e(e + y_m) \tag{1.27}$$

where σ is a positive design parameter. In view of (1.27) the equations describing the stability properties of the tracking problem in the presence of parasitics are

$$\dot{e} = -a_m e - (K(t) - K^*)(e + y_m) + 2\eta \tag{1.28}$$

$$\mu\dot{\eta} = -\eta + \mu[\gamma e(e+y_m)^2 - K(K-a_p)(e+y_m) + 2K\eta + Kr - \dot{r}] \tag{1.29}$$

$$\dot{K} = -\sigma K + \gamma e(e + y_m) \tag{1.30}$$

Theorem 2: Let the reference input r(t) satisfy

$$|\, r(t) \,| < r_1 \;, \quad |\, \dot{r}(t) \,| < r_2 \quad \forall\, t > 0 \tag{1.31}$$

where $r_1$, $r_2$ are given positive constants. Then there exist positive constants $t_1$, $\mu^*$, σ, $\alpha < 1/2$, $c_1$ to $c_5$ such that for $\mu \in (0,\mu^*]$ every solution of (1.28) to (1.30) starting at t = 0 from the set

$$\mathcal{D}(\mu) = \{e,K,\eta\colon |e| + |K| < c_1\mu^{-\alpha} \;,\; |\eta| < c_2\mu^{-\alpha}\} \tag{1.32}$$

enters the residual set

$$\mathcal{D}_o(\mu) = \{e,\eta,K\colon (e,\eta) \in \mathcal{B}(\mu),\ K \in \mathcal{K}\} \tag{1.33}$$

where

$$\mathcal{B}(\mu) = \{e,\eta\colon |e| + |\eta| \leq c_3\sqrt{\sigma}\} \text{ and } \mathcal{K} = \{K\colon |K| \leq c_4\} \tag{1.34}$$

at $t = t_1$ and remains in $\mathcal{D}_o(\mu)$ for all $t > t_1$. Furthermore $c_5 > \sigma > \mu$.

<u>Proof</u>: Choosing the function

$$V(y_p,\eta,K) = \frac{e^2}{2} + \frac{(K-K^*)^2}{2\gamma} + \frac{\mu}{2}(\eta + 2e)^2 \tag{1.35}$$

we can see that for each $\mu > 0$, $c_o > 0$, $\alpha > 0$ the equality

$$V(e,\eta,K) = c_o\mu^{-2\alpha} \tag{1.36}$$

defines a closed surface $S(\mu,\alpha,c_o)$ in $R^3$ space. The derivative of V along the solution of (1.28) to (1.30) is

$$\dot{V} = -a_m e^2 - \frac{\sigma}{\gamma}K(K-K^*) - \frac{\eta^2}{2} + \mu(\eta + 2e)[\gamma e(e+y_m)^2 - K(K-a_p)(e+y_m)$$
$$+ 2K\eta + Kr - 2a_m e - \dot{r} - 2(K-K^*)(e+y_m) + 2\eta] \tag{1.37}$$

Hence

$$\dot{V} \leq -a_m e^2 - \frac{\sigma}{\gamma}\left|K - \frac{K^*}{2}\right|^2 - \frac{\eta^2}{2} + \frac{\sigma}{\gamma}\left|\frac{K^*}{4}\right|^2 + \mu|\eta + 2e|\{|\gamma e(e+y_m)^2|$$
$$+ |K(K-a_p)(e+y_m)| + 2|K\eta| + 2|K\eta| + 2|(K-K^*)(e+y_m)|$$
$$+ 2|\eta| + 2a_m|e| + |K|r_1 + r_2\} \tag{1.38}$$

A detailed analysis of (1.38) shows that there exists positive constants $c_o$, $\alpha < 1/2$, $c_1$ to $c_5$, and $\mu^*$ such that for all $\mu \in (0,\mu^*]$, and $c_5 > \sigma > \mu$ the sets $\mathcal{D}(\mu)$, $\mathcal{D}_o(\mu)$ given by (1.32) and (1.33), respectively, are enclosed by $S(\mu,\alpha,c_o)$ and $\dot{V} < 0$ everywhere inside $S(\mu,\alpha,c_o)$, except possibly in $\mathcal{D}_o(\mu)$. Furthermore set $\mathcal{D}_o(\mu)$ is closed and bounded, $\mathcal{D}_o(\mu) \subset \mathcal{D}(\mu)$ and $\mathcal{D}(\mu)/\mathcal{D}_o(\mu)$ is a non-empty set. Thus every solution of (1.28), (1.29), (1.30) starting at $t = 0$ from $\mathcal{D}_o(\mu)$ will remain in $\mathcal{D}_o(\mu)$. Also every solution starting at $t = 0$ from $\mathcal{D}(\mu)/\mathcal{D}_o(\mu)$ will enter $\mathcal{D}_o(\mu)$ at $t = t_1$ and remain in $\mathcal{D}_o(\mu)$ thereafter.

Remark 5. Constants $c_i$ $i = 0,1..4$ depend on $r_1$ and $r_2$ which characterize the magnitude and frequency content of the reference input signal. A further analysis of (1.38) indicates that for a given $\mu$ an increase in $r_1$ or $r_2$ can no longer guarantee that $\dot{V} < 0$ everywhere in $\mathcal{D}(\mu)/\mathcal{D}_o(\mu)$. For this reason our formulation excludes high frequency or high amplitude reference input signals such as square or random waveforms, the traditional favorites of the adaptive control literature.

Remark 6. It can also be shown that increasing the adaptive gain $\gamma$ for given $\mu$, $r_1$ and $r_2$ reduces the size of the domain $\mathcal{D}(\mu)$. For $\gamma \geq O(1/\mu)$ the stability properties of Theorem 2 can no longer be guaranteed.

Remark 7. The use of $\sigma$ is found to be essential in obtaining sufficient conditions for boundedness in the presence of parasitics. However in the absence of parasitics ($\mu = 0$), $\sigma > 0$ causes an output error of $O(\sqrt{\sigma})$. This is a trade-off between boundedness of all signals in the presence of parasitics and the loss of exact convergence of the output error to zero in the absence of parasitics. The size of $\sigma$ reflects our ignorance about $\mu$. If an upper bound of $\mu$ is known $\sigma$ can be set equal to this upper bound. For high frequency parasitics $\mu$ is small and therefore $\sigma$ can be small.

