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DECENTRALIZED STACKELBERG STRATEGIES FOR INTERCONNECTED STOCHASTIC DYNAMIC SYSTEMS

SUVALAI PRATISHTHANANANDA GLANKWAMDEE

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by

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TABLE OF CONTENTS

1. INTRODUCTION

1.1 Introduction

A multi-level structure for a large scale system appears rather naturally in practice. It is the consequence of an effort toward efficient utilization of the available resources or the inherent limitations of the elements out of which the system is built. An interconnected power system provides an important example of a class of large-scale systems.

A significant development in large-scale system theory is the concept of multi-person stochastic games with nonclassical information patterns and their implications on decentralized and hierarchical control strategies [1,4,12-15,42,43,45,60,62]. It is evident that a theory of coordination using the bargaining approach [15] is an important and interesting avenue for new research.

The main object of this thesis is to investigate Stackelberg coordination for decentralized stochastic control. A strong motivation for this study is its potential application to decentralize control problems such as those found in an interconnected power system which can be described as a collection of subsystems, each of which is called a control area. Each area is responsible for meeting its obligation to maintain the appropriate system frequency and supply its own load demand. Also, each area provides mutual assistance to its neighbours in accordance with the basic operating policy of interconnected power systems [23]. When the interconnected network is small centralized techniques can be used quite effectively [19,20,23,25,33,39,57]. However, in the more general case the communications/computational costs involved in implementing a centralized controller often become prohibitive and decentralized of some sort becomes essential.

We will investigate both the theoretical framework and a potential practical application of Stackelberg coordination for decentralized stochastic control of general organizational forms of large scale system. These systems may be controlled by multiple decision makers having different models, different information sets and different objective functionals. Our approach will be based on differential games [16-18,47-53], stochastic control [2-3,5,30,37,38,42,54,63,64] and electric power system control [19,20,23,25,33,39,57]

1.2 Literature Survey

The design of large, complex systems invariably involves decomposition of the system into a number of smaller subsystems each with its own objective functions and constraints [40]. The resulting interconnection of subsystems may take on many forms, but one of the most common is the hierarchical form in which a given level subsystem controls or coordinates the subsystems on the level below it and in turn is controlled by the subsystems on the level above it. The information available to a subsystem on a given level and the way such a subsystem can make use of the information to influence or control another subsystem has been the object of much study.

Decentralized information among decision makers was first studied in the static team theory of Radner [44]. For the dynamic cases, H.S. Witsenhausen [61-63] was the first who showed that the linear quadratic Gaussian problem is nontrivial when the information pattern is nonclassical. Chong and Athans [16] imposed constraints on the control structure of the LQG system having different information sets. They showed that the parameter matrices of each dynamic controller could then be globally optimized by solving a deterministic matrix optimal control problem. Ho and Chu [13,27] have demonstrated that certain nonclassical stochastic control problems admit a linear solution. Sandell and Athans [45] have shown that LQG problems with a unit time delay of information exchange admit a linear optimal decision rule, which can be calculated explicitly. The results appeared to be promising as far as their applicability to decentralized control theory is concerned. With decentralized information, there is a trade-off between information efficiency and computation efficiency. Chong and Athans [14] assumed that the "coordinator" was allowed to "interfere" only once in a while. When the coordinator is acting open-loop the lower level problems can be decomposed completely.

Although different infomation sets are available to each controller, there is cooperation among the different controllers because they all try to minimize the same cost functional in the framework of team theory. This type of a situation can be described as the "cooperative and partially decentralized case" in large scale system theory.

It appears that a theory of coordination using the bargaining approach could also be developed using the same framework. It certainly represents an important and interesting avenue for new research. This has not been attempted until very recently. Cruz [15] proposed the extension of Stackelberg strategies to the coordination of several subsystems.

One could naturally expect that game theory is of considerable use in bargaining. In fact, game theory has already been used to study bargaining type situation between organizations in an economy or a sociaty [58,59]. The idea of using control theory to solve games with dynamic evoluation was initiated by Isaacs [29]. The games Isaacs studied were primarily deterministic zero-sum games. Later a more general concept of differential games known as the theory of N-player differential games has been introduced. Starr and Ho [47,48] considered non-zero sum differential games with solution concepts or rationales such as Nash, Pareto and minimax in a dynamic sense. The concepts of closed-loop and open-loop solutions were adapted from modern control theory to dynamic game theory, and relates to the class of admissible strategies. In particular, interest has been focused on the determination of Nash equilibrium strategies for deterministic linear-quadratic nonzero-sum differential games with dynamic information structures [45-46]. Most of the equilibrium solutions found in the literatures for such games have been linear in the information available to each player. Only Recently, T. Basar [8] has shown, via a counterexample, that when at least one of the players has access to closed-loop information, such games admit non-unique and nonlinear equilibrium solutions. Recently, Cruz et al. [16,49-53] have introduced the Stackelberg strategy developed in static games [58] to dynamic games. The feedback Stackelberg solution concept [17] has been extended to a class of stochastic games by Castanon and Athans [18].

The theory of stochastic dynamic games is based on the works of Witsenhausen [61-64], but earlier, Rhodes and Luenberger [42,43], and Behn and Ho [4] considered the problem of zero-sum dynamic games with imperfect information. The restriction of the transfer of information through decision was discussed by Aoki [1] while considering equilibria in Nash games.

Interconnected electric energy systems provide an important example of a class of large-scale systems. In several papers [19,20,23,25,33,39,57], attempts have been made to analyze the load frequency controller of an interconnected power system via modern optimal control theory. Since the solution proposed by Elgerd [25] is based on the standard linear regulator theory for disturbance free dynamic systems, it neither eliminates the steady-state errors of frequency and tie-line flows, caused by system load disturbance, nor provides the desired generation distribution. However, a resonable dynamic model was given. A new design procedure for load and frequency control was developed later by Calovic [19,20] which avoids all the short comings of previous solutions. The procedure used is to adjoin the integral of each area control error (ACE) to the system state variables. These new state variables as well as the original system state variables are included in the cost functional. As a result, all areas capable of doing so will drive their area control errors to zero in steady-state provided the system is stable. Recently, Kwatny [33] suggested that when energy source response limitations are recognized, the load frequency control (LFC) problem should be viewed as a "tracking" problem rather than a "regulator" problem. The estimation and prediction of load are used to coordinate generation in each area so as to regulate power flows and frequencies.

1.3 Problem Area and Methodology

The coordination of a large scale system, which has the following characteristics [15]: 1. two or more decision makers having different models, 2. different information sets available to the decision makers, and 3. different objective functionals, using differential games approach represents an important and interesting area for research. We will investigate, in details, Stackelberg Strategies for multilevel systems. The leader who acts as a coordinator and other decision makers who are viewed as followers assume different models of the same system. Several classes of information structures available to the decision makers will be discussed.

First, we consider an interconnection of M discrete-time linear stochastic subsystems and associate with each subsystem a decision-maker, a quadratic performance criterion, and a linear noisy measurement. Superimposed on the interconnection is an addition decision maker called the coordinator acting through an additional discrete-time linear stochastic subsystem, with a separate quadratic performance criterion and a separate linear noisy measurement. The coordinator is viewed as a leader and the other decision-makers as followers assuming Nash rationale or Pareto rationale among themselves. The Stackelberg equilibrium strategy [17] is extended to fit this situation when there is one leader and many followers.

When all decision makers have perfect system measurement, or when all the information of all the followers are identical and the coordinator's information contains the followers' information, feedback control structure will be sought based on the stochastic Stackelberg equilibrium strategy [18]. The following special cases will also be examined: 1. when the coordinator has perfect measurement and all the followers have indentical noisy measurement, and 2. when the coordinator has noisy measurement and all the followers have no measurement.

The classes of information structure are not too realistic but they provide some insight into the more complex and realistic cases treated subsequently. Satisfactory control of a high order system may often be achieved using relatively few measurements and a controller of relatively low order. This has been the motivation for a number of design procedures using output feedback or dynamic controllers of a specified order [31,32,38,54,63,64]. Although, the assumption of linearity in the class of instantaneous feedback control laws might lead to results far from optimal which was pointed out by Witsenhausen [61] and Basar [8], the practical need for simplifying approximations becomes more acute in decentralized control when there are many separate controllers. Decentralized Stackelberg strategies which are constrained to be linear dynamic controllers of specified orders, will be determined. This control policy has the obvious advantage of being structurally simpler to implement since it does not require memory of past meausurements. However, there exist, at present, no stability results for this algorithm.

Finally, decentralized Stackelberg strategies will be used to develop a decentralized controller for a three-area electric power system. This design procedure meets all the performance requirements of load and frequency control, i.e. control law independent of disturbance, zero steady-state offsets of frequency and tie-lie exchange variations and optmal transient performance. The dynamic model, developed by Elgerd and Fosha [25] and Calovic [19] will be used. To overcome the problem of zero steady state offsets of frequency and tie-lie exchange variations, the integral of each area control error (ACE) is adjoined to the system equations. These new state variables are included in the cost functional. So as Stackelberg decentralized are concerned, each control area is constrained to feedback only its own measurement and they have their own choice of cost functional. The area which has superiority in computing his strategy/collecting information, will be declared as a coordinator who coordinates the other areas which are viewed as followers. When the lower-level subsystems desire to cooperate among themselves a Pareto optimal solution will be chosen, otherwise Nash equilibrium solution will be chosen. The algorithm for obtaining decentralized controllers is developed and applied to load-frequency control of interconnected power systems. The computational algorithm suggested can not guarantee satisfactory results. However, in practice the algorithm has exhibited rapid convergence.

1.4 Organization of the Work

In Chapter 2, three important strategies in Games theory, i.e. Nash equilibrium, Pareto optimal and Stackelberg equilibrium are discussed. The necessary conditions for the three strategies applied to a linear quadratic Gaussian discrete game are reviewed.

Chapters 3 and 4 deal with Stackelberg coordination. Centralized and decentralized information structure are studied in this context. Decentralized structure is more attractive since the control sequences are function of the measurable output only. The general approach is to designate one subsystem to be a coordinator or leader who coordinates the rest of the subsystems who are viewed as followers. Among the followers a Pareto optimal or Nash equilibrium solution is selected according to their decisions to cooperate or not. These concepts along with the solutions are derived.

In Chapter 5, the algorithm to solve the decentralized stochastic Stackelberg coordination suggested in Section 3.4 is investigated further. A three-area interconnected power system, which is a class of large scale system, is selected as our example. The design procedure emphasizes proportional-plus-integral feedback control. A simulation study is presented.

2. LINEAR QUADRATIC DIFFERENTIAL GAMES

2.1 Introduction

In this chapter, some important aspects of nonzero-sum games that are pertinent to this work are reviewed. We will consider a special class of differential games, where the system is linear and the cost functions are quadratic functions of the state vectors and controls, which is probably the only non-trivial class of differential games in which solutions based on any rationale can be obtained analytically without difficulty.

In differential games, one must choose a solution concept such as, Nash equilibrium, noninferiority, Stackelberg equilibrium etc.. One must also specify what information is available to each player during the course of the game. Extensive work has been done on deterministic nonzero-sum differential games with particular emphasis given on two-person games of linear quadratic form [35,36,47-53]. Results available in the literatures indicate that the solutions of interest, Nash equilibrium, Pareto equilibrium and Stackelberg equilibrium, for this class of games, and for different a priori fixed strategy spaces, is an affine policy for each player, provided that certain existence conditions are satisfied. T. Basar [8] has given a counterexample to show that a two-person nonzero-sum game problem admits a nonlinear Nash solution. He has also shown that it is possible to obtain a robust solution which is globally unique by including an additive zero mean white noise in the state dynamics. To present the idea without loss of conceptual generality a two-person stochastic nonzero-sum game with perfect measurement is considered. Three types of strategies are reviewed, the Nash equilibrium strategy, the Stackelberg equilibrium strategy and the Pareto optimal strategy.

2.2 Problem Formulation

A general formulation of the two-person discrete-time linear quadratic stochastic differential game is given as follows:

$$x(k+1) = Ax(k) + Bu(k) + Cv(k) + w(k) \qquad (2.1)$$

$$y^1(k) = H^1x(k) + \omega^1(k) \qquad (2.2)$$

$$y^2(k) = H^2x(k) + \omega^2(k) \qquad (2.3)$$

where x(k) is the n-dimensional state vector, u(k) is the m-dimensional control vector of player 1, v(k) is the l-dimensional control vector of playeyr 2, $y^i(k)$ is the p^i-dimensional measured output vector for the i-th player. The vector w(k), $\omega^i(k)$ and x(0) are independent Gaussian random vectors for all k, where $x(0) = N(0,X(0))$; $w(k) = N(0,\Phi(k))$; $\omega^i(k) = N(0,\Omega^i(k))$. Each player i chooses a control vector from a set of admissible control U^i to minimize the expected value of cost function J^i, where

$$J^i(x,u,v,k) = x^T(N)Q^i(N)x(N) + \sum_{k=o}^{N-1}[x^T(k)Q^i(k)x(k)$$
$$+u^T(k)R^i(k)u(k)+v^T(k)S^i(k)v(k)] \quad i=1,2 \quad (2.4)$$

Because there are more than one cost functional in differential games, optimality is defined in terms of the rationality assumed by the players in computing their controls. The most commonly known rationale are the Nash, Pareto and Stackelberg solutions which are reviewed in the following section. These are discussed in detail in [35,36,47,53].

At each stage of the game, each player will have access to some information I^i about the present and/or past value of the state vector, its own cost function as well as those of the other players, and control strategies of the other players. Each player i has a control strategy which is a mapping from the information set I^i to the control space U^i.

2.3 Nash Equilibrium Strategy

The Nash equilibrium strategy which is secure against unilateral deviations by any one player, depends on what information is available to the players during the course of play: for example, the 'closed-loop' and 'open-loop' assumptions lead to entirely different costs and controls. It is important to indicate that all the cost function mappings are included in each information I^i. Furthermore, all players' decisions are announced simultaneously. The Nash equilibrium strategy is reasonable when cooperation or coalition can not be guaranteed and the information structure is as stated above.

In this section, we review the necessary conditions for obtaining Nash equilibrium strategies for discrete-time dynamic games (2.1) with perfect information, i.e. $y^i(k) = x(k)$, via dynamic programming.

At stage k:

$$u^*(k) = \arg\min_{u(k)} E\{x^T(k)Q^1(k)x(k) + u^T(k)R^1(k)u(k) + v^{*T}(k)S^1(k)v^*(k) + J^{1*}(k+1)/I^1(k)\} \quad (2.5)$$

$$v^*(k) = \arg\min_{v(k)} E\{x^T(k)Q^2(k)x(k) + u^{*T}(k)R^2(k)u^*(k) + v^T(k)S^2(k)v(k) + J^{2*}(k+1)/I^2(k)\} \quad (2.6)$$

When $u^*(k)$ and $v^*(k)$ satisfy (2.5) and (2.6) simultaneously, a pair $(u^*(k), v^*(k))$ constitutes a Nash equilibrium solution. The Nash optimal strategies for (2.1) are:

$$u^*(k) = -\triangle^1(k)x(k) \quad (2.7)$$

$$v^*(k) = -\triangle^2(k)x(k) \quad (2.8)$$

where

$$\triangle^1(k) = [R^1+K^1B]^{-1}K^1A$$

$$\triangle^2(k) = [S^2+K^2C]^{-1}K^2A$$

$$K^1 = B^TP^1(k+1)[I-C(S^2+C^TP^2(k+1)C)^{-1}C^TP^2(k+1)] \quad (2.9)$$

$$K^2 = C^TP^2(k+1)[I-B(R^1+B^TP^1(k+1)B)^{-1}B^TP^1(k+1)] \quad (2.10)$$

The optimal cost-to-go are

$$J^{1*}(k) = x^T(k)P^1(k)x(k) + \pi^1(k) \quad (2.11)$$

$$J^{2*}(k) = x^T(k)P^2(k)x(k) + \pi^2(k) \quad (2.12)$$

where

$$P^1(k) = Q^1 + \triangle^{1T}R^1\triangle^1+\triangle^{2T}S^1\triangle^2 + (A-B\triangle^1-C\triangle^2)^TP^1(k+1)(A-B\triangle^1-C\triangle^2) \quad (2.13)$$

$$P^1(N) = Q^1(N)$$

$$\pi^1(k) = \pi^1(k+1) + tr\{\Phi(k)P^1(k+1)\}; \quad \pi^1(N)=0 \quad (2.14)$$

$$P^2(k) = Q^2 + \triangle^{1T}R^2\triangle^1+\triangle^{2T}S^2\triangle^2 + (A-B\triangle^1-C\triangle^2)^TP^2(k+1)(A-B\triangle^1-C\triangle^2) \quad (2.15)$$

$$P^2(N) = Q^2(N)$$

$$\pi^2(k) = \pi^2(k+1) + \mathrm{tr}\{\Phi(k)P^2(k+1)\};\ \pi^2(N)=0 \qquad (2.16)$$

These equation can be solve backwards in time using the given final conditions. Sufficient conditions for the existence of the solution given by T. Basar [10], is that $[R^1+K^1B]$ and $[S^2+K^2C]$ are non singular.

2.4 Pareto Optimal Strategy

If it is possible for all players in a differential game to agree, prior to the starting time, to coordinate their strategies, then the resulting set of control should be chosen from the Pareto set of solutions. No other feasible choice of controls could decrease the costs incurred by one or more players without increasing the costs incurred by the others. The selection of a particular solution in the Pareto set is generally made subjectively based upon negotiation among the players. Finding the Pareto set for a differential game is equivalent to solving an optimal control problem with a vector cost function. When appropriate convexity conditions are satisfied [47,48] the problem is equivalent to solving an N-1 parameter family of optimal control problems with scalar cost criteria

$$\begin{aligned} J &= \sum_{i=1}^{2} \alpha_i J^i \\ &= \sum_{i=1}^{2} \alpha_i [x^T(N)Q^i(N)x(N)+\sum_{K=0}^{N-1} x^T(k)Q^i(k)x(k) \\ &\qquad +u^T(k)R^i(k)u(k)+v^T(k)S^i(k)v(k)] \end{aligned} \qquad (2.17).$$

$$\alpha_i \geqslant 0,\ \sum_{i=1}^{2} \alpha_i = 1$$

The components of α are interpreted as the relative weights placed on the interests of the players entering the agreement. For any given weighting vector α , the Pareto optimal solution is found by solving a linear quadratic optimal control problem. The controls corresponding to this solution are:

$$U^*(\alpha) = -[R+D^TP(k+1)D]^{-1}D^TP(k+1)Ax(k) \tag{2.18}$$

where

$$D = [B \quad C] ; \qquad U = \begin{bmatrix} u \\ v \end{bmatrix}$$

$$Q = \sum_{i=1}^{2} \alpha_i Q^i ; \qquad R = \begin{bmatrix} \sum_{i=1}^{2} \alpha_i R^i \\ \sum_{i=1}^{2} \alpha_i S^i \end{bmatrix}$$

$$P(k) = Q + K^TRK + (A-DK)^TP(k+1)(A-DK) \tag{2.19}$$

$$P(N) = Q(N)$$

A sufficient condition for the solution to be exist is that the matrix to be inverted is positive definite. These equations can be solved backwards in time using given final conditions.

The cost-to-go incurred when the players use arbitrary linear feedback control of the form

$$u_i(k) = K_i x(k) \qquad i=1,2 \tag{2.20}$$

$$J^i(k) = x^T(k)P^i(k)x(k) + \pi^i(k) \tag{2.21}$$

where

$$P^i(k) = Q^i + K^{iT}R^iK^i$$
$$+ (A+BK^1+CK^2)^TP^i(k+1)(A+BK^1+CK^2) \quad (2.22)$$

$$P^i(N) = Q^i(N)$$

$$\pi^i(k) = \pi^i(k+1) + trP^i(k+1)\Phi(k);\ \pi^i(N)=0 \quad (2.23)$$

2.5 Stackelberg Equilibrium Strategy

In this section we consider two-person games, where one player is called the leader and the other is called the follower. In the Stackelberg solution concepts there is a difference in information between two players. The leader, who acts first, knows the cost function mapping of the follower but the follower may or may not know the cost function mapping of the leader. However, the follower, who acts second, knows the value of the first player's decisions and take this into account in computing his strategy. Within the dynamic game context, three types of solution concepts are important in Stackelberg games: open-loop, closed-loop and equilibrium solutions. In this thesis we consider only the Stackelberg equilibrium strategy which satisfies the principle of optimality. For a discrete time system (2.1) with perfect information, u(k) represents the decision of the leader, v(k) the decision of the follower. Using dynamic programming at stage k,

$$v_o(u,k) = \arg\min_{v(k)} E\{x^T(k)Q^2(k)x(k) + u^T(k)R^2(k)u(k)$$
$$+ v^T(k)S^2(k)v(k)+J^{2*}(k+1)/I^2(k)\} \quad (2.24)$$

$$u^*(k) = \underset{u(k)}{\arg\min}\, E\{x^T(k)Q^1(k)x(k) + u^T(k)R^1(k)u(k) + v_o(k)S^1(k)v_o(k) + J^{1*}(k+1)/I^1(k)\} \quad (2.25)$$

$v_o(u,k)$ is the follower's optimal reaction to a decision $u(k)$ by the leader. The optimal strategies are:

$$u^*(k) = -W^{-1}(k)Y(k)x(k) \quad (2.26)$$

$$v_o(u,k) = -\Delta(k)[Ax(k)+Bu(k)] \quad (2.27)$$

where

$$W(k) = R^1 + B^T\Delta^T S^1\Delta B + B^T(I-C\Delta)^T K^1(k+1)(I-C\Delta)B$$

$$Y(k) = B^T\Delta^T S^1\Delta A + B^T(I-C\Delta)^T K^1(k+1)(I-C\Delta)A$$

$$\Delta(k) = [S^2+C^T K^2(k+1)C]^{-1}C^T K^2(k+1)$$

$$L(k) = Q^1 + A^T\Delta^T S^1\Delta A + A^T(I-C\Delta)^T K^1(k+1)(I-C\Delta)A$$

The optimal cost to go are

$$J^{1*}(k) = x^T(k)K^1(k)x(k) + \pi^1(k) \quad (2.28)$$

$$J^{2*}(k) = x^T(k)K^2(k)x(k) + \pi^2(k) \quad (2.29)$$

where

$$K^1(k) = L(k) - Y^T(k)W^{-1}(k)Y(k) \quad (2.30)$$

$$K^1(N) = Q^1(N)$$

$$\pi^1(k) = \pi^1(k+1) + \mathrm{tr}[\Phi(k)K^1(k+1)] \quad (2.31)$$

$$\pi^1(N) = 0$$

$$K^2(k) = Q^2 + (A-BW^{-1}Y)^T K^2(k+1)(I-C\Delta)(A-BW^{-1}Y) + Y^T W^{-1}R^2 W^{-1}Y \quad (2.32)$$

$$K^2(N) = Q^2(N)$$

$$\pi^2(k) = \pi^2(k+1) + \mathrm{tr}[\Phi(k)K^2(k+1)] \quad (2.33)$$

$$\pi^2(N) = 0$$

These equations can be solved backwards in time using the given final conditions. Sufficient conditions for the existence of the inverse and the minimum are that Q^1, S^1, Q^2 and R^2 are positive semidefinite and R^1 and S^2 are positive definite. In addition, these conditions ensure that the functionals minimized are strictly convex over the whole space, so the minima are unique.

