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FLOW RELIABILITY OF COMMUNICATION NET

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ABSTRACT

In a communication net, each edge has a capacity called an edge capacity and as long as a given flow Ψ_{ij} is not larger than the terminal capacity t_{ij}, Ψ_{ij} can be transmitted via the net. When edges have non-zero probabilities of failure to handle flow, in addition to edge capacities, then we may not be able to transmit a flow Ψ_{ij}, via a net all the time. However there is a high probability that a portion of flow Ψ_{ij} can be transmitted. To indicate how much of Ψ_{ij} can be transmitted under such circumstances, a flow reliability is introduced. Then several ways of increasing the flow reliability of a given communication net such as reduction of probabilities of failure of edges, reduction of a given flow, and modifying a given net will be discussed.

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INTRODUCTION

In order to represent a communication system having imperfect components by a linear graph [1] probabilities will be assigned to edges and (or) vertices representing such imperfect components. For convenience, we will limit ourselves to the case where only edges represent imperfect components of a system so that in addition to capacity c(e) [2,3,4,5], probability f(e) of failure will be assigned to each edge e. Note that failure of a component will correspond to deletion of an edge.

Let Ψ_{ij} be messages per unit of time transmitted via a communication system. Suppose some components of the system are defected, then we may not be able to maintain the transmission of Ψ_{ij}. However, we may be able to maintain the transmission of most of Ψ_{ij} by the efficient use of the remaining system. The largest portion of Ψ_{ij} which can be expected to be transmitted under existence of failure in the system is called the expected flow.

When Ψ_{ij} is very small for a system, then even if several components are down, we may be able to transmit all of Ψ_{ij} via the remaining system. On the other hand, if Ψ_{ij} is the maximum for a system, failure of one component may make the remaining system unable to transmit Ψ_{ij}. For this case, the expected flow would be rather small compared with Ψ_{ij}.

The ratio of the expected flow and a given flow Ψ_{ij} is the "flow reliability."

FLOW RELIABILITY

Consider that a communication net G [6,7,8] represents a practical system such as a data communication system. Then flow Ψ_{ij} indicates an amount of data transmitted from i to j <u>per unit of time</u>. Suppose flow Ψ_{ij} is transmitted via G for T period of time. Then the total flow Φ_{ij} handled by G during T will be

$$\Phi_{ij} = T\Psi_{ij}. \qquad (1)$$

If Ψ_{ij} is equal to the maximum flow from i to j in G known as the terminal capacity t_{ij}, then

$$\Phi_{ij} = Tt_{ij} \qquad (2)$$

is clearly the maximum amount of flow from i to j which can be handled by G during T.

In practical system, components may break down in a finite time. If some components are down during an operating period T, then clearly the total flow Φ_{ij} will be less than Tt_{ij}.

To calculate total flow Φ_{ij} under such circumstance, we assume that edge (component) e is failed to operate for ΔT during T. However all other edges are operating properly during the same period T. Then the total flow $\Phi_{ij}(e)$ under the continuous transmission of the maximum flow will be

$$\Phi_{ij}(e) = Tt_{ij} - \Delta T[t_{ij} - t_{ij}(e)] \qquad (3)$$

where $t_{ij}(e)$ is the maximum flow from i to j in G when edge e is absent. The ratio $R(t_{ij})$ of the total flow $\Phi_{ij}(e)$ with failure of edge e during ΔT and that Φ_{ij} without any failure will be

$$R(t_{ij}) = 1 - \frac{\Delta T}{T}\left[1 - \frac{t_{ij}(e)}{t_{ij}}\right] \tag{4}$$

which is called the "flow reliability" under the maximum flow.

Example 1: Consider a communication net in Fig. 1.

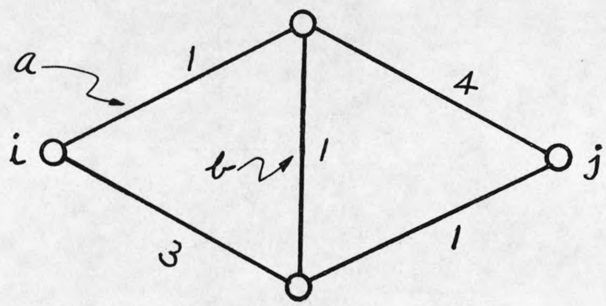

Fig. 1. Communication net.