It is of interest to examine whether for initial conditions outside the set $\mathcal{D}(\mu)$ we can loose boundedness. Simulation results with $a_p = 4$, $a_m = 3$ and $\gamma = 5$ are summarized in Figures 5 to 12. Plots of the output error and the function V versus time are obtained for different initial conditions, $\mu$, $\sigma$ and reference input characteristics. In Fig. 5a,b the output error e and function V are plotted against time for $\mu = 0.01$, $e(0) = 1$, $\eta(0) = 1$, $K(0) = 3$, $\sigma = 0.06$ and $r(t) = 3\sin 2t$. The output error decreases and remains close to zero and function V is strictly decreasing for $V > 0.05$, but $\dot{V}$ changes sign in the region $V < 0.05$ as shown in Fig. 5b. Keeping the same conditions as in Fig. 5a,b but increasing $\mu$ from 0.01 to 0.05 we can still achieve similar results as shown in Fig. 6a,b. However in this case the steady state error is bigger and $\dot{V}$ changes sign for $V < 0.4$. Increasing the value of $\mu$ from 0.05 to 0.08 the output error becomes unbounded for all $\sigma \geq 0$ as indicated in Fig. 7. The effects of the input characteristics are summarized in Fig. 8,9 and 10. In Fig. 8, $\mu = 0.05$, $e(0) = 1$, $\eta(0) = 1$, $K(0) = 3$, $\sigma = 0$ and $r(t) = 3\sin 10t$ results into an unbounded output error due to the increase of the frequency of r(t) from 2 to 10. The same instability result has been observed for $\sigma = 0.02$, 0.06. However for $\sigma = 0.08$ the output error became bounded as shown in Fig. 9 indicating the beneficial effects of $\sigma$ when parasitics are present. The effect of the amplitude of the reference input r(t) is shown in Fig. 10. With $\mu = 0.05$, $\sigma = 0$, 0.06 and the same initial conditions as before but with $r(t) = 15\sin 2t$ the output error goes unbounded. Fig. 11 shows the effect of initial conditions on boundedness. By increasing e(0) from 1 to 2.5 and keeping $\mu = 0.05$, $\eta(0) = 1$, $K(0) = 3$, and $r(t) = 3\sin 2t$ the output error becomes unbounded for all $\sigma \geq 0$. In Fig. 12a,b we show the loss of exact convergence of the output error to zero in the absence of parasitics ($\mu$=0) due to the design parameter $\sigma$.

## II. Adaptive Control of a SISO Plant in the Presence of Parasitics

We now consider the general problem of adaptive control of a SISO time-invariant plant of order $n+m$ where $n$ is the order of the dominant part of the plant and $m$ is the order of the parasitics. The plant is assumed to possess slow and fast parts and is represented in the explicit singular perturbation form*

$$\dot{x} = A_{11}x + A_{12}z + b_1u \tag{2.1}$$

$$\mu\dot{z} = A_{21}x + A_{22}z + b_2u \ , \qquad \mathrm{Re}\lambda(A_{22}) < 0 \tag{2.2}$$

$$y = c_o x \tag{2.3}$$

where $x$, $z$ are $n$ and $m$ vectors respectively and $u$, $y$ are the scalar input and output of the plant respectively. State $z$ is formed of a "fast transient" and a "quasi-steady state" defined as the solution of (2.2) with $\mu\dot{z} = 0$. This motivates the definition of the fast parasitic state as

$$\eta = z + A_{22}^{-1}(A_{21}x + b_2u) \tag{2.4}$$

Defining

$$\begin{aligned} &A_o = A_{11} - A_{12}A_{22}^{-1}A_{21}, \quad b_o = b_1 - A_{12}A_{22}^{-1}b_2, \quad A_1 = A_{22}^{-1}A_{21}A_o \\ &A_2 = A_{22}^{-1}A_{21}b_o, \quad A_3 = A_{22}^{-1}A_{21}A_{12}, \quad A_4 = A_{22}^{-1}b_2 \end{aligned} \tag{2.5}$$

and substituting (2.4) into (2.1), (2.2) we obtain a representation of (2.1),

---

*Note that the parasitics are only weakly observable from (2.3), that is the dependence of $y$ on the parasitic modes is $O(\mu)$. In the case of strongly observable parasitics, $y = c_1x + c_2z$, constant output feedback can lead into instability in general [16] and this case is of no interest to us at this moment. Further discussions on strongly observable parasitics can be found in [9].

(2.2), (2.3) with the dominant part (2.6) and the parasitic part (2.7) appearing explicitly

$$\dot{x} = A_o x + b_o u + A_{12}\eta \tag{2.6}$$

$$\mu\dot{\eta} = A_{22}\eta + \mu(A_1 x + A_2 u + A_3\eta + A_4\dot{u}) \tag{2.7}$$

$$y = c_o x \tag{2.8}$$

The output y of the system (2.6) to (2.8) is required to track the output $y_m$ of an n-th order reference model

$$\dot{x}_m = A_m x_m + b_m r \tag{2.9}$$

$$y_m = c_m^T x_m \tag{2.10}$$

whose transfer function $W_m(s)$

$$W_m(s) = c_m^T(sI - A_m)^{-1} b_m = K_m \frac{Z_m(s)}{R_m(s)} \tag{2.11}$$

is chosen to be strictly positive real and r(t) is a uniformly bounded reference input signal.

The reduced order plant obtained by setting $\mu = 0$ in (2.6)-(2.8) is assumed to satisfy the following conditions:

(i) The triple $(A_o, b_o, c_o)$ is completely controllable and observable.

(ii) In the transfer function

$$W_o(s) = c_o^T(sI - A_o)^{-1} b_o \triangleq K_p \frac{N(s)}{D(s)} \tag{2.12}$$

N(s) is a monic Hurwitz polynomial of degree n-1 and D(s) is a monic polynomial of degree n. For ease of exposition we assume that $K_p = K_m = 1$.

The controller structure has the same form as that used in [1] for the parasitic -free plant that is for $\mu = 0$ in (2.6) to (2.8). In this controller the plant input u and measured output y are used to generate a (2n-2) dimensional auxiliary vector v as

$$\dot{v}_1 = \Lambda v_1 + gu$$

$$W_1 = c^T v_1 \tag{2.13}$$

$$\dot{v}_2 = \Lambda v_2 + gy$$

$$W_2 = d_o y + d^T v_2 \tag{2.14}$$

where $\Lambda$ is an (n-1)x(n-1) stable matrix and $(\Lambda, g)$ is a controllable pair. The plant input is given by

$$u = r + \theta^T \omega \tag{2.15}$$

where $\omega^T = [v_1^T, v_2^T, y]$ and $\theta(t)$ is a (2n-1) dimensional adjustable parameter vector. It has been shown in [1] that a constant vector $\theta^*$ exists such that for $\theta(t) = \theta^*$ the transfer function of the parasitic -free plant (2.12) with controller (2.13) to (2.15) matches that of the model (2.11).

