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A DYNAMIC PROGRAMMING
OF OPTIMUM FLOWS
IN LOSSY COMMUNICATION NETS

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I. Introduction

A lossy communication net [2], [4], [5] is a weighted linear graph $N = \{\omega_c, \omega_p, G, \Omega, f\}$ where ω_c is the set of the edge capacities, ω_p is the set of the edge efficiency factors, G is the edge set, Ω is the node (vertex) set, and f is the mapping function such that if an edge e is issuing from node x and entering into node y , then $f(e) = (x, y)$.

A flow ψ in a lossy communication net is a function of two variables, one is the edge variable whose range is the edge set G of the net N , and the other is the node variables whose range is the node set, Ω . If x is one of the nodes of the edge e (that is, the initial or terminal node of the edge e), then $\psi(e, x)$ is amount of the flow passing through edge e at node x in the direction of the edge orientation. If x is not one of the nodes of the edge e , $\psi(e, x)$ is zero. A flow must satisfy the following conditions (the axioms of the flow):

(i) The flow is conserved at internal nodes, that is

$$\sum_{e_i \in A_x} \psi(e_i, x) - \sum_{e_j \in B_x} \psi(e_j, x) = \begin{cases} \bar{t} & \text{if } x \text{ is the source} \\ 0 & \text{if otherwise} \\ -\underline{t} & \text{if } x \text{ is the sink} \end{cases}$$

where A_x is the set of edges issuing from node x ; B_x is the set of edges entering into node x , \bar{t} is the sending flow value at the source and \underline{t} is the receiving flow value at the sink.

(ii) If $f(e) = (x, y)$, then we have

$$\psi(e, y) = \rho_e \cdot \psi(e, x)$$

$$0 \leq \psi(e, x) \leq C_e$$

where ρ_e and C_e is the edge efficiency factor ($0 \leq \rho_e \leq 1$) and the edge capacity of edge e , respectively.

Definition 1: An edge in an edge sequence is called forward (or backward) if the edge orientation and the edge sequence orientation agree (or do not agree).

If a flow ψ_0 with the sending flow value \bar{t} and the receiving flow value \underline{t} is already assigned to the net N from the source v to the sink v' , then increment Δ' with sending flow value \mathcal{E} (we assume that \mathcal{E} is a small positive number and we call Δ' \mathcal{E} -increment) is assigned to an edge sequence $S_{vv'}$ from v to v' by the following recurrence equations. Let $S_{vv'} = \{v = x'_0, e'_1, x'_1, e'_2, x'_2, \dots, e'_k, x'_k = v'\}$.

$$(i) \quad \Delta'(e'_1, x'_0) = \mathcal{E}$$

$$(ii) \quad \begin{cases} \Delta'(e'_m, x'_m) = \rho_{e'_m} \cdot \Delta'(e'_m, x'_{m-1}) & \text{if } f(e'_m) = (x'_{m-1}, x'_m) \\ \Delta'(e'_m, x'_m) = \frac{1}{\rho_{e'_m}} \cdot \Delta'(e'_m, x'_{m-1}) & \text{if } f(e'_m) = (x'_m, x'_{m-1}) \end{cases}$$

$$(iii) \quad \begin{cases} \Delta'(e'_m, x'_m), \Delta'(e'_m, x'_{m-1}) > 0 & \text{if } f(e'_m) = (x'_{m-1}, x'_m) \text{ (} e'_m \text{ is forward)} \\ \Delta'(e'_m, x'_m), \Delta'(e'_m, x'_{m-1}) < 0 & \text{if } f(e'_m) = (x'_m, x'_{m-1}) \text{ (} e'_m \text{ is backward)} \end{cases}$$

$$(iv) \quad \text{For } m = 1, 2, \dots, k-1$$

$$\Delta'(e'_{m+1}, x'_m) + \Delta'(e'_m, x'_m) = 0$$

(v) $\Delta'(e', x') = 0$ if e' is not in the sequence $S_{vv'}$.

From \mathcal{E} -increment Δ' we define Δ by

$$\Delta(e, x) = \sum_{\substack{x=x'_i \\ e=e'_j}} \Delta'(e'_j, x'_i) .$$

If the sum $\psi_1 = \psi_0 + \Delta$ satisfies the axioms of the flow, then ψ_1 is called a sum flow or a resultant flow and Δ is called a flow increment to the edge sequence $S_{vv'}$. Conversely an edge sequence is called a flow sequence if an \mathcal{E} -increment along the edge sequence produces a flow increment.

Definition 2: A flow path from v to v' is a flow sequence from v to v' which is a path if the edge orientation is disregarded. A flow path is called either a directed flow path or an augmented flow path depending on whether the edge sequence is a directed path or not.

It is easily seen that an edge sequence is a flow path if (1) it is a path when the edge orientation is disregarded, and (2) it is composed of non-saturated forward edges and non-zero-flow backward edges.