In the Stackelberg equilibrium solution concept, decisions are not enacted simultaneously at each stage. Hence, it is attractive when one player has enough information to be a leader. In some multicriteria optimization problems the Stackelberg equilibrium strategy appears to be appropriate, although the Nash equilibrium strategy could also be justified.

3 STACKELBERG COORDINATION WITH NASH RATIONALE AMONG LOWER-LEVEL SUBSYSTEMS

3.1 Introduction

In this chapter we investigate a sequential decision approach to the control of an interconnection of several subsystems. Associated with each subsystem is a decision maker or a performance criterion function or cost function. A framework for studying strategies for the control of such systems is non-zero N-person differential games [35,36,47,48]. Various solution concepts for defining optimality have been proposed and examined. One of the most widely studied solution concepts is the Cournot or Nash strategy [47,48] whereby the decision-makers simultaneously minimize their respective cost functions with respect to their individual controls. At equilibrium when all the decision-makers apply their Nash strategies, the cost function of any subsystem is at minimum with respect to the control for that subsystem.

A sequential decision solution concept was first studied by Stackelberg [58] in the context of a static economic problem with two decision-makers. In [16,51,52] the Stackelberg concept was developed for two-person dynamic games with perfect information. Three types of Stackelberg strategies were investigated in [16,51,52]: open-loop, closed-loop, and feedback. In general, the open-loop and closed-loop Stackelberg strategies do not satisfy the principle of optimality but the feedback strategy and the more general equilibrium strategy [17] are defined to satisfy the principle of optimality. Open-loop Stackelberg strategies were considered in [53] for two groups of players where the player in each group use Nash strategies with respect to each other but each group plays according to the open-loop Strackelberg concept with respect to other groups. All these strategies are for deterministic dynamic games. In [18] the feedback Stackelberg solution concept is extended to stochastic two-person dynamic games.

The approach to be explicitly developed in this chapter is based on the coordination solution concept suggested in [15] for deterministic systems. We allow stochastic disturbances in the dynamic process model and in the measurement model, as in [18], but several second-level decision makers or followers are presented as in [15]. Several types of information structure are considered. Explicit recursion formulas for the design of the feedback Stackelberg controllers for the coordinator and the followers are presented. The strategies are adaptive to changes in information available at each stage and they satisfy the principle of optimality. The strategies of the second level decision-makers are equilibrium Nash strategies with respect to each other and in addition, they take into account the known strategy of the coordinator. The coordinator chooses his strategy with the full anticipation that the other decision makers will take the coordinator strategy into account in mimizing their individual cost functions.

3.2 Problem Formulation

Consider M discrete-time linear subsystems, each modeled by

$$x^i(k+1) = A^{io}(k)x^o(k) + A^{ii}(k)x^i(k) + \sum_{\substack{i=1 \\ i\neq j}}^{M} A^{ij}(k)x^j(k) + B^i(k)u^i(k) + \theta^i(k) \qquad (3.1)$$

The measurement of each subsystem is given by

$$z^i(k) = H^{io}(k)x^o(k) + H^{ii}(k)x^i(k) \quad , \quad + \sum_{\substack{j=1 \\ i\neq j}}^{M} H^{ij}(k)x^j(k) + \xi^i(k) \qquad i=1,\ldots,M; \qquad (3.2)$$

where x^i is the n^i-dimensional state vector of the i-th subsystem, u^i is the m^i-dimensional local control vector of the decision maker DM^i for the i-th subsystem, z^i is the l^i-dimensional measured output vector for the i-th subsystem. The vector $x^i(0)$; $\theta^i(k) \in R^{ni}$; $\xi^i(k) \in R^{li}$; $i=1,\ldots,M$; are mutually independent Gaussian random vactors for all k with known means and covariences.

$$E\{x^i(0)\} = 0 \ ; \qquad Cov\{x^i(0)\} = \Sigma^i(0)$$

$$E\{\theta^i(k)\} = 0 \ ; \qquad Cov\{\theta^i(k)\} = \Theta^i(k)$$

$$E\{\xi^i(k)\} = 0 \ ; \qquad Cov\{\xi^i(k)\} = \Xi^i(k)$$

Each subsystem seeks to minimize the expected value of its cost function

$$J^i(u^i) = \frac{1}{2} x^{iT}(N)Q^{ii}(N)x^i(N) + \frac{1}{2}\sum_{k=o}^{N-1}[x^{iT}(k)Q^{ii}(k)x^i(k)+u^{iT}(k)R^{ii}(k)u^i(k)] \qquad i=1,\ldots,M \qquad (3.3)$$

In addition to the M-subsystems, we assume that we have a coordinator subsystem modeled by

$$x^{o}(k+1) = A^{o}(k)x^{0}(k) + \sum_{i=1}^{M} A^{oi}(k)x^{i}(k) + \theta^{o}(k) \tag{3.4}$$

and the measurement of the coordinator subsystem is given by

$$z^{o}(k) = H^{o}(k)x^{o}(k) + \sum_{i=1}^{M} H^{oi}(k)x^{i}(k) + \xi^{o}(k) \tag{3.5}$$

where x^{o} is the n^{o}-dimensional state vector of the coordinator subsystem, u^{o} is an m^{o}-dimensional control vector chosen by the coordinator DM^{o}, z^{o} is the l^{o}-dimensional measured output vector of the coordinateor subsystem. $\{x^{o}(0);\ \theta^{o}(k) \in R^{no};\ \xi^{o}(k) \in R^{lo};\ k=0,\dots,N-1\}$ are mutually independent with the random vector of each subsystem.

$$E\{x^{o}(0)\} = 0 \; ; \qquad Cov\{x^{o}(0)\} = \Sigma^{o}(0)$$

$$E\{\theta^{o}(k)\} = 0 \; ; \qquad Cov\{\theta^{o}(k)\} = \Theta^{o}(k)$$

$$E\{\xi^{o}(k)\} = 0 \; ; \qquad Cov\{\xi^{o}(k)\} = \Xi^{o}(k)$$

The coordinator chooses u^{o} to minimize the expected value of the cost function

$$\begin{aligned} J^{o}(u^{o}) = {} & \tfrac{1}{2}x^{oT}(N)Q^{o}(N)x^{o}(N) + \tfrac{1}{2}\sum_{i=1}^{M} x^{iT}(N)Q^{oi}(N)x^{i}(N) \\ & + \tfrac{1}{2}\sum_{k=o}^{N-1} [x^{oT}(k)Q^{o}(k)x^{o}(k)+u^{oT}(k)R^{o}(k)u^{o}(k) \\ & + \sum_{i=1}^{M} x^{iT}(k)Q^{oi}(k)x^{i}(k)] \end{aligned} \tag{3.6}$$

where Q^{o}, Q^{oi}, R^{o} are all positive definite.

The Stackelberg approach [15] to the coordination of the subsystems is to consider DM^o as a leader and DM^i as followers. We imagine that DM^o provides DM^i exact knowledge of all decisions made by the coordinator and each DM^i minimizes J^i with respect to u^i for each given decision of DM^o assuming that the other subsystems will do the same. With this assumption, the subsystems play Nash among themselves. The coordinator then minimizes J^o with respect to u^o, considering that the decision from the subsystems result from choices of u^i which minimize J^i for $i=1,\ldots,M$. Additionally, the information sets include exact knowledge of the system dynamic DM^o, DM^i, the measurements and the cost functionals. The statistics of the random elements for all k are also included.

The optimal feedback Stackelberg approach to the 2-level coordination of the subsystems [15] is described by the following procedure: At each stage, the coordinator computes the subsystems' expected reaction to his decision, based on minimizing the subsystems' expected cost-to-go assuming that all second level decision makers will use their optimal feedback Stackelberg strategies in the future. The coordinator then seeks to minimize his expected cost-to-go assuming that the subsystems will respond as expected. Each subsystem then uses the coordinator's decision to compute his optimal decision, assuming that other subsystems will do the same. These expectations are conditioned on the information sets available to each subsystem.

The information set consists of exact knowledge of the system dynamics, the measurement rules and the cost functionals of all decision makers. Additionally, it includes exact knowledge of all decisions made by each player up to stage k-1 and the statistics of random elements $\theta^i(k)$, $\xi^i(k)$, i=0,...,M for all k. Also the Stackelberg nature of the game implies that the followers' information contains the exact value of the leader's decision at time k, $u^o(k)$.

Let arg min f(k) denote the value of u at which f(k) achieves its absolute minimum. Then the equations that define these optimal solutions are as follows:

$$u_o^i(u^o,k) = \arg\min_{u^i} E\{J^i(u^i,x^i,k)/z^i(k)\} \tag{3.7}$$

$$u^{o*}(k) = \arg\min_{u^i} E\{J^o(u^o,x^o,x^i,k)/z^o(k)\} \tag{3.8}$$

$$u^{i*}(k) = u^i(u^{o*},k) \tag{3.9}$$

The optimal cost-to-go at each stage are

$$J^{i*}(k) = E\{J^i(u^i,x^i,k)/z^i(k),u^i=u^{i*},u^o=u^{o*}\} \quad i=1,...,M \tag{3.10}$$

$$J^{o*}(k) = E\{J^o(u^o,x^o,x^i,k)/z^o(k),u^o=u^{o*},u^i=u^{i*}\} \quad i=1,...,M \tag{3.11}$$

Stochastic dynamic programming can be used to obtain the solutions.

Two possible cases will be considered in this chapter. First, when the information is centralized, several classes of information structures are discussed. One is when all decision makers have perfect system state measurement. Another is when the information of all the followers are identical and the coordinator's information contains the followers' information. Second, we will constrain each controller to be in decentralized structure and the i-th subsystem including the coordinator knows only its own measurement.

3.3 Coordination with Centralized Information

In general, the coordinator has some information from each subsystem and, in turn makes some decisions that will influence the dynamic response of the lower-level subsystems. By definition of Stackelberg strategies [52], all decisions made by the coordinator are known to the second level decision makers. However, some information may or may not be available to the coordinator and lower-level subsystems. When the information sets are centralized, either the coordinator and the lower-level subsystems have perfect information of state, or the lower-level subsystems have the same measurement but the information set of the coordinator consists of his own measurement and the lower level subsystems' measurement. Several particular cases of this problem are examined. Let us examine a system with one coordinator and two second level decision makers.

Consider the augmented system

$$x(k+1) = A(k)x(k) + B^o(k)u^o(k) + B^1(k)u^1(k) + B^2(k)u^2(k) + v(k) \qquad (3.12)$$

where $x^T(k) = [x^{oT}(k) \quad x^{1T}(k) \quad x^{2T}(k)]$

$$v^T(k) = [\theta^{oT}(k) \quad \theta^{1T}(k) \quad \theta^{2T}(k)]$$

x(0) and v(k) are Gaussian random vectors with zero mean and covariance

$\Sigma(0)$ and $\Lambda(k)$, and the measurement of each subsystem is

$$z^i(k) = H^i(k)x(k) + \xi^i(k) \qquad i=0,1,2 \tag{3.13}$$

The quadratic cost is

$$J^i(u^i) = \tfrac{1}{2}x^T(N)Q^i(N)x(N) + \tfrac{1}{2}\sum_{k=0}^{N-1}[x^T(k)Q^i(k)x(k) + u^{iT}(k)R^i(k)u^i(k)] \qquad i=0,1,2 \tag{3.14}$$

3.3.1 Perfect Information

Suppose all subsystems have perfect information of the states, i.e., $z^i(k)=x(k)$, $i=0,1,2$. Assume that the expected cost-to-go at stage k is

$$V^i(k) = \tfrac{1}{2}x^T(k)S^i(k)x(k) + \tfrac{1}{2}\gamma^i(k), \qquad i=0,1,2 \tag{3.15}$$

for some deterministic matrix $S^i(k)$ and scalar function $\gamma^i(k)$. Using dynamic programming as shown in Appendix 1, the optimum strategies are

$$u^{o*}(k) = -L^o(k)x(k) \tag{3.16}$$

$$u^i_o(k) = -\Delta^i(k)[A(k)x(k) + B^o(k)u^o(k)], \qquad i=1,2 \tag{3.17}$$

where

$$L^o(k) = [R^o(k) + \hat{B}^T(k)S^o(k+1)\hat{B}(k)]^{-1}\hat{B}^T(k)S^o(k+1)\hat{A}(k)$$

$$\Delta^i(k) = [I - L^i(k)B^j(k)L^j(k)B^i(k)]^{-1}(L^i(k)-L^i(k)B^j(k)L^j(k))$$

$$i=1,2,\ j=1,2,\ i\neq j$$

$$\hat{A}(k) = A(k) - B^1(k)\Delta^1(k)A(k) - B^2(k)\Delta^2(k)A(k)$$

$$\hat{B}(k) = B^o(k) - B^1(k)\Delta^1(k)B^o(k) - B^2(k)\Delta^2(k)B^o(k)$$

$$L^i(k) = [R^i(k) + B^{iT}(k)S^i(k+1)B^i(k)]^{-1}B^{iT}(k)S^i(k+1)$$

Assuming that the indicated inverses exist, the other quantities are obtained from

$$S^{o}(k) = Q^{o}(k) + \hat{A}^{T}(k)S^{o}(k+1)\hat{A}(k)$$

$$- L^{oT}(k)[R^{o}(k) + \hat{B}^{T}(k)S^{o}(k+1)\hat{B}(k)]L^{o}(k) \qquad (3.18)$$

$$S^{O}(N) = Q^{o}(N) \qquad (3.19)$$

$$\gamma^{o}(k) = \gamma^{o}(k+1) + trS^{o}(k+1)\Lambda(k) \qquad (3.20)$$

$$\gamma^{o}(N) = 0 \qquad (3.21)$$

$$S^{i}(k) = Q^{i}(k)$$

$$+ [A(k)-B^{o}(k)L^{o}(k)]^{T}\Delta^{iT}(k)R^{i}(k)\Delta^{i}(k)[A(k)-B^{o}(k)L^{o}(k)]$$

$$+ [\hat{A}(k)-\hat{B}(k)L^{o}(k)]^{T}S^{i}(k+1)[\hat{A}(k)-\hat{B}(k)L^{o}(k)] \quad i=1,2 \qquad (3.22)$$

$$S^{i}(N) = Q^{i}(N) \qquad i=1,2 \qquad (3.23)$$

$$\gamma^{i}(k) = \gamma^{i}(k+1) + trS^{i}(k+1)\Lambda(k) \qquad i=1,2 \qquad (3.24)$$

$$\gamma^{i}(N) = 0 \qquad i=1,2 \qquad (3.25)$$

These equations can be solved backwards in time. In summary, we have the following calculations: Starting at k=N-1; $S^{o}(N)$, $S^{i}(N)$ i=1,2, are given.

1. Compute $L^{i}(k)$, i=1,2
2. Compute $\Delta^{i}(k)$, i=1,2
3. Compute $\hat{A}(k)$, $\hat{B}(k)$
4. Compute $L^{o}(k)$
5. Compute $S^{o}(k)$, $S^{i}(k)$, i=1,2
6. $k \rightarrow k-1$ and go to 1. Stop when k=0.

Note that the control laws for the coordinator and the i-th subsystem involve perfect measurement of the state.

Illustrative Example

Consider a linear system described by the difference equation:

$$x_1(k+1) = 0.75x_1(k) + 0.9x_2(k) + 0.9x_3(k) + u_1(k) + w_1(k)$$

$$x_2(k+1) = 0.3x_1(k) + 0.8x_2(k) + 0.2x_3(k) + u_2(k) + w_2(k)$$

$$x_3(k+1) = 0.3x_1(k) + 0.2x_2(k) + 0.8x_3(k) + u_3(k) + w_3(k)$$

where u_i are the controls of players i; i=1,2,3 respectively. $\{w_i(k); i=1,2,3\}$ are mutually independent Gaussian random vectors with zero means and known covariances. Let the cost functions be of the form:

$$J_i = \frac{1}{2}(x_i(N)-p_i)^2 + \frac{1}{2}\sum_{k=1}^{N-1}[(x_i(k)-y_i)^2 + u_i^2(k)] \qquad i=1,2,3$$

where p_i; i=1,2,3 and y_i; i=1,2,3 are constants. This problem is similar to a tracking problem where the players are trying to force the states to be as close as possible to some prespecified values while investing a minimal amount of energy.

Assume that player 1 is the coordinator or leader and players 2 and 3 are followers. Every players will seek control policies which are functions of states. Stackelberg coordination for an interconnected system with player 1 as the coordinator and players 2 and 3 as followers, who assume Nash rationale between themselves is sought. The parameters in the cost functional have the following values: $p_i = 0$; i=1,2,3 and $y_i = 0$; i=1,2,3 and N = 10. Fig 3.1 shows the trajectory and control policies of the system.

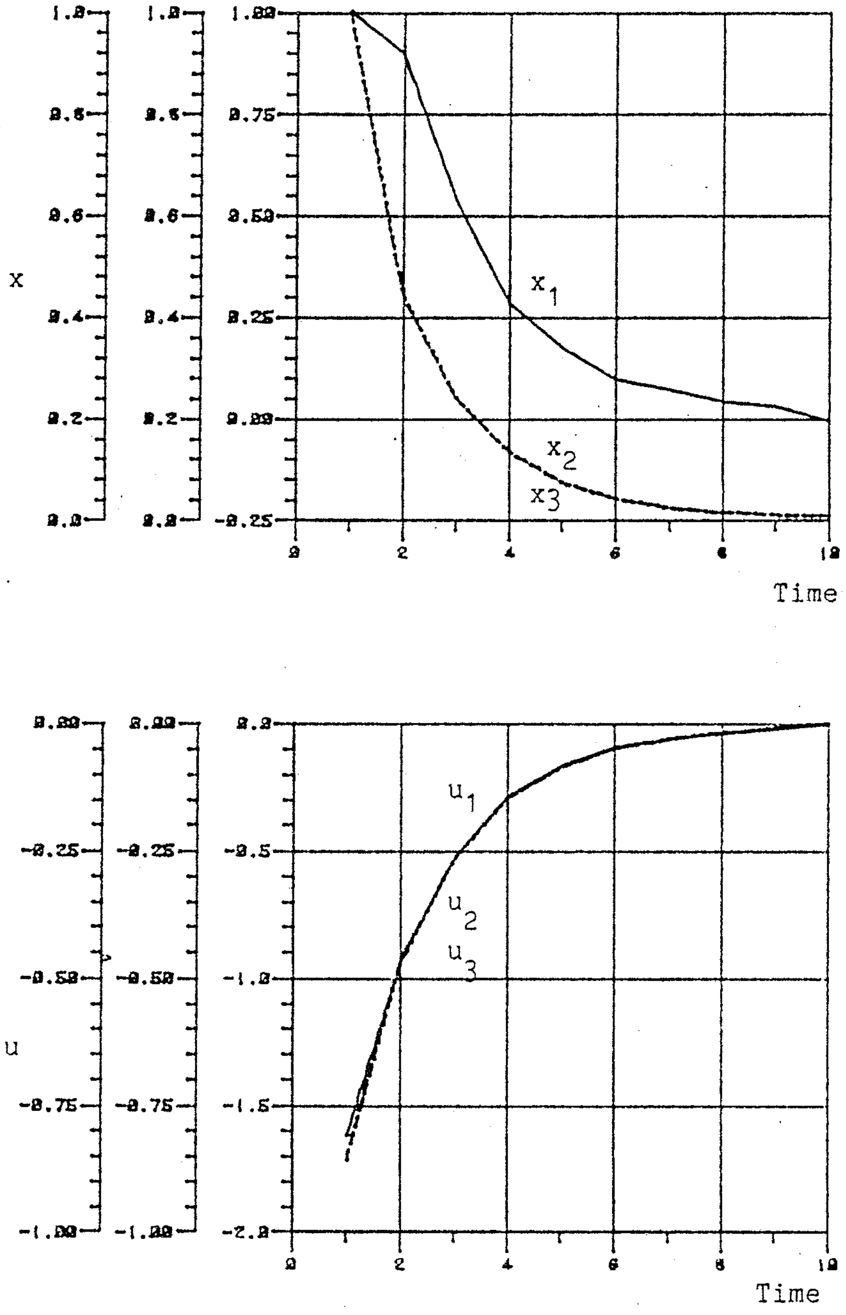

Fig. 3.1 The trajectories and controls of Illustrative Example

3.3.2 Coordination with Nested Information Structure

1. Incomplete Information for Coordinator and Subsystem

Consider the case where the information of the state is incomplete. At each stage, in addition to their own estimates, the optimal strategies would include terms involving an estimate of the other subsystems' estimates of the state in the future. This leads to estimators of much larger dimension than the system itself. For a special case of the stochastic problem, consider the case where each subsystem has the same measurement

$$z^1(k) = z^2(k) = z(k) = H(k)x(k) + \xi(k)$$

and the coordinator knows both his measurement and all subsystem measurements. So for any k, $z^o(k) \supset z(k)$, implying that the information sets are nested. We also have to assume that there is no information transfer among subsystems through their controls [18]. The optimum strategies for this case are derived in Appendix 2 as

$$u^i(k) = -\Delta^i(k)(A(k)\hat{x}(k) + B^o(k)u^o(k)) \qquad i=1,2 \tag{3.26}$$

$$u^{o*}(k) = -\Delta^o(k)Y(k)\hat{x}^o(k) - \Delta^o(k)M(k)[\hat{x}(k)-\hat{x}^o(k)] \tag{3.27}$$

$$J^{o*}(k) = \frac{1}{2}\begin{bmatrix} \hat{x}^o(k) \\ \hat{x}(k)-\hat{x}^o(k) \end{bmatrix}^T \begin{bmatrix} S^A(k) & S^B(k) \\ S^{BT}(k) & S^C(k) \end{bmatrix} \begin{bmatrix} \hat{x}^o(k) \\ \hat{x}(k)-\hat{x}^o(k) \end{bmatrix} + \frac{1}{2}\gamma^o(k) \tag{3.28}$$

$$J^{i*}(k) = \frac{1}{2}\hat{x}^T(k)S^i(k)\hat{x}(k) + \frac{1}{2}\gamma^i(k) \qquad i=1,2 \tag{3.29}$$

where $\hat{x}(k) = E\{x(k)/z^i(k)\}$, $\hat{x}^o(k) = E\{x(k)/z^o(k)\}$

$\Delta^i(k)$, $\hat{A}(k)$, $\hat{B}(k)$, and $L^i(k)$ are defined in the perfect information case with S^A replacing $S^o(k)$. In addition we have

$$S^A(k) = Q^0(k) + A^T(k)(I-G(k))^T S^A(k+1)(I-G(k))A(k) - Y^T(k)\Delta^o(k)Y(k) \tag{3.30}$$

$$\begin{aligned} S^B(k) = {} & A^T(k)(I-G(k))^T S^B(k+1)(I-G(k))A(k) \\ & + A^T(k)(I-G(k))^T(S^B(k+1)-S^A(k+1))G(k))A(k) \\ & - A^T(k)(I-G(k))^T S^B(k+1)K(k+1)H(k+1)A(k) \\ & - Y^T(k)\Delta^o(k)M(k) \end{aligned} \tag{3.31}$$

$$\begin{aligned} S^C(k) = {} & - M^T(k)\Delta^o(k)M(k) + A^T(k)G^T(k)S^A(k+1)G(k)A(k) \\ & + A^T(k)[I - K(k+1)H(k+1)]^T S^C(k+1)[I - K(k+1)H(k+1)]A(k) \\ & + A^T(k)(S^B(k+1)K(k+1)H(k+1)-S^B(k+1))G(k)A(k) \\ & - A^T(k)G^T(k)(S^B(k+1)-S^B(k+1)K(k+1)H(k+1))A(k) \end{aligned} \tag{3.32}$$

$$Y(k) = \hat{B}(k)S^A(k+1)[I-G(k)]A(k)$$

$$\begin{aligned} M(k) = {} & \hat{B}(k)S^A(k+1)G(k)A(k) + \hat{B}^T(k)(S^B(k+1)-S^A(k+1))A(k) \\ & - \hat{B}^T(k)S^B(k+1)K(k+1)H(k+1)A(k) \end{aligned}$$

$$G(k) = B^1(k)\Delta^1(k) + B^2(k)\Delta^2(k)$$

$$\Delta^o(k) = [R^o(k) + \hat{B}(k)S^A(k+1)\hat{B}(k)]^{-1}$$

$$K^i(k+1) = P^i(k+1/k)H^{iT}(k+1)[H^i(k+1)P^i(k+1/k)H^{iT}(k+1)+\Xi^i(k+1)]^{-1}$$

$$P^i(k+1/k) = A(k+1)P^i(k/k)A^T(k+1) + A(k)$$

$$P^i(k+1/k+1) = [I - K^i(k+1)H^i(k+1)]P^i(k+1/k)$$

$$P^i(0/0) = \Sigma(0)$$

for $i=0,1,2$ and where $H^i=H$ for $i=1,2$.