If we apply to the plant with parasitics (2.6) to (2.8) the controller described by (2.13) to (2.15) we obtain the following set of equations for the overall feedback system

$$\begin{bmatrix} \dot{x} \\ \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} A_o & 0 & 0 \\ 0 & \Lambda & 0 \\ gc_o & 0 & \Lambda \end{bmatrix} \begin{bmatrix} x \\ v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} b_o \\ g \\ 0 \end{bmatrix} (\theta^T \omega + r) + \begin{bmatrix} A_{12} \\ 0 \\ 0 \end{bmatrix} \eta \tag{2.16}$$

$$\mu\dot{\eta} = A_{22}\eta + \mu(A_1 x + A_2\theta^T\omega + A_2 r + A_3\eta + A_4\dot{\theta}^T\omega + A_4\theta^T\dot{\omega} + A_4\dot{v}) \tag{2.17}$$

$$y = c_o x \tag{2.18}$$

Introducing $\theta^*$, $Y^T = [x^T, v_1^T, v_2^T]$ and

$$A_c = \left[\begin{array}{c|c|c} A_o + d_o^* b_o c_o & b_o c^{*T} & b_o d^{*T} \\ \hline g d_o^* c_o & \Lambda + g c^{*T} & g d^{*T} \\ \hline g c_o & 0 & \Lambda \end{array}\right], \quad b_c = \left[\begin{array}{c} b_o \\ \hline g \\ \hline 0 \end{array}\right] \tag{2.19}$$

we rewrite (2.16) to (2.17) in a form convenient for our stability analysis

$$\dot{Y} = A_c Y + b_c((\theta - \theta^*)^T\omega + r) + \bar{A}_{12}\eta \tag{2.20}$$

$$\mu\dot{\eta} = A_{22}\eta + \mu(\bar{A}_1 Y + A_2\theta^T\omega + A_2 r + A_3\eta + A_4\dot{\theta}^T\omega + A_4\theta^T\dot{\omega} + A_4\dot{r}) \tag{2.21}$$

where $\bar{A}_{12} = [A_{12}^T \quad 0 \quad 0]^T$ and $\bar{A}_1 = [A_1^T \quad 0 \quad 0]^T$. An advantage of this form is that for $\theta(t) = \theta^*$ in the parasitic -free case (2.20) becomes a non-minimal representation of the reference model

$$\dot{x}_{mc} = A_c x_{mc} + b_c r, \qquad x_{mc} = [x_m^T, v_{1m}^T, v_{2m}^T]^T \tag{2.22}$$

The equations for the error $e \triangleq Y - x_{mc}$ can be expressed as

$$\dot{e} = A_c e + b_c(\theta - \theta^*)^T(\bar{e} + \bar{x}_{mc}) + \bar{A}_{12}\eta \tag{2.23}$$

$$\mu\dot{\eta} = A_{22}\eta + \mu[\bar{A}_1(e + x_{mc}) + A_2\theta^T(\bar{e} + \bar{x}_{mc}) + A_2 r + A_3\eta + A_4\dot{\theta}^T(\bar{e} + \bar{x}_{mc})$$
$$+ A_4\theta^T f(\theta, e, \eta, r) + A_4\dot{r}] \tag{2.24}$$

$$e_1 = h^T e = [1 \quad 0 \; . \; . \; 0]e \tag{2.25}$$

where

$$\bar{e} \triangleq [v_1^T, v_2^T, y]^T - [v_{1m}^T, v_{2m}^T, y_m]^T, \quad \bar{x}_{mc} \triangleq [v_{1m}^T, v_{2m}^T, y_m]^T \tag{2.26}$$

$$f(\theta, e, \eta, r) = \begin{bmatrix} \Lambda(e^{(1)} + v_{1m}) + gr + g\theta^T(\bar{e} + \bar{x}_{mc}) \\ \Lambda(e^{(2)} + v_{2m}) + g(e_1 + y_m) \\ c_o A_o(e^{(0)} + x_m) + c_o b_o(r + \theta^T(\bar{e} + \bar{x}_{mc})) + A_{12}\eta \end{bmatrix} \tag{2.27}$$

$$e^{(0)} \triangleq x - x_m, \quad e^{(1)} \triangleq v_1 - v_{1m}, \quad e^{(2)} \triangleq v_2 - v_{2m} \tag{2.28}$$

we now need to design an adaptive law for updating the parameter vector $\theta(t)$. For the parasitic-free case [1] the adaptive law

$$\dot{\theta} = -\Gamma e_1 \omega = -\Gamma e_1(\bar{e} + \bar{x}_{mc}), \quad \Gamma = \Gamma^T > 0 \tag{2.29}$$

guarantees that the output error goes to zero as $t \to \infty$ and the signals in the close loop remain bounded for any uniformly bounded reference input $r(t)$. As demonstrated in Section I for the scalar tracking problem, the best we can expect in the presence of parasitics is to guarantee that the solutions starting in a domain $\mathcal{D}(\mu)$ remain bounded and converge to a disk $\mathcal{B}(\mu)$ around the origin in the $e$, $\eta$ plane. To achieve this we modify the adaptive law (2.29) as

$$\dot{\theta} = -\sigma\theta - \Gamma e_1(\bar{e} + \bar{x}_{mc}) \tag{2.30}$$

where $\sigma$ is a design scalar parameter. The resulting adaptive control system with parasitics is described by

$$\dot{e} = A_c e + b_c(\theta - \theta^*)^T(\bar{e} + \bar{x}_{mc}) + \bar{A}_{12}\eta \tag{2.31}$$

$$\mu\dot{\eta} = A_{22}\eta + \mu[\bar{A}_1(e + x_{mc}) + A_2\theta^T(\bar{e} + \bar{x}_{mc}) + A_2 r + A_3\eta - \sigma A_4\theta^T(\bar{e} + \bar{x}_{mc})$$

$$- A_4(\bar{e} + \bar{x}_{mc})^T\Gamma(\bar{e} + \bar{x}_{mc})e_1 + A_4\theta^T f(\theta, e, \eta, r) + A_4\dot{r}] \tag{2.32}$$

$$e_1 = [1\ 0\ .\ .\ 0]e \tag{2.33}$$

$$\dot{\theta} = -\sigma\theta - \Gamma e_1(\bar{e} + \bar{x}_{mc}) \tag{2.34}$$

Theorem 3. Let the reference input $r(t)$ satisfy

$$|r(t)| < r_1, \quad |\dot{r}(t)| < r_2 \quad \forall\ t > 0 \tag{2.35}$$

for some given positive constants $r_1$, $r_2$. Then there exists positive constants $\mu^*$, $\sigma$, $\alpha < 1/2$, $c_1$ to $c_5$ and $t_1$ such that for each $\mu \in (0, \mu^*]$ every solution of (2.31) to (2.34) starting at $t = 0$ from the set

$$\mathcal{D}(\mu) = \{e, \eta, \theta: \|e\| + \|\theta\| < c_1\mu^{-\alpha}, \|\eta\| < c_2\mu^{-\alpha}\} \tag{2.36}$$

enters the residual set

$$\mathcal{D}_o(\mu) = \{e, \eta, \theta: (e, \eta) \in \mathcal{B}(\mu), \theta \in \mathcal{K}\} \tag{2.37}$$

where

$$\mathcal{B}(\mu) = \{e, \eta: \|e\| + \|\eta\| \le c_3\sqrt{\sigma}\}, \ \mathcal{K} = \{\theta: \|\theta\| \le c_4\} \tag{2.38}$$

at $t = t_1$ and remains in $\mathcal{D}_o(\mu)$ for all $t > t_1$. Furthermore $c_5 > \sigma > \mu$.

Corollary 1. Assume $r(t) = 0$. Then there exists a $\mu^*$ such that for all $\mu \in (0,\mu^*]$ and $\sigma = 0$ in (2.34) any solution $e(t)$, $\eta(t)$, $\theta(t)$ of (2.31) to (2.34) which starts from $\mathcal{D}(\mu)$ given by (2.36) is bounded and $\|e\| \to 0$, $\|\eta\| \to 0$, $\|\theta\| \to$ constant as $t \to \infty$.