Definition 3: A flow sequence is called saturated if its \mathcal{E} -increment no longer produces a flow increment.

We notice that a flow path is saturated if one of its forward edges is saturated and/or one of its backward edges has zero flow.

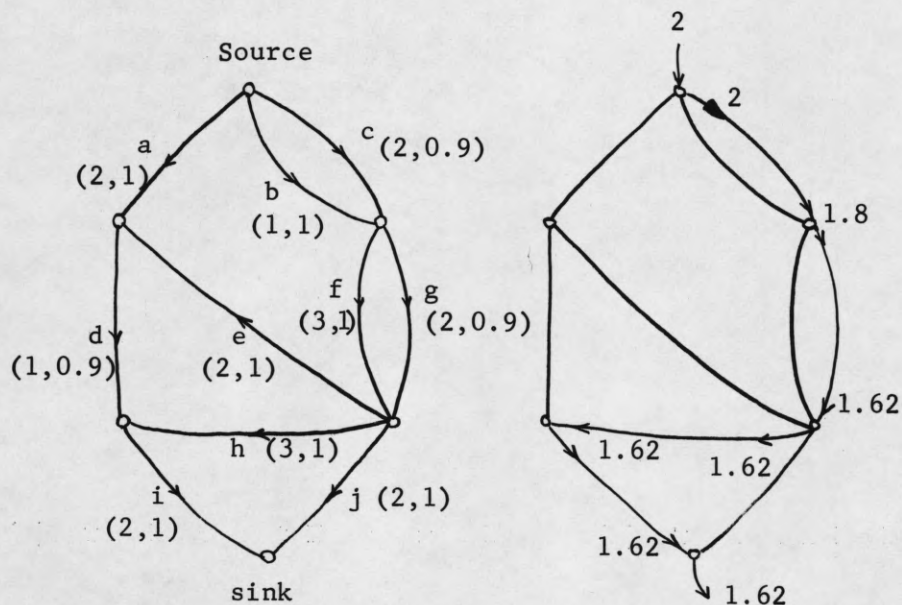
Definition 4: A flow circuit is a flow sequence of non-saturated forward edges and non-zero-flow backward edges, which is a circuit if the edge orientation is disregarded.

Definition 5: A flow return sequence is a flow sequence which starts from the source and returns to the source.

A flow increment Δ to a flow return sequence is defined exactly the same as for a flow increment to a flow sequence. The only exception is that additional flow is sent from the source and is ultimately returned to the source.

In order to make the definitions above more understandable we shall give examples of flow increments to flow sequences and flow return sequences.

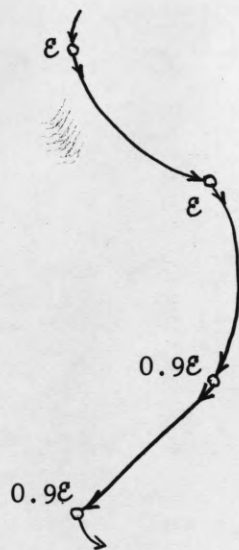
Example: The edge capacity and the edge efficiency factor of an edge in the net N are given by a pair of numbers: the first number is the edge capacity and the second number is the edge efficiency factor, see (A). The initial flow ψ_0 is shown in (B) and flow values at the end points of the edges are also given. Flow saturation is indicated by solid arrowheads.



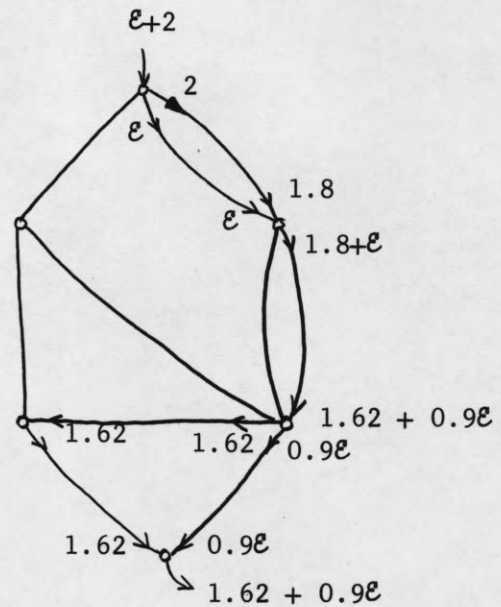
(A) The lossy net N for example

(B) The initial flow ψ_0

[1] (b, g, j) is a directed flow path.