$$\begin{aligned} \gamma^o(k) = {} & \gamma^o(k+1) + \mathrm{tr}Q^o(k)P^o(k/k) + \mathrm{tr}\{K^o(k+1)[H^o(k+1)P^o(k+1/k)H^{oT}(k+1) \\ & + \Xi^o(k)]K^{oT}(k+1)(S^A(k+1) + S^C(k+1) - 2S^B(k+1))\} \\ & + 2\mathrm{tr}P^o(k+1/k)K(k+1)H(k+1)(S^B(k+1)-S^C(k+1)) \\ & + \mathrm{tr}K(k+1)[H(k+1)P^o(k+1/k)H^T(k+1)+\Xi(k+1)]K^T(k+1)S^C(k+1) \end{aligned} \tag{3.33}$$

$$\begin{aligned} S^i(k) = {} & Q^i(k) + (A(k)+\hat{B}(k)\Delta^o(k)Y(k))^T S^i(k+1)(A(k)+\hat{B}(k)\Delta^o(k)Y(k)) \\ & + (\Delta^i(k)A(k)+B^o(k)\Delta^o(k)Y(k))^T R^i(k)(\Delta^i(k)A(k)+B^o(k)\Delta^o(k)Y(k)) \\ & \qquad i=1,2 \end{aligned} \tag{3.34}$$

$$\begin{aligned}\gamma^i(k) = {} & \gamma^i(k+1) + trQ^i(k)P(k/k) + trS^i(k+1)K(k+1) \\ & + trS^i(k+1)K(k+1)[H(k+1)P(k+1/k)H^T(k+1)+\Xi(k+1)]K^T(k+1) \\ & + tr[P(k/k) - P^o(k/k)](M(k)-Y(k))^T\Delta^{oT}(k)(B^{oT}(k)R^i(k)B^o(k) \\ & + \hat{B}^T(k)S^i(k+1)\hat{B}(k))\Delta^o(k)(M(k)-Y(k))\end{aligned} \tag{3.35}$$

The recursive equations (3.30) and (3.34) are identical to equations (3.18) and (3.22) in the perfect information case, with the same initial conditions, so that the solution $S^A(k)$ and $S^i(k)$ in (3.30) and (3.34) are equal to $S^o(k)$ and $S^i(k)$ in (3.18) and (3.22). Thus, as far as the followers are concerned, they play a "separation principle" strategy which consists of the optimal deterministic feedback law of their best estimate of the state. The leader strategy includes his own estimate and a term involving a difference in estimates. When both estimates are the same, the leader also plays as in the "separation principle".

2. Perfect Information for Coordinator

Consider the problem in which the coordinator has perfect state measurement while the lower level subsystems have available only noisy output measurements. In addition, we assume that conditions are such that the coordinator can deduce exactly the lower level subsystems' state estimators, and the lower level subsystems have the same noisy measurement, i.e., $z^1(k) = z^2(k) = z(k)$.

When the coordinator has perfect state measurement and can deduce exactly the state of the lower level subsystems' state estimator, i.e., $H^o(k)\equiv I$ and $\xi^o(k)=0$, also $z^o(k) \supset z(k)$. The problem is of "nested information" type except the coordinator does not have to estimate its own state ($E[x(k)/z^o(k)]=x(k)$).

The control law of the coordinator is

$$u^{o*}(k) = -\Delta^o(k)Y(k)x(k) - \Delta^o(k)M(k)(\hat{x}(k)-x(k)) \tag{3.36}$$

and the control laws of the lower level subsystems are

$$u_0^i(k) = -\Delta^i(k)[A(k)\hat{x}(k) + B^o(k)u^o(k)] \qquad i=1,2 \tag{3.37}$$

where $E[x(k)/z(k)]=\hat{x}(k)$. The optimum cost-to-go is

$$J^{o*}(k) = \frac{1}{2}\begin{bmatrix} x(k) \\ \hat{x}(k)-x(k) \end{bmatrix}^T \begin{bmatrix} S^A(k) & S^B(k) \\ S^{BT}(k) & S^C(k) \end{bmatrix} \begin{bmatrix} x(k) \\ \hat{x}(k)-x(k) \end{bmatrix} + \frac{1}{2}\gamma^o(k) \tag{3.38}$$

$$J^{i*}(k) = \frac{1}{2}\hat{x}^T(k)S^i(k)\hat{x}(k) + \frac{1}{2}\gamma^i(k) \qquad i=1,2 \tag{3.39}$$

where all matrices are the same as in Section 3.2.1.

3. No Measurements for Subsystems

Consider the problem in which the coordinator has a noisy measurement, while the lower level subsystems have no measurement available to them and are restricted to using only a priori information.

When the lower level subsystems have no measurements, i.e., $H^i(k)\equiv 0$ (null matrix) and $z^i(k)\equiv z^i(0)$ for all k, the problem is also of nested information type. The control law of the coordinator is

$$u^o(k) = -\Delta^o(k)Y(k)\hat{x}^o(k) - \Delta^o(k)M(k)(\hat{x}(k)-\hat{x}^o(k)) \tag{3.40}$$

and the control laws of the lower level subsystems is

$$u_0^i(k) = -\Delta^i(k)[A(k)\hat{x}(k) + B^o(k)u^o(k)], \qquad i=1,2 \tag{3.41}$$

where $E[x(k)/z^o(k)] = \hat{x}^o(k)$, $E[x(k)/z(k)] = \hat{x}(k)$.

The optimum cost-to-go is

$$J^{o*}(k) = \frac{1}{2}\begin{bmatrix} \hat{x}^o(k) \\ \hat{x}(k)-\hat{x}^o(k) \end{bmatrix}^T \begin{bmatrix} S^A(k) & S^B(k) \\ S^{BT}(k) & S^C(k) \end{bmatrix} \begin{bmatrix} \hat{x}^o(k) \\ \hat{x}(k)-\hat{x}^o(k) \end{bmatrix} + \frac{1}{2}\gamma^o(k) \tag{3.42}$$

$$J^{i*}(k) = \frac{1}{2}\hat{x}^T(k)S^i(k)\hat{x}(k) + \frac{1}{2}\gamma^i(k) \qquad i=1,2 \tag{3.43}$$

where all matrices are the same as in Section 3.2.1.

Substitution of (3.40) and (3.41) into the system equation gives

$$\begin{aligned} x(k+1) = {} & A(k)x(k) - (B^1(k)\Delta^1(k)A(k) - B^2(k)\Delta^2(k)A(k)\hat{x}(k) \\ & - (B^o(k) - B^1(k)\Delta^1(k)B^o(k) - B^2(k)\Delta^2(k)B^o(k))\Delta^o(k)Y(k)\hat{x}(k) \\ & - (B^o(k) - B^1(k)\Delta^1(k)B^o(k) - B^2(k)\Delta^2(k)B^o(k))\Delta^o(k)Y(k) \\ & (\hat{x}(k)-\hat{x}^o(k)) \end{aligned} \tag{3.44}$$

It follows that the optimal estimate of the states by the lower level subsystems, given only a priori information, i.e., no output measurement, is given by

$$\begin{aligned} \hat{x}(k+1) = {} & [A(k)-B^1(k)\Delta^1(k)A(k)-B^2(k)\Delta^2(k)A(k) \\ & -(B^o(k)-B^1(k)\Delta^1(k)B^o(k)-B^2(k)\Delta^2(k)B^o(k))\Delta^o(k)Y(k)]\hat{x}(k) \end{aligned} \tag{3.45}$$

with initial condition $\hat{x}(0/0)=\bar{x}(0)$.

In addition, when $\bar{x}(0)=0$, then $\hat{x}(k/k)=0$ so that

$$u^{o*}(k) = -\Delta^o(k)[Y(k) - M(k)]\hat{x}^o(k) \tag{3.46}$$

and $$u_o^i(k) = -\Delta^i(k)B^o(k)u^o(k), \qquad i=1,2 \tag{3.47}$$

3.4 Constrained Decentralized Structure

It may be desirable to have a control policy that is simpler to implement than the optimal policy. Satisfactory control of a high-order linear system may often be achieved using relatively fewer system measurements and a controller of low order. This has been the motivation for a number of optimal designs, using output feedback or dynamic controllers of a specified order. For recent work in this field we refer the reader to [31,32,34,37,38,54].

3.4.1 Decentralized Control with Instantaneous Output Feedback

Consider the stochastic problem where a restriction is placed on the control of the i-th subsystem and the coordinator at any instant to be a linear transformation of the measurement at that instant. Also, there is no information transfer among subsystems through their controls. This simplifies the problem since a filter is no longer used to estimate the state. Then

$$u^i(k) = F^i(k)z^i(k), \quad i=0,1,2, \quad k=0,1,\ldots,N-1 \tag{3.48}$$

where $F^i(k)$ is to be determined to minimize the expected value of $J^i(u^i)$.

Consider the augmented system (3.12) and the measurement

$$z^i(k) = H^i(k)x(k) + \xi^i(k), \quad i=0,1,2 \tag{3.49}$$

Then

$$u^i(k) = F^i(k)H^i(k)x(k) + F^i(k)\xi^i(k), \quad i=0,1,2 \tag{3.50}$$

and

$$\begin{aligned} x(k+1) = {} & (A(k) + \sum_{i=0}^{2} B^i(k)F^i(k)H^i(k))x(k) \\ & + \sum_{i=0}^{2} B^i(k)F^i(k)\xi^i(k) + v_k \end{aligned} \tag{3.51}$$

Define $P(k) = E\{x(k)x^T(k)\}$ and note that $x(k)$ depends on $\xi^i(k)$ for $i=0,1,\ldots k-1$ only, implying that $E\{x(k)v^T(k)\}=0$. Then the recursive equation for $P(k)$ is

$$P(k+1) = [A(k)+\sum_{i=o}^{2} B^i(k)F^i(k)H^i(k)]P(k)[A(k)+\sum_{i=o}^{2} B^i(k)F^i(k)H^i(k)]^T$$
$$+\sum_{i=o}^{2} B^i(k)F^i(k)\Xi^i(K)F^{iT}(k)B^{iT}(k)+\Lambda(k) \qquad (3.52)$$

<u>Lemma 3.1</u> If the linear system described by (3.12) is controlled using a linear control policy (3.48), i=1,2 then the expected cost (3.14) i=1,2 can be expressed as

$$E[J^i(k)] = \tfrac{1}{2}E[x^T(k)S^i(k)x(k)] + \tfrac{1}{2}\sum_{l=k+1}^{N} trS^i(l)\Lambda(l-1)$$
$$+ \tfrac{1}{2}\sum_{l=k+1}^{N} \{trF^{iT}(l-1)[R^i(l-1)+B^{iT}(l-1)S^i(l)B^i(l-1)]F^i(l-1)\Xi^i(l-1)$$
$$+\sum_{\substack{j=o\\ i\neq j}}^{2} trF^{jT}(l-1)B^{jT}(l-1)S^i(l)B^j(l-1)F^j(l-1)\Xi^j(l-1)\} \qquad i=1,2 \qquad (3.53)$$

where $S^i(k) = Q^i(k)+H^{iT}(k)F^{iT}(k)R^i(k)F^i(k)H^i(k)$

$$+[A(k)+\sum_{j=o}^{2} B^j(K)F^j(k)H^j(k)]^T S^i(k+1)[A(k)+\sum_{j=o}^{2} B^j(k)F^j(k)H^j(k)] \qquad i=1,2 \qquad (3.54)$$

$$S^i(N) = Q^i(N) \qquad (3.55)$$

<u>Proof</u> The proof is by induction.

Consider the augmented system (3.12) and the cost criterion (3.14). The assumption obviously holds for k=N. For any k

$$E[J^i(k)] = \tfrac{1}{2}E\{\sum_{l=k}^{N-1}[x^T(l)Q^i(l)x(l)+u^{iT}(l)R^i(l)u^i(l)]\}+\tfrac{1}{2}E\{x^T(N)Q^i(N)x(N)\}$$

$$= E[J^i(k+1)] + E\tfrac{1}{2}\{x^T(k)Q^i(k)x(k)+u^{iT}(k)R^i(k)u^i(k)\}$$

$$i=1,2 \qquad (3.56)$$

with k=k+1 using (3.53) in (3.56) and after some algebra the assumption holds for k=k+1. Thus (3.53) holds for k=0,1,...,n. The necessary condition for a minimum at each step is that the derivative of the remaining cost with respect to $F^i(k)$; i=1,2 must equal zero.

$$F^{1*}(k) = -[R^1+B^{1T}S^1(k+1)B^1]^{-1}B^{1T}S^1(k+1)[A+B^oF^o(k)H^o+B^2F^2(k)H^2]$$

$$P(k)H^{1T}[H^1P(k)H^{1T}+\Xi^1]^{-1} \qquad (3.57)$$

$$F^{2*}(k) = -[R^2+B^{2T}S^2(k+1)B^2]^{-1}B^{2T}S^2(k+1)[A+B^oF^o(k)H^o+B^1F^1(k)H^1]$$

$$P(k)H^{2T}[H^2P(k)H^{2T}+\Xi^2]^{-1} \qquad (3.58)$$

or

$$F^{1*}(k) = \Gamma^1(k)[A(k)+B^o(k)F^o(k)H^o(k)]T^1(k) \qquad (3.59)$$

$$F^{2*}(k) = \Gamma^2(k)[A(k)+B^o(k)F^o(k)H^o(k)]T^2(k) \qquad (3.60)$$

where

$$\Gamma^i(k) = [I-M^i(k)B^j(k)M^j(k)B^i(k)]^{-1}[M^i(k)+M^i(k)B^j(k)M^j(k)]$$

$$i=1,2,\ j=1,2,\ i\neq j$$

$$T^i(k) = [Y^i(k)+Y^j(k)H^j(k)Y^i(k)][I-H^i(k)Y^j(k)H^j(k)Y^i(k)]^{-1}$$

$$i=1,2,\ j=1,2,\ i\neq j$$

$$M^i(k) = -[R^i(k)+B^{iT}(k)S^i(k+1)B^i(k)]^{-1}B^{iT}(k)S^i(k+1) \qquad i=1,2$$

$$Y^i(k) = P(k)H^{iT}(k)[H^i(k)P(k)H^{iT}(k)+\Xi^i(k)]^{-1} \qquad i=1,2$$

Lemma 3.2 If a linear system described by (3.12) is controlled using a linear control policy (3.48) for i=0 then the expected cost (3.14) for i=0 is expressed as

$$\begin{aligned}E[J^o(k)] = \tfrac{1}{2}E[x^T(k)S^o(k)x(k)] + \tfrac{1}{2}[\sum_{i=k+1}^{N} & trS^o(i)\Lambda(i-1) \\ & + trF^{oT}(i-1)[R^o(i-1)+B^{oT}(i-1)S^o(i)B^o(i-1)]F^o(i-1)\Xi^o(i-1) \\ & + \sum_{j=1}^{2} trF^{j*T}(i-1)B^{jT}(i-1)S^o(i)B^j(i-1)F^{j*}(i-1)\Xi^j(i-1)]\end{aligned} \tag{3.61}$$

where

$$\begin{aligned}S^o(k) = Q^o(k) & + H^{oT}(k)F^{oT}(k)R^o(k)F^o(k)H^o(k) \\ & + [A(k)+B^o(k)F^o(k)H^o(k)+\sum_{i=1}^{2} B^i(k)F^{i*}(k)H^i(K)]^T S^o(k+1) \\ & \quad [A(k)+B^o(k)F^o(k)H^o(k)+\sum_{i=1}^{2} B^i(k)F^{i*}(k)H^i(k)]\end{aligned} \tag{3.62}$$

$$S^o(N) = Q^o(N) \tag{3.63}$$

At each step the necessary condition for a minimum is that the derivative of the remaining cost with respect to each element of $F^o(k)$ must equal zero.

$$\begin{aligned}F^o(k) = -[& R^o(k)+(B^o(k)+B^1(k)\Gamma^1(k)B^o(k)+B^2(k)\Gamma^2(k)B^o(k))^T S^o(k+1) \\ & (B^o(k)+B^1(k)\Gamma^1(k)B^o(k)+B^2(k)\Gamma^2(k)B^o(k))]^{-1} \\ & \{(B^o(k)+B^1(k)\Gamma^1(k)B^o(k)+B^2(k)\Gamma^2(k)B^o(k))^T S^o(k+1) \\ & [A(k)+B^1(k)\Gamma^1(k)A(k)T^1(k)H^1(k)+B^2(k)\Gamma^2(k)A(k)T^2(k)H^2(k)] \\ & P(k)[H^o(k)+H^o(k)T^1(k)H^1(k)+H^o(k)T^2(k)H^2(k)]^T \\ & +[B^1(k)\Gamma^1(k)B^o(k)+B^2(k)\Gamma^2(k)B^o(k)]^T S^o(k+1) \\ & [B^1(k)\Gamma^1(k)A(k)T^1(k)\Xi^1(k)T^{1T}(k)H^{oT}(k) \\ & +B^2(k)\Gamma^2(k)A(k)T^2(k)\Xi^2(k)T^{2T}(k)H^{oT}(k)]\} \\ & [H^o(k)P(k)H^{oT}(k)+(H^o(k)T^1(k)H^1(k)+H^o(k)T^2(k)H^2(k)) \\ & P(k)(H^o(k)+H^o(k)T^1(k)H^1(k)+H^o(k)T^2(k)H^2(k))^T\end{aligned}$$

$$+(\Xi^{o}(k)+H^{o}(k)T^{1}(k)\Xi^{1}(k)T^{1T}(k)H^{oT}(k)$$

$$+H^{o}(k)T^{2}(k)\Xi^{2}(k)T^{2T}(k)H^{oT}(k))]^{-1} \qquad (3.64)$$

<u>Theorem 3.1</u> The sequences $\{F^{i}(k)\}$ $i=0,1,2;$ $k=0,1,\ldots,N-1$ of the i-th subsystem that minimizes $E\{J^{i}(u^{i})\}$ $i=0,1,2$ subject to the constraint (3.48) are given by the equations (3.59), (3.60) and (3.64) where it is assumed that the required inversed exist and

1. $$P(k+1) = [A(k)+\sum_{i=o}^{2} B^{i}(k)F^{i}(k)H^{i}(k)]P(k)[A+\sum_{i=o}^{2} B^{i}(k)F^{i}(k)H^{i}(k)]^{T}$$

$$+\sum_{i=o}^{2} B^{i}(k)F^{i}(k)\Xi^{i}(k)F^{iT}(k)B^{iT}(k)+\Lambda(k) \qquad (3.65)$$

   P(0) is given.

