



# Molecular spectroscopy from first principles

Sergey Yurchenko

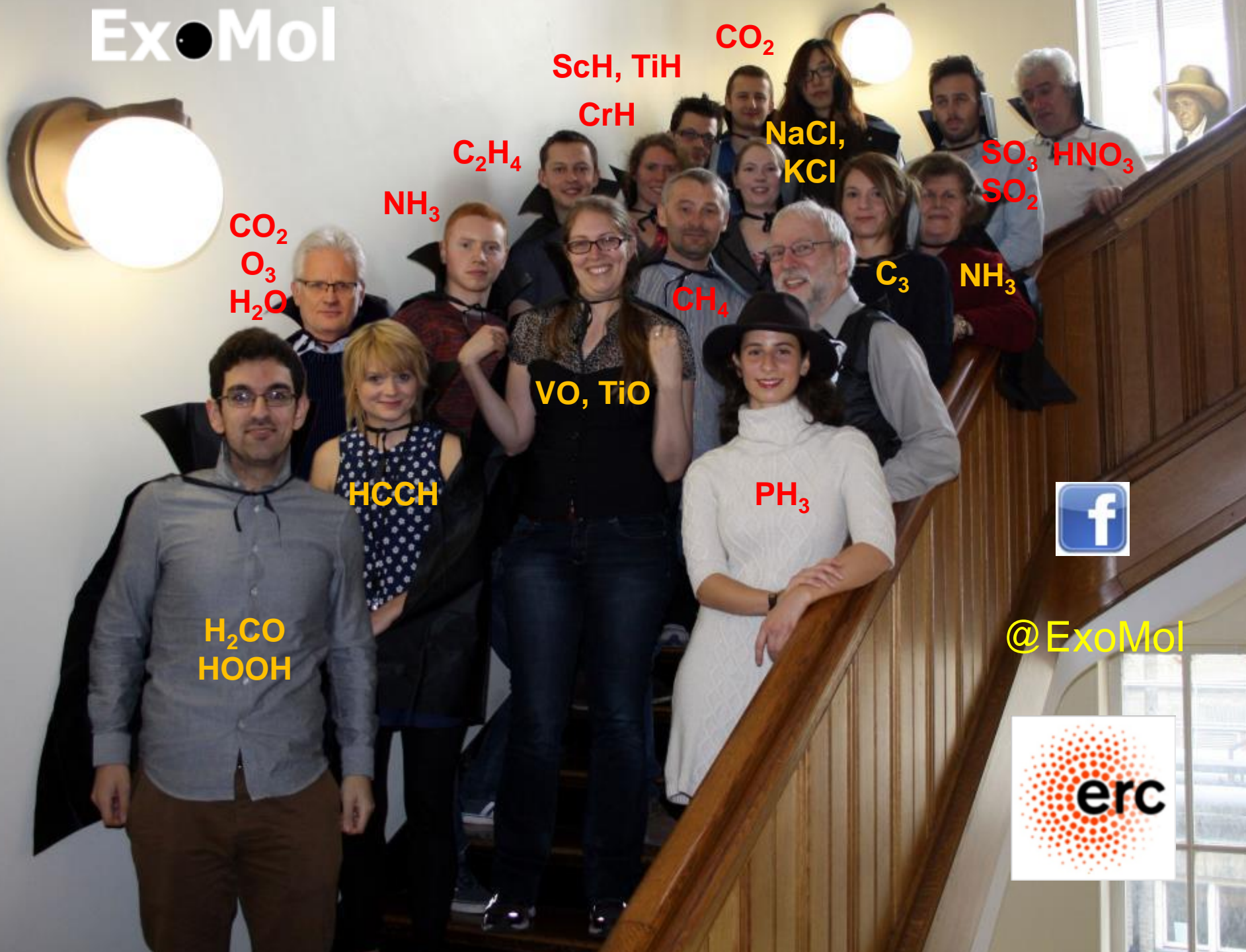


**Science & Technology**  
Facilities Council



@ExoMol





$\text{CO}_2$   
 $\text{O}_3$   
 $\text{H}_2\text{O}$

$\text{NH}_3$

$\text{C}_2\text{H}_4$

$\text{ScH}$ ,  $\text{TiH}$

$\text{CrH}$

$\text{CO}_2$

$\text{NaCl}$ ,  
 $\text{KCl}$

$\text{SO}_3$   $\text{HNO}_3$   
 $\text{SO}_2$

$\text{C}_3$

$\text{NH}_3$

$\text{CH}_4$

$\text{VO}$ ,  $\text{TiO}$

$\text{HCCH}$

$\text{PH}_3$

$\text{H}_2\text{CO}$   
 $\text{HOOH}$



@ExoMol





ExoMol

Emma Barton

Emil Zak

"Zoe" Na

Anatoly Pavlyuchko

Lorenzo Lodi

Maire Gorman

Andrey Yachmenev

Phillip Coles

Dan Underwood

Afaf Al-Derzi

Oleg Polyansky

Sergey Yurchenko

Renia Diamantopoulou

Laura McKemmish

Katy Chubb

Clara Sousa-Silva

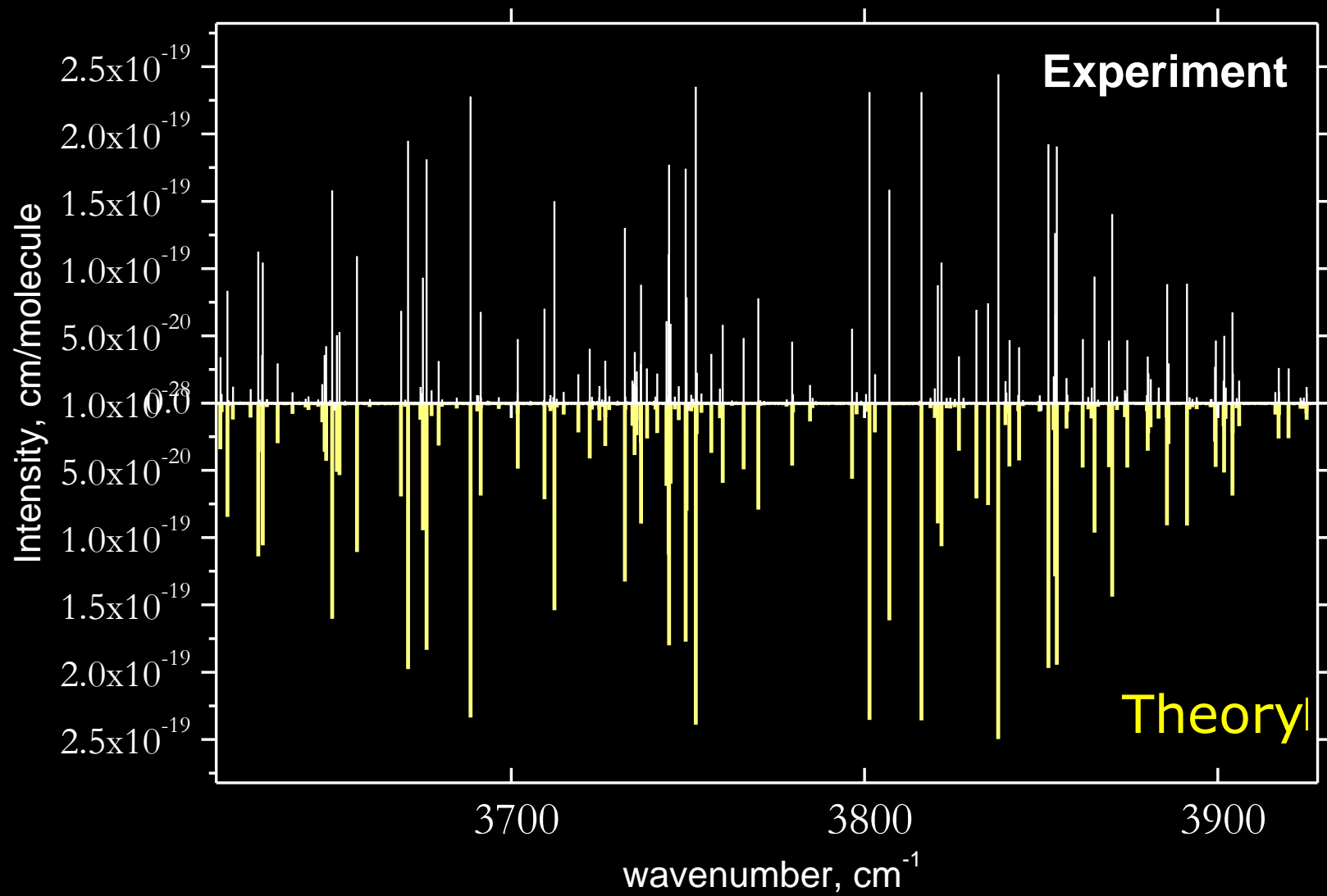
Ahmed Al-Refaie



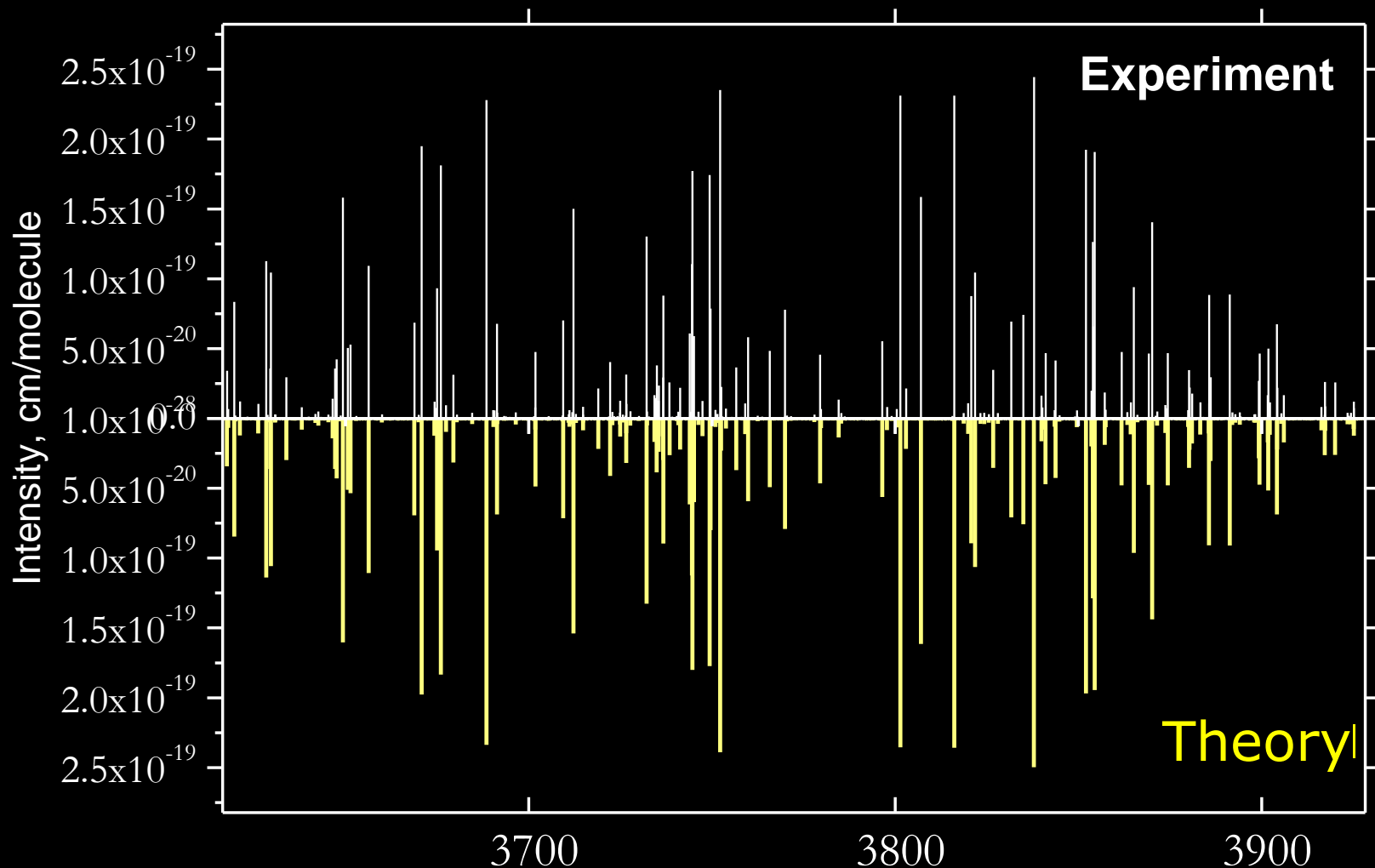
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# Theoretical spectroscopy



# Theoretical spectroscopy



This is our 2018-line list for water

Recipe



How to make do accurate  
ab initio spectra

Electronic structure = solving  
Schrödinger equation for  
electrons

Electronic structure = solving  
Schrödinger equation for  
electrons

in the Born-Oppenheimer  
approximation

*Really* ab initio can be only  
microwave spectra

Accurate equilibrium structure  
using ab initio and vibrational  
corrections



This is for (almost only) pure  
rotational spectra of  
molecules, even very large  
molecules

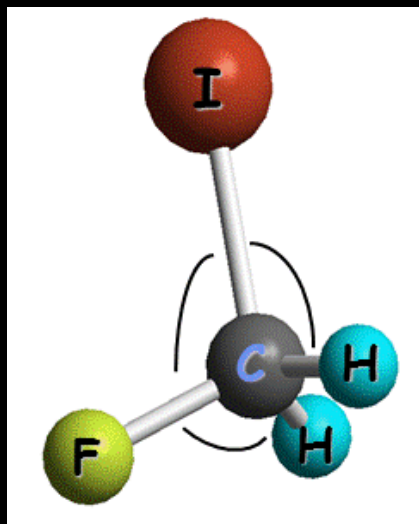
... where you start from  
accurate equilibrium structure

... which provides  $A_e$ ,  $B_e$ ,  $C_e$

... and estimate centrifugal distortion constants  $\alpha_e$  as vibrational corrections

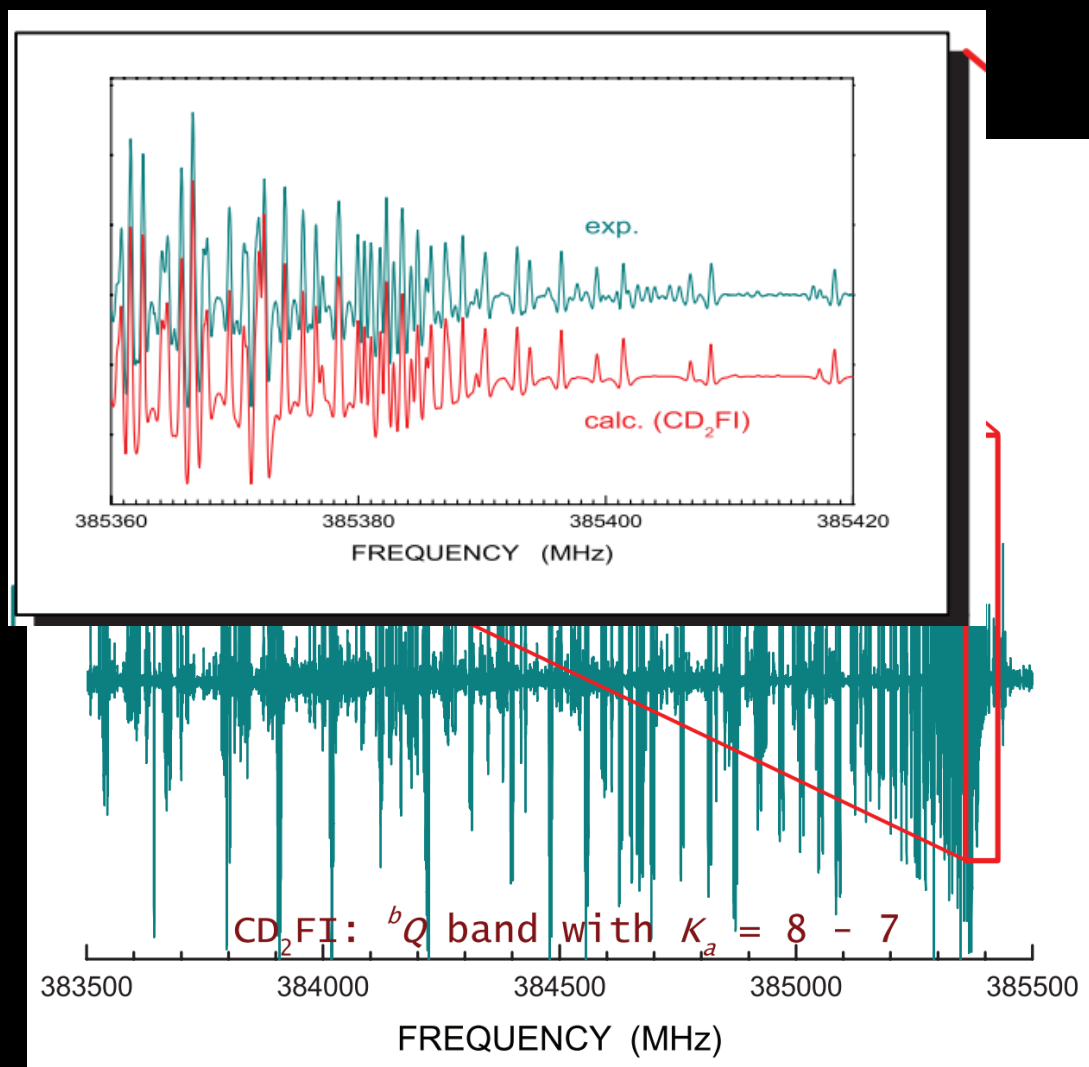
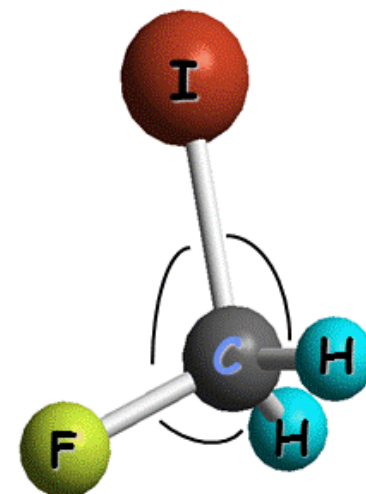
$$E_{Jv} = B_e J(J + 1) - \alpha_e J(J + 1) \left( v + \frac{1}{2} \right) + \dots$$

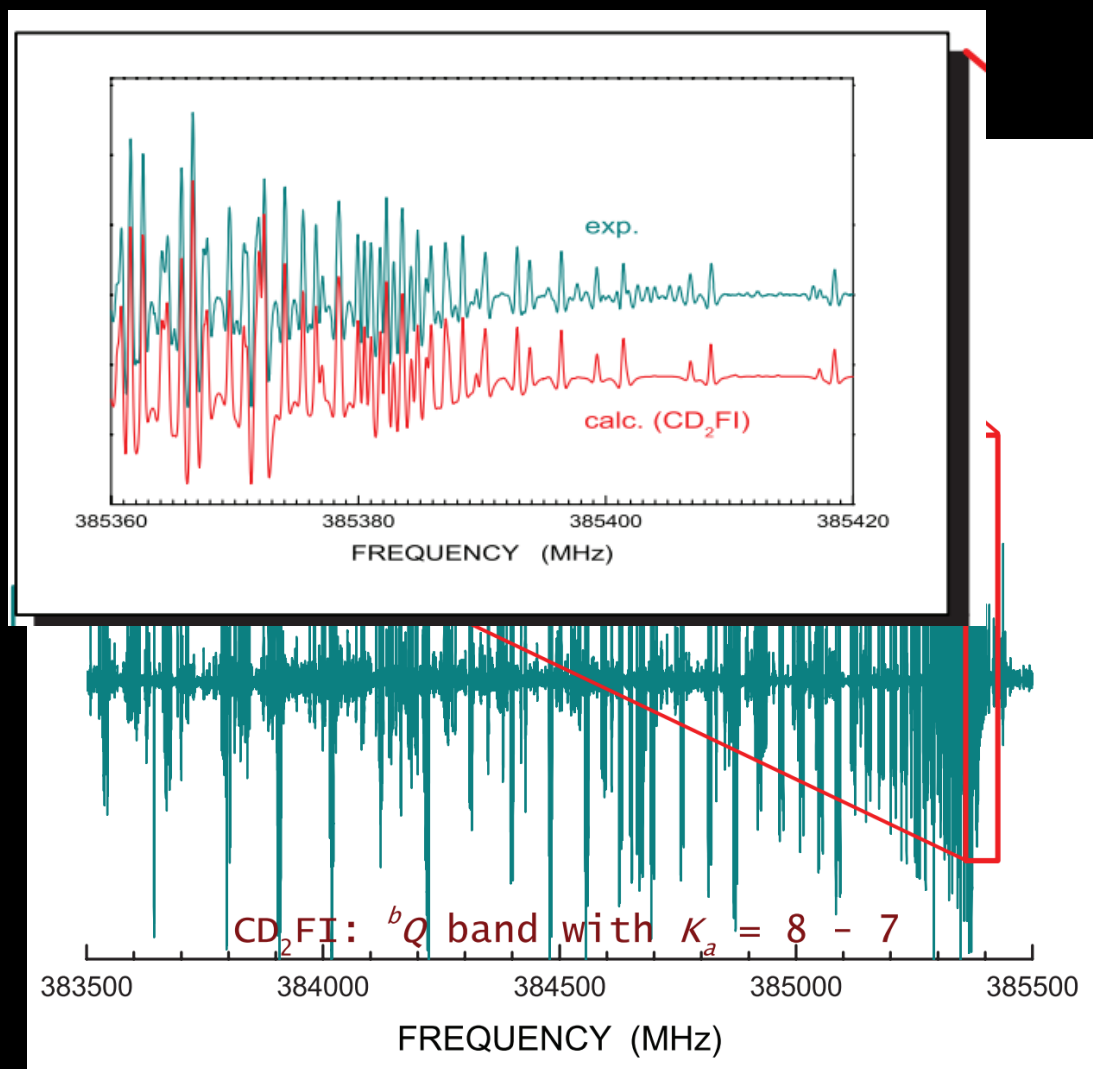
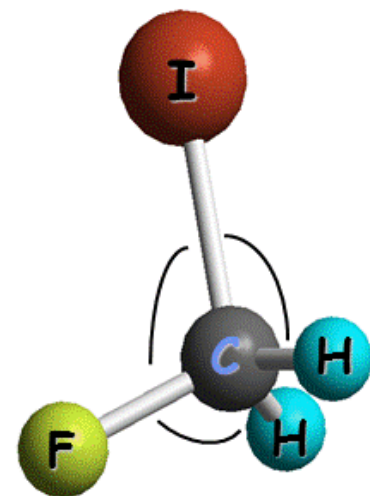
# CH<sub>2</sub>FI: Microwave spectrum



C. Puzzarini .... J. Gauss et al. J. Chem Phys. 134, 174312 (2011)







**Level of ab initio theory**  
 CCSD(T)/cc-pwCVQZ(-PP) level  
 augmented by vibrational  
 corrections at the MP2/cc-  
 pVTZ(-PP) level, quartic, and  
 sextic centrifugal-distortion  
 constants at the MP2/cc-pVTZ(-  
 PP) level, nuclear quadrupole-  
 coupling constants at the SFDC-  
 CCSD(T)/unc-ANO-RCC level,  
 and spin-rotation constants at  
 the MP2/unc-ANO-RCC level of  
 theory

No cheating!

... the equilibrium structure  
(A,B,C) can be refined using  
experimental values ...

... leads to amazing quality of  
the ab initio rotational  
spectroscopy

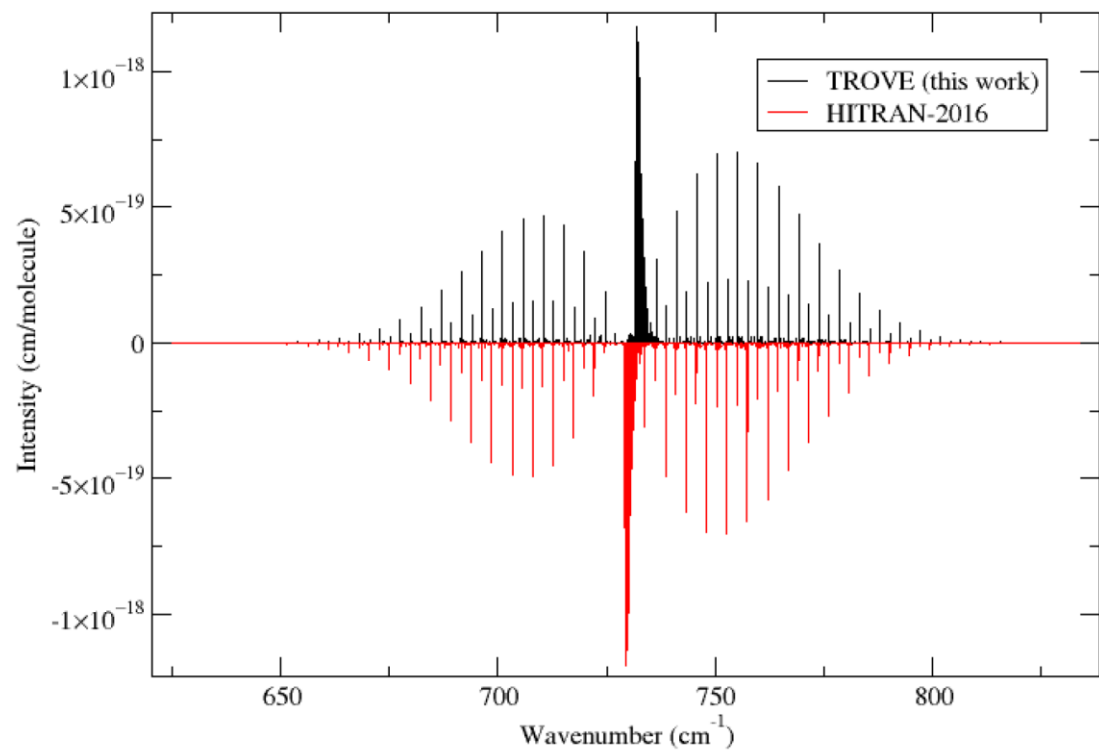


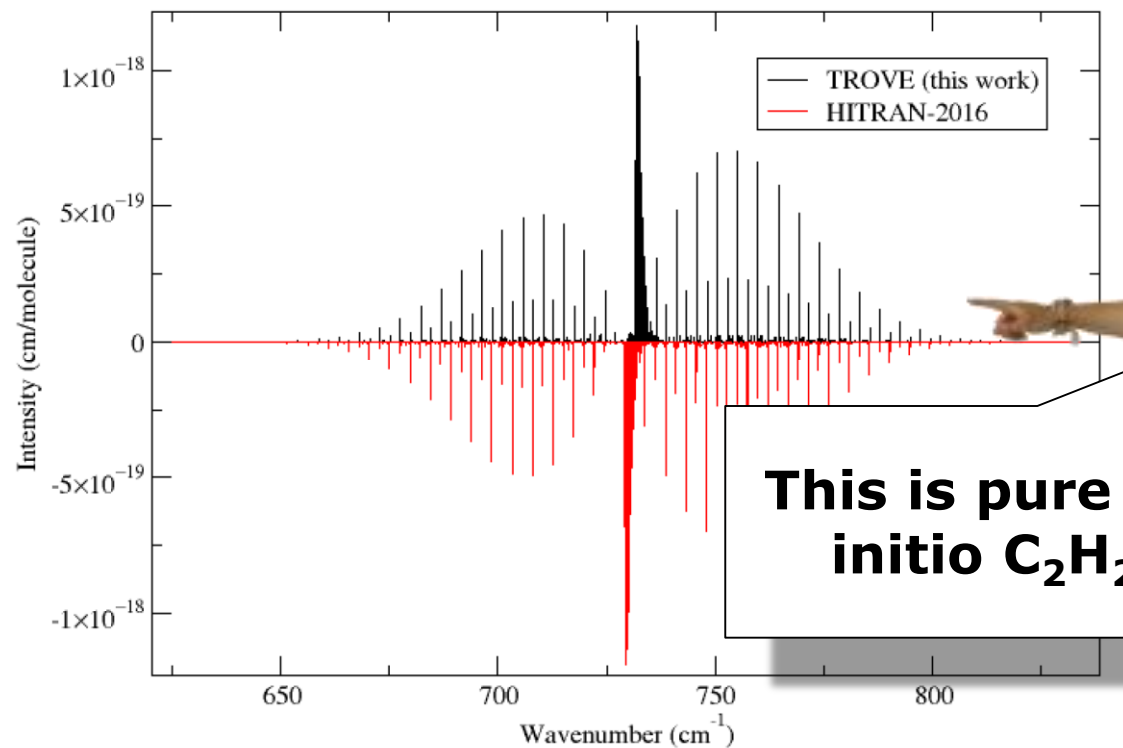
Vibrational motion however is  
much more complicated for ab  
initio methods

Qualitative description of ro-  
vibrational spectra by  
standard ab initio methods is  
reasonable

Qualitative description of ro-  
vibrational spectra by  
standard ab initio methods is  
reasonable

... but not perfect







How accurate ab initio line  
positions?

As accurate as ab initio PESs

Typically  $1\text{-}5\text{ cm}^{-1}$   
for the fundamentals  
using standard CCSD(T)-F12  
methods

For example CH<sub>4</sub>

## A highly accurate *ab initio* potential energy surface for methane

Alec Owens,<sup>1,2,a)</sup> Sergei N. Yurchenko,<sup>1</sup> Andrey Yachmenev,<sup>1</sup> Jonathan Tennyson,<sup>1</sup>  
and Walter Thiel<sup>2</sup>

<sup>1</sup>*Department of Physics and Astronomy, University College London, Gower Street,  
WC1E 6BT London, United Kingdom*

<sup>2</sup>*Max-Planck-Institut für Kohlenforschung, Kaiser-Wilhelm-Platz 1, 45470 Mülheim an der Ruhr, Germany*

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Mode	Sym.	Experiment	Calculated	Obs—calc
$\nu_4^1$	$F_2$	1310.76	1310.24	0.52
$\nu_2^1$	$E$	1533.33	1533.04	0.29
$2\nu_4^0$	$A_1$	2587.04	2585.74	1.30
$2\nu_4^2$	$F_2$	2614.26	2613.04	1.22
$2\nu_4^2$	$E$	2624.62	2624.08	0.54
$\nu_2^1 + \nu_4^1$	$F_2$	2830.32	2829.71	0.61
$\nu_2^1 + \nu_4^1$	$F_1$	2846.07	2845.44	0.63
$\nu_1$	$A_1$	2916.48	2917.16	-0.68
$\nu_3^1$	$F_2$	3019.49	3020.57	-1.08
$2\nu_2^0$	$A_1$	3063.65	3063.04	0.61
$2\nu_2^2$	$E$	3065.14	3064.53	0.61
$3\nu_4^1$	$F_2$	3870.49	3869.18	1.31
$3\nu_4^1$	$A_1$	3909.20	3907.11	2.09

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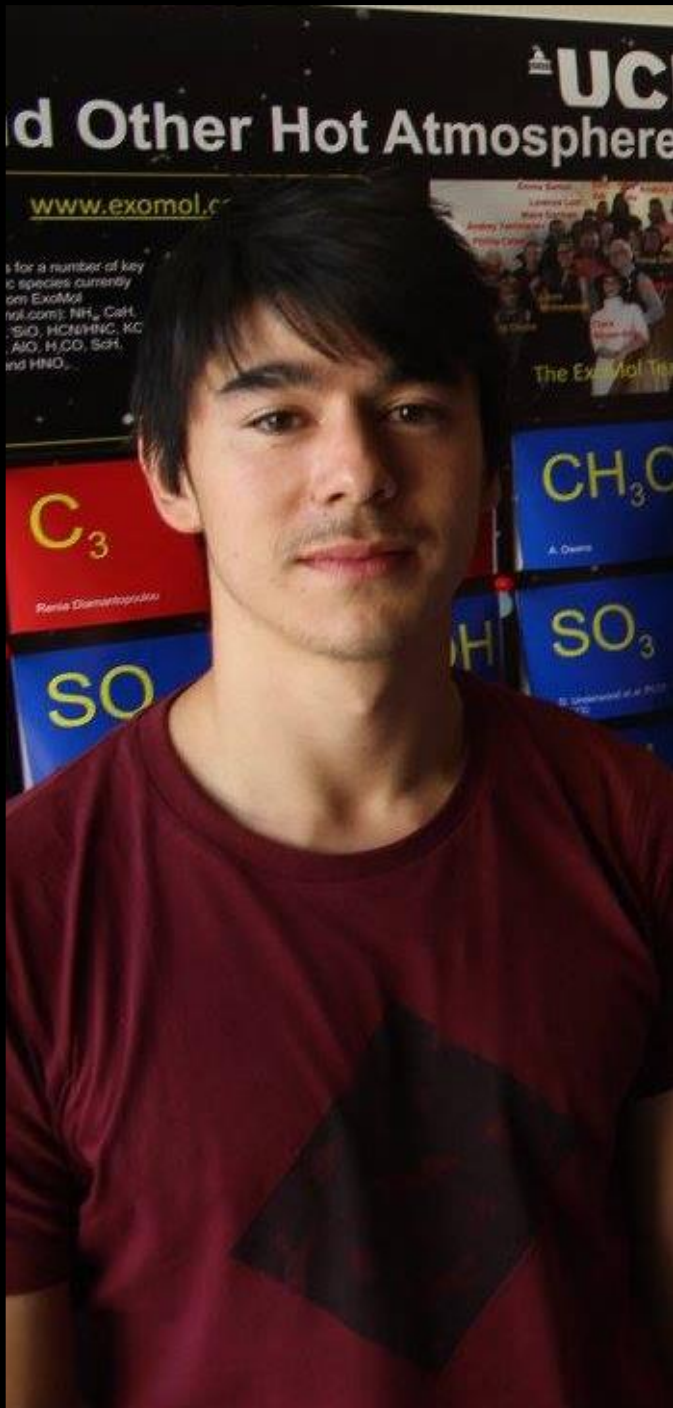


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This is still not good enough

You can do a better ab initio if  
you push really hard ...



... for example pure ab initio (no cheating) spectrum of CH<sub>3</sub>F

Ab initio PES

# Ab initio PES

CCSD(T)-F12b

# Ab initio PES

**Basic level**

CCSD(T)-F12b



# Ab initio PES

**Basic level**

CCSD(T)-F12b

Douglas-Kroll-Hess



# Ab initio PES

**Basic level**

CCSD(T)-F12b

Douglas-Kroll-Hess

$$E_{\text{tot}} = E_{\text{CBS}} + \Delta E_{\text{CV}} + \Delta E_{\text{HO}} + \Delta E_{\text{SR}} + \Delta E_{\text{DBOC}}.$$





# Ab initio PES

High Order

CCSD(T)-F12b

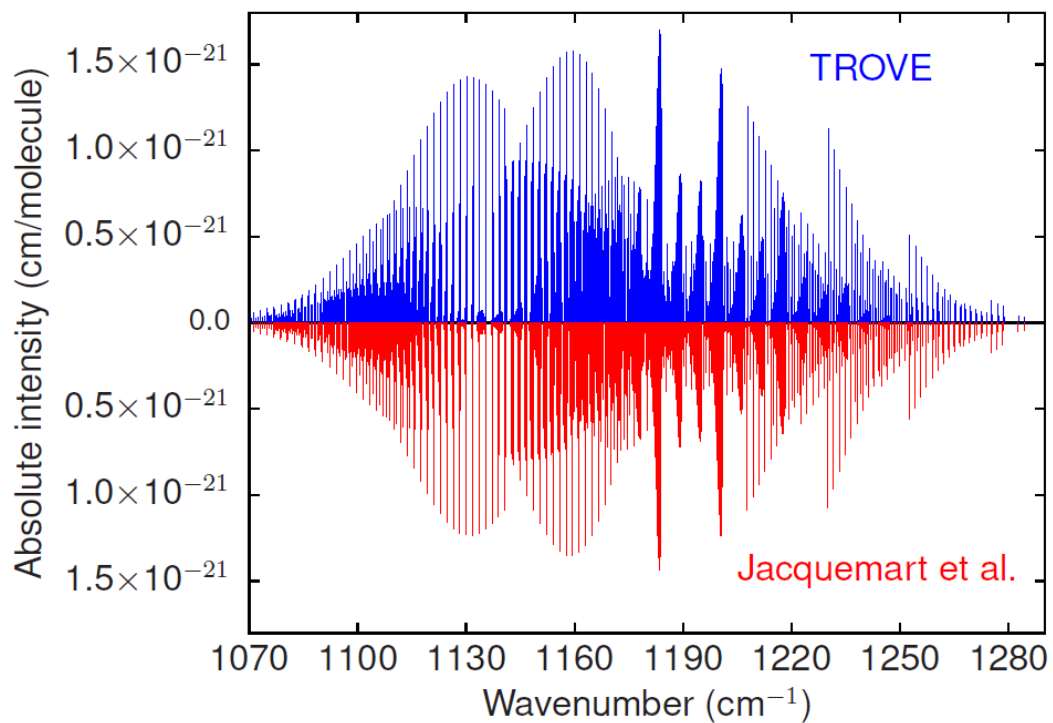
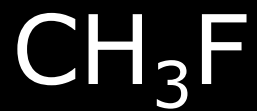
Douglas-Kroll-Hess

CCSD(T), CCSDT, and CCSDT(Q)

$$E_{\text{tot}} = E_{\text{CBS}} + \Delta E_{\text{CV}} + \Delta E_{\text{HO}} + \Delta E_{\text{SR}} + \Delta E_{\text{DBOC}}.$$



Then ab initio results can be  
really good



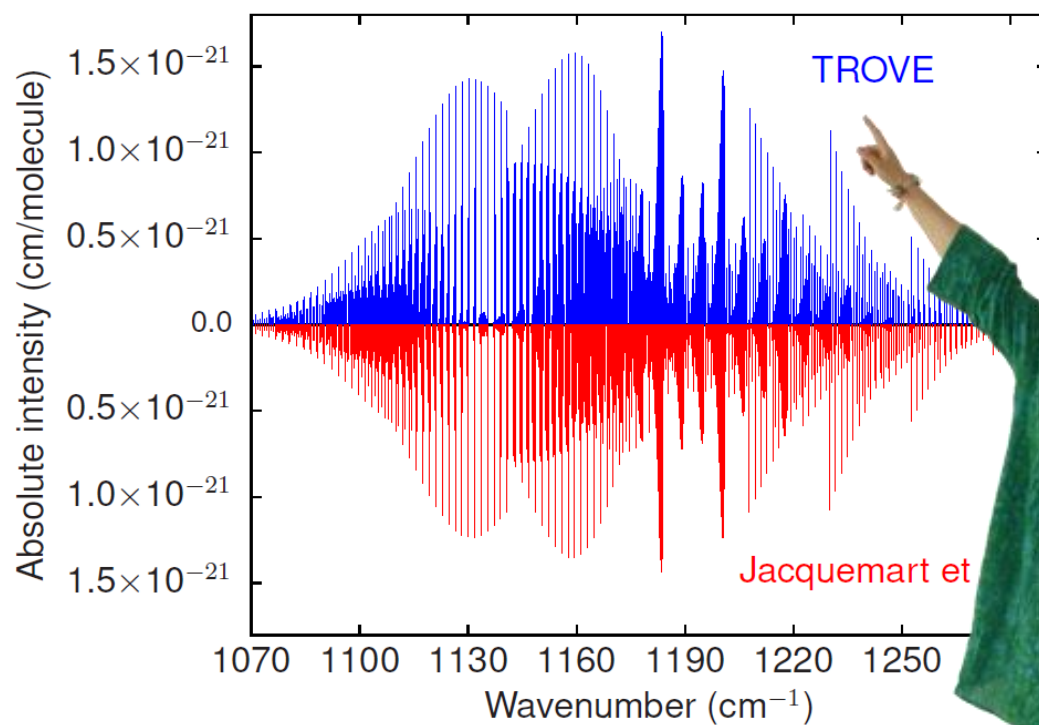
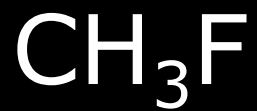
PCCP

PAPER

## The rotation–vibration spectrum of methyl fluoride from first principles†

Cite this: DOI: 10.1039/c8cp01721b

Alec Owens,<sup>a</sup> Andrey Yachmenev,<sup>ab</sup> Jochen Küpper,<sup>b</sup> Sergei N. Yurchenko<sup>d</sup> and Walter Thiel<sup>e</sup>



Theory

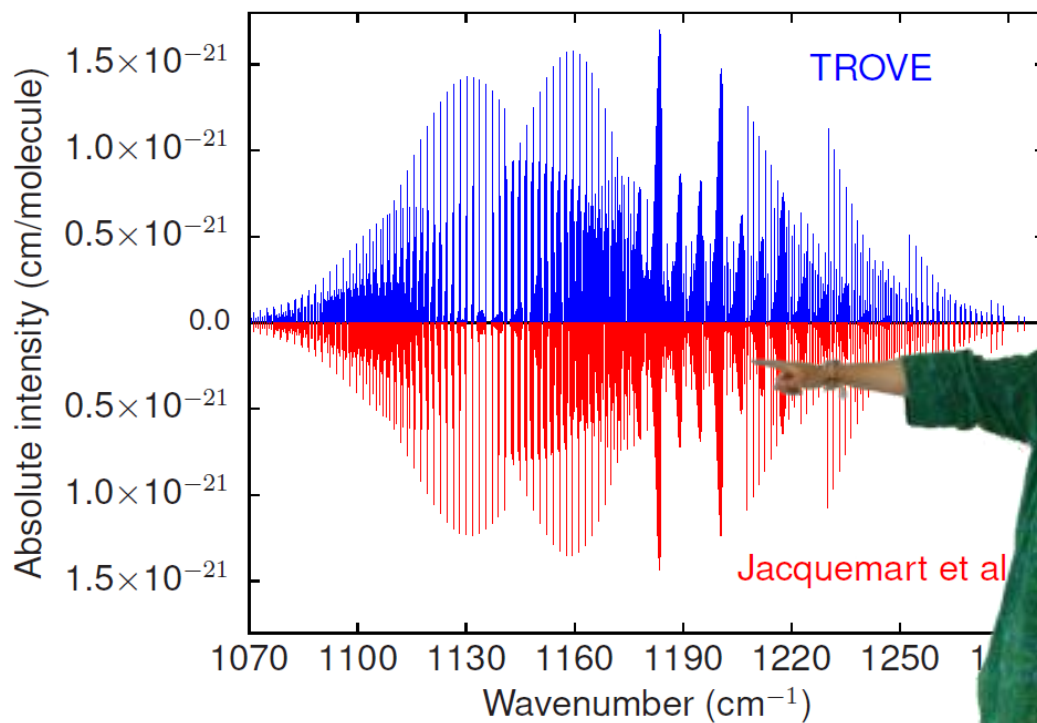
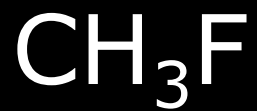
PCCP

PAPER

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Experiment

PCCP

PAPER

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DMS is also ab initio

DMS is also ab initio

CCSD(T)-F12b/aug-cc-pVTZ

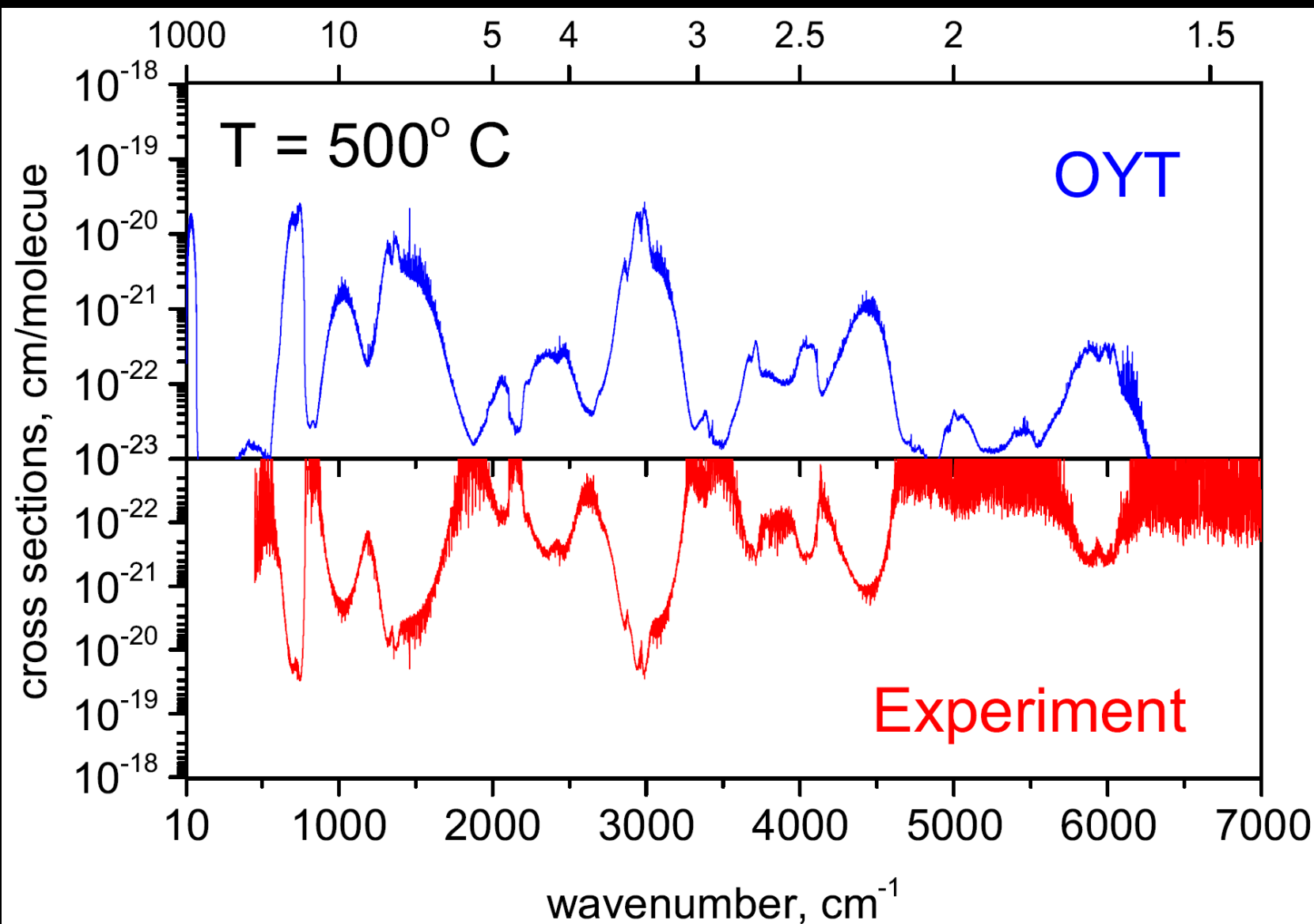
In fact intensities are (almost)  
always ab initio!

