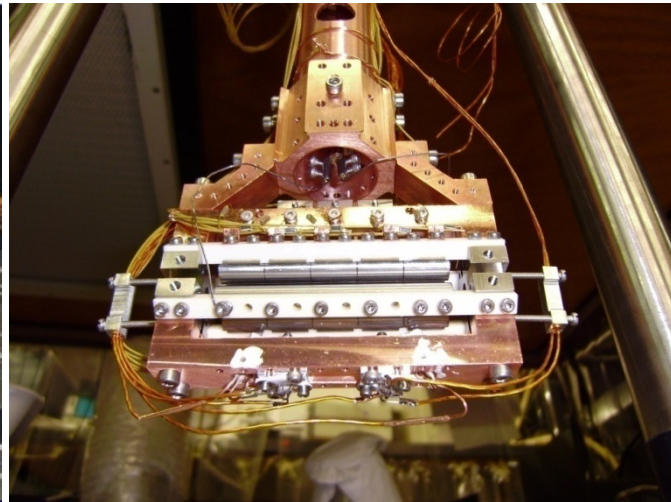
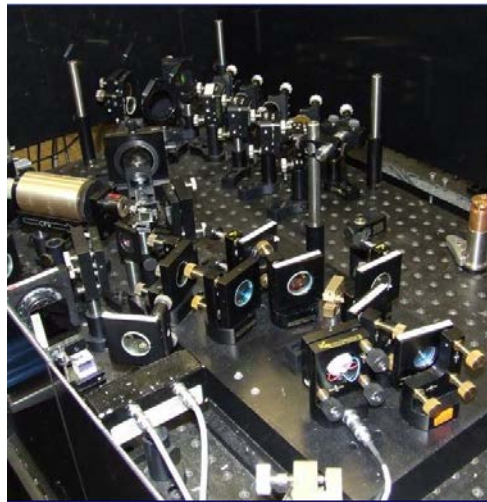
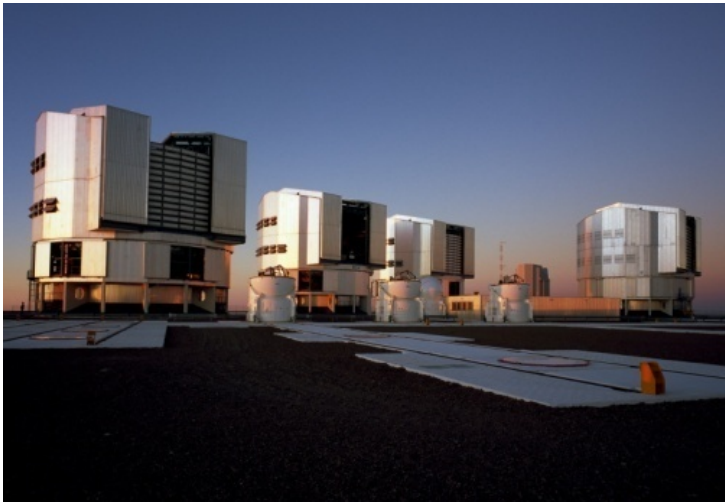


Precision spectroscopy of molecular hydrogen and the search for new physics



Wim Ubachs
Vrije Universiteit Amsterdam

International Symposium on Molecular Spectroscopy
Urbana-Champaign
22 June 2017

The Standard Model of Physics

What do we know ?

		Fermions			Bosons
Quarks	u up	c charm	t top	γ photon	
	d down	s strange	b bottom	g gluon	
Leptons	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	Z Z boson	
	e electron	μ muon	τ tau	W W boson	
I II III Three generations of matter				H Higgs boson	

What do we not know ?

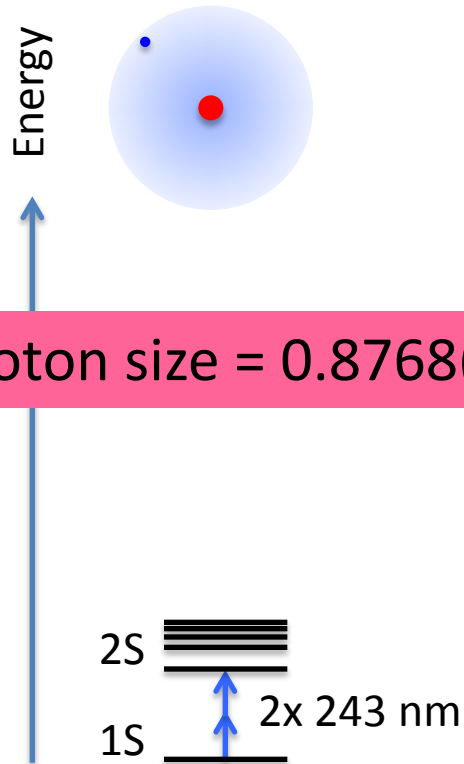
- Dark Matter
- Dark Energy
- How does Gravity fit to SM ?
- Why is Gravity so weak ?
- Are there only 3+1 dimensions ?
- Are there only 4 forces ?

Fundamental constants drift ?

- Variation on cosmological scale
- Dependencies on fields as indicators ?

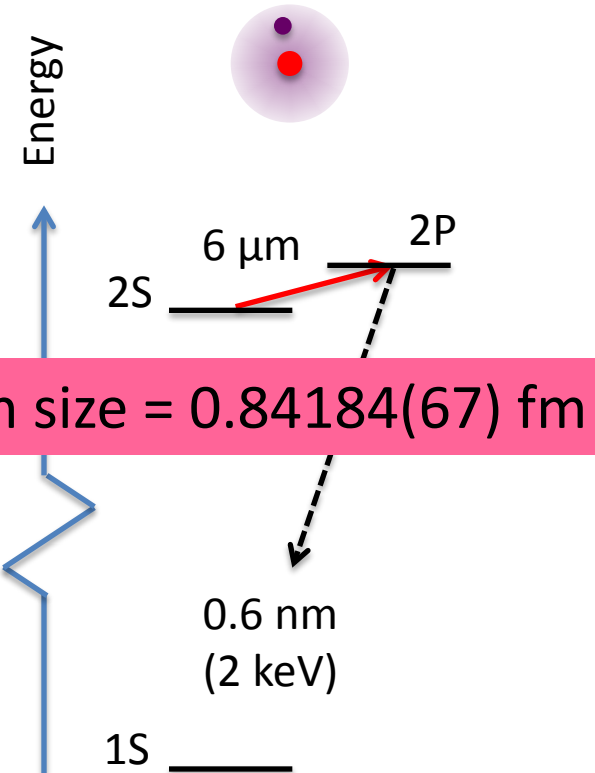
New urgent question: proton-radius puzzle

Hydrogen (H)



C.G. Parthey et al.,
PRL **107**, 203001 (2011)

Muonic hydrogen (μH)

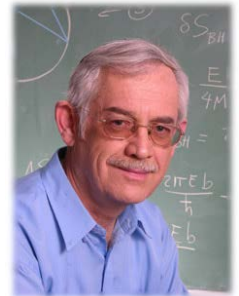


R. Pohl et al,
Nature, vol. **466**, pp. 213-216 (2010)
Science **339**, 417-420 (2013).

Varying Constants of Nature ?

Coupling constants are free parameters in Standard Model
But cannot be varied at will

$$S = \int \left(L_{mat} + \frac{j_\mu}{c} A^\mu - \frac{\epsilon_0}{4} F_{\mu\nu} F^{\mu\nu} e^{-2\phi} - \frac{\hbar c}{2l^2} \partial_\mu \partial^\mu \phi \right) d\Omega$$



Jacob Bekenstein

Bekenstein – Barrow – Flambaum : consistent models

$$\phi \rightarrow \Delta\alpha, \Delta\mu$$

1) Variation on cosmological
time scales
“Connection to
Dark Energy scenarios”

2) Coupling to environment
-> “chameleons”
Dependence on local density
Dependence on gravity

Empirical search for a change in μ

Compare H₂ in different epochs

Lab

today

QSO

12 Gyr ago

90-112 nm

~300-500 nm



Cosmological redshift

$$\frac{\lambda_i^z}{\lambda_i^0} \equiv 1 + z_i$$

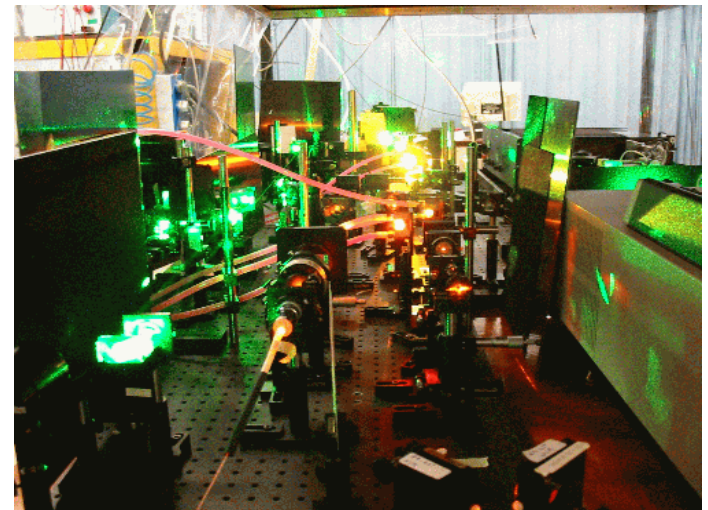
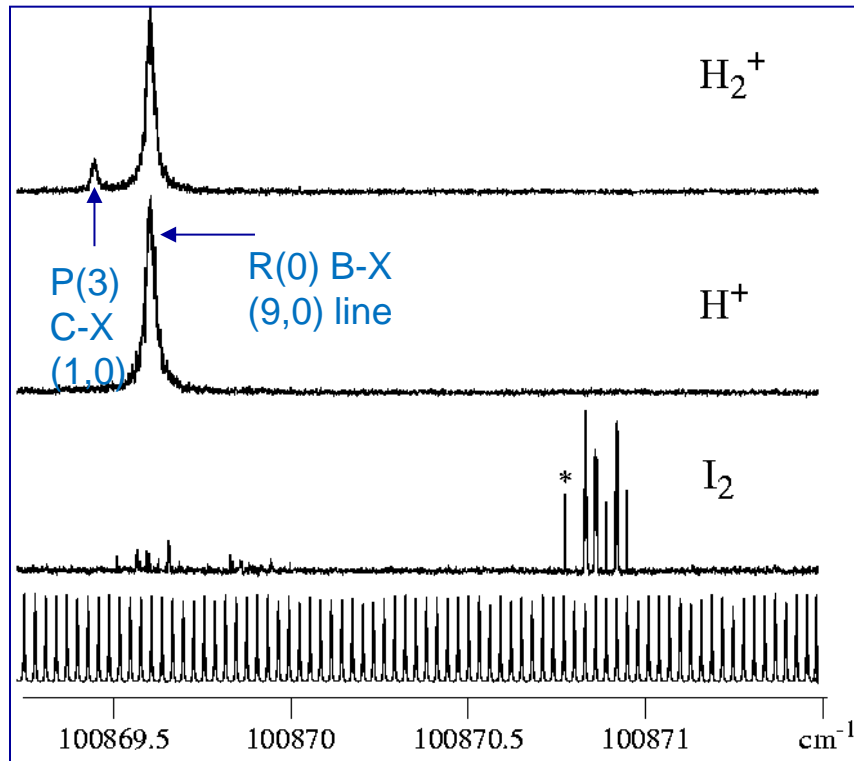
$$T = T_0 \left[1 - \frac{1}{(1 + z_{abs})^{3/2}} \right]$$

Practical: atmospheric transmission only for $z > 2$

H₂ laboratory wavelengths

The Amsterdam “XUV-laser”

XUV-laser excitation (90-115 nm)



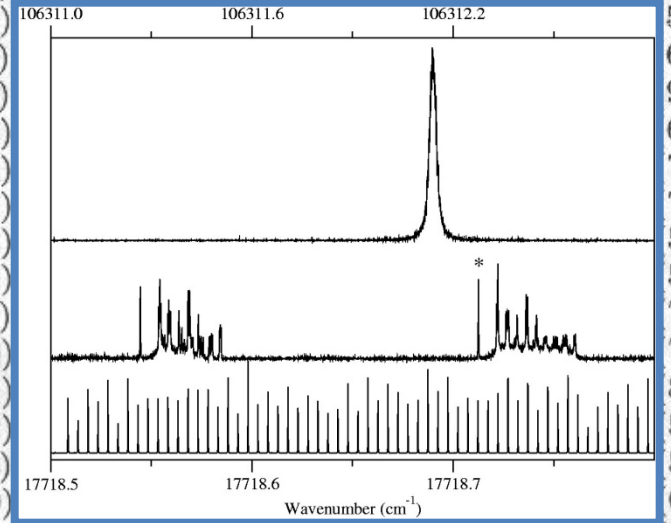
Ubachs, Phys. Rev. Lett. (2004)
 Reinhold et al, Phys. Rev. Lett. (2006)

For HD

Ivanov et al., Phys. Rev. Lett. (2008)

TABLE I: Comprehensive list of measured transition wavelengths of the Lyman (L) and Werner (W) lines using the ultranarrowband XUV laser source in Amsterdam. Values in nm.