2. $$S^{i}(k) = Q^{i}(k)+H^{iT}(k)F^{iT}(k)R^{i}(k)F^{i}(k)H^{i}(k)$$

$$+[A(k)+\sum_{i=o}^{2} B^{i}(k)F^{i}(k)H^{i}(k)]^{T}S^{i}(k+1)[A(k)+\sum_{i=o}^{2} B^{i}(k)F^{i}(k)H^{i}(k)]$$

$$i=1,2 \qquad (3.66)$$

$$S^{i}(N) = Q^{i}(N) \qquad i=1,2 \qquad (3.67)$$

3. $$S^{o}(k) = Q^{o}(k) + H^{oT}(k)F^{oT}(k)R^{o}(k)F^{o}(k)H^{o}(k)$$

$$+[A(k)+B^{o}(k)F^{o}(k)H^{o}(k)+\sum_{i=1}^{2} B^{i}(k)F^{i*}(k)H^{i}(k)]^{T}S^{o}(k+1)$$

$$[A(k)+B^{o}(k)F^{o}(k)H^{o}(k)+\sum_{i=1}^{2} B^{i}(k)F^{i*}(k)H^{i}(k)] \qquad (3.68)$$

$$S^{o}(N) = Q^{o}(N) \qquad (3.69)$$

The sequence $\{F^{i}(k)\}$, $i=0,1,2;$ $k=0,1,\ldots,N-1$ of the coordinator and the i-th subsystem are the solution to the discrete two-point boundary value problem. Note that (3.65), (3.66) and (3.67) are recursive relationships for generating P(k) and $S^{i}(k)$, $i=0,1,2$ except

(3.65) which is a forward equation and (3.66) and (3.67) which are backward equations, and all depend on the sequence $\{F^i(k)\}$ or $\{S^i(k)\}$ and $\{P(k)\}$ are known no simple calculation will solve the problem. We suggest the following simple procedure to solve the equations:

1. Make an initial guess for the gain $\{F_j^o(k)\}$ and $\{F_j^i(k)\}$ i=1,2; k=0,1,...,N-1. Let j=0.
2. Use $\{F_j^o(k)\}$ and $\{F_j^i(k)\}$ to solve (3.65) forward in time to determine $\{P_j(k)\}$ with $P_j(0)=\Sigma(0)$.
3. Use $\{F_j^o(k)\}$ and $\{F_j^i(k)\}$ to solve (3.66) and (3.68) backward in time to determine $\{S_j^i(k)\}$, i=1,2 and $\{S_j^o(k)\}$ with $S^i(N)=Q^i(N)$, i=0,1,2
4. Use $\{P_j(k)\}$ and $\{S_j^o(k)\}$ in (3.64) to determine $\{F_{j+1}^o(k)\}$.
5. Use $\{P_j(k)\}$, $\{S_j^i(k)\}$, i=1,2 and $\{F_{j+1}^o(k)\}$ in (3.59) and (3.60) to determine $\{F_{j+1}^i(k)\}$, i=1,2. Let j=j+1.
6. Repeat (2)-(5) until the desired degree of convergence is reached.

So far no convergence conditions for this algorithm have been found, but as with most algorithms of this type it is expected that convergence depends on the initial guess.

3.4.2 Decentralized control with dynamic output feedback

Consider the stochastic problem where a dynamic controller of a specified order for the i-th subsystem and the coordinator described by

$$w^i(k+1) = D^i(k)w^i(k)+\bar{M}^i(k)z^i(k) \quad i=0,1,2 \qquad (3.70)$$

where $w^i \in R^{si}$ is the state vector of the controllers used, then

$$u^i(k) = N^i(k)w^i(k)+F^i(k)z^i(k) \quad i=0,1,2 \qquad (3.71)$$

also

$$z^i(k) = H^i(k)x(k)+\xi^i(k) \qquad i=0,1,2 \tag{3.72}$$

For a given integer s^i ($0 \leq s^i \leq n$) find matrices $N^i(k)$, $F^i(k)$, $D^i(k)$ and $\overline{M}^i(k)$ such that the corresponding expected cost $E\{J^i(u^i)\}$ will be minimum. Note that if $s^i = 0$ the controller is reduced to

$$u^i(k) = F^i(k)z^i(k) \qquad i=0,1,2$$

and if $s^i = n$, an optimal solution is obtained. The cost functional to be consider is the same as in Section 3.4.1.

Consider the augmented state vector

$$\tilde{x}^T(k) = [\; x^T(k) \quad w^{oT}(k) \quad w^{1T}(k) \quad w^{2T}(k) \;]$$

then

$$\tilde{x}(k+1) = (\tilde{A}(k)+\sum_{i=o}^{2}\tilde{B}^i(k)\tilde{F}^i(k)\tilde{H}^i(k))\tilde{x}(k) + \sum_{i=o}^{2}\tilde{B}^i(k)\tilde{F}^i(k)\tilde{I}^i(k)\xi^i(k)+\tilde{I}v(k) \tag{3.73}$$

where

$$\tilde{F}^i(k) = \begin{bmatrix} F^i(k) & N^i(k) \\ \overline{M}^i(k) & D^i(k) \end{bmatrix}$$

$$\tilde{I} = \left[\begin{array}{c} I \\ \hline 0 \end{array}\right] \begin{array}{l} \updownarrow n \\ \updownarrow \sum_{i=o}^{2} si \end{array}$$

and

$$\tilde{u}^i(k) = T\tilde{F}^i(k)\tilde{H}^i(k)\tilde{x}(k)+T\tilde{F}^i(k)\tilde{I}^i\xi^i(k) \qquad i=0,1,2 \tag{3.74}$$

where $T = [\, \underset{n}{\underleftrightarrow{I}} \,|\, \underset{\sum_{i=o}^{2} si}{\underleftrightarrow{0}} \,]$

Let $P(k) = E[x(k)x^T(k)]$ and the cost functional of the coordinator is

$$J^{o} = \frac{1}{2}\tilde{x}^{T}(N)\tilde{Q}^{o}(N)\tilde{x}(N) + \frac{1}{2}\sum_{k=o}^{N-1}[\tilde{x}^{T}(k)\tilde{Q}^{o}(k)\tilde{x}(k)+\tilde{u}^{oT}(k)\tilde{R}^{o}(k)\tilde{u}^{o}(k)] \quad (3.75)$$

Also, the cost functional of the lower-level subsystem is

$$J^{i} = \frac{1}{2}\tilde{x}^{T}(N)\tilde{Q}(N)\tilde{x}(N) + \frac{1}{2}\sum_{k=o}^{N-1}[\tilde{x}^{T}(k)\tilde{Q}^{i}(k)\tilde{x}(k)+\tilde{u}^{iT}(k)\tilde{R}^{i}(k)\tilde{u}^{i}(k)] \quad (3.76)$$

The augmented system (3.73) and controller (3.74) are of the same form as (3.51) and (3.49). Also the cost functionals are the same. The following theorem can be derived using the same argument as Theorem 3.1.

<u>Theorem 3.2</u> The sequences $\{\tilde{F}^{i}(k)\}$ $i=0,1,2;$ $k=0,1,\ldots,N-1$ of the coordinator and the i-th subsystem that minimize $E\{J^{i}(u^{i})\}$ $i=0,1,2$ subject to the constraint (3.44) are given by

$$\begin{aligned}\tilde{F}^{o}(k) = & -[\tilde{R}^{o}+(\tilde{B}^{o}+\tilde{B}^{1}\Gamma^{1}\tilde{B}^{o}+\tilde{B}^{2}\Gamma^{2}\tilde{B}^{o})^{T}\tilde{S}^{o}(k+1)(\tilde{B}^{o}+\tilde{B}^{1}\Gamma^{1}\tilde{B}^{o}+\tilde{B}^{2}\Gamma^{2}\tilde{B}^{o})]^{-1}\\ & \{(\tilde{B}^{o}+\tilde{B}^{1}\Gamma^{1}\tilde{B}^{o}+\tilde{B}^{2}\Gamma^{2}\tilde{B}^{o})^{T}\tilde{S}^{o}(k+1)[\tilde{A}+\tilde{B}^{1}\Gamma^{1}\tilde{A}T^{1}\tilde{H}\\ & +\tilde{B}^{2}\Gamma^{2}\tilde{A}T^{2}\tilde{H}^{2}]\tilde{P}[\tilde{H}^{o}+\tilde{H}^{o}T^{1}\tilde{H}^{1}+\tilde{H}^{o}T^{2}\tilde{H}^{2}]^{T}\\ & +[\tilde{B}^{1}\Gamma^{1}\tilde{B}^{o}+\tilde{B}^{2}\Gamma^{2}\tilde{B}^{o}]^{T}\tilde{S}^{o}(k+1)[\tilde{B}^{1}\Gamma^{1}\tilde{A}T^{1}\Xi^{1}T^{1T}\tilde{H}^{oT}\\ & +\tilde{B}^{2}\Gamma^{2}\tilde{A}T^{2}\Xi^{2}T^{2T}\tilde{H}^{oT}]\}\\ & [\tilde{H}^{o}\tilde{P}\tilde{H}^{oT}+(\tilde{H}^{o}T^{1}\tilde{H}^{1}+\tilde{H}^{o}T^{2}\tilde{H}^{2})\tilde{P}(\tilde{H}^{o}+\tilde{H}^{o}T^{1}\tilde{H}^{1}+\tilde{H}^{o}T^{2}\tilde{H}^{2})\\ & +(\Xi^{o}+\tilde{H}^{o}T^{1}\Xi^{1}T^{1T}\tilde{H}^{oT}+\tilde{H}^{o}T^{2}\Xi^{2}T^{2T}\tilde{H}^{oT})]^{-1} \qquad (3.77)\end{aligned}$$

$$\tilde{F}^{i*}(k) = \Gamma^{i}(k)[\tilde{A}(k)+\tilde{B}^{o}(k)\tilde{F}^{o}(k)\tilde{H}^{o}(k)]T^{i}(k) \quad i=1,2 \qquad (3.78)$$

where

$$\Gamma^{i} = [I-M^{i}\tilde{B}^{j}M^{j}\tilde{B}^{i}]^{-1}[M^{i}+M^{i}\tilde{B}^{j}M^{j}] \quad i=1,2, j=1,2, i\neq j$$

$$T^{i} = [Y^{i}+Y^{j}\tilde{H}^{j}Y^{i}][I-\tilde{H}^{i}Y^{j}\tilde{H}^{j}Y^{i}]^{-1} \quad i=1,2,\ j=1,2,\ i\neq j$$

$$M^{i} = -[\tilde{R}^{i}+\tilde{B}^{iT}\tilde{S}^{i}(k+1)\tilde{B}^{i}]^{-1}\tilde{B}^{iT}\tilde{S}^{i}(k+1) \quad i=1,2$$

$$Y^{i} = \tilde{P}(k)\tilde{H}^{iT}[\tilde{H}^{i}\tilde{P}(k)\tilde{H}^{iT}+\Xi^{i}]^{-1} \quad i=1,2$$

It is assumed that the required inverse matrices exist and where

$$1.\ \tilde{P}(k+1) = [\tilde{A}+\sum_{i=o}^{2}\tilde{B}^{i}\tilde{F}^{i}(k)\tilde{H}^{i}]\tilde{P}(k)[\tilde{A}+\sum_{i=o}^{2}\tilde{B}^{i}\tilde{F}^{i}(k)\tilde{H}^{i}]^{T} + \sum_{i=o}^{2}\tilde{B}^{i}\tilde{F}^{i}(K)\Xi^{i}\tilde{F}^{iT}(k)\tilde{B}^{iT}+\tilde{\Lambda}(k) \qquad (3.79)$$

$\tilde{P}(0)$ is given.

$$2.\ \tilde{S}^{i}(k) = \tilde{Q}^{i}+ \tilde{H}^{iT}\tilde{F}^{iT}(k)\tilde{R}^{i}\tilde{F}^{i}(k)\tilde{H}^{i} + [\tilde{A}+\sum_{i=o}^{2}\tilde{B}^{i}\tilde{F}^{i}(K)\tilde{H}^{i}]^{T}\tilde{S}^{i}(k+1)[\tilde{A}+\sum_{i=o}^{2}\tilde{B}^{i}\tilde{F}^{i}(k)\tilde{H}^{i}] \qquad (3.80)$$

$\tilde{S}^{i}(N) = \tilde{Q}^{i}(N)$

$$3.\ \tilde{S}^{o}(k) = \tilde{Q}^{o} + \tilde{H}^{oT}\tilde{F}^{oT}(k)\tilde{R}^{o}\tilde{F}^{o}(k)\tilde{H}^{o} + [\tilde{A}+\tilde{B}^{o}\tilde{F}^{o}(k)\tilde{H}^{o}+\sum_{i=1}^{2}\tilde{B}^{i}\tilde{F}^{i*}(k)\tilde{H}^{i}]^{T} \tilde{S}^{o}(k+1)[\tilde{A}+\tilde{B}^{o}\tilde{F}^{o}(k)\tilde{H}^{o}+\sum_{i=1}^{2}\tilde{B}^{i}\tilde{F}^{i*}(k)\tilde{H}^{i}] \qquad (3.81)$$

$\tilde{S}^{o}(N) = \tilde{Q}^{o}(N)$

Again the sequence $\{\tilde{F}^{i}(k)\}$ $i=0,1,2;$ $k=0,1,\ldots,N-1$ of the coordinator and the i-th subsystem are the solution to the discrete two-point boundary value problem as the previous one but are more complicated to solve.

In the case where either the coordinator has noise in its measurement or the lower-level subsystems have no noise in their measurement, and want to use output feedback, they can do so by reducing the dimension of their controller to zero.

3.5 Conclusions

The control of an interconnected set of linear discrete time stochastic systems has been considered. The organizational form of the system permits one decision maker to be the coordinator or leader and the decision makers for the other subsystems are all followers with respect to the coordinator, but they use the Nash strategy with respect to other second level decision makers. Both centralized and decentralized control structure were considered. As in single decision maker control problems with output feedback constraints, decentralization constraints generally lead to two-point boundary value problems. Explicit recursive formulas for these two-point boundary value problems have been derived. The sequential decision approach seems to be a natural one when the cost function associated with one decision maker has a more global significance compared to the others. This decision maker takes the role of a coordinator and leader.

4. STACKELBERG COORDINATION WITH PARETO RATIONALE AMONG LOWER-LEVEL SUBSYSTEMS

4.1 Introduction

In the previous chapter, a sequential decision approach to the control of an interconnected system, where the lower-level subsystems choose to play Nash rationale among themselves, has been obtained. An extension of this sequential decision, is to consider the problem when the lower-level subsystems choose to play Pareto optimal. It is possible that the lower-level subsystems desire to cooperate within their group. Then the resulting set of controls should be chosen from the Pareto optimal set of solutions. In this chapter, we will investigate the Stackelberg coordination of a discrete linear quadratic Gaussian problem, when the lower-level subsystems cooperate among themselves. Several types of information structure are considered. The main ideas in this chapter are basically derived from Chapter 3. To avoid repetition of identical arguments, a more compact treatment is presented.

4.2 Problem Formulation

Consider the M discrete time linear system described in Section 3.2, the Stackelberg approach [15] to the coordination of the subsystems is to consider DM^o as a leader and DM^i as followers. DM^o provides DM^i exact knowledge of all decisions made by the coordinator and each DM^i

minimizes J^i with respect to u^i for each given decision of DM^o, assuming that all the followers agree on an cooperation. With this assumption the subsystems use Pareto optimal strategies among themselves. The coordinator then minimizes J^o with respect to u^o, considering that the decisions from the subsystems result from choices of u^i which minimize J^i for $i=1,\ldots,M$. Additionally, the information sets include exact knowledge of the system dynamic DM^o, DM^i, the measurements and the cost functionals. The statistics of the random elements for all k are also included. Consider the augmented system

$$x(k+1) = Ax(k) + B^o u^o(k) + Bu(k) + v(k) \tag{4.1}$$

where

$$x^T(k) = [\ x^{oT}(k)\ x^{1T}(k)\ x^{2T}(k)\ ]$$

$$v^T(k) = [\ \theta^{oT}(k)\ \theta^{1T}(k)\ \theta^{2T}(k)\ ]$$

$$u^T(k) = [\ u^{1T}(k)\ u^{2T}(k)\ ]$$

$x(0)$ and $v(k)$ are Gaussian random vectors with zero means and covariance $\Sigma(0)$ and $\Lambda(k)$. The measurement equation of each subsystem is given by

$$z^i(k) = H^i(k)x(k) + \xi^i(k) \qquad i=0,1,2 \tag{4.2}$$

The quadratic cost of the lower-level subsystems becomes

$$J(u) = \sum_{i=1}^{2}\alpha^i J^i(u^i); \quad \alpha^i \geq 0; \quad \sum_{i=1}^{2}\alpha^i = 1 \tag{4.3}$$

$$= \tfrac{1}{2}x^T(N)Q(N)x(N) + \tfrac{1}{2}\sum_{k=o}^{N-1}[x^T(k)Q(k)x(k)+u^T(k)R(k)u(k)] \tag{4.4}$$

Also, the quadratic cost of the coordinator becomes

$$J^o(u^o) = \tfrac{1}{2}x^T(N)Q^o(N)x(N)$$

$$+ \tfrac{1}{2}\sum_{k=o}^{N-1}[x^T(k)Q^o(k)x(k) + u^{oT}(k)R^o(k)u^o(k)] \tag{4.5}$$

The equations that define the optimal solutions are as follows:

$$u_o(u^o,k) = \arg\min_{u} E\{J(u,x,k)/z(k)\}$$

$$u^{o*}(k) = \arg\min_{u^o} E\{J^o(u^o,x,k)/z^o(k)\}$$

$$u^*(k) = u_o(u^{o*},k)$$

The optimal cost-to-go at each stage are

$$J^*(k) = E\{J(u,x,k)/z(k), u = u^*, u^o = u^{o*}\}$$

$$J^{o*}(k) = E\{J^o(u^o,x,k)/z^o(k), u^o = u^{o*}, u = u^*\}$$

Centralized and decentralized structure of information are investigated in the following section.

4.3 Coordination with Centralized Information

In this section, two cases of centralized information is considered, perfect information and nested information. Recursive equations for the design of feedback controllers for the coordination and the followers are obtained. For simplicity, a system with one coordinator and two second-level decision makers is examined.

4.3.1 Perfect Information

Suppose all subsystems have perfect information of the state i.e. $z^i(k)=x(k)$. Assume that the expected cost-to-go of the lower level at stage k is

$$V(k) = \tfrac{1}{2}x^T(k)S(k)x(k) + \tfrac{1}{2}\beta(k) \qquad (4.6)$$

for some deterministic matrix S(k) and function $\beta(k)$. Also the expected cost-to-go of the coordinator at stage k is:

$$V^{o}(k) = \frac{1}{2}x^{T}(k)S^{o}(k)x(k) + \frac{1}{2}\beta^{o}(k) \tag{4.7}$$

for some deterministic matrix $S^{o}(k)$ and function $\beta^{o}(k)$. The optimal strategies are derived using dynamic programming:

$$u^{o*}(k) = -\Delta^{o}(k)\hat{B}^{T}(k)S^{o}(k+1)\hat{A}(k)x(k) \tag{4.8}$$

$$u_{o}(k) = -\Delta(k)B^{T}(k)S(k+1)[A(k)x(k) + B^{o}(k)u^{o}(k)] \tag{4.9}$$

where

$$\Delta(k) = [R(k) + B^{T}(k)S(k+1)B(k)]^{-1}$$

$$\Delta^{o}(k) = [R^{o}(k) + \hat{B}^{T}(k)S^{o}(k+1)\hat{B}(k)]^{-1}$$

$$\hat{A}(k) = [A(k)-B(k)\Delta(k)B^{T}(k)S(k+1)A(k)]$$

$$\hat{B}(k) = [B^{o}(k)-B(k)\Delta(k)B^{T}(k)S(k+1)B^{o}(k)]$$

Assume all the required inverse matrices exist and

$$\begin{aligned} S^{o}(k) = {} & Q^{o}(k) + \hat{A}^{T}(k)S^{o}(k+1)\hat{A}(k) \\ & + \hat{A}^{T}(k)S^{o}(k+1)\hat{B}(k)\Delta^{o}(k)\hat{B}^{T}(k)S^{o}(k+1)\hat{A}(k) \end{aligned} \tag{4.10}$$

$$S^{o}(N) = Q^{o}(N) \tag{4.11}$$

$$\beta^{o}(k) = \beta^{o}(k+1) + \mathrm{tr}S^{o}(k+1)\Lambda(k) \tag{4.12}$$

$$\beta^{o}(N) = 0 \tag{4.13}$$

$$\begin{aligned} S(k) = {} & Q(k) + M^{T}(k)R(k)M(k) \\ & + [\hat{A}(k)-\hat{B}(k)\Delta^{o}(k)\hat{B}^{T}(k)S^{o}(k+1)\hat{A}(k)]^{T} \\ & \quad S(k+1)[\hat{A}(k)-\hat{B}(k)\Delta^{o}(k)\hat{B}^{T}(k)S^{o}(k+1)\hat{A}(k)] \end{aligned} \tag{4.14}$$

$$S(N) = Q(N) \tag{4.15}$$

$$M(k) = \Delta(k)\hat{B}^{T}(k)S(k+1)[A(k)-B^{o}(k)\Delta^{o}(k)\hat{B}^{T}(k)S^{o}(k+1)\hat{A}(k)]$$

$$\beta(k) = \beta(k+1) + \mathrm{tr}S(k+1)\Lambda(k) \tag{4.16}$$

$$\beta(N) = 0 \tag{4.17}$$

These equations can be solved backwards in time with the given final conditions. The condition for the existence of the solutions is that the matrices to be inverted are nonsingular.