Proof of Theorem 3. Choose the function

$$V(e,\eta,\theta) = \frac{e^T Pe}{2} + \frac{(\theta - \theta^*)^T \Gamma^{-1}(\theta - \theta^*)}{2} + \frac{\mu}{2}[\eta - P_1^{-1}(e^T P\bar{A}_{12}A_{22})^T]^T P_1[\eta - P_1^{-1}(e^T P\bar{A}_{12}A_{22})^T] \tag{2.39}$$

where $P$ satisfies

$$A_c^T P + PA_c = -qq^T - \epsilon L \tag{2.40}$$

$$Pb_c = h \tag{2.41}$$

for some vector $q$, matrix $L = L^T > 0$ and $\epsilon > 0$, and $P_1$ satisfies

$$P_1 A_{22} + A_{22}^T P_1 = -Q_1 , \quad Q_1 = Q_1^T > 0 \tag{2.42}$$

Equations (2.40), (2.41) follow from the fact that $h^T(sI - A_c)b_c$ is strictly positive real [1] and (2.42) follows from the assumption that $\mathrm{Re}\lambda(A_{22}) < 0$.

Observe that for each $\mu > 0$, $d_o > 0$, $\alpha < 1/2$ the equality

$$V(e,\eta,\theta) = d_o \mu^{-2\alpha} \tag{2.43}$$

defines a closed surface $S(\mu,\alpha,d_o)$ in $R^{6n+m-2}$. The derivative of $V$ along the solution of (2.31) to (2.34) is

$$\dot{V} = -\frac{1}{2} e^T(qq^T + \epsilon L)e - \sigma(\theta - \theta^*)^T\Gamma^{-1}\theta - \frac{1}{2}\eta^T Q_1\eta + \mu[\eta$$

$$- P_1^{-1}(e^T P\bar{A}_{12}A_{22}^{-1})^T]^T P_1[\bar{A}_1(e + x_{mc}) + A_2\theta^T(\bar{e} + \bar{x}_{mc}) + A_2 r + A_3\eta$$

$$- \sigma A_4\theta^T(\bar{e} + \bar{x}_{mc}) - A_4(\bar{e} + \bar{x}_{mc})^T\Gamma(\bar{e} + \bar{x}_{mc})e_1 + A_4\theta^T f(\theta, e, \eta, r) + A_4\dot{r}$$

$$- P_1^{-1}A_{22}^{-1T}\bar{A}_{12}^T P(A_c e + b_c(\theta - \theta^*)^T(\bar{e} + \bar{x}_{mc}) + \bar{A}_{12}\eta)] \quad (2.44)$$

Let

$$\lambda_1 = \min \lambda(L), \ \lambda_2 = \min \lambda(\Gamma^{-1}), \quad \lambda_3 = \min \lambda(Q_1) \quad (2.45)$$

Then

$$\dot{V} \leq -\frac{1}{2}\lambda_1\|e\|^2 - \sigma\lambda_2[\|\theta\| - \frac{\|\Gamma^{-1}\|}{2\lambda_2}\|\theta^*\|]^2 - \frac{1}{2}\lambda_3\|\eta\|^2 + \frac{\sigma\|\Gamma^{-1}\|^2\|\theta^*\|^2}{4\lambda_2}$$

$$+ \mu\|\eta - P_1^{-1}(e^T P\bar{A}_{12}A_{22}^{-1})^T\|[\alpha_1\|e\|^3 + \alpha_2\|e\|^2 + \alpha_3\|\theta\|^2\|e\| + \alpha_4\|\theta\|\|e\|$$

$$+ \alpha_5\|\theta\|^2 + \alpha_6\|\theta\| + \alpha_7\|\theta\|\|\eta\| + \alpha_8\|\eta\| + \|A_4\|r_2] \quad (2.46)$$

where $\alpha_1$ to $\alpha_8$ are positive constants determined from $r_1$, and the norms of the system and the reference model matrices. A detailed analysis of (2.46) shows that there exists positive constants $\sigma$, $\alpha < 1/2$, $c_i$ $i = 1,..c_5$ and $\mu^*$ such that for all $\mu \in (0,\mu^*]$ and $c_5 > \sigma > \mu$, $\mathcal{D}(\mu)$, $\mathcal{D}_o(\mu)$ defined by (2.36) to (2.38) are enclosed by $S(\mu,\alpha,d_o)$ and $\dot{V} < 0$ everywhere inside $S(\mu,\alpha,d_o)$ except possibly in $\mathcal{D}_o(\mu)$. The set $\mathcal{D}_o(\mu)$ is closed and bounded, $\mathcal{D}_o(\mu) \subset \mathcal{D}(\mu)$ and $\mathcal{D}(\mu)/\mathcal{D}_o(\mu)$ is a non-empty set. Every solution of (2.31) to (2.34) starting at $t = 0$ from $\mathcal{D}_o(\mu)$ will remain in $\mathcal{D}_o(\mu)$. Since in $\mathcal{D}(\mu)/\mathcal{D}_o(\mu)$, V is strictly decreasing any solution starting at $t = 0$ from $\mathcal{D}(\mu)/\mathcal{D}_o(\mu)$ will enter $\mathcal{D}_o(\mu)$ at $t = t_1$ and remain in $\mathcal{D}_o(\mu)$ thereafter.

Proof of Corollary 1. The proof of Corollary 1 follows directly from the proof of Theorem 3 by noting that when $r(t) = 0$, $x_m = 0$ and $\sigma = 0$, the disk $\mathcal{B}(\mu)$ reduces to the origin $e = 0, \eta = 0$ i.e. in (2.38) $c_3 = 0$.

In Theorem 3 and Corollary 1 is assumed that $\mu^* < \mu_1$ where $\mu_1$ is defined in the following lemma.

Lemma 3. There exists a $\mu_1 > 0$ such that constant output feedback $u = \theta_o^T \omega$ stabilizes (2.16) to (2.18) for all $\mu \in (0,\mu_1]$.

The proof of Lemma 3 is more complicated than that of Lemma 2 and can be found in [17] where an explicit expression has been obtained for $\mu_1$.

Remark 9. The dependence of constants $c_i$ $i = 1,\ldots,4$ on $r_1$, $r_2$ shows that for a given $\mu$ a reference input signal with high magnitude or high frequencies can no longer guarantee that $\dot{V} < 0$ everywhere in $\mathcal{D}(\mu)/\mathcal{D}_o(\mu)$. Such a reference signal introduces frequencies in the input control signal which are in the parasitic range. Thus the control signal is no longer dominantly rich [6] and, hence, it excites the parasitics considerably and leads to instability. This explains the instability phenomena observed by other authors in simulations such as [18] where a square wave was used as a reference input signal.