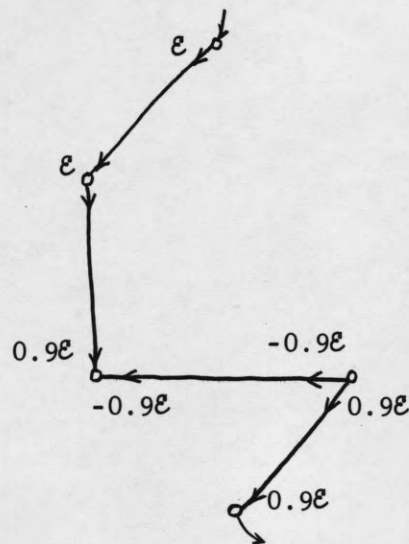


(C) ϵ -increment $\Delta' = \text{flow increment } \Delta$

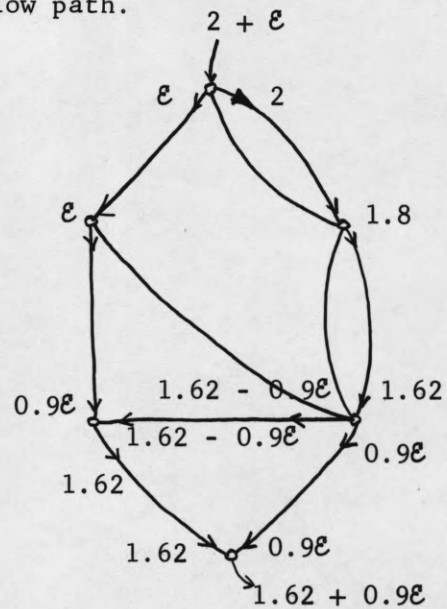


(D) Resultant flow $\psi_1 = \psi_0 + \Delta$

[2] (a, d, h, j) is an augmented flow path.

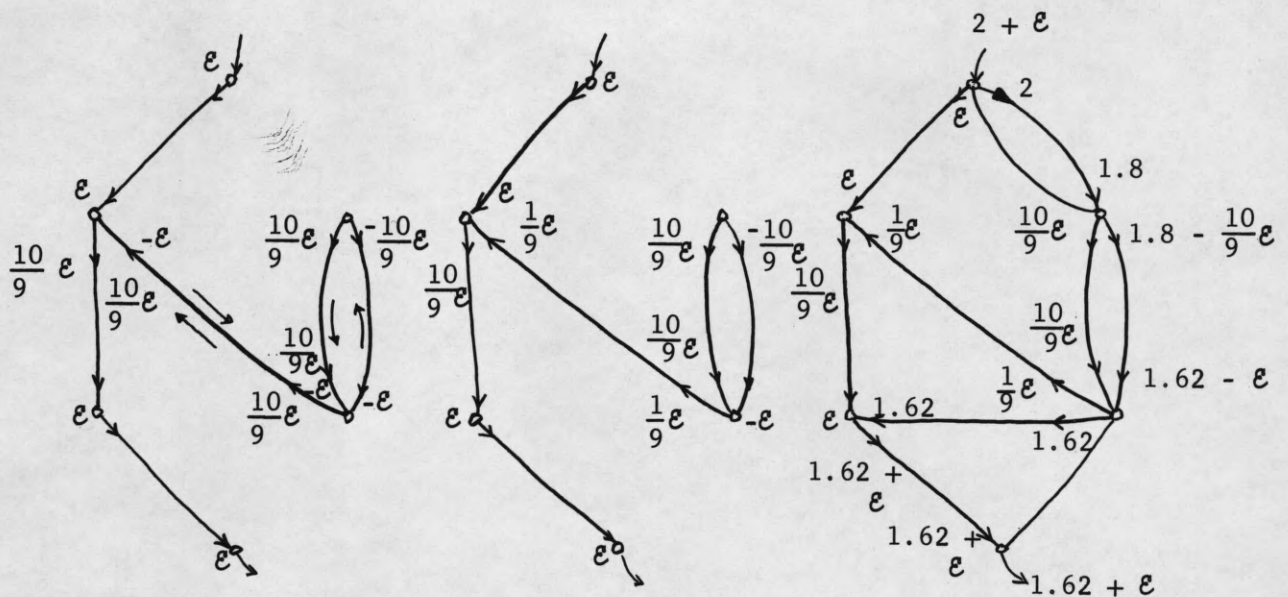


(E) ϵ -increment $\Delta' = \text{flow increment } \Delta$



(F) Resultant flow $\psi_1 = \psi_0 + \Delta$

[3] (a, e, g, f, e, d, i) is a flow sequence.