4.3.2 Coordination With Nested Information Structure

Consider the case when the information of the states is incomplete but the lower-level subsystems know the same measurement i.e. $z^1(k) = z^2(k) = z(k) = H(k)x(k) + \xi(k)$ and the coordinator knows both his measurement and all the subsystems measurement, $z^o(k) \supset z(k)$. Assume no information transfer among subsystems through their controls. The optimal strategies are derived using dynamic programming:

$$u_o(k) = -\Delta(k)B^T(k)S(k+1)[A(k)\hat{x}(k) + B^o(k)u^o(k)] \qquad (4.18)$$

$$u^{o*}(k) = -\Delta^o(k)Y(k)\hat{x}^o(k) - \Delta^o(k)M(k)[\hat{x}(k) - \hat{x}^o(k)] \qquad (4.19)$$

where $\hat{x}(k) = E[x(k)/z(k)]$, $\hat{x}^o(k) = E[x(k)/z^o(k)]$, $\Delta(k)$ and $\Delta^o(k)$ are defined as in Section 4.3.1 and

$$Y(k) = B^{oT}(I-BG)^T S^A(k+1)(I-BG)A$$

$$M(k) = B^{oT}(I-BG)^T[S^B(k+1) - S^A(k+1)]A + Y(k)$$
$$\qquad - B^{oT}(I-BG)^T S^B(k+1)K(k+1)HA$$

$$G(k) = \Delta(k)B^T S(k+1)$$

The optimal cost-to-go at each stage for the coordinator and the lower-level subsystems are:

$$J^{o*}(k) = \frac{1}{2}\begin{bmatrix} \hat{x}^o(k) \\ \hat{x}(k) - \hat{x}^o(k) \end{bmatrix}^T \begin{bmatrix} S^A(k) & S^B(k) \\ S^{BT}(k) & S^C(k) \end{bmatrix} \cdot \begin{bmatrix} \hat{x}^o(k) \\ \hat{x}(k) - \hat{x}^o(k) \end{bmatrix} + \frac{1}{2}\beta^o(k) \qquad (4.20)$$

$$J^*(k) = \frac{1}{2}\hat{x}^T(k)S(k)\hat{x}(k) + \frac{1}{2}\beta(k) \qquad (4.21)$$

where

$$S^A(k) = Q^o + A^T(I-BG)^T S^A(k+1)(I-BG)A - Y^T\triangle^o Y \tag{4.22}$$

$$\begin{aligned} S^B(k) = {} & A^T(I-BG)^T S^A(k+1)(I-BG)A + A^T(I-BG)[S^B(k+1)-S^A(k+1)]A \\ & - A^T(I-BG)^T S^B(k+1)K(k+1)HA - Y^T\triangle^o M \end{aligned} \tag{4.23}$$

$$\begin{aligned} S^C(k) = {} & - M^T\triangle^o M + A^T G^T B^T S^A(k+1)BGA \\ & + A^T[I-K(k+1)H]^T S^C(k+1)[I-K(k+1)H]A \\ & + A^T[S^B(k+1)K(k+1)H-S^B(k+1)]^T BGA \\ & - A^T G^T B^T[S^B(k+1)K(k+1)H-S^B(k+1)]A \end{aligned} \tag{4.24}$$

$$\begin{aligned} \beta^o(k) = {} & \beta^o(k+1) + \mathrm{tr}Q^o P^o(k) \\ & + \mathrm{tr}\{K^o(k+1)[H^o(k+1)P^o(k+1/k)H^{oT}(k+1)+\Xi^o(k+1)] \\ & \quad K^o(k+1)[S^A(k+1) + S^C(k+1)-2S^B(k+1)]\} \\ & + 2\mathrm{tr}\{P^o(k+1/k)K(k+1)H(k+1)[S^B(k+1)-S^C(k+1)]\} \\ & + \mathrm{tr}K(k+1)[H(k+1)P^o(k+1/k)H^T(k+1)+\Xi(k+1)] \\ & \qquad K^T(k+1)S^C(k+1) \end{aligned} \tag{4.25}$$

$$\begin{aligned} S(k) = {} & Q + [A-B\triangle^o Y]^T[S(k+1)-G^T B^T S(k+1)][A-B\triangle^o Y] \\ & + Y^T\triangle^o R\triangle^o Y \end{aligned} \tag{4.26}$$

$$\begin{aligned} \beta(k) = {} & \beta(k+1) + \mathrm{tr}QP(k) + \mathrm{tr}S(k+1)K(k+1)[H(k+1)P(k+1/k)H^T(k+1) \\ & + \Xi(k+1)]K^T(k+1) + \mathrm{tr}\{[P(k/k)-P^o(k/k)][M-Y]^T\triangle^o \\ & \qquad [R + B^T S(k+1)[I-BG]B]\triangle^o[M-Y]\} \end{aligned} \tag{4.27}$$

$$K(k+1) = P(k+1/k)H^T(k+1)[H(k+1)P(k+1/k)H^T(k+1)+ \ (k+1)]^{-1} \tag{4.28}$$

$$P(k+1/k) = A(k+1)P(k/k)A^T(k+1) + \Lambda(k) \tag{4.29}$$

$$P(k+1/k+1) = [I-K(k+1)H(k+1)]P(k+1/k) \tag{4.30}$$

$$P(0/0) = \Sigma(0)$$

All these recursive equations can be solved with given initial or final conditions. The existence condition of the solutions is that the matrices to be inverted are nonsingular.

4.4 Constrained Decentralized Structure

Section 3.4 describes why output feedback and dynamic output feedback are more desirable in practical applications. In this section we will derive the necessary conditions for Stackelberg coordination when the lower-levels choose to use Pareto optimal solution with constraint being placed on the controls. The cost functional of the lower-level is

$$J(k) = \sum_{i=1}^{2} \alpha^i J^i; \quad \alpha^i \geq 0, \quad \sum_{i=1}^{2} \alpha^i = 1$$

$$J(k) = \tfrac{1}{2} x^T(N)Q(N)x(N)$$

$$+ \tfrac{1}{2}\sum_{k=o}^{N-1} [x^T(k)Q(k)x(k) + \tfrac{1}{2}\sum_{i=1}^{2}\alpha^i u^{iT}(k)R^i(k)u^i(k)] \qquad (4.31)$$

where $Q = \sum_{i=1}^{2} \alpha^i Q^i$

4.4.1 Decentralized Control with Instantaneous Output Feedback

When the controls are constrained to be a linear transformation of measurement at that instant and there is no information transfer through the control, then

$$u^i(k) = F^i(k)z^i(k) \qquad i=0,1,2 \qquad (4.32)$$

and

$$z^i(k) = H^i(k)x(k) + \xi^i(k) \qquad i=0,1,2 \qquad (4.33)$$

where $F^i(k)$ is to be determined to minimize the expected value of $J^i(u^i)$.

Consider the augmented system (4.1) and the measurement (4.33). Then

$$u^i(k) = F^i(k)H^i(k)x(k) + F^i(k)\xi^i(k) \qquad i=0,1,2$$

and

$$x(k+1) = [A(k) + \sum_{i=o}^{2} B^i(k)F^i(k)H^i(k)]x(k) + \sum_{i=o}^{2} B^i(k)F^i(k)\xi^i(k) + v(k) \tag{4.34}$$

Then the recursive equation for $P(k)=E\{x(k)x^T(k)\}$ is given by

$$P(k+1) = [A(k)+\sum_{i=o}^{2} B^i(k)F^i(k)H^i(k)]P(k)[A(k)+\sum_{i=o}^{2} B^i(k)F^i(k)H^i(k)]^T + \sum_{i=o}^{2} B^i(k)F^i(k)\Xi^i(K)F^{iT}(k)B^{iT}(k) + \Lambda(k) \tag{4.35}$$

<u>Lemma 4.1</u> If the linear system described by (4.1) is controlled using a linear control policy (4.32), i=1,2 then the expected cost (4.31) i=1,2 can be expressed as

$$\begin{aligned} E[J(k)] = {} & \tfrac{1}{2}E[x^T(k)S(k)x(k)] + \tfrac{1}{2}\sum_{l=k+1}^{N} trS(l)\Lambda(l-1) \\ & + \tfrac{1}{2}\sum_{l=k+1}^{N} \{\sum_{i=1}^{2} trF^{iT}(l-1)[\alpha^i R^i(l-1) \\ & + B^{iT}(l-1)S(l)B^i(l-1)]F^i(l-1)\Xi^i(l-1) \\ & + trF^{oT}(l-1)B^{oT}(l-1)S(l)B^o(l-1)F^o(l-1)\Xi^o(l-1)\} \end{aligned} \tag{4.36}$$

where

$$\begin{aligned} S(k) = {} & Q(k) + \sum_{i=1}^{2} \alpha^i H^{iT}(k)F^{iT}(k)R^i(k)F^i(k)H^i(k) \\ & + [A(k)+\sum_{i=o}^{2} B^i(K)F^i(k)H^i(k)]^T S(k+1) \\ & [A(k)+\sum_{i=o}^{2} B^i(k)F^i(k)H^i(k)] \end{aligned} \tag{4.37}$$

$$S(N) = Q(N)$$

The necessary condition for a minimum at each step is that the derivative of the remaining cost with respect to $F^i(k)$; $i=1,2$ must equal zero.

$$F^{1*}(k) = -[R^1+B^{1T}S(k+1)B^1]^{-1}B^{1T}S(k+1)[A+B^oF^o(k)H^o+B^2F^2(k)H^2]$$
$$P(k)H^{1T}[H^1P(k)H^{1T}+\Xi^1]^{-1} \tag{4.38}$$
$$F^{2*}(k) = -[R^2+B^{2T}S(k+1)B^2]^{-1}B^{2T}S(k+1)[A+B^oF^o(k)H^o+B^1F^1(k)H^1]$$
$$P(k)H^{2T}[H^2P(k)H^{2T}+\Xi^2]^{-1} \tag{4.39}$$

or

$$F^{1*}(k) = \Gamma^1[A + B^oF^o(k)H^o]T^1 \tag{4.40}$$
$$F^{2*}(k) = \Gamma^2[A + B^oF^o(k)H^o]T^2 \tag{4.41}$$

where

$$\Gamma^i = [I-M^iB^jM^jB^i]^{-1}[M^i + M^iB^jM^j] \quad i=1,2,\ j=1,2,\ i\neq j$$
$$T^i = [Y^i + Y^jH^jY^i][I-H^iY^jH^jY^i]^{-1} \quad i=1,2,\ j=1,2,\ i\neq j$$
$$M^i = -[\alpha^iR^i + B^{iT}S(k+1)B^i]^{-1}B^{iT}S(k+1) \quad i=1,2$$
$$Y^i = P(k)H^{iT}[H^iP(k)H^{iT} + \Xi^i]^{-1} \quad i=1,2$$

<u>Lemma 4.2</u> If a linear system described by (4.1) is controlled using a linear control policy (4.32) i=0 then the expected cost (4.31) i=0 is expressed as

$$E[J^o(k)] = \tfrac{1}{2}E[x^T(k)S^o(k)x(k)] + \tfrac{1}{2}\{\sum_{i=k+1}^{N} trS^o(i)\Lambda(i-1)$$
$$+ trF^{oT}(i-1)[R^o(i)+B^{oT}(i)S^o(i)B^o(i)]F^o(i-1)\Xi^o(i-1)$$
$$+ \sum_{j=1}^{2} trF^{j*T}(i-1)B^{jT}(i-1)S^o(i)B^j(i-1)F^{j*}(i-1)\Xi^j(i-1)\}$$
$$(4.42)$$

where

$$S^{o}(k) = Q^{o}(k) + H^{oT}(k)F^{oT}(k)R^{o}(k)F^{o}(k)H^{o}(k)$$

$$+ [A(k) + B^{o}(k)F^{o}(k)H^{o}(k) + \sum_{i=1}^{2} B^{i}(k)F^{i*}(k)H^{i}(K)]^{T}S^{o}(k+1)$$

$$[A(k) + B^{o}(k)F^{o}(k)H^{o}(k) + \sum_{i=1}^{2} B^{i}(k)F^{i*}(k)H^{i}(k)] \quad (4.43)$$

$$S^{o}(N) = Q^{o}(N)$$

At each step the ncessary condition for a minimum is that the derivative of the remaining cost with respect to each element of $F^{o}(k)$ must equal zero.

$$F^{o}(k) = -[R^{o} + (B^{o}+B^{1}\Gamma^{1}B^{o}+B^{2}\Gamma^{2}B^{o})^{T}S^{o}(k+1)(B^{o}+B^{1}\Gamma^{1}B^{o}+B^{2}\Gamma^{2}B^{o})]^{-1}$$

$$\{(B^{o}+B^{1}\Gamma^{1}B^{o}+B^{2}\Gamma^{2}B^{o})^{T}S^{o}(k+1)[A+B^{1}\Gamma^{1}AT^{1}H^{1}+B^{2}\Gamma^{2}AT^{2}H^{2}]P$$

$$[H^{o}+H^{o}T^{1}H^{1}+H^{o}T^{2}H^{2}]^{T}+[B^{1}\Gamma^{1}B^{o}+B^{2}\Gamma^{2}B^{o}]^{T}S^{o}(k+1)$$

$$[B^{1}\Gamma^{1}AT^{1}\Xi^{1}T^{1T}H^{oT}+B^{2}\Gamma^{2}AT^{2}\Xi^{2}T^{2T}H^{oT}]\}$$

$$[H^{o}PH^{oT}+(H^{o}T^{1}H^{1}+H^{o}T^{2}H^{2})P(H^{o}+H^{o}T^{1}H^{1}+H^{o}T^{2}H^{2})$$

$$+(\Xi^{o}+H^{o}T^{1}\Xi^{1}T^{1T}H^{oT}+H^{o}T^{2}\Xi^{2}T^{2T}H^{oT})]^{-1} \quad (4.44)$$

<u>Theorem</u> <u>4.1</u> The sequence $\{F^{i}(k)\}$ $i=0,1,2;$ $k=0,1,\ldots,n-1$ of the i-th subsystem that minimizes $E\{J^{i}(k)\}$ $i=0,1,2$ subject to the constraint (4.32) are given by the equations (4.44), (4.40) and (4.41) where it is assumed that the required inverses exist and

1. $$P(k+1) = [A + \sum_{i=o}^{2} B^{i}F^{i}(k)H^{i}]P(k)[A + \sum_{i=o}^{2} B^{i}F^{i}(k)H^{i}]^{T}$$

$$+ \sum_{i=1}^{2} B^{i}F^{i}(K)\Xi^{i}F^{iT}(k)B^{iT} + \Lambda(k) \quad (4.45)$$

P(0) is given.

2. $$S(k) = Q + \sum_{i=1}^{2} \alpha^i H^{iT} F^{iT}(k) R^i F^i(k) H^i$$

$$+ [A + \sum_{i=o}^{2} B^i F^i(K) H^i]^T S(k+1) [A + \sum_{i=o}^{2} B^i F^i(k) H^i] \qquad (4.46)$$

$$S(N) = Q(N)$$

3. $$S^o(k) = Q^o + H^{oT} F^{oT}(k) R^o F^o(k) H^o$$

$$+ [A + B^o F^o(k) H^o + \sum_{i=1}^{2} B^i F^{i*}(k) H^i]^T S^o(k+1)$$

$$[A + B^o F^o(k) H^o + \sum_{i=1}^{2} B^i F^{i*}(k) H^i] \qquad (4.47)$$

$$S^o(N) = Q^o(N)$$

To compute the cost incurred when the players use arbitrary linear output feedback control of the form

$$u^i(k) = K^i(k) z^i(k) \qquad (4.48)$$

the cost-to-go at stage k is

$$J^i(k) = \tfrac{1}{2} E[x^T(k) S^i(k) x(k)] + \tfrac{1}{2} \mathrm{tr} S^i(k+1) \Lambda(k)$$

$$+ \tfrac{1}{2} \sum_{l=k+1}^{N} \{ \mathrm{tr} K^{iT}(l-1) [R^i(l-1) + B^{iT}(l-1) S^i(l) B^i(l-1)]$$

$$K^i(l-1) \Xi^i(l-1) + \sum_{\substack{j=o \\ i \neq j}}^{2} \mathrm{tr} K^{jT}(l-1) B^{jT}(l-1)$$

$$S^i(l) B^j(l-1) K^j(l-1) \Xi^j(l-1) \} \qquad (4.49)$$

where

$$S^i(k) = Q^i(k) + H^{iT} K^{iT}(k) R^i K^i(k) H^i$$

$$+ [A + \sum_{i=o}^{2} B^i K^i(k) H^i]^T S^i(k+1) [A + \sum_{i=o}^{2} B^i K^i(k) H^i] \qquad (4.50)$$

$$S^i(N) = Q^i(N)$$

The sequence $\{F^i(k)\}$ $i=0,1,2$; $k=0,1,\ldots,n-1$ of the coordinator and the i-th subsystem are the solution to the discrete two-point boundary value problem. The simple procedure to solve the equations suggested in Section 3.4.1 is also recommended here.

4.4.2 Decentralized control with dynamic output feedback

When the controls are constrained to be a linear dynamic output feedback where a dynamic controller of a specified order for the i-th subsystem and the coordinator described by

$$w^i(k+1) = D^i(k)w^i(k) + \bar{M}^i(k)z^i(k) \qquad i=0,1,2 \qquad (4.51)$$

where $w^i \in R^{si}$ is the state vector of the controllers used, then

$$u^i(k) \quad = N^i(k)w^i(k) + F^i(k)z^i(k) \qquad =0,1,2 \qquad (4.52)$$

also

$$z^i(k) \quad = H^i(k)x(k) + \xi^i(k) \qquad i=0,1,2 \qquad (4.53)$$

For a given integer s^i $(0 \leq s^i \leq n)$ find matrices $N^i(k)$, $F^i(k)$, $D^i(k)$ and $\bar{M}^i(k)$ such that the corresponding expected cost $E\{J^i(k)\}$ will be minimum. The cost functional to be consider is the same as in Section 4.4.1.

Consider the augmented state vector

$$\tilde{x}^T(k) \quad = [\ x^T(k) \quad w^{oT}(k) \quad w^{1T}(k) \quad w^{2T}(k)\ ]$$

then

$$\tilde{x}(k+1) = (\tilde{A} + \sum_{i=o}^{2}\tilde{B}^i\tilde{F}^i\tilde{H}^i)\tilde{x}(k) + \sum_{i=o}^{2}\tilde{B}^i\tilde{F}^i\tilde{I}^i\xi^i(k) + \tilde{I}v(k) \tag{4.54}$$

where

$$\tilde{F}^i(k) = \begin{bmatrix} F^i(k) & N^i(k) \\ \bar{M}^i(k) & D^i(k) \end{bmatrix}$$

$$\tilde{I} = \begin{bmatrix} I \\ \hdashline 0 \end{bmatrix} \begin{array}{l} \updownarrow n \\ \updownarrow \sum_{i=o}^{2} si \end{array}$$

and

$$\tilde{u}^i(k) = T\tilde{F}^i(k)\tilde{H}^i\tilde{x}(k) + T\tilde{F}^i(k)\tilde{I}^i\xi^i(k) \qquad i=0,1,2 \tag{4.55}$$

where

$$T = [\ \underbrace{I}_{n}\ |\ \underbrace{0}_{\sum_{i=o}^{2} si}\ ]$$

Let $\tilde{P}(k) = E[\tilde{x}(k)\tilde{x}^T(k)]$ and the cost functional of the coordinator is

$$J^o = \tilde{x}^T(N)\tilde{Q}^o(N)\tilde{x}(N) + \frac{1}{2}\sum_{k=o}^{N}[\tilde{x}^T(k)\tilde{Q}^o(k)\tilde{x}(k) + \tilde{u}^{oT}(k)\tilde{R}^o(k)\tilde{u}^o(k)] \tag{4.56}$$

Also, the cost functional of the lower-level subsystem is

$$J = \sum_{i=1}^{2}\alpha^i J^i; \quad \alpha^i \geq 0, \ \sum_{i=1}^{2}\alpha^i = 1$$

$$= \frac{1}{2}\tilde{x}^T(N)\tilde{Q}(N)\tilde{x}(N) + \frac{1}{2}\sum_{k=o}^{N-1}[\tilde{x}^T(k)\tilde{Q}(k)\tilde{x}(k) + \sum_{i=1}^{2}\alpha^i\tilde{u}^{iT}(k)\tilde{R}^i(k)\tilde{u}^i(k)] \tag{4.57}$$

The augmented system (4.51) and controller (4.55) are of the same form as (4.34) and (4.32). Also the cost functional are the same. The following theorem can be derived using the same argument as Theorem 4.1.

<u>Theorem</u> <u>4.2</u> The sequences $\{\tilde{F}^{i}(k)\}$ $i=0,1,2;$ $k=0,1,\ldots,n-1$ of the coordinator and the i-th subsystem that minimize $E\{J^{i}(k)\}$ $i=0,1,2$ subject to the constraint (4.55) are given by

$$\begin{aligned}\tilde{F}^{o}(k) = &-[\tilde{R}^{o} + (\tilde{B}^{o}+\tilde{B}^{1}\Gamma^{1}\tilde{B}^{o}+\tilde{B}^{2}\Gamma^{2}\tilde{B}^{o})^{T}\tilde{S}^{o}(k+1)(\tilde{B}^{o}+\tilde{B}^{1}\Gamma^{1}\tilde{B}^{o}+\tilde{B}^{2}\Gamma^{2}\tilde{B}^{o})]^{-1}\\ &\{(\tilde{B}^{o}+\tilde{B}^{1}\Gamma^{1}\tilde{B}^{o}+\tilde{B}^{2}\Gamma^{2}\tilde{B}^{o})^{T}\tilde{S}^{o}(k+1)[\tilde{A}+\tilde{B}^{1}\Gamma^{1}\tilde{A}T^{1}\tilde{H}^{1}+\tilde{B}^{2}\Gamma^{2}\tilde{A}T^{2}\tilde{H}^{2}]\tilde{P}\\ &[\tilde{H}^{o}+\tilde{H}^{o}T^{1}\tilde{H}^{1}+\tilde{H}^{o}T^{2}\tilde{H}^{2}]^{T}+[\tilde{B}^{1}\Gamma^{1}\tilde{B}^{o}+\tilde{B}^{2}\Gamma^{2}\tilde{B}^{o}]^{T}\tilde{S}^{o}(k+1)\\ &[\tilde{B}^{1}\Gamma^{1}\tilde{A}T^{1}\Xi^{1}T^{1T}\tilde{H}^{oT}+\tilde{B}^{2}\Gamma^{2}\tilde{A}T^{2}\Xi^{2}T^{2T}\tilde{H}^{oT}]\}\\ &[\tilde{H}^{o}\tilde{P}H^{oT}+(\tilde{H}^{o}+\tilde{H}^{o}T^{1}\tilde{H}^{1}+\tilde{H}^{o}T^{2}\tilde{H}^{2})\tilde{P}(\tilde{H}^{o}+\tilde{H}^{o}T^{1}\tilde{H}^{1}+\tilde{H}^{o}T^{2}\tilde{H}^{2})\\ &+(\Xi^{o}+\tilde{H}^{o}T^{1}\Xi^{1}T^{1T}\tilde{H}^{oT}+\tilde{H}^{o}T^{2}\Xi^{2}T^{2T}\tilde{H}^{oT})]^{-1}\end{aligned} \qquad (4.58)$$

$$\tilde{F}^{i}(k) = \Gamma^{i}(k)[\tilde{A}(k)+\tilde{B}^{o}(k)\tilde{F}^{o}(k)\tilde{H}^{o}(k)]T^{i}(k) \qquad i=1,2 \qquad (4.59)$$

where

$$\Gamma^{i} = [I-M^{i}\tilde{B}^{j}M^{j}\tilde{B}^{i}]^{-1}[M^{i}+M^{i}\tilde{B}^{j}M^{j}] \quad i=1,2, j=1,2, i\neq j$$

$$T^{i} = [Y^{i}+Y^{j}\tilde{H}^{j}Y^{i}][I-\tilde{H}^{i}Y^{j}\tilde{H}^{j}Y^{i}]^{-1} \quad i=1,2,\ j=1,2,\ i\neq j$$

$$M^{i} = -[\alpha^{i}\tilde{R}^{i}+\tilde{B}^{iT}\tilde{S}(k+1)\tilde{B}^{i}]^{-1}\tilde{B}^{iT}\tilde{S}(k+1) \quad i=1,2$$

$$Y^{i} = \tilde{P}(k)\tilde{H}^{iT}[\tilde{H}^{i}\tilde{P}(k)\tilde{H}^{iT}+\Xi^{i}]^{-1} \quad i=1,2$$

It is assumed that the required inverse matrices exist where

1. $$\begin{aligned}\tilde{P}(k+1) = &[\tilde{A} + \sum_{i=o}^{2}\tilde{B}^{i}\tilde{F}^{i}(k)\tilde{H}^{i}]\tilde{P}(k)[\tilde{A} + \sum_{i=o}^{2}\tilde{B}^{i}\tilde{F}^{i}(k)\tilde{H}^{i}]^{T}\\ &+ \sum_{i=o}^{2}\tilde{B}^{i}\tilde{F}^{i}(K)\Xi^{i}\tilde{F}^{iT}(k)\tilde{B}^{iT} + \tilde{\Lambda}(k)\end{aligned} \qquad (4.60)$$

   $\tilde{P}(0)$ is given.