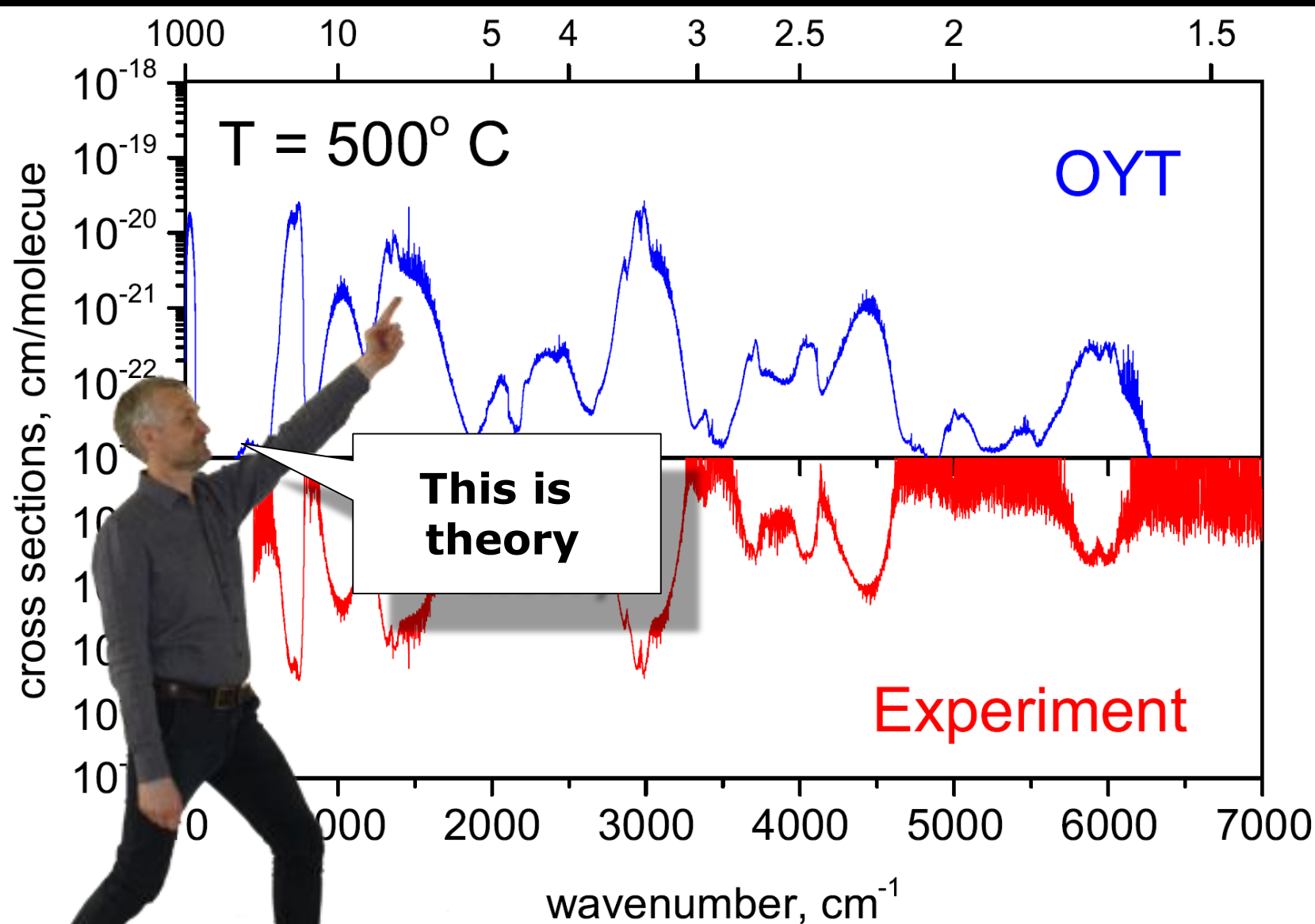


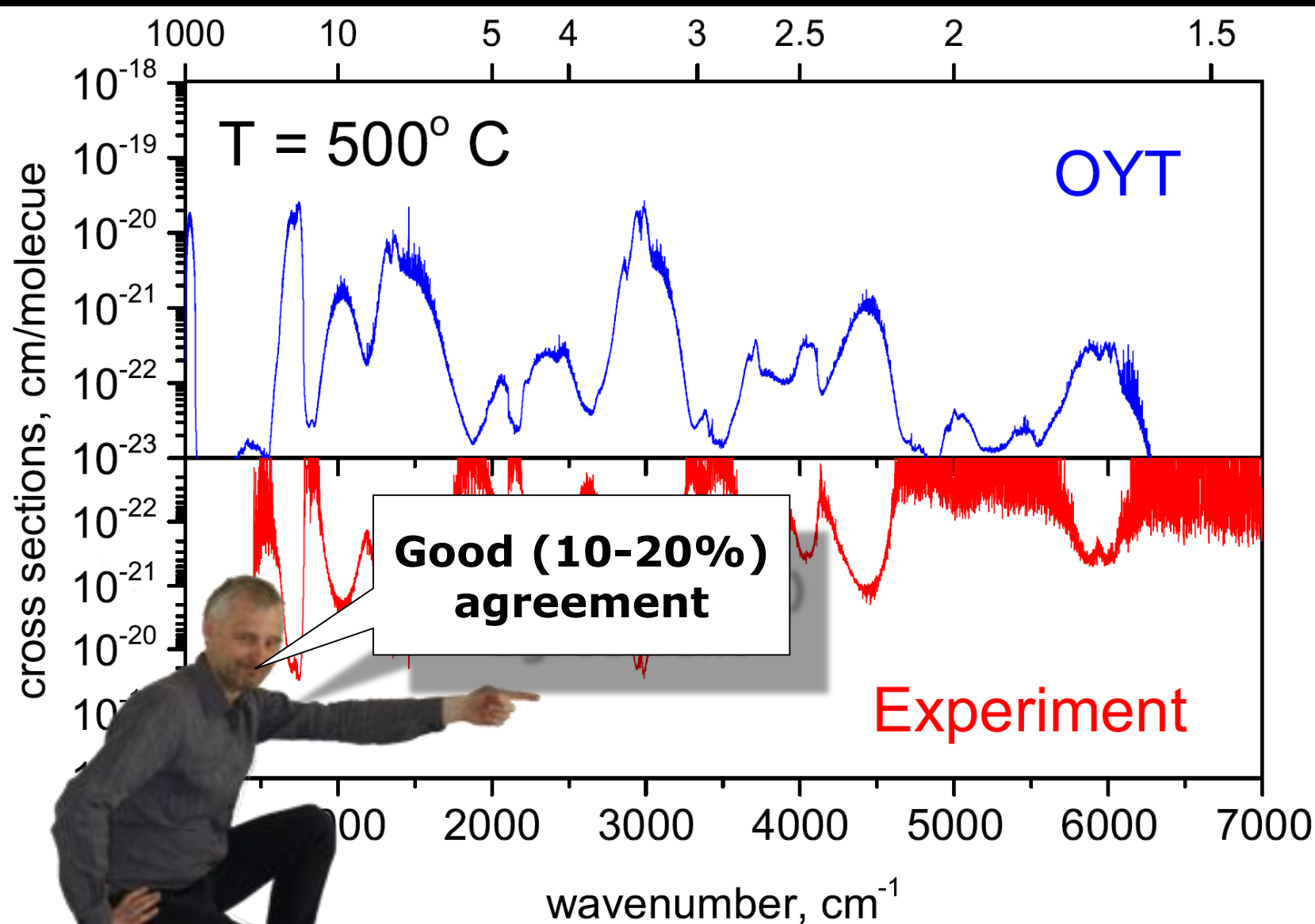
Accuracy of 10-20% is  
common for a reasonable level  
of theory

Ab initio dipole with the  
standard level of theory  
CCSD(T)/aug-cc-pVQZ

Comparing to high  
temperature experiments  
 $T = 500^{\circ}\text{C}$








You can do much better than  
this with the right levels of ab  
initio theory



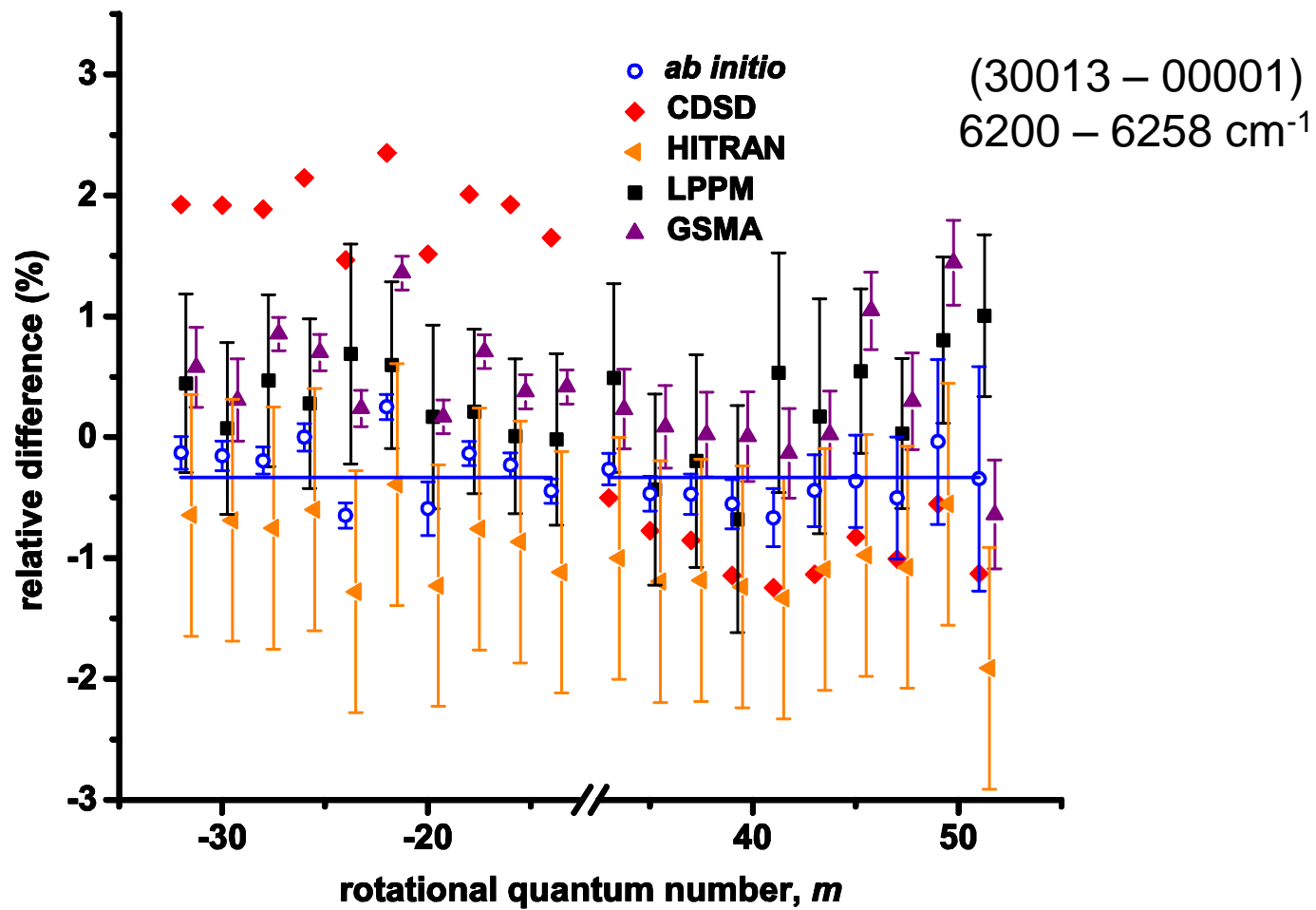


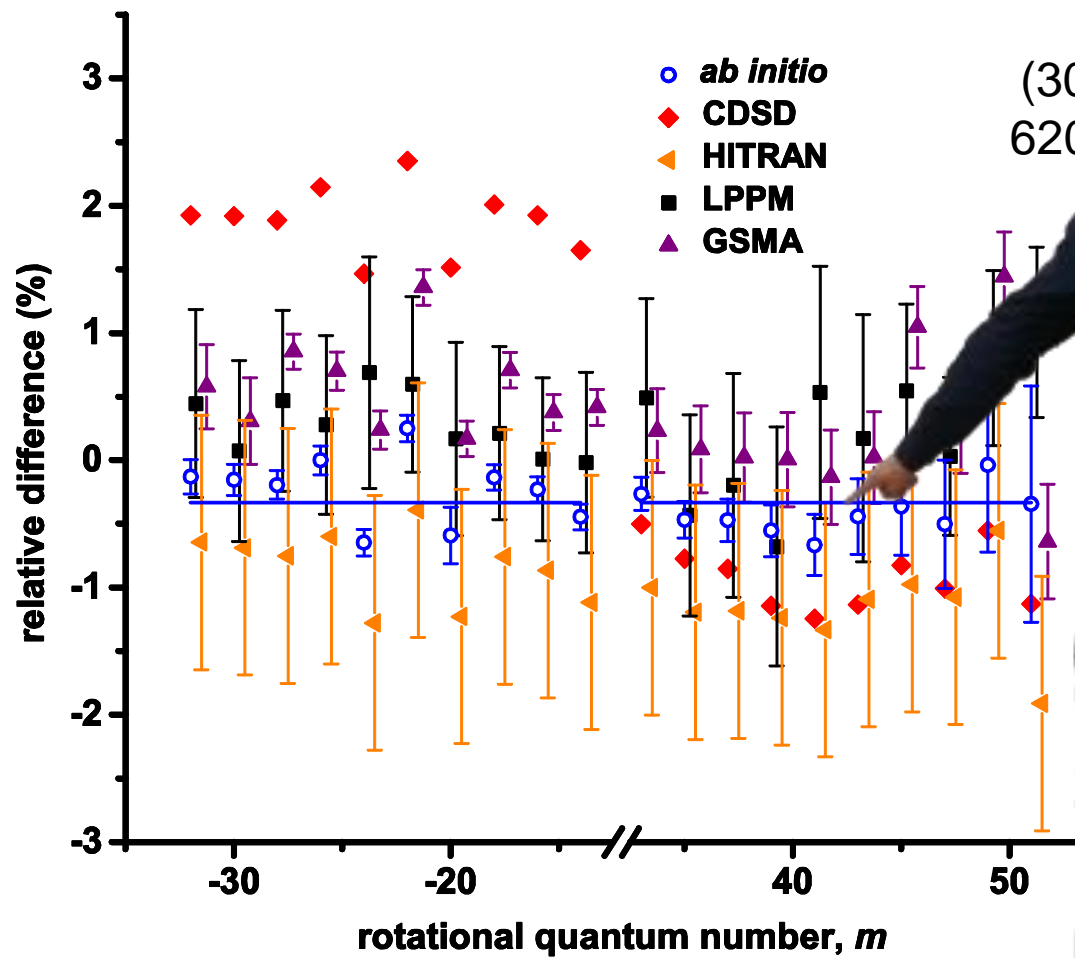
Even a  
sub-percent is  
possible

You can do much better than  
this with the right levels of ab  
initio theory



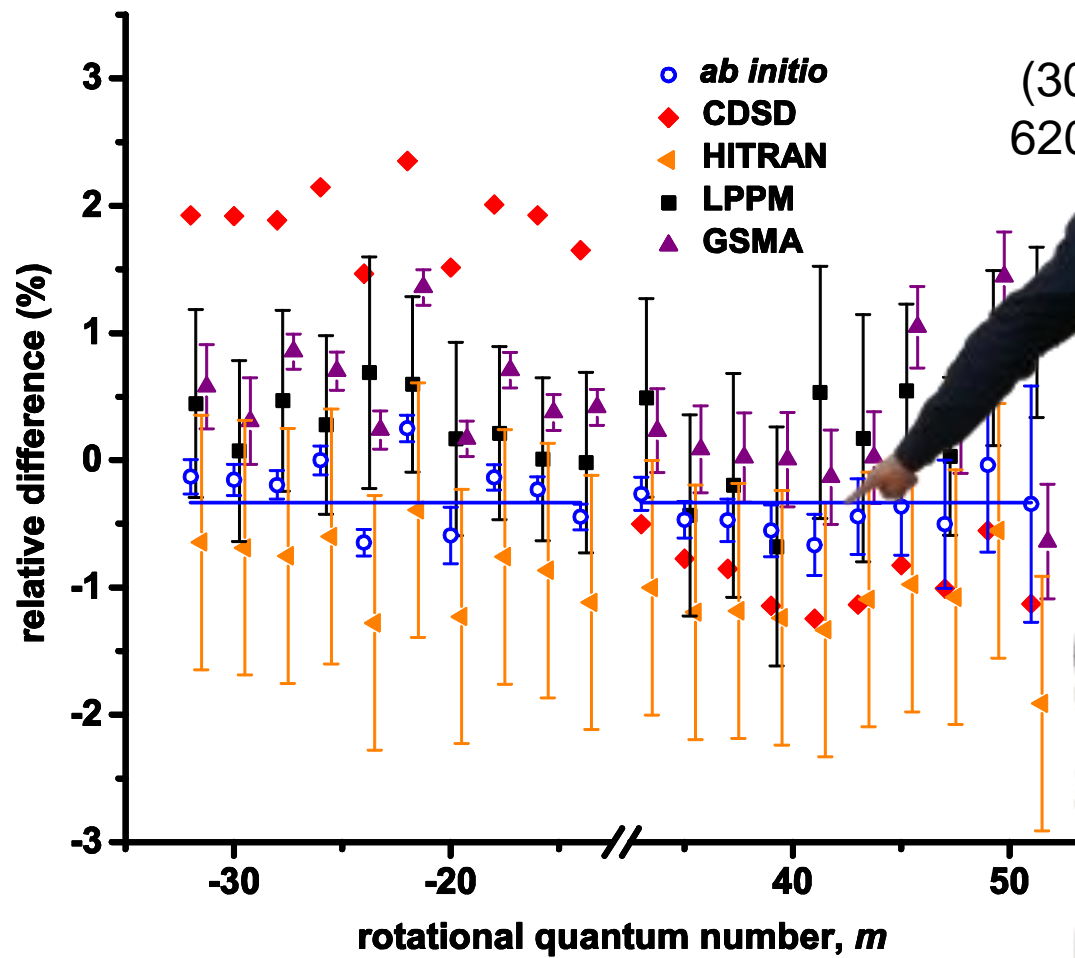
Example:  
Carbon dioxide intensities  
from Polyansky, Tennyson,  
Zak and CO<sub>2</sub>-mpany





(30013-0001)  
6200 - 6000  $\text{cm}^{-1}$

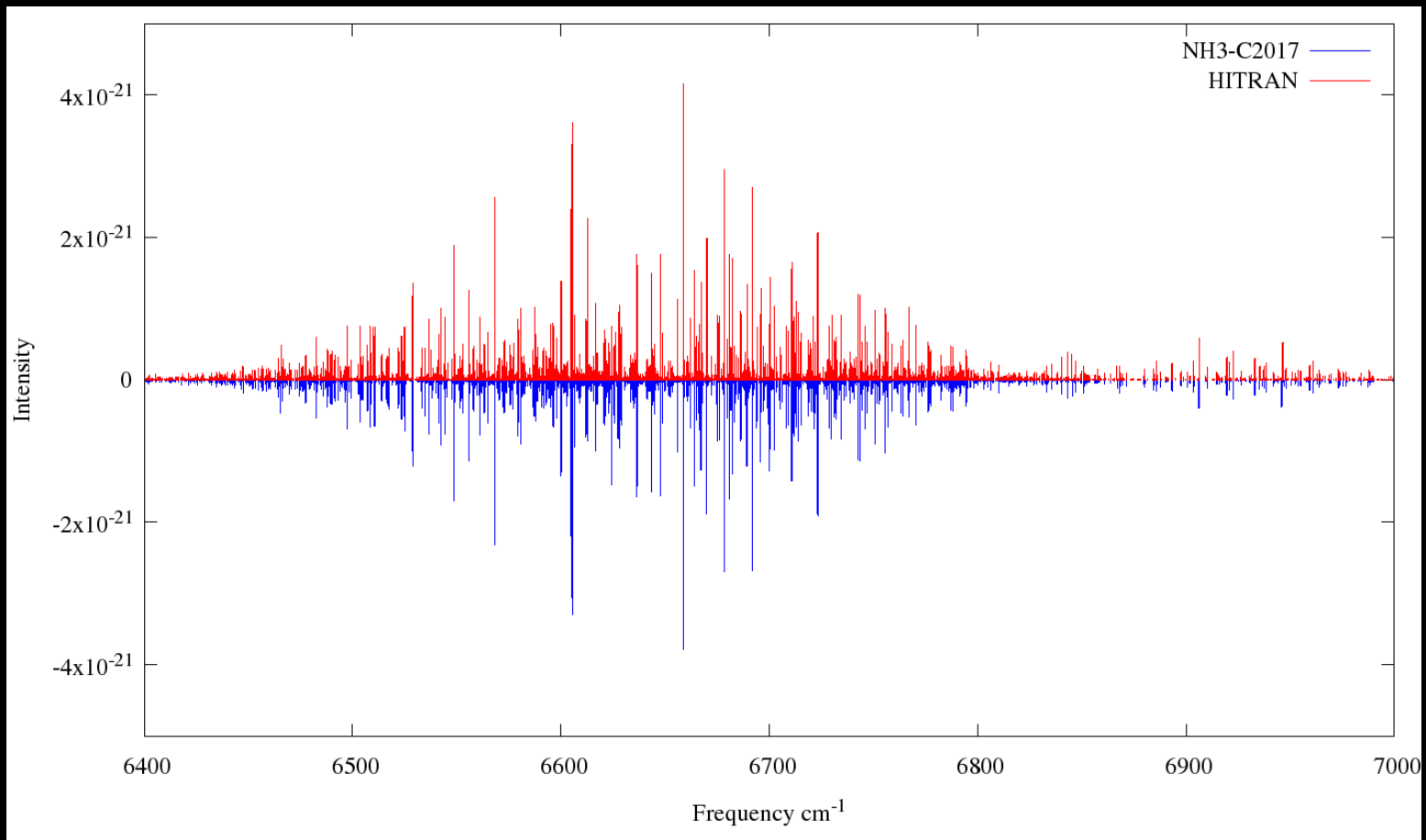
Blue is *ab initio*  
intensities

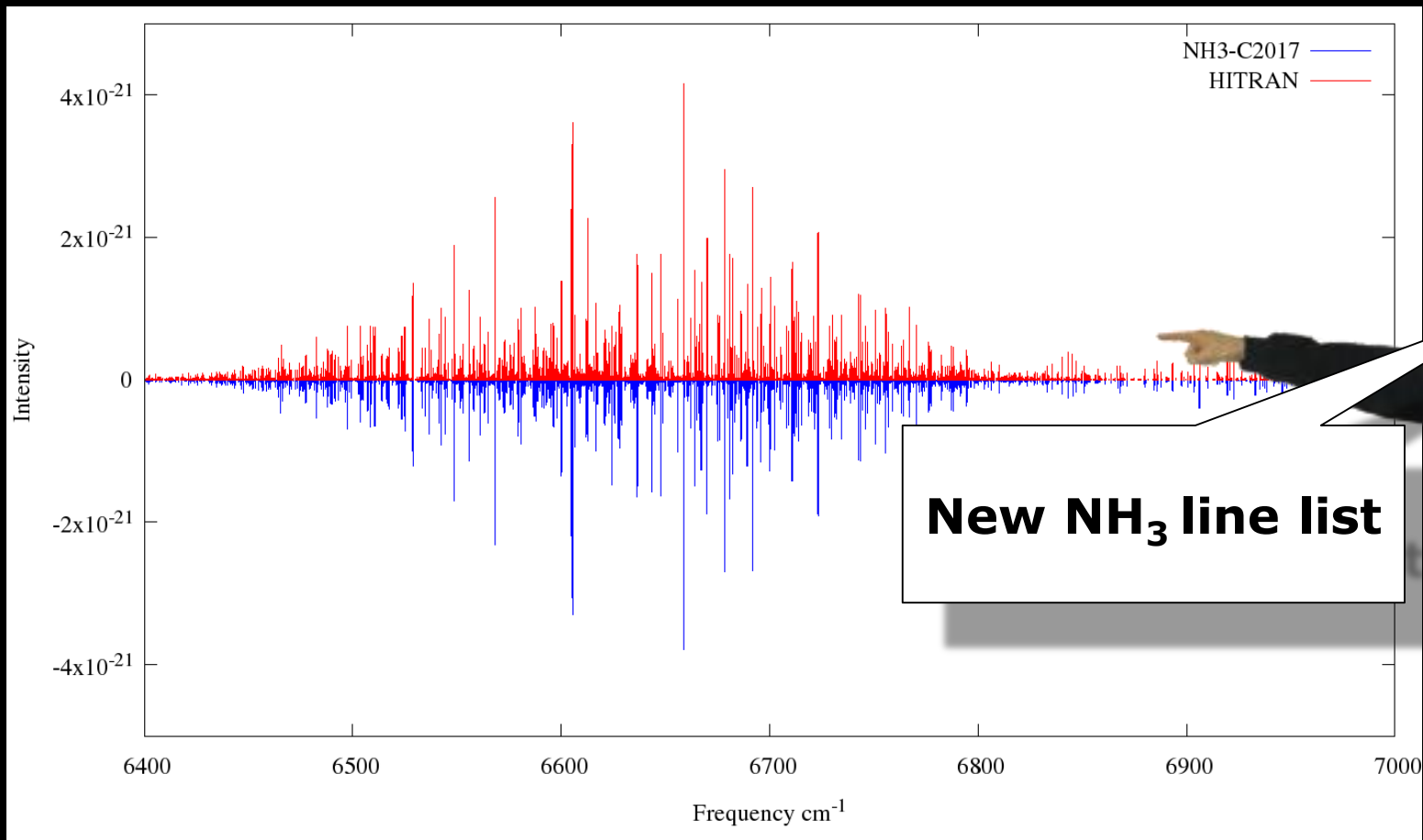


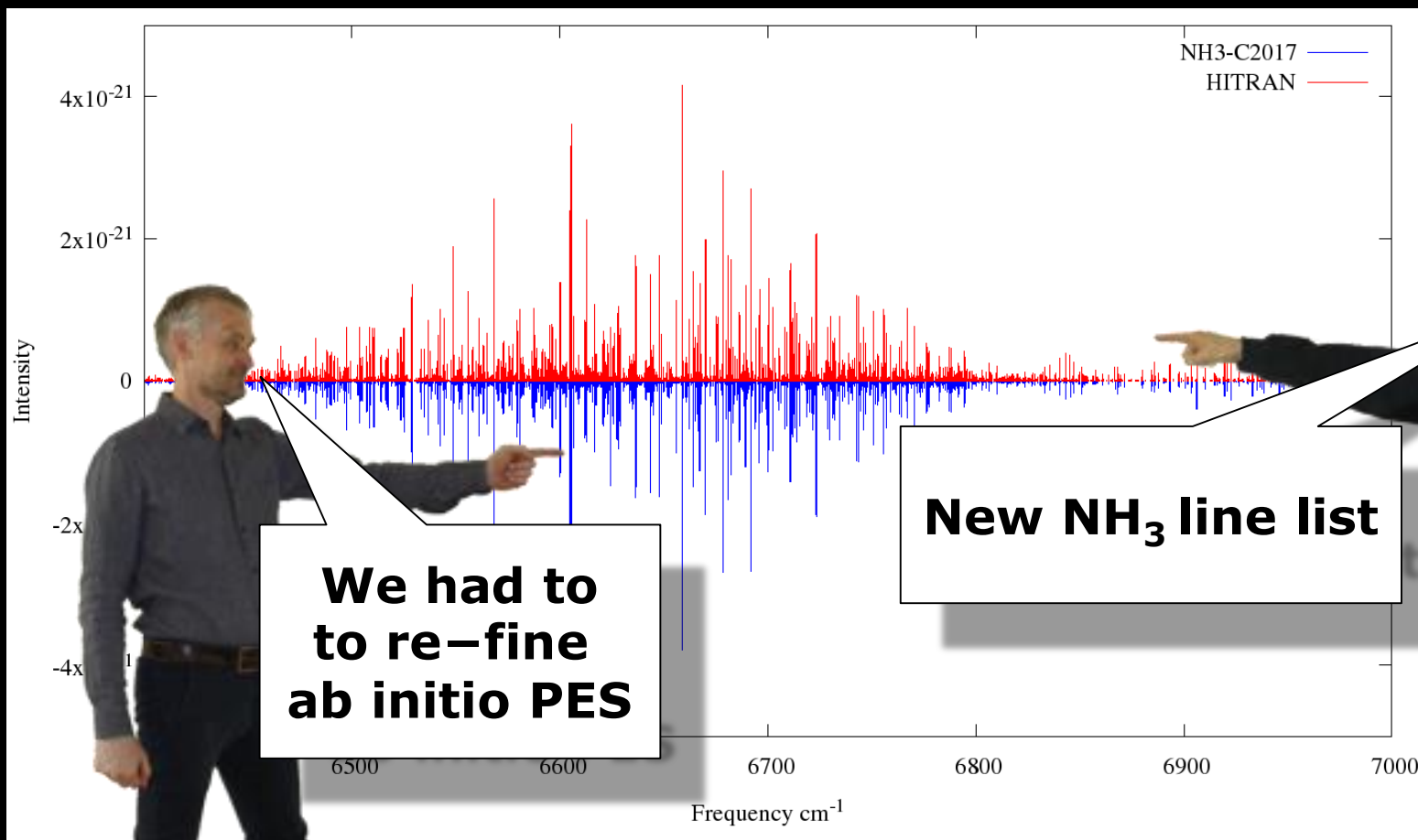
**Amaizing  
Sub-percents**

**Blue is ab  
initio  
intensiies**

Accurate ro-vibrational line  
positions?









# Method

Method

**DIATOMICS**

Solve Schrödinger equations

Solve Schrödinger equations

.... using Born-Oppenheimer  
approximation for motions of  
electrons and nuclei

# Single electronic state example

# Ab initio Spectra (SiO)

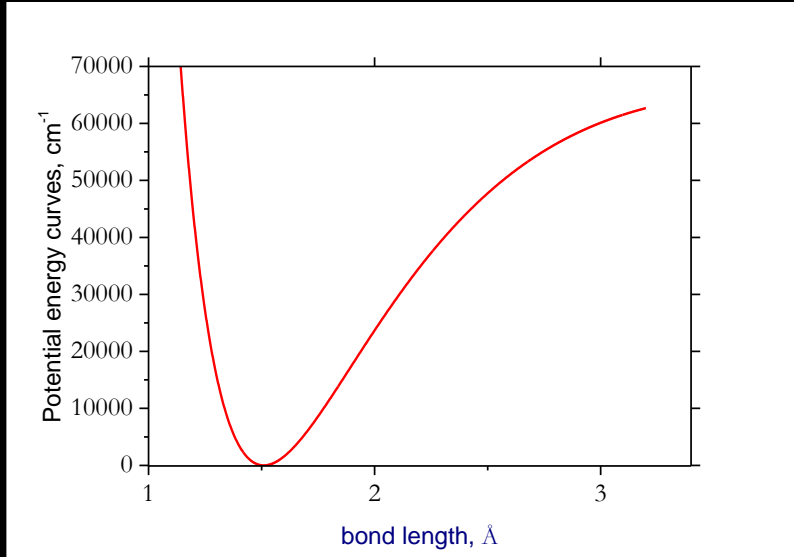
1. Ab initio Potential  
Energy Curve

2. Ab initio Dipole Moment  
Curve

3. Solution of Schrödinger  
equation (Energies and  
Wavefunctions)

4. Calculation of Intensities  
(Einstein coefficients)

# Ab initio Spectra (SiO)

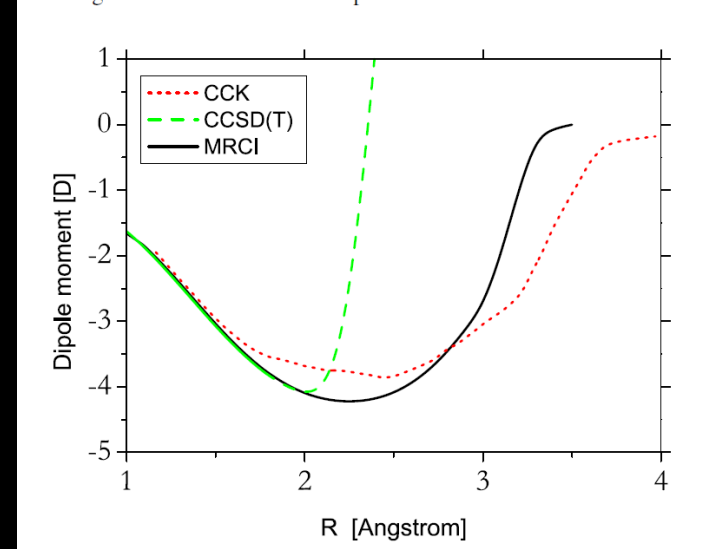
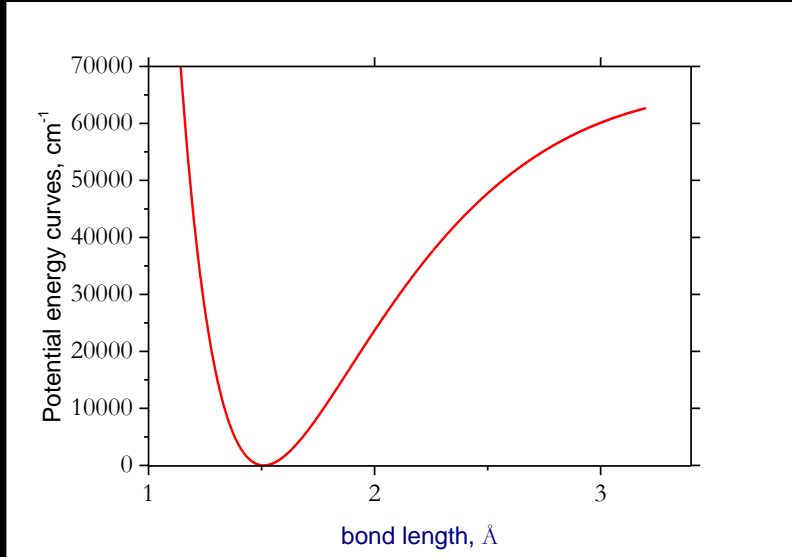


## 2. Ab initio Dipole Moment Curve

## 3. Solution of Schrödinger equation (Energies and Wavefunctions)

## 4. Calculation of Intensities (Einstein coefficients)

# Ab initio Spectra (SiO)

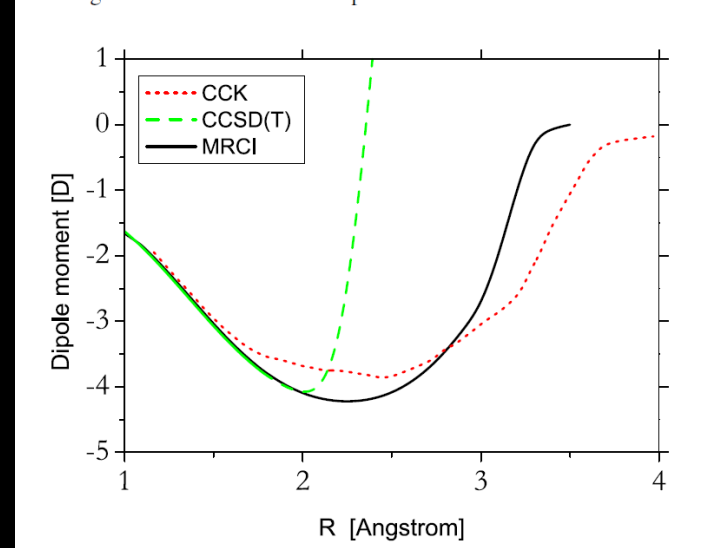
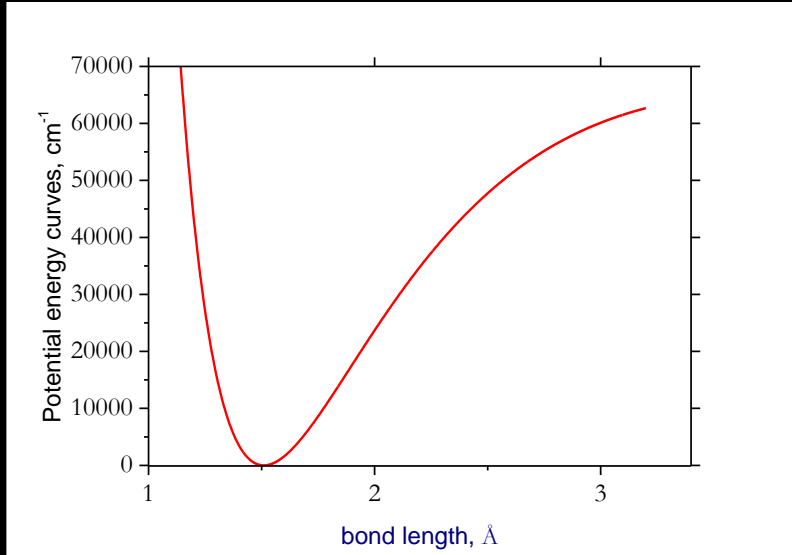