Suppose $\Psi_{ij} = t_{ij} = 3$ is transmitted during $T = 10$. Then the total flow $\Phi_{ij} = 30$. Suppose edge a is down for $\Delta T = 3$ during the period T. Then the total flow $\Phi_{ij}(a)$ will be $\Phi_{ij}(a) = 27$. The flow reliability for this case will be $R(t_{ij}) = 0.9$.

It can be shown that $\Delta T/T$ in Eq. 4 will approach the probability f(e) of failure of edge e for large T. ($1 - \Delta T/T$ will approach the reliability r(e) of edge e.) Hence, replacing $\Delta T/T$ by f(e), Eq. 4 becomes

$$R(t_{ij}) = 1 - f(e)[1 - T(e)] \quad (5)$$

where

$$T(e) = t_{ij}(e)/t_{ij} \quad (6)$$

is called the "threshold level" under failure of edge e. The symbol $T(e_{p_1} e_{p_2} \ldots e_{p_k})$ will be used for indicating, for convenience, the threshold level under failure of edges $e_{p_1}, e_{p_2}, \ldots,$ and e_{p_k} defined by

$$T(e_{p_1} e_{p_2} \ldots e_{p_k}) = t_{ij}(e_{p_1} e_{p_2} \ldots e_{p_k})/t_{ij} \quad (7)$$

where $t_{ij}(e_{p_1} e_{p_2} \ldots e_{p_k})$ is the maximum flow from i to j when edges e_{p_1}, e_{p_2}, ... and e_{p_k} are deleted. For example, the threshold level T(a) under failure of edge a in Fig. 1 will be T(a) = 2/3. In the same figure, the threshold level under the failure of edges a and b will be T(ab) = 1/3.

When each edge e_p in net G has a probability $f(e_p)$ of failure (or a reliability $r(e_p) = 1 - f(e_p)$), then the flow reliability $R(t_{ij})$ under the maximum flow will be

$$R(t_{ij}) = 1 - \Sigma F(e_{p_1} e_{p_2} \ldots e_{p_k})[1 - T(e_{p_1} e_{p_2} \ldots e_{p_k})]. \quad (8)$$

where $F(e_{p_1} e_{p_2} \ldots e_{p_k})$ is the probability that exactly edges $e_{p_1}, e_{p_2} \ldots$ and e_{p_k} fail and "Σ" means that the sum of all possible cases of failure.

Example 2: Consider a communication net in Fig. 2.

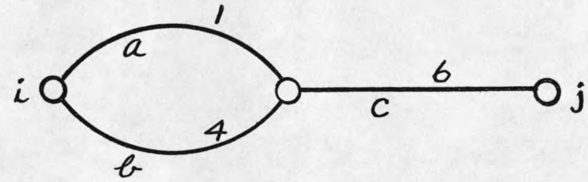

Fig. 2. Communication net.

For input i and output j, the threshold levels are

$$T(a) = 1/5, \quad T(b) = 4/5, \quad T(c) = 0, \quad T(ab) = 0$$

$$T(ac) = 0, \quad T(bc) = 0, \quad \text{and } T(abc) = 0.$$

Let the probability of failure of each edge be

$$f(a) = .1, \quad f(b) = .2, \quad \text{and } f(c) = .3.$$

Then the probabilities of failure appearing in Eq. 8 will be

$$F(a) = .056, \quad F(b) = .126, \quad F(c) = .216$$

$$F(ab) = .014, \quad F(ac) = .024, \quad F(bc) = .054 \quad \text{and } F(abc) = .006.$$