2. $$\begin{aligned}\tilde{S}(k) = &\tilde{Q} + \sum_{i=1}^{2}\alpha^{i}\tilde{H}^{iT}\tilde{F}^{iT}(k)\tilde{R}^{i}\tilde{F}^{i}(k)\tilde{H}^{i}\\ &+ [\tilde{A} + \sum_{i=o}^{2}\tilde{B}^{i}\tilde{F}^{i}(k)\tilde{H}^{i}]^{T}\tilde{S}(k+1)[\tilde{A} + \sum_{i=o}^{2}\tilde{B}^{i}\tilde{F}^{i}(k)\tilde{H}^{i}]\end{aligned} \qquad (4.61)$$

$$\tilde{S}(N) = \tilde{Q}(N)$$

$$3.\ \tilde{S}^{o}(k) = \tilde{Q}^{o} + \tilde{H}^{oT}\tilde{F}^{oT}(k)\tilde{R}^{o}\tilde{F}^{o}(k)\tilde{H}^{o}$$

$$+ [\tilde{A} + \tilde{B}^{o}\tilde{F}^{o}(k)\tilde{H}^{o} + \sum_{i=1}^{2}\tilde{B}^{i}\tilde{F}^{i*}(k)\tilde{H}^{i}]^{T}\tilde{S}^{o}(k+1)$$

$$[\tilde{A} + \tilde{B}^{o}\tilde{F}^{o}(k)\tilde{H}^{o} + \sum_{i=1}^{2}\tilde{B}^{i}\tilde{F}^{i*}(k)\tilde{H}^{i}] \qquad (4.62)$$

$$\tilde{S}^{o}(N) = \tilde{Q}^{o}(N)$$

Again the sequence $\{\tilde{F}^{i}(k)\}$ $i=0,1,2;$ $k=0,1,\ldots,N-1$ of the coordinator and the i-th subsystem are the solution to the discrete two-point boundary value problem. The procedure used in Section 4.4.1 can be used to solve for the solutions.

4.5 Conclusions

As in all nonzero-sum differential games, there are a variety of "optimal solutions", since the lower-level may or may not cooperate within their group. When the lower-level subsystems, which are all followers with respect to the coordinator or leader, desire to cooperate within their group, the Pareto optimal solutions are obtained. Both centralized and decentralized control structures were considered. The main idea is the same as in Chapter 3.

5. DECENTRALIZED STOCHASTIC STACKELBERG COORDINATION IN AUTOMATIC GENERATION CONTROL OF INTERCONNECTED POWER SYSTEM

5.1 Introduction

An interconnected electric energy system can be described as a collection of subsystems, each of which is called a control area. Each area is responsible for meeting its obligation to maintain the appropriate system frequency and supply its own load demand. Also, each area provides mutual assistance to its neighbors in accordance with the basic operating policy of interconnected power systems [23]. Two of the most important aspects of system control involve the regulation of system frequency and net power interchange. When the interconnected network is small centralized techniques can be used quite effectively [19,20,33]. However, in the more general case the communication/computational costs involved in implementing a centralized controller often become essential. Furthermore, the trend in the utility industry is strongly to digital control, using the digital computer for calculating generation changes etc.. A discrete formulation of this problem would thus seem of more practical interest.

Interest in the dynamical aspects of load frequency control has stimulated the application of modern control techniques to this problem, particularly the theory of optimal linear regulator [25]. Calovic [19,20] was the first to cleary distinguish the steady state problem from the transient problem. The procedure used is to adjoin the integral of each area control error ($ACE_i = B_i \Delta f_i + \Delta P_{tie,i}$ where B_i is the tie line bias constant specified as the area frequency characteristic.) to the system equations (Fosha and Elgerd [25] adjoin integrals of frequency and tie line flow errors). These new state variables as well as the original system state variables are included in the cost functional. As a result, all areas capable of doing so will drive their area control errors to zero in steady state provided the system is stable. It is not clear from the control equations what control actions would be taken in each area if any area is not able to control optimally.

Decentralized Stackelberg strategies will be used to developed a decentralized controller for a three area electric power system. This new design procedure is based on a stochastic Stackelberg strategy extended by introducing optimal regulation with individual choice of cost functional to each control area. The problem now becomes a multicriteria problem with multi-decision makers. This is where differential games theory is relevent to define "optimality". Once the optimality is defined, we can calculate K_I (K_I is the controller gain used by the area to accomplish the required control action on the error ACE) which vary between areas because of differences in dynamics and disturbance. The control laws are linear functions of measurable output for each control area and do not require measurement of disturbance. This new decentralized Stackelberg coordination is investigated for a three-area interconnected power system. Optimal solutions, suboptimal simplifications and simulation results are presented.

5.2 Power System Dynamic Model

A power system dynamic model was developed in [20,25]. where it is assumed that area buses are stiffly interconnected, and that the deviations in frequency and scheduled power interchange are caused solely by the load disturbances. If each area is modeled as an equivalent electric generating system wherein a nonreheat steam turbine is employed, then the following equations represent the interconnected power system linearized about a given nominal operating point:

$$\frac{d}{dt}(\Delta f_i) = \frac{1D_i}{2H_i} f^* \Delta f_i + \frac{1f^*}{2H_i}(\Delta P_{ti} - \Delta P_{tie,i} - \Delta P_{Li}) \tag{5.1}$$

$$\frac{d}{dt}(\Delta P_{ti}) = \frac{1}{Tti}(\Delta S_{gi} - \Delta P_{ti}) \tag{5.2}$$

$$\frac{d}{dt}(\Delta S_{gi}) = \frac{1}{T_{qi}}(\Delta P_{ci} - \Delta S_{gi} - \frac{1}{Ri}\Delta f_i) \tag{5.3}$$

$$\frac{d}{dt}(\Delta P_{tie,i}) = \sum_{j=1}^{N} Tij(\Delta f_i - \Delta fj) \tag{5.4}$$

$$ACE_i = b_i \Delta f_i + \Delta P_{tie,i} \tag{5.5}$$

The symbols are defined as follows:

f^*	nominal system frequency
H_i	inertia constant
D_i	system damping
T_t	turbine time constant
T_g	governor time constant
T_{ij}	transmission constant
R_i	speed droop
b_i	frequency bias constant

Δf_i	frequency deviation
ΔP_{ti}	turbine output deviation
ΔS_{gi}	governor position deviation
$\Delta P_{tie,i}$	net power flow deviation
ΔP_{ci}	control signal-command to speed changer
ΔP_{Li}	load disturbance
ϵ_{ik}	plant noise

The values of the parameters are as follows:

$P_{ro} = P_{r1} = P_{r2} =$ 2000 MW

$H_o = H_1 = H_2 =$ 5 seconds

$D_o = D_1 = D_2 = 8.33\text{x}10^{-3}$ pu MW/Hz

$T_{to} = T_{t1} = T_{t2} =$ 0.3 sec.

$T_{go} = T_{g1} = T_{g2} =$ 0.08 sec.

$R_o = R_1 = R_2 =$ 2.4 Hz/pu MW

$P_{tie,max} =$ 200 MW

$\delta_i^* - \delta_j^* =$ 30 degrees

$T_{ij} =$ 0.545 pu MW

$b_i = 0.425$

For more complete definition of the model and terms see [20,25].

An appropriate formalization of this problem involves defining the following linear quadratic regulator problem:

state equation $\quad x(t) = Ax(t) + Bu(t) + Dv(t) + \xi(t) \qquad (5.6)$

output equation $\quad y(t) = Hx(t) + \eta(t) \qquad (5.7)$

cost function $\quad J = \int_0^\infty [x^TQx + u^TRu]dt \qquad (5.8)$

where, for each control area the state vector

$x^i = (\Delta f_i, \Delta P_{ti}, \Delta S_{gi}, \Delta P_{tie,i}, IACE_i)$ $(IACE_i = \int ACE_i dt)$. These new state variables are included for the purpose of inducing the steady state errors [20]), the control vector $u_i = \Delta P_{ci}$, and the disturbance vector $v_i = \Delta P_{Li}$ with $i=1,\ldots,n$. The plant and measurement noise vector $\xi(t)$ and $\eta(t)$ respectively, are modeled as zero mean mutually independent stationary white Gaussian processes. The matrices R and Q in the cost functional are selected in such a way that emphasizes the ACE_i. For simplicity we choose R = I.Here it is assumed that each area has only one plant.

5.3 Stackelberg Coordination

In accordance with the basic operating policy, the desired goal is to regulate each area control error, ACE_i, to zero without using excessive control effort. Each control area problem can be formulated as a linear regulator problem with a cost functional of its own. Decision making by any area to obtain optimum control performance for its area will effect other areas. With multicriteria and multidecision making we have to define "optimality". In differential games theory "optimality" is defined in terms of the rationality assumed by the decision makers in computing their controls. Each area can choose a strategy depending on the dynamics of its system, its information and its computational capability. Since we have more than two areas, it seems appropriate to apply Stackelberg coordination for decentralized control to this problem. Designate an area to be a coordinator who coordinates the other areas which are viewed as followers. The coordinator chooses a leader Stackelberg strategy to play with the lower level subsystems. The lower level subsystems may or may not cooperate among themselves so they can either choose Nash rationale or Pareto rationale to play between them.

The controllers are constrained to be of the form

$$u^i(t) \quad = \quad F^i(t)y^i(t) \qquad i=0,1,2 \tag{5.9}$$

where $y^i(t)$ is the measurable output of each area and $F^i(t)$ is chosen so as to minimize the cost functions. The resulting necessary conditions for optimality of F^i, for discrete system, are derived in Section 3.4 and Section 4.4. A simple approximation computational algorithm is also suggested, but there is no guarantee that the algorithm will converge.

5.4 Design and Simulation Study

A three-area power system with numerical constant as given in [24] was chosen as the basis for this study. In discretization of the system, LINSYS [11] was used. Since we are only interested in load-frequency control, we can consider the turbine controller fast relative to the rest of the system. By assumption above the time constant of the system is approximately 1 sec. [24], so we chose a discretization interval of 0.2 sec.. After discretization LINSYS was used to determine the eigenvalues, controllability, and observability of the discrete-time system. The discrete-time system with discretization interval 0.2 sec. is stable and controllable.

Consider a discrete version of a three area interconnected power system:

state equation $$x(k+1) = Ax(k) + B^o u^o(k) + B^1 u^1(k) + B^2 u^2(k) + Ew(k) + v(k) \qquad (5.10)$$

measurement equation $$y^i(k) = H^i x(k) + \eta^i(k) \qquad i=0,1,2 \qquad (5.11)$$

cost function $$J^i = x^T(n)Q^i(n)x(n) + \sum_{K=0}^{n-1} [x^T(k)Q^i x(k) + u^{iT}(k)R^i u^i(k)] \qquad i=0,1,2 \qquad (5.12)$$

where for each area the state vector is

$$x^i(k) = (x^i_1, \ldots, x^i_4)$$
$$= (\Delta f_i, \Delta P_{ti}, \Delta S_{gi}, \Delta P_{tie,i})$$

The control vector is $u_i = \Delta P_{ci}$ and the disturbance vector is $w_i = \Delta P_{Li}$ where $i=0,1,2$. The plant and measurement noise vectors $v(k)$ and $\eta^i(k)$ are zero mean mutually independent stationary white Gaussian processes with 0.001 per unit standard deviation. The matrices appearing in the cost function are defined as in the continuous case. The measurable output vector is formed as a linear combination of states required to have zero steady-state values $\Delta f_i, \Delta P_{tie,i}$. The numerical value of the element of matrices appearing in (5.10) are given in Appendix 3. The object is to design a linear feedback control $u^i(k)$ $i=0,1,2$ to compensate the effect of constant or slowly varying disturbance $w(k)$ using only the output $y^i(k)$. For any constant or slowly varying disturbance $w(k)$, using the Smith/Davidson [55] approach, consider the augmented system:

$$\hat{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}^1\hat{u}^1(k) + \hat{B}^2\hat{u}^2(k) + \hat{B}^o\hat{u}^o(k) + \hat{v}(k) \qquad (5.13)$$

$$\hat{y}^i(k) = \hat{H}^i\hat{x}(k) + \hat{\eta}^i(k) \qquad i=0,1,2 \qquad (5.14)$$

where

$$\hat{x}(k) = \begin{bmatrix} x(k+1)-x(k) \\ y(k) \end{bmatrix}, \quad \hat{u}^i(k) = [u^i(k+1)-u^i(k)],$$

$$\hat{y}^i(k) = \begin{bmatrix} y^i(k+1)-y^i(k) \\ y^i(k) \end{bmatrix}$$

The linear control law $u^i(k)$ is

$$\hat{u}^i(k) = F^i(k)\hat{y}^i(k) \qquad i=0,1,2 \qquad (5.15)$$

where $F^i(k)$ is determined using the decentralized stochastic Stackelberg method.

Area 0 is chosen to be coordinator or leader. Then area 1 and 2 are followers with respect to area 0. When the lower level subsystems choose to play Nash rationale the resulting controllers are as defined in Section 3.4.1. When the lower level subsystems choose to play Pareto rationale the resulting controllers are as defined in Section 4.4.1. The matrices R^i and Q^i appearing in the cost functional (5.12) are selected in such a way that the cost function for each area is

$$J^i = \sum_{k=1}^{N} \Delta f_i^2(k+1) + \Delta P_{tie,i}^2(k+1) + ACE_i^2(k+1) + u_i^2(k) \qquad i=0,1,2$$

5.5 Discussion on Algorithm and Results

So far no convergence conditions for this algorithm have been found, but as with most algorithms of this type it is felt that convergence depends on the initial guess. A test for satisfactory convergence in cost is inserted when the computational procedure is implemented. The iterative procedure converged in cost. From the test results, one might hope that it would always converge to the 'optimal solution'. Unfortunately for certain systems the limiting values produced depended on the initial guess. In these cases, the algorithm converged to a solution to a two point boundary value problem one of whose solutions is the optimal. It is the nature of specific optimal problems to have local as well as global minima. Since uniqueness has not been proved, all solutions to the boundary value problem must be found to determine the global minimum. This difficulty with uniqueness could be anticipated since the necessary conditions are local. One must therefore find a good starting point if the procedure is to converge to the optimum.

The computational algorithm for the solution of this problem suggested in this work can not guarantee satisfactory results. For this particular example the algorithm has exhibited rapid convergence so no more exotic techniques have been tried. The method developed in this work is suitable for solving finite time problems. Unnecessary complexity is particularly burdensome in these problems as the time records of all the controller gains must be stored. The algorithm proposed can provide solutions for many problems at a reasonable cost, but it should be noted that the computer time will increase as the state dimension of the system, the number of gains and the number of time intervals increase.

Fig 5.1 shows the curves of frequency and tie-lie variations for a free system response upon a 1% step-load change in area 0. Fig 5.2 shows the system response under output feedback Stackelberg coordination with the lower-level using Nash rationale within their group. The diturbance is the same as in Fig 5.1. Fig 5.3 shows the system response under output feedback Stackelberg coordination with the lower-level using Pareto optimal within their group, α^i is chosen to be 0.5. The disturbance is the same as in Fig 5.1.

From the results of the computer simulation study, it is concluded that in this particular example decentralized stochastic Stackelberg coordination retains favourable transient features. However, the disturbed area still has a small steady-state error in deviation of frequency (.006 Hz.). A ratio of the coefficient of weighting matrices Q^i and R^i plays an important role in system response. It should be noted that improper choice of R^i and Q^i can make the system unstable or this algorithm may not give desired system response. However, a good choice of R^i and Q^i depends on the system. By trial and error the suitable values can be selected. However, the implementation of these control sequences in practice is complex, since the controls vary with time. Therefore we suggest a suboptimal simplification of the control. These suboptimal simplifications are selected from the constant part of each control sequences, respectively, and are used throughout the entire period. Fig. 5.4 shows the plots of the optimal gains of area 0,1,and 2. The constant gains of each area are chosen to be (-.1,-.39), (-.09,-.6) and (-.09,-.6) respectively. Fig. 5.5 shows the system

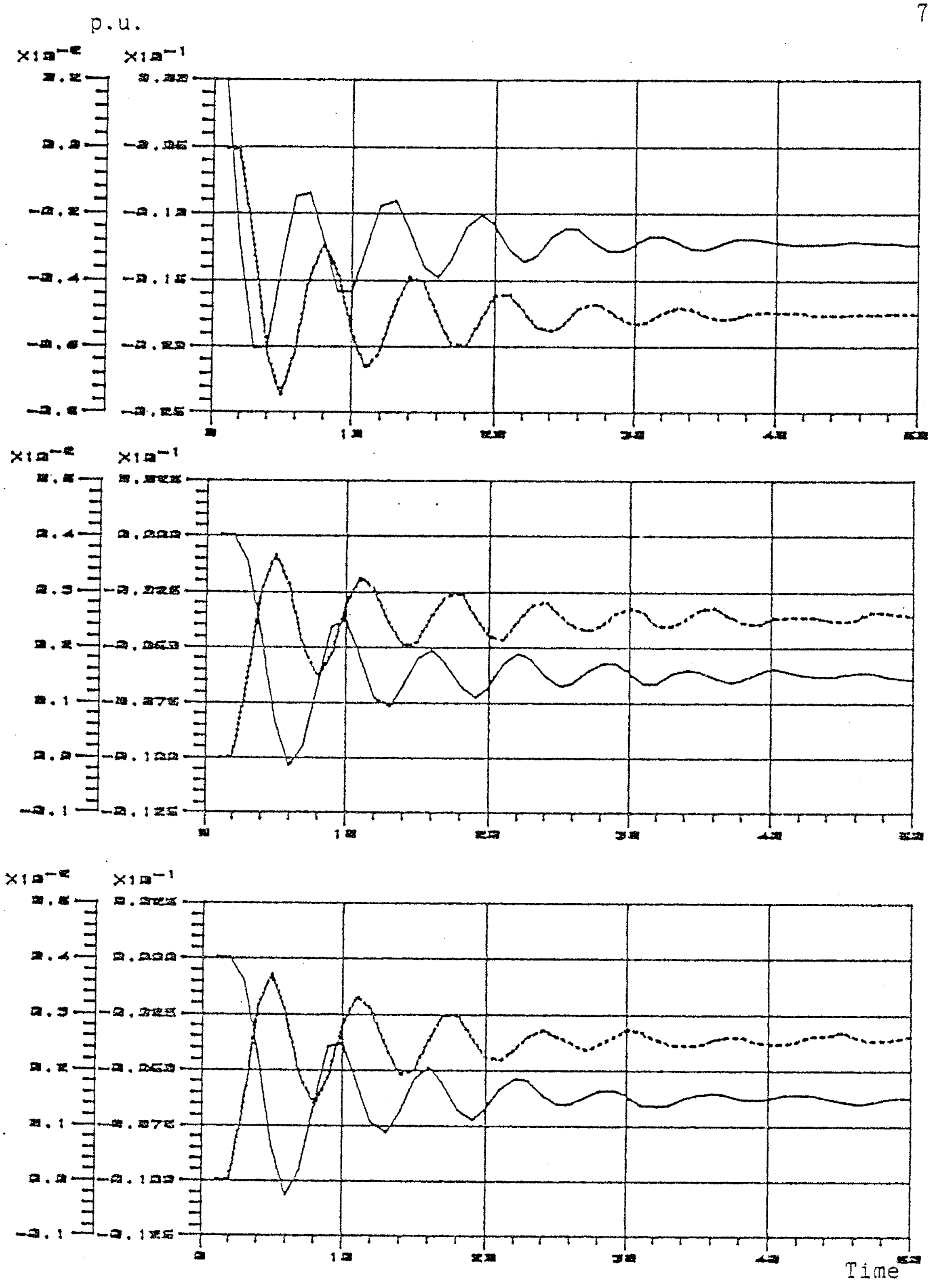

Fig. 5.1 Free system response of area 0,1,2

—— Δf_i, ... $\Delta P_{tie,i}$

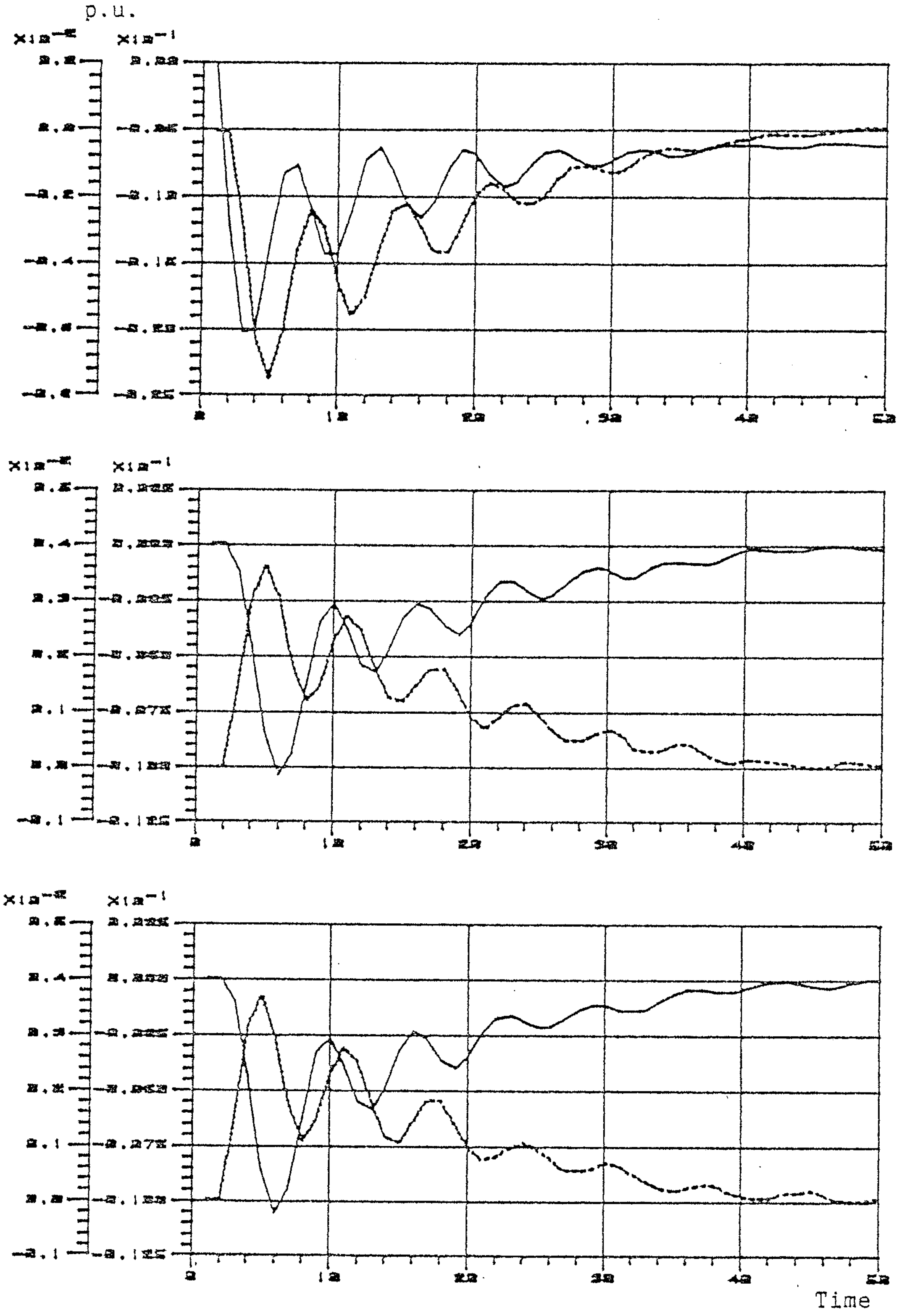

Fig 5.2 The system response of area 0,1,2 under decentralized Stackelberg coordination with Nash rationale among lower-level subsystems.