3. Solution of Schrödinger equation (Energies and Wavefunctions)

4. Calculation of Intensities (Einstein coefficients)



# Ab initio Spectra (SiO)



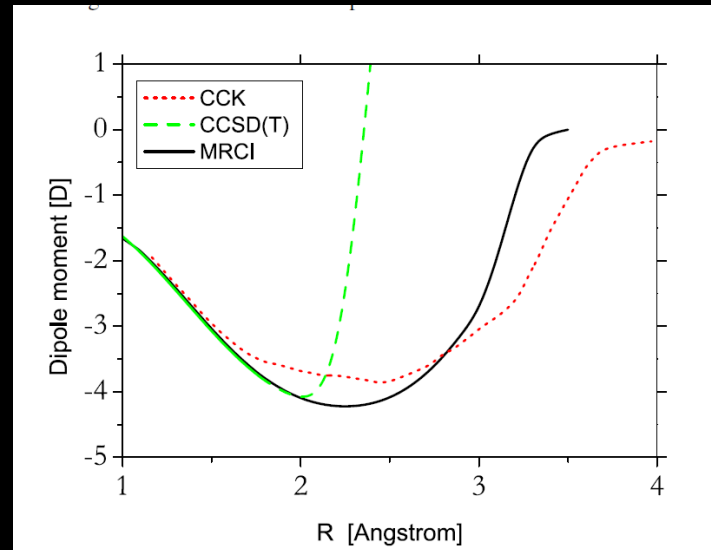
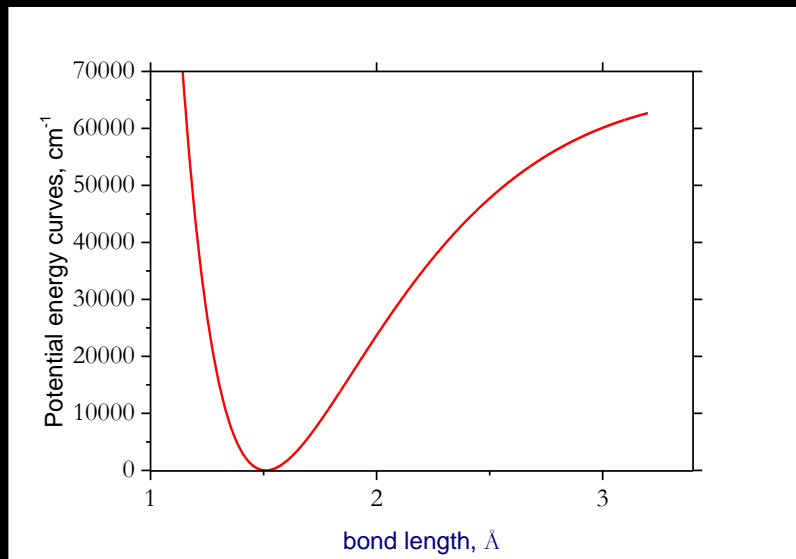
$$\frac{\hbar^2}{2\mu r^2} J(J+1) - \frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial r^2} \Psi + V(r)\Psi = E\Psi$$

## LEVEL 8.0

R. Le Roy, Waterloo, Canada  
Our Duo program

## 4. Calculation of Intensities (Einstein coefficients)

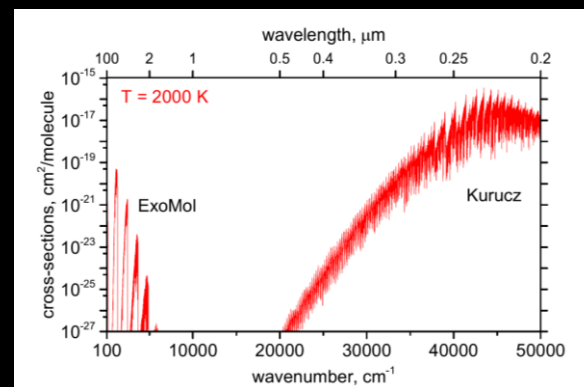
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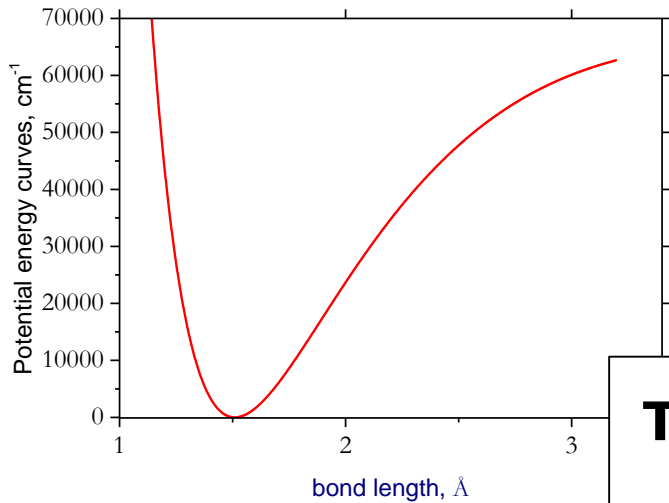
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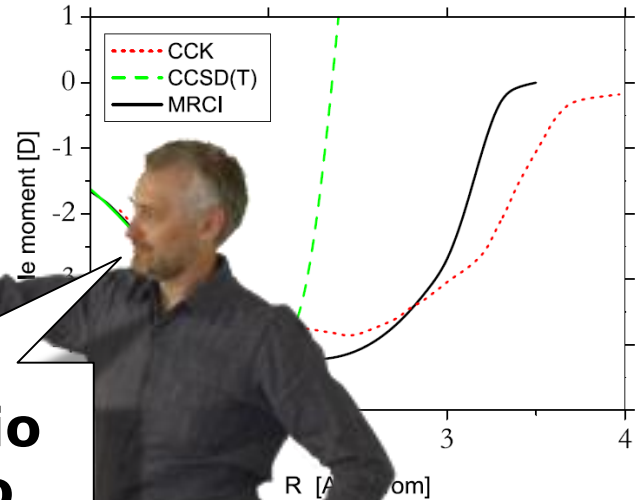
R. Le Roy, Waterloo, Canada  
Our Duo program



# Ab initio Spectra (SiO)



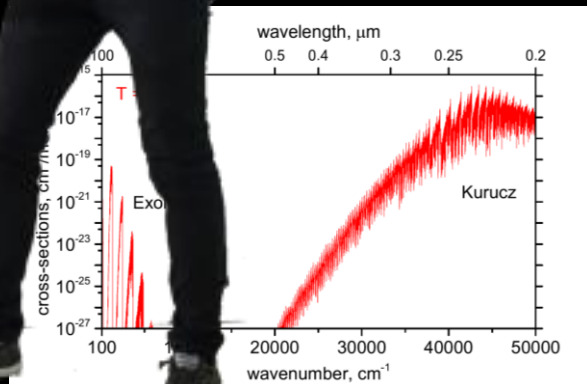
The ab initio  
PEC has to  
be refined



$$\frac{\hbar^2}{2\mu r^2} J(J+1) - \frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial r^2} \Psi + V(r)\Psi = E\Psi$$

## LEVEL 8.0

R. Le Roy, Waterloo, Canada  
Our Duo program



Most of the systems are not  
single-state

# Ab initio spectrum (MgO)

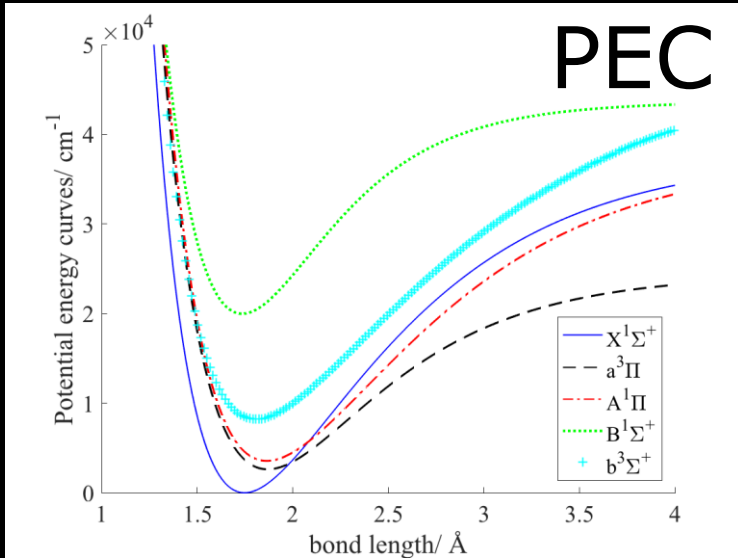
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Energy Curves

2. Ab initio Dipole Moment  
Curves

3. Spin-Orbit couplings

4. Electronic angular  
momenta couplings

# Ab initio spectrum (MgO)

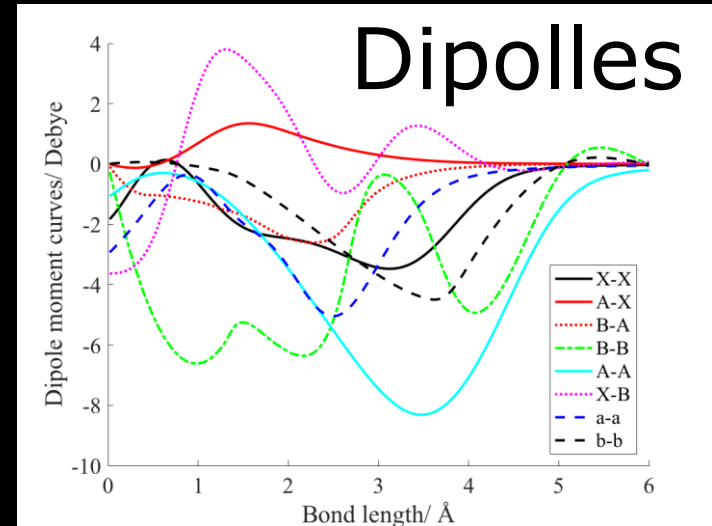
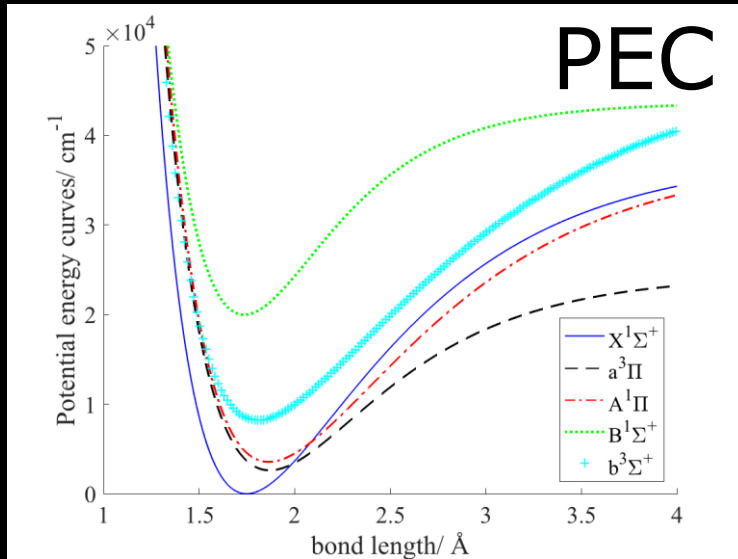


## 2. Ab initio Dipole Moment Curves

## 3. Spin-Orbit couplings

## 4. Electronic angular momenta couplings

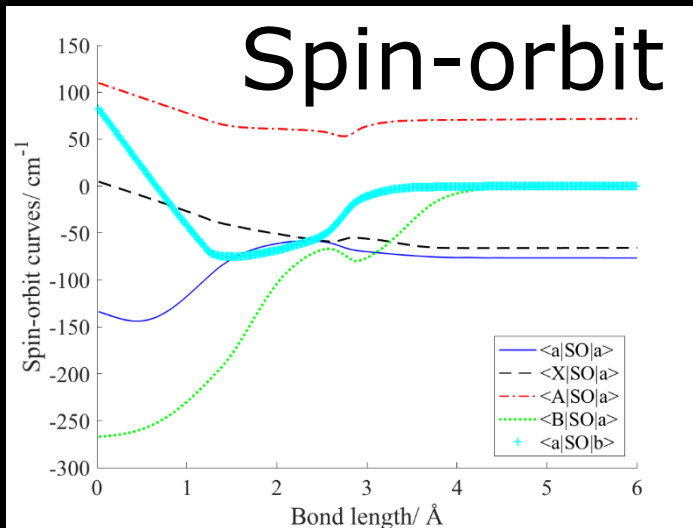
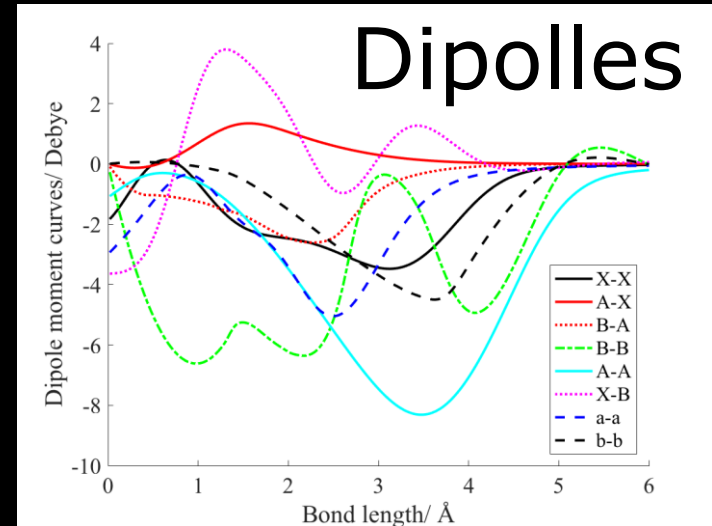
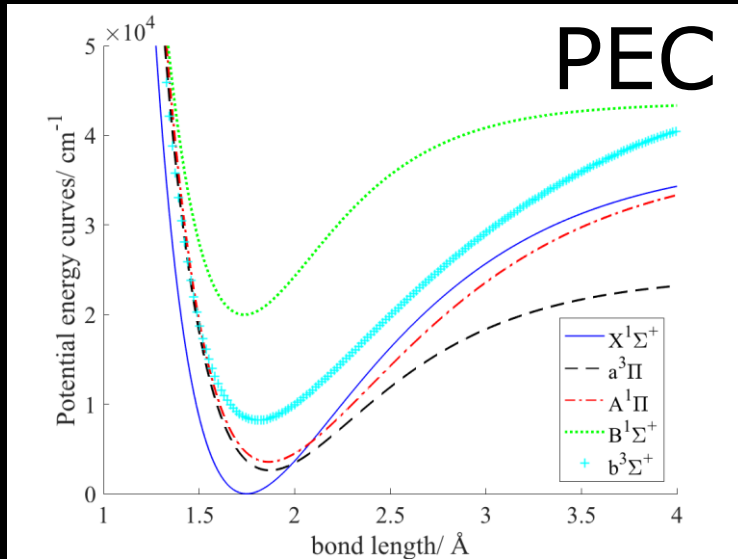
# Ab initio spectrum (MgO)



3. Spin-Orbit couplings

4. Electronic angular momenta couplings

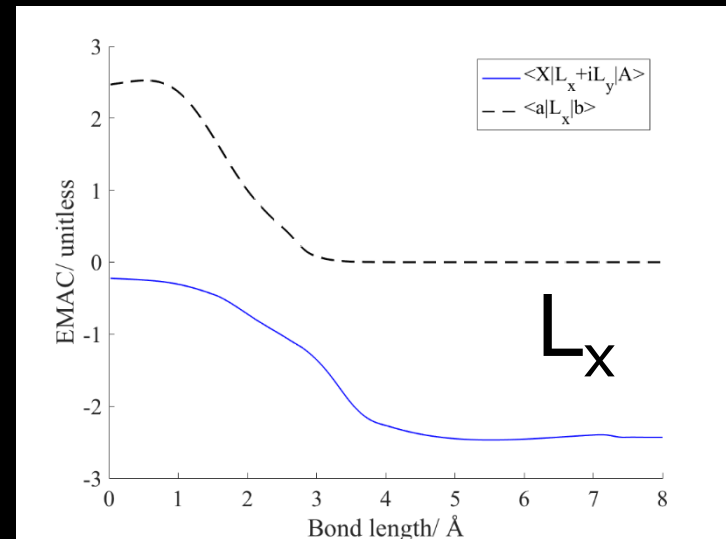
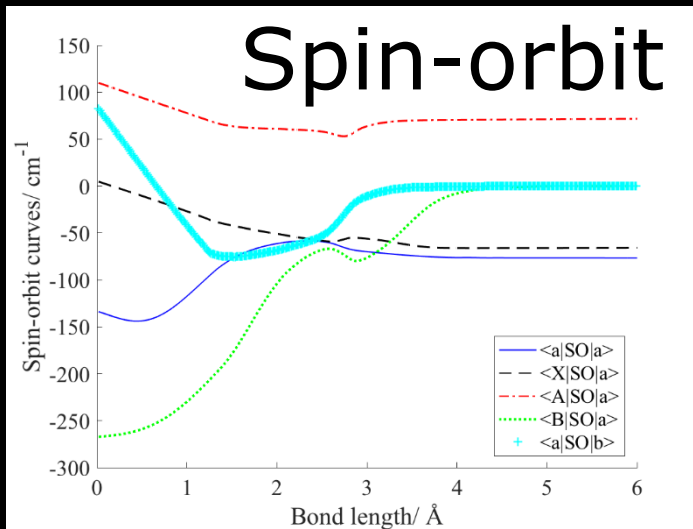
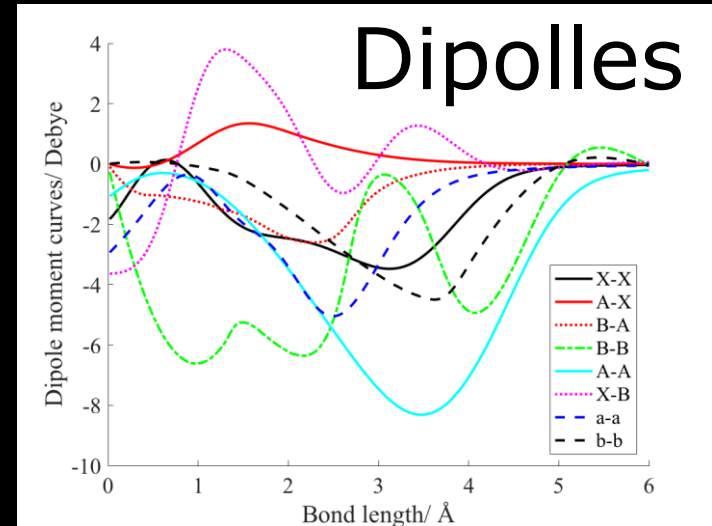
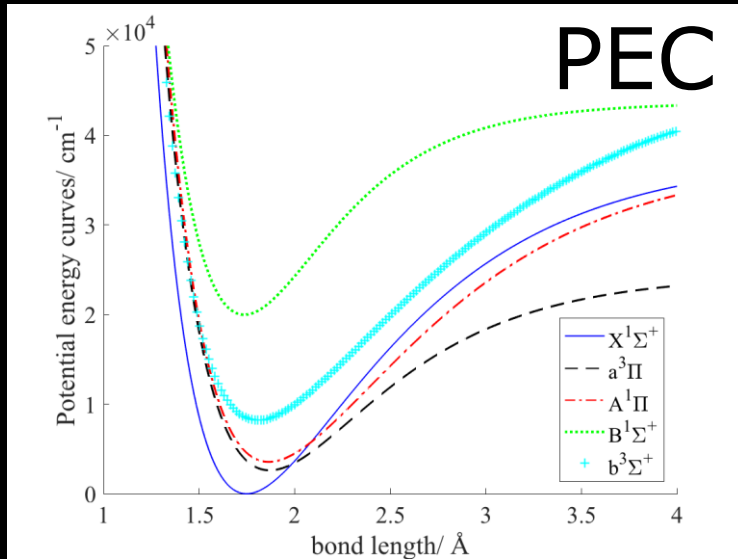
# Ab initio spectrum (MgO)



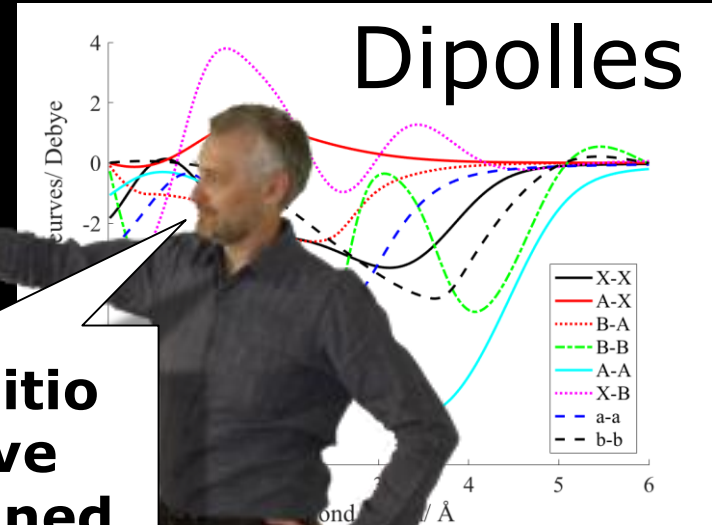
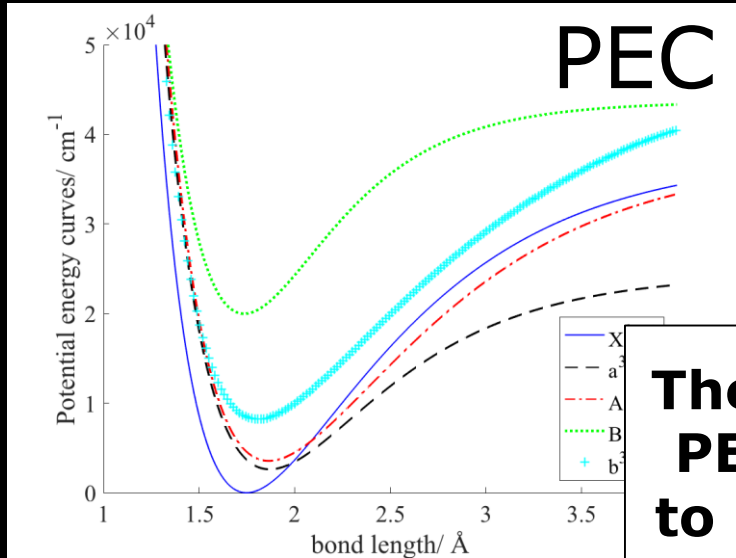
## 4. Electronic angular momenta couplings



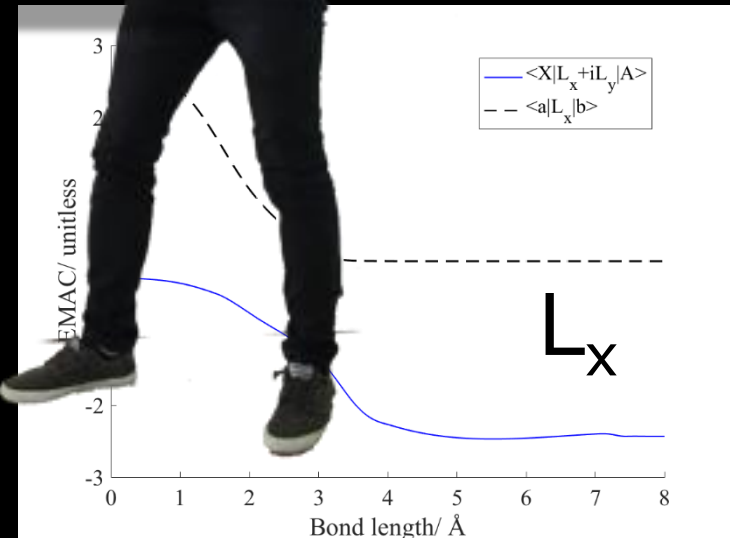
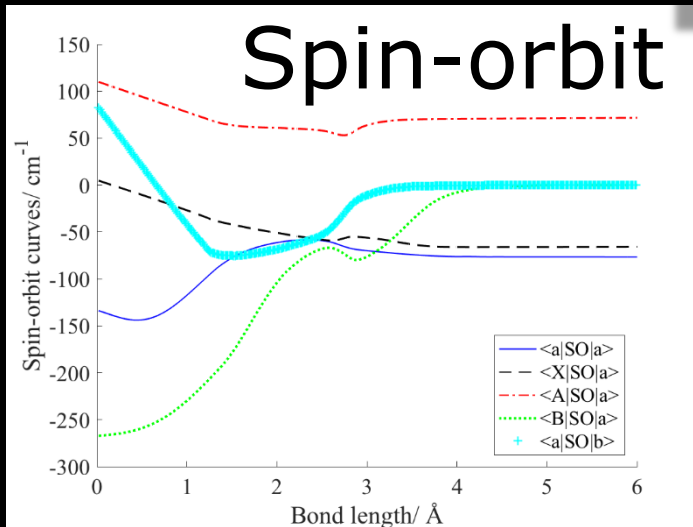
# Ab initio spectrum (MgO)



# Ab initio spectrum (MgO)

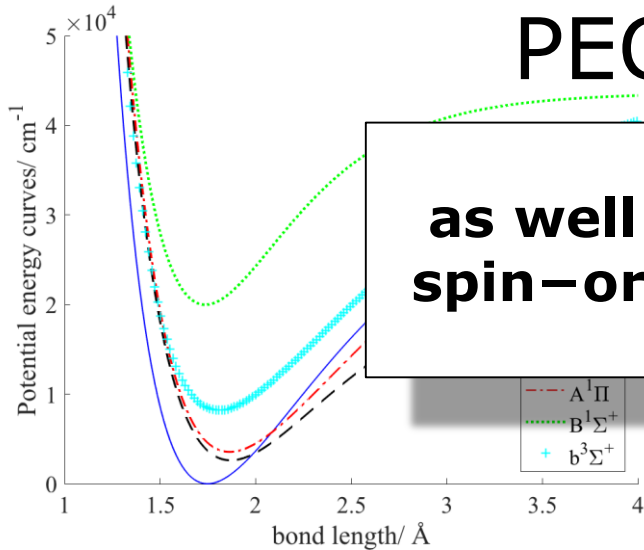


**The ab initio  
PECs have  
to be refined**



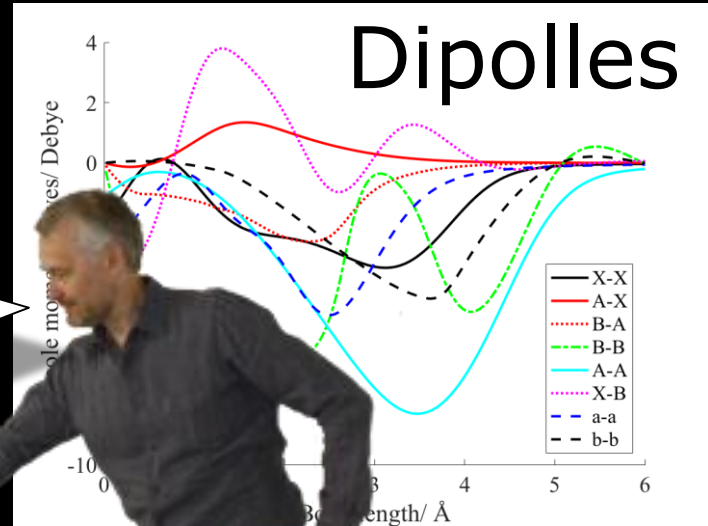
# Ab initio spectrum (MgO)

PEC

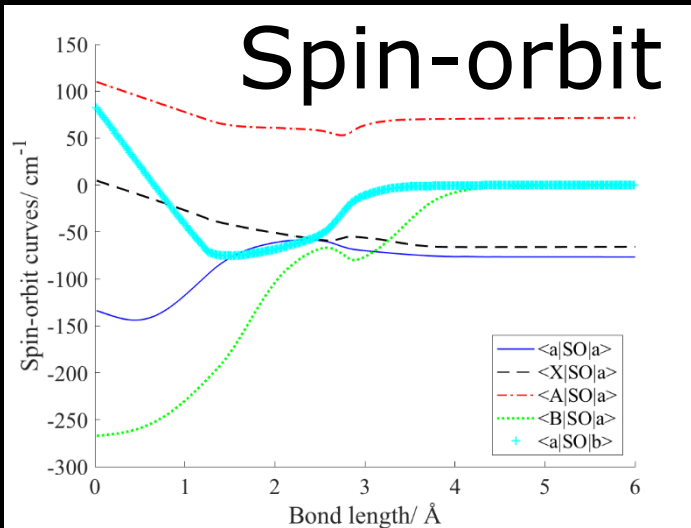


as well as  
spin-orbits

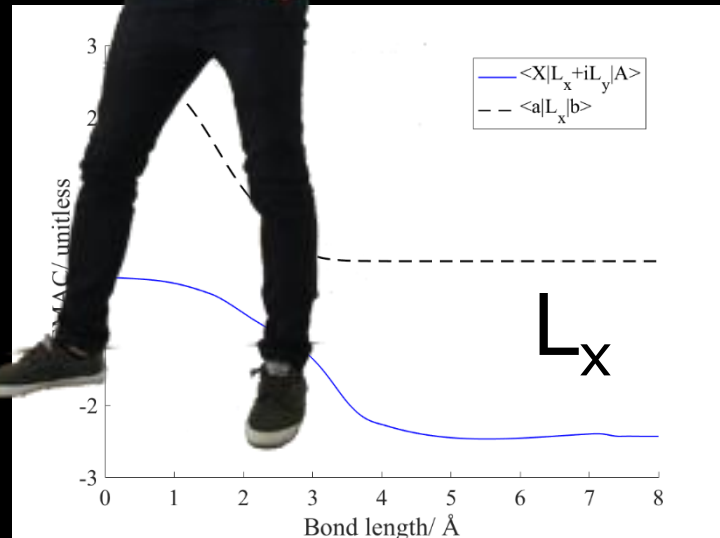
Dipolles



Spin-orbit



$L_x$



A more sophisticated code for  
a coupled system is required

$$\frac{\hbar^2}{2\mu r^2} \hat{R}^2 - \frac{\hbar^2}{2\mu} \frac{\partial^2}{\partial r^2} + V(r)\Psi = E\Psi$$

**Our code Duo**

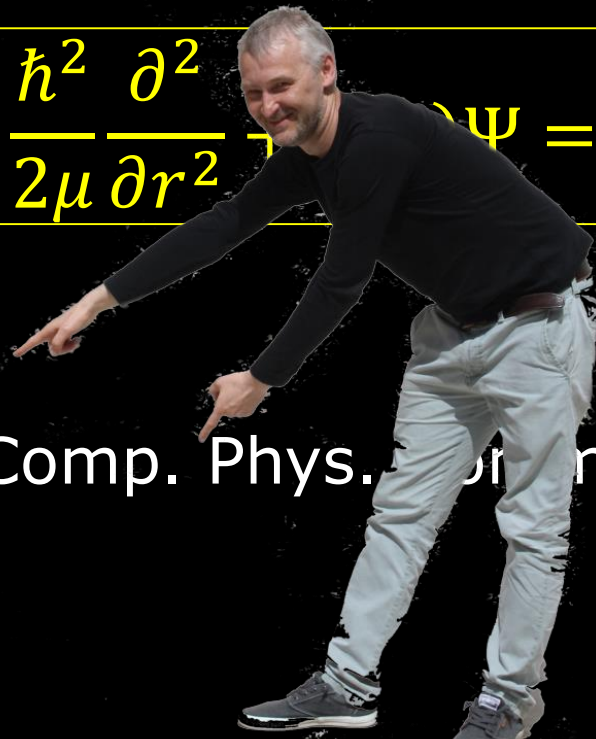
Yurchenko et al Comp. Phys. Comm. (2016)

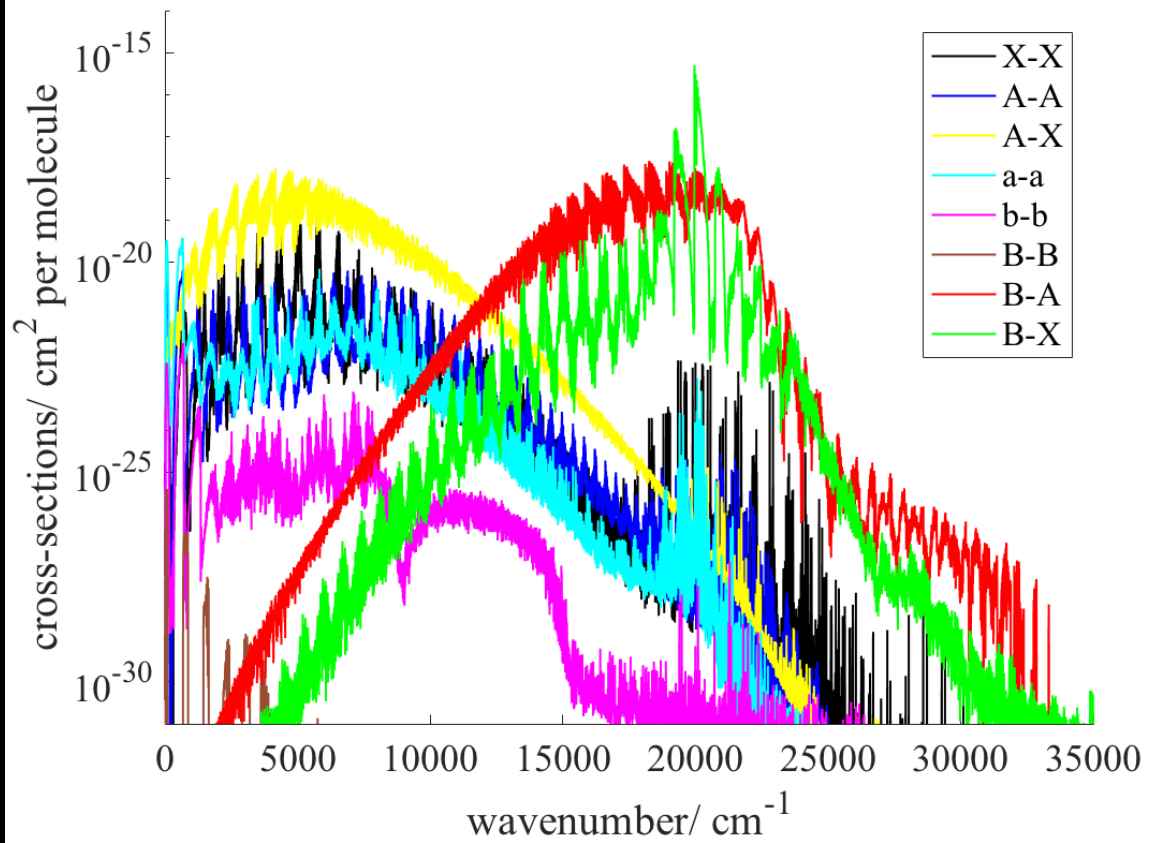
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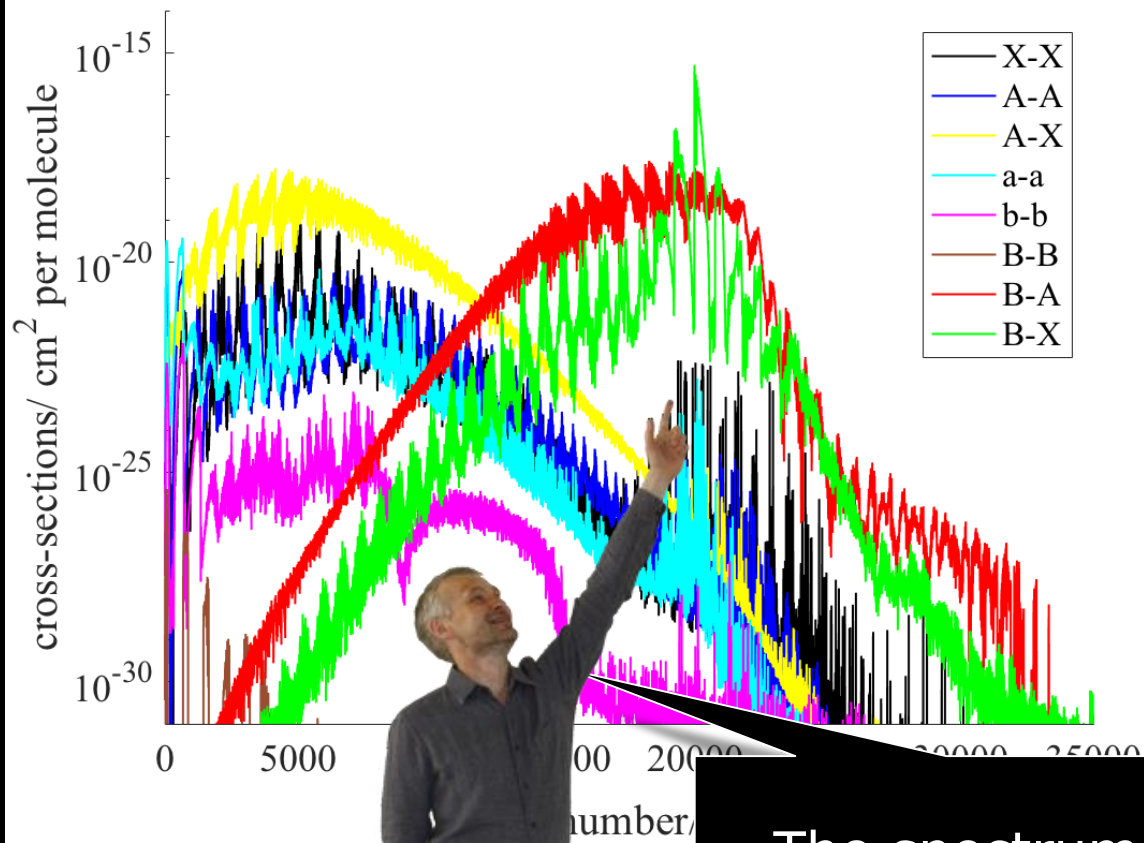
**Our code Duo**

Yurchenko et al Comp. Phys. Commun. (2016)





MgO



MgO

The spectrum is more complex, with multiple interacting electronic bands

In fact:  
for diatomics pure ab initio  
calculations is almost never  
good enough



In fact:  
for diatomics pure ab initio  
calculations is almost never  
good enough