Line	λ_0	Line	λ_0	Line	λ_0	Line	λ_0
L0 P(1)	111.006 251 (6)	L8 P(3)	100.838 615 (6)	L13 R(3)	95.894 665 (6)	W1 P(3)	99.138 046 (8)
L0 R(0)	110.812 733 (7)	L8 R(0)	100.182 387 (5)	L13 R(4)	96.215 297 (6)	W1 Q(1)	98.679 800 (5)
L0 R(1)	110.863 326 (7)	L8 R(1)	100.245 210 (5)	L14 P(1)	94.751 403 (10)	W1 Q(2)	98.797 445 (6)
L1 P(1)	109.405 198 (6)	L8 R(2)	100.398 545 (5)	L14 R(0)	94.616 931 (10)	W1 Q(3)	98.972 929 (8)
L1 P(2)	109.643 894 (6)	L8 R(3)	100.641 416 (6)	L14 R(1)	94.698 040 (10)	W1 R(0)	98.563 371 (5)
L1 P(3)	109.978 718 (7)	L9 P(1)	99.280 968 (5)	L14 R(2)			
L1 R(0)	109.219 523 (6)	L9 R(0)	99.137 891 (5)	L15 P(1)			
L1 R(1)	109.273 243 (6)	L9 R(1)	99.201 637 (5)	L15 P(3)			
L1 R(2)	109.424 460 (6)	L9 R(2)	99.355 061 (9)	L15 R(0)			
L1 R(3)	109.672 534 (6)	L9 R(3)	99.597 278 (20)	L15 R(1)			
L2 P(1)	107.892 547 (5)	L10 P(1)	98.283 533 (5)	L15 R(2)			
L2 R(0)	107.713 874 (5)	L10 P(2)	98.486 398 (5)	L15 R(3)			
L2 R(1)	107.769 894 (5)	L10 P(3)	98.776 882 (6)	L15 R(4)			
L2 R(2)	107.922 542 (6)	L10 R(0)	98.143 871 (5)	L16 P(1)			
L2 R(3)	108.171 124 (7)	L10 R(1)	98.207 427 (5)	L16 R(0)			
L2 R(4)	108.514 554 (6)	L10 R(2)	98.359 107 (5)	L16 R(1)			
L3 P(1)	106.460 539 (5)	L10 R(3)	98.596 279 (6)	L16 R(2)			
L3 P(2)	106.690 068 (5)	L11 P(1)	97.334 458 (5)	L17 P(1)			
L3 R(0)	106.288 214 (5)	L11 P(2)	97.534 576 (5)	L17 R(0)			
L3 R(1)	106.346 014 (5)	L11 P(3)	97.821 804 (6)	L17 R(1)	92.464 326 (9)	W2 R(3)	96.678 035 (7)
L3 R(2)	106.499 481 (5)	L11 R(0)	97.198 623 (5)	L18 P(1)	91.841 331 (9)	W3 P(2)	94.961 045 (5)
L3 R(3)	106.747 855 (5)	L11 R(1)	97.263 275 (5)	L18 R(0)			
L4 P(1)	105.103 253 (4)	L11 R(2)	97.415 791 (5)	L18 R(1)			
L4 R(0)	104.936 744 (4)	L11 R(3)	97.655 283 (6)	L18 R(2)			
L4 R(1)	104.995 976 (4)	L11 R(4)	97.980 512 (7)	L19 P(1)			
L4 R(2)	105.149 857 (5)	L11 R(5)	98.389 896 (7)	L19 P(2)			
L4 R(3)	105.397 610 (4)	L12 P(1)	96.431 064 (5)	L19 P(3)			
L5 P(1)	103.815 713 (4)	L12 P(2)	96.627 550 (5)	L19 R(0)			
L5 R(0)	103.654 581 (4)	L12 P(3)	96.908 984 (6)	L19 R(1)			
L5 R(1)	103.714 992 (4)	L12 R(0)	96.297 800 (5)	L19 R(2)	91.295 107 (17)	W3 R(4)	95.031 536 (5)
L5 R(2)	103.869 027 (4)	L12 R(1)	96.360 800 (5)	L19 R(3)	91.521 225 (17)	W4 P(2)	93.260 468 (10)
L5 R(3)	104.115 892 (4)	L12 R(2)	96.504 574 (5)	W0 P(2)	101.216 942 (6)	W4 P(3)	93.479 006 (10)
L6 P(1)	102.593 517 (8)	L12 R(3)	96.767 695 (6)	W0 P(3)	101.450 423 (6)	W4 Q(1)	93.057 708 (10)
L6 R(0)	102.437 395 (8)	L12 R(4)	97.083 820 (8)	W0 Q(1)	100.977 088 (5)	W4 Q(2)	93.178 086 (10)
L6 R(1)	102.498 790 (8)	L12 R(5)	97.488 649 (9)	W0 Q(2)	101.093 845 (6)	W4 Q(3)	93.357 794 (10)

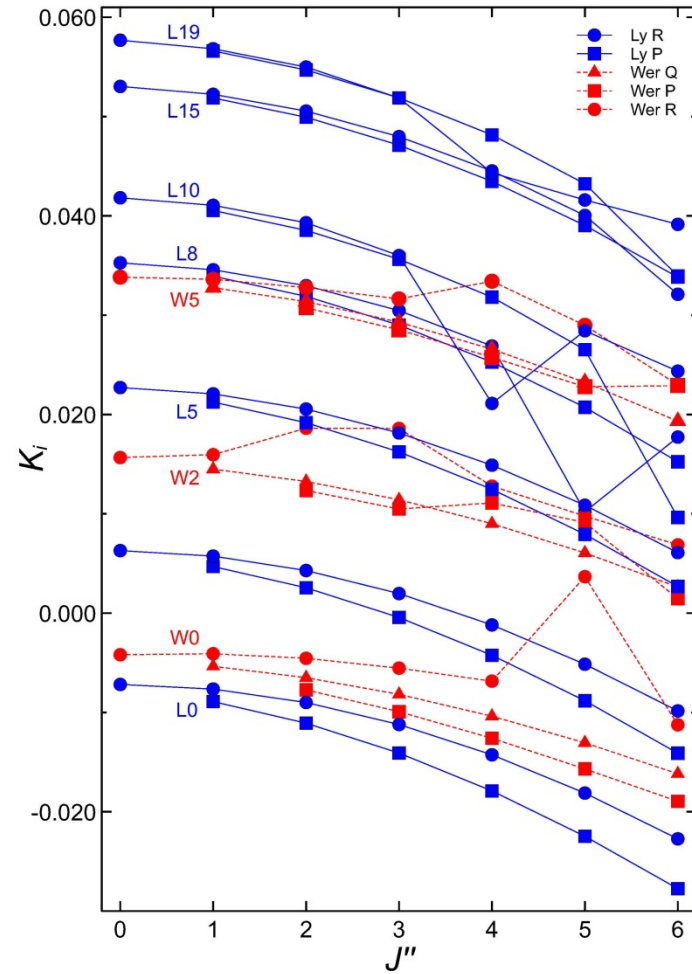


>160 lines measured
at $\sim 5 \times 10^{-8}$

Some lines at $< 1 \times 10^{-8}$

“Sensitivity”

$$K_i = \frac{d \ln \lambda_i}{d \ln \mu}$$



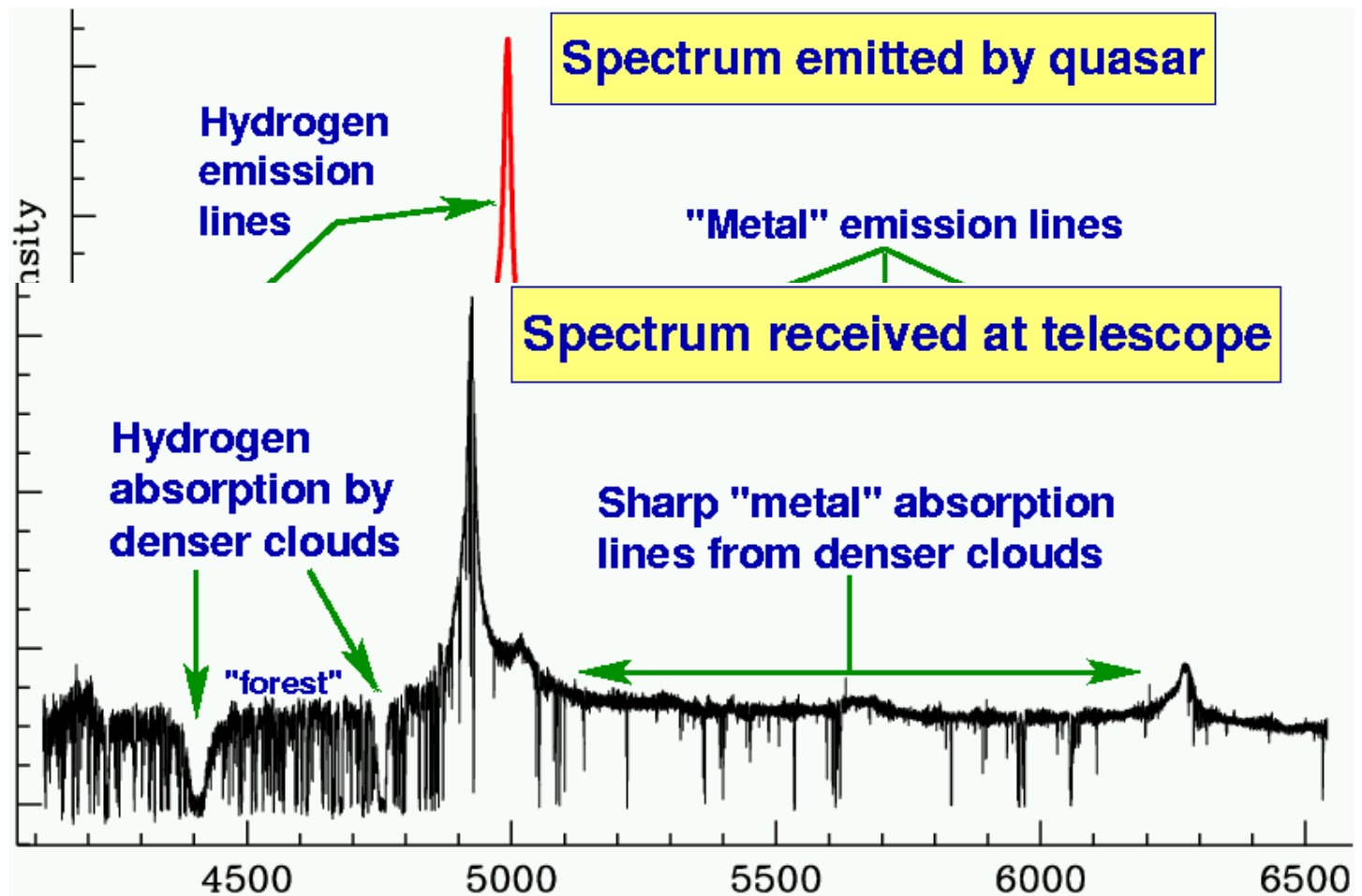
$$\frac{\lambda_i^z}{\lambda_i^0} \equiv 1 + z_i = (1 + z_{abs}) \left(1 + K_i \frac{\Delta \mu}{\mu} \right)$$

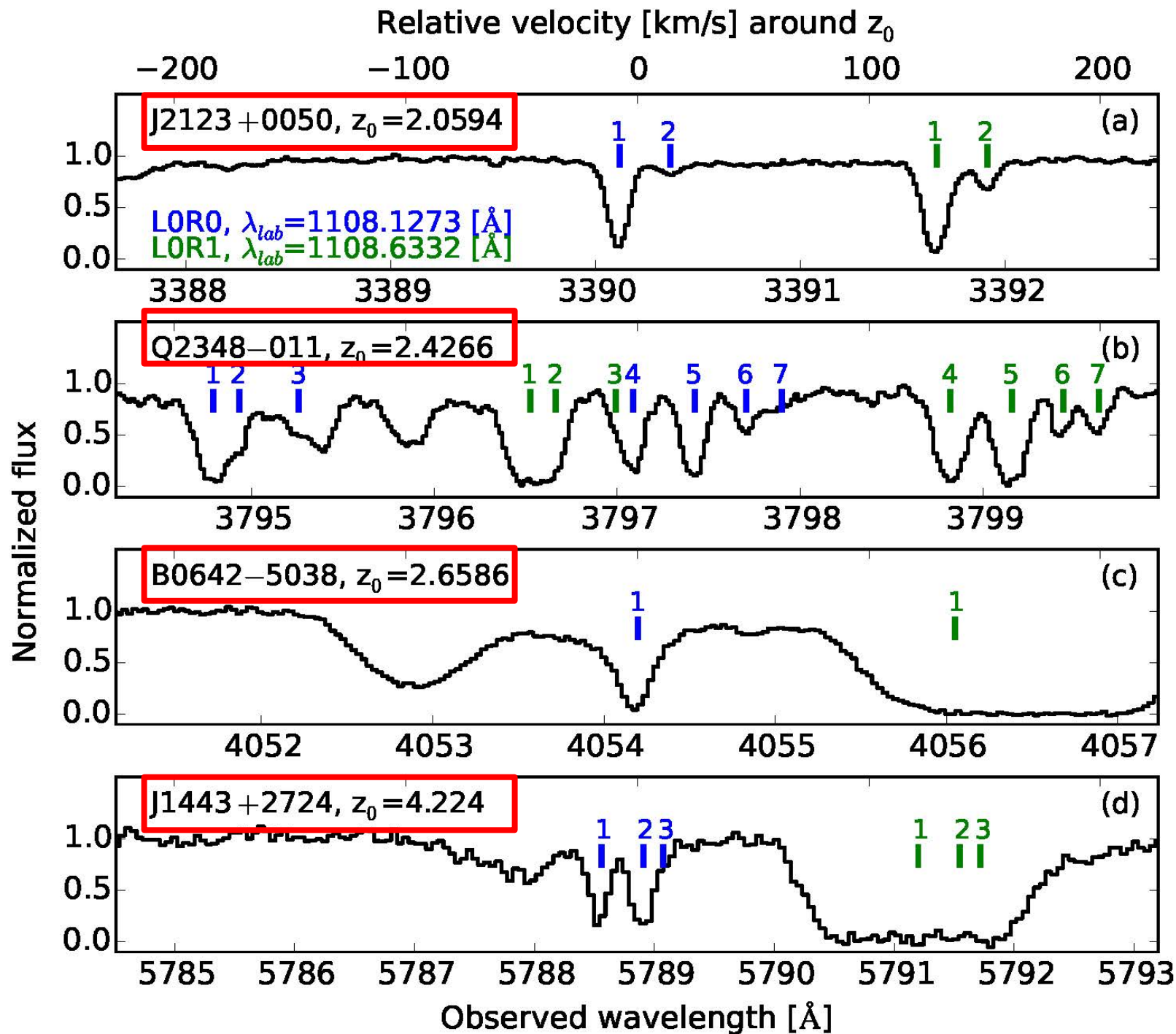


VLT – UVES
Paranal, Chili

Keck – HIRES
Hawaii







Analysis method: “comprehensive fitting”

Produce molecular fingerprint

λ_i – set of accurate wavelengths

f_i – set of line oscillator strengths (from ab initio theory)

Γ_i – set of damping coefficients (from ab initio theory)

Astrophysical conditions

b – Doppler width parameter

z – red shift

N_j – column densities

Fit equation onto spectrum

“Treat” H I and metal lines

Multiple velocity components (?)

$$\frac{\lambda_i^z}{\lambda_i^0} \equiv 1 + z_i = (1 + z_{abs}) \left(1 + K_i \frac{\Delta\mu}{\mu} \right)$$