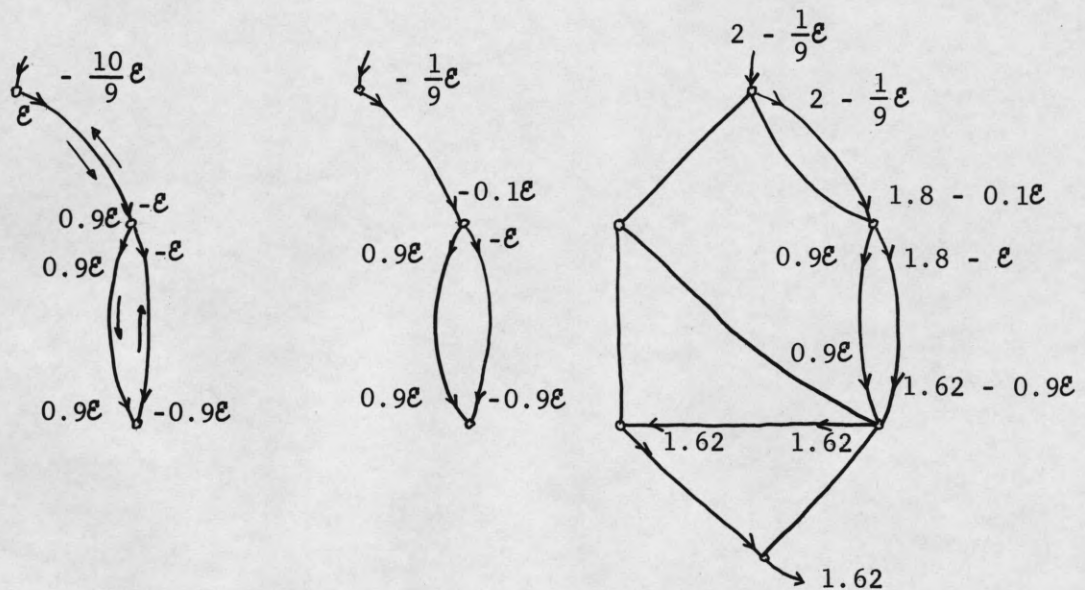
(G) ϵ -increment Δ'

(H) Flow increment Δ

(I) Resultant flow

$$\psi_1 = \psi_0 + \Delta$$

[4] (c, f, g, c) is a flow return sequence.



(J) ϵ -increment Δ'

(K) Flow increment Δ

(L) Resultant flow

$$\psi_1 = \psi_0 + \Delta$$

II. Characterization of the optimum flow

A flow ψ_0 with the sending flow value \bar{t} and the receiving flow value \underline{t} is said to be optimum if no other flows has less sending flow value than \bar{t} while having the same receiving flow value \underline{t} .

If a flow ψ_0 with \bar{t} and \underline{t} is already assigned to the net N and if an increment Δ' with sending flow value $\mathcal{E}(> 0)$ is assigned to a flow return sequence, then it is easily seen that the resultant flow $\psi_1 = \psi_0 + \Delta$ has the same receiving flow value \underline{t} as flow ψ_0 and that the sending flow value of ψ_1 is increased by amount $\mathcal{E} - \mathcal{E}'$, where \mathcal{E}' is amount of flow returned to the source.

Definition 6: The return ratio, r , of a flow return sequence is a ratio of the sending flow value \mathcal{E} of Δ assigned to the flow return sequence to the returning flow value \mathcal{E}' of Δ . That is $r = \mathcal{E}'/\mathcal{E}$.

If F is the set of forward edges in the flow return sequence and if B is the set of backward edges in the flow return sequence, then \mathcal{E}' is expressed as the product of \mathcal{E} and the efficiency factor product of the flow return sequence, that is

$$\mathcal{E}' = \mathcal{E} \cdot \prod_{e_m \in F} \rho_{e_m} \cdot \prod_{e_n \in B} \left(\frac{1}{\rho_{e_n}} \right).$$

Hence the return ratio r is equal to the efficiency product of the flow return sequence, $\prod_{e_m \in F} \rho_{e_m} \cdot \prod_{e_n \in B} \frac{1}{\rho_{e_n}}$.

We note that if the return ratio r of a flow return sequence is greater than unity, flow increment along the flow return sequence improves economy,

for the resultant flow has less sending flow value \bar{t}' ($\bar{t}' = \bar{t} + (1-r)\epsilon < \bar{t}$) and have the same receiving flow value \underline{t} . This observation leads us to the following theorem.

Theorem 1: A flow is optimum if and only if no flow return sequence has the return ratio greater than unity.

Proof: Necessity: Suppose that a flow ψ with the sending flow value \bar{t} and the receiving flow value \underline{t} has a flow return sequence whose return ratio is greater than unity. If amount of flow ϵ (a small positive number) is sent from the source along the flow return sequence, then amount of flow $r\epsilon$ will be returned to the source. Consequently the resultant flow possesses less sending flow value $\bar{t}' = \bar{t} - (r-1)\epsilon$ and the same receiving flow value \underline{t} , hence the flow ψ is not optimum.