... at least for practical applications

Example: CrH ab initio spectra

# CrH ab initio absorption at $T=2000$ K

---

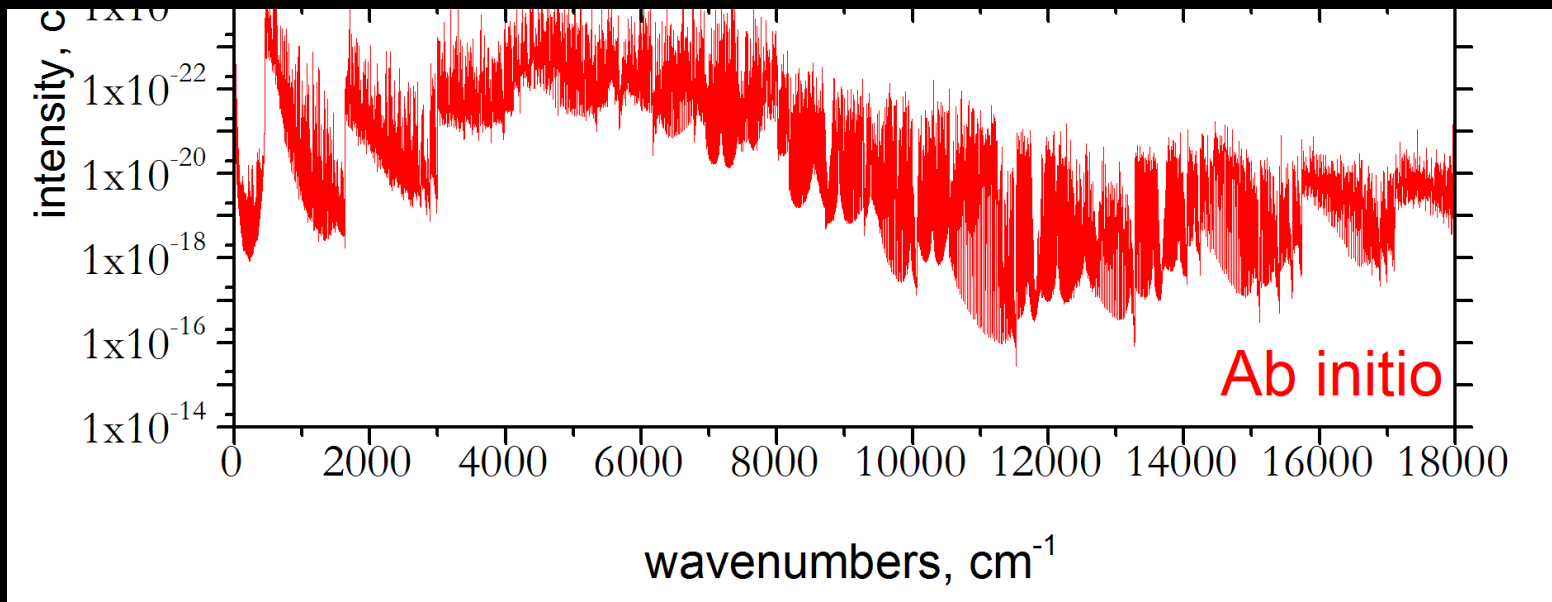
# CrH ab initio absorption at T=2000 K

---

High level ab initio  
ic-MRCI/aug-cc-pV5Z

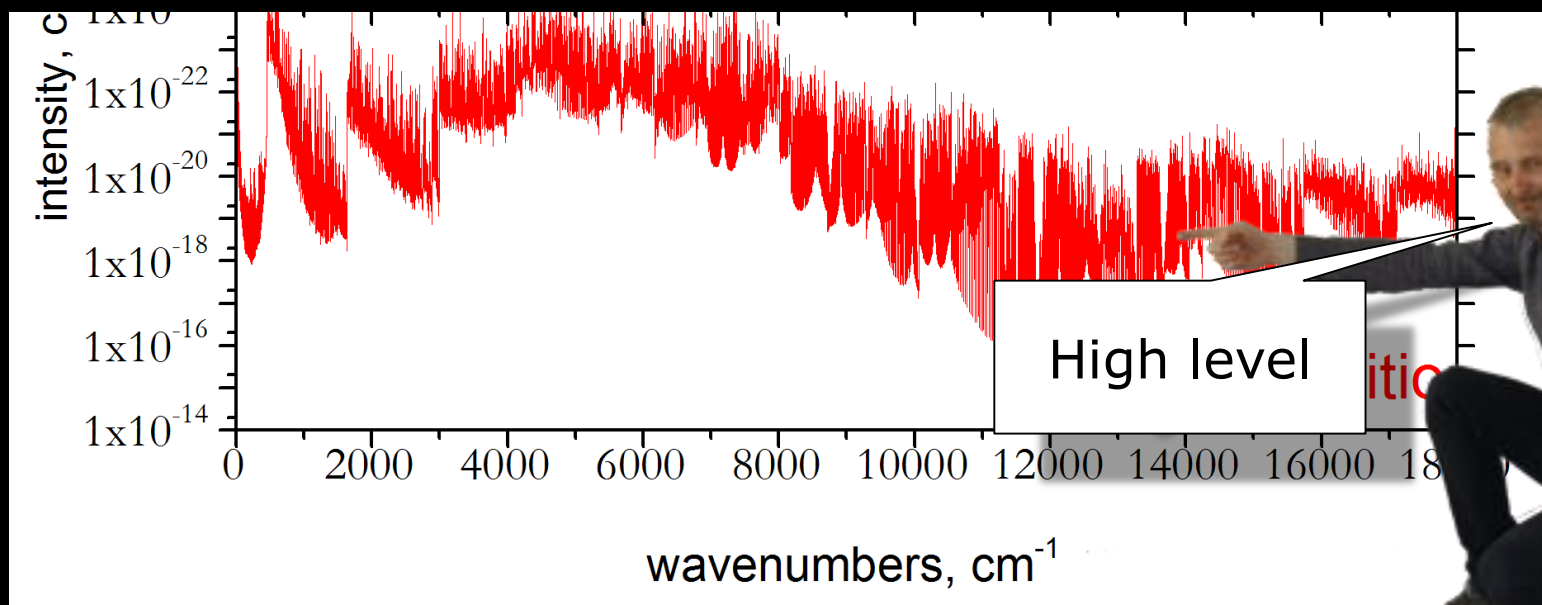
# CrH ab initio absorption at T=2000 K

High level ab initio  
ic-MRCI/aug-cc-pV5Z

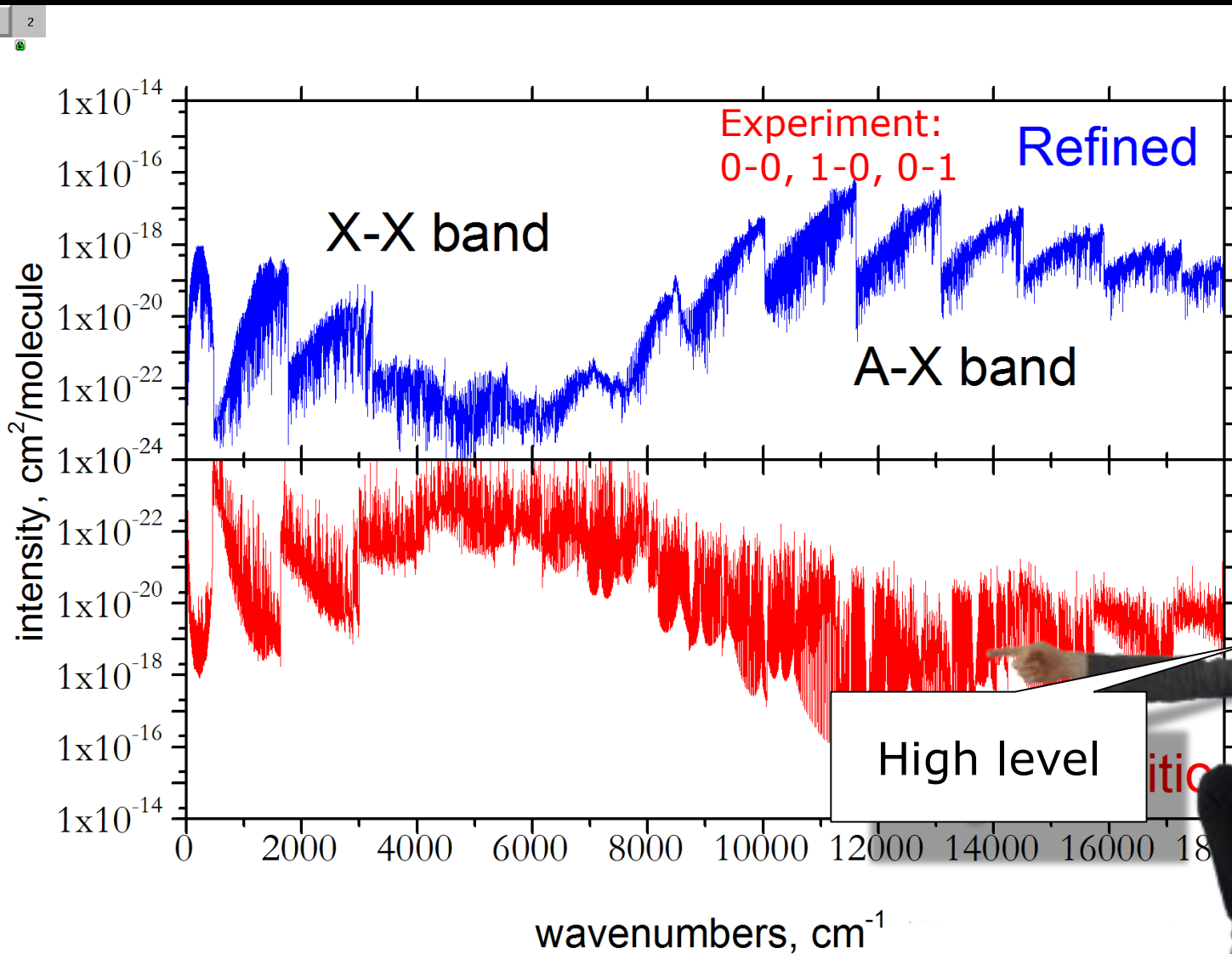


# CrH ab initio absorption at T=2000 K

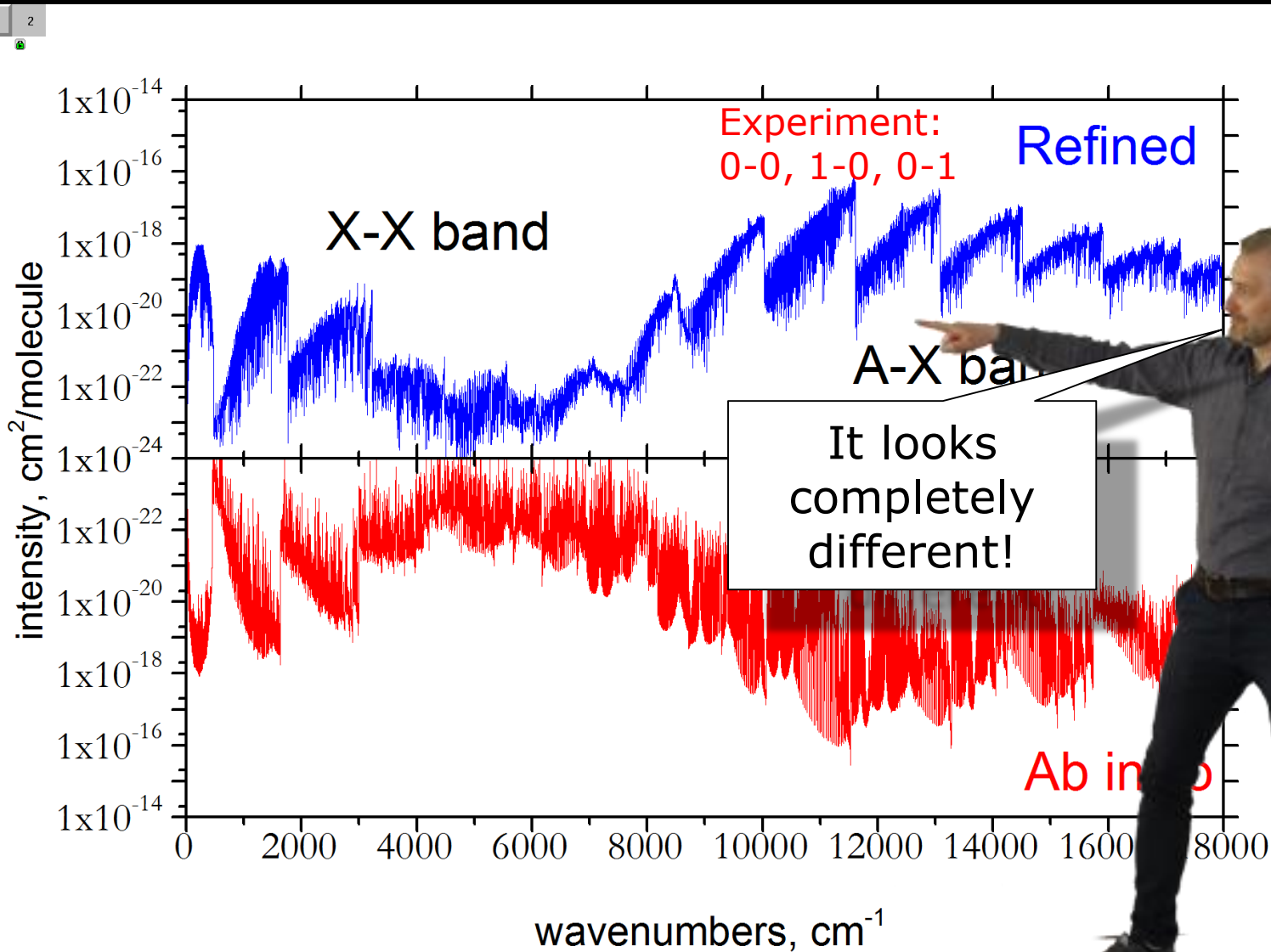
High level ab initio  
ic-MRCI/aug-cc-pV5Z



# CrH ab initio absorption at T=2000 K



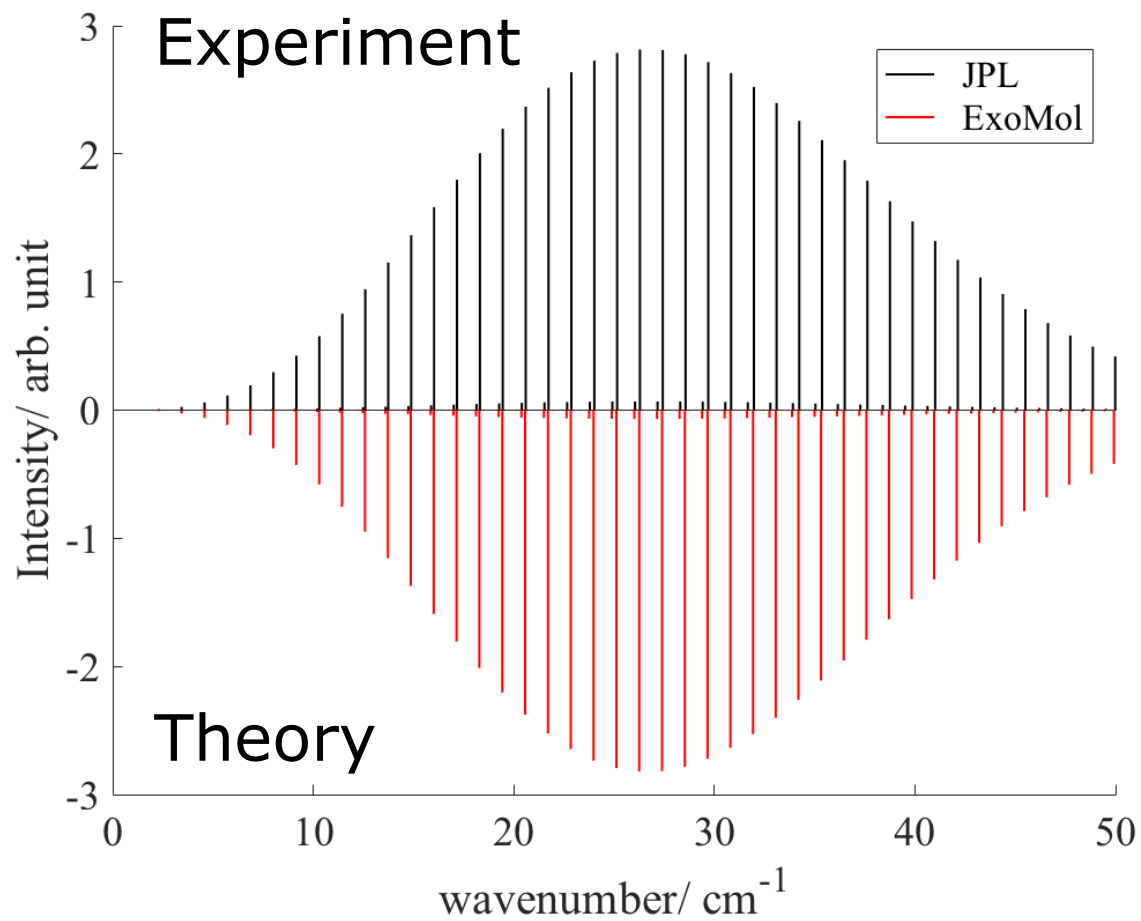
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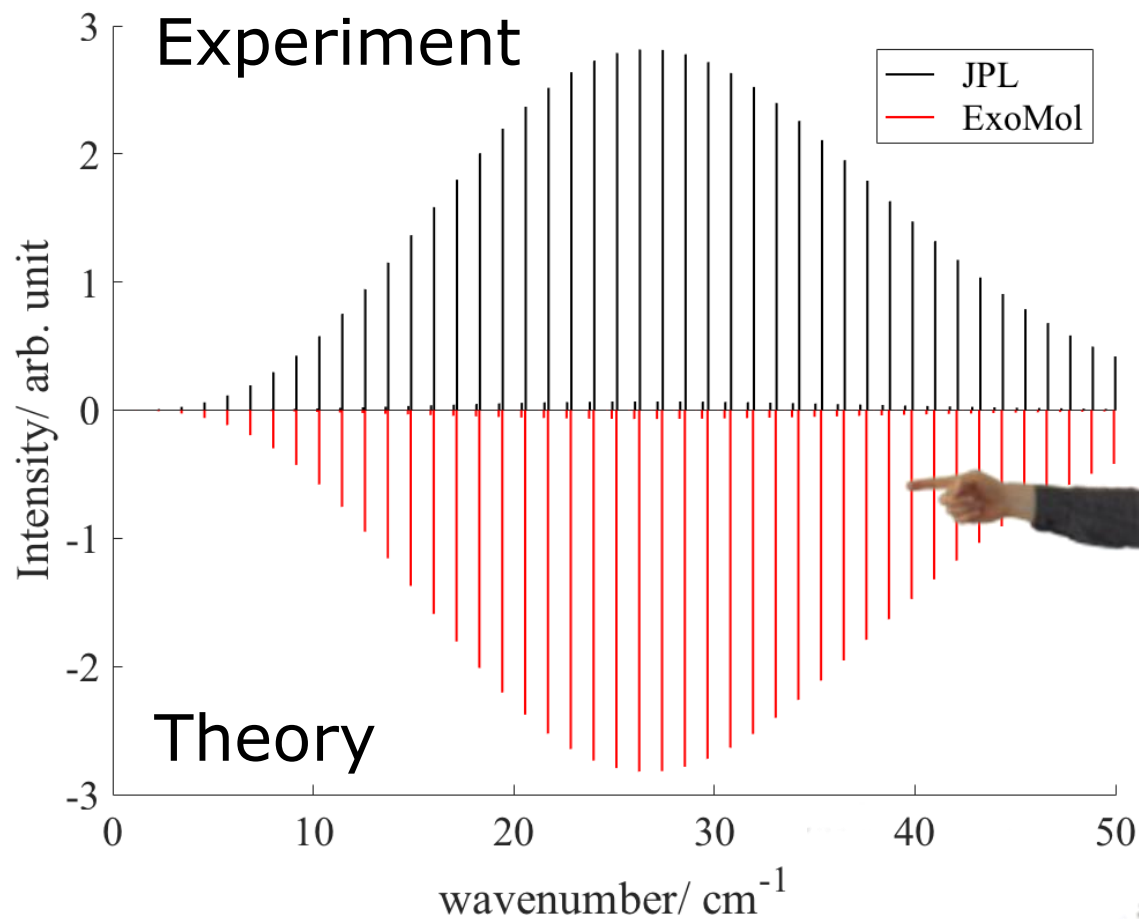




Refinement of our pure  
theoretical model to experiment  
is always mandatory

How accurate are we?





**MgO: Only  
after  
refinement**

For most of hot diatomics  
no experimental intensities  
or lifetimes are available  
but hugely needed!

More on diatomics: see my  
presentation this afternoon

More on diatomics: see my  
presentation this afternoon

WH05 (small molecules) at 14:53



Polyatomic molecules

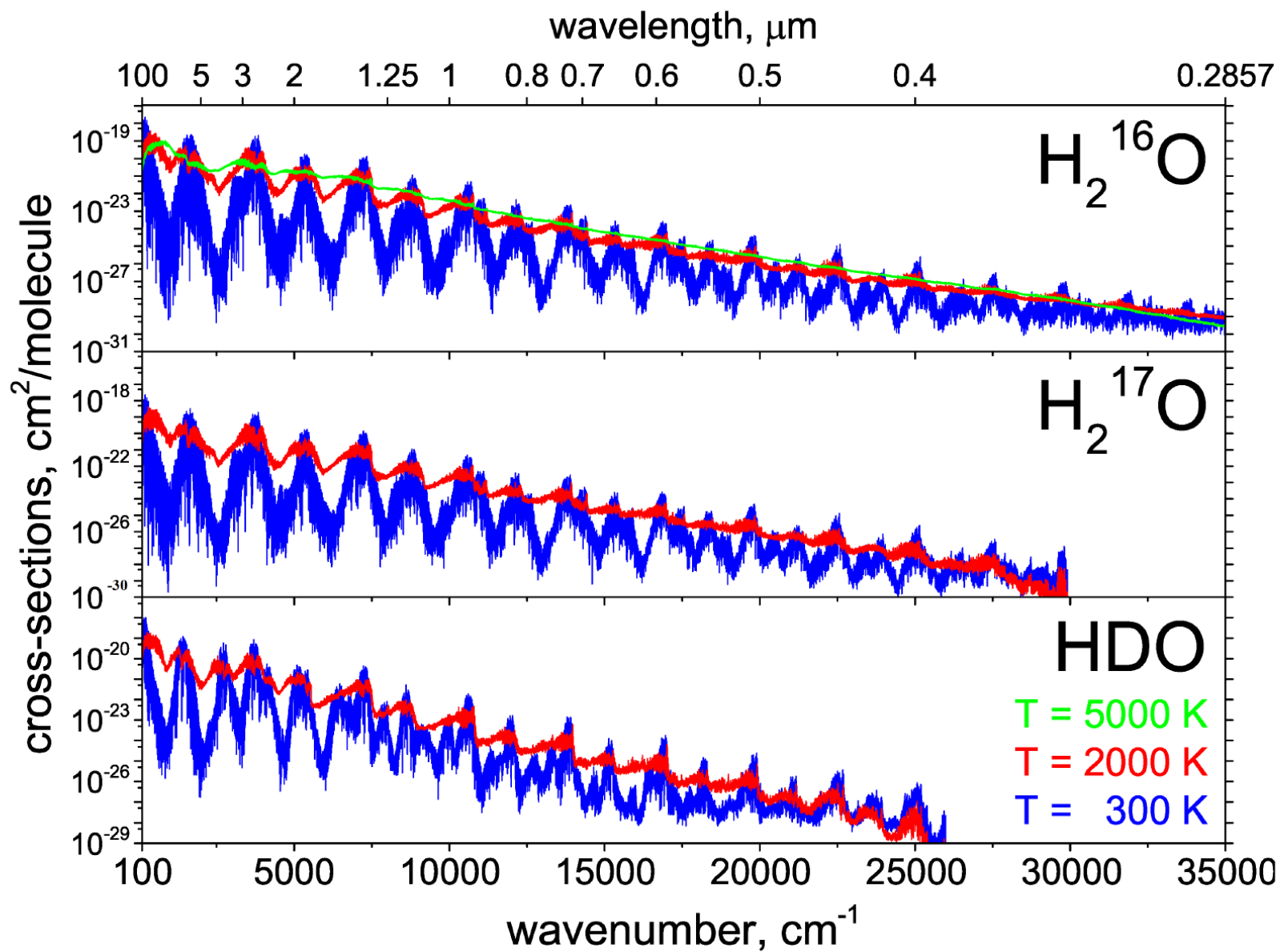


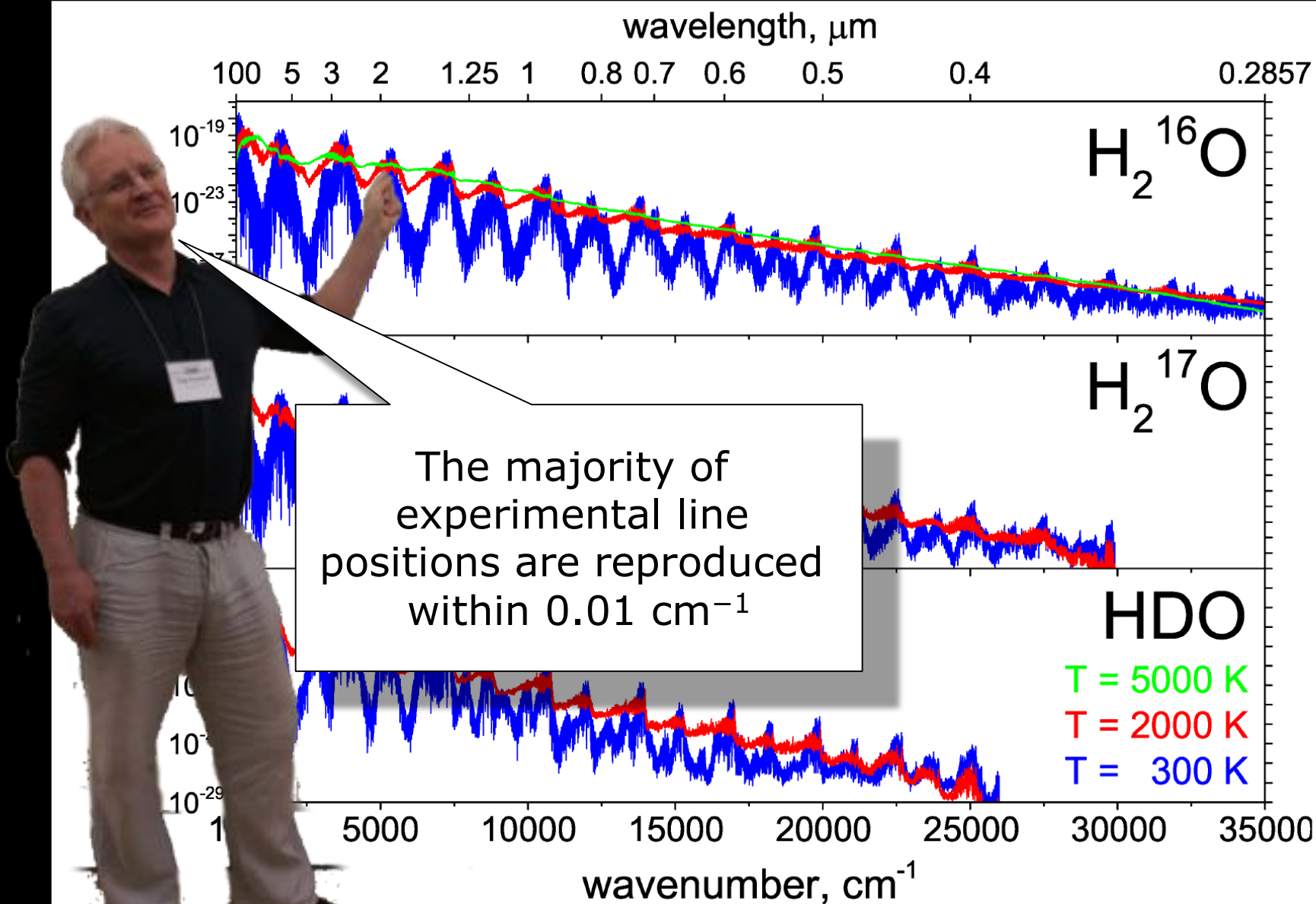
We have produced a new  
water line list POKAZATEL,  
fully complete and more  
accurate

Oleg Polyansky et al MNRAS  
re-submitted (2018)

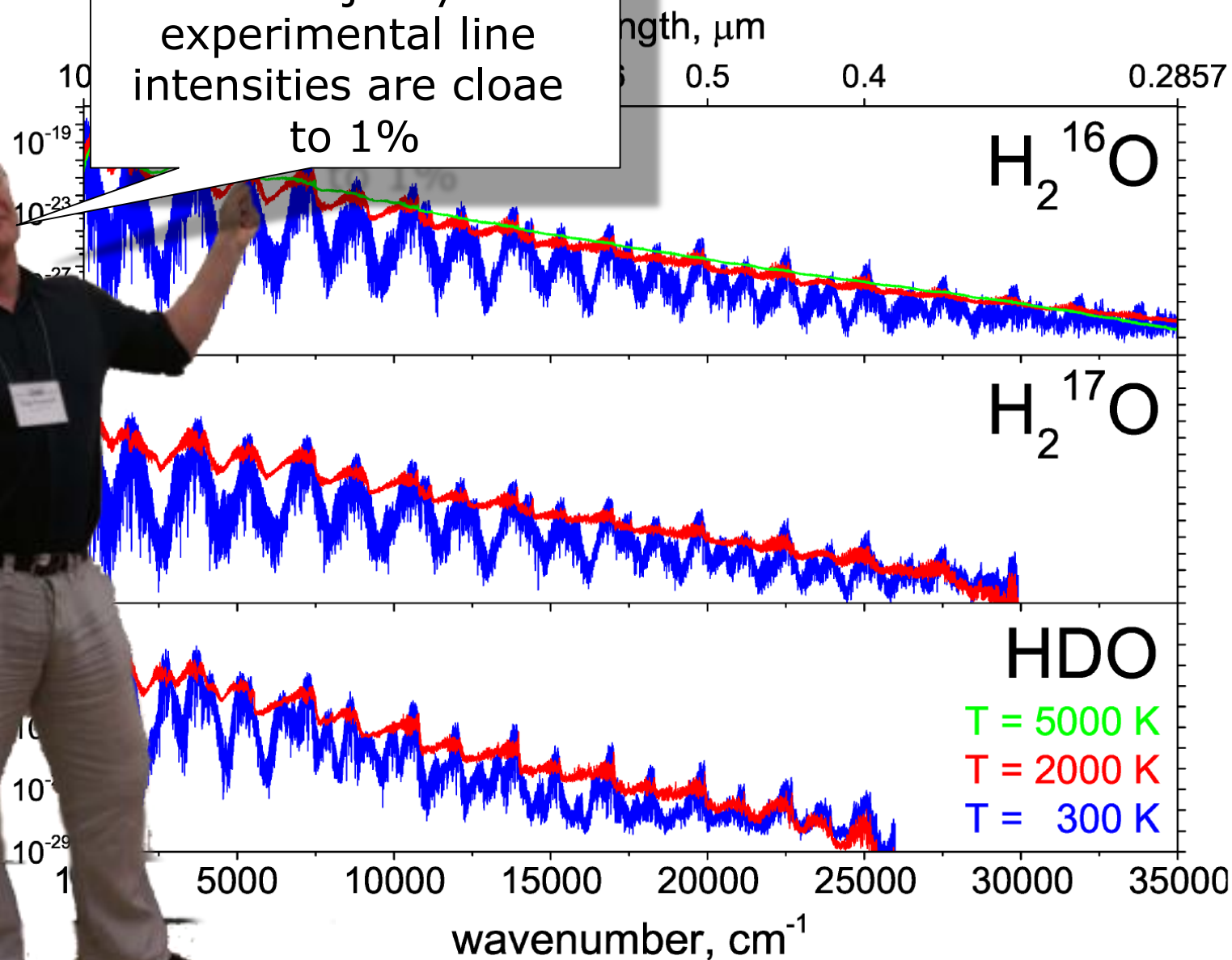


All bound states up to  
dissociation





The majority of  
experimental line  
intensities are close  
to 1%



This summarises our main  
goals...

Accurate line positions

Accurate line intensities



To be complete for high  
excitations and hot spectra

These goals however are not  
always compatible

It is too difficult or even  
impossible to be accurate and  
complete

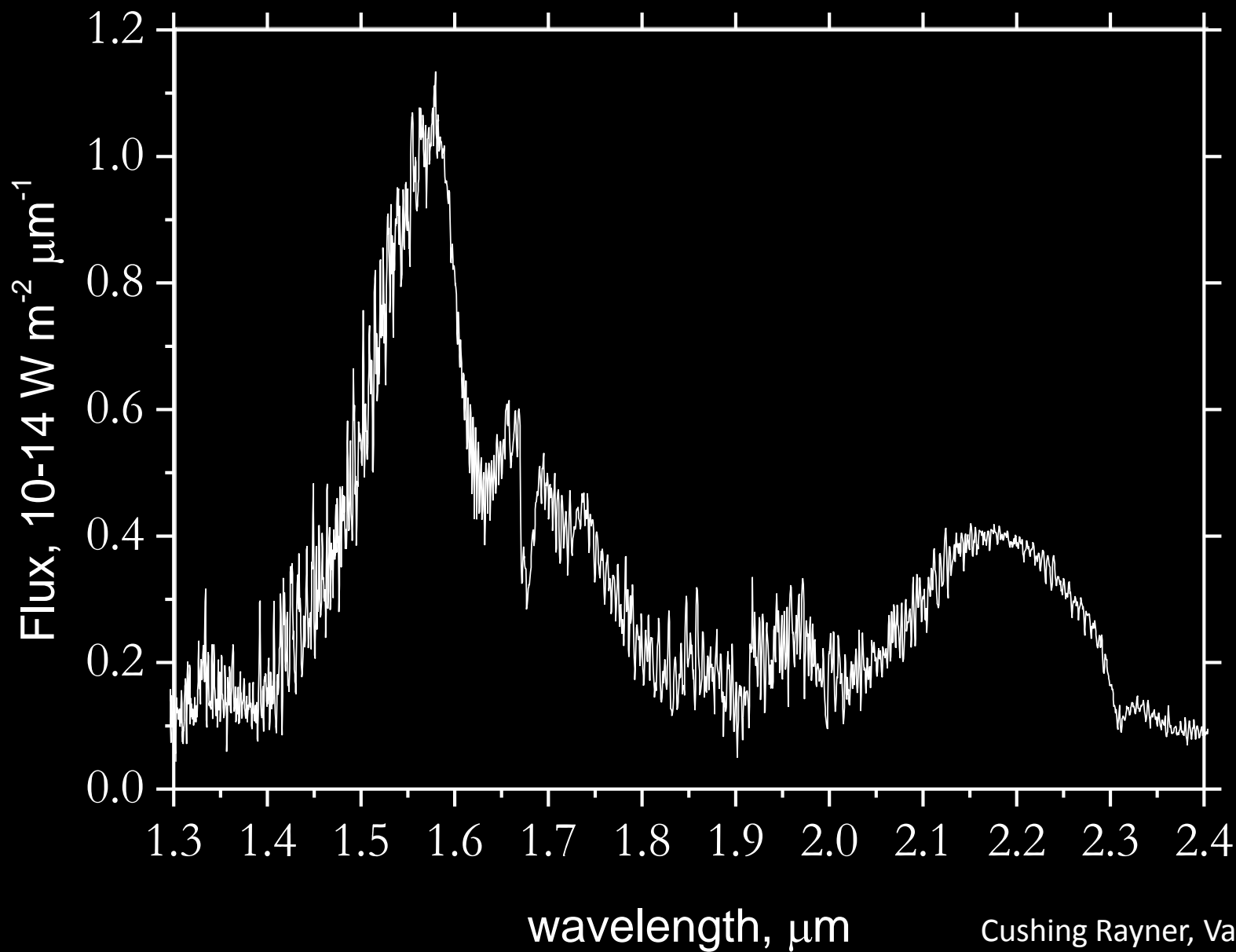
Luckily, most of applications  
require to be either accurate  
or complete

For example, the microwave  
spectral applications must be  
accurate but not so much  
complete

Most of the opacity  
applications (atmospheric  
retrievals) require to be  
complete not so much  
accurate

Example: brown dwarf spectra

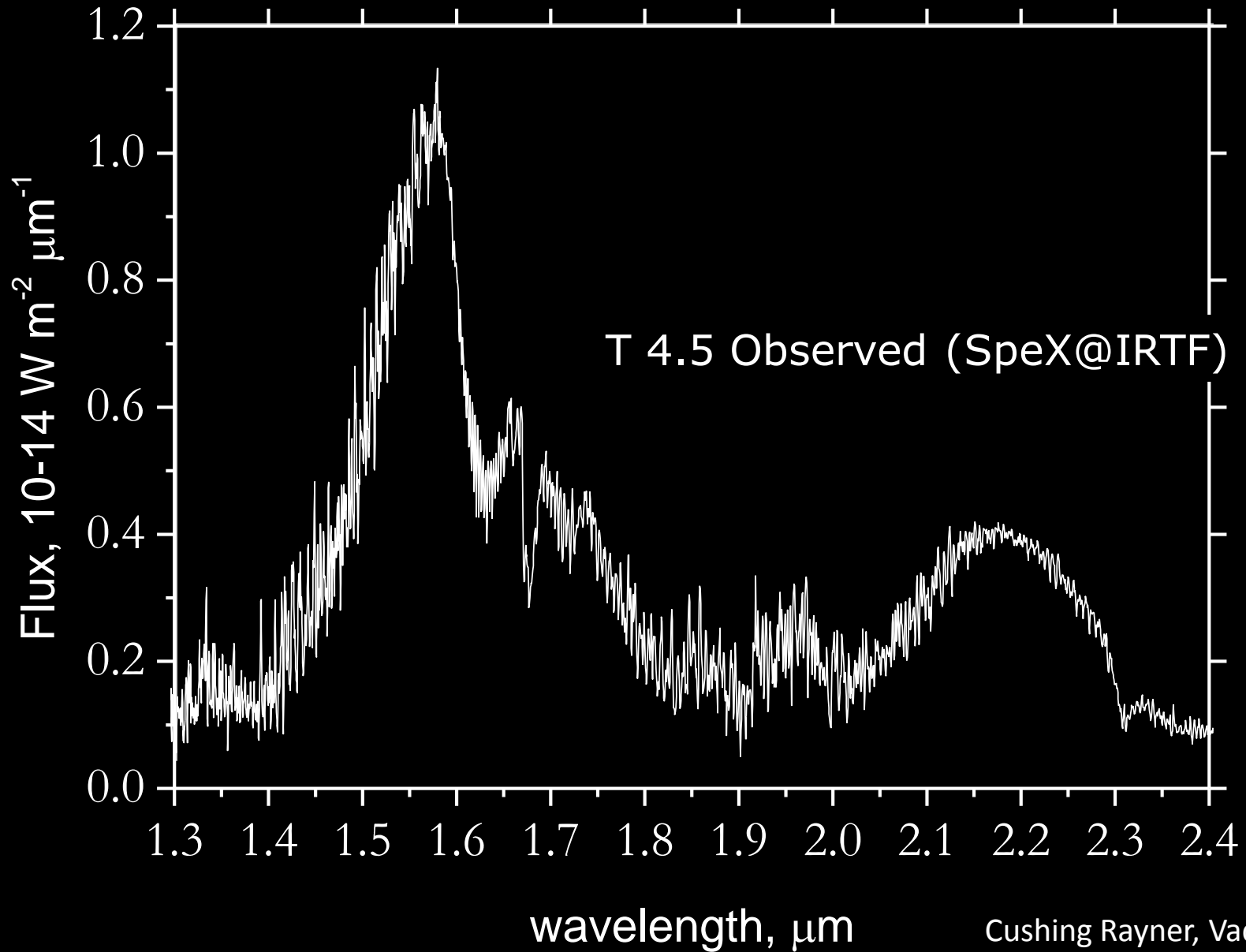
# T4.5 Brown dwarf 2MASS 0559-14



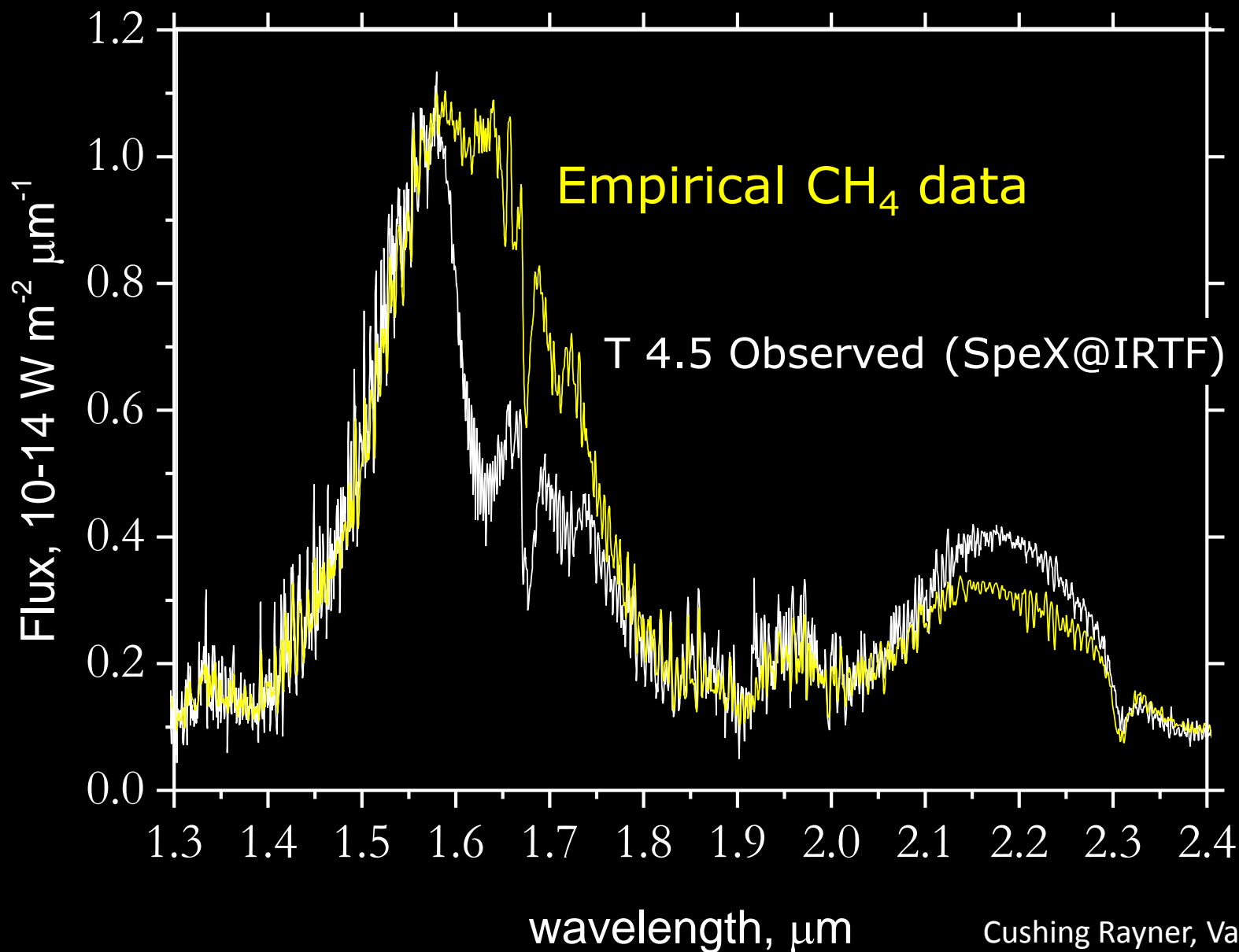
Cushing Rayner, Vacca (2005)