K_i – set of sensitivity coefficients



H₂ high redshift absorption systems

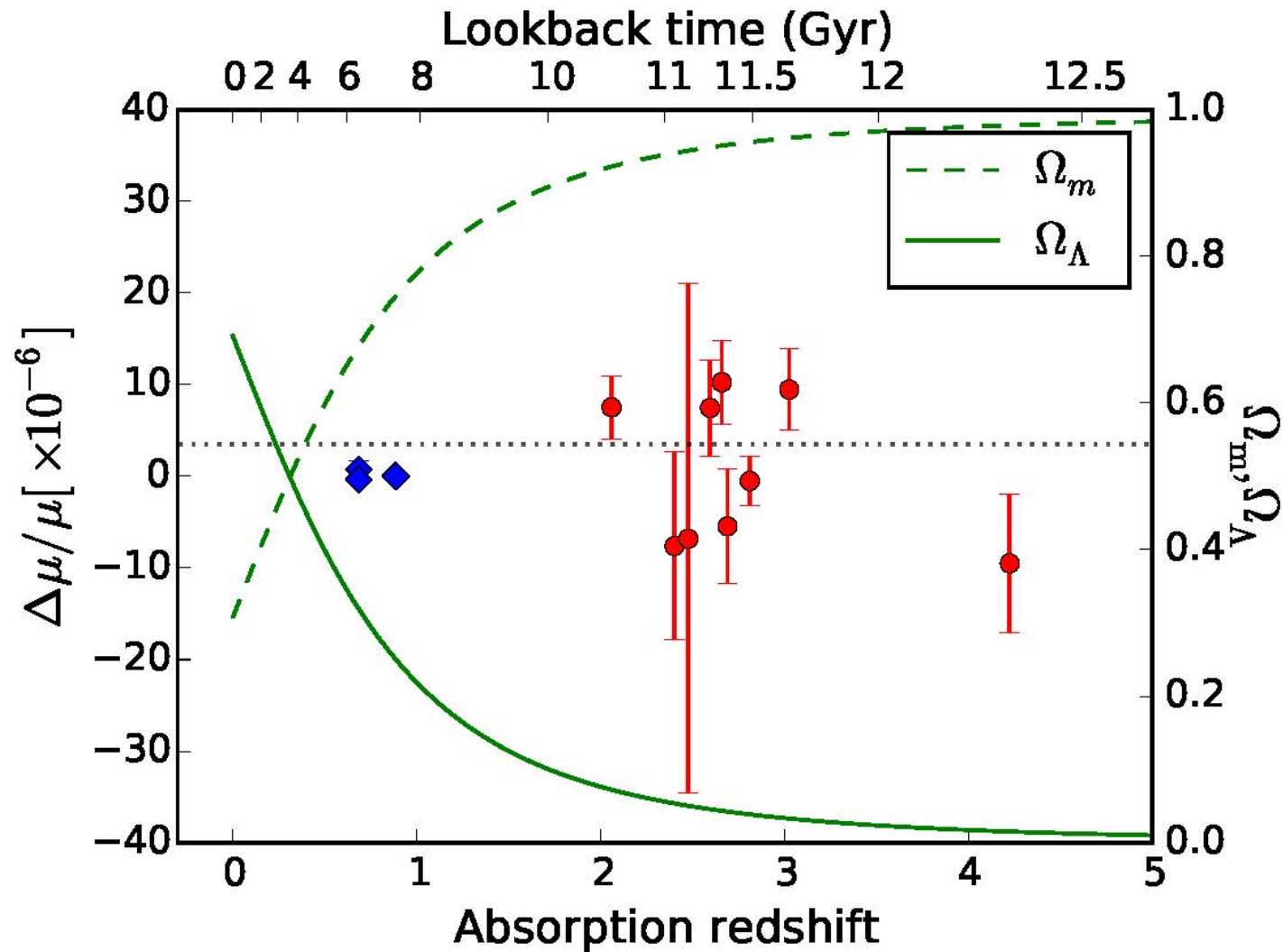
10 H₂ proper absorption systems towards quasars analyzed:

Quasar	z_{abs}	z_{em}	RA(J2000)	Decl.(J2000)	$N(\text{H}_2)$	$N(\text{HD})$	$N(\text{CO})$	$N(\text{HI})$	R_{mag}
HE0027–1836	2.42	2.55	00:30:23.62	–18:19:56.0	17.3			21.7	17.37
Q0347–383	3.02	3.21	03:49:43.64	–38:10:30.6	14.5			20.6	17.48
Q0405–443	2.59	3.00	04:07:18.08	–44:10:13.9	18.2			20.9	17.34
Q0528–250	2.81	2.81	05:30:07.95	–25:03:29.7	18.2	13.3		21.1	17.37
B0642–5038	2.66	3.09	06:43:26.99	–50:41:12.7	18.4			21.0	18.06
Q1232+082	2.34	2.57	12:34:37.58	+07:58:43.6	19.7	15.5		20.9	18.40
J1237+064	2.69	2.78	12:37:14.60	+06:47:59.5	19.2	14.5	14.2	20.0	18.21
J1443+2724	4.22	4.42	14:43:31.18	+27:24:36.4	18.3			21.0	18.81
J2123–0050	2.06	2.26	21:23:29.46	–00:50:52.9	17.6	13.8		19.2	15.83
Q2348–011	2.42	3.02	23:50:57.87	–00:52:09.9	18.4			20.5	18.31

+ 23 additional H₂ absorption systems towards quasars known
+ some 20 tentative detections [Balashev et al (2014)]

Status/Review

$$|\Delta\mu/\mu| = (3.1 \pm 1.6) \times 10^{-6}$$



W. Ubachs, J. Bagdonaite, E.J. Salumbides, M.T. Murphy, L. Kaper

[Search for a drifting proton-electron mass ratio from H₂](#)

Rev. Mod. Phys. 88, 021003 (2016)

Take
Home

$|\Delta\mu/\mu| < 5 \times 10^{-6}$ at 3σ level for redshifts $z = 2-4$

Perspectives

Wavelength distortions ThAr
relate to beam pointing.

Solutions for 'ESPRESSO'

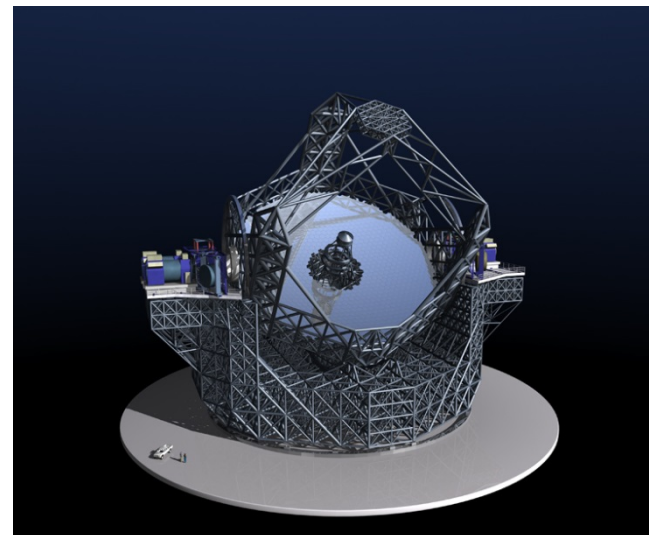
- Frequency comb calibration
- Fiber feeding
- Fibers designed for $\lambda > 3700 \text{ \AA}$

Problematic for H_2 studies,
only $z > 3$

EELT

More light collection (39 m dish)

Improved signal to noise ratio



“Chameleon” Scenario

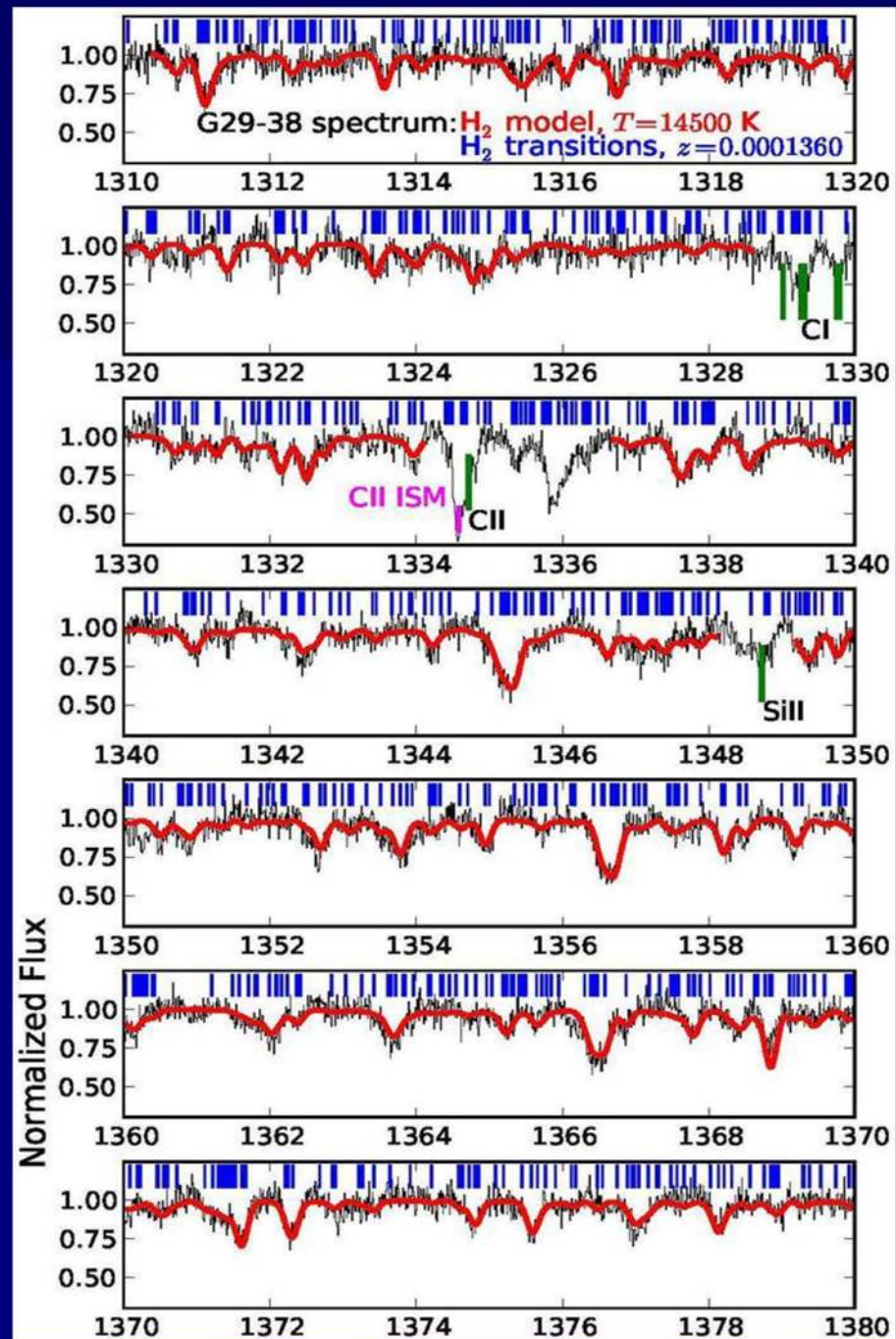
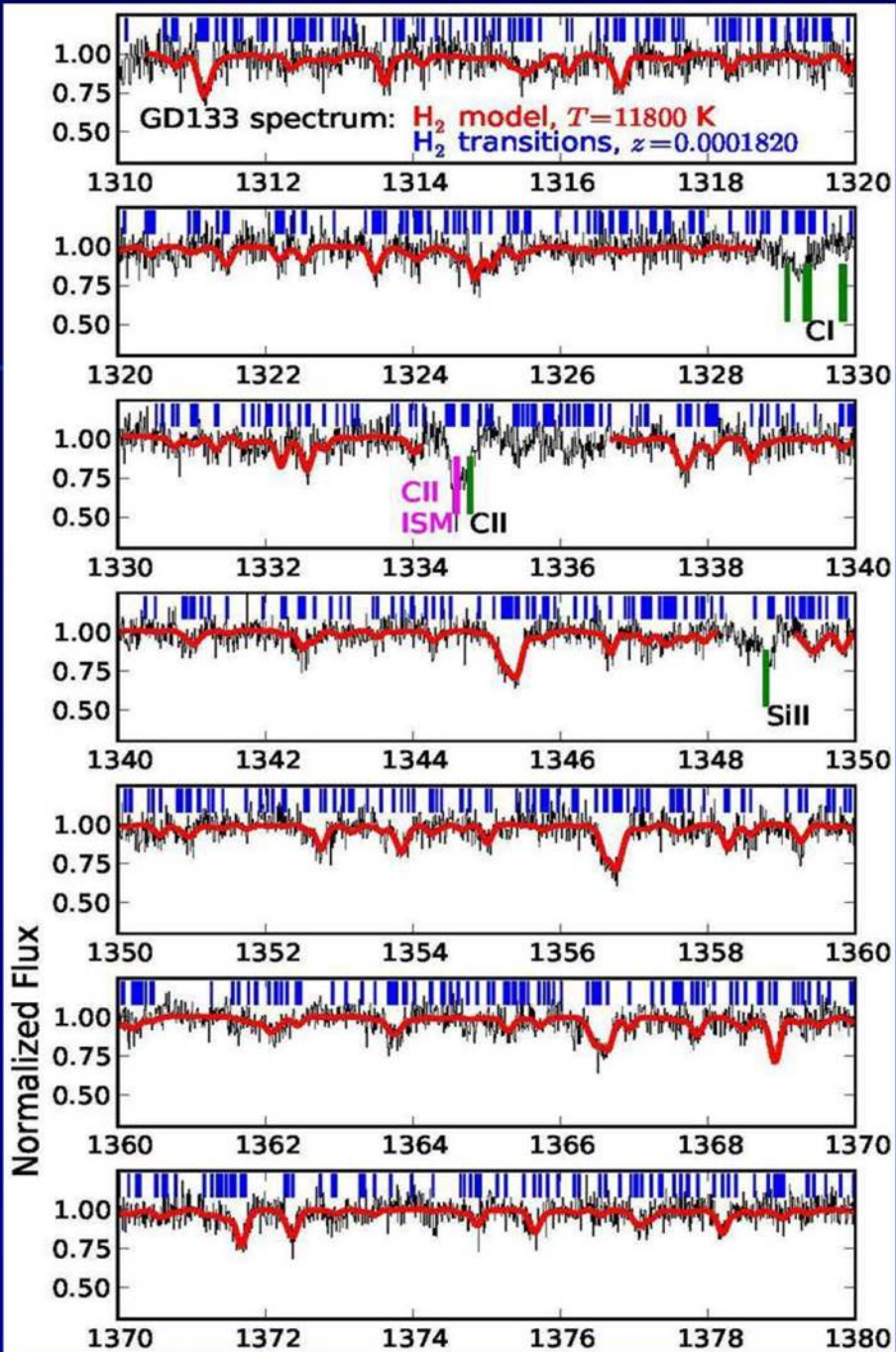
Dependence of μ
on gravitational field ?

$$\phi_{WD} = \frac{GM}{Rc^2} \sim 10^4 \times \phi_{Earth}$$

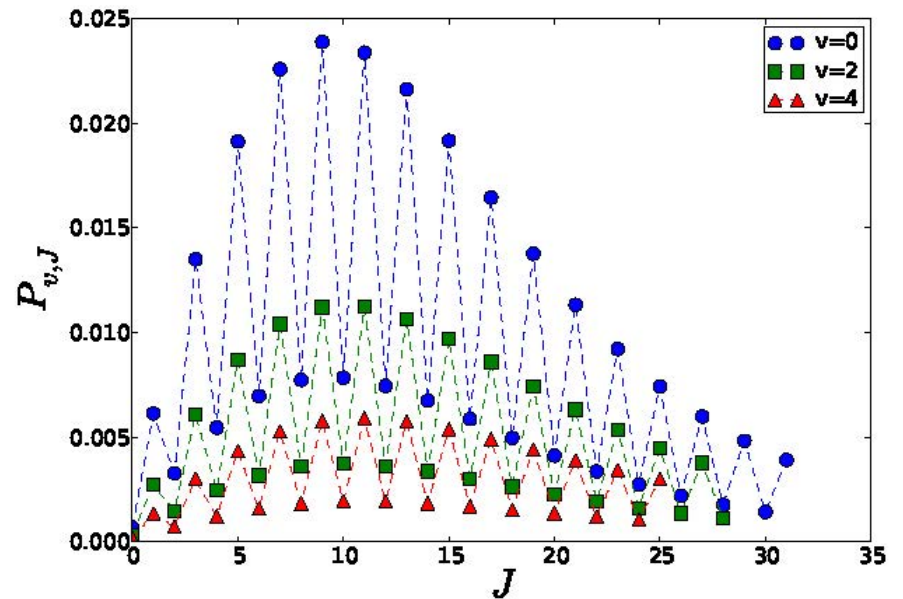
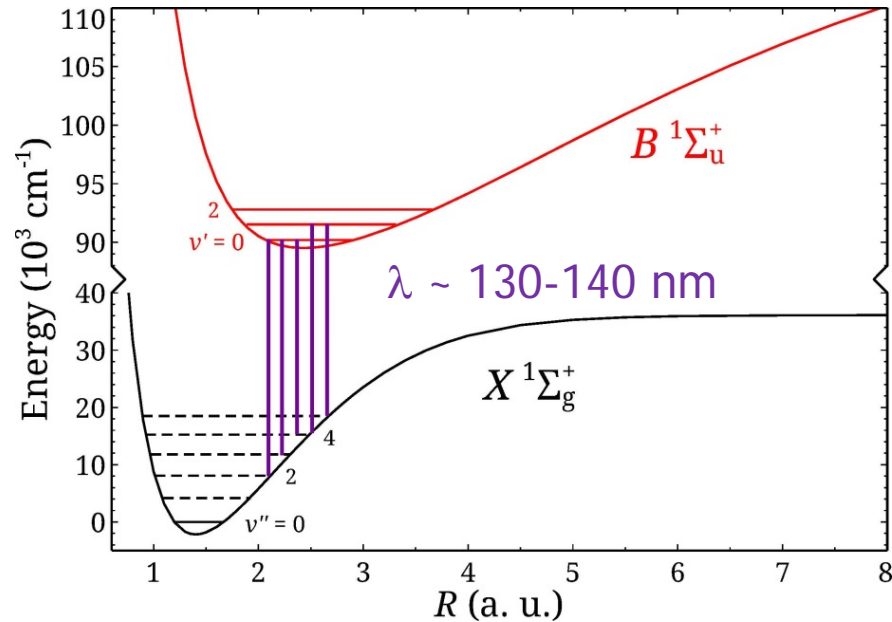
Spectrum of GD-133 and GD29-38
Photosphere of White Dwarf stars

In the Galaxy !