Sufficiency:^{*} Let $P = \{p_1, p_2, p_3, \dots\}$ be the class of all directed pathes from the source to the sink. The flow ψ can be decomposed into the sum of directed path flows ψ_{p_i}

$$\psi = \sum_{p_i \in P} \psi_{p_i}.$$

Suppose that flow ψ with \bar{t} and \underline{t} has no return ratio greater than unity, and that there is a flow ψ' such that it has less sending flow value $\bar{\omega}$ ($\bar{\omega} < \bar{t}$) and the same receiving flow value $\underline{\omega}$ ($\underline{\omega} = \underline{t}$). The inequality $\bar{t} > \bar{\omega}$ implies that there is at least one directed path, p_m such that the sending flow value \bar{t}_m of ψ_{p_m} is greater than the sending flow value $\bar{\omega}_m$ of ψ'_{p_m} and consequently the receiving flow value \underline{t}_m of ψ_{p_m} is greater than $\underline{\omega}_m$ of ψ'_{p_m} . The equality $\underline{t} = \underline{\omega}$ and the inequality $\underline{t}_m > \underline{\omega}_m$ implies that there is at least one directed

*The proof of sufficiency in this form was suggested to the author by Professor W. Mayeda, Coordinated Science Laboratory, University of Illinois.

path p_n such that $\bar{t}_n < \bar{\omega}_n$ and $\underline{t}_n < \underline{\omega}_n$. Hence the classes, $D = \{p_m : \bar{t}_m > \bar{\omega}_m\}$ and $E = \{p_n : \bar{t}_n < \bar{\omega}_n\}$ are not empty. It is easily seen that an edge sequence which traverses p_n in forward direction from the source to the sink, and traverses p_m in backward direction to the source constitutes a flow return sequence of the flow ψ . Flow ψ can be transformed into flow ψ' by assigning flows along flow return sequences mentioned above until the classes D and E become empty. If amount of flow ϵ_j at the source is sent along j^{th} flow return sequence with return ratio r_j , then we have

$$\bar{\omega} = \bar{t} + \sum_j (1-r_j) \epsilon_j.$$

Since $\bar{t} > \bar{\omega}$, we must have $r_j > 1$ for some j . This is a contradiction to the hypothesis that the original flow ψ has no return ratio greater than unity. Therefore, the classes D and E must have been empty, that is, flow ψ must have been optimum. Q.E.D.

Corollary: A flow is optimum if and only if no flow circuit has the efficiency factor product greater than unity.

Proof: If the return ratio of a flow return sequence is greater than unity, then it must contain a flow circuit whose efficiency factor product is greater than unity. Since there always exists a directed flow path from the source to a node in the flow circuit, it is easy to see that the edge sequence which traverses the directed path from the source to the flow circuit, circles the flow circuit and returns to the source via the directed path constitutes a flow return sequence if the flow circuit has the efficiency factor product greater than unity. It is obvious that the return ratio is greater than unity. Q.E.D.

III. A Dynamic Programming of Optimum Flows

Theorem 1 or its corollary provides a good criterion for checking the optimality of a given flow, but they do not offer a constructive procedure to achieve optimum flows. A flow assignment algorithm which yields optimum flows successively, (a dynamic programming), is achieved by the following Theorem.

Theorem 2: Suppose that an optimum flow ψ_0 has been assigned to the net N. If flow increment Δ with sending flow value $\ell > 0$ is assigned to a flow path p which has the largest efficiency factor product in the net N with flow ψ_0 , then the resultant flow $\psi_0 + \Delta$ is optimum.

Proof: Suppose that $\psi_0 + \Delta$ were not optimum. By the corollary to Theorem 1, there exists a flow circuit, 0, whose efficiency factor product is greater than unity. Since the subnet 0 is not a flow circuit of ψ_0 , the flow path p must interfere with 0 in such a manner that 0 becomes a flow circuit of $\psi_0 + \Delta$. One example is presented in Fig. 2 where the saturated forward edges of 0 become unsaturated and the zero-flow backward edges become non-zero-flow edges. Suppose that the flow path p encounters the flow circuit 0 at node x_1 for the first time and ultimately leaves the flow circuit 0 at node x_{2k} , and that p interferes 0 with flow sequences $s_1, s_3, s_5, \dots, s_{2k-3}$, and s_{2k-1} . Let a part of the flow path p be

$$q' = (x_1, s_1, x_2, s_2', x_3, s_3, \dots, s_{2k-2}', x_{2k-1}, s_{2k-1}, x_{2k})$$

and let the flow circuit 0 be divided into flow sequences $s_1, s_2, s_3, \dots, s_{2k-1}$ and s_{2k} .

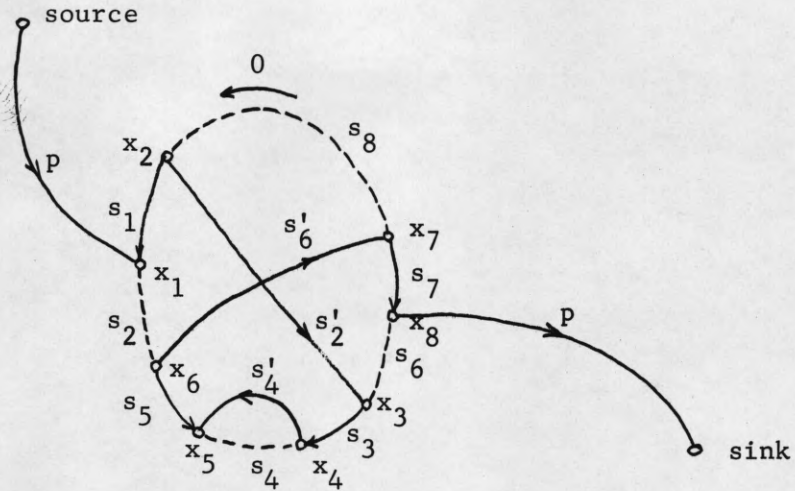


Fig. 2 Interference between a flow path p and a flow circuit 0 . $\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 \alpha_6 \alpha_7 \alpha_8 > 1$,