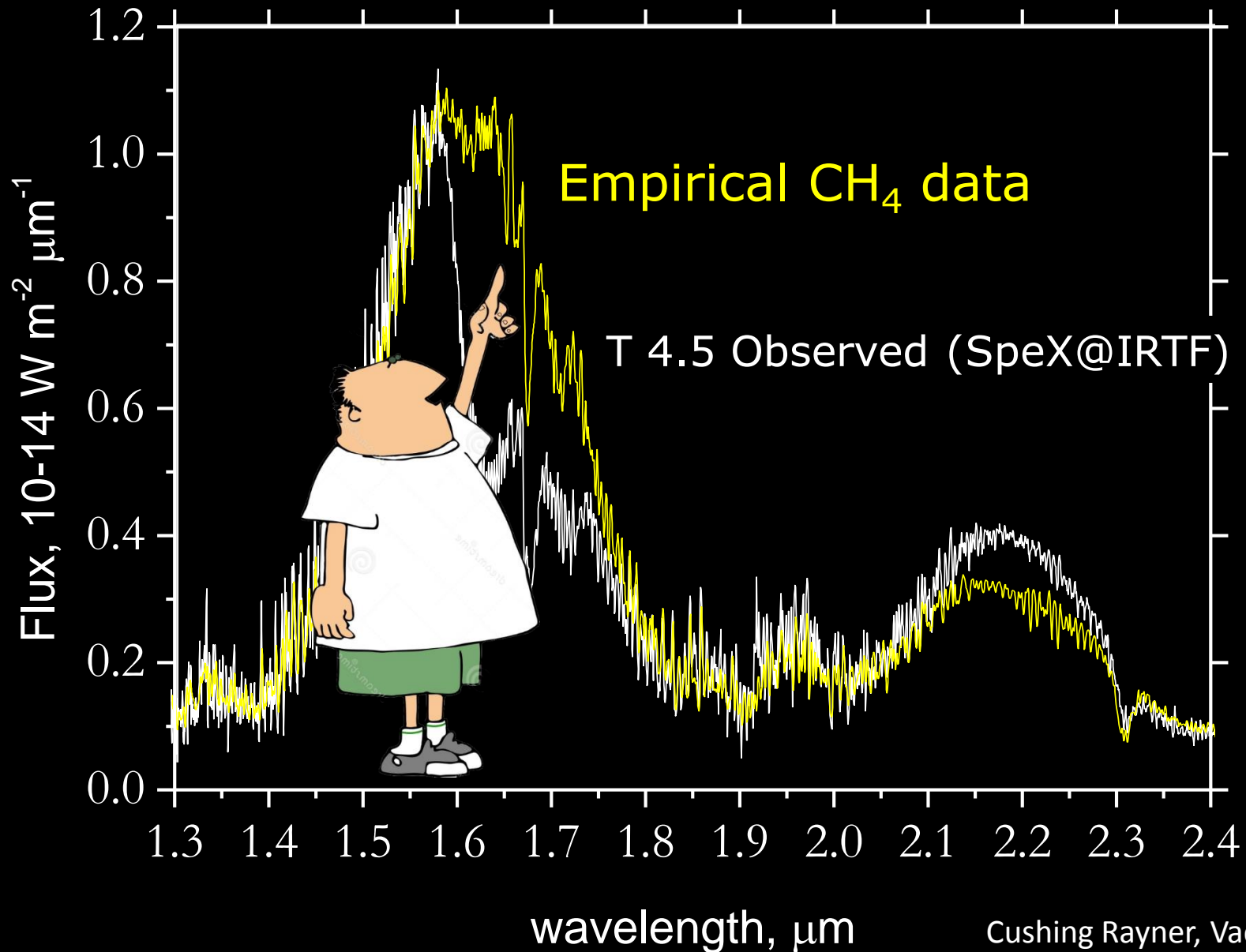
# T4.5 Brown dwarf 2MASS 0559-14



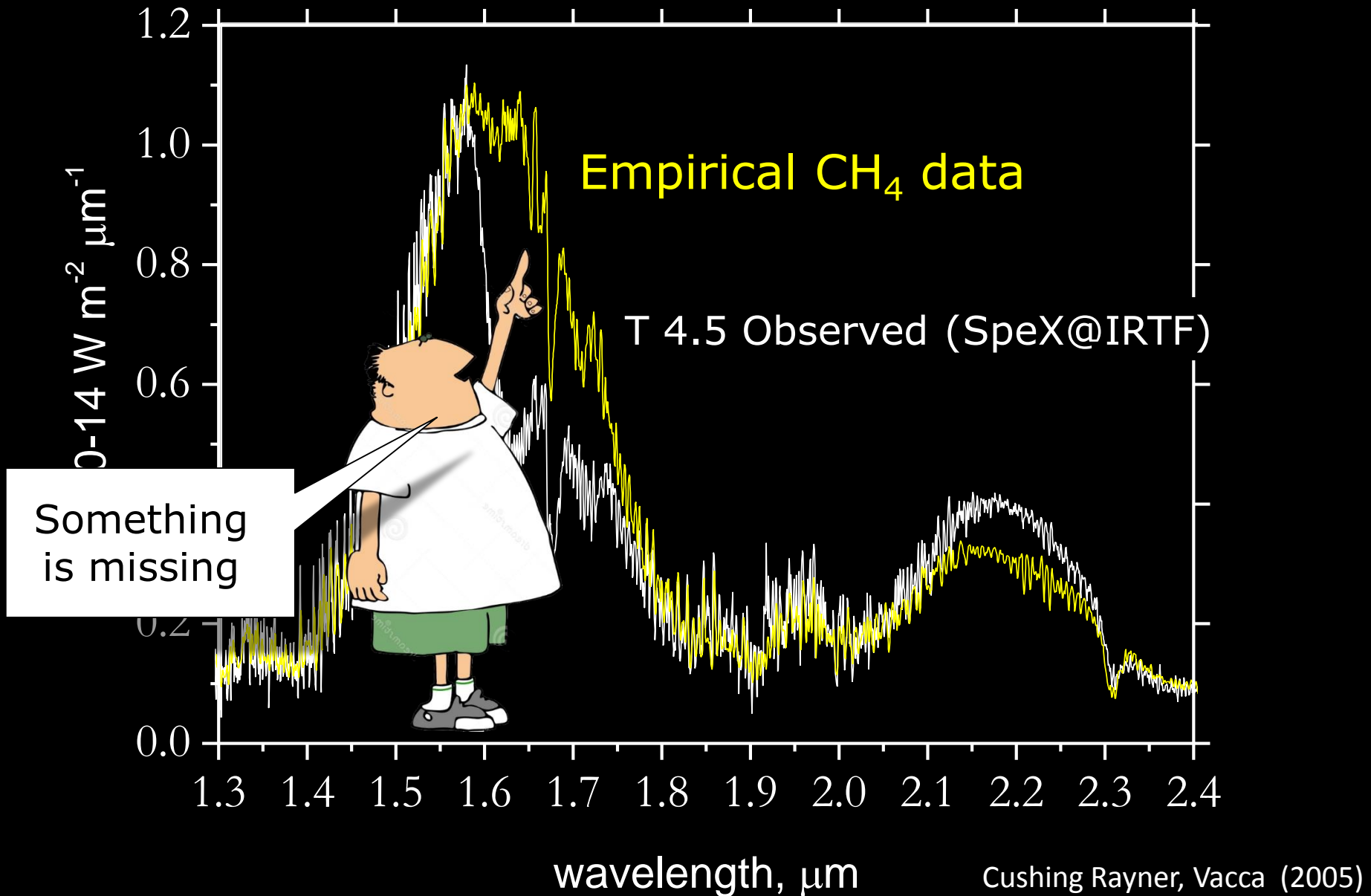
# T4.5 Brown dwarf 2MASS 0559-14



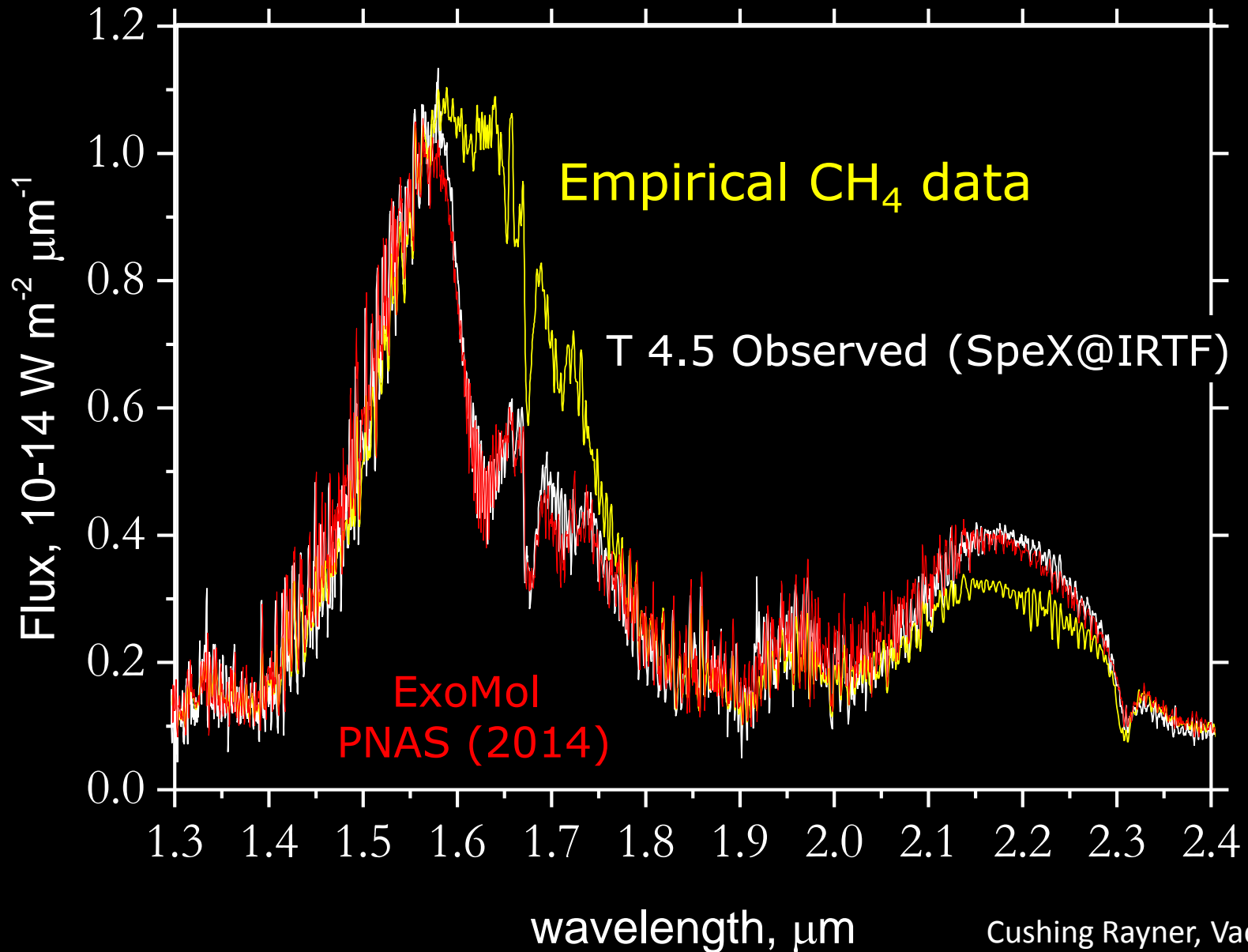
# T4.5 Brown dwarf 2MASS 0559-14



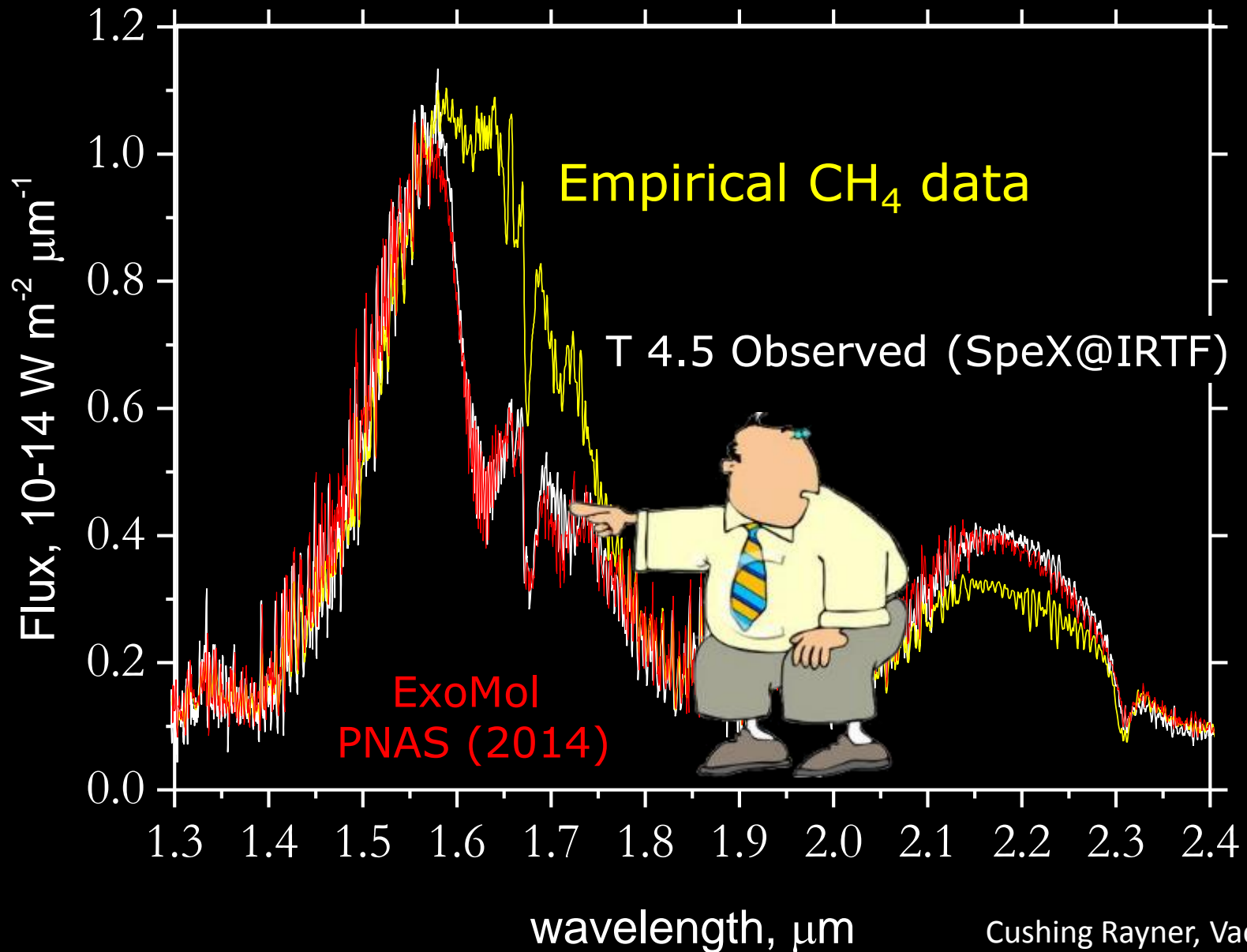
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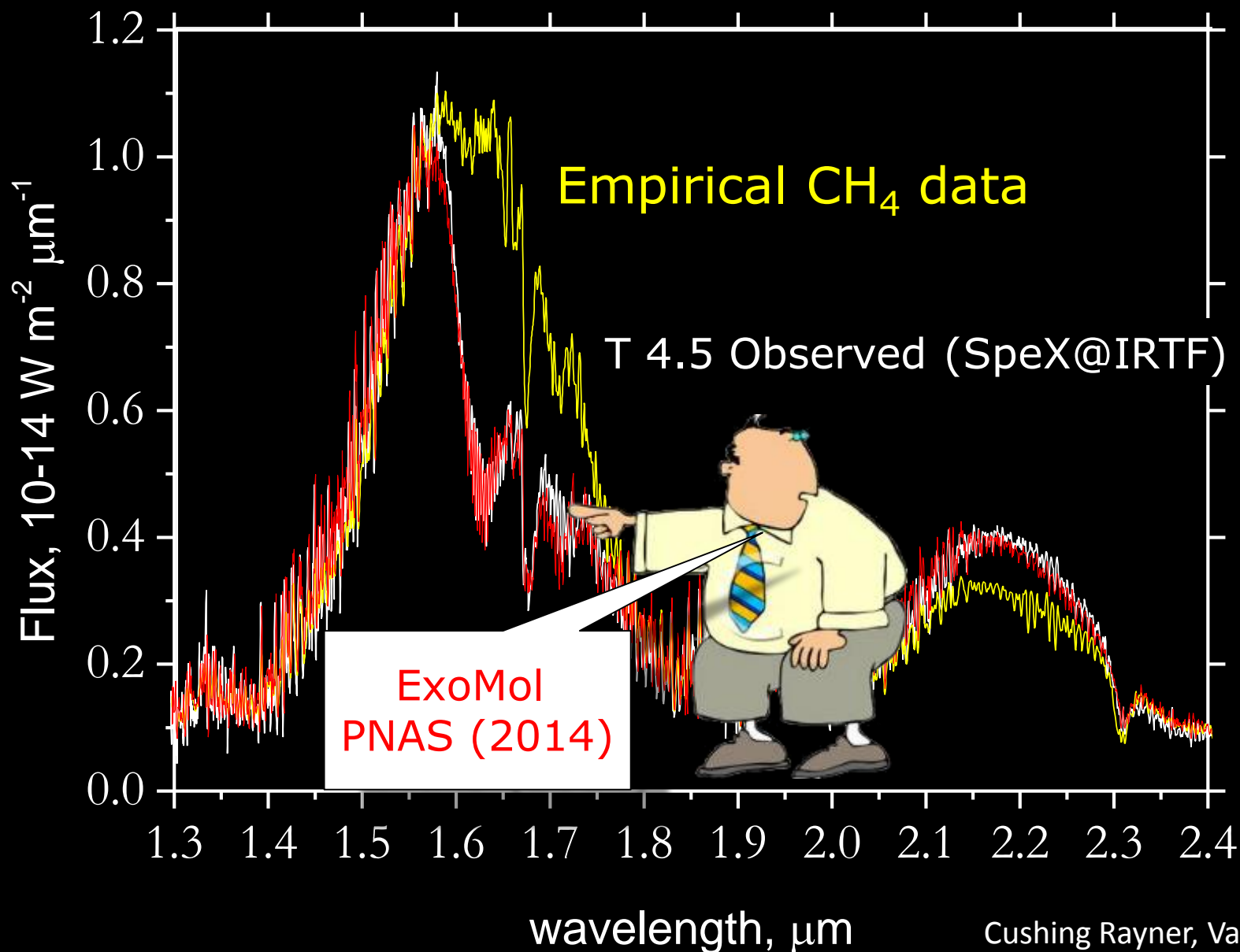
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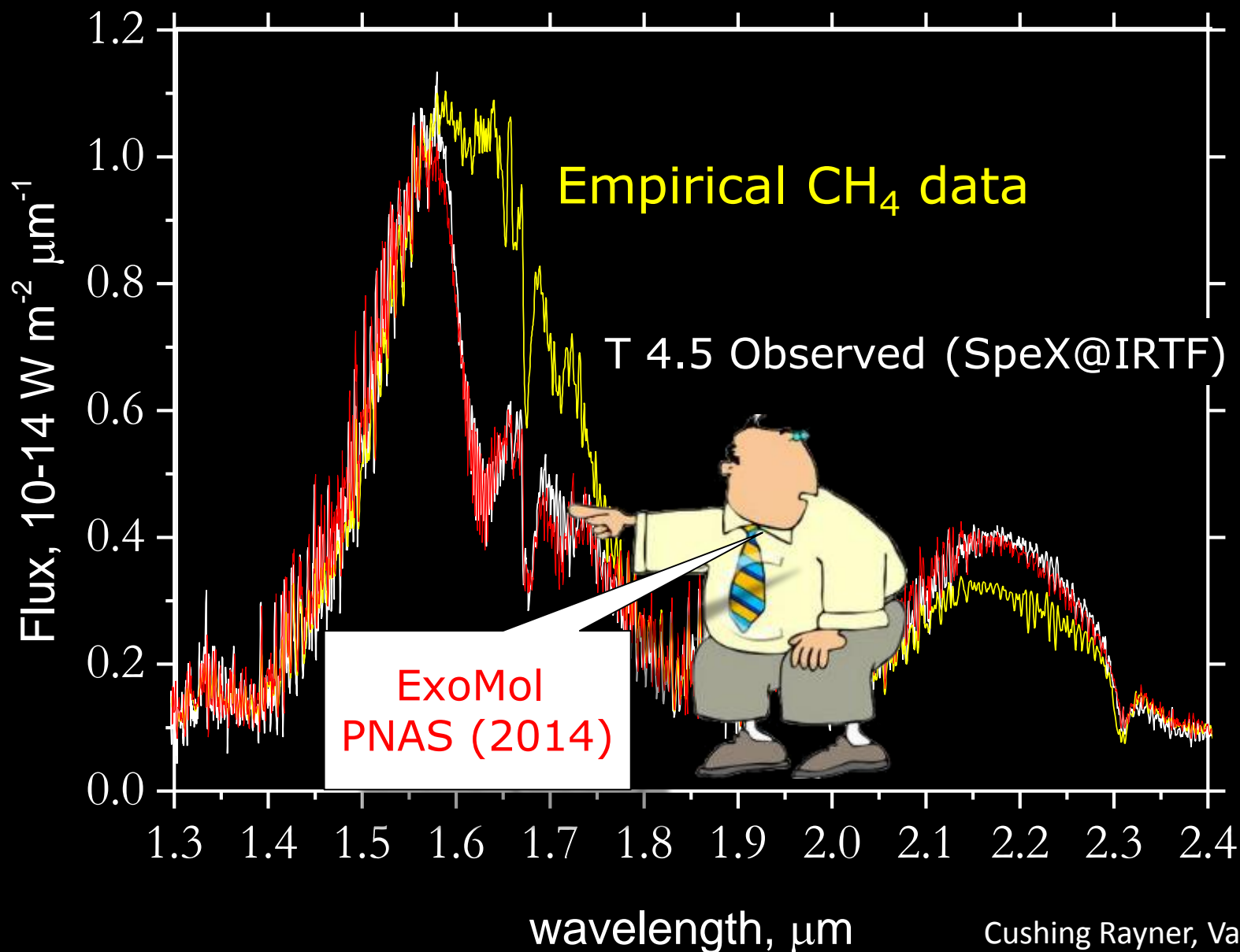
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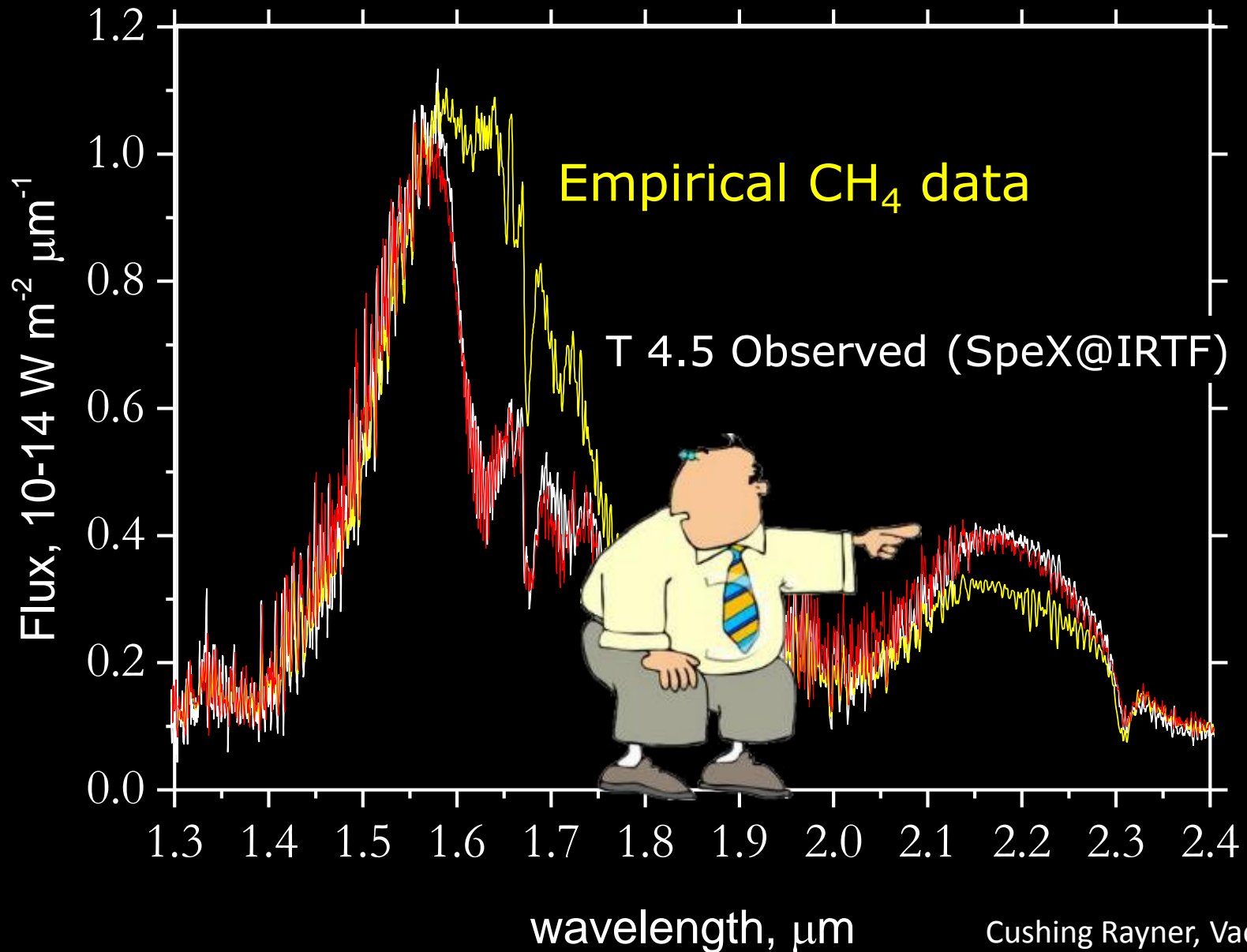


# T4.5 Brown dwarf 2MASS 0559-14

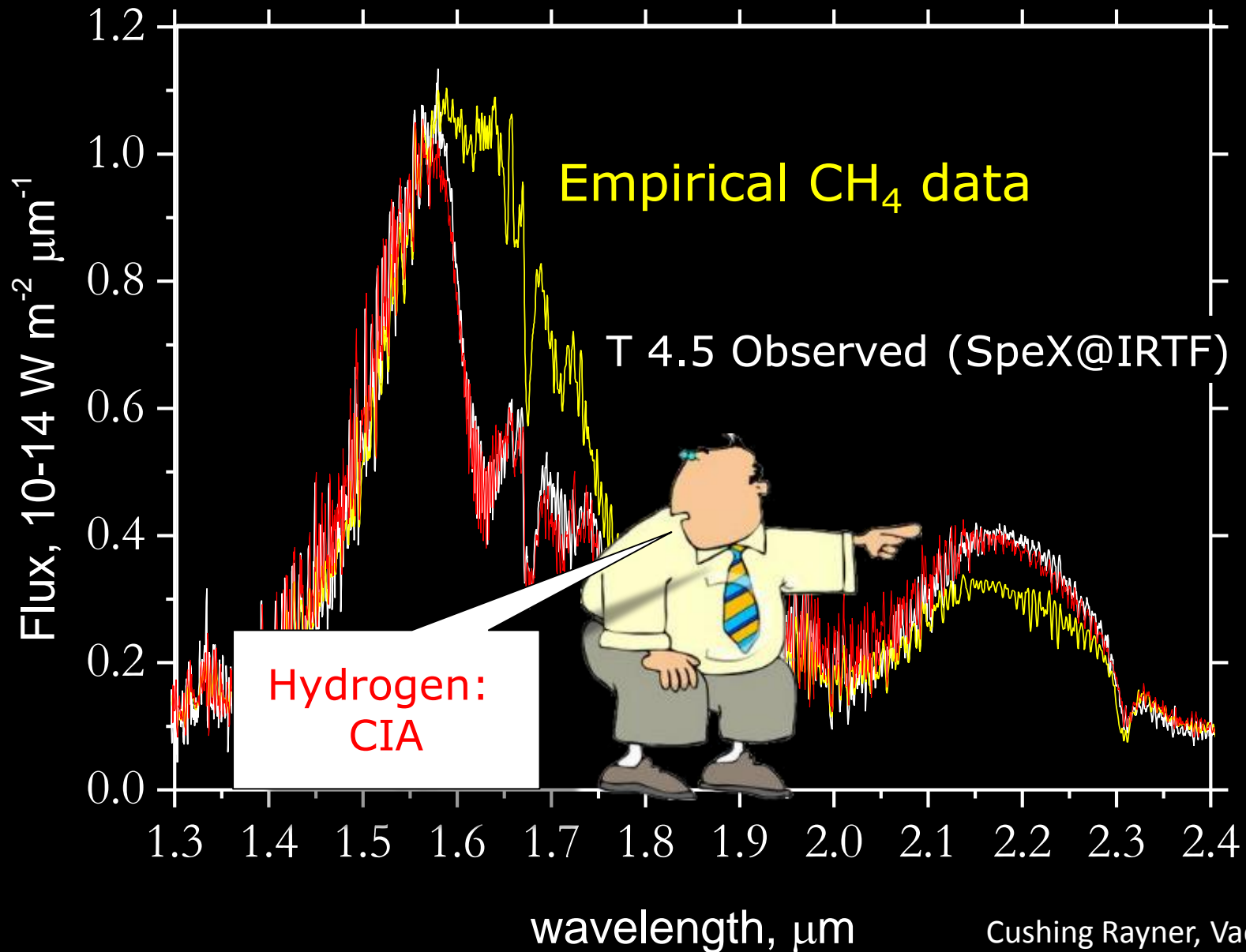


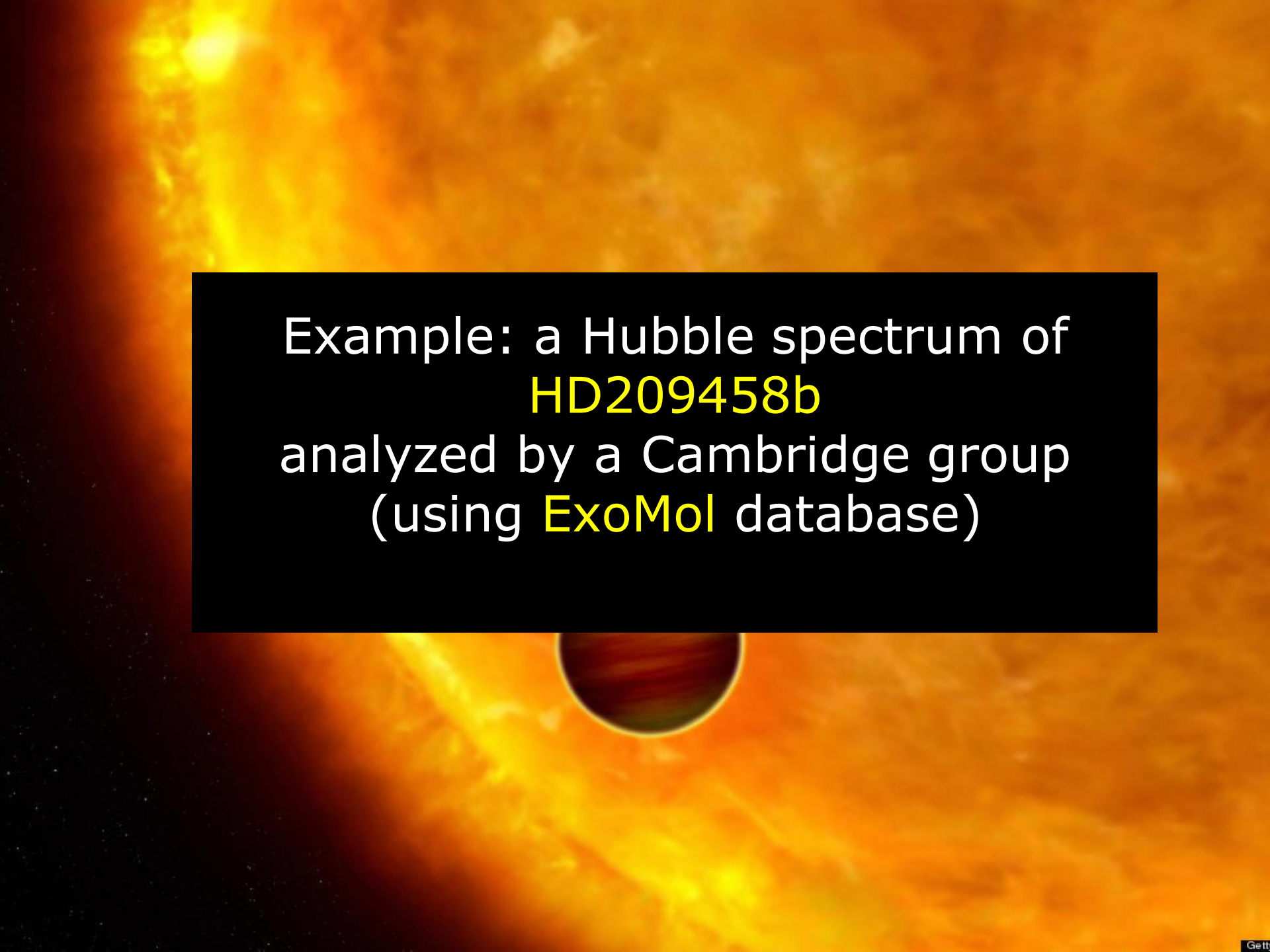


# T4.5 Brown dwarf 2MASS 0559-14

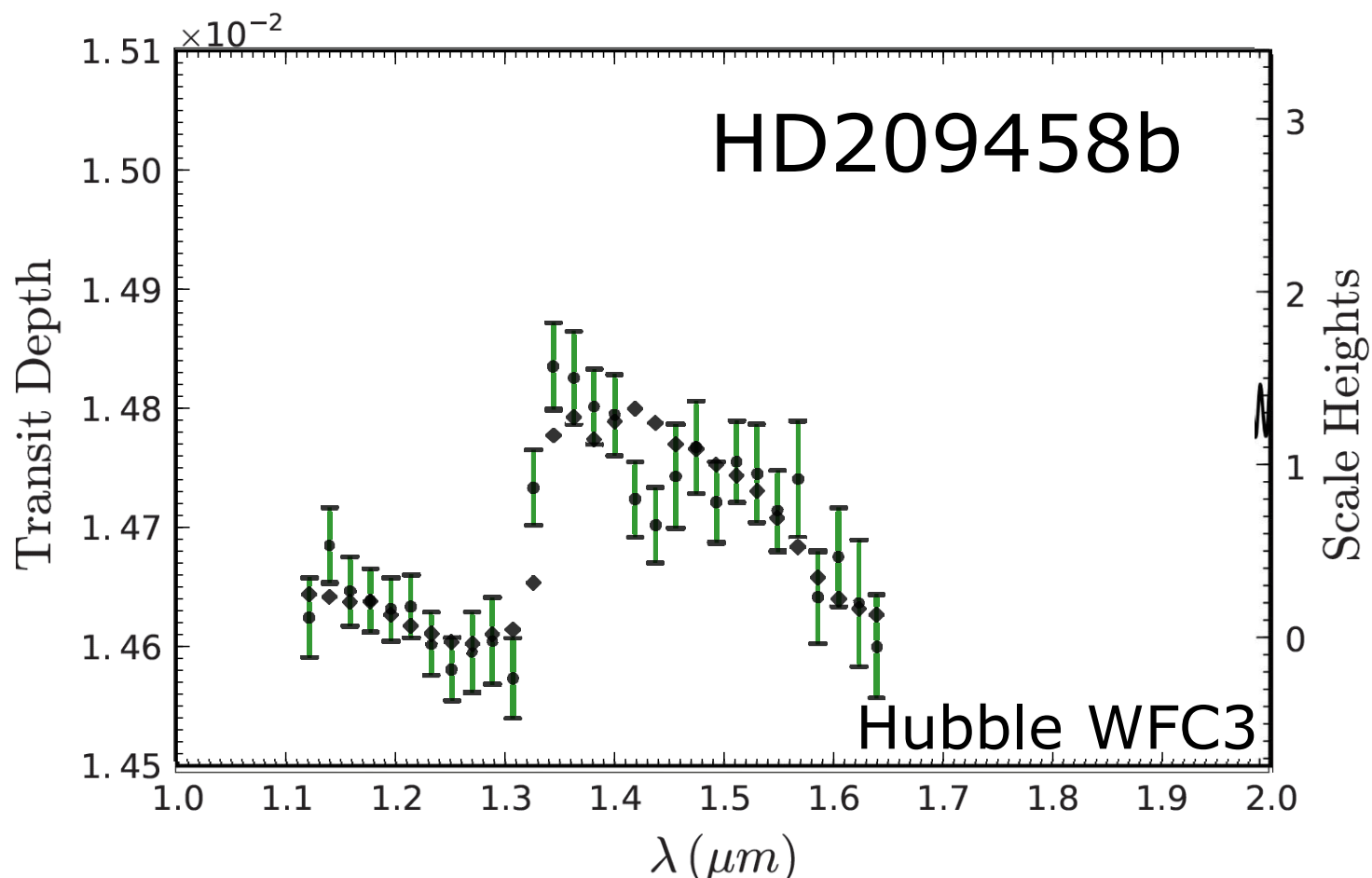


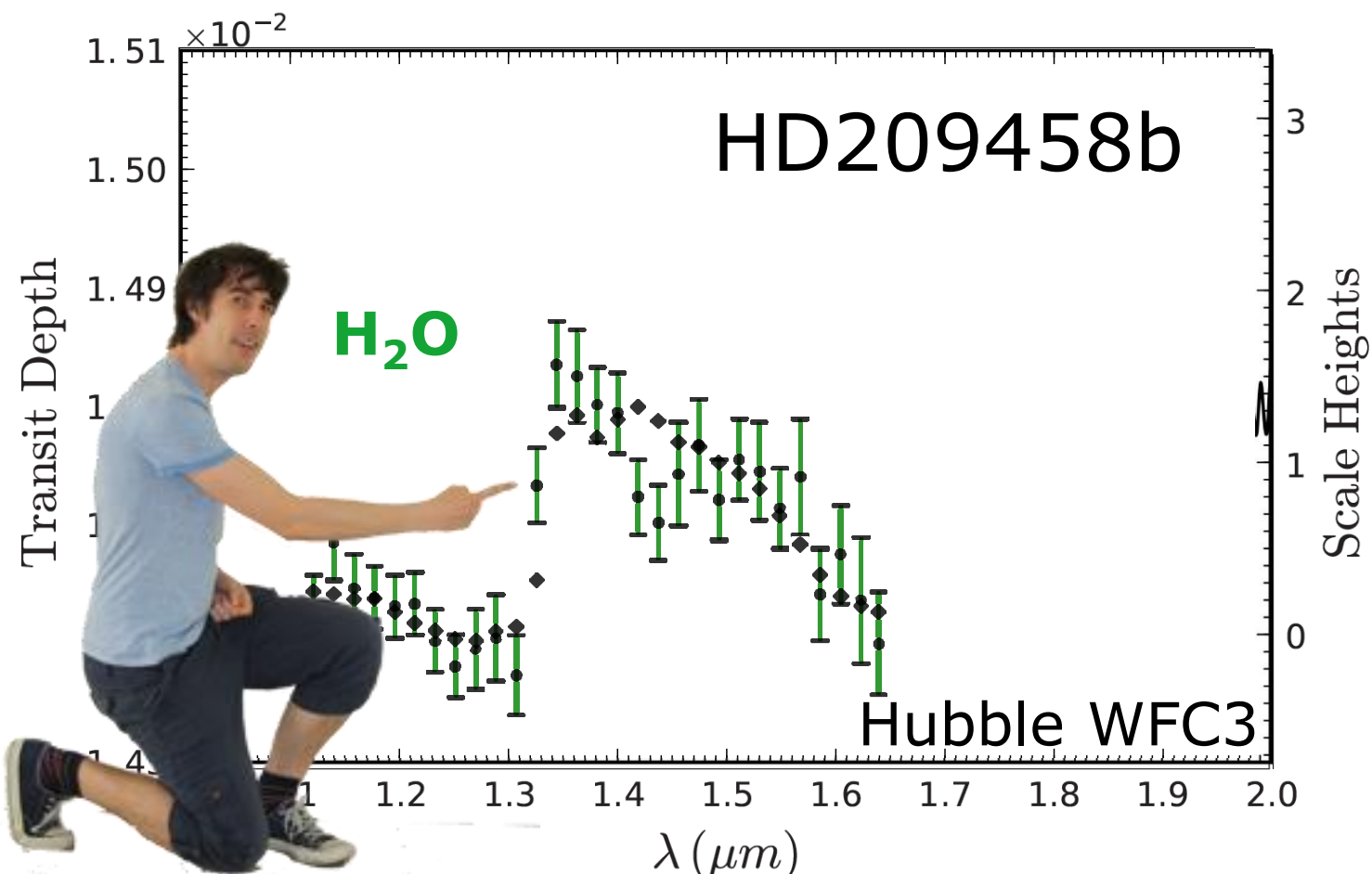
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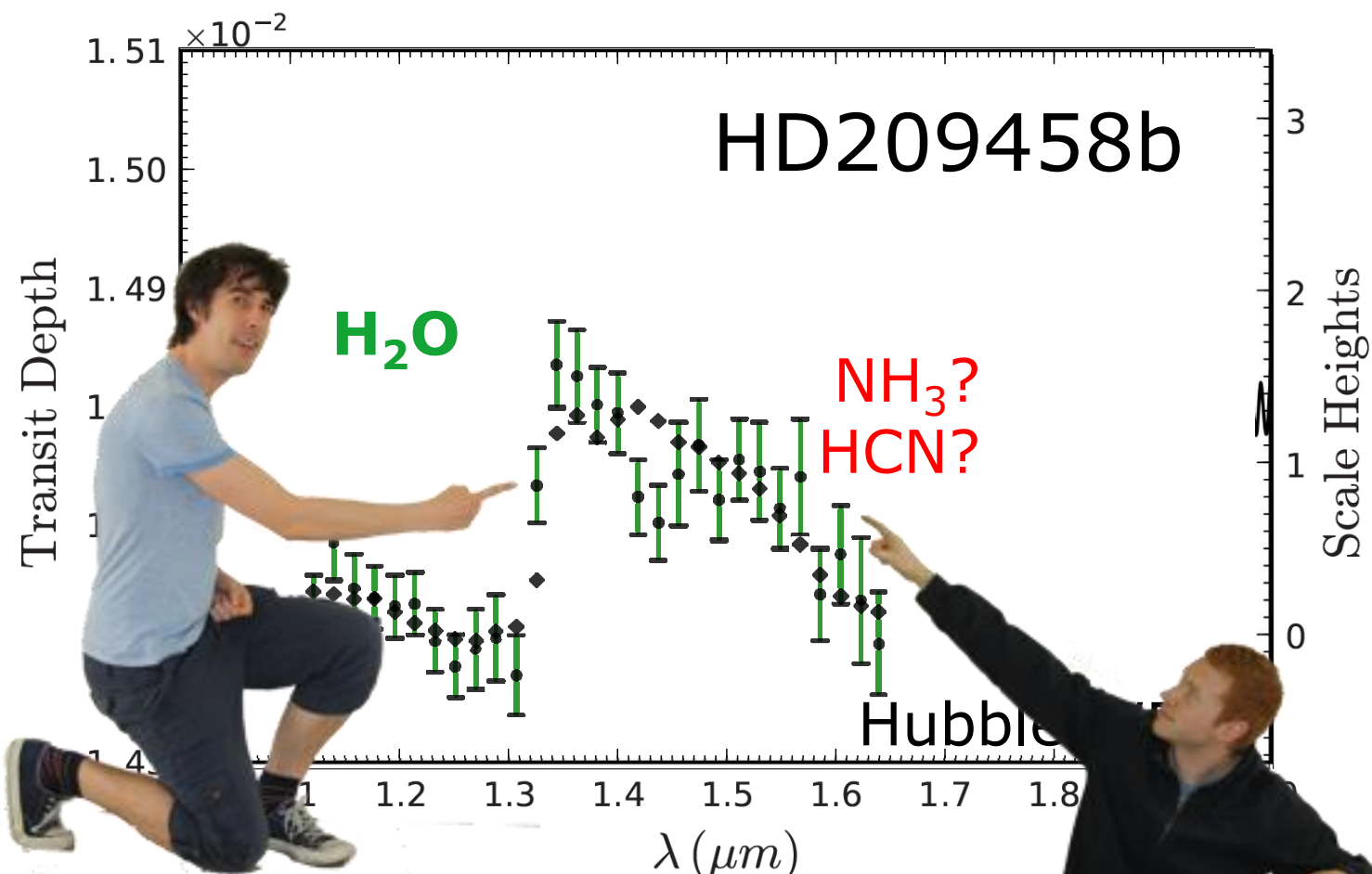


The background of the slide features a vibrant, orange-yellow star with a bright, glowing core and a textured, fiery surface. In the lower center, a portion of a planet is visible, showing horizontal bands of brown and tan, suggesting a gas giant like Jupiter. The overall scene is set against a dark, star-speckled space background.