Hence the flow reliability under the maximum flow $\Psi_{ij} = t_{ij} = 5$ will be

$$R(t_{ij}) = .616.$$

Instead of transmitting the maximum flow, suppose we transmit flow $\Psi_{ij} < t_{ij}$. Also suppose only one edge e fails for ΔT. Then Eq. 3 will become

$$\Phi_{ij}(e) = T\Psi_{ij} - \Delta T[\Psi_{ij} - \Psi_{ij}(e)] \tag{9}$$

where

$$\Psi_{ij}(e) = \min\{\Psi_{ij}, t_{ij}(e)\}. \tag{10}$$

This is true because if $\Psi_{ij} \leq t_{ij}(e)$, then the failure of edge e will not influence the transmission of Ψ_{ij}. On the other hand, if $\Psi_{ij} > t_{ij}(e)$, the maximum amount of flow which the remaining net can handle is clearly $t_{ij}(e)$. This result is under the assumption that there are no storage places for flow. In other words, if there are storage such that a part of flow, which can not be transmitted because of failure of edge e, can be stored until edge e is fixed, then it will be possible to transmit these stored flow after edge e being fixed as long as $\Psi_{ij} < t_{ij}$. Hence the total flow $\Phi_{ij}(e)$ with storage places will be larger than that given by Eq. 9. However, in practical cases, the amount of flow $\Delta T[\Psi_{ij} - \Psi_{ij}(e)]$ to be stored would be too large to handle. Also if $t_{ij} - \Psi_{ij}$ is small, to transmit $\Delta T[\Psi_{ij} - \Psi_{ij}(e)]$ may take a long time. Thus in this paper, we will assume that there are no storage places. By this assumption, the flow reliability $R(\Psi_{ij})$ under a given flow Ψ_{ij} will be

$$R(\Psi_{ij}) = 1 - \frac{\Delta T}{T}[1 - \min\{1, t_{ij}(e)/\Psi_{ij}\}]. \quad (11)$$

With the probabilities of failure of edges, the above equation becomes as

$$R(\Psi_{ij}) = 1 - \Sigma F(e_{p_1} e_{p_2} \ldots e_{p_k})[1 - \min\{1, (t_{ij}/\Psi_{ij})T(e_{p_1} e_{p_2} \ldots e_{p_k})\}]. \quad (12)$$

For example, if $\Psi_{ij} = 4$, the flow reliability $R(\Psi_{ij})$ of a communication net in Fig. 2 will be

$$R(\Psi_{ij}) = .644$$

which is higher than that of the maximum flow.

The flow reliability will increase if we do the following:

(1) decreasing the probabilities of failure of edges,

(2) decreasing a flow to be transmitted, and

(3) changing its configuration.

The reason for the first way is obvious from Eq. 12. However, in general, the cost of a component will increase exponentially as the probability of failure decreases. Hence it would not be practical to use the first way to increase its flow reliability. Since $\min\{1,(t_{ij}/\Psi_{ij})T(e_{p_1}e_{p_2}\ldots e_{p_k})\} \geq T(e_{p_1}e_{p_2}\ldots e_{p_k})$, the second way of increasing its flow reliability is true. However, decreasing a flow to be transmitted means to increase the cost of transmission per unit flow. Hence it may not be economical to employ the second way of increasing its flow reliability. The last way can be divided into two cases, one of which is to change its topology so that the threshold levels $T(e_{p_1}e_{p_2}\ldots e_{p_k})$ will increase and the other is to interchange edges or one of two weights, f(e) and c(e), of edges so that $\Sigma F(e_{p_1}\ldots e_{p_k})$ $[1-\min\{1,(t_{ij}/\Psi_{ij})T(e_{p_1}\ldots e_{p_k})\}]$ will decrease. The first case may be useful when a system is at the designing stage. However reconstruction of an entire system may not be practical. The second case is equivalent to interchange of locations of components (equipments) of a similar kind which may be able to do without too much trouble. However, as we will see in the next section, there is an upper bound of the flow reliability which can be obtained just by interchanging components and if any further increase of its flow reliability is required, we must employ some other ways.

DISTRIBUTION OF PROBABILITIES OF FAILURE OF EDGES AND THE MAXIMUM FLOW RELIABILITY

Here, we will investigate the effect of interchanging the probabilities of failure of edges on the flow reliability. Let $f(e_1)$ and $f(e_2)$ be the probabilities of failure of edges e_1 and e_2 in a communication net G_o respectively. Suppose $f(e_1) = q_1$ and $f(e_2) = q_2$ where $q_1 < q_2$. Let $R_o(\Psi_{ij})$ be the flow reliability of G_o. If we interchange these probabilities so that $f(e_1) = q_2$ and $f(e_2) = q_1$, can we have a higher flow reliability? Suppose interchange of any probabilities of failure of edges is permissible. Then how can we obtain the largest flow reliability? Note that interchange of probabilities is equivalent to reassignment of these probabilities to edges. An answer to the above question may not be simple. However, if we assume that these probabilities are very small, an answer becomes very simple as one given by the following theorem