$$q'' = \{x_1, s_2, x_6, s'_6, x_7, s_8, x_2, s'_2, x_3, s_6, x_8\}$$

If we designate the efficiency factor product of the edge sequence s_i by α_i , we can express the efficiency factor products, K and π , of the flow circuit 0 and a flow path q' , respectively by

$$K = \alpha_1 \alpha_2 \alpha_3 \dots \alpha_{2k} > 1$$

$$\pi = \frac{\alpha'_2 \alpha'_4 \dots \alpha'_{2k-2}}{\alpha_1 \alpha_3 \dots \alpha_{2k-1}}$$

Hence multiplying π to both sides of $K > 1$, we get $K\pi > \pi$

$$\alpha_2 \alpha'_2 \alpha_4 \alpha'_4 \dots \alpha_{2k-2} \alpha'_{2k-2} \alpha_{2k} > \frac{\alpha'_2 \alpha'_4 \dots \alpha'_{2k-2}}{\alpha_1 \alpha_3 \dots \alpha_{2k-1}}.$$

Now delete the interfering edge sequences $s_1, s_3, s_5 \dots s_{2k-1}$ from the subnet $0 + q'$. At every node except x_1 and x_{2k} , exactly one flow sequence enters and leaves the node, hence there is a flow path q'' from node x_1 to node x_{2k} in the new subnet $0 + q' - \{s_1, s_3, s_5, \dots, s_{2k-1}\}$. If there are flow sequences not employed in q'' , they constitute a union of flow circuits which do not touch the flow path q'' . The efficiency factor product of the union of flow circuits must be less than unity by optimality hypothesis of the given flow ψ_0 . Hence

$$(\text{the efficiency factor product of } q'') \geq \alpha_2 \alpha'_2 \dots \alpha_{2k-2} \alpha'_{2k-2} \alpha_{2k} >$$

(the efficiency factor product of q').

Therefore our conclusion must be that there is a flow path p' whose efficiency factor product is greater than that of the original flow path p . This is a contradiction to the maximality assumption of the efficiency factor product of p . Q.E.D.

IV. An Illustrative Example

The dynamic programming for obtaining optimum flows from v to v' in the given lossy net, [A] Fig. 3 is demonstrated in this section. The edge capacity and the edge efficiency factor are given for each edge by a pair of two numbers: the first number is the edge capacity and the second number is the edge efficiency factor. The edge orientation is indicated by an arrowhead. The dynamic programming is performed in [B], Fig. 3.

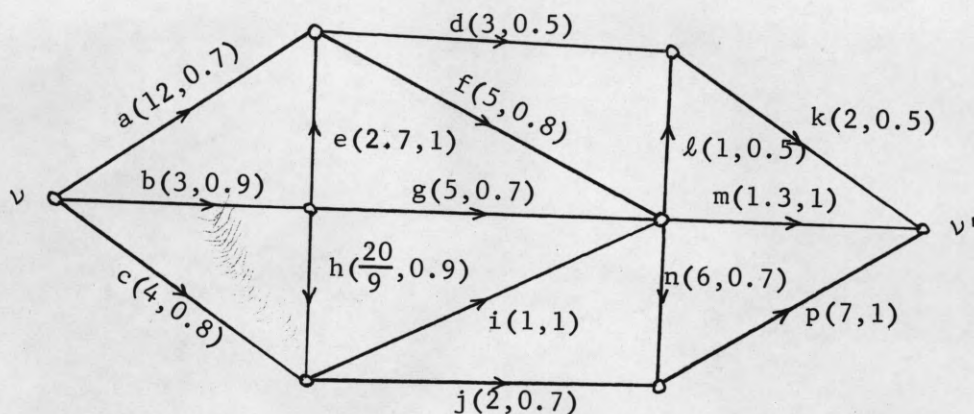
Program 1: Assign flow along a flow path (b, h, i, m) , since the flow path has the largest efficiency factor product (if there exist several such flow pathes, choose one of them). The flow path (b, h, i, m) is labeled by 1 in [B].

Program 2: When edge i becomes saturated, assign flow along the next flow path (b,e,f,m) which has the second largest efficiency factor product and is labeled by number 2 in $[B]$. The flow saturation is indicated by a solid arrowhead.