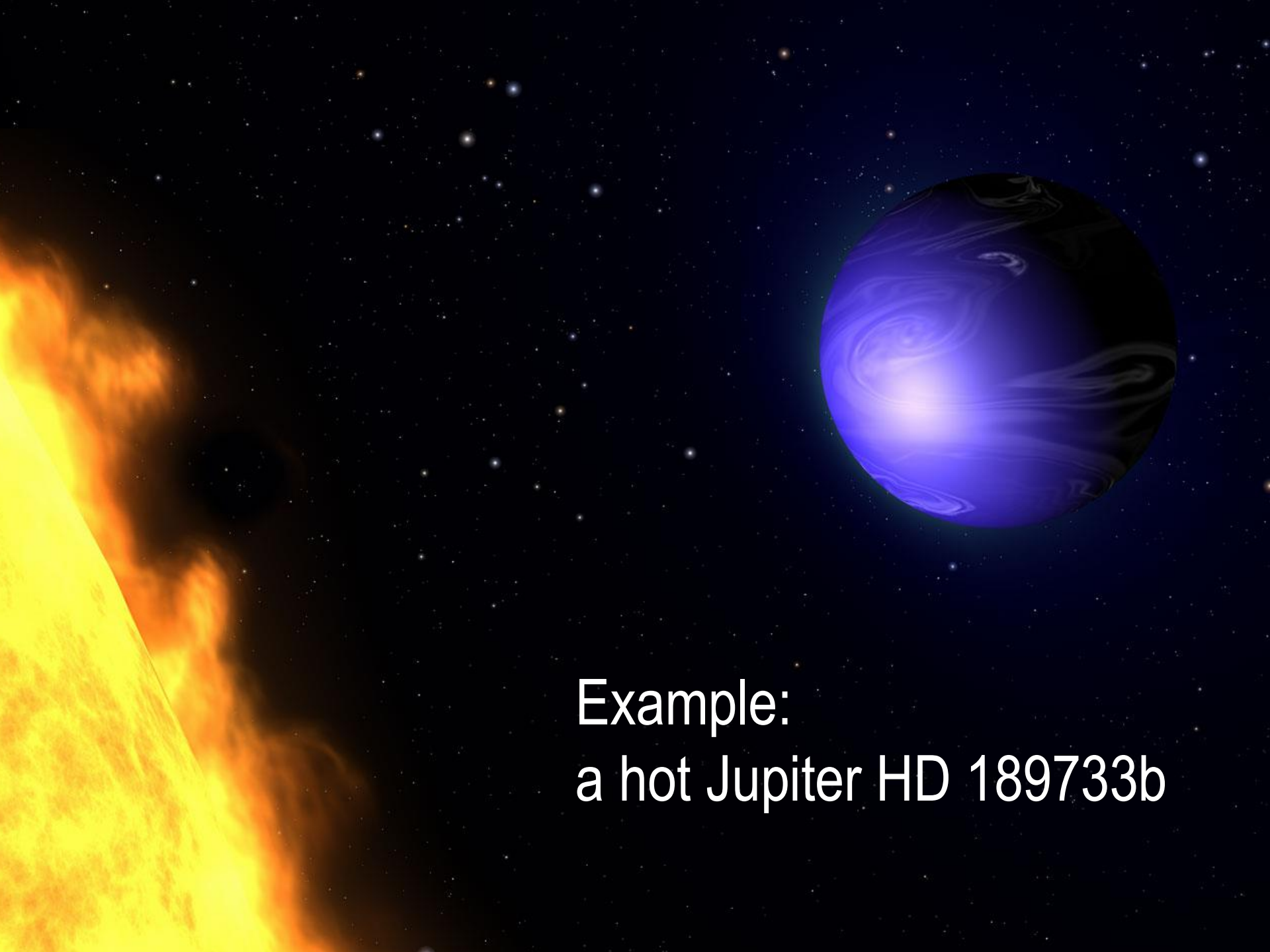
Example: a Hubble spectrum of  
**HD209458b**  
analyzed by a Cambridge group  
(using **ExoMol** database)





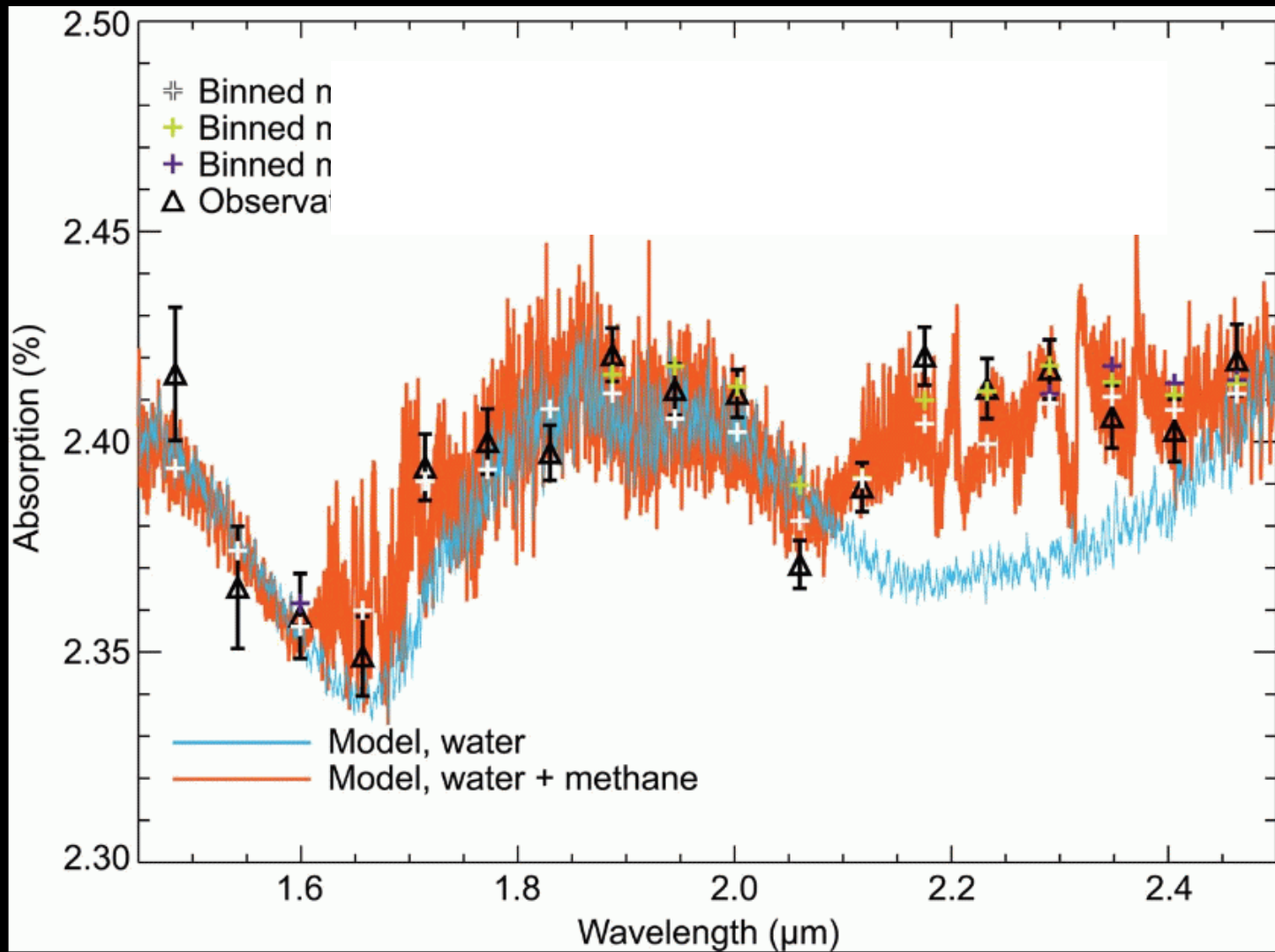


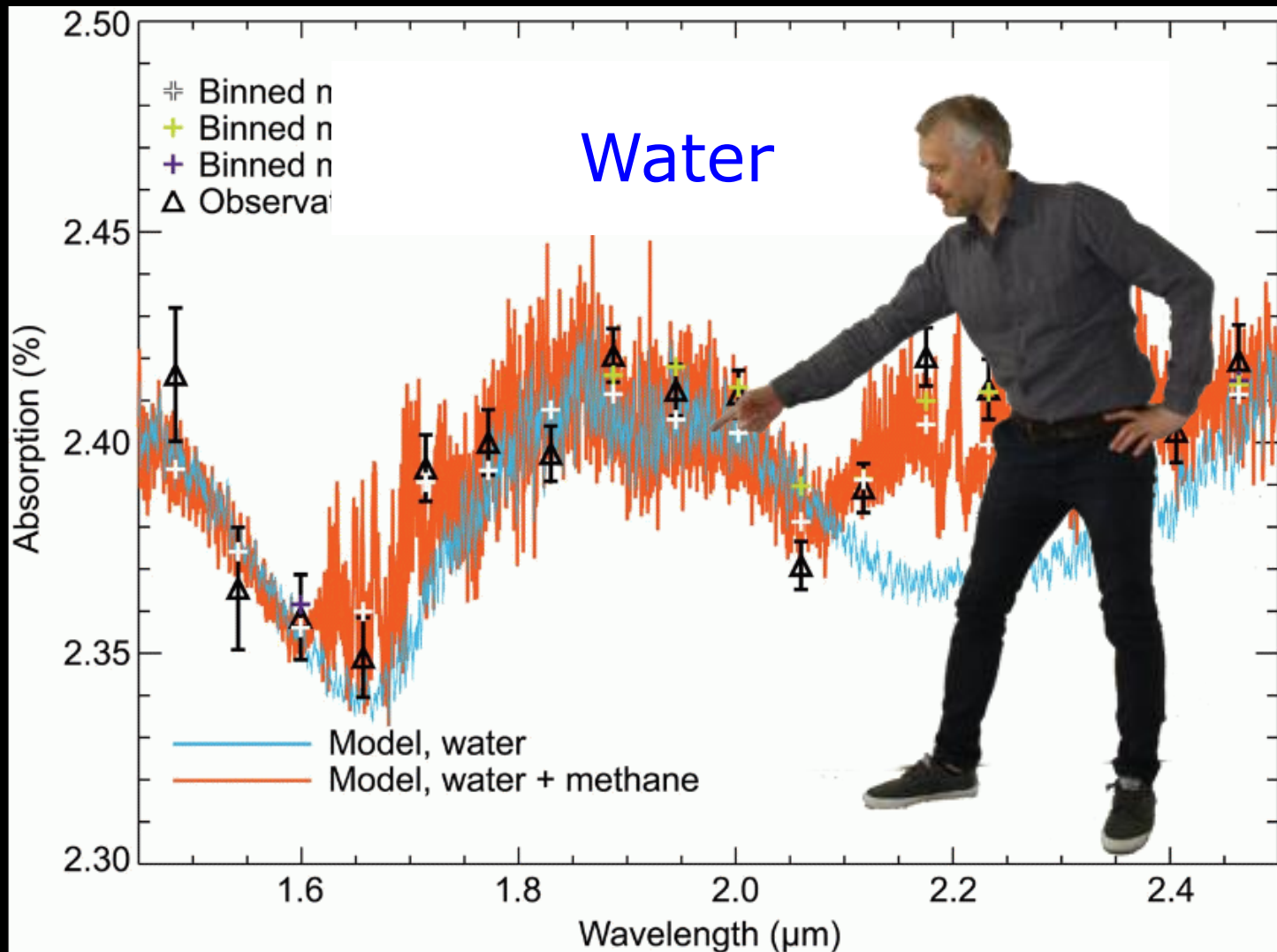
Water and Methane are  
usually first to find

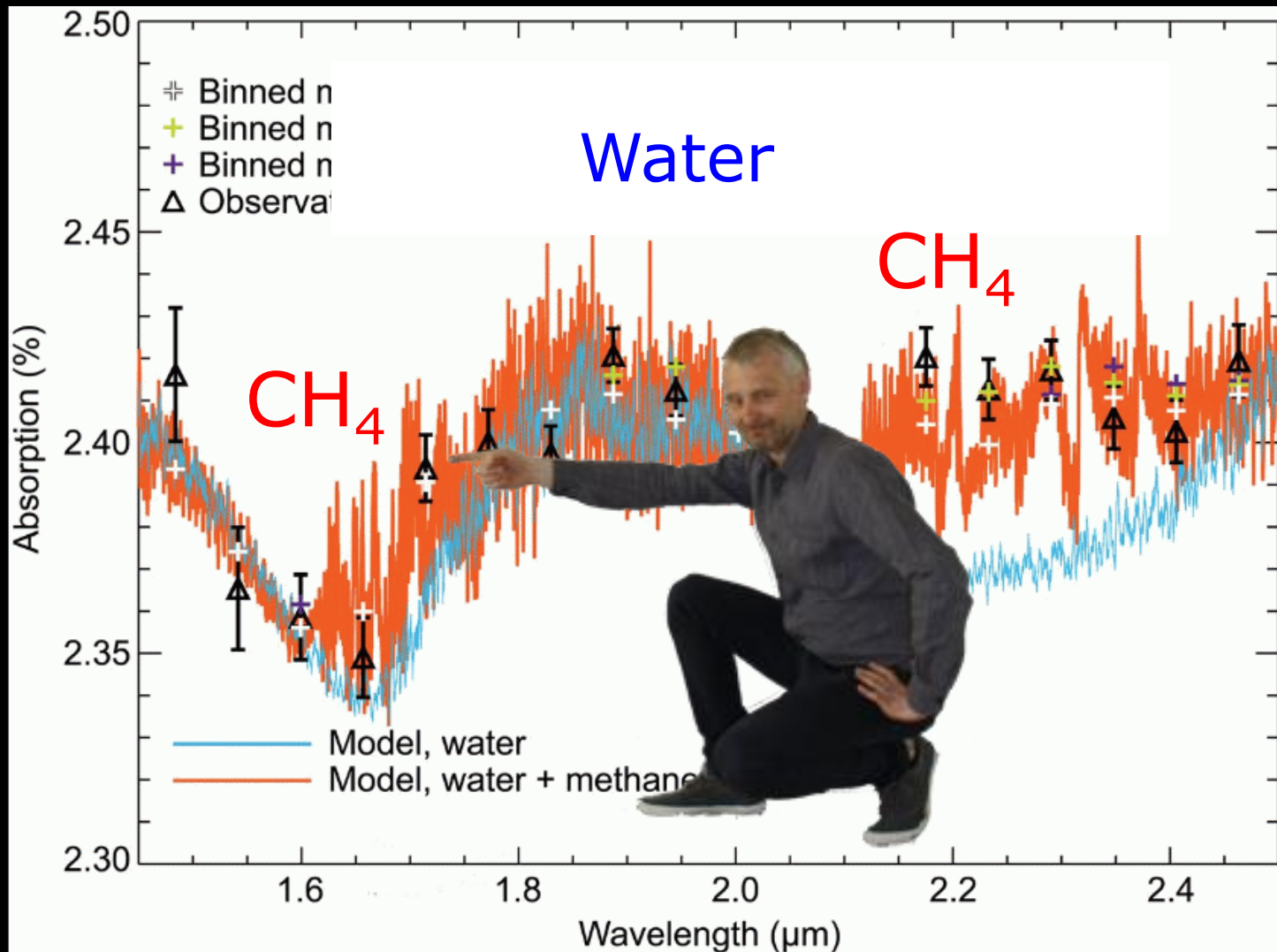


Example:  
a hot Jupiter HD 189733b









# Polyatomics: Method for solving the nuclear motion Schrödinger equation

# Polyatomics: Method for solving the nuclear motion Schrödinger equation

**This is waht I  
do for living**



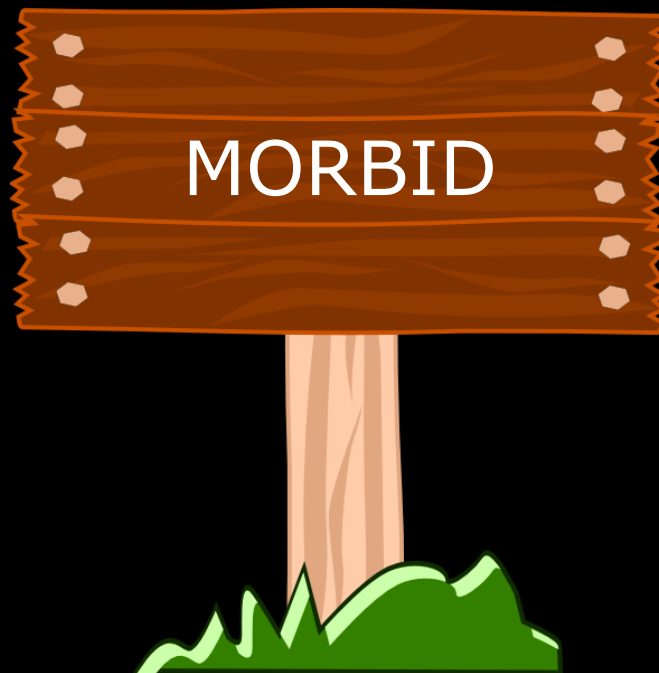
# Polyatomics: Method for solving the nuclear motion Schrödinger equation

TROVE



A wooden signpost with a horizontal sign and a vertical post, set against a black background. The sign is made of three horizontal wooden planks with visible grain and is held together by screws. The word "MORBID" is written in white, bold, sans-serif capital letters on the middle plank. The signpost is mounted on a small patch of green grass.

MORBID





**DVR3D**

**MORBID**



Ingredients and calculation  
steps are essentially the same  
as for diatomics

1. Accurate ab initio  
potential energy surface

...computed with  
CCSD(T)-F12b/aug-cc-pVQZ-DK  
MOLPRO

+ corrections:  
cc-pCVTZ, CCSD(T),  
CCSDT, and CCSDT(Q)  
CFOUR

## 2. Accurate ab initio Dipole Moment Surface

## 2. Accurate ab initio Dipole Moment Surface

Typical Dipoles:  
CCSD(T)/aug-cc-pVQZ  
using finite fields

3: Solve Schrödinger equation

$$H\Psi = E \Psi$$

3: Solve Schrödinger equation



... variationally in the matrix  
representation:

$$\langle \phi_1 \phi_2 \phi_3 \dots | H | \phi'_1 \phi'_2 \phi'_3 \dots \rangle$$

The Hamiltonian has to be transferred to the body-fixed system which makes complicated

$$\begin{aligned}
\hat{H} = & \sum_{i,j} \frac{\partial}{\partial q_i} G_{ij}(\bar{q}) \frac{\partial}{\partial q_j} + \\
& + \sum_{i,\alpha} \frac{\partial}{\partial q_i} G_{i\alpha}(\bar{q}) \hat{J}_\alpha - \hat{J}_\alpha G_{i\alpha}(\bar{q}) \frac{\partial}{\partial q_i} + \\
& + \sum_{\alpha,\beta} \hat{J}_\alpha G_{\alpha\beta}(\bar{q}) \hat{J}_\beta + \\
& + U(\bar{q}) + V(\bar{q})
\end{aligned}$$

The most known and widely  
used is Watsonian

# Simplification of the molecular vibration–rotation hamiltonian

JAMES K. G. WATSON

Department of Chemistry, The University, Reading, Berkshire, England

(Received 5 September 1968)

By use of the commutation relations and sum rules, the Darling–Dennison vibration–rotation hamiltonian for a nonlinear molecule is rearranged to the form:

$$H = \frac{1}{2} \sum_{\alpha\beta} (\Pi_\alpha - \pi_\alpha) \mu_{\alpha\beta} (\Pi_\beta - \pi_\beta) + \frac{1}{2} \sum_k P_k^2 - \frac{1}{8} \hbar^2 \sum_\alpha \mu_{\alpha\alpha} + V.$$

The order of the factors in the first term is immaterial, on account of the relation:

$$\sum_\alpha \pi_\alpha \mu_{\alpha\beta} = \sum_\alpha \mu_{\alpha\beta} \pi_\alpha.$$

A simple expansion is given for the  $\mu_{\alpha\beta}$  tensor in terms of the normal coordinates.

## 1. Introduction

On the basis of the classical expression given by Wilson and Howard [1], Darling and Dennison [2],

$$U = -\frac{1}{8} \hbar^2 \sum_\alpha \mu_{\alpha\alpha}. \quad (3)$$

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TROVE approach is to  
*numerically* expand the kinetic  
energy operator in the sum-  
of-product form

$$G_{ij}(\bar{q}) = \sum_{n_1 n_2 n_3} G_{n_1 n_2 n_3}^{(ij)} q_1^{n_1} q_2^{n_2} q_3^{n_3}$$



$$G_{ij}(\bar{q}) = \sum_{n_1 n_2 n_3} G_{n_1 n_2 n_3}^{(ij)} q_1^{n_1} q_2^{n_2} q_3^{n_3}$$

Obtained numerically

$$G_{ij}(\bar{q}) = \sum_{n_1 n_2 n_3} G_{n_1 n_2 n_3}^{(ij)} q_1^{n_1} q_2^{n_2} q_3^{n_3}$$

Obtained numerically

$$V(\bar{q}) = \sum_{n_1 n_2 n_3} F_{n_1 n_2 n_3} q_1^{n_1} q_2^{n_2} q_3^{n_3}$$

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Obtained numerically

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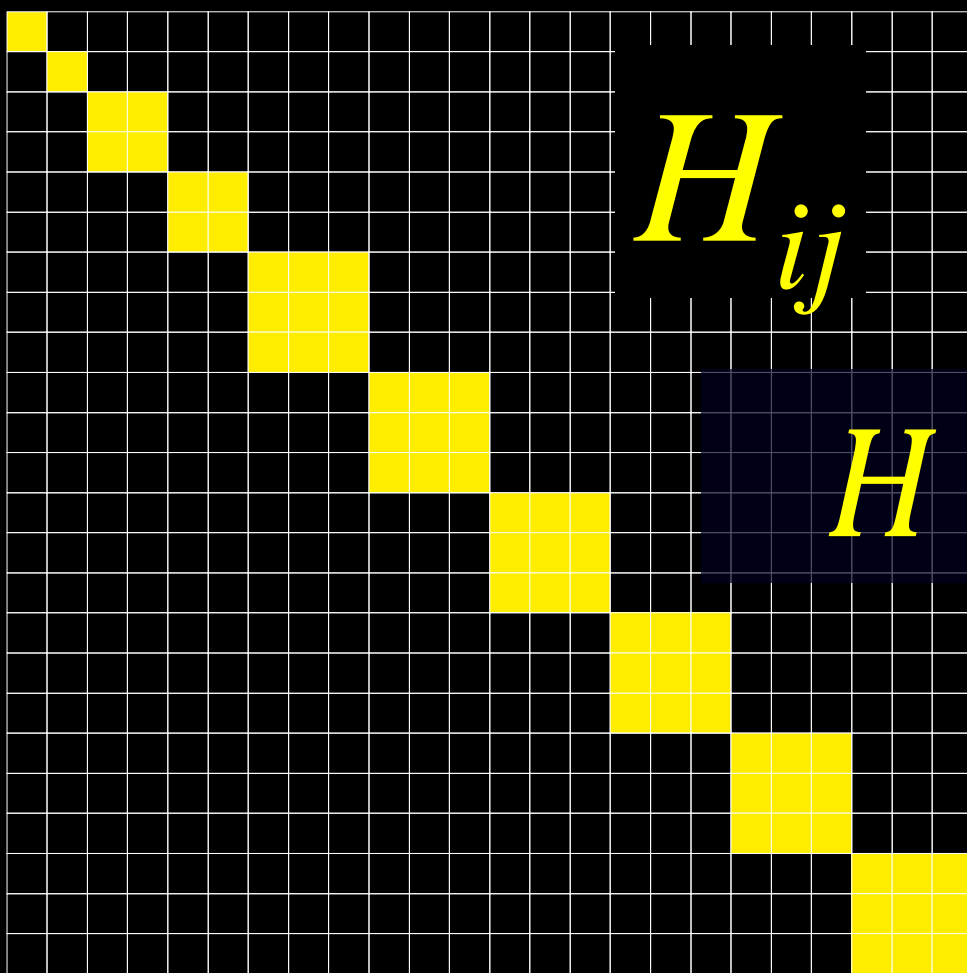
Obtained numerically

... which is good for product-type  
basis set representations

... which is good for product-type  
basis set representations

$$\Phi = |J, k, m\rangle |v_1\rangle |v_2\rangle |v_3\rangle \dots$$

We use a symmetry  
adapted basis so that the  
Hamiltonian matrix is  
block-diagonal



$H_{ij}$



$C_i$



$C_i$

$$H \Psi = E \Psi$$

In fact last year we published  
a paper on an automatic  
symmetrisation procedure we  
use



# Symmetry-Adapted Ro-vibrational Basis Functions for Variational Nuclear Motion Calculations: TROVE Approach

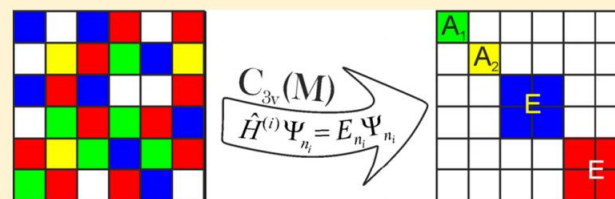
Sergei N. Yurchenko,<sup>\*,†,‡</sup> Andrey Yachmenev,<sup>‡</sup> and Roman I. Ovsyannikov<sup>¶</sup>

<sup>†</sup>Department of Physics and Astronomy, University College London, London, WC1E 6BT, United Kingdom

<sup>‡</sup>Center for Free-Electron Laser Science (CFEL), DESY, Notkestrasse 85, 22607 Hamburg, Germany

<sup>¶</sup>Institute of Applied Physics, Russian Academy of Sciences, Ulyanov Street 46, Nizhny Novgorod, Russia 603950

**ABSTRACT:** We present a general, numerically motivated approach to the construction of symmetry-adapted basis functions for solving ro-vibrational Schrödinger equations. The approach is based on the property of the Hamiltonian operator to commute with the complete set of symmetry operators and, hence, to reflect the symmetry of the system. The symmetry-adapted ro-vibrational basis set is constructed numerically by solving a set of reduced vibrational eigenvalue



Symmetry adaptation without symmetry operations

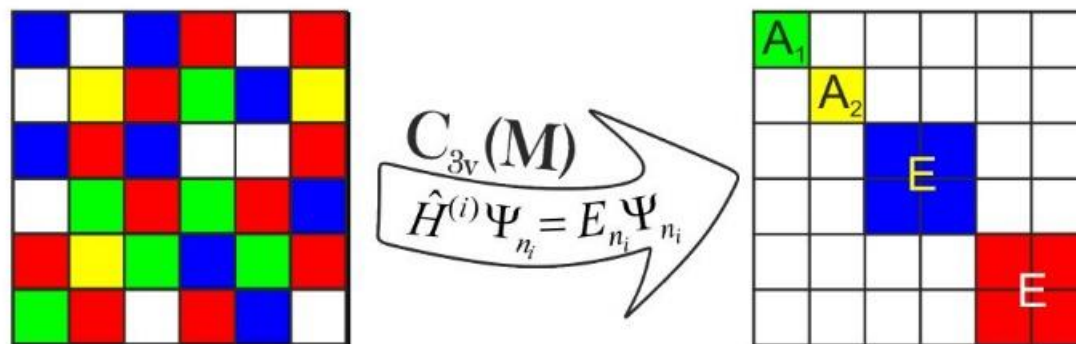
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**Symmetry adaptation without symmetry operations**

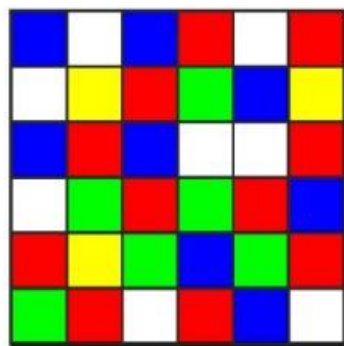
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$$C_{3v}(M) \rightarrow \hat{H}^{(i)} \Psi_{n_i} = E_{n_i} \Psi_{n_i}$$



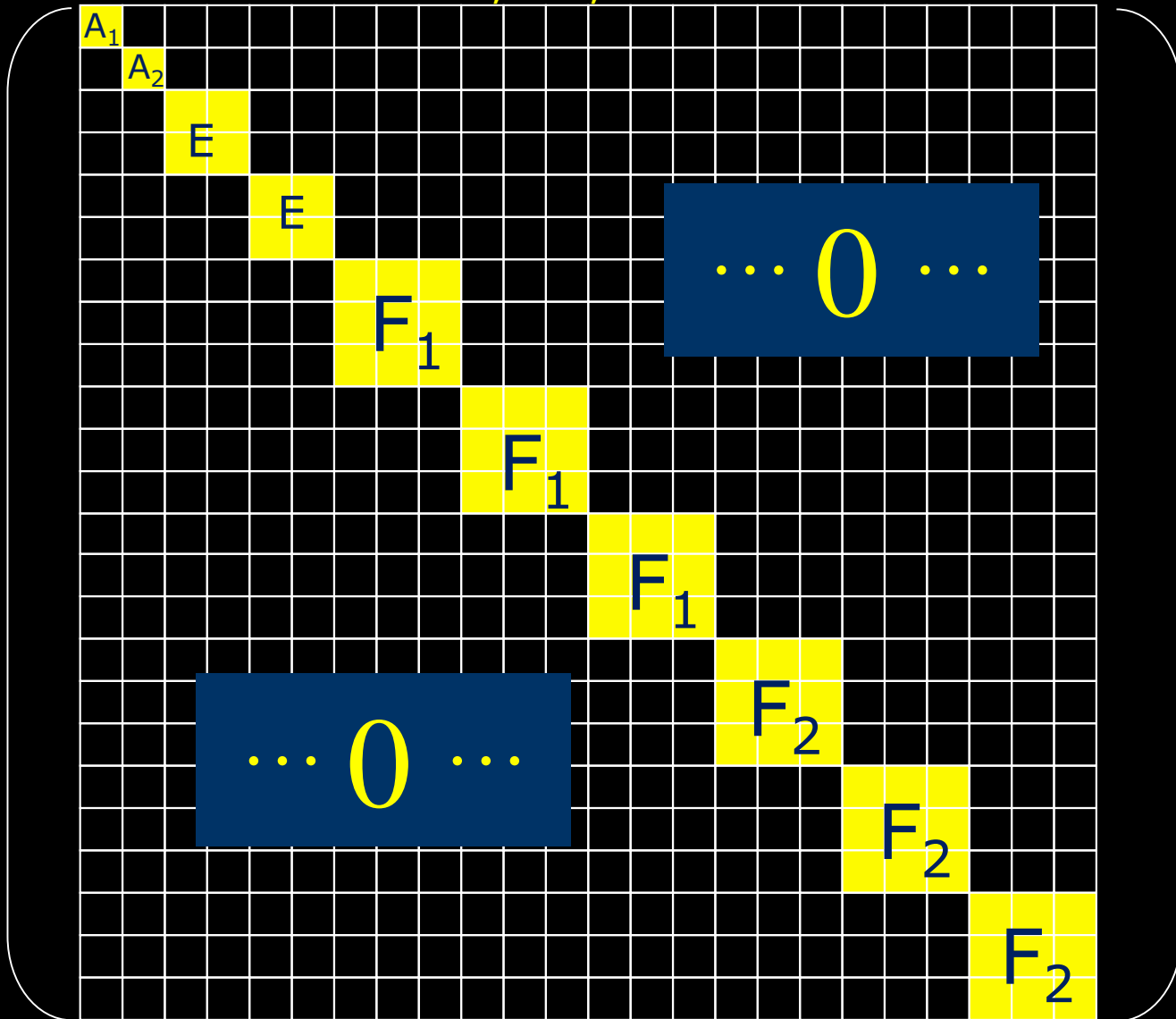
Symmetry adaptation without symmetry operations

Symmetries help to reduce  
the dimension

$$T_d(M)$$

2,000,000 elements

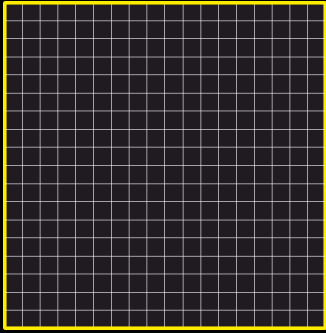
2,000,000 elements



1/8

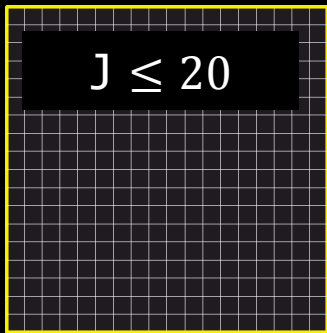
... and obligatory for intensity  
calculations

Then we diagonalize the  
Hamiltonian matrices

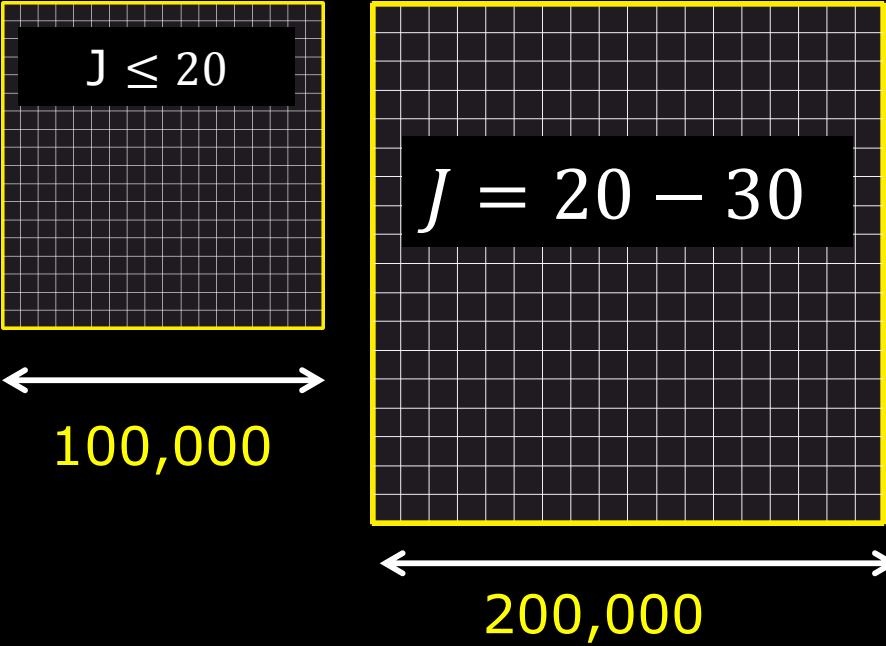


100,000





100,000

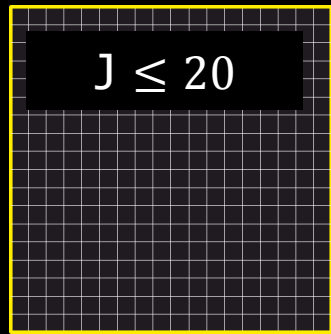

$$J \leq 20$$

100,000

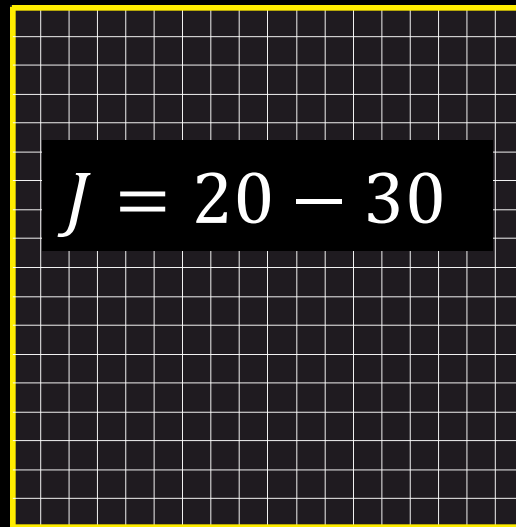
$$J = 20 - 30$$

200,000

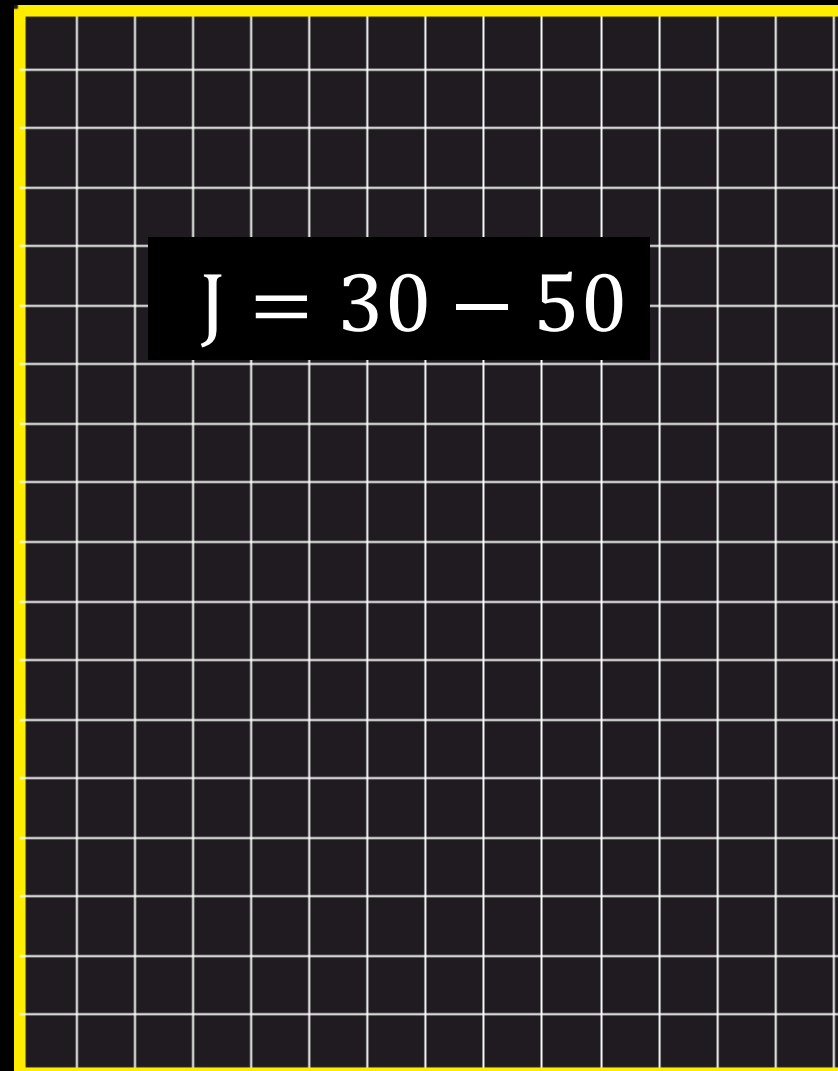
1-2 M CPUh total



100,000



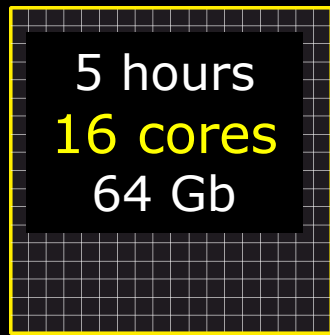
200,000



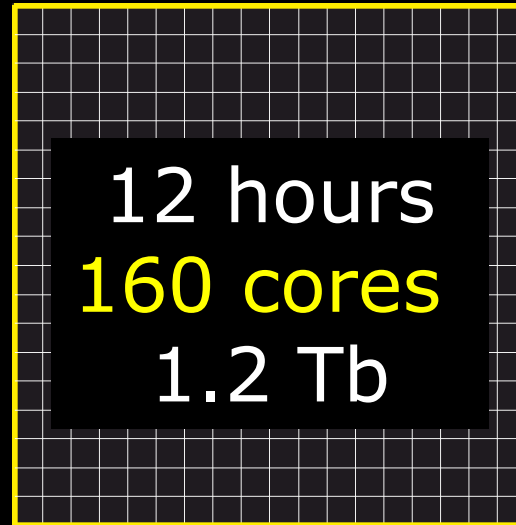
400,000

# Diagonalization: Time vs size

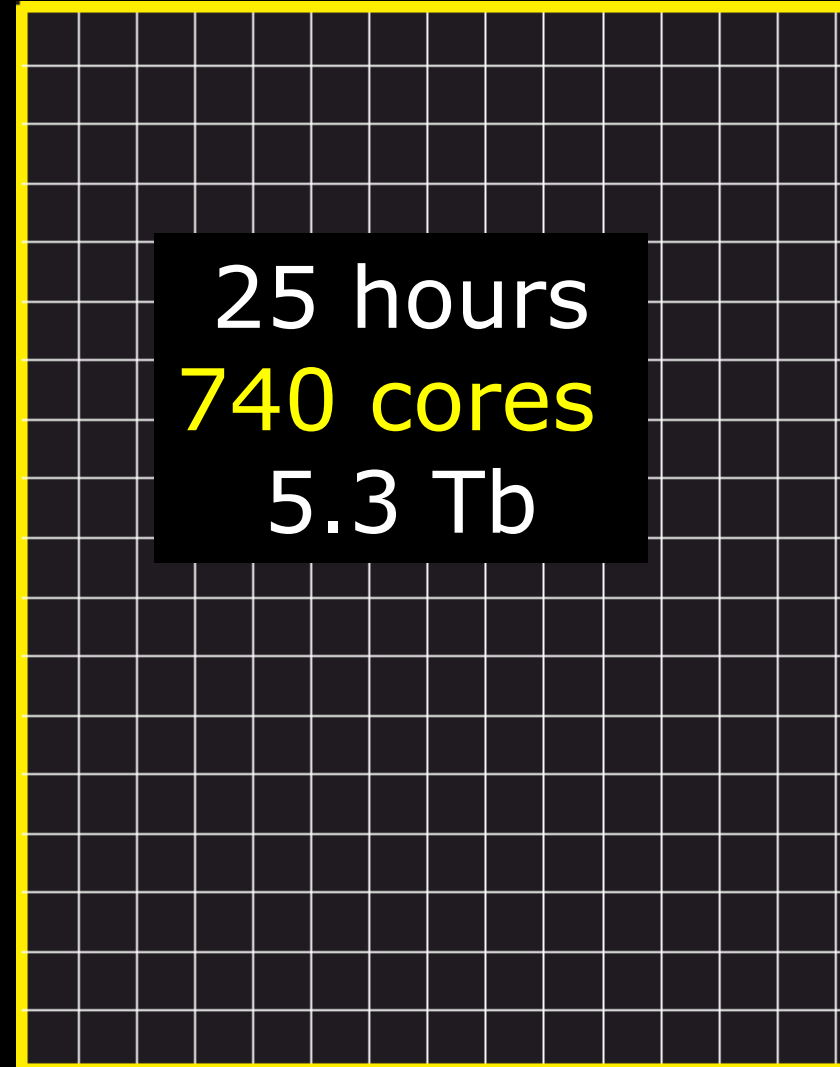
1-2 M CPUh total



100,000



200,000



400,000

And sometimes ( $J \sim 200$ )