## Conclusion

We have analyzed reduced-order adaptive control schemes in which reference models can match the dominant part of the plant, while the model-plant mismatch is caused by the neglected high frequency parasitic modes. In presence of parasitics the global stability properties of the parasitic-free schemes can be lost. However, we have shown that in the regulation problem a region of attraction exists for exact adaptive regulation. This region is a function of the adaptive gains and the speed ratio $\mu$, and as $\mu \to o$, it becomes the whole space. Thus the adaptive regulation problem is well posed with respect to parasitics. In the case of tracking we proposed a more robust adaptive law. The new scheme guarantees the existence of a region of attraction from which all signals converge to a residual set which contains the equilibrium for exact tracking. The dependence of the size of this set on design parameters indicates that a trade-off can be made sacrifizing some of the ideal parasitic-free properties, in order to achieve robustness in presence of parasitics. The crucial effects of the frequency range of parasitics, the adaptive gains and the reference input signal characteristics on the stability properties of adaptive control schemes, explain the undesirable phenomena observed in [12,13]. The results of this paper are obtained for a continuous-time SISO adaptive control scheme where the transfer function of the dominant part of the plant has a relative degree of one. The same methodology can be extended to more complicated continuous and discrete-time adaptive control problems.

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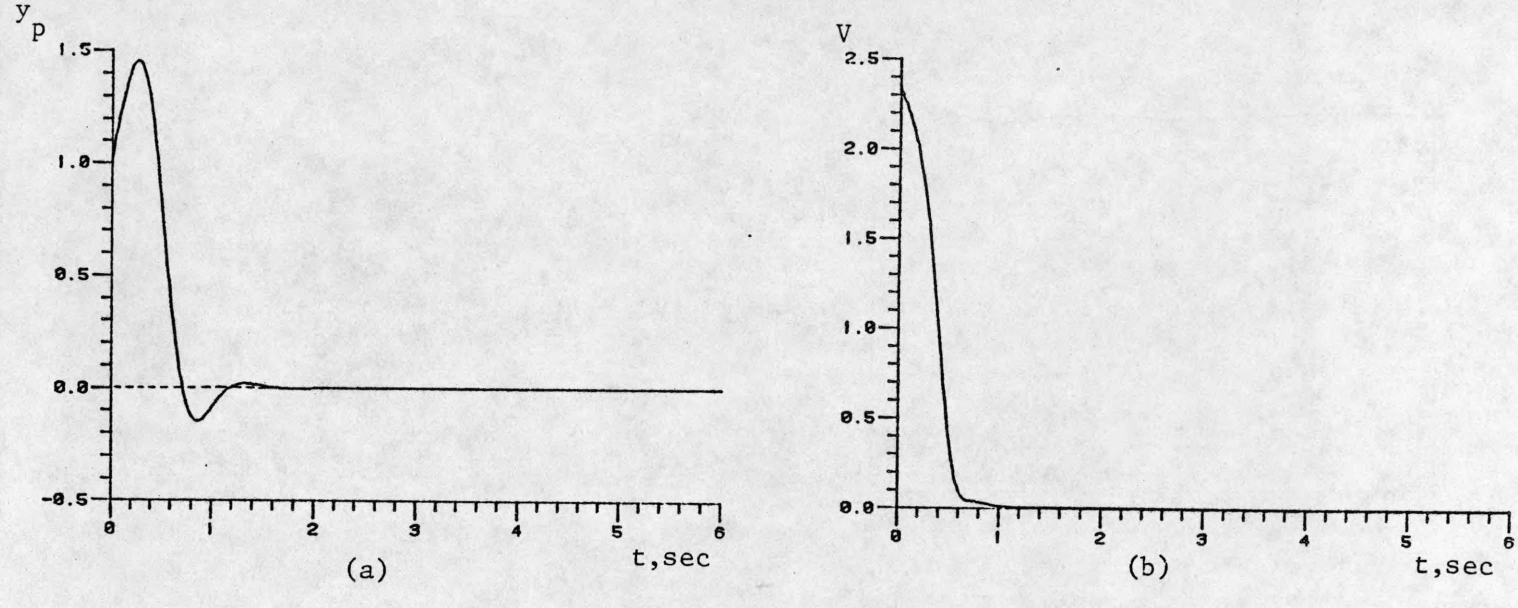


Fig. 1 $\mu=0.05$, $y_p(0)=1.$, $\eta(0)=1$, $K(0)=3$, $\gamma=5$

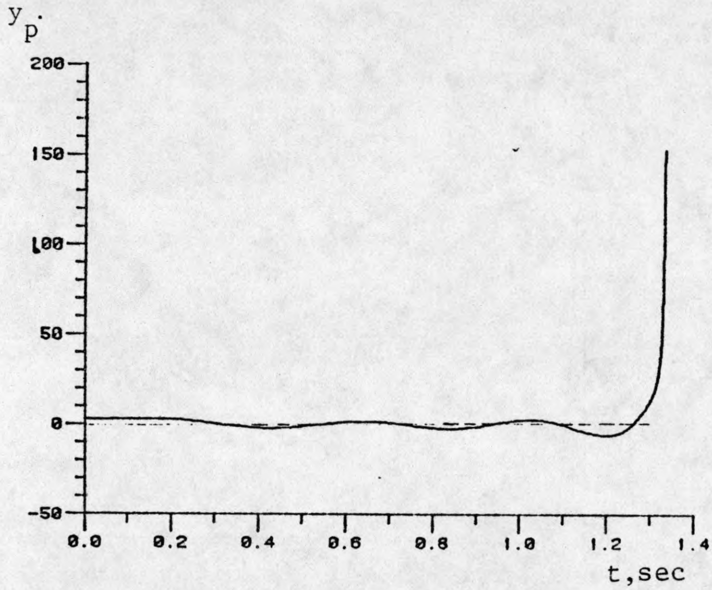


Fig. 2 $\mu=0.05$, $y_p(0)=2.4$, $\eta(0)=1.$, $K(0)=3.$, $\gamma=5.$

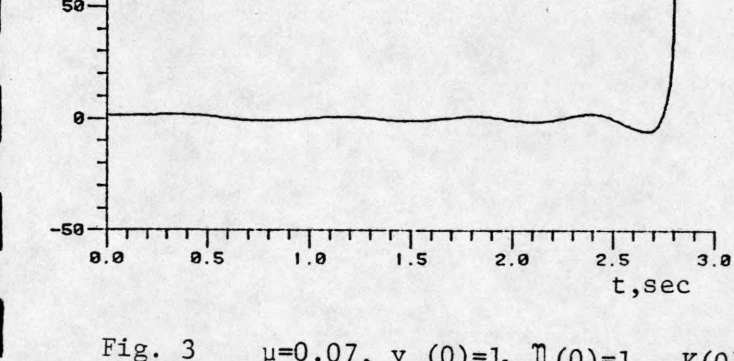


Fig. 3 $\mu=0.07$, $y_p(0)=1$, $\eta(0)=1.$, $K(0)=3$; $\gamma=5$.