—— Δf_i, ... $\Delta P_{tie,i}$

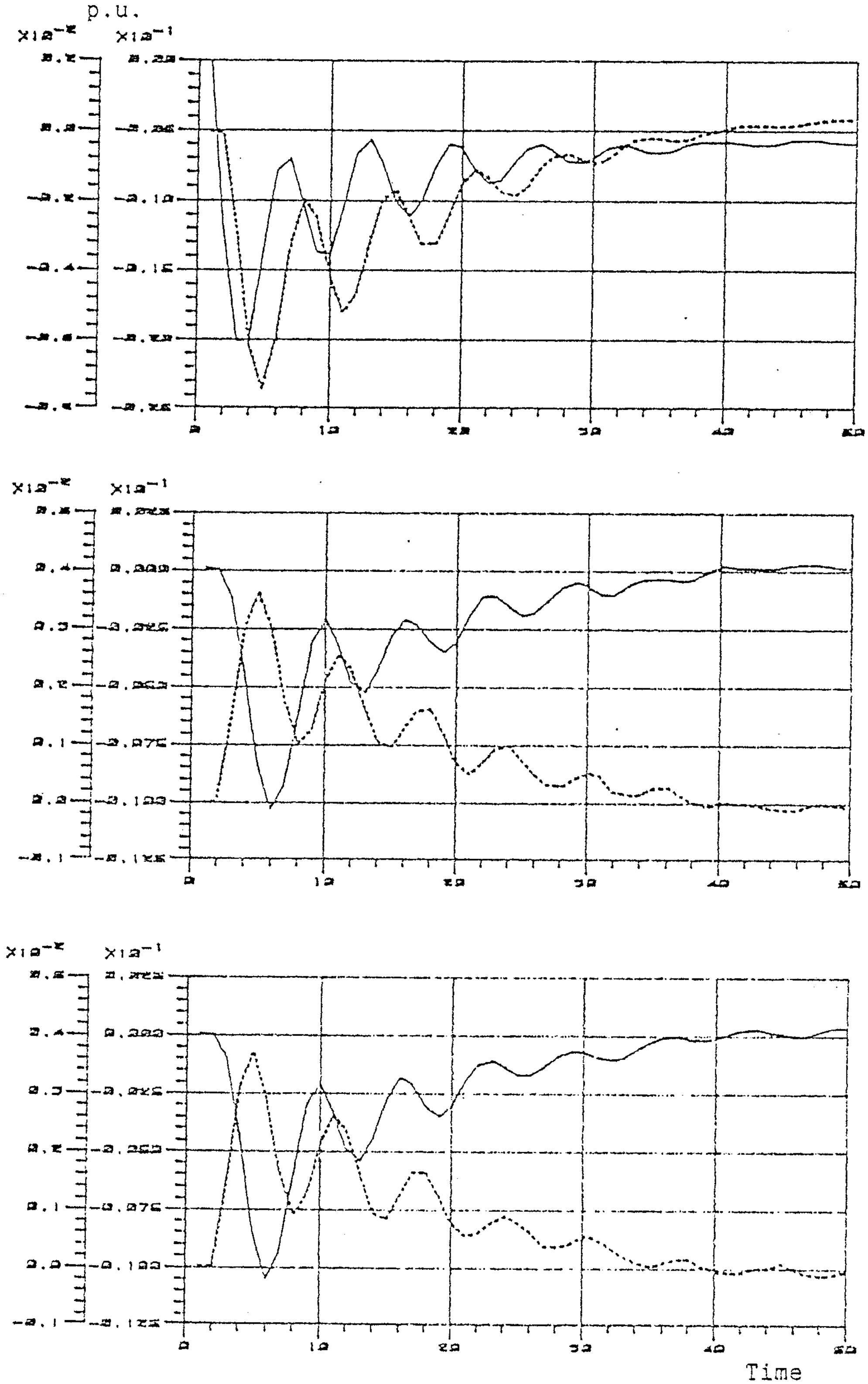

Fig. 5.3 The system response of area 0,1,2 under decentralized Stackelberg coordination with Pareto rationale among lower-level subsystems.

—— Δf_i, ... $\Delta P_{tie,i}$

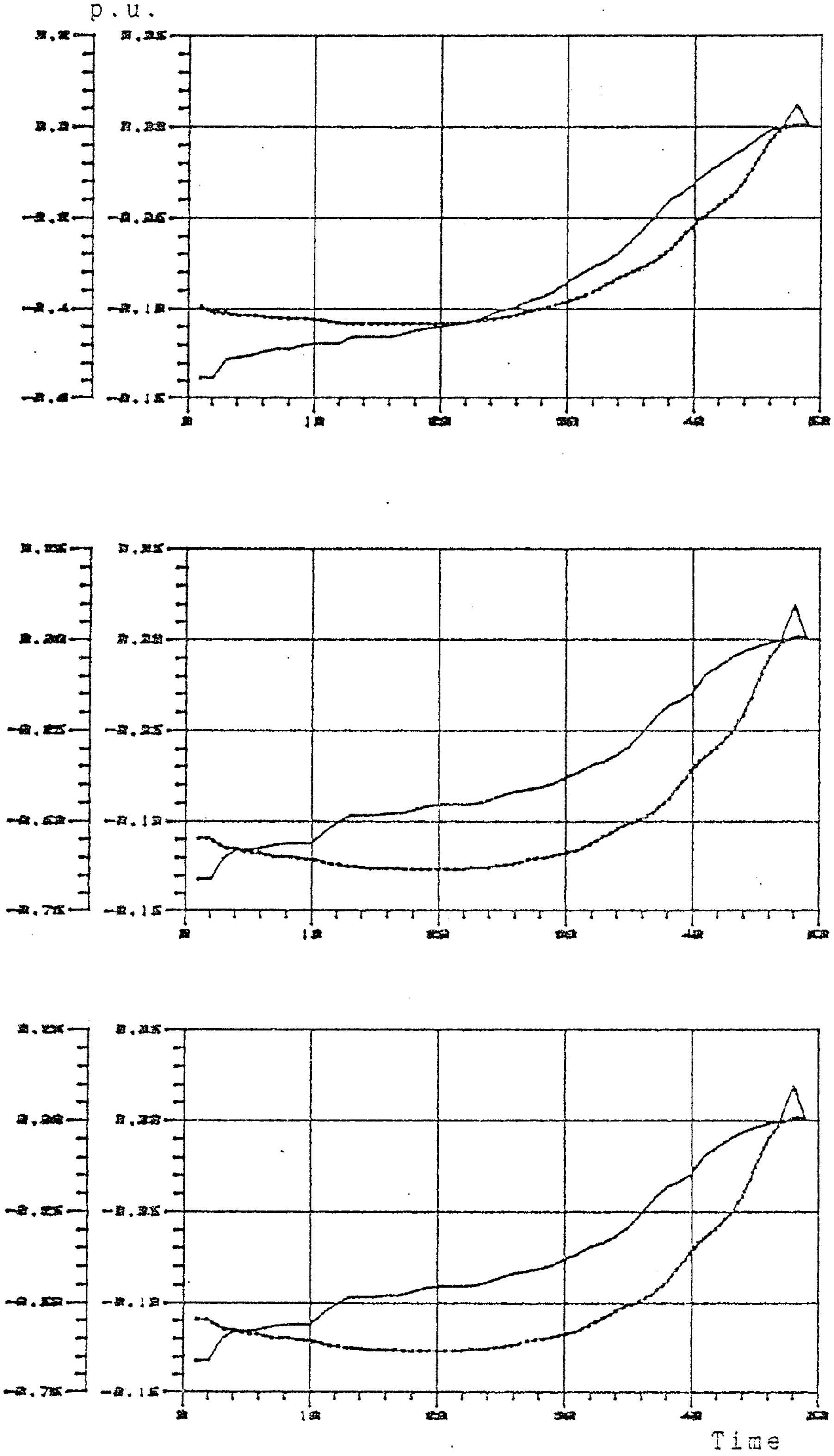

Fig. 5.4 The optimal gain of area 0,1,2 under decentralized Stackelberg coordination

—— $F^i(1)$, ... $F^i(2)$

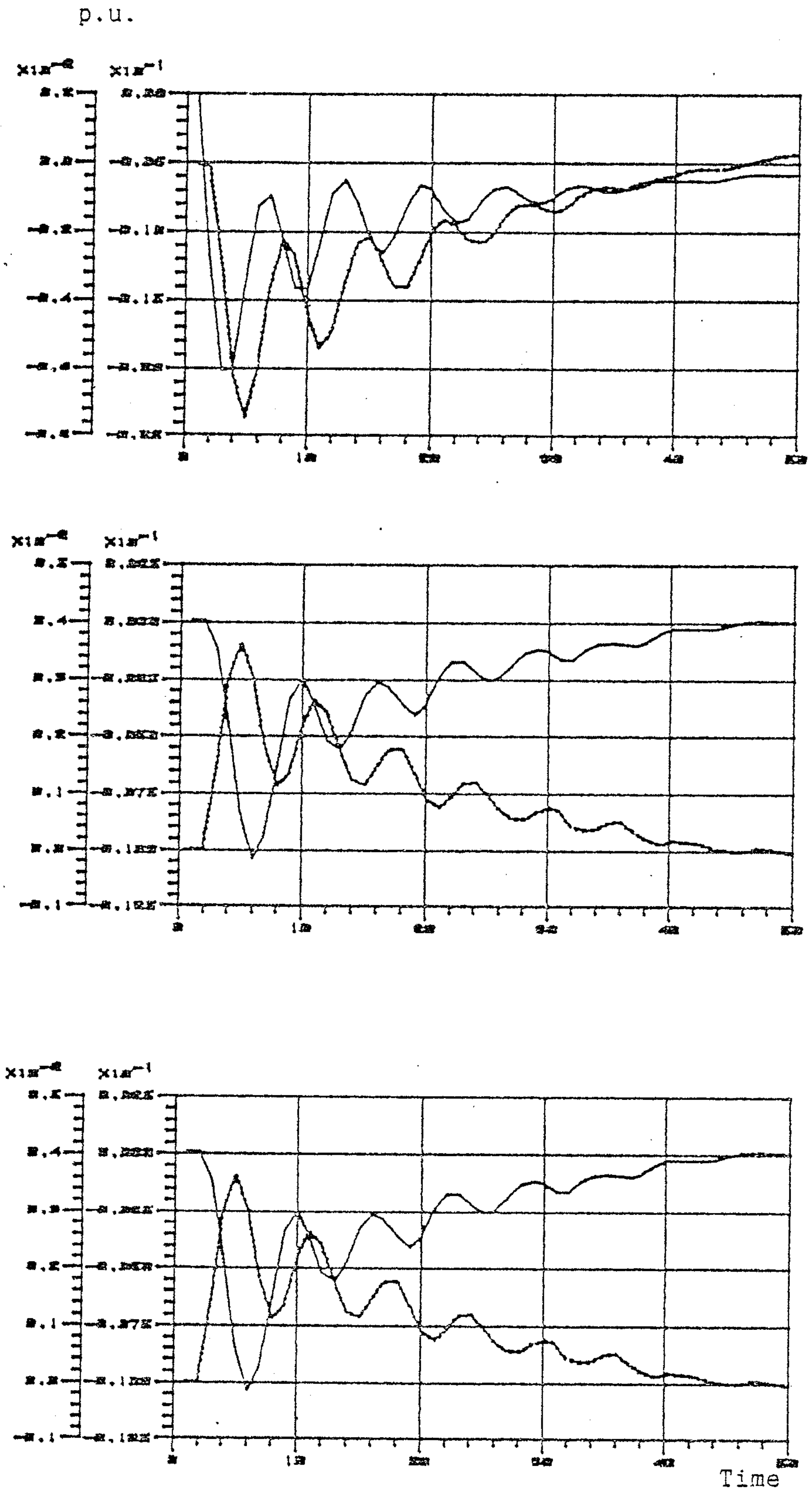

Fig.5.5 The system response of area 0,1,2 under suboptimal simplification of control.

—— Δf_i, ... $\Delta P_{tie,i}$

responses under suboptimal simplification. The responses do not have significant difference from the responses under optimal solution.

5.6 Conclusions

In this chapter, an attempt to develop a new decentralized linear regulator approach for load-frequency control in a three-area interconnected power system has been discussed. The method is based on decentralized stochastic Stackelberg coordination. Each control area uses a feedback control based only on measurements from its own area. Also, the area is free to select an appropriate cost function. The extended theory is applied to a discrete model of a three-area interconnected power system. A numerical design method utilizing a proportional-plus-integral control structure is suggested. From the studied example, this method gives satisfactory results. The adjustment of a desired speed in dynamic response is possible by adjusting the elements of the weighting matrices Q^i and R^i. Unfortunately the stability and convergence of the procedure has not been established yet. Since constant control laws are preferable in practice, we also suggest a suboptimal simplification in the controls which performs quite well in our particular example.

6. CONCLUSIONS

In the first part of this thesis, we have reviewed the equilibrium solutions of a two-person LQNZSDG in which we have modelled the effect of random disturbances by including an additive zero mean white noise in the state dynamics, whose statistics are not necessarily known to the players. Both cooperative and noncooperative solution concepts, i.e. Pareto optimal, Nash equilibrium and Stackelberg equilibrium, are examined. Results available in the literatures indicate that solutions for this class of game, and for different strategies, are affine for each player.

In the second part of this thesis, an interconnected set of linear discrete-time stochastic systems, where N decision-makers try to minimize different criteria, was introduced as an extension of differential game theory. The organizational form of the system permits one decision maker to be the coordinator or leader and the decision makers for the other subsystems are all followers with respect to the coordinator. The followers may or may not cooperate among themselves, so they can select Nash strategy or Pareto optimal with respect to the other second level decision makers. Centralized and decentralized control structures were considered. A decentralized structure is more realizable since the control sequences are functions of measurable output only. The equilibrium solutions are obtained via dynamic programming. The solutions of the centralized structure , both perfect and nested information, can be obtained backwards in time with given final conditions. But decentralized constraints lead to a discrete two-point boundary value problems. A simple procedure to solve this problem is suggested but the conditions for convergence are not yet available. As with most problems of this type, the solutions depend very much on the initial guess.

Finally, decentralized Stackelberg coordination is applied to a three-area interconnected power system. This method allows each control area to select an appropriate cost function and feedback only its own area measurement which is more realistic in practical situation. The design procedure is emphasis on the proportional plus integral feedback control. The study gave a satisfactory results.

Further study of decentralized Stackelberg coordination should include the stability and convergence condition of the procedure. Comparison of this control with other controls is also suggested. Another interesting extension of this work would be to investigate the stochastic Stackelberg coordination of nonlinear systems. Since the differential dynamic programming failed to obtain the solutions to N-person nonzero-sum Nash equilibrium solution, the same problem still exists for using this method to solve nonlinear stochastic Stackelberg coordination.

APPENDIX 1

Consider augmented system (3.12)

$$x(k+1) = A(k)x(k) + B^{o}(k)u^{o}(k) + B^{1}(k)u^{1}(k) + B^{2}(k)u^{2}(k) + v(k) \qquad (A.1.1)$$

then

$$E[x(k+1)/z(k)=x] = Ax(k) + B^{o}u^{o}(k) + B^{1}u^{1}(k) + B^{2}u^{2}(k) \qquad (A.1.2)$$

and quadratic cost (3.14)

$$J^{i}(u^{i}) = \tfrac{1}{2}x^{T}(N)Q^{i}(N)x(N) + \tfrac{1}{2}\sum_{k=o}^{N-1}[x^{T}(k)Q^{i}(k)x(k)+u^{iT}(k)R^{i}(k)u^{i}(k)] \qquad (A.1.3)$$

Assume that the expected cost-to-go at stage k is

$$E[V^{i}(k)/x(k)] = \tfrac{1}{2}x^{T}(k)S^{i}(k)x(k) + \tfrac{1}{2}\gamma^{i}(k) \qquad i=1,2 \qquad (A.1.4)$$

then

$$E[V^{i}(k)/x(k)] = \min_{u^{i}} E[\tfrac{1}{2}x^{T}(k)Q^{i}(k)x(k)+\tfrac{1}{2}u^{iT}(k)R^{i}(k)u^{i}(k)+V^{i}(k+1)/x(k)]$$

$$= \min_{u^{i}} [\tfrac{1}{2}x^{T}(k)Q^{i}(k)x(k)+\tfrac{1}{2}u^{iT}(k)R^{i}(k)u^{i}(k)+E[V^{i}(k+1)/x(k)] \qquad i=1,2 \qquad (A.1.5)$$

when k=N

$$V^{i}(N) = \tfrac{1}{2}x^{T}(n)Q^{i}(N)x(N) \qquad i=1,2 \qquad (A.1.6)$$

when k=k+1

$$E[V^{i}(k+1)/x(k)] = \tfrac{1}{2}(Ax(k)+B^{i}u^{i}(k)+B^{j}u^{j}(k)+B^{o}u^{o}(k))^{T}S^{i}(k+1)(Ax(k)+B^{i}u^{i}(k)+B^{j}u^{j}(k)+B^{o}u^{o}(k)) + \tfrac{1}{2}\mathrm{tr}S^{i}(k+1)\Lambda(k) + \tfrac{1}{2}\gamma^{i}(k+1) \qquad i=1,2 \quad i\neq j \qquad (A.1.7)$$

Using (A.1.7) in (A.1.5) to obtain $u^i(k)$ that minimize the expected value of the cost function

$$u^i(k) = -[R^i+B^{iT}S^i(k+1)B^i]^{-1}B^{iT}S^i(k+1)[Ax(k)+B^ju^j(k)+B^ou^o(k)] \quad i=1,2 \quad i\neq j \qquad (A.1.8)$$

Let

$$L^i(k) = [R^i+B^{iT}S^i(k+1)B^i]^{-1}B^{iT}S^i(k+1) \qquad (A.1.9)$$

Then (A.1.8) becomes

$$u^i(k) = -L^i(k)[Ax(k)+B^ju^j(k)+B^ou^o(k)] \quad i=1,2 \quad i\neq j \qquad (A.1.10)$$

For 2-subsystems solve for $u^1(k)$ and $u^2(k)$

$$u^1(k) = -\Delta^1(k)(Ax(k)+B^ou^o(k)) \qquad (A.1.11)$$

and

$$u^2(k) = -\Delta^2(k)(Ax(k)+B^ou^o(k)) \qquad (A.1.12)$$

where

$$\Delta^i(k) = [I-L^iB^jL^jB^i]^{-1}[L^i-L^iB^jL^i] \quad i=1,2 \quad i\neq j \qquad (A.1.13)$$

Using (A.1.11) and (A.1.12) in (A.1.1) and defining

$$\hat{A}(k) = A + B^1\Delta^1A + B^2\Delta^2A \qquad (A.1.14)$$

$$\hat{B}(k) = B^o - B^1\Delta^1B^o - B^2\Delta^2B^o \qquad (A.1.15)$$

We have

$$x(k+1) = \hat{A}(k)x(k) + \hat{B}(k)u^o(k) + v(k) \qquad (A.1.16)$$

Now

$$E[V^o(k)x(k)] = \tfrac{1}{2}x^T(k)S^o(k)x(k) + \tfrac{1}{2}\gamma^o(k) \qquad (A.1.17)$$

Then

$$E[V^o(k)/x(k)] = \min_{u^o(k)} E[\tfrac{1}{2}x^T(k)Q^o(k)x(k)+\tfrac{1}{2}u^{oT}(k)R^o(k)u^o(k)+V^o(k+1)/x(k)]$$

$$= \min_{u^o(k)} [\tfrac{1}{2}x^T(k)Q^o(k)x(k)+\tfrac{1}{2}u^{oT}(k)R^o(k)u^o(k)+E[V^o(k+1)/x(k)]]$$

(A.1.18)

At k=N,

$$V^{o}(N) = \frac{1}{2}x^{T}(N)Q^{o}(N)x(N) \tag{A.1.19}$$

at k=k+1

$$E[V^{o}(k+1)/x(k)] = \frac{1}{2}(A(k)x(k)+B(k)u^{o}(k))^{T}S^{o}(k+1)(A(k)x(k)+B(k)u^{o}(k)) + \frac{1}{2}\mathrm{tr}S^{o}(k+1)\Lambda(k)+\frac{1}{2}\gamma^{o}(k+1) \tag{A.1.20}$$

Using (A.1.20) in (A.1.18) we obtain

$$u^{o*}(k) = -[R^{o}+\hat{B}^{T}S^{o}(k+1)\hat{B}]^{-1}\hat{B}^{T}S^{o}(k+1)\hat{A}x(k) \tag{A.1.21}$$

Let

$$L^{o}(k) = [R^{o}+\hat{B}^{T}S^{o}(k+1)\hat{B}]^{-1}\hat{B}^{T}S^{o}(k+1)\hat{A} \tag{A.1.22}$$