Contributions of many lines in the B-X Lyman system



High temperatures
High v populated
Franck-Condon factors

Dependence of $\Delta\mu/\mu$ on gravitational field

Invoke partition function:

$$P_{vJ}(T) = \frac{g_I(J)(2J+1)\exp\left(\frac{-E_{vJ}}{kT}\right)}{\sum_{v=0}^{v_{\max}} \sum_{J=0}^{J_{\max}(v)} g_I(J)(2J+1)\exp\left(\frac{-E_{vJ}}{kT}\right)}$$

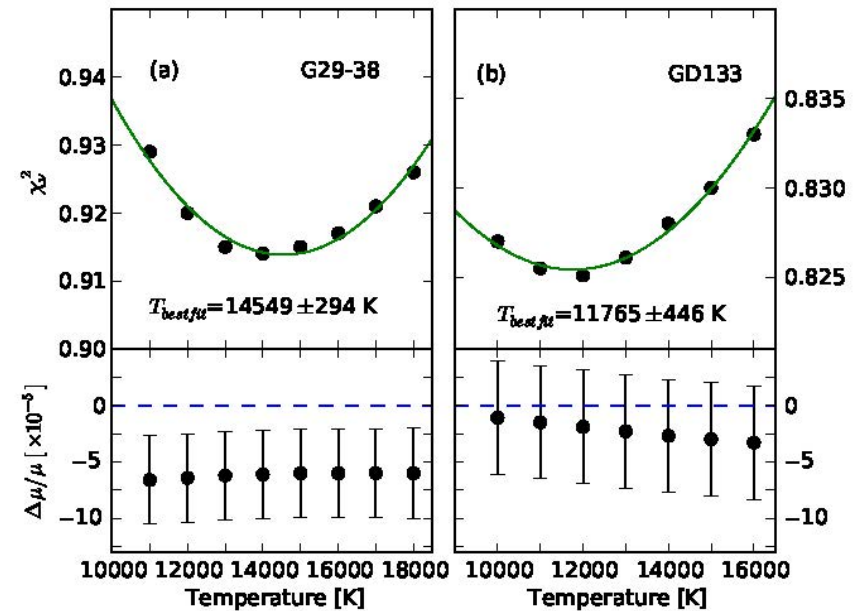
Invoke intensities (1500 lines):

$$I_i = N_{col} f_{v'v''J'J''} P_{v''J''}(T)$$

Lines

$$\frac{\lambda_i^z}{\lambda_i^0} \equiv 1 + z_i = (1 + z_{abs}) \left(1 + K_i \frac{\Delta\mu}{\mu} \right)$$

Fit T and $\Delta\mu/\mu$



GD133: $\Delta\mu/\mu = (-2.7 \pm 4.7) \times 10^{-5}$

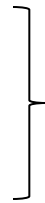
GD29-38: $\Delta\mu/\mu = (-5.9 \pm 3.8) \times 10^{-5}$

Molecules as a metrology test system

Search for BSM-physics from **laboratory** spectroscopy experiment

$$\Delta E = E_{\text{exp}} - E_{\text{theory}}$$

$$\delta E = \sqrt{\delta E_{\text{exp}}^2 + \delta E_{\text{theory}}^2}$$



$$\Delta E < \delta E$$

Validate theory (QED)

$$\Delta E > \delta E$$

New Physics:

Theory is needed – only for “calculable” systems

H₂ – Krzysztof Pachucki & team ;

H₂⁺ - Jean-Philippe Karr, Laurent Hilico, Vladimir Korobov

Discover new physics

$$\langle \Delta V_{\text{new}} \rangle > \delta E$$

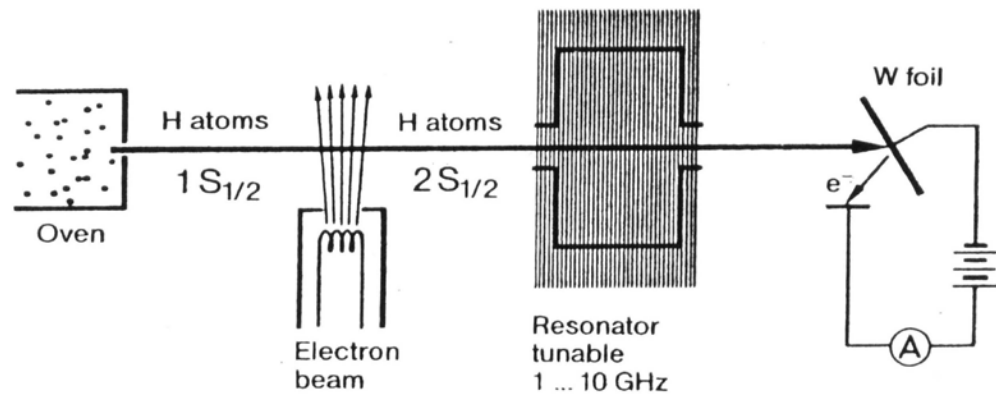
Constrain new physics

$$\langle \Delta V_{\text{new}} \rangle < \delta E$$

Historical Inspiration



Willis E Lamb

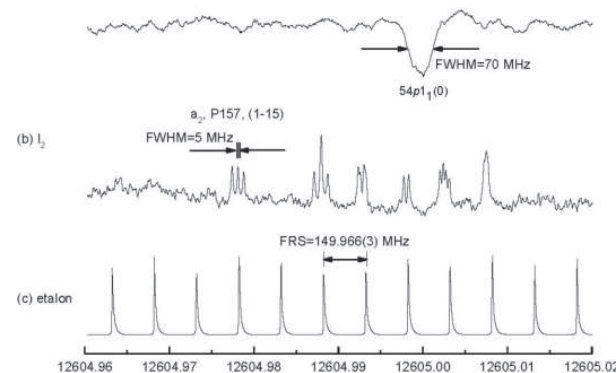
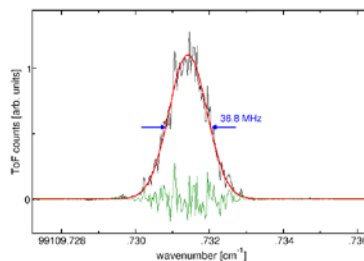
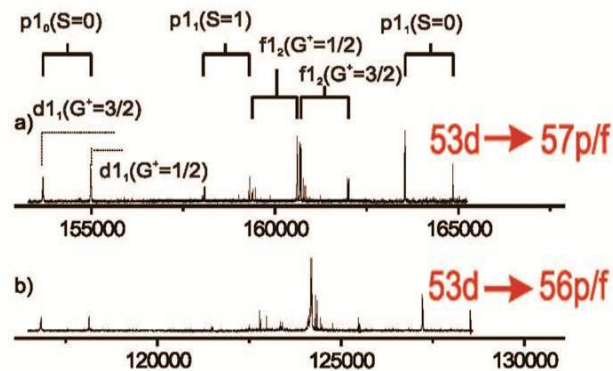
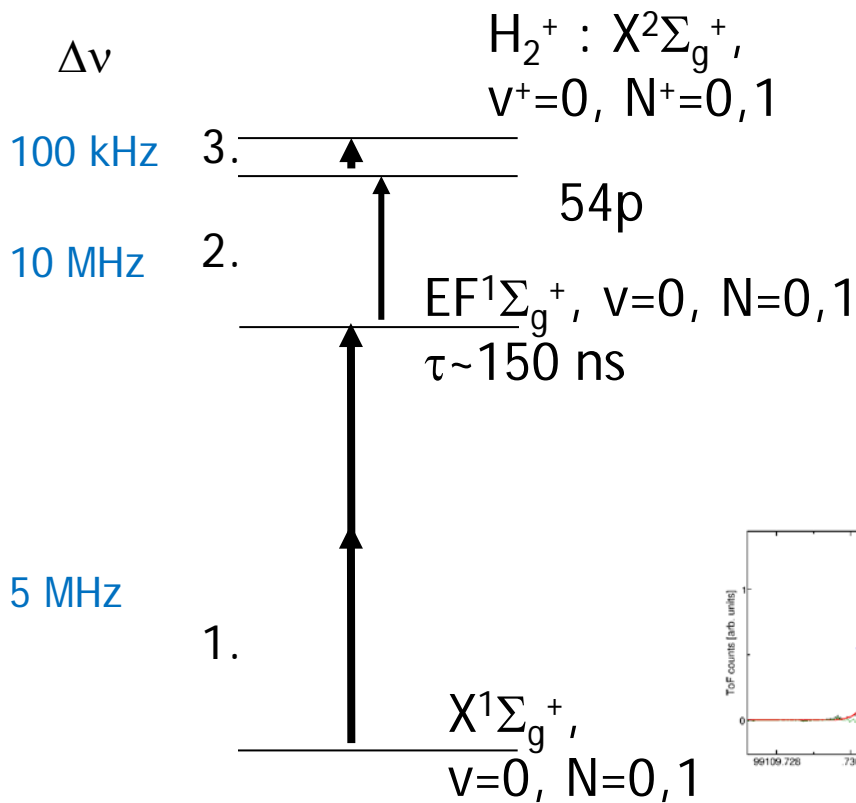


Measurement of the tiny $2S_{1/2} - 2P_{1/2}$ splitting

Breakdown of the Dirac theory of the electron
The advent of Quantum Electro Dynamics

Measurement of IP in H₂

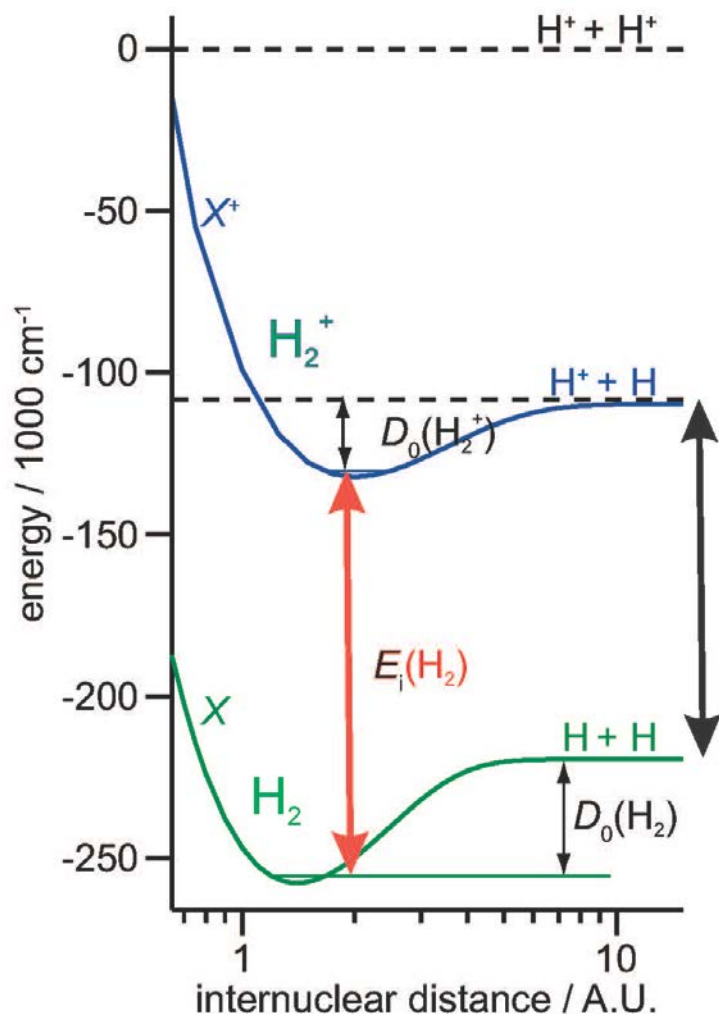
3 step approach (Zürich-Amsterdam collaboration)



$$E_i (\text{ortho}) = 124\,357.237\,97\,(36) \text{ cm}^{-1}$$

$$E_i (\text{para}) = 124\,417.491\,13\,(37) \text{ cm}^{-1}$$

Benchmark: Dissociation energy H_2



$$D_0(H_2) = E_{IP}(H_2) + D_0(H_2^+) - E_{IP}(H)$$

$$D_0(H_2^+) = 21379.350232(50) \text{ cm}^{-1}$$

$$E_{IP}(H) = 109678.7717426(10) \text{ cm}^{-1}$$

$$E_{IP}(H_2) \rightarrow D_0(H_2)$$

Comparison Theory/Experiment



(Theory: Pachucki, Komasa, et al.: 2010 values)

$D_0(\text{H}_2)$: Experiment [1] 36118.0696(4) cm⁻¹

Theory [2]:

Born–Oppenheimer 36112.5927(1) cm⁻¹

adiabatic + 5.7711(1) cm⁻¹

nonadiabatic + 0.4339(2) cm⁻¹

total α^0 36118.7978(2) cm⁻¹

α^2 all relativistic – 0.5319(5) cm⁻¹

α^3 all QED – 0.1948(3) cm⁻¹

α^4 one-loop term – 0.0016(8) cm⁻¹

Total theory 36118.0695(10) cm⁻¹

QED

$D_0(\text{D}_2)$: Total theory [2] 36748.3633(9) cm⁻¹

Experiment [4] 36748.3629(7) cm⁻¹

Fundamental vibration in H₂

Collision-free measurement

Features

Narrowband UV sources

Absolute frequency calibration

2-photon Doppler-free REMPI

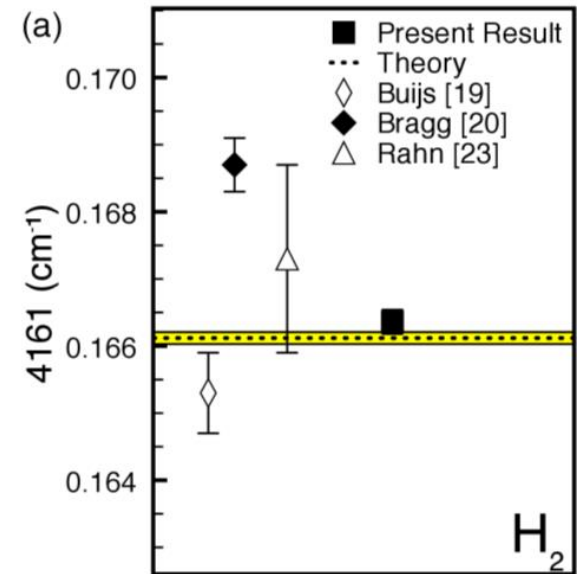
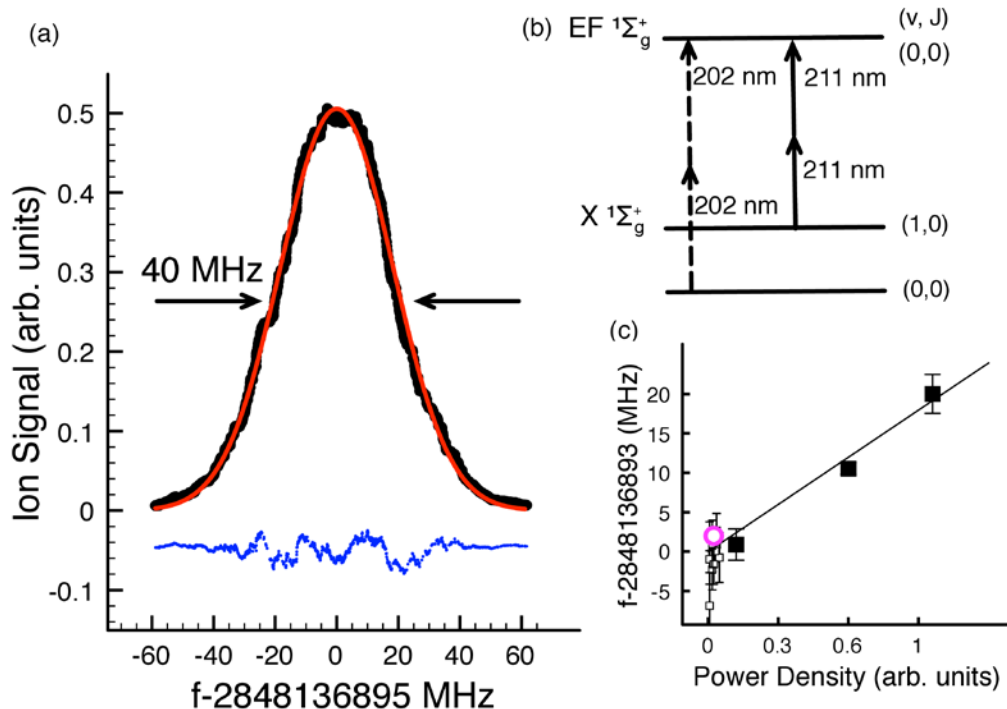
Sagnac alignment

Delayed ionisation

ac-Stark extrapolation

$$E = 4161.16632(18) \text{ cm}^{-1}$$

$$\delta E_{\text{exp}} \sim 1.8 \times 10^{-4} \text{ cm}^{-1}$$



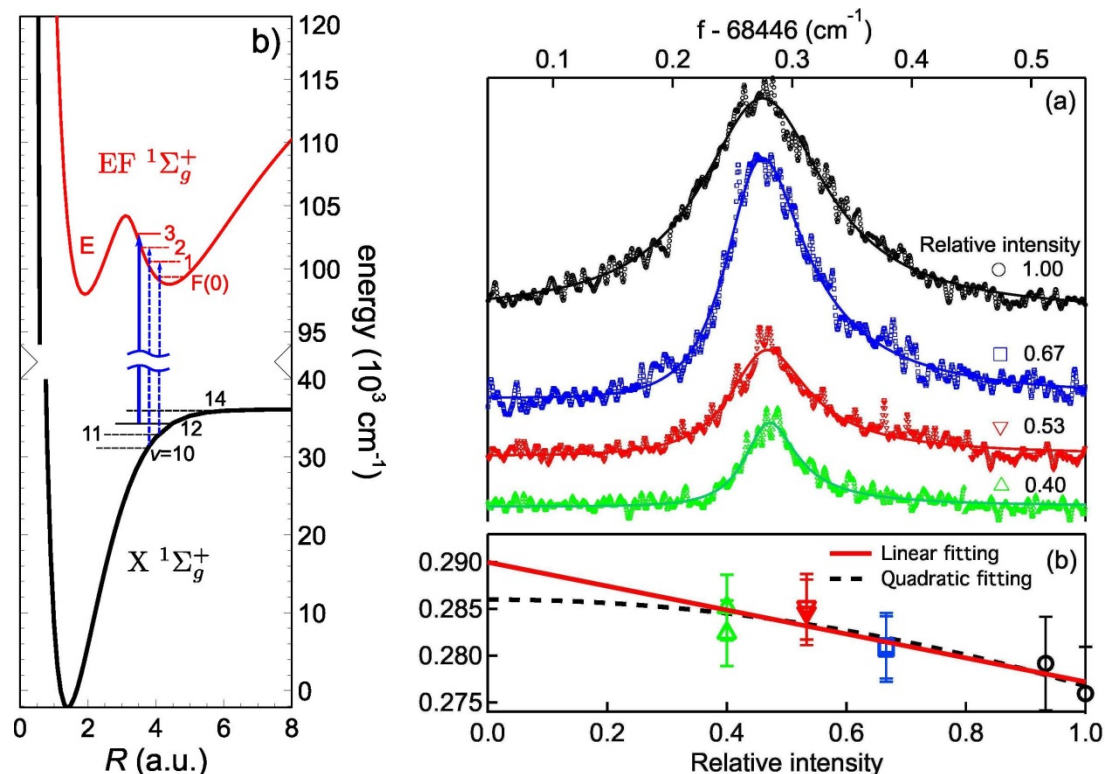
Precision study of $\text{H}_2 \text{X}^1\Sigma_g^+ v=12, 11$

Production of H_2 , v
 Photolysis of H_2S
 Steadman & Baer (1989)

Now:
 Three independent lasers

$V=12$: Niu et al.
 JCP Comm 142 (2015) 081102

$V=11$: New results
 Trivikram et al.
 Appl. Phys. B 122, 294 (2016)



J''	E_{exp}	E_{the}	$\Delta E_{\text{exp-the}}$
0	34 302.1823 (35)	34 302.1741 (47)	0.008 (6)
1	34 343.8531 (35)	34 343.8483 (46)	0.005 (6)
2	34 426.2216 (35)	34 426.2179 (46)	0.004 (6)
3	34 547.3362 (35)	34 547.3332 (45)	0.003 (6)

Progress in theory

PHYSICAL REVIEW A **82**, 032509 (2010)

Born-Oppenheimer potential for H₂

Krzysztof Pachucki*

In the whole range of internuclear distance, about 10^{-15} precision is achieved; as an example, at the equilibrium distance $r = 1.4011$ a.u., the Born-Oppenheimer potential amounts to $-1.174\,475\,931\,400\,216\,7(3)$.

THE JOURNAL OF CHEMICAL PHYSICS **141**, 224103 (2014)

Accurate adiabatic correction in the hydrogen molecule

Krzysztof Pachucki^{1,a)} and Jacek Komasa^{2,b)}

¹*Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*

²*Faculty of Chemistry, Adam Mickiewicz University, Umultowska 89b, 61-614 Poznań, Poland*

(Received 8 October 2014; accepted 18 November 2014; published online 8 December 2014)

For the ground state of H₂ the estimated precision is $3 \times 10^{-7} \text{ cm}^{-1}$, which is almost three orders of magnitude higher than that of the best previous result.

THE JOURNAL OF CHEMICAL PHYSICS **143**, 034111 (2015)

Leading order nonadiabatic corrections to rovibrational levels of H₂, D₂, and T₂

Krzysztof Pachucki^{1,a)} and Jacek Komasa^{2,b)}

¹*Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*

²*Faculty of Chemistry, Adam Mickiewicz University, Umultowska 89b, 61-614 Poznań, Poland*

(Received 22 May 2015; accepted 8 July 2015; published online 21 July 2015)

An estimated accuracy of the leading nonadiabatic correction to the rovibrational energy levels is of the order of 10^{-7} cm^{-1} .

Progress in theory

Complete $\alpha^6 m$ corrections to the ground state of H_2

Mariusz Puchalski and Jacek Komasa

Faculty of Chemistry, Adam Mickiewicz University, Umultowska 89b, 61-614 Poznań, Poland

QED

Paweł Czachorowski and Krzysztof Pachucki

Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland

(Dated: July 29, 2016)

Phys. Rev. Lett. 117, 263002 (2016)

REL

PHYSICAL REVIEW A 95, 052506 (2017)



Relativistic corrections for the ground electronic state of molecular hydrogen

Mariusz Puchalski and Jacek Komasa

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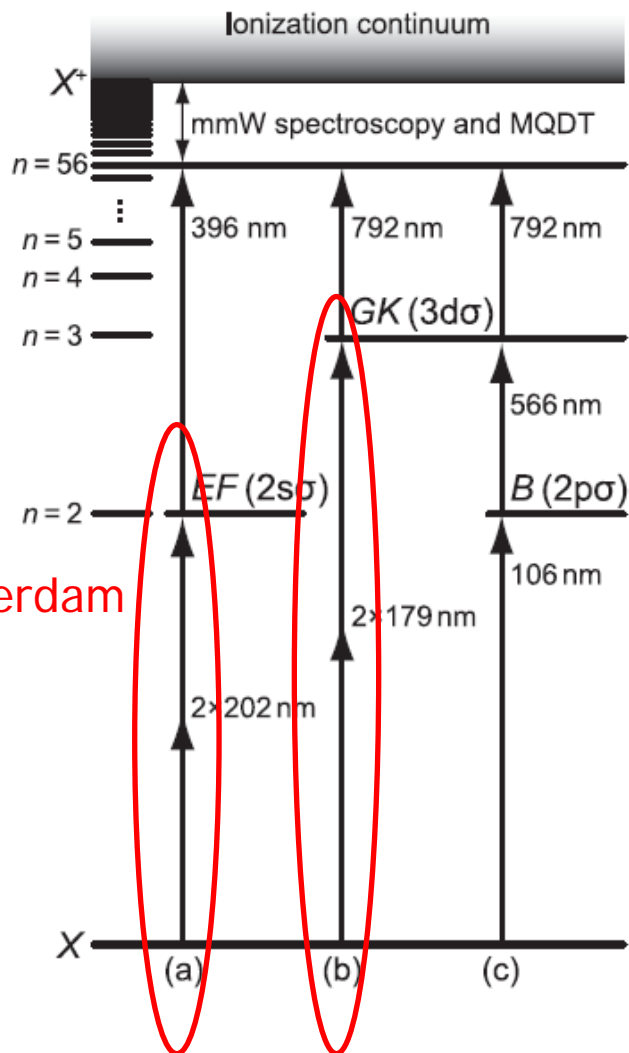
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(Received 13 April 2017; published 25 May 2017)

More importantly, the updated theoretical energies became discrepant with the known experimental values and we conclude that the yet unknown relativistic recoil corrections might be larger than previously anticipated.

Important estimate: 10 kHz on rovibrational energies ~ proton radius at 1%

Strategies to improve the experimental side



Amsterdam

Three step approach to IP
has been probed via nanosecond excitation
Accuracy: 11 MHz

THE JOURNAL OF CHEMICAL PHYSICS 130, 174306 (2009)

Determination of the ionization and dissociation energies of the hydrogen molecule

Jinjun Liu,¹ Edcel J. Salumbides,² Urs Hollenstein,¹ Jeroen C. J. Koelemeij,²
Kjeld S. E. Eikema,² Wim Ubachs,² and Frédéric Merkt^{1,a)}

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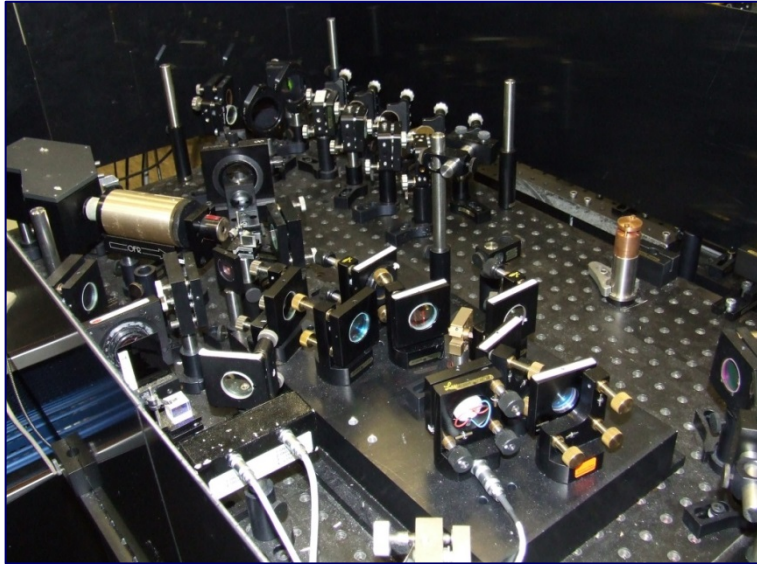
GK level splitting to IP is known to 1.2 MHz

Sprecher, Beyer, Merkt
Mol. Phys. 111, 2100 (2011)

Improvements ongoing (2017) GK and EF to IP

Max Beyer, Frederic Merkt

Measurement of the GK-X transition - ongoing

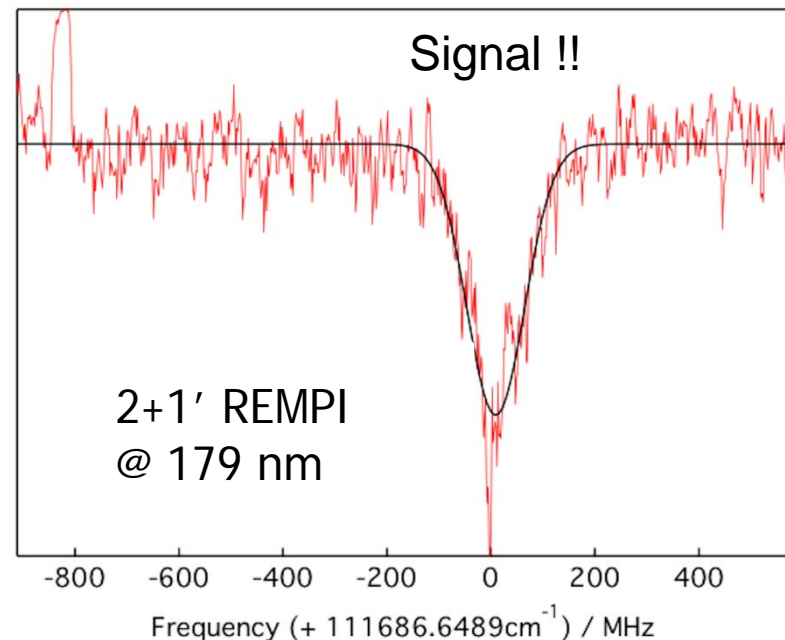
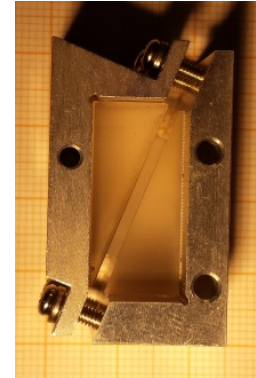