← 1,000,000 →

And sometimes (J~200)

**This is too  
much!**

← 1,000,000 →

We have constructed an efficient  
machinery for accurate production  
of molecular spectra

We have constructed an efficient  
machinery for accurate production  
of molecular spectra

for atmospheric  
applications (stars and  
exoplanets)



This is where we started





2018

ExoMol

H <sub>2</sub>	PH <sub>3</sub>	AlO	AlH	CS	HNO <sub>3</sub>	PN	H <sub>2</sub> S	CrH	ScH	Done
LiH	OH	SO <sub>2</sub>	CH <sub>3</sub> Cl	C <sub>2</sub>	BeH	PS	KCl	HCN	HNC	
HeH <sup>+</sup>	NO	SH	HCl	CH <sub>4</sub>	NaCl	SiO	MgH	CH	CN	
H <sub>3</sub> <sup>+</sup>	O <sub>3</sub>	H <sub>2</sub> CO	HDO	H <sub>2</sub> O	NH <sub>3</sub>	CaH	SO <sub>3</sub>	CO	CO <sub>2</sub>	
H <sub>2</sub> D <sup>+</sup>	O <sub>2</sub>	HOOH	CH <sub>3</sub> F	TiO	VO	FeH	CaO	C <sub>3</sub>	C <sub>2</sub> H <sub>2</sub>	To-Do
NS	NaH	BaO	VN	CH <sub>3</sub> D	YO	SiH <sub>4</sub>	PH	SH	C <sub>2</sub> H <sub>4</sub>	
ZnS	P <sub>2</sub> H <sub>2</sub>	SO	SiH	SiS	NiH	TiH	MgO	CH <sub>3</sub> Cl	C <sub>2</sub> H <sub>6</sub>	
		PO				SiC		PS	C <sub>3</sub> H <sub>8</sub>	

2018

# ExoMol

H <sub>2</sub>	PH <sub>3</sub>	AlO	AlH	CS	HNO <sub>3</sub>	PN	H <sub>2</sub> S	CrH	ScH	Done
LiH	OH	SO <sub>2</sub>	CH <sub>3</sub> Cl	C <sub>2</sub>	BeH	PS	KCl	HCN	HNC	
HeH <sup>+</sup>	NO	SH	HCl	CH <sub>4</sub>	NaCl	SiO	MgH	CH	CN	
H <sub>3</sub> <sup>+</sup>	O <sub>3</sub>	H <sub>2</sub> CO	H <sub>2</sub> O	NH <sub>3</sub>	CaH	SO <sub>3</sub>	CO	CO <sub>2</sub>		
H <sub>2</sub> D <sup>+</sup>	HO	FeH	CaO	C <sub>3</sub>	C <sub>2</sub> H <sub>2</sub>	To-Do				
NS	SiH <sub>4</sub>	PH	SH	C <sub>2</sub> H <sub>4</sub>						
Zn	TiH	MgO	CH <sub>3</sub> Cl	C <sub>2</sub> H <sub>6</sub>						
	SiC		PS	C <sub>3</sub> H <sub>8</sub>						

This is now

Not all of them are actively  
used for retrievals yet

# ExoMol 2018

H <sub>2</sub>	PH <sub>3</sub>	AlO	AlH	CS	HNO <sub>3</sub>	PN	H <sub>2</sub> S	CrH	ScH	Done
LiH	OH	SO <sub>2</sub>	CH <sub>3</sub> Cl	C <sub>2</sub>	BeH	PS	KCl	HCN	HNC	
H <sub>2</sub> O	NO	SH	HCl	CH <sub>4</sub>	NaCl	SiO	MgH	CH	CN	
CO	H <sub>2</sub> CO	HDO	H <sub>2</sub> O	NH <sub>3</sub>	CaH	SO <sub>3</sub>	CO	CO <sub>2</sub>		
O <sub>2</sub>	HCN	CH <sub>3</sub> F	TiO	VO	FeH	CaO	C <sub>3</sub>	C <sub>2</sub> H <sub>2</sub>	To-Do	
NaH				YO	SiH <sub>4</sub>	PH	SH	C <sub>2</sub> H <sub>4</sub>		
C <sub>2</sub> H <sub>2</sub>				NiH	TiH	MgO	CH <sub>3</sub> Cl	C <sub>2</sub> H <sub>6</sub>		
								C <sub>3</sub> H <sub>8</sub>		
		PO				SiC		PS		

Only these

Larger molecules



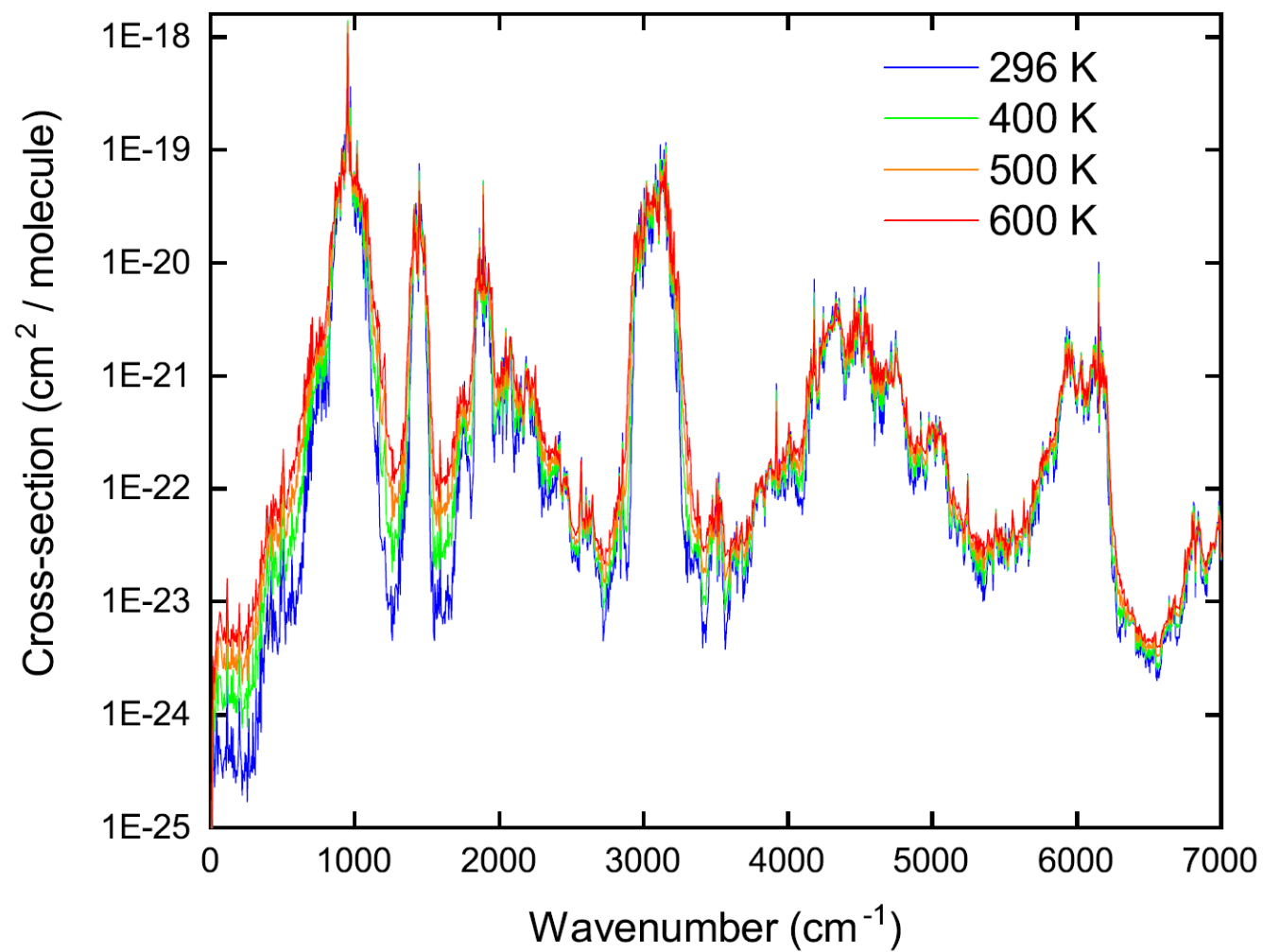
Barry Mant

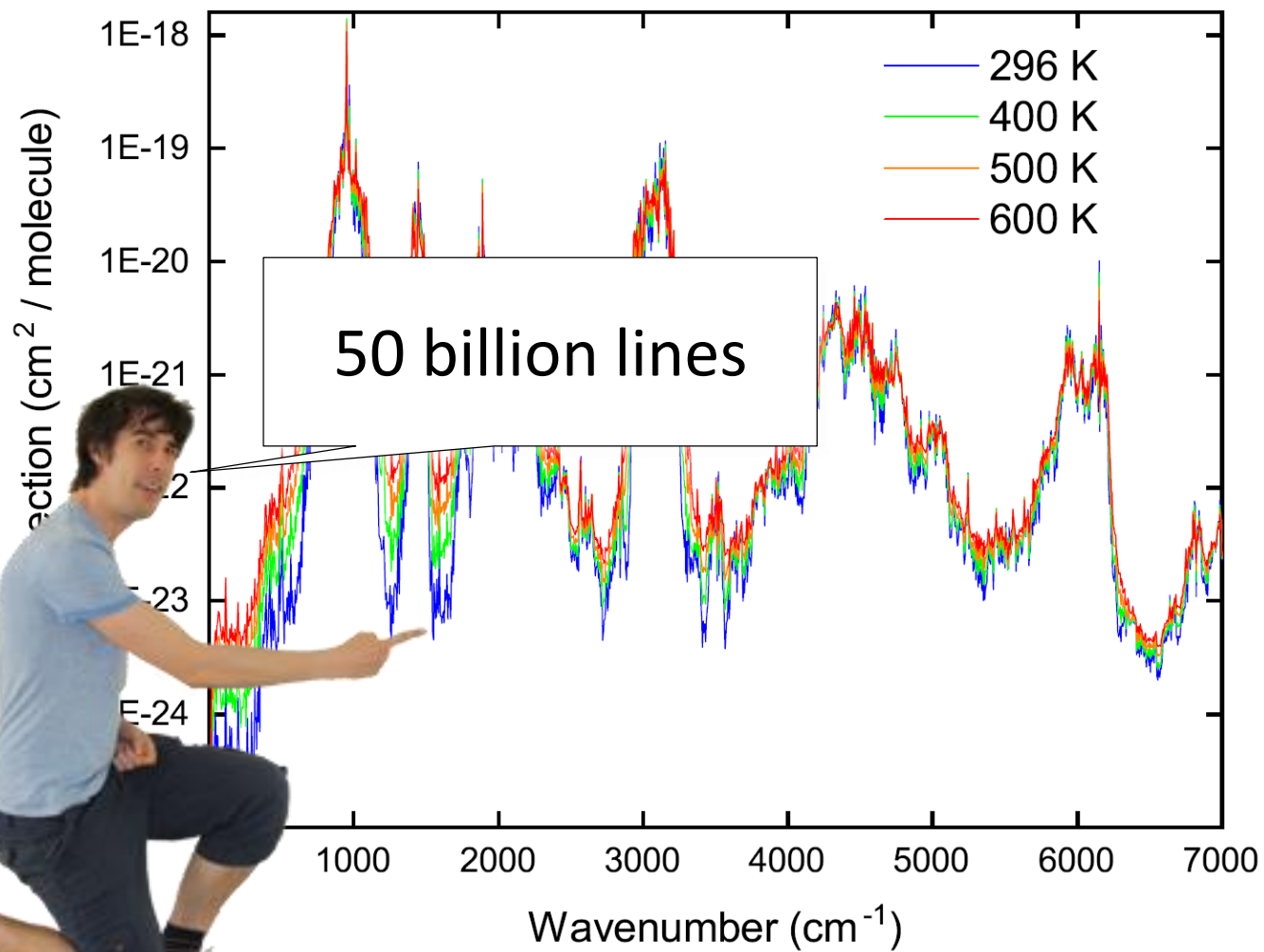


Ethylene:  
new ExoMol song line list

Barry Mant

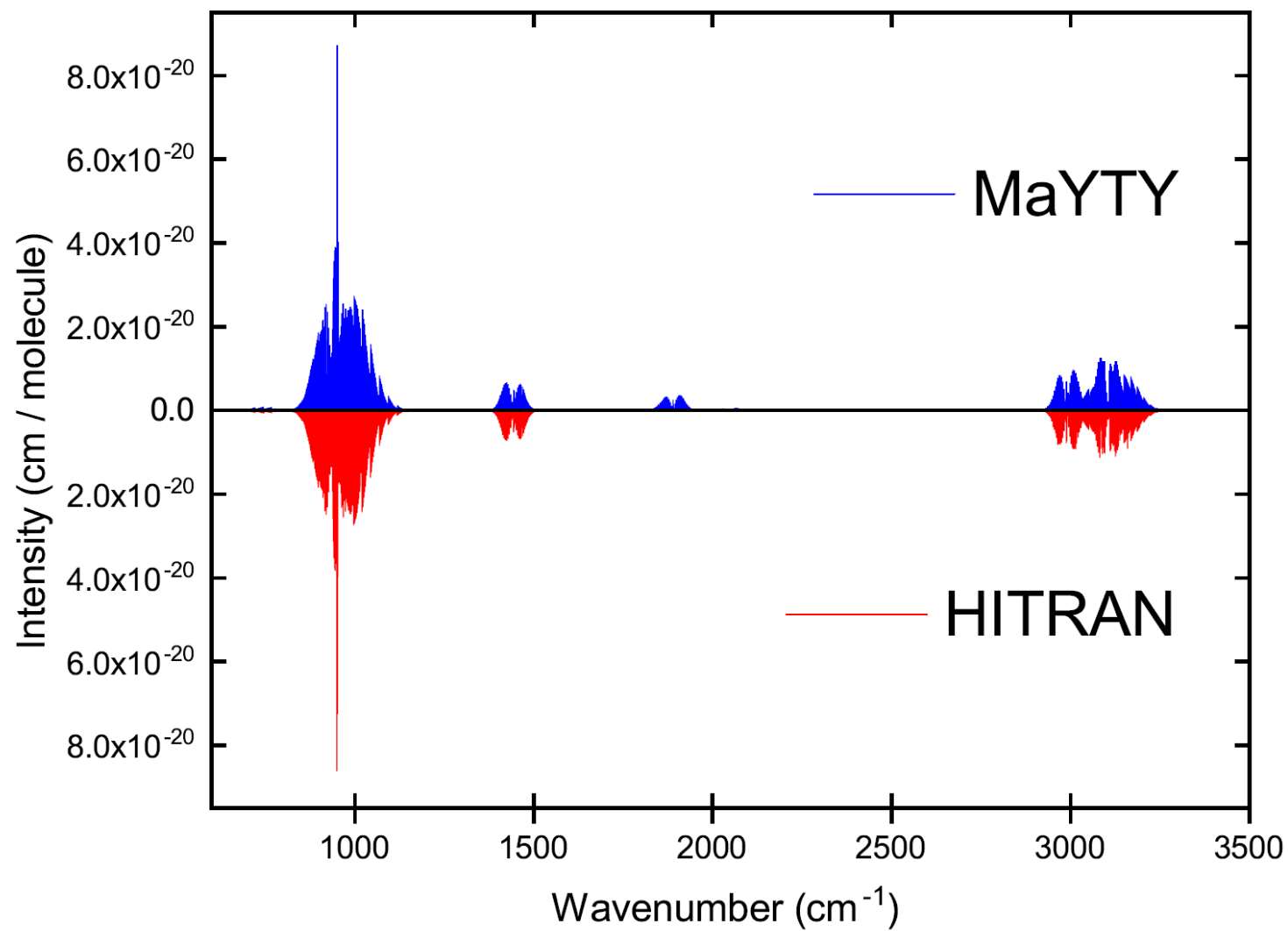


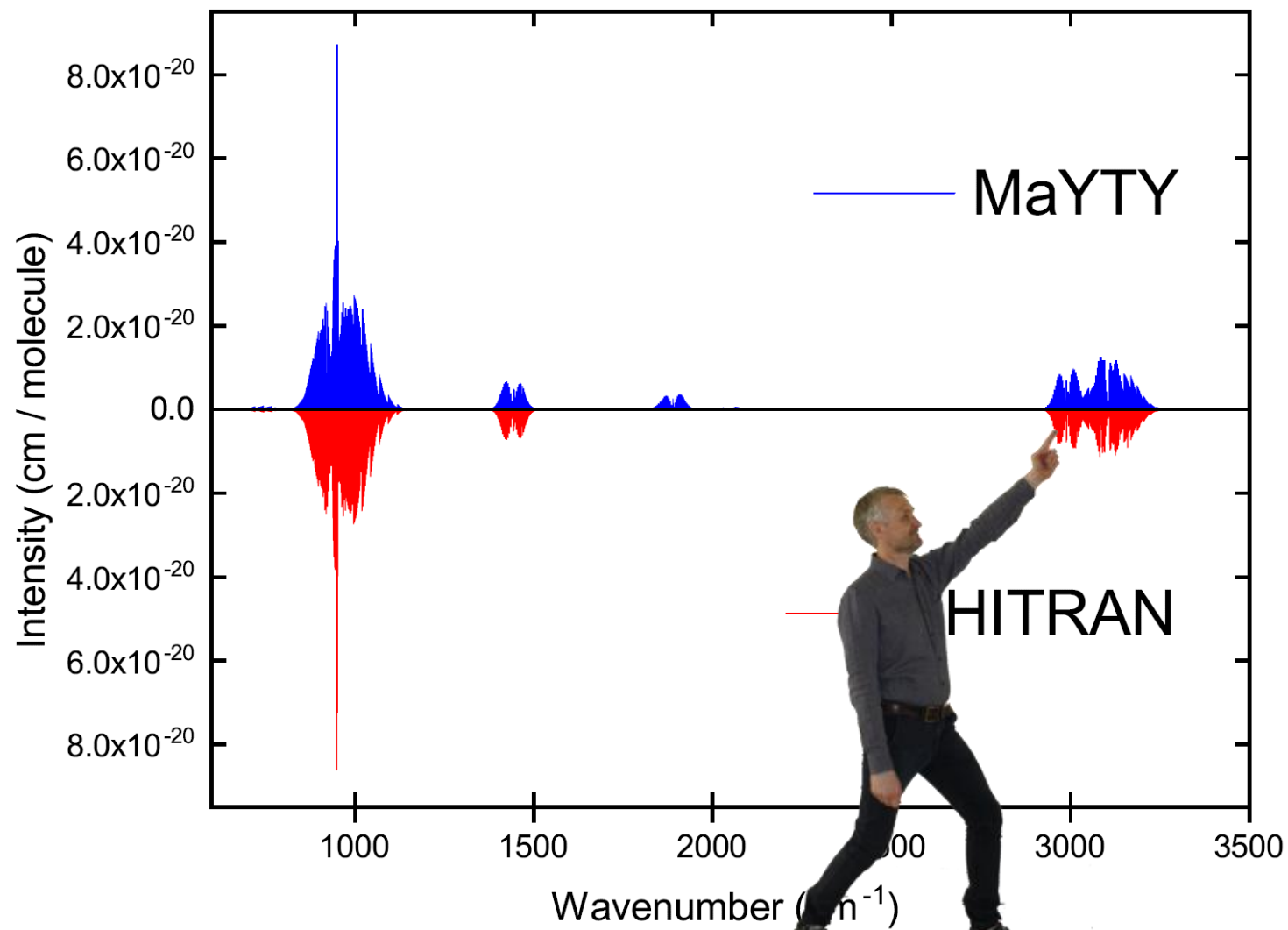


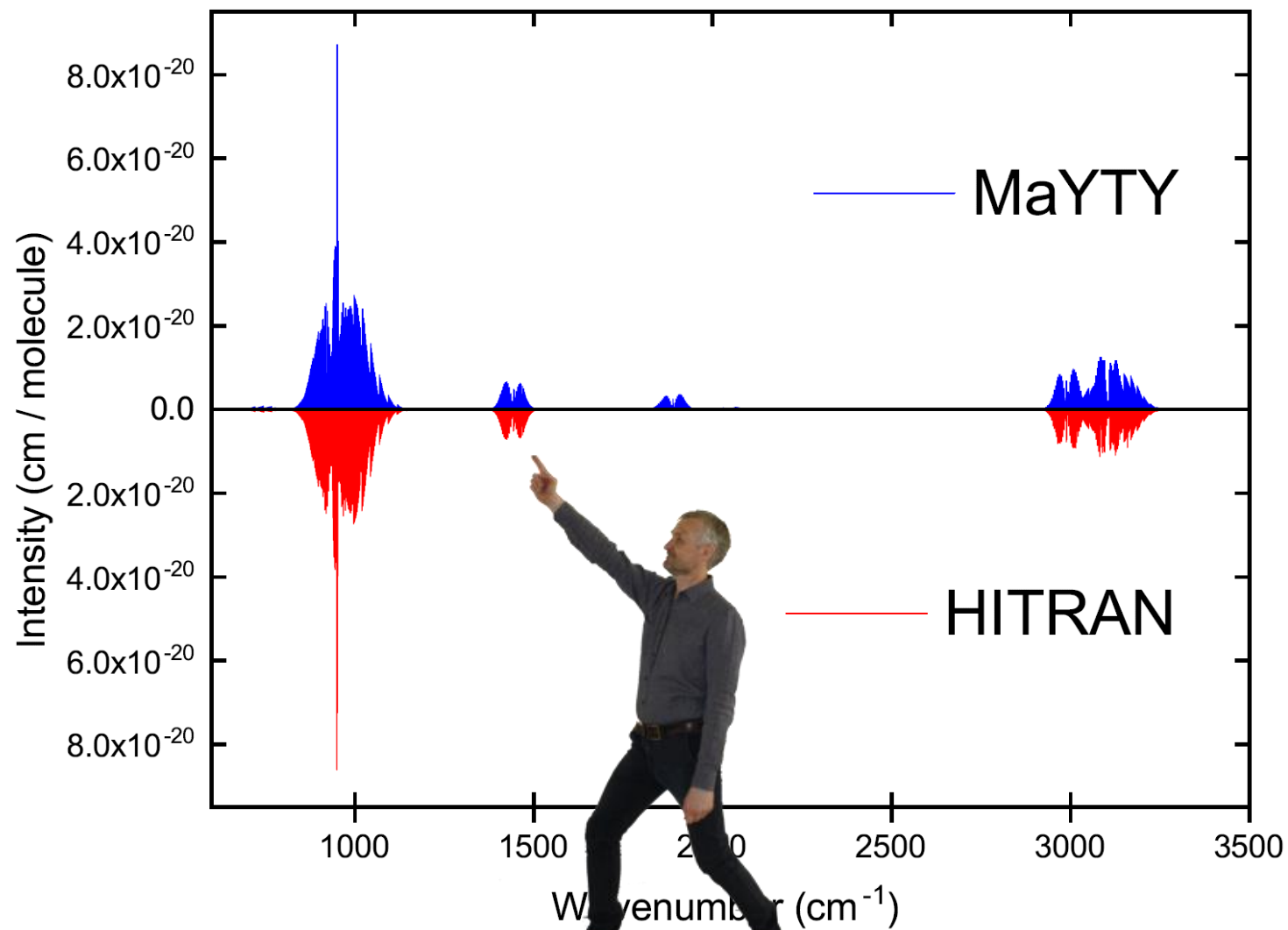




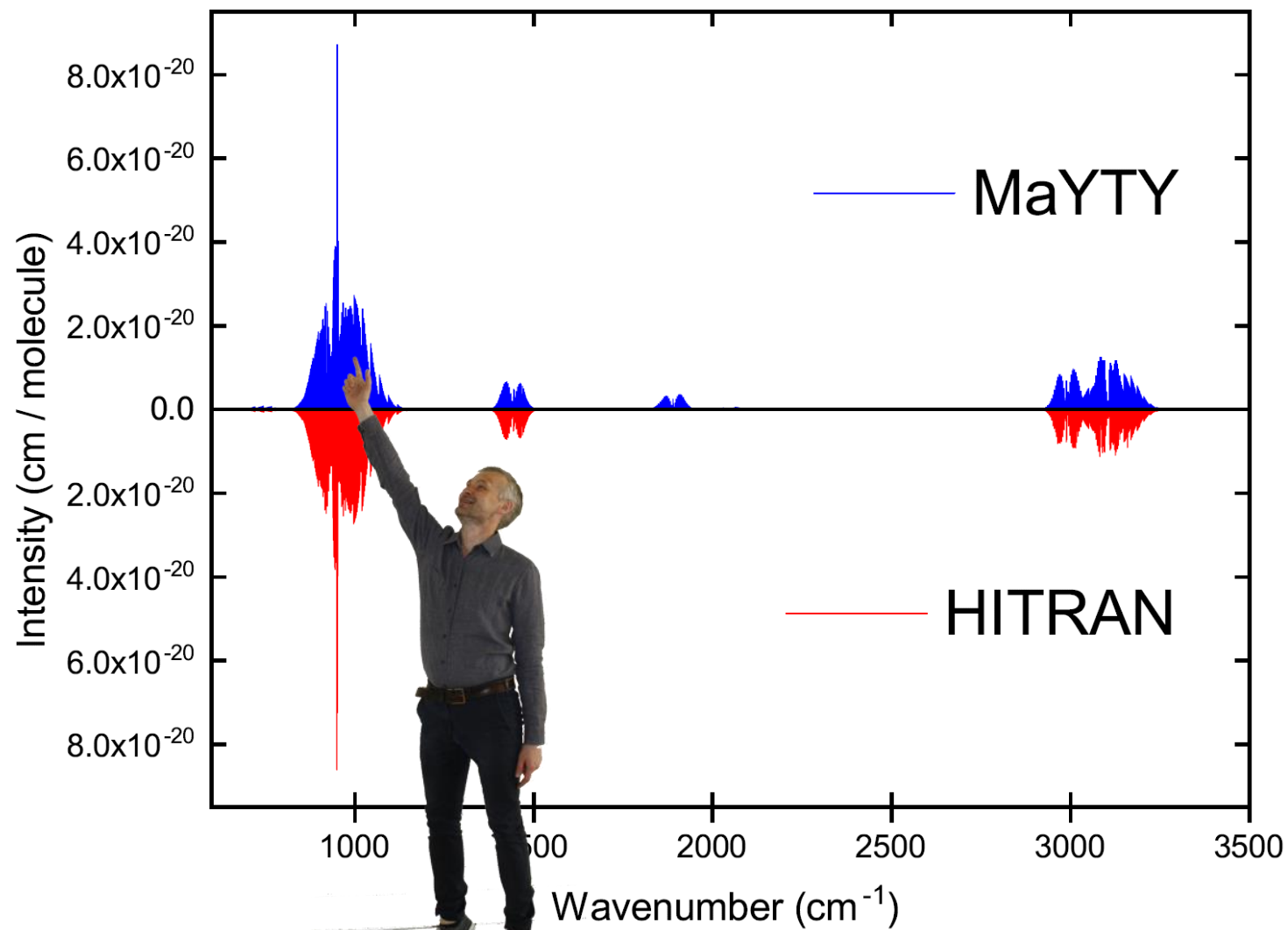
3 band model:  
Vibrational spectrum of  $\text{C}_2\text{H}_4$



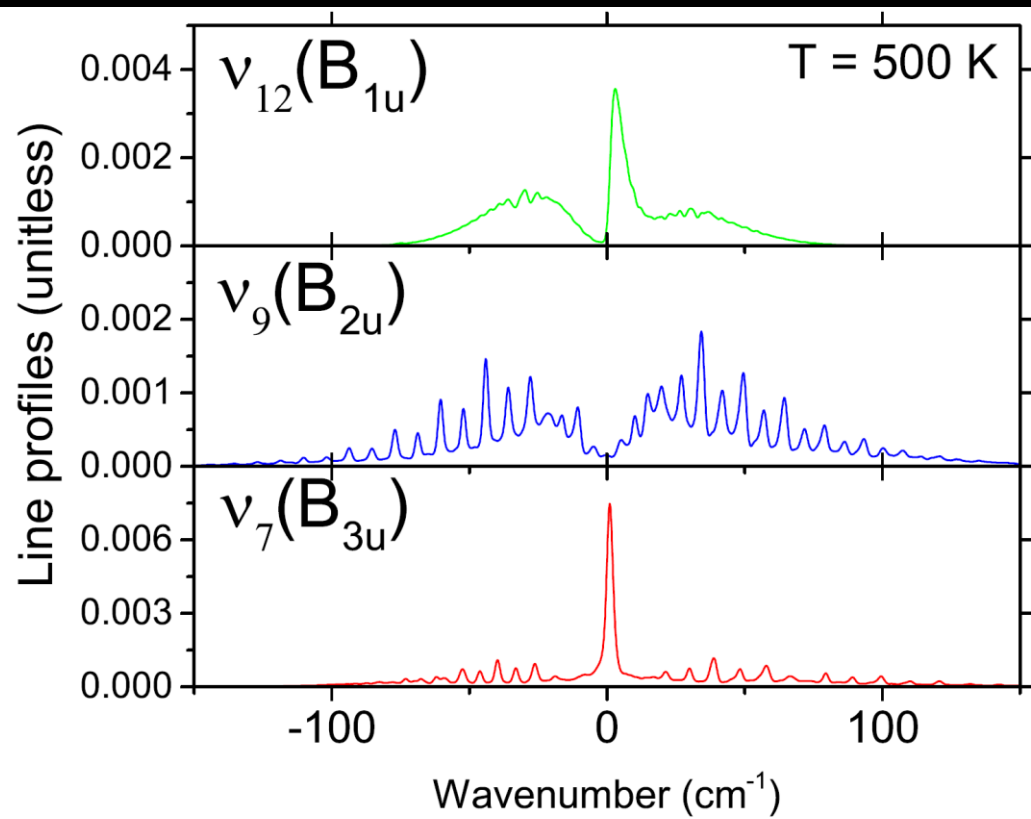


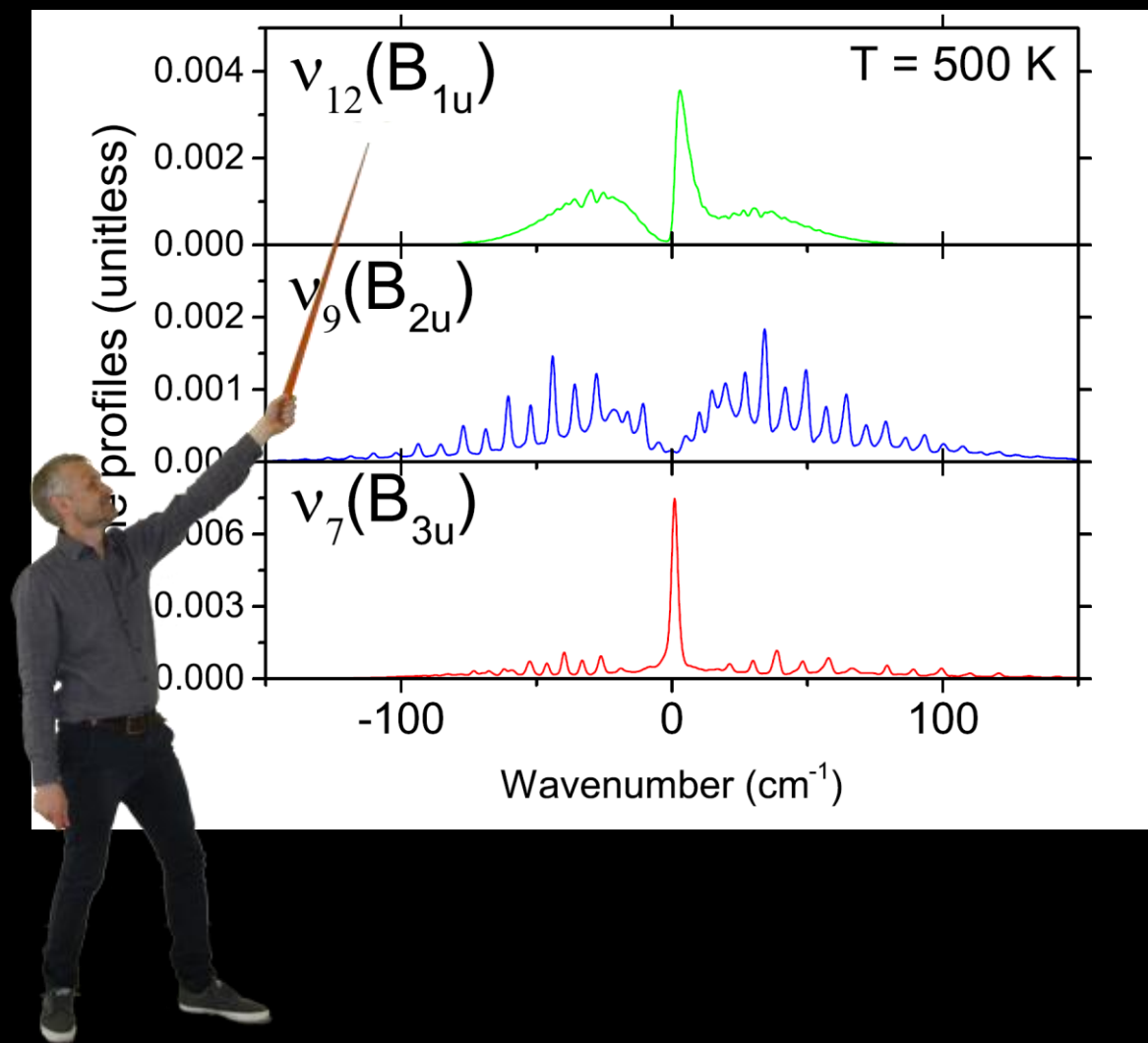


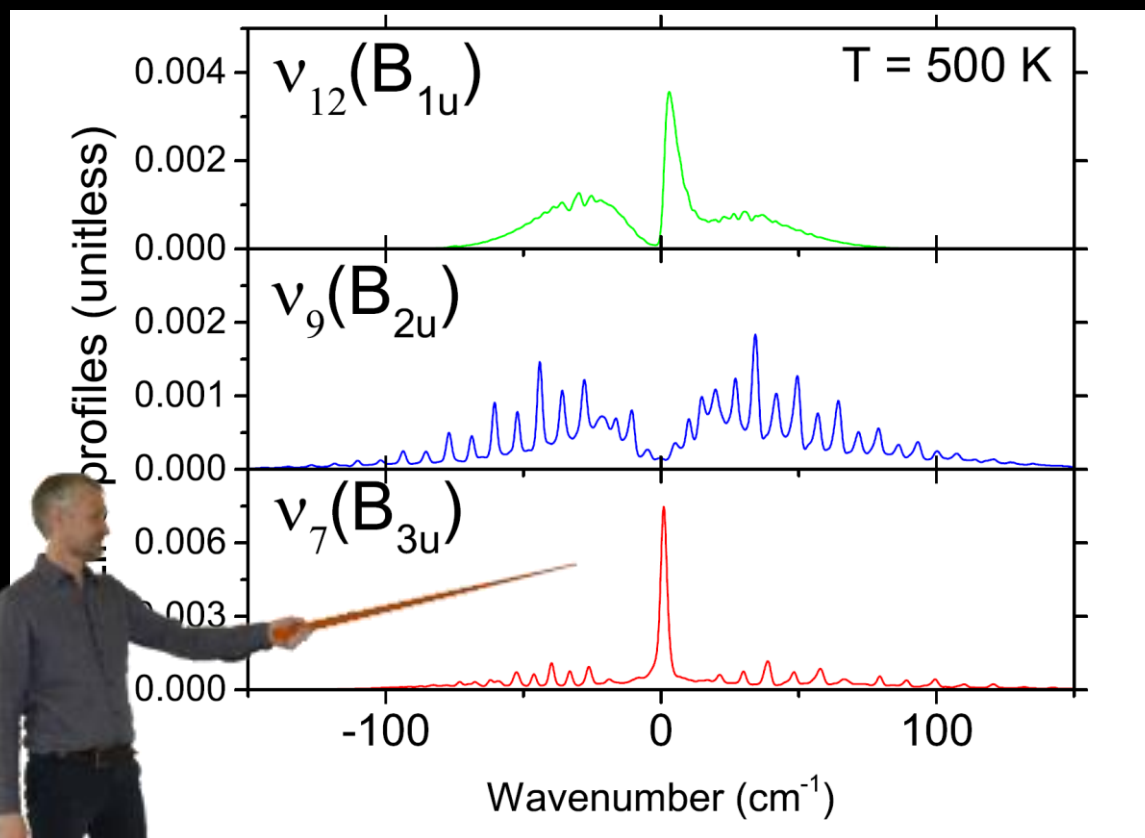




Extract their shapes







... and apply to all other bands

... and apply to all other bands

Low cost vibrational  
band intensities

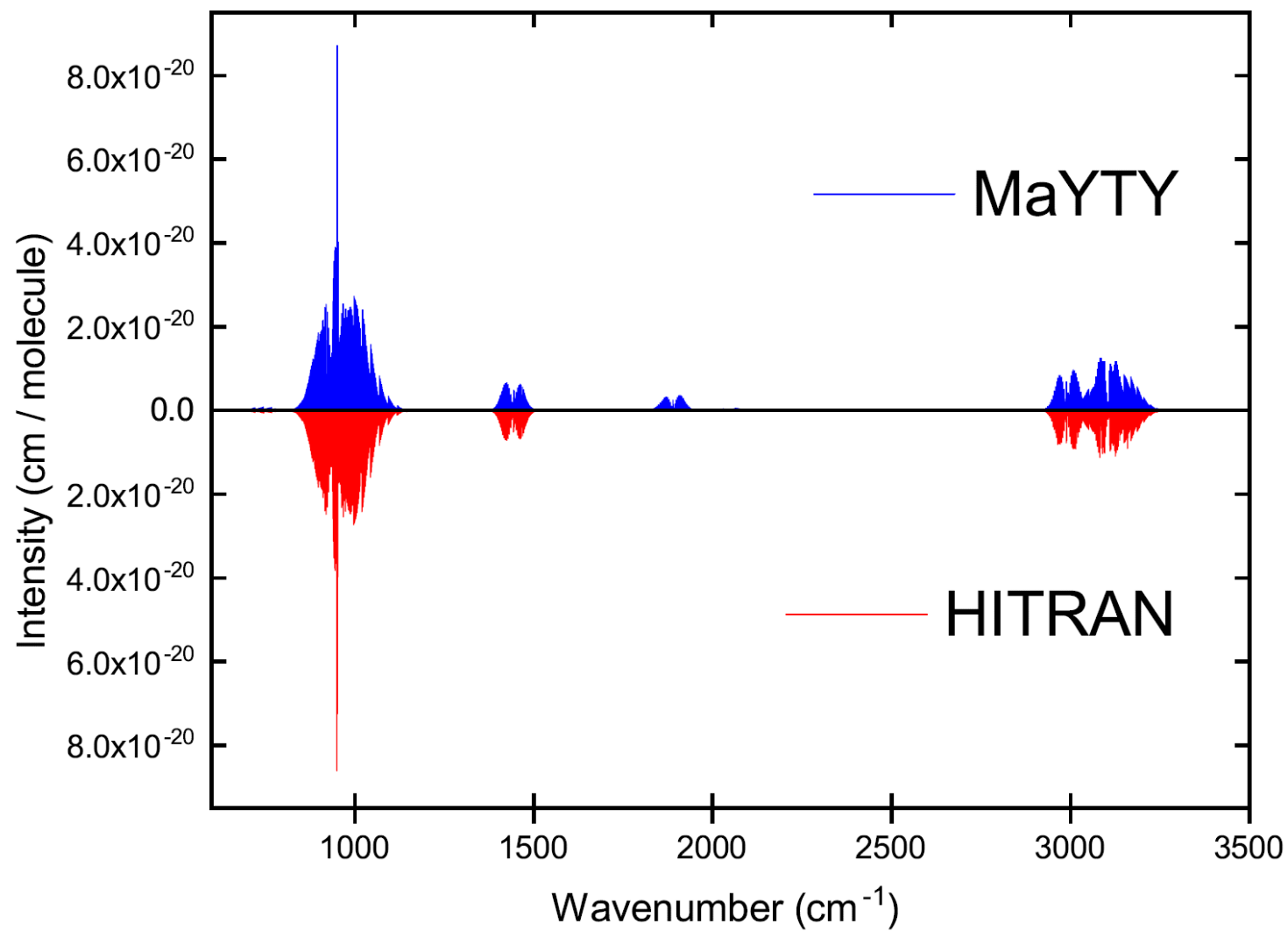
**Cheating on  
variational  
calculations**