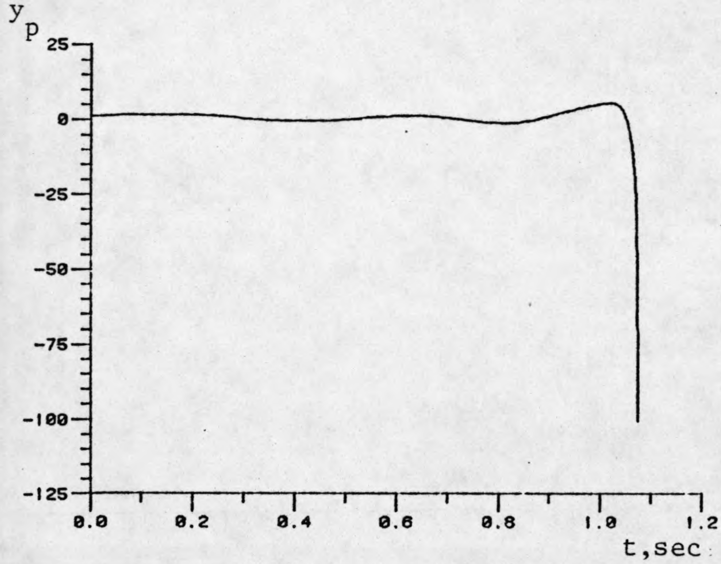


Fig. 4 $\mu=0.05$, $y_p(0)=1.$, $\eta(0)=1.$, $K(0)=3.$, $\gamma=30$

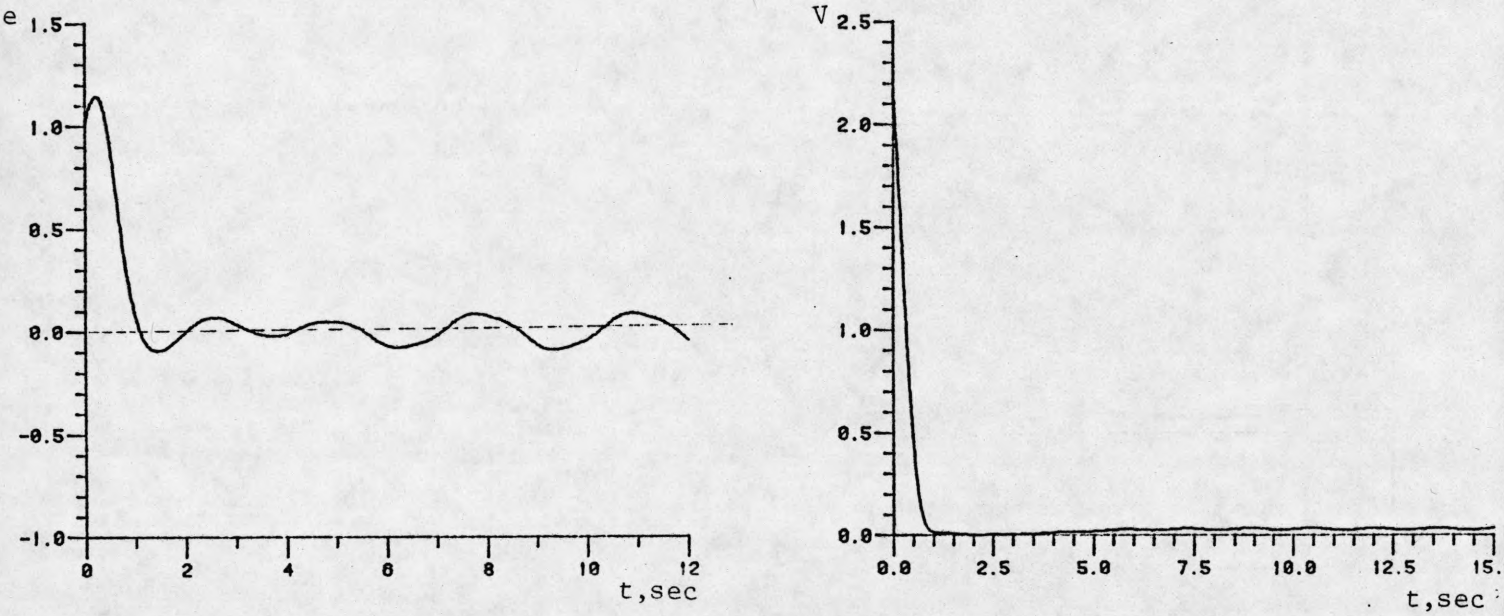


Fig. 5 $\mu=0.01$, $e(0)=1.$, $\eta(0)=1.$, $K(0)=3.$, $\sigma=.06$, $r(t)=3\sin 2t$

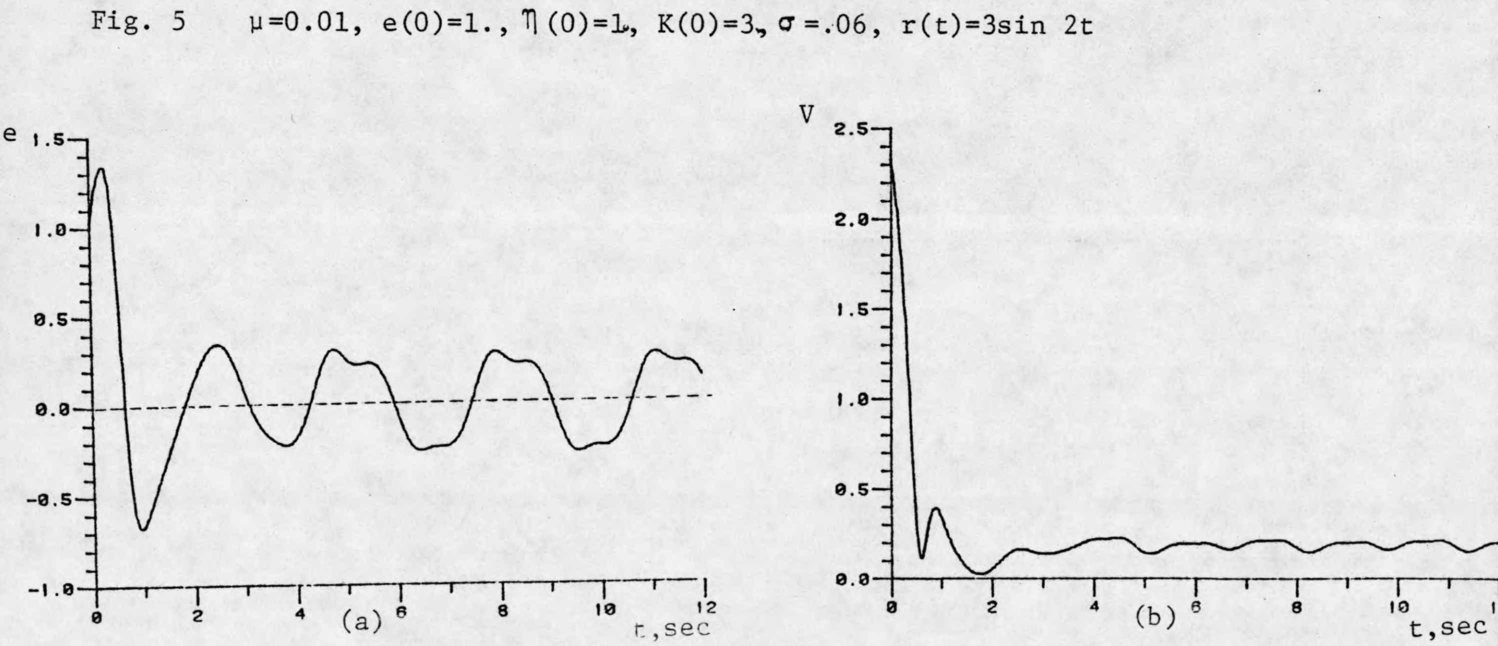


Fig. 6 $\mu=0.05$, $e(0)=1.$, $\eta(0)=1.$, $K(0)=3.$, $\sigma=0.06$, $r(t)=3\sin 2t$