Home-built long-pulse narrowband
Ti:Sa laser @ 716 nm

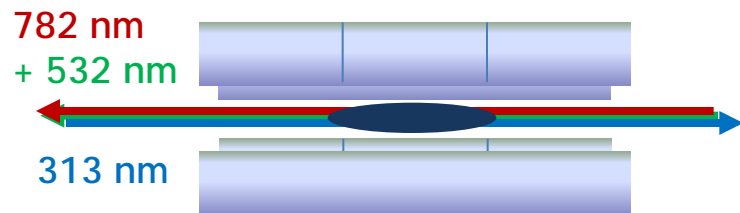
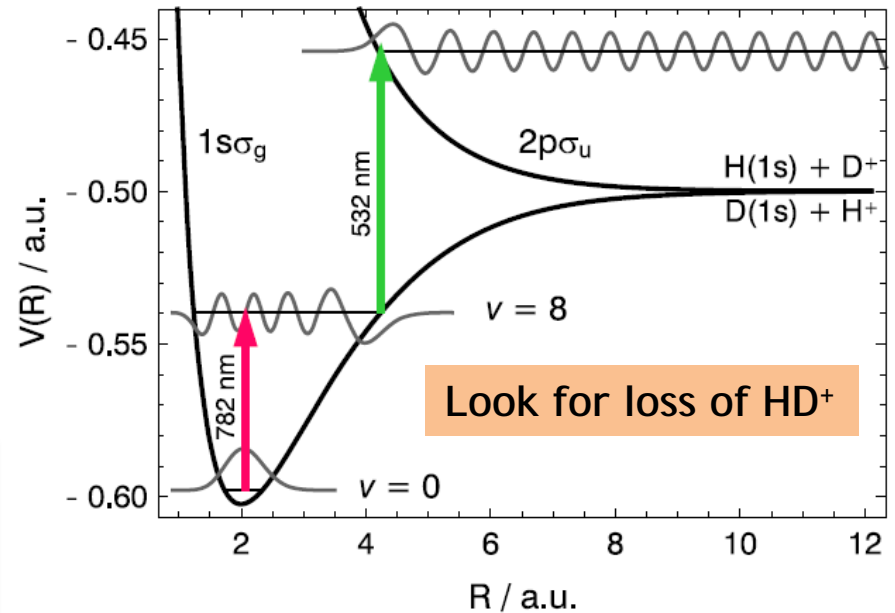
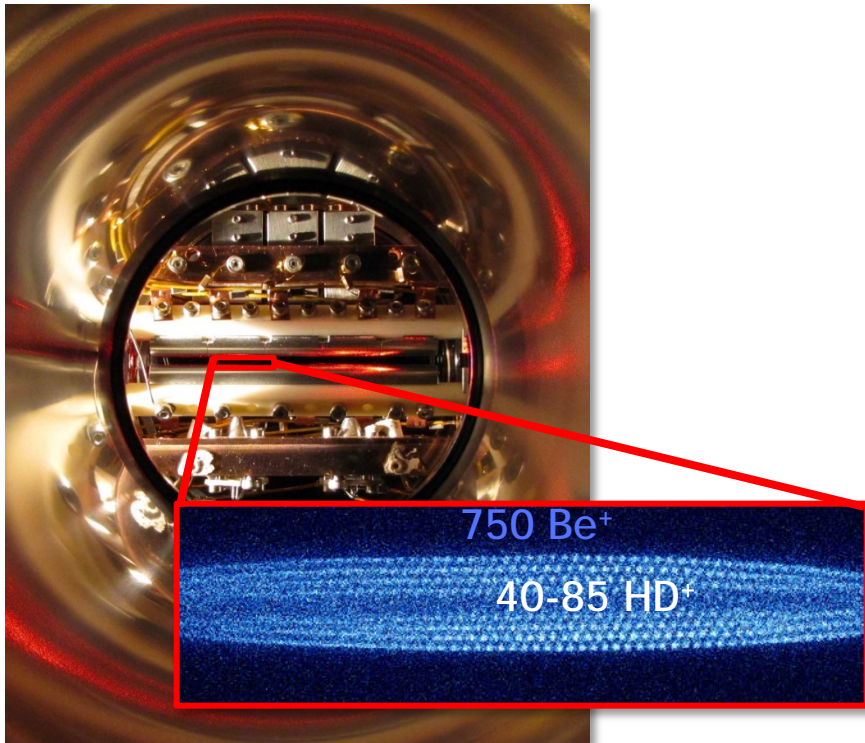
Adjustable pulse duration (match $\tau=30$ ns)
Two-photon Doppler-free
Study of auto-ionization
Improved molecular beam source
Chirp measurement
AC Stark studies
Seed-laser locked to Freq. comb

KBBF crystal for
doubling to 179 nm

VUV mirror for
retroreflect

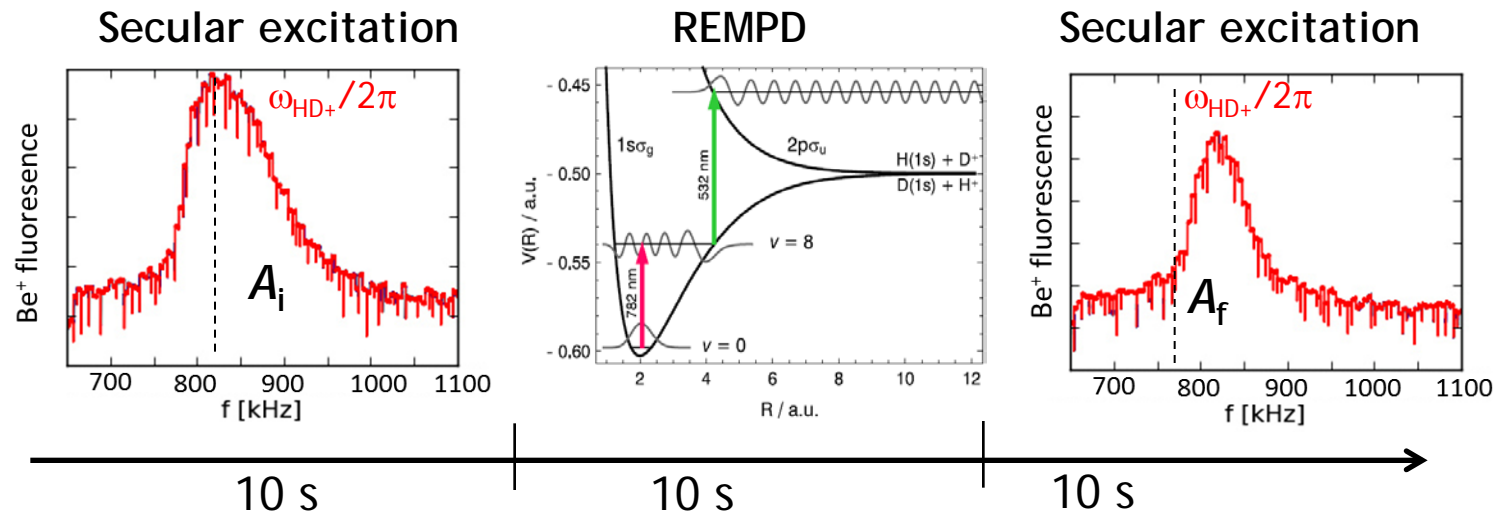


HD⁺ ions in a trap; measurement of (8,0)



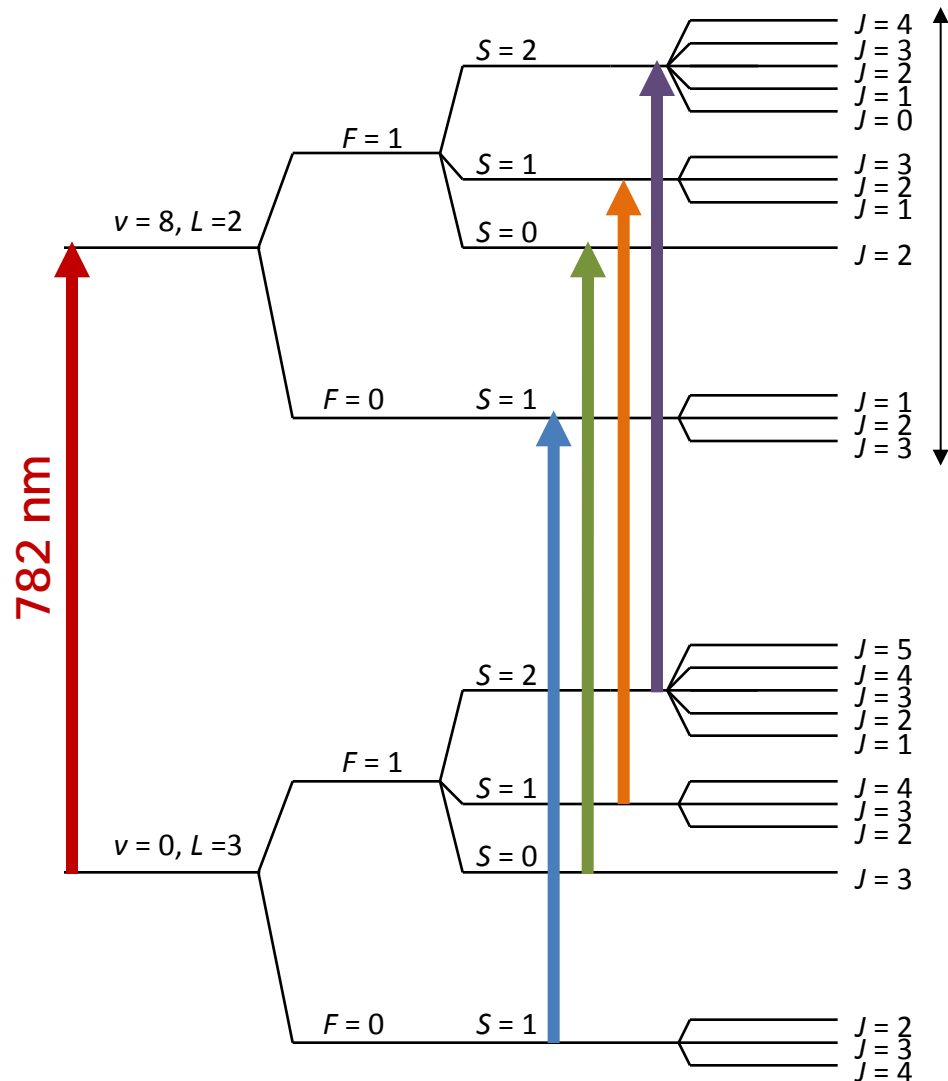
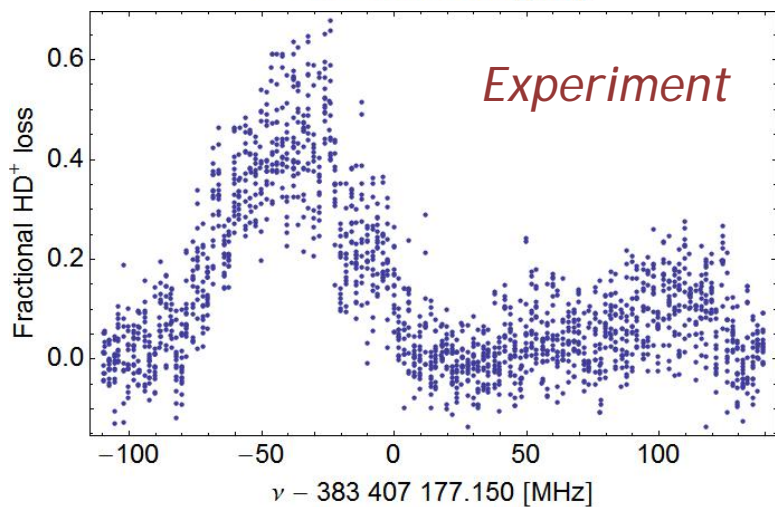
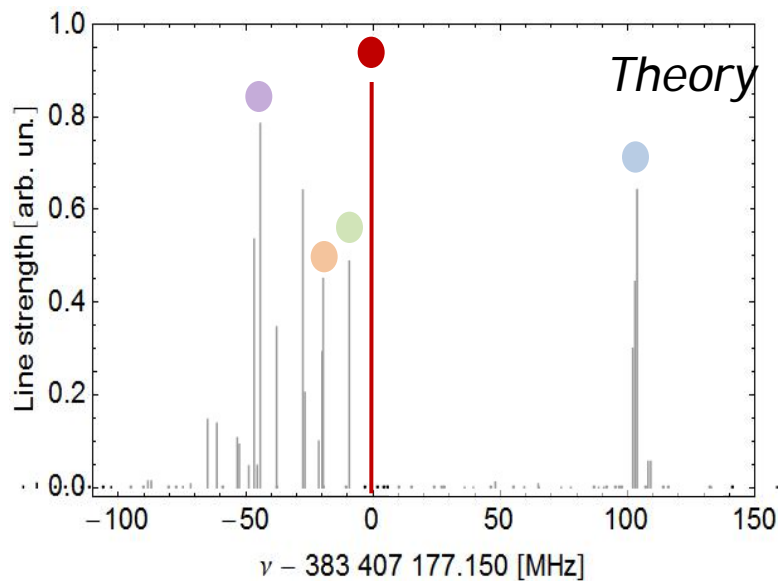
Signal detection by
REMPD

REMPD detection cycle:



- Signal = fractional loss of ions
- During secular excitation, T_{Be^+} rises to T_{max}
- $A_i \propto T \propto N_{\text{HD}^+}$ for $T_{\text{max}} < 400\text{ mK}$
- BUT in practice $T_{\text{max}} \approx 4\text{ K}$...
- Be⁺ fluorescence modeled (non-linear)

HD⁺ spectrum



Experiment: 383,407,177.38(41) MHz
 Theory*: 383,407,177.150(15) MHz

*Korobov, Hilico, Karr, Phys. Rev. A 89, 032511 (2014)