Theorem: Let e_r be an edge for $r = 1,2,\ldots,n$ in a communication net. Suppose the probabilities $f(e_r)$ of failure of edges are very small. Then the flow reliability $R(\Psi_{ij})$ is the maximum if

$$f(e_1) \leq f(e_1) \leq \ldots \leq f(e_n)$$

implies

$$T(e_1) \leq T(e_2) \leq \ldots \leq T(e_n).$$

Example 3: Consider a communication net in Fig. 3.

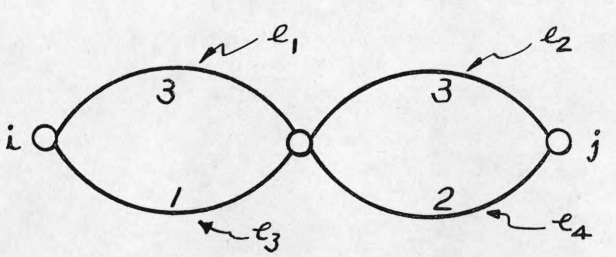

Fig. 3. Communication net.

Let $f(e_1) = .1$, $f(e_2) = .15$, $f(e_3) = .2$ and $f(e_4) = .25$. Hence $f(e_1) \leq f(e_2) \leq f(e_3) \leq f(e_4)$. The threshold levels are $T(e_1) = 1/4$,

$T(e_2) = 1/2$, $T(e_3) = 3/4$ and $T(e_4) = 3/4$. Thus by the above theorem, interchange of probabilities $f(e_1)$, $f(e_2)$, $f(e_3)$ and $f(e_4)$ will not increase the flow reliability. For example, if $\Psi_{ij} = 3$, the flow reliability $R(\Psi_{ij})$ will be .939. If we interchange probabilities of failure of e_1 and e_2 as $f(e_1) = .15$ and $f(e_2) = .1$, the flow reliability $R(\Psi_{ij})$ will be reduced to .929.

Proof of the above theorem: By the assumption that $f(e_r)$ is very small, Eq. (12) can be changed to

$$R(\Psi_{ij}) = 1 - \sum_{(p)} F(e_p)[1 - Q(e_p)] \tag{13}$$

where

$$F(e_p) = f(e_p) \prod_{\substack{r=1 \\ r \neq p}}^{n} [1 - f(e_r)] \tag{14}$$

and

$$Q(e_p) = \min\{1 , [t_{ij}(e_p)/\Psi_{ij}]T(e_p)\} . \tag{15}$$

It is clear from Eq. (14) that the relationship,

$$f(e_1) \leq f(e_2) \leq \cdots \leq f(e_n),$$

will give

$$F(e_1) \leq F(e_2) \leq \cdots \leq F(e_n).$$

Also it is obvious from Eq. (13) that smaller $\Sigma F(e_p)[1 - Q(e_p)]$ will give larger $R(\Psi_{ij})$. Hence the theorem will be proven if we can show that $\Sigma F(e_p)[1 - Q(e_p)]$ is the smallest when $T(e_1) \leq T(e_2) \leq \cdots \leq T(e_n)$ is satisfied.

Consider two sets of positive numbers $\{a_p;\ p = 1,2,\ldots,n\}$ and $\{b_p;\ p = 1,2,\ldots,n\}$. Let $a_1 \leq a_2 \leq \cdots \leq a_n$ and $b_1 \geq b_2 \geq \cdots \geq b_n$.