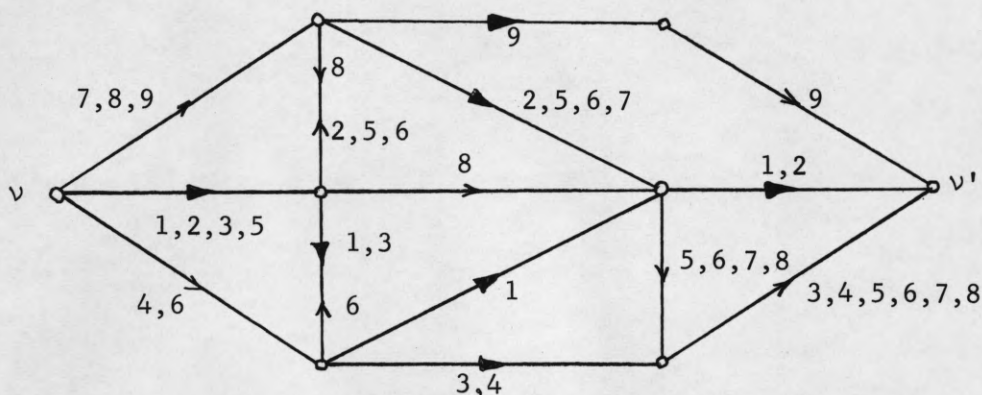
Program 3: When edge m becomes saturated, assign flow along the next flow path (b,h,j,p) which has the third largest efficiency factor product and is labeled by number 3 in $[B]$, until edge h is saturated.

Program 4, 5, 6, 7, 8, 9 can be stated similarly and the program will halt at program 10 when there is no flow path from v to v' . Consequently we will have an optimum maximum flow, see $[C]$ Fig. 3.

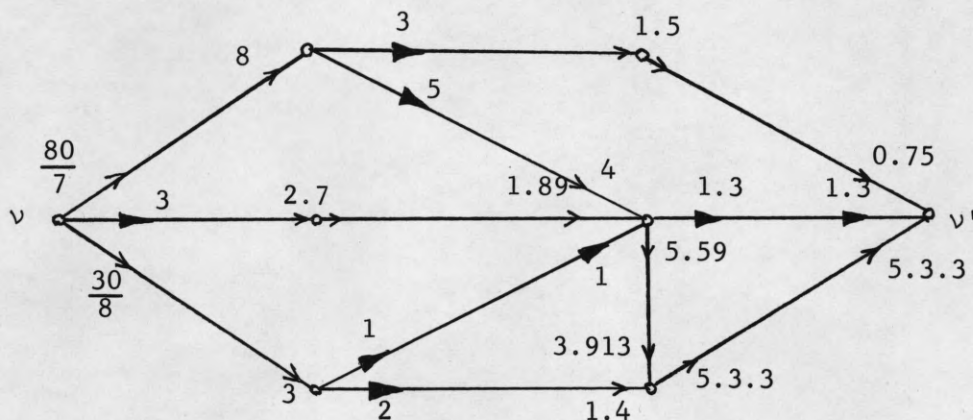
The result of the dynamic program for optimum flows from v to v' is summarized in Table 1 and the curve of the sending flow value \bar{t} versus the receiving flow value \underline{t} of optimum flows is depicted in Fig. 4.



[A] The lossy net for illustrating the dynamic programming of optimum flows from the source v to the sink v' .



[B] The dynamic program of optimum flows from v to v' .



[C] The optimum maximum flow from v to v' .