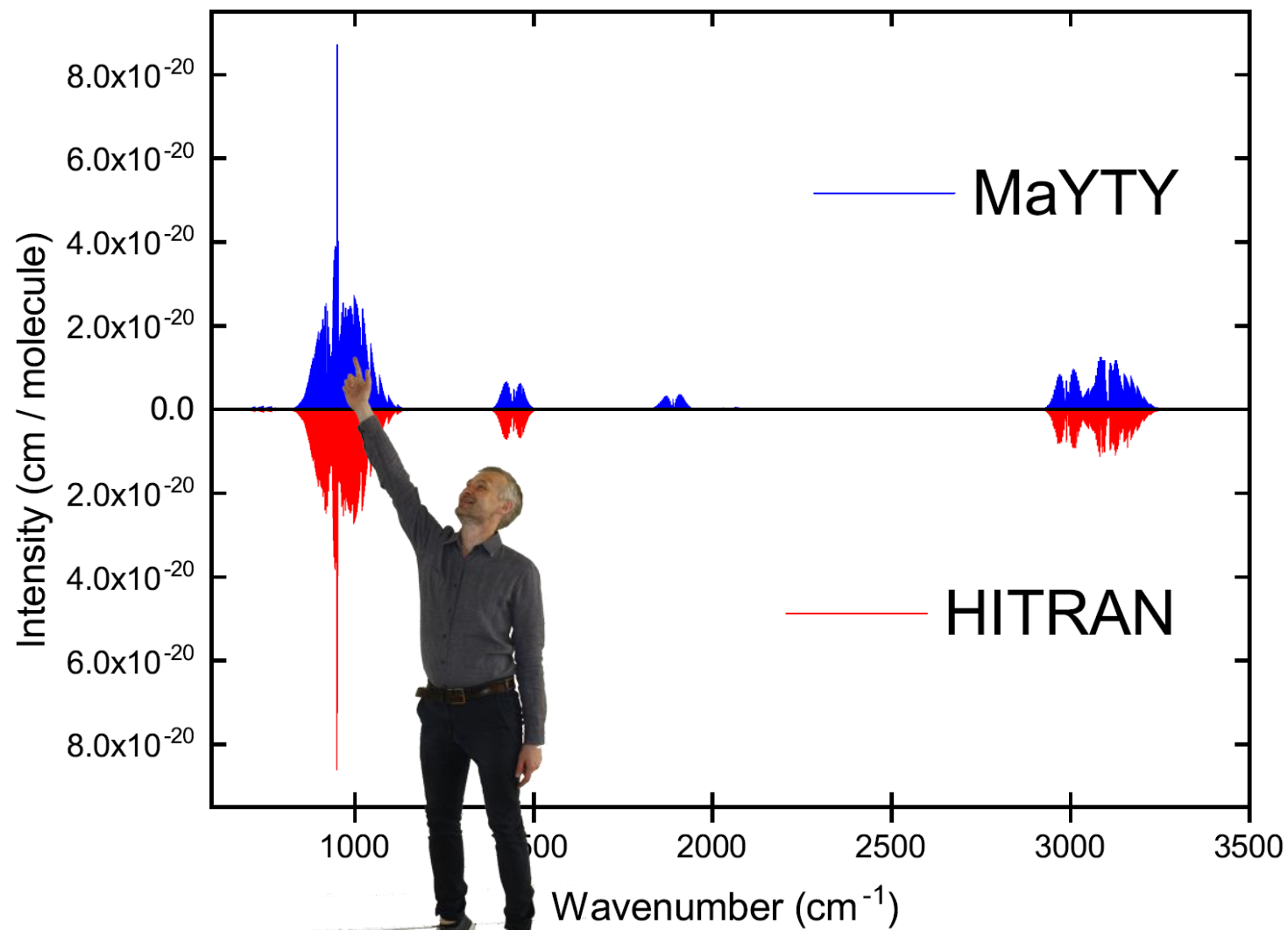
... and apply to all other bands

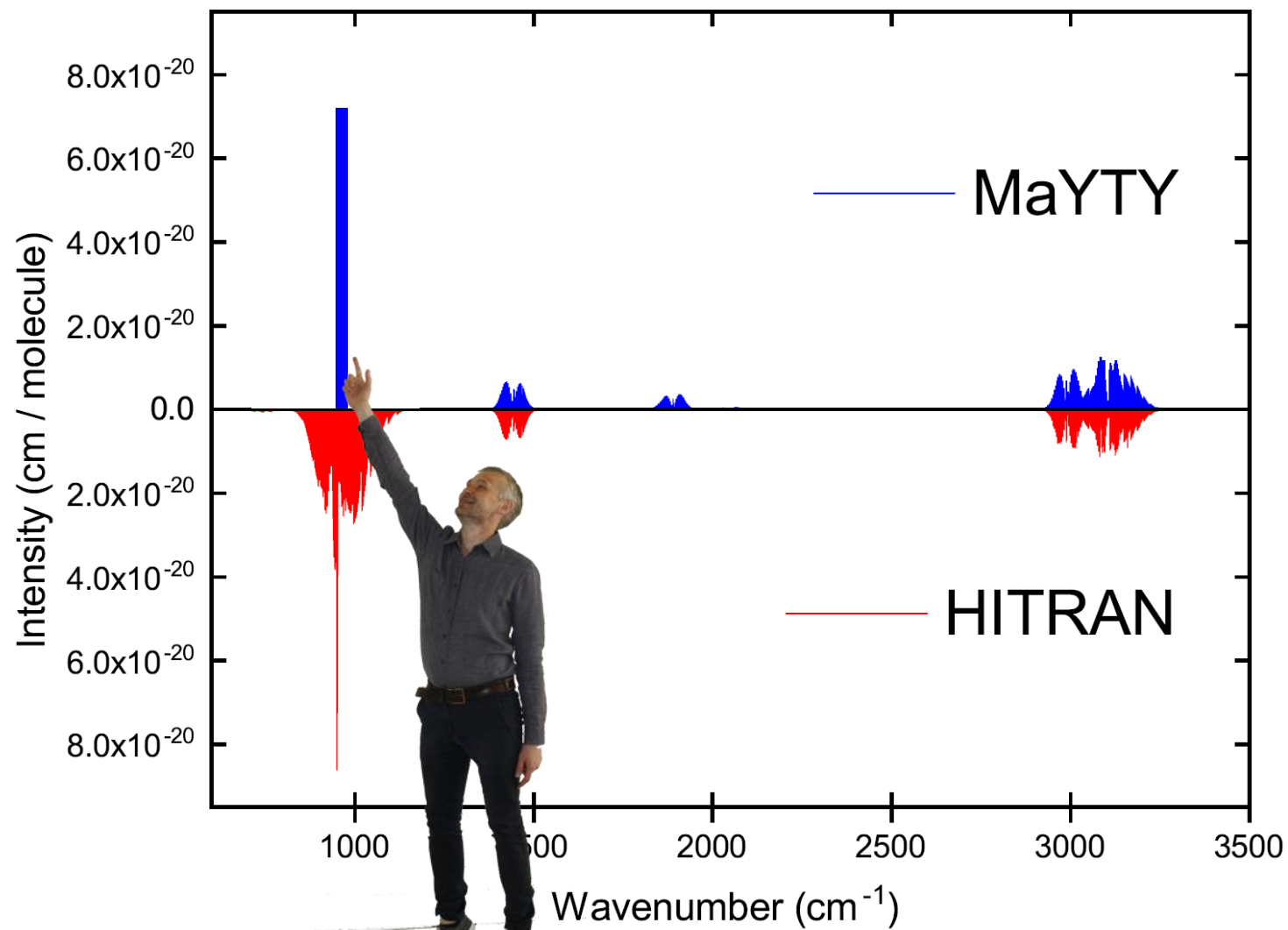
Low cost vibrational  
band intensities

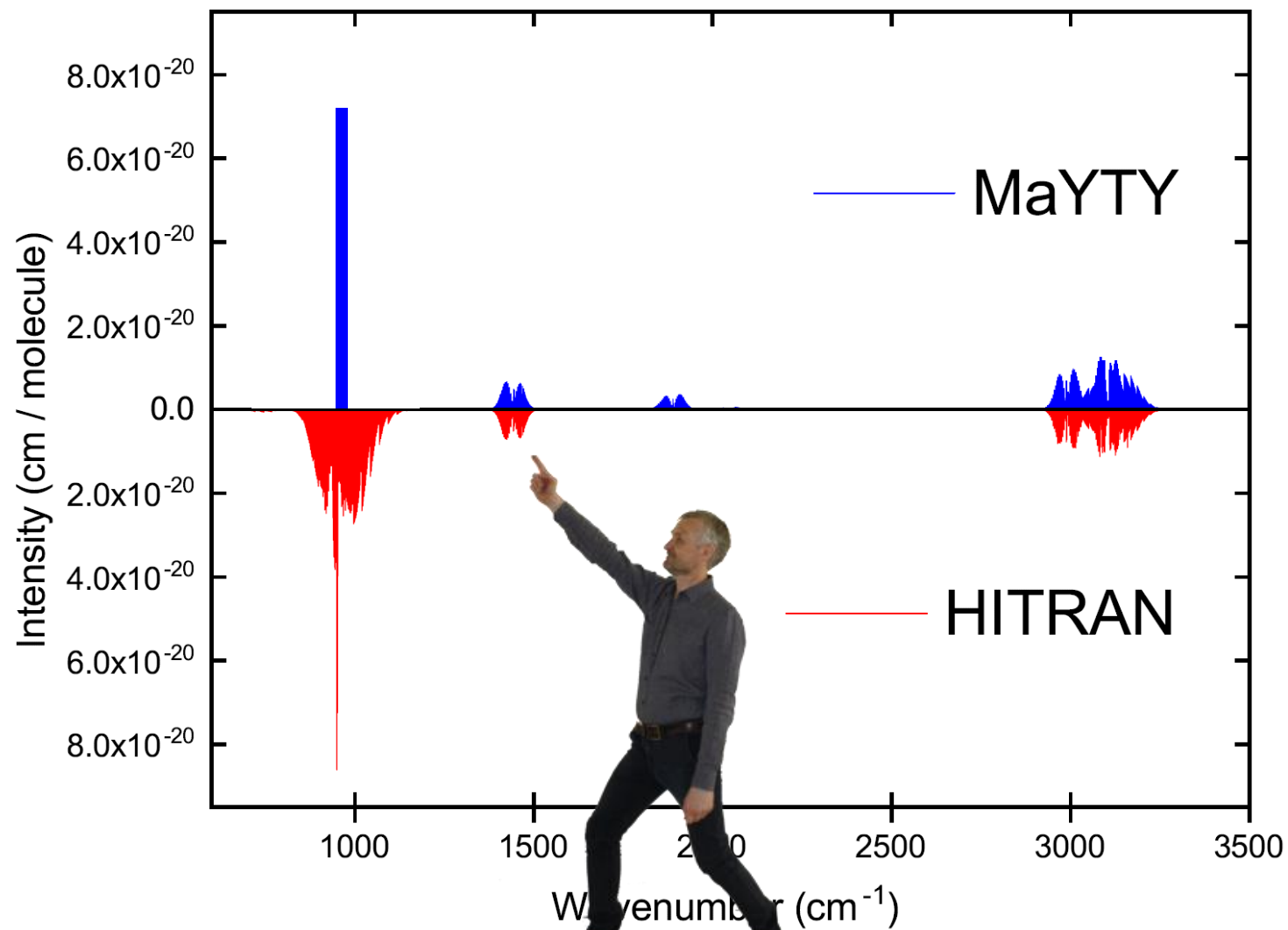


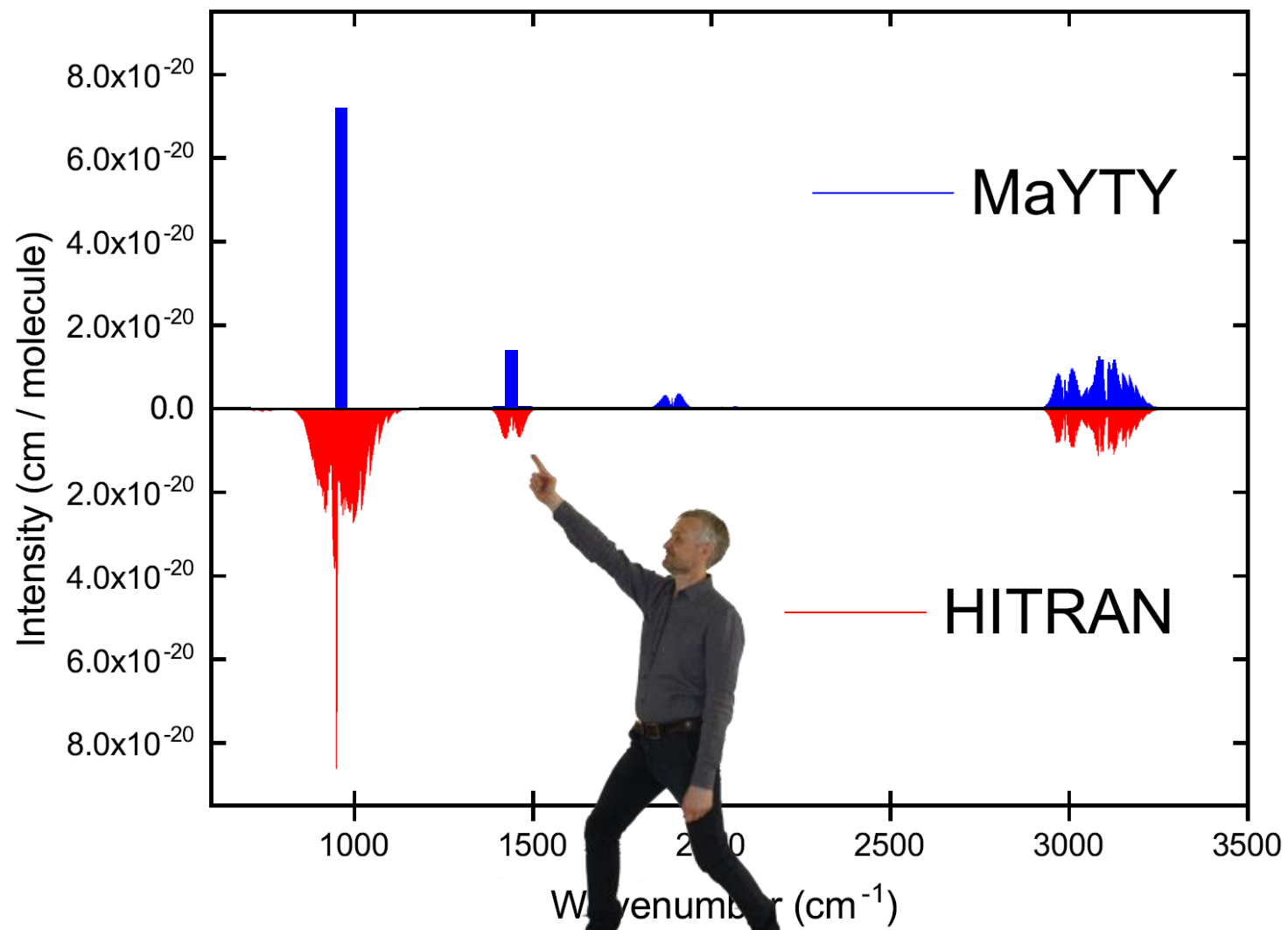


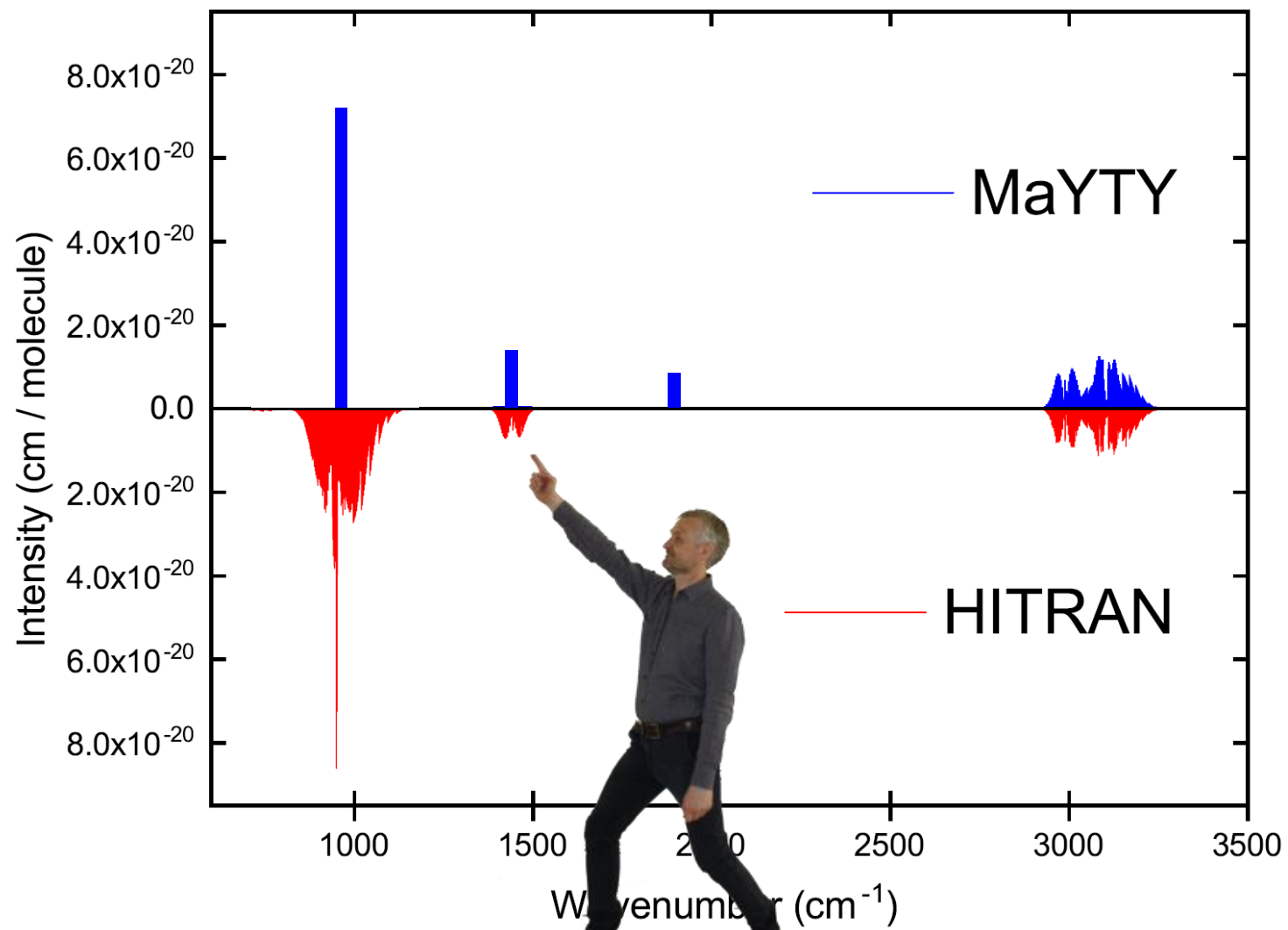


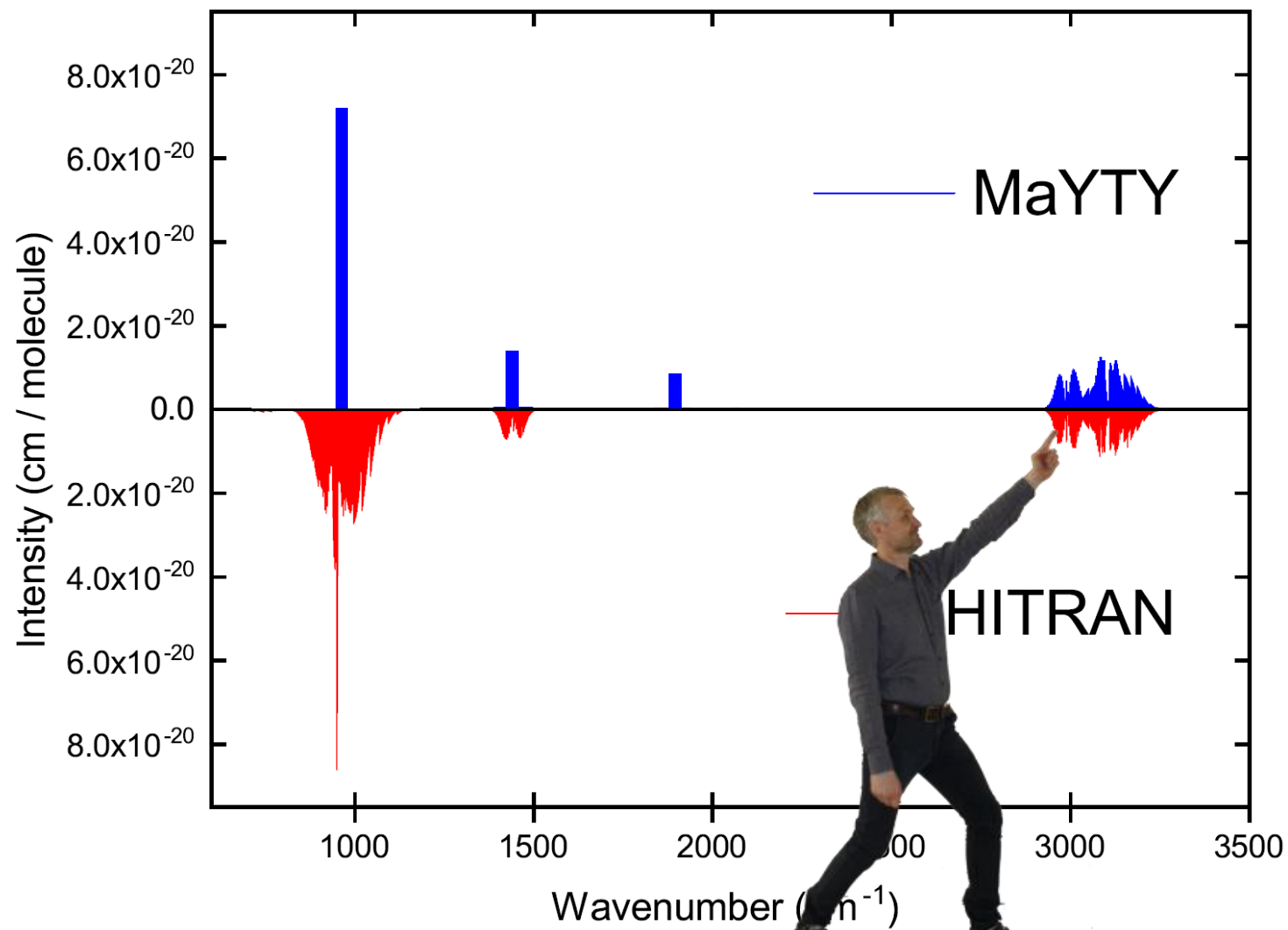


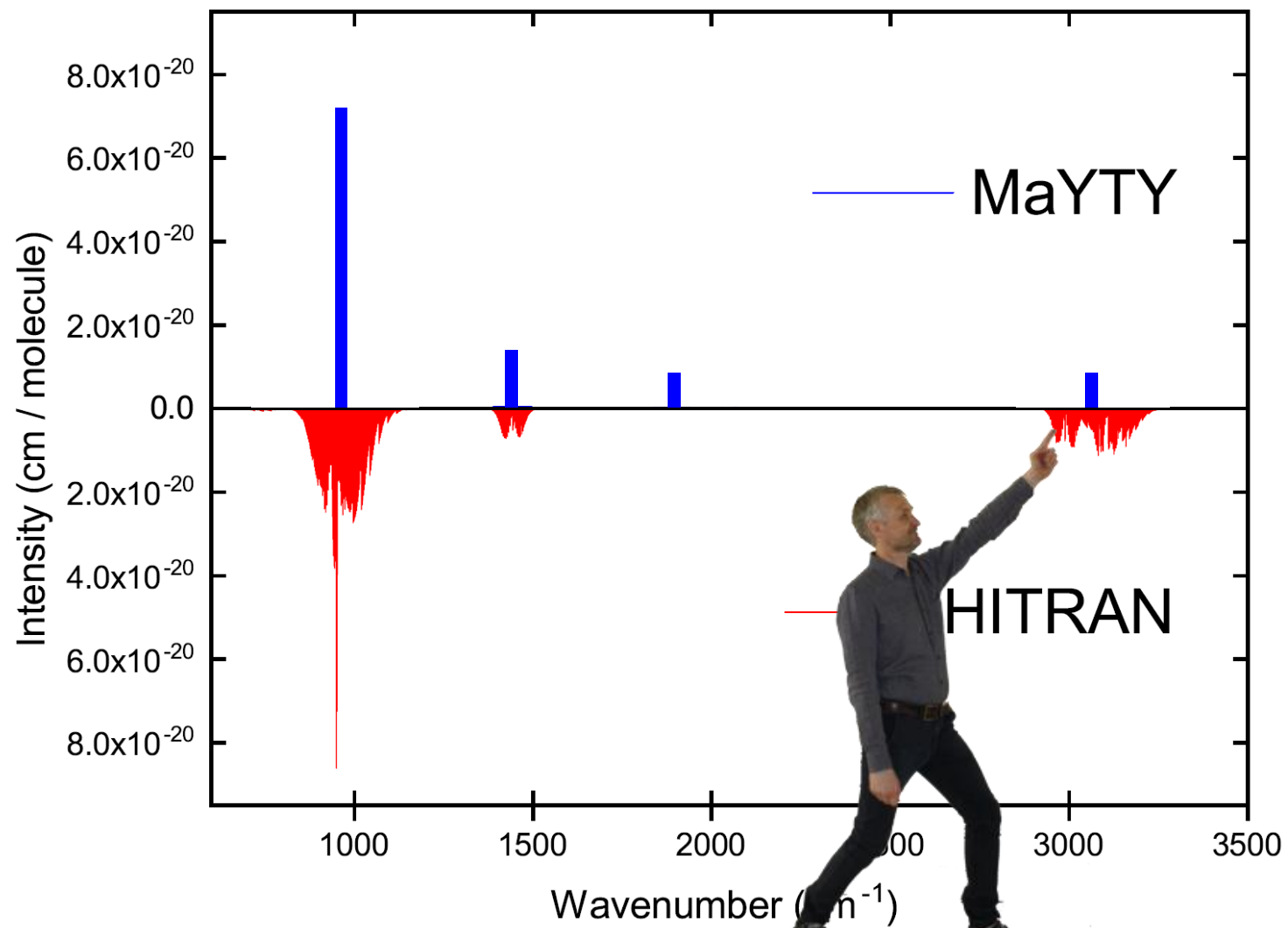




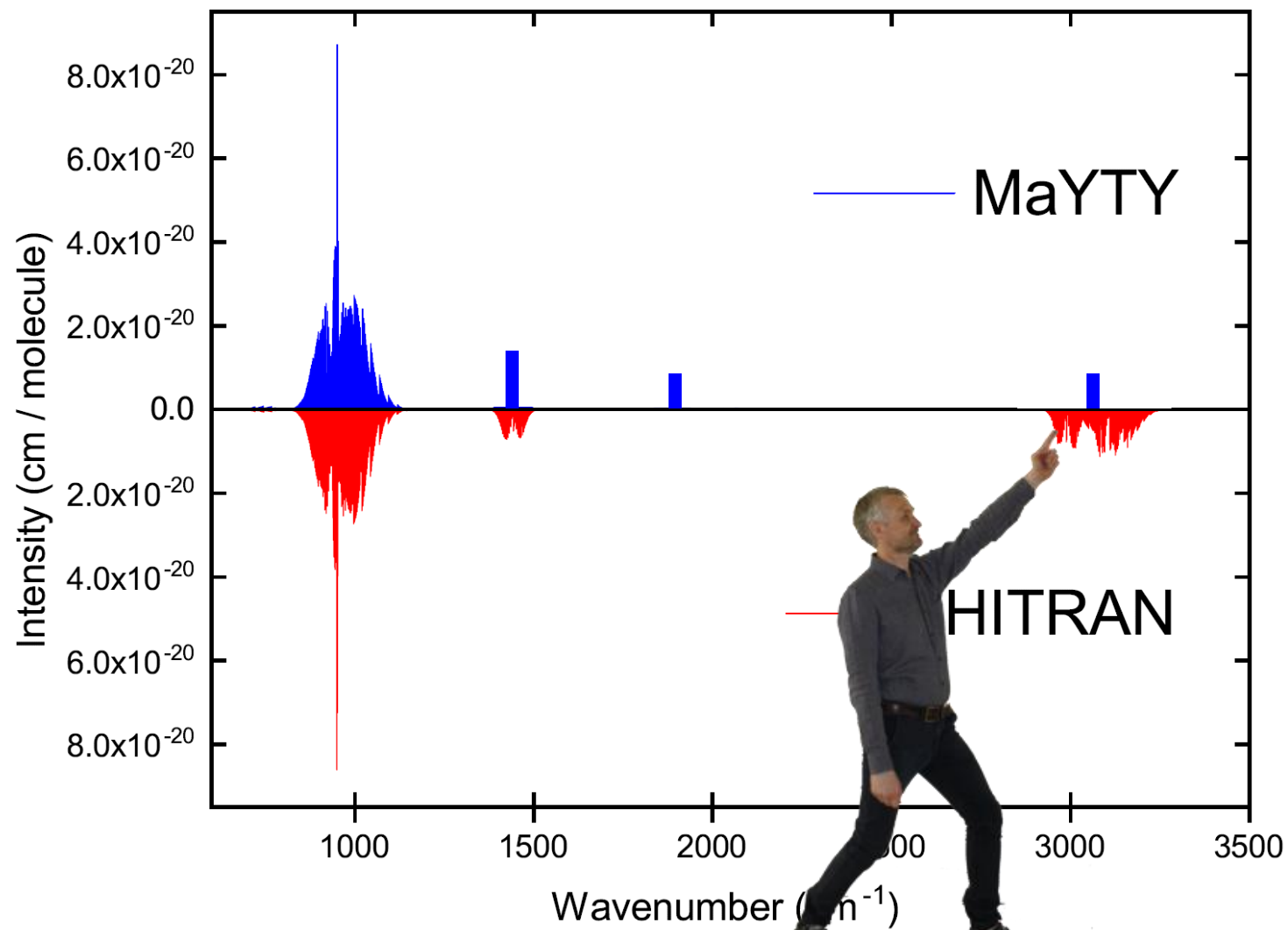


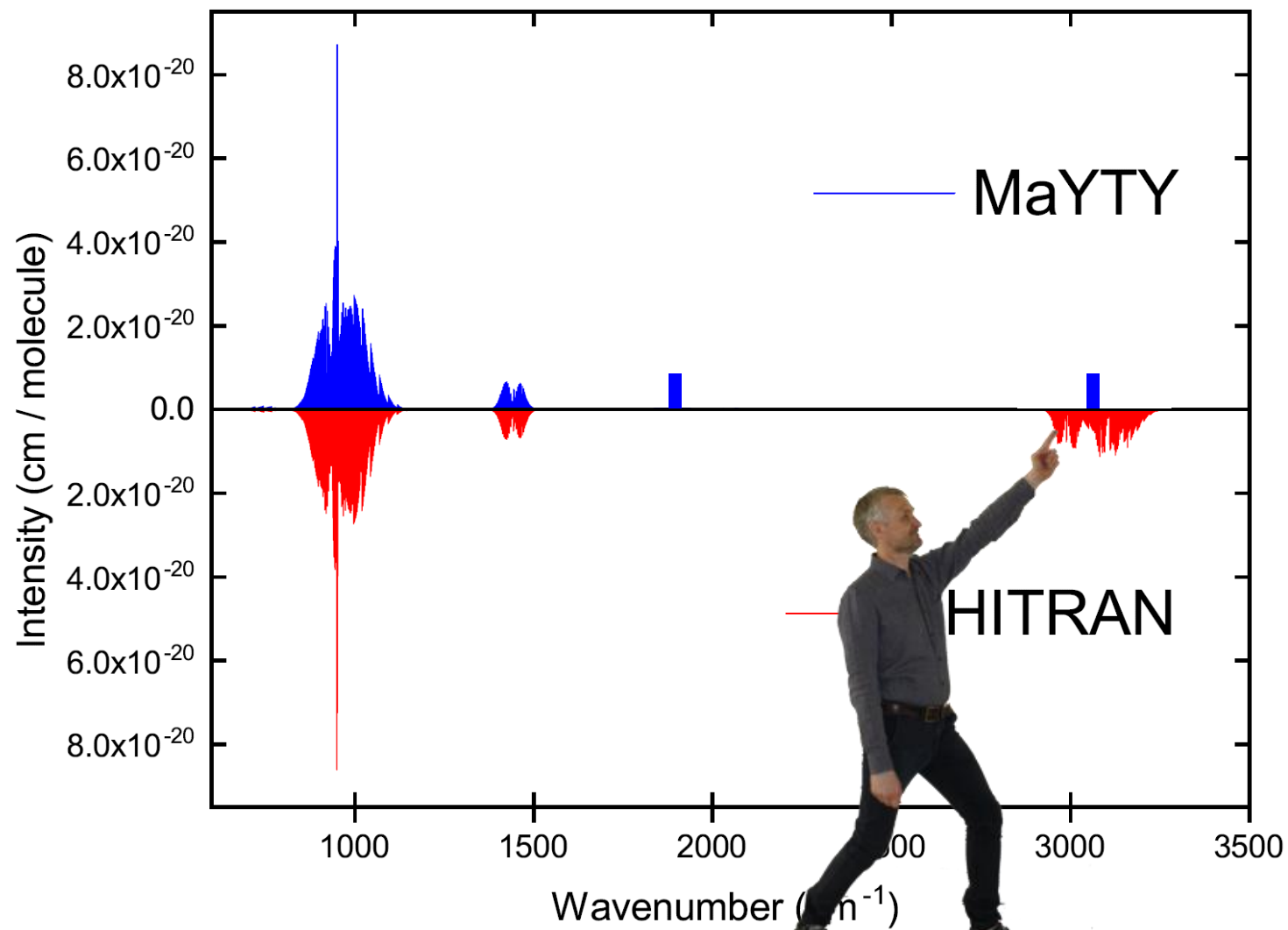


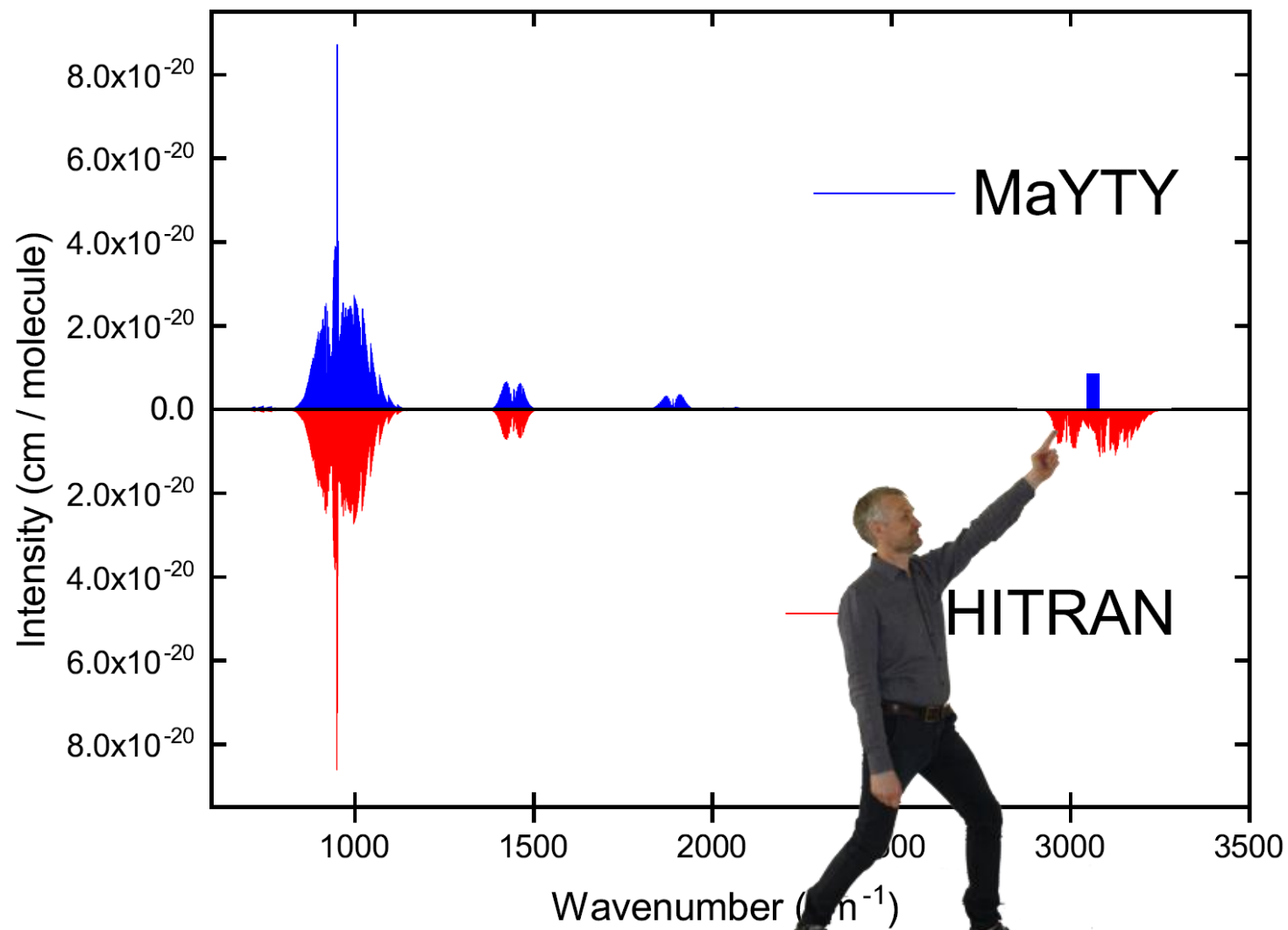


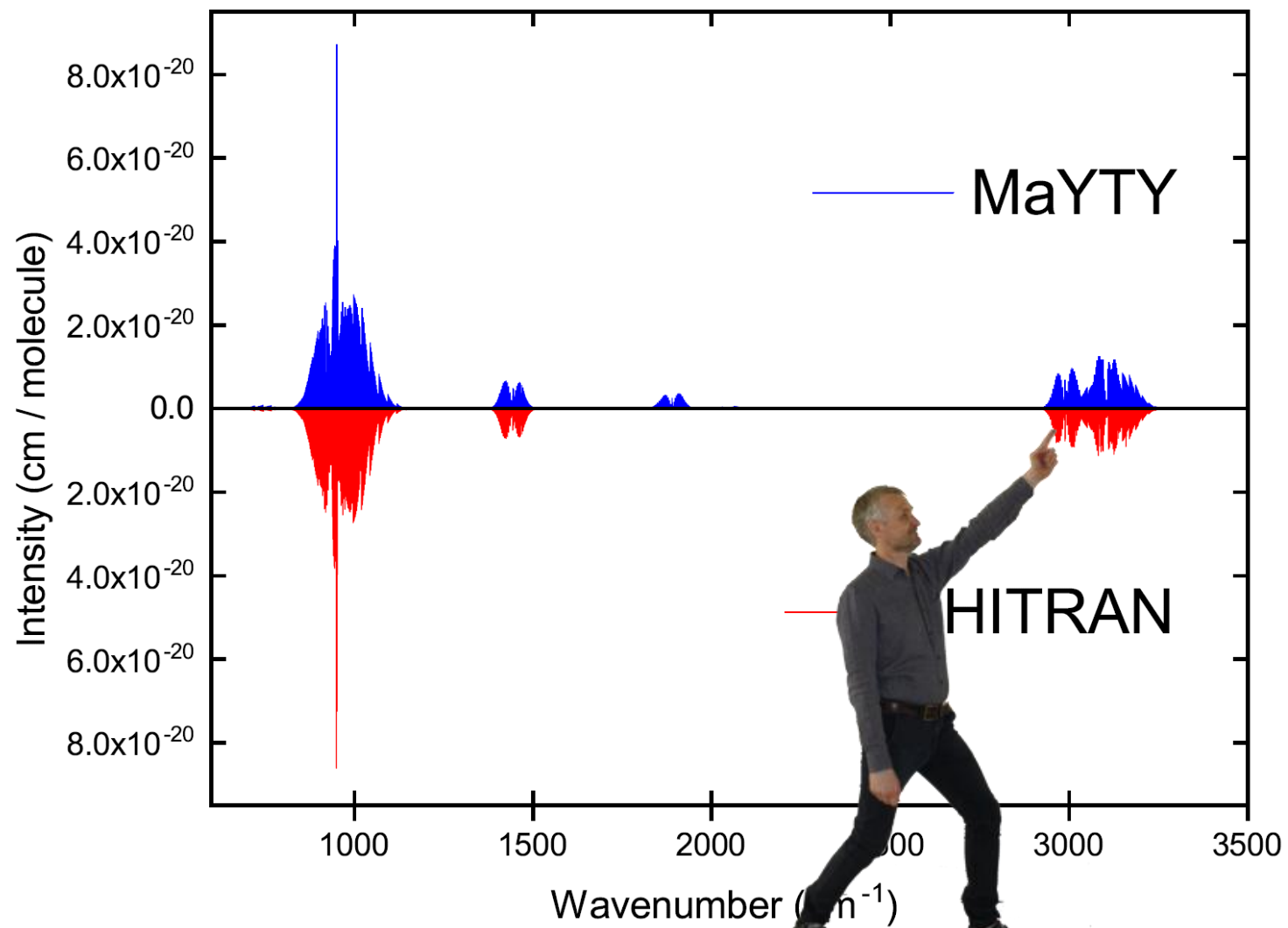