Let sum S_j be defined as

$$S_j = \sum_{r=1}^{n} a_{j_r} b_r \tag{16}$$

where $(j_1\ j_2\ \ldots\ j_n)$ is a permutation of $(1,2,\ldots,n)$. Let $\{S\}$ be a set of sums S_j produced by employing all possible permutations of $(1,2,\ldots,n)$. Also let sum S_o be

$$S_o = \sum_{r=1}^{n} a_r b_r \ . \tag{17}$$

It can be seen that any sum S_p in $\{S_j\}$ other than S_o can be expressed as

$$S_p = \sum_{r=1}^{m} a_{j_r} b_r + \sum_{r=m+1}^{n} a_r b_r \tag{18}$$

where $(j_1\ j_2\ \ldots\ j_m)$ is a permutation of $(1,2,\ldots,m)$ and $j_m \neq m$. When $m = n$, the second part in the right hand side of Eq. 18 will be absent. Let j_t $(1 \leq t < m)$ be m. Then S_p can be rewritten as

$$S_p = \sum_{\substack{r=1 \\ r\neq t}}^{m-1} a_{j_r} b_r + a_m b_t + a_{j_m} b_m + \sum_{r=m+1}^{n} a_r b_r \ . \tag{19}$$

By interchanging a_{j_m} and a_m, we will have a new sum S'_p as

$$S'_p = \sum_{\substack{r=1 \\ r\neq t}}^{m-1} a_{j_r} b_r + a_{j_m} b_t + a_m b_m + \sum_{r=m+1}^{n} a_r b_r \ . \tag{20}$$

Since $a_{j_m} \leq a_m$ and $b_t \geq b_m$, it is clear that $S'_p \leq S_p$. Similarly, if S'_p is not S_o, we can obtain another sum S''_p such that $S''_p \leq S'_p$. Hence, we can state that any sum S_j in $\{S_j\}$ satisfies that $S_j \geq S_o$. In other words, sum S_o is the smallest in $\{S_j\}$.

By substituting $a_r = F(e_r)$ and $b_r = [1 - Q(e_r)]$, we can state that $\Sigma F(e_r)[1 - Q(e_r)]$ is minimum when

$$[1 - Q(e_1)] \geq [1 - Q(e_2)] \geq \cdots \geq [1 - Q(e_n)].$$

Since $[1 - Q(e_r)]$ is non-negative, the above result gives

$$Q(e_1) \leq Q(e_2) \leq \cdots \leq Q(e_n).$$

From the definition of $Q(e_r)$ in Eq. (15), this relationship gives

$$T(e_1) \leq T(e_2) \leq \cdots \leq T(e_n)$$

which proves the theorem. Q.E.D.

By this theorem, we can assign probabilities of failure of edges so that the flow reliability becomes maximum. In practical systems, not all probabilities of failure of components can be interchanged. However, this theorem will hold by considering only those interchangeable components. For example, if components $e_1, e_2, \ldots,$ and e_k can interchange their probabilities and components $e_1', e_2', \ldots,$ and e_m' have interchangeable probabilities among themselves, then we can interchange probabilities of $e_1, e_2, \ldots,$ and e_k to obtain the highest available flow reliability first, and we interchange probabilities of $e_1', e_2', \ldots,$ and e_m' to obtain the maximum flow reliability.

CONCLUSIONS AND FUTURE PROBLEMS

We can easily agree that flow reliabilities are interesting merit to judge qualities of communication systems. We should notice that, just by considering e_r in the paper as a vertex, the definitions and the theorem can be applied to communication nets where vertices represent components in a physical system. A purpose of this paper is to introduce a concept of a flow reliability. There are several future problems associated with flow reliabilities such as those given below:

(1) What is a relationship between capacities of edges and the flow reliability?

(2) How can we synthesize communication nets satisfying a given flow reliability?

(3) Suppose we define a flow reliability matrix to indicate the flow reliability between all pairs of vertices. Then what are properties of such a matrix?

(4) It is possible to synthesize a communication net which satisfies both a terminal capacity matrix and a flow reliability matrix?

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13. ABSTRACT

In a communication net, each edge has a capacity called an edge capacity and as long as a given flow Ψ_{ij} is not larger than the terminal capacity t_{ij}, Ψ_{ij} can be transmitted via the net. When edges have non-zero probabilities of failure to handle flow, in addition to edge capacities, then we may not be able to transmit a flow Ψ_{ij}, via a net all the time. However there is a high probability that a portion of flow Ψ_{ij} can be transmitted. To indicate how much of Ψ_{ij} can be transmitted under such circumstances, a flow reliability is introduced. Then several ways of increasing the flow reliability of a given communication net such as reduction of probabilities of failure of edges, reduction of a given flow, and modifying a given net will be discussed.

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