Tests of QED in molecules

$$\Delta E < \delta E$$

Species	Splitting	δE_{exp}	Ref.	δE_{calc}	δE	ΔE
H ₂	$\nu = 0, J = 6 - 12$	150 ^c	[56]	12	150 ^c	20
	$\nu = 0, J = 13 - 16$	300 ^c	[56]	27	300 ^c	90
	$\nu = 0 \rightarrow 1$	4.5 ^a	[54]	2.7	5	7
	$\nu = 0 \rightarrow 2$	30	[57]	50	60	12
	$\nu = 0 \rightarrow 3$	1.3	[58]	75	75	10
	$\nu = 0 \rightarrow 12$	105	[59]	140	170	150 ^b
	D_0	12	[44]	30	3	36
HD	$\nu = 0 \rightarrow 1$	7 ^a	[54]	2.4	8	4
	D_0	11	[49]	30	27	32
D ₂	$\nu = 0 \rightarrow 1$	4.5 ^a	[54]	2.1	5	-0.6
	$\nu = 0 \rightarrow 2$	30	[60]	12	30	-12
	D_0	21	[48]	27	30	12
H ₂ ⁺	$\nu = 0, J = 0 \rightarrow 2$	2.3	[61]	0.003	2.3	-1.0
HD ⁺	$\nu = 0 \rightarrow 1$	0.064	[62]	0.002	0.064	-0.156
	$\nu = 0 \rightarrow 4$	0.50	[63]	0.008	0.50	-0.35
	$\nu = 0 \rightarrow 8$	0.41	[64]	0.015	0.41	0.22

Fifth-force searches

Assume: Extra *hadron-hadron* interaction

Parametrize (quantum field theory) as:

Yukawa potential

$$V_5(r) = N_1 N_2 \left\{ \alpha_5 \frac{\exp(-r/\lambda)}{r} \right\} \hbar c$$

Strength: α_5

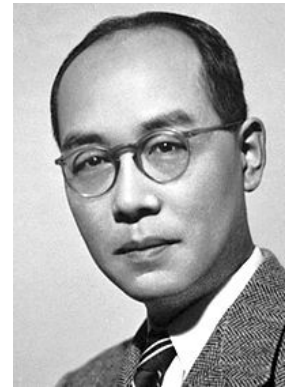
Range: $\lambda = \hbar / m_5 c$

Mass of force carrying particle: m_5

Hadron numbers: N_1, N_2

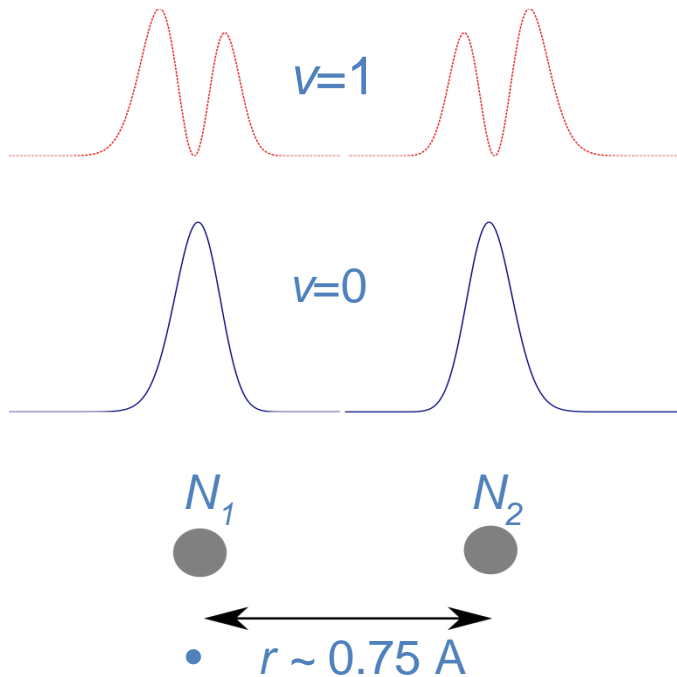
See for analysis:

Salumbides, Koelemeij, Komasa, Pachucki, Eikema, Ubachs,
Phys Rev D **87**, 112008 (2013).



Hideki Yukawa

Calculate the expectation value of the energy operator



Level shifts:

$$\langle \Psi_1 | V_5 | \Psi_1 \rangle; \langle \Psi_0 | V_5 | \Psi_0 \rangle$$

Transition shift:

$$\langle \Psi_1 | V_5 | \Psi_1 \rangle - \langle \Psi_0 | V_5 | \Psi_0 \rangle$$

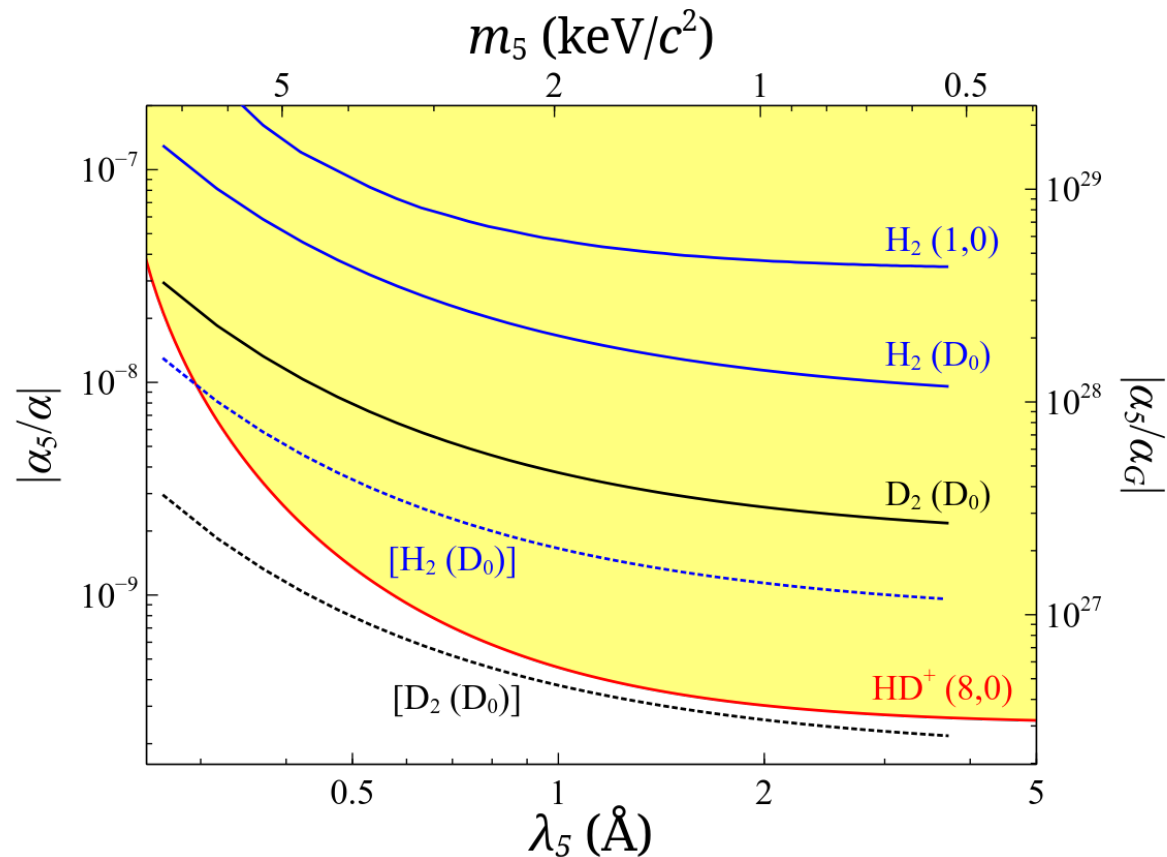
Differential effect larger for very high v 's (D_0 limit)

$$\langle \Delta V_{5,\lambda} \rangle = \alpha_5 N_1 N_2 \hbar c \left\{ \left\langle \Psi_{v',J'}(r) \left| \frac{\exp(-r/\lambda)}{r} \right| \Psi_{v',J'}(r) \right\rangle - \left\langle \Psi_{v'',J''}(r) \left| \frac{\exp(-r/\lambda)}{r} \right| \Psi_{v'',J''}(r) \right\rangle \right\}$$

Parameters α_5 and λ

Impose constraints on 5th force from spectroscopy HD⁺/H₂

$$\langle \Delta V_5 \rangle < \delta E \quad \text{hence} \quad \alpha_5 < \frac{\delta E}{N_1 N_2 \hbar c \langle \Delta \Psi \rangle} (\lambda)$$



Physics of extra spatial dimensions

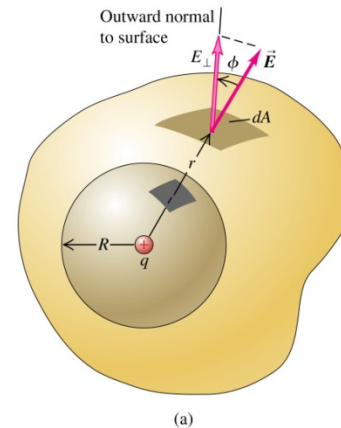
Immanuel Kant

Number of dimensions consequence
of Newton's Universal law of gravitation

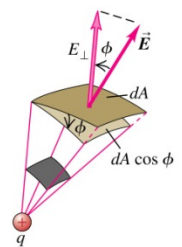
$$\oint_V \vec{F} \cdot d\vec{A} = kQ_{encl}$$

3-dim: $A_V \propto r^2 \Rightarrow F \propto \frac{1}{r^2}$

N-dim: $A_V \propto r^{N-1} \Rightarrow F \propto \frac{1}{r^{N-1}}$



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(b)

Physics of extra spatial dimensions

Immanuel Kant

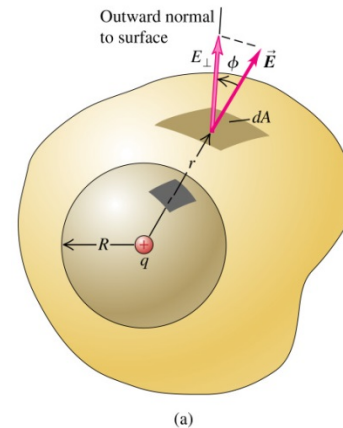
Number of dimensions consequence
of Newton's Universal law of gravitation



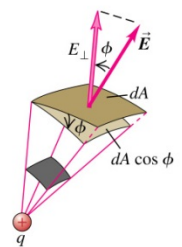
$$\oint_V \frac{\vec{F}}{m} \cdot d\vec{A}_n = -\hat{A}_n G_n M$$

$$n\text{-dim: } A_n = \hat{A}_n r^{n-1} = \left[\frac{n\pi^{n/2}}{\Gamma\left(\frac{n}{2} + 1\right)} \right] r^{n-1} \Rightarrow F \propto \frac{1}{r^{n-1}}$$

Gravitational attraction
depends on dimensionality



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(b)

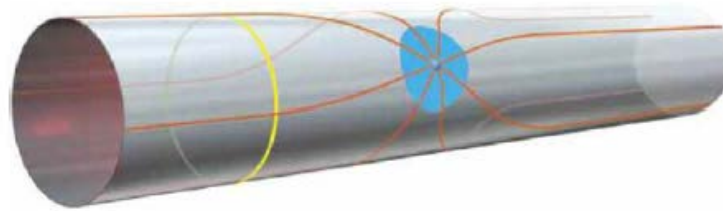
“Compactification”

Theory of consistent EM + Gravity in 5 dimensions (Kaluza)

Extra dimensions are not observed in the macroscopic world
They may be compactified: rolled up (Klein 1926)



Oscar Klein



String theory: “M-Theory” (Witten) is consistent in 11 dimensions



ADD-theory and Large Extra Dimensions

Arkani-Hamed, Dimopoulos, Dvali

Phys. Lett. B **429**, 263–272 (1998)



Hierarchy Problem:

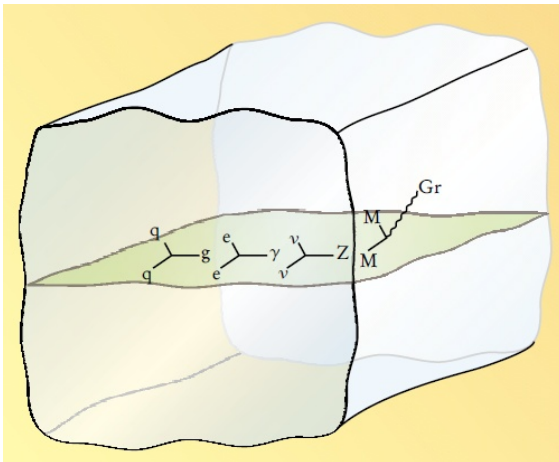
Why is gravity so much weaker ?

For protons

$$\frac{V_G}{V_{em}} = 8 \times 10^{-37}$$

Why is the Planck mass so much bigger
then SM masses ?

$$\frac{M_{Pl}}{M_Z} \sim 10^{17}$$

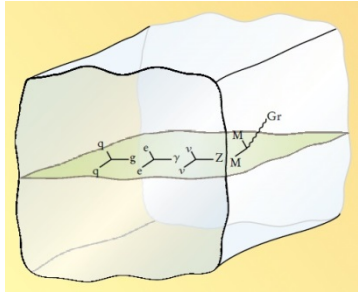


Solution :

Gravity “escapes” into higher dimensions

3-brane (SM) and “Bulk” (gravity)

Large Extra Dimensions (compactified n extra)



$$V_{ADD}(r) = -G_{(3+n)} \frac{m_1 m_2}{R_{comp}^n} \frac{1}{r} \quad \text{for } r > R_n$$

Gravity outside Klein radius

$$V_{Newton}(r) = -G_3 m_1 m_2 \frac{1}{r}$$

$$V_{ADD}(r) = -G_{(3+n)} m_1 m_2 \frac{1}{r^{n+1}} \quad \text{for } r < R_n$$

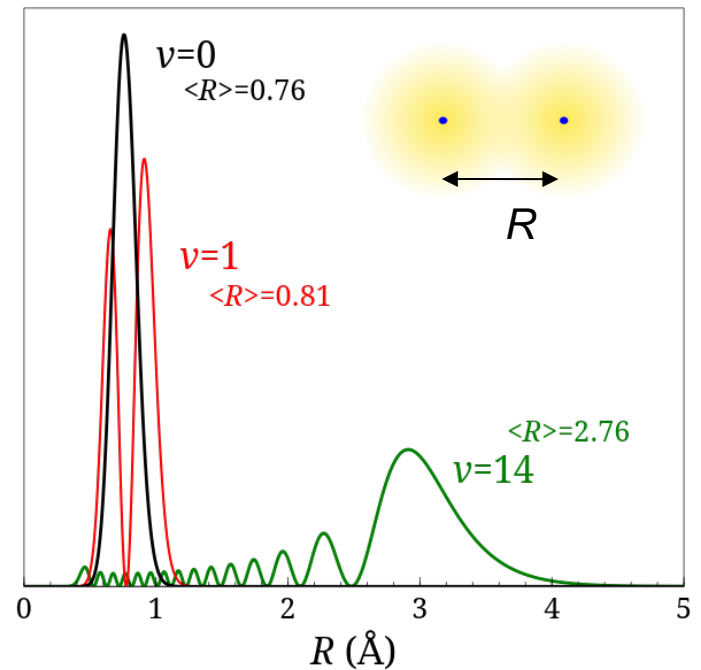
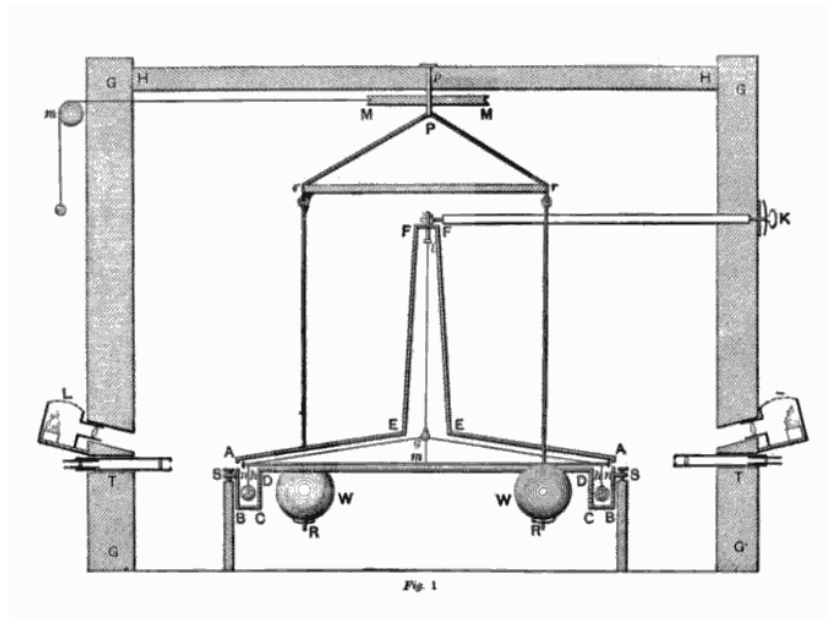
Gravity inside Klein radius

derive $G_{(3+n)} = \left(R_{comp}\right)^n G_3$

$$V_{ADD}(r) = -G_3 \frac{m_1 m_2}{r} \left(\frac{R_{comp}}{r} \right)^n$$

Enhancement factor
for gravity in n extra
dimensions

A Cavendish torsion balance at 1 Å distance



Two protons act as
Cavendish gravitating balls

ADD in Molecules

Expectation value for the ADD-compactification in a molecule:

$$\langle V_{ADD}(r) \rangle_{\text{state}} = \alpha_G N_1 N_2 \left[R_n^n \int_0^{R_n} \Psi^*(r) \frac{1}{r^{n+1}} \Psi(r) r^2 dr + \int_{R_n}^{\infty} \Psi^*(r) \frac{1}{r} \Psi(r) r^2 dr \right]$$