Then

$$u^{o*}(k) = -L^{o}(k)x(k) \tag{A.1.23}$$

To obtain recursive equation for $S^{o}(k)$, use (A.1.23) in (A.1.18) and after some algebra

$$S^{o}(k) = Q^{o}(k) + \hat{A}^{T}S^{o}(k+1)\hat{A}-L^{oT}[R^{o}+\hat{B}^{T}S^{o}(k+1)\hat{B}]L^{o} \tag{A.1.24}$$

$$S^{o}(N) = Q^{o}(N) \tag{A.1.25}$$

$$\gamma^{o}(k) = \gamma^{o}(k+1) + \mathrm{tr}\, S^{o}(k+1)\Lambda(k) \tag{A.1.26}$$

$$\gamma^{o}(N) = 0 \tag{A.1.27}$$

To obtain recursive equations for $S^{i}(k)$ $i=1,2$, use (A.1.23), (A.1.11), (A.1.12), and (A.1.5). after some algebra

$$S^{i}(k) = Q^{i}(k) + (A-B^{o}L^{o})^{T}\Delta^{iT}(k)R^{i}\Delta^{i}(k)(A-B^{o}L^{o}) + (\hat{A}-\hat{B}L^{o})S^{i}(k+1)(\hat{A}-\hat{B}L^{o}) \qquad i=1,2 \tag{A.1.28}$$

$$S^{i}(N) = Q^{i}(N) \qquad i=1,2 \tag{A.1.29}$$

$$\gamma^{i}(k) = \gamma^{i}(k+1) + \mathrm{tr}S^{i}(k+1)\Lambda(k) \qquad i=1,2 \tag{A.1.30}$$

$$\gamma^{i}(N) = 0 \qquad i=1,2 \tag{A.1.31}$$

APPENDIX 2

Given a stochastic Markov sequence of state vector $\{x(k)\}$

$$x(k+1) = A(k)x(k) + B^{o}(k)u^{o}(k) + B^{1}(k)u^{1}(k) + B^{2}(k)u^{2}(k) + v(k) \qquad (A.2.1)$$

where $u^{i}(k)$, $i=0,1,2$ are deterministic inputs, $v(k)$ random, and measurements given by

$$z^{1}(k) = z^{2}(k) = H(k)x(k) + \xi(k) \qquad (A.2.2)$$

$$z^{o}(k) \supset z1(k); z^{o}(k) = H^{o}(k)x(k) + \xi^{o}(k) \qquad (A.2.3)$$

The assumptions are the same as given in Section 3.2. Define

$$z^{*}(k) = [z^{1T}(0),\ldots,z^{1T}(k)]^{T} \qquad (A.2.4)$$

$$z^{o*}(k) = [z^{oT}(0),\ldots,z^{oT}(k)]^{T} \qquad (A.2.5)$$

$$\hat{x}(k) = E[x(k)/z^{*}(k)] \qquad (A.2.6)$$

$$\hat{x}^{o}(k) = E[x(k)/z^{o*}(k)] \qquad (A.2.7)$$

$$P(k/k) = E\{(x(k)-\hat{x}(k))(x(k)-\hat{x}(k))^{T}/z^{*}(k)\} \qquad (A.2.8)$$

$$\hat{x}(k+1/k) = E[x(k+1)/z^{*}(k)]. \qquad (A.2.9)$$

The recursive relations define the conditional expectations for lower level assumptions given by

$$\hat{x}(k+1/k) = A(k)\hat{x}(k) + B^{o}(k)u^{o}(k) + B^{1}(k)u^{1}(k) + B^{2}(k)u^{2}(k) \quad (A.2.10)$$

$$P(k+1/k) = A(k+1)P(k/k)A^{T}(k+1) + \Lambda(k) \qquad (A.2.11)$$

$$\hat{x}(k+1) = \hat{x}(k+1/k) + K(k+1)[z(k+1)-H(k+1)\hat{x}(k+1/k)] \qquad (A.2.12)$$

$$K(k+1) = P(k+1/k)H^{T}(k+1)[H(k+1)P(k+1)H^{T}(k+1)+\Xi(k+1)]^{-1} \quad (A.2.13)$$

$$P(k+1/k+1) = [I-K(k+1)H(k+1)]P(k+1/k) \qquad (A.2.14)$$

$$P(0/0) = \Sigma(0). \qquad (A.2.15)$$

Also

$$E[\hat{x}(k+1)/z^{*}(k)] = \bar{x}(k+1) = A\hat{x}(k) + B^{o}u^{o}(k) + B^{1}u^{1}(k) + B^{2}u^{2}(k) \qquad (A.2.16)$$

$$\mathrm{Cov}[\hat{x}(k+1)/z^*(k)] = K(k+1)[H(k+1)P(k+1/k)H^T(k+1)+\Xi(k+1)]K^T(k+1) \tag{A.2.17}$$

The recursive relation defining the conditional expectation for the coordinator subsystem is given by

$$\begin{aligned}
\hat{x}^o(k+1) &= \hat{x}^o(k+1/k) + K^o(k+1)[z^o(k+1)-H^o(k+1)\hat{x}^o(k+1/k)] \\
K^o(k+1) &= P^o(k+1/k)H^{oT}(k+1)[H^o(k+1)P^o(k+1/k)H^{oT}(k+1)+\Xi(k+1)] \\
P^o(k+1/k) &= A(k+1)P^o(k/k)A^T(k+1) + \Lambda(k) \\
P^o(k+1/k+1) &= [I-K^o(k+1)]P^o(k+1/k) \\
P^o(0/0) &= \Sigma(0)
\end{aligned}$$

Also

$$\begin{aligned}
E[\hat{x}^o(k+1)/z^*(k)] &= A\hat{x}^o(k) + B^o u^o(k) + B^1 u^1(k) + B^2 u^2(k) \\
\mathrm{Cov}[x^o(k+1)/z^*(k)] &= K^o(k+1)[H^o(k+1)P(k+1/k)H^{oT}(k+1)+\Xi(k+1)]K^{oT}(k+1)
\end{aligned}$$

Assume at stage k the cost-to-go for the i-th subsystem is

$$J^{i*}(k) = \tfrac{1}{2}\hat{x}^T(k)S^i(k)\hat{x}(k) + \tfrac{1}{2}\gamma^i(k) \tag{A.2.18}$$

The optimal strategies for subsystem i are given by

$$u^i(k) = \arg\min_{u^i(k)} E[\tfrac{1}{2}x^T(k)Q^i(k)x(k)+\tfrac{1}{2}u^{iT}(k)R^i(k)u^i(k)+J^{i*}(k+1)/z^*(k)] \tag{A.2.19}$$

At k=N

$$\begin{aligned}
J^{i*}(N) &= E[\tfrac{1}{2}x^T(N)Q^i(N)x(N)/z^*(N)] \\
&= \tfrac{1}{2}\hat{x}^T(N)Q^i(N)\hat{x}(N) + \tfrac{1}{2}\mathrm{tr}Q^i(N)P^i(N)
\end{aligned} \tag{A.2.20}$$

$$\begin{aligned}
u^i(k) = \arg\min_{u^i(k)} \Big[&\tfrac{1}{2}\hat{x}^T(k)Q^i(k)\hat{x}(k)+\tfrac{1}{2}\mathrm{tr}Q^i(k)P^i(k)+\tfrac{1}{2}u^{iT}(k)R^i(k)u^i(k) \\
&+ \tfrac{1}{2}[A\hat{x}(k)+B^o u^o(k)+B^j u^j(k)]^T S^i(k+1) \\
&\quad [A\hat{x}(k)+B^o u^o(k)+B^i u^i(k)+B^j u^j(k)] \\
&+ \tfrac{1}{2}\mathrm{tr}S^i(k+1)K^i(k+1)[H^i(k+1)P^i(k+1/k)H^{iT}(k+1)+\Xi^i(k+1)]K^{iT}(k+1) \\
&+ \tfrac{1}{2}\gamma^i(k+1)\Big]
\end{aligned} \tag{A.2.21}$$

The minimizing control $u^i(k)$ is

$$u^i(k) = -[R^i(k)+B^{iT}S^i(k+1)B^i]^{-1}B^{iT}S^i(k+1)[A\hat{x}(k)+B^o u^o(k)+B^j u^j(k)] \tag{A.2.22}$$

Recall the difinition of $L^i(k)$ in (A.1.9)

$$L^i(k) = [R^i(k)+B^{iT}S^i(k+1)B^i]^{-1}B^{iT}S^i(k+1) \tag{A.2.23}$$

Then

$$u^i(k) = -L^i(k)[A\hat{x}(k)+B^o u^o(k)+B^j u^j(k)] \tag{A.2.24}$$

For 2-subsystem solve for $u^1(k)$ and $u^2(k)$

$$u^1(k) = -\Delta^1(k)[A\hat{x}(k)+B^o u^o(k)] \tag{A.2.25}$$

$$u^2(k) = -\Delta^2(k)[A\hat{x}(k)+B^o u^o(k)] \tag{A.2.26}$$

where

$$\Delta^i(k) = [I-L^iB^jL^jB^i]^{-1}[L^i-L^iB^jL^i] \quad i=1,2 \quad i\neq j \tag{A.2.27}$$

Assume that at stage k the cost-to-go for the coordinator subsystem is

$$J^{o*}(k) = \frac{1}{2}\begin{bmatrix} \hat{x}^o(k) \\ \hat{x}(k)-\hat{x}^o(k) \end{bmatrix}^T \begin{bmatrix} S^A(k) & S^B(k) \\ S^{BT}(k) & S^C(k) \end{bmatrix} \begin{bmatrix} \hat{x}^o(k) \\ \hat{x}(k)-\hat{x}^o(k) \end{bmatrix} + \frac{1}{2}\tau^o(k) \tag{A.2.28}$$

At k=N

$$u^{o*}(k) = \underset{u^o(k)}{\arg\min}\, E[\tfrac{1}{2}x^T(k)Q^o(k)x(k)+\tfrac{1}{2}u^{oT}(k)R^o(k)u^o(k)+J^{o*}(k+1)/z^{o*}(k)] \tag{A.2.29}$$

For any matrix Γ [3.12]

$$\begin{aligned} &E\{\hat{x}^{oT}(k+1)\Gamma\hat{x}(k+1)/z^{o*}(k)\} \\ &\quad = E\{[\hat{x}^o(k+1/k)+\tilde{K}(k+1)[z(k+1)-H(k+1)\hat{x}^o(k+1/k)]]^T \\ &\qquad \Gamma[\hat{x}(k+1/k)+K(k+1)[z(k+1)-H(k+1)\hat{x}(k+1)/k)]]/z^{o*}(k)\} \end{aligned} \tag{A.2.30}$$

where

$$\tilde{K}(k+1) = P^o(k+1/k)H^T(k+1)[H(k+1)P^o(k+1/k)H^T(k+1)+\Xi(k+1)]^{-1} \quad (A.2.31)$$

$$\begin{aligned} E\{\hat{x}^{oT}(k+1)\Gamma\hat{x}(k+1)/z^{o*}(k)\} = {} & \bar{x}^o(k+1)^T\Gamma\bar{x}(k+1) + \bar{x}^{oT}(k+1)\Gamma K(k+1)H(k+1) \\ & (\bar{x}^o(k+1)-\bar{x}(k+1)) + trP^o(k+1/k)\Gamma K(k+1)H(k+1) \end{aligned} \quad (A.2.32)$$

$$\begin{aligned} & E[\hat{x}(k+1)\Gamma\hat{x}(k+1)/z^{o*}(k)] \\ & = E\{[\hat{x}(k+1/k)+K(k+1)[z(k+1)-H(k+1)\hat{x}(k+1/k)]]^T \\ & \quad [\hat{x}(k+1/k)+K(k+1)[z(k+1)-H(k+1)\hat{x}(k+1/k)]]/z^{o*}(k)\} \\ & = \bar{x}^T(k+1)\Gamma\bar{x}(k+1) + 2\bar{x}^T(k+1)\Gamma K(k+1)H(k+1)(\bar{x}^o(k+1)-\bar{x}(k+1)) \\ & \quad + tr\{\Gamma K(k+1)[H(k+1)P^o(k+1/k)H^T(k+1)+\Xi(k+1)]K^T(k+1) \\ & \quad + (\bar{x}(k+1)\bar{x}^o(k+1)^T H^T(k+1)K^T(k+1)\Gamma K(k+1)H(k+1)(\bar{x}(k+1)-\bar{x}^o(k+1)) \end{aligned} \quad (A.2.33)$$

Expand (A.2.29) using (A.2.32) and (A.2.33)

$$\begin{aligned} u^{o*}(k) = \arg\min_{u^o(k)} & [\tfrac{1}{2}\hat{x}^{oT}(k)Q^o(k)\hat{x}^o(k)+\tfrac{1}{2}u^{oT}(k)R^o(k)u^o(k)+\tfrac{1}{2}trQ^o(k)P^o(k) \\ & + \tfrac{1}{2}\bar{x}^{oT}(k+1)(S^A-AC-2S^B)\bar{x}^o(k+1) + \bar{x}^{oT}(k+1)(S^B-S^C)\bar{x}(k+1) \\ & + \bar{x}^{oT}(k+1)(S^B-S^C)K(k+1)H(k+1)(\bar{x}^o(k+1)-\bar{x}(k+1)) \\ & + \bar{x}^T(k+1)S^C\bar{x}(k+1) + \bar{x}^T(k+1)S^CK(k+1)H(k+1)(\bar{x}^o(k+1)-\bar{x}(k+1)) \\ & + \tfrac{1}{2}(\bar{x}(k+1)-\bar{x}^o(k+1))^TH^T(k+1)K^T(k+1)S^CK(k+1)H(K+1) \\ & \quad (\bar{x}(k+1)-\bar{x}^o(k+1)) + \tfrac{1}{2}\gamma^o(k) \\ & + \tfrac{1}{2}tr\{K^o(k+1)[H^o(k+1)P^o(k+1/k)H^{oT}(k+1)+\Xi^o(k+1)]K^{oT}(k+1) \\ & \qquad (S^A+S^C-2S^B)\} + tr2P^o(k+1/k)K(k+1)H(k+1)(S^B-S^C) \\ & + \tfrac{1}{2}trK(k+1)[H(k+1)P^o(k+1/k)H^T(k+1)+\Xi(k+1)]K^T(k+1)S^C] \end{aligned} \quad (A.2.34)$$

Recall that

$$\bar{x}^o(k+1) = A(k)\hat{x}^o(k) - (B^1(k)\Delta^1(k)A(k)+B^2\Delta^2(k)A(k))\hat{x}(k) + B(k)u^o(k) \quad (A.2.35)$$

where

$$\hat{B}(k) = B^{o}(k) - B^{1}(k)\Delta^{1}(k)B^{o}(k) - B^{2}(k)\Delta^{2}(k)B^{o}(k) \tag{A.2.36}$$

Let

$$G(k) = B^{1}(k)\Delta^{1}(k) + B^{2}(k)\Delta^{2}(k) \tag{A.2.37}$$

Then (A.2.35) becomes

$$\bar{x}^{o}(k+1) = (I-G(k))A(k)\hat{x}^{o}(k) - G(k)A(k)(\hat{x}(k)-\hat{x}^{o}(k)) + \hat{B}(k)u^{o}(k) \tag{A.2.38}$$

and

$$\bar{x}(k+1) = (I-G(k))A(k)\hat{x}^{o}(k) - (I-G(k))A(k)(\hat{x}(k)-\hat{x}^{o}(k)) + \hat{B}(k)u^{o}(k) \tag{A.2.39}$$

$$\bar{x}(k+1)-\bar{x}^{o}(k+1) = A(k)(\hat{x}(k)-\hat{x}^{o}(k)) \tag{A.2.40}$$

Substitue (A.2.40) in (A.2.34) and differentiating $u^{o*}(k)$ is given by

$$u^{o*}(k) = -\Delta^{o}(k)Y(k)\hat{x}^{o}(k) - \Delta^{o}(k)M(k)[\hat{x}(k)-\hat{x}^{o}(k)] \tag{A.2.41}$$

where

$$\Delta^{o}(k) = [R^{o}(k)+\hat{B}^{T}(k)S^{A}(k+1)\hat{B}(k)]^{-1}$$

$$Y(k) = \hat{B}(k)S^{A}(k+1)[I-G(k)]A(k)$$

$$\begin{aligned} M(k) &= \hat{B}^{T}(k)S^{A}(k+1)G(k)A(k) + \hat{B}^{T}(k)(S^{B}(k+1)-S^{A}(k+1))A(k) \\ &\quad - \hat{B}^{T}(k)S^{B}(k+1)K(k+1)H(k+1)A(k) \end{aligned}$$

The recursive equations for S^{A}, S^{B}, S^{C}, $\gamma^{o}(k)$ are obtained by substituting $u^{o*}(k)$ back in (A.2.40)

$$S^{A}(k) = Q^{o}(k) + A^{T}(k)(I-G(k))^{T}S^{A}(k+1)(I-G(k))A(k) - Y^{T}(k)\Delta^{o}(k)Y(k) \tag{A.2.42}$$

$$\begin{aligned} S^{B}(k) &= A^{T}(k)(I-G(k))^{T}S^{B}(k+1)(I-G(k))A(k) \\ &\quad + A^{T}(k)(I-G(k))^{T}(S^{B}(k+1)-S^{A}(k+1))G(k)A(k) \\ &\quad - A^{T}(k)(I-G(k))^{T}S^{B}(k+1)K(k+1)H(k+1)A(k) - Y^{T}(k)\Delta^{o}(k)M(k) \end{aligned} \tag{A.2.43}$$

$$\begin{aligned} S^C(k) = & - M^T(k)\Delta^o(k)M(k) + A^T(k)G^T(k)S^A(k+1)G(k)A(k) \\ & + A^T(k)[I-K(k+1)H(k+1)]^T S^C(k+1)[I-K(k+1)H(k+1)]A(k) \\ & + A^T(k)(S^B(k+1)K(k+1)H(k+1)-S^B(k+1))G(k)A(k) \\ & - A^T(k)G^T(k)(S^B(k+1)-S^B(k+1)K(k+1)H(k+1))A(k) \end{aligned} \qquad (A.2.44)$$

$$\begin{aligned} \gamma^o(k) = & \gamma^o(k+1) + trQ^o(k)P^o(k) \\ & + tr[K^o(k+1)[H^o(k+1)P^o(k+1/k)H^{oT}(k+1)+\Xi^o(k)]K^{oT}(k+1) \\ & \quad (S^A(k+1)+S^C(k+1)-2S^B(k+1))] \\ & + 2trP^o(k+1/k)K(k+1)H(k+1)(S^B(k+1)-S^C(k+1)) \\ & + trK(k+1)[H(k+1)P^o(k+1/k)H^T(k+1)+\Xi(k+1)]K^T(k+1)S^C(k+1) \end{aligned}$$

(A.2.45)

To obtain the recursive equation for $S^i(k)$ of the i-th subsystem, substitue $u^{o*}(k)$, $u^i(k)$ back in (A.2.21)

$$\begin{aligned} S^i(k) = & Q^i(k) + (A(k)+\hat{B}(k)\Delta^o(k)Y(k))^T S^i(k+1)(A(k)+\hat{B}(k)\Delta^o(k)Y(k)) \\ & + (\Delta^i(k)A(k)+B^o(k)\Delta^o(k)Y(k))^T R^i(k)(\Delta^i(k)A(k)+B^o(k)\Delta^o(k)Y(k)) \\ & \qquad i=1,2 \end{aligned} \qquad (A.2.46)$$

$$\begin{aligned} \gamma^i(k) = & \gamma^i(k+1) + trQ^i(k)P(k) + trS^i(k+1)K(k+1) \\ & + trS^i(k+1)K(k+1)[H(k+1)P(k+1/k)H^T(k+1)+\Xi(k+1)]K^T(k+1) \\ & + tr[P(k/k)-P^o(k/k)](M(k)-Y(k))^T\Delta^{oT}(k) \\ & \quad (B^{oT}(k)R^i(k)B^o(k)+\hat{B}^T S^i(k+1)\hat{B})\Delta^o(k)(M(k)-Y(k)) \end{aligned} \qquad (A.2.47)$$

APPENDIX 3

$$A = \begin{bmatrix}
.68 & .89 & .19 & -1.34 & .09 & .04 & .00 & -.05 & .09 & .04 & .00 & -.05 & .00 & .00 & .00 \\
-.16 & .35 & .13 & .11 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\
-.33 & -.29 & .00 & .40 & -.02 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\
.24 & .15 & .02 & .81 & -.12 & -.07 & -.01 & .09 & -.12 & -.07 & -.01 & .09 & .00 & .00 & .00 \\
.09 & .04 & .00 & -.05 & .68 & .89 & .19 & -1.34 & .09 & .04 & .00 & -.05 & .00 & .00 & .00 \\
.00 & .00 & .00 & .00 & -.16 & .35 & .13 & .11 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\
-.02 & .00 & .00 & .00 & -.33 & -.29 & .00 & .40 & -.02 & .00 & .00 & .00 & .00 & .00 & .00 \\
-.11 & -.07 & -.01 & .09 & .24 & .15 & .02 & .81 & -.12 & -.07 & -.01 & .09 & .00 & .00 & .00 \\
.09 & .03 & .00 & -.05 & .09 & .04 & .00 & -.04 & .68 & .89 & .18 & -1.34 & .00 & .00 & .00 \\
.00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & -.16 & .35 & .12 & -.02 & .00 & .00 & .00 \\
-.12 & -.07 & -.01 & .09 & -.11 & -.07 & -.01 & .09 & .23 & .15 & .02 & .80 & .00 & .00 & .00 \\
.43 & .00 & .00 & 1.00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & 1.00 & .00 & .00 \\
.00 & .00 & .00 & .00 & .43 & .00 & .00 & 1.00 & .00 & .00 & .00 & .00 & .00 & 1.00 & .00 \\
.00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .43 & .00 & .00 & 1.00 & .00 & .00 & 1.00
\end{bmatrix}$$

$$B^{o} = \begin{bmatrix} .00 \\ .00 \\ .20 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \end{bmatrix} \quad B^{1} = \begin{bmatrix} .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .20 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \end{bmatrix} \quad B^{2} = \begin{bmatrix} .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .00 \\ .20 \\ .00 \\ .00 \\ .00 \\ .00 \end{bmatrix} \quad E = \begin{bmatrix} -1.34 & .00 & .00 \\ .00 & .00 & .00 \\ .00 & .00 & .00 \\ .00 & .00 & .00 \\ .00 & -1.34 & .00 \\ .00 & .00 & .00 \\ .00 & .00 & .00 \\ .00 & .00 & .00 \\ .00 & .00 & -1.34 \\ .00 & .00 & .00 \\ .00 & .00 & .00 \\ .00 & .00 & .00 \end{bmatrix}$$

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