Fig. 3 The Illustrative Example

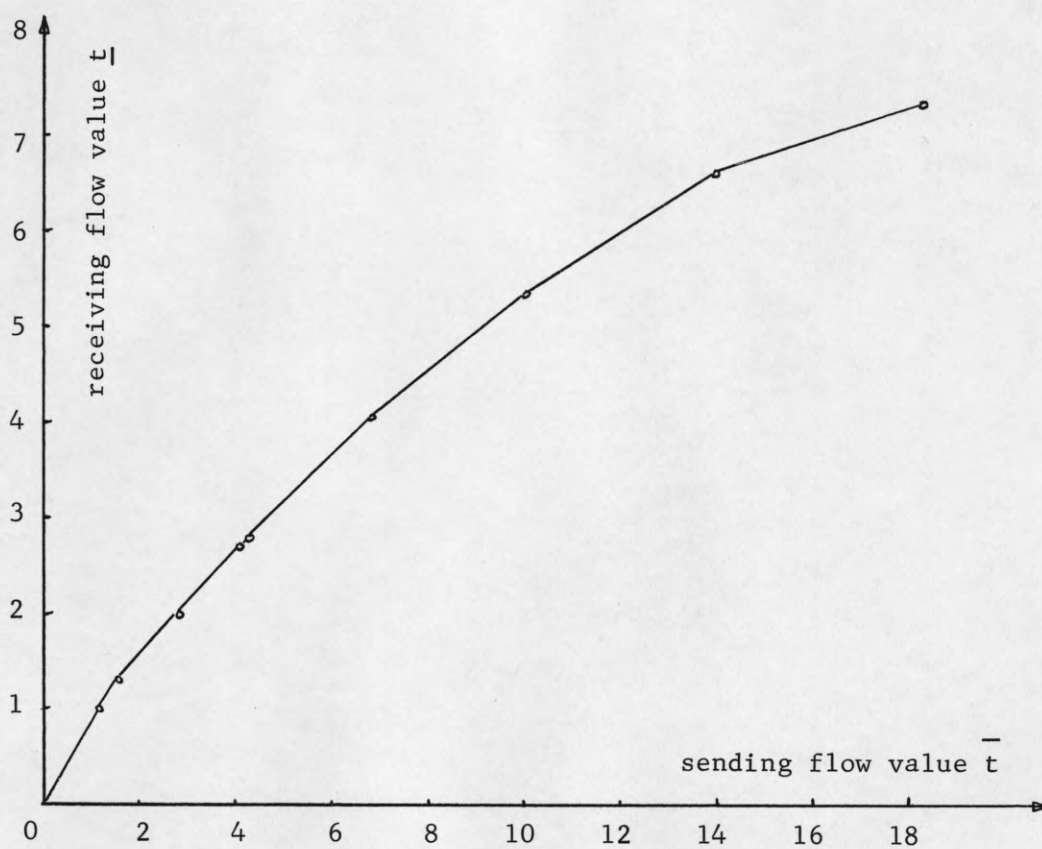


Fig. 4 The curve of the sending flow value - receiving flow value of optimum flows from ν to ν'

Table 1. The Dynamic Program for Optimum Flows from v to v'

Program No.	Flow Path	Eff. Factor Product	Sending Value \bar{t}_m	$\Sigma \bar{t}_m$	Receiving Value t_m	Σt_m
1	(b,h,i,m)	0.81	1.2	1.2	1.0	1.0
2	(b,e,f,m)	0.72	0.45	1.65	0.3	1.3
3	(b,h,j,p)	0.567	1.17	2.82	0.7	2.0
4	(c,j,p)	0.56	1.30	4.12	0.7	2.7
5	(b,e,f,n,p)	0.504	0.11	4.23	0.117	2.817
6	(c,h,e,f,n,p)	0.498	2.52	6.75	1.253	4.16
7	(a,f,n,p)	0.392	3.28	10.03	1.288	5.348
8	(a,e,g,n,p)	0.343	3.87	13.9	1.265	6.613
9	(a,d,k)	0.175	4.28	18.18	0.75	7.363
10	none	program	halts			

V. Conclusions

Characterization and a dynamic programming of optimum flows in lossy communication nets have been obtained and illustrated. Theorem I and 2 have a striking similarity with the counterpart of the minimum transportation cost problem of flows in lossless communication nets [1], [3]. In fact, if we regard flows in the net as flows of the values of the commodity rather than as flows of the commodity itself, then the loss factor of an edge, $(1-\rho)$, is equivalent to the edge cost factor of transportation, provided that the value of a unit commodity is 1. In the case of a direct path (e_1, e_2, \dots, e_k) the total cost for flow ψ due to transportation is the sum of the edge costs $C_i \cdot \psi$ of the path, $\psi \cdot (\sum C_i)$, while the cost due to loss of flow is equal to $(1-\rho_1)\psi + (1-\rho_2)\rho_1\psi + \dots + (1-\rho_k)\rho_1\rho_2\dots\rho_{k-1} \cdot \psi = \psi \cdot (1-\rho_1\rho_2\dots\rho_k)$.

One of the important future problems is to obtain analytical properties and dynamic programmings of flows that minimize the cost due to transportation and loss of flows, for the fixed receiving flow quantity \underline{t} .

Acknowledgment

The author wishes to acknowledge the helpful discussions with Professor W. Mayeda, University of Illinois.

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Footnote

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1. Here we are paying a particular attention to edge sequences of the circuit 0 encountered by the path p in the reverse direction with respect to the circuit orientation. Even when s'_{2n} and s_{2m} (where $n, m = 1, 2, \dots, k-1$) share some edges in the same direction, the proof would be affected little because s'_{2n} and s_{2m} can be considered edge disjoint in the context of the proof.

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<p>A flow ψ with the sending flow value \bar{t} at the source and the receiving flow value \underline{t} at the sink is said to be optimum if there is no flow which has less sending flow value than \bar{t} while having the same receiving value \underline{t}. A necessary and sufficient condition of optimality is obtained. Based on this characterization a dynamic programming of optimum flows in a lossy communication net is devised and demonstrated by an example.</p>			

KEY WORDS	LINK A		LINK B		LINK C	
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<p>Optimum flows</p> <p>Lossy communication nets</p>						

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