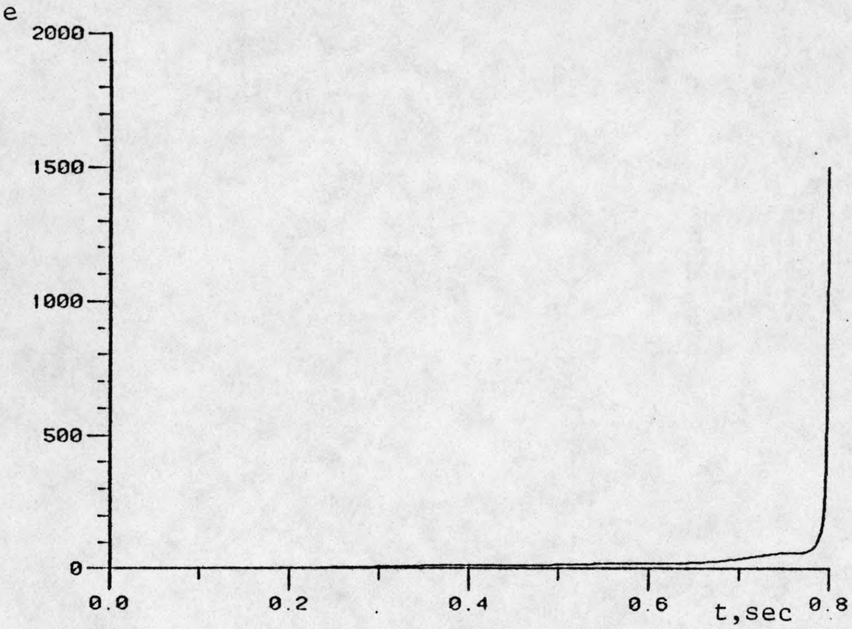


Fig. 7 $\mu=0.08$, $e(0)=1$, $\eta(0)=1$, $K(0)=3$, $\sigma\geq 0.0$, $r(t)=3\sin 2t$

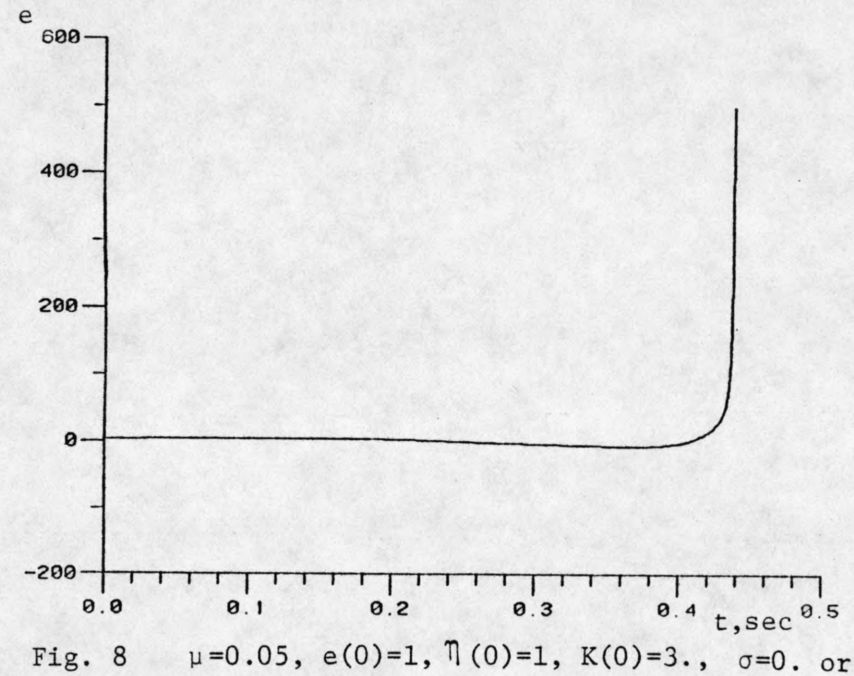


Fig. 8 $\mu=0.05$, $e(0)=1$, $\eta(0)=1$, $K(0)=3.$, $\sigma=0.$ or $0.06$, $r(t)=3\sin 10t$

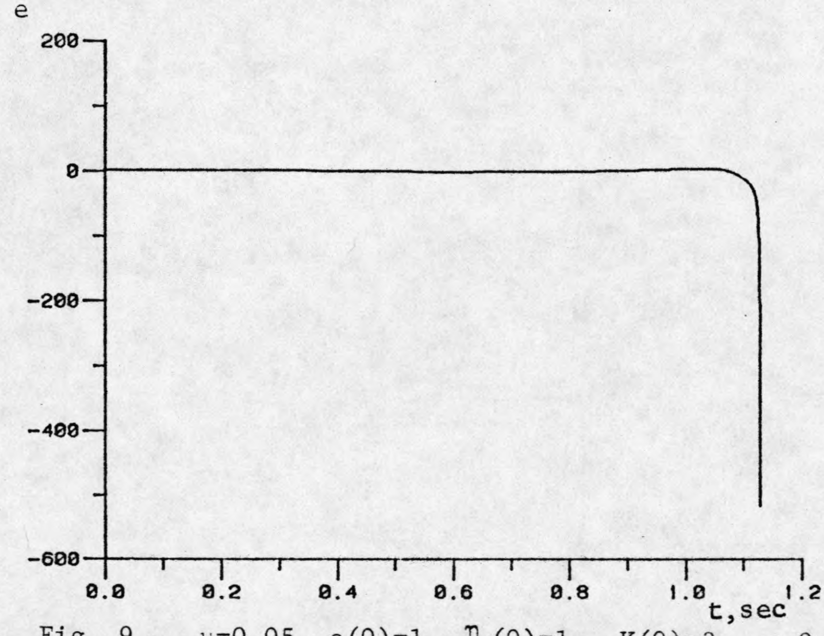


Fig. 9 $\mu=0.05$, $e(0)=1.$, $\eta(0)=1.$, $K(0)=3.$, $\sigma=0.08$, $r(t)=3\sin 10t$