Difference between two quantum states:

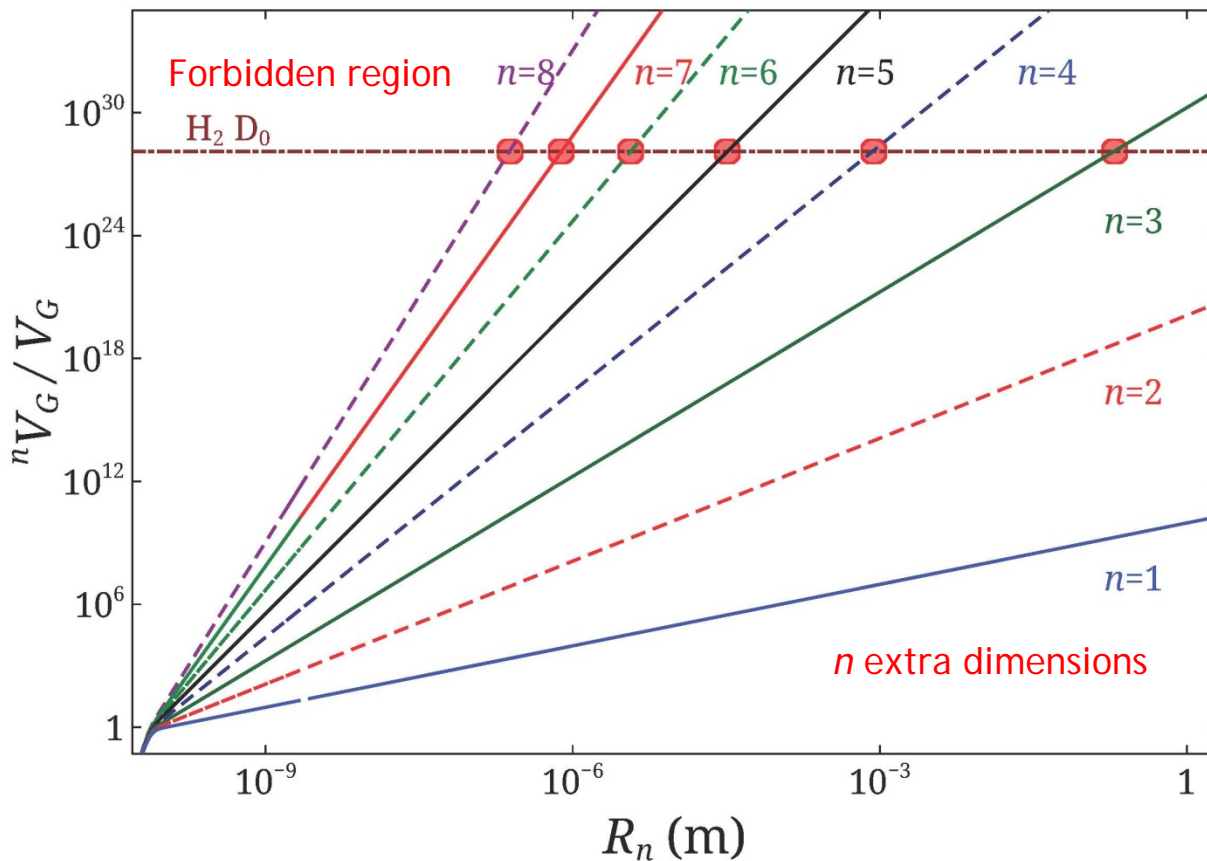
$$\langle \Delta V_{ADD}(r) \rangle_{\text{transition}} = \alpha_G N_1 N_2 \left[\left\langle \frac{1}{r^{n+1}} \right\rangle_{\Psi_1} - \left\langle \frac{1}{r^{n+1}} \right\rangle_{\Psi_0} \right]$$

Test for:

$$\langle \Delta V_{ADD} \rangle_{\text{transition}} < \delta E$$

Constraints from $\text{H}_2 \text{D}_0$

$$\left(R_{\text{comp}}\right)^n < \frac{\delta E}{\alpha_G \hbar c N_1 N_2 \Delta \left\langle r^{-(n+1)} \right\rangle}$$



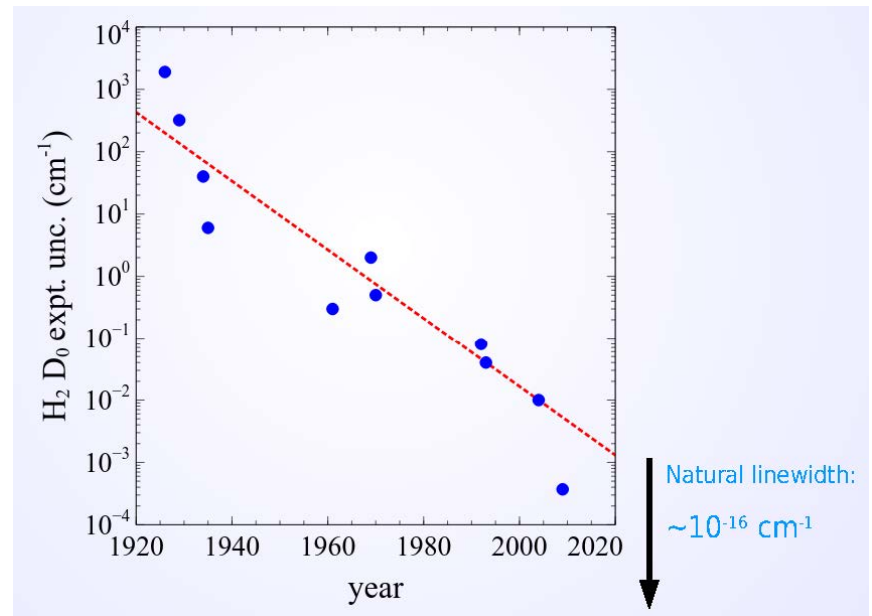
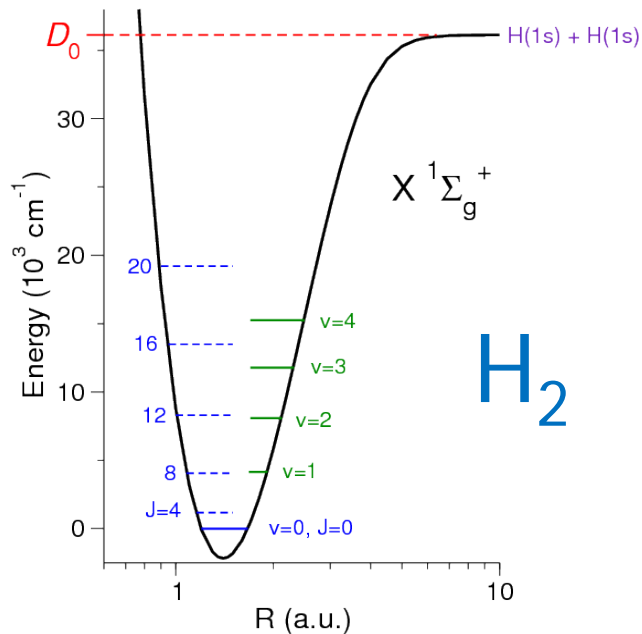
$\text{H}_2 \text{D}_0$	
n	$R_n \text{ (m)}$
2	2.2×10^4
3	1.9×10^{-1}
4	8.5×10^{-4}
5	3.2×10^{-5}
6	3.7×10^{-6}

M-theory (10 dim):
Compactification on
 μm scale !!

$$R_c < 0.6 \mu\text{m}$$

OUTLOOK:

A future molecular test system for physics



Lifetimes 10^6 seconds (!)

Quadrupole transitions $\sim 10^{14}$ Hz

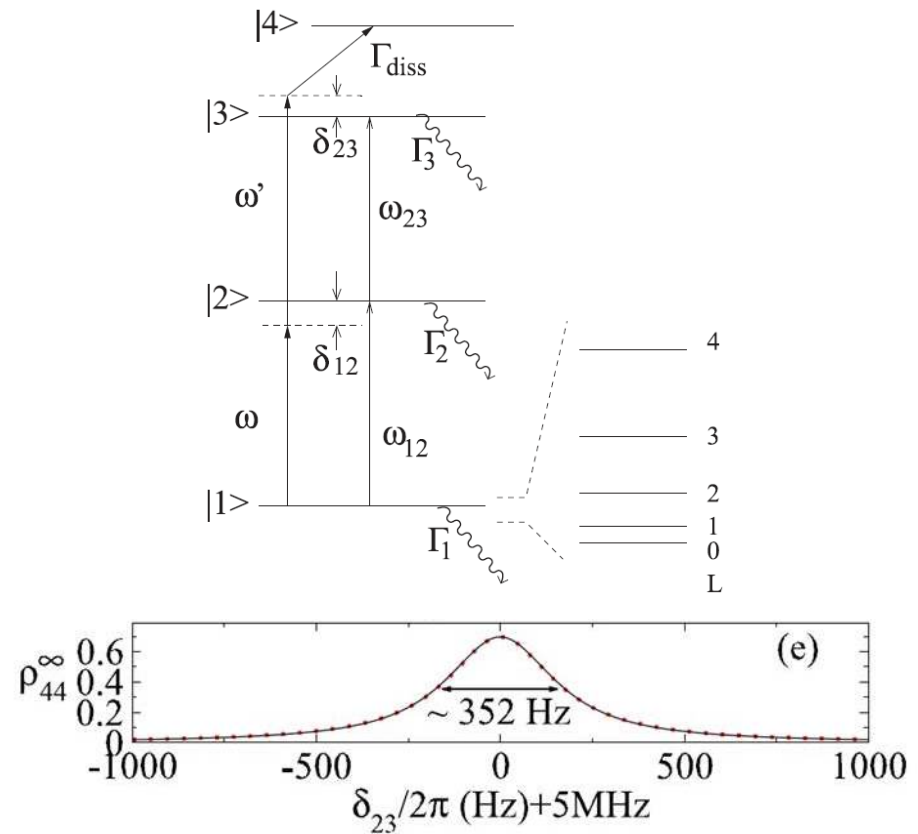
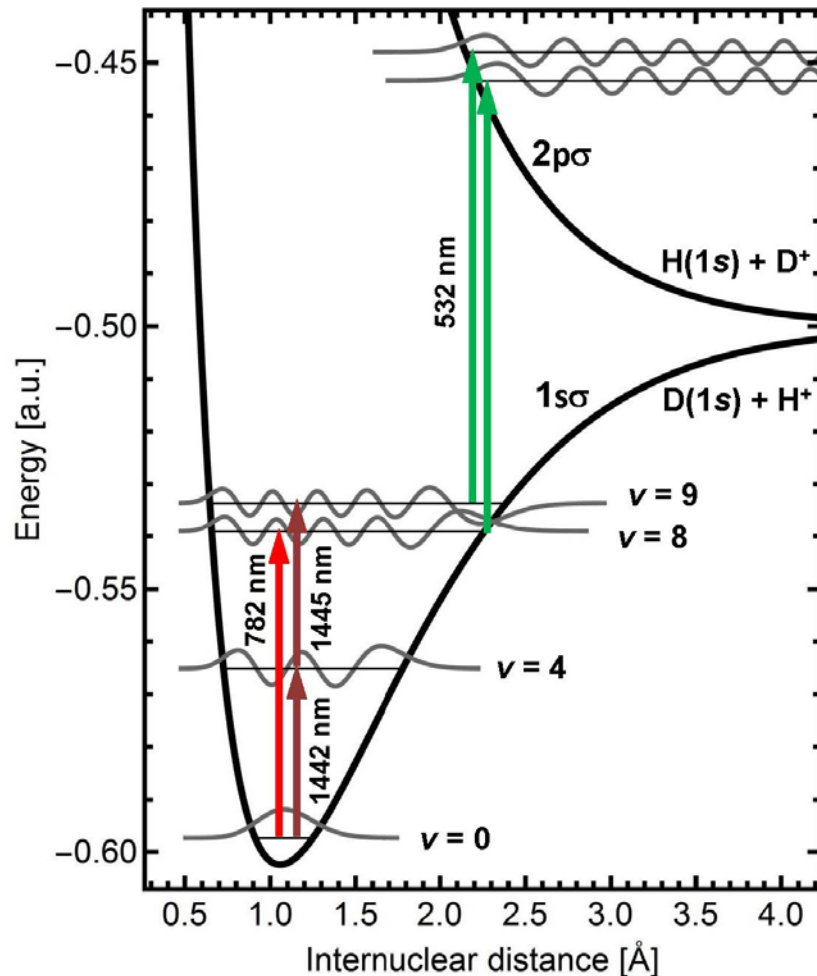
Possible precision 20-digit



There is room at the bottom guys

OUTLOOK

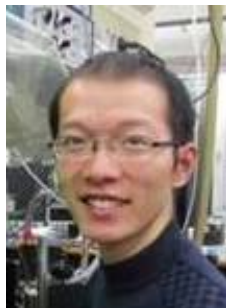
HD⁺: Doppler-free spectroscopy in Lamb-Dicke regime



Thanks & Acknowledgement



Edcel
Salumbides



MingLi
Niu



Julija
Bagdonaite



Madhu T
Trivikram



Jurriaan
Biesheuvel



Robert
Altman



Frederic
Merkt



Michael
Murphy



Krzysztof
Pachucki



Magnus
Schloesser



Jeroen
Koelemeij



Kjeld
Eikema