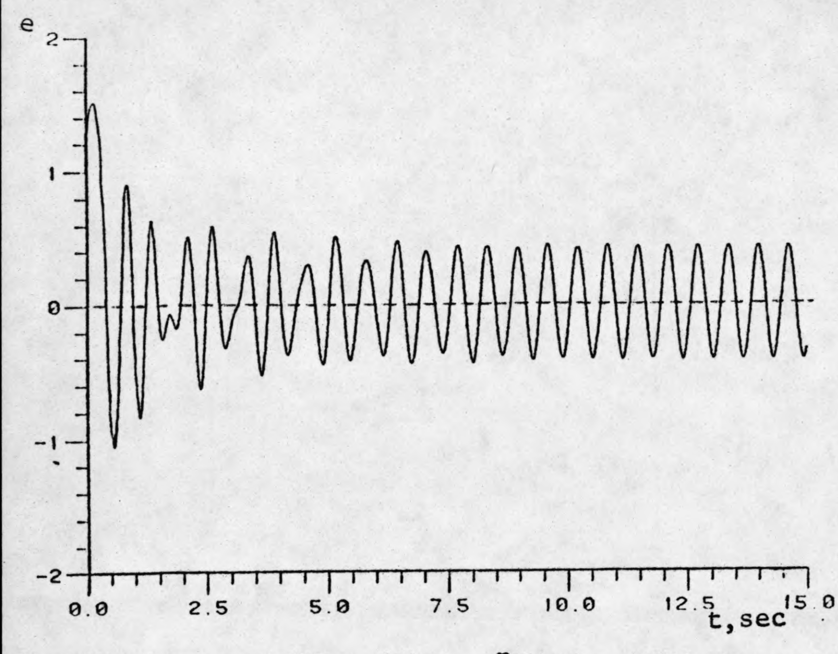


Fig. 10 $\mu=0.05$, $e(0)=1$, $\eta(0)=1.$, $K(0)=3.$, $\sigma=0.$ or $0.06$, $r(t)=15\sin 2t$

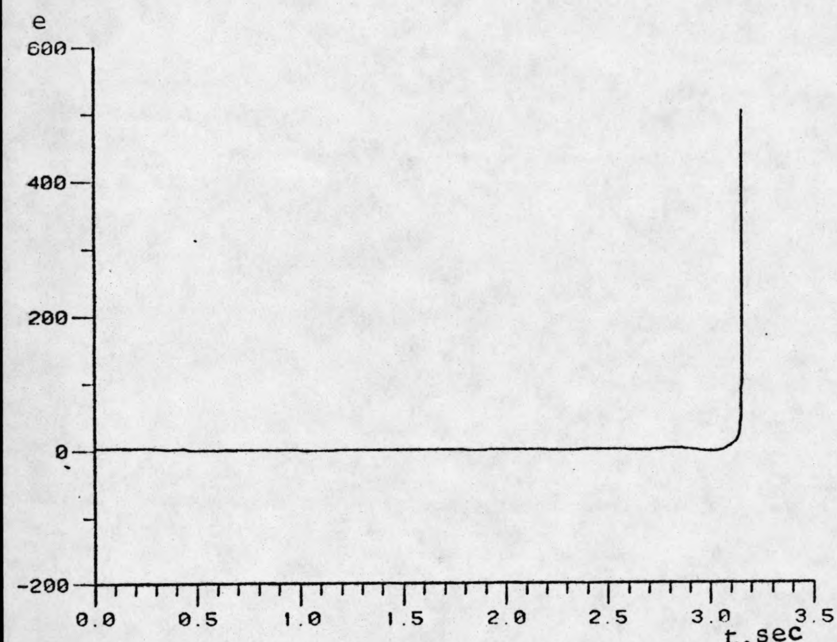


Fig. 11 $\mu=0.05$, $e(0)=2.5$, $\eta(0)=1.$, $K(0)=3.$, $\sigma\geq 0.$, $r(t)=3\sin 2t$

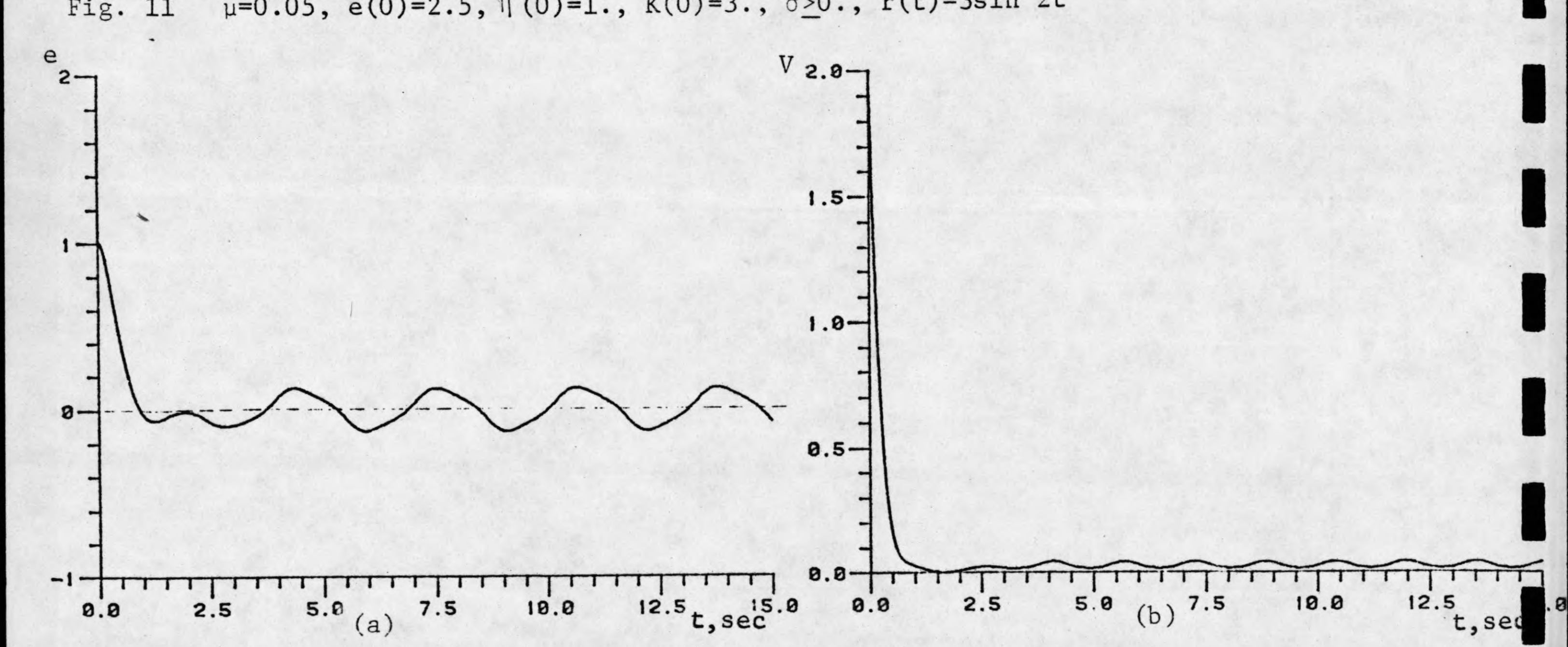


Fig. 12 $\mu=0$, $e(0)=1.$, $\eta(0)=1.$, $K(0)=3.$, $\sigma=0.08$, $r(t)=3\sin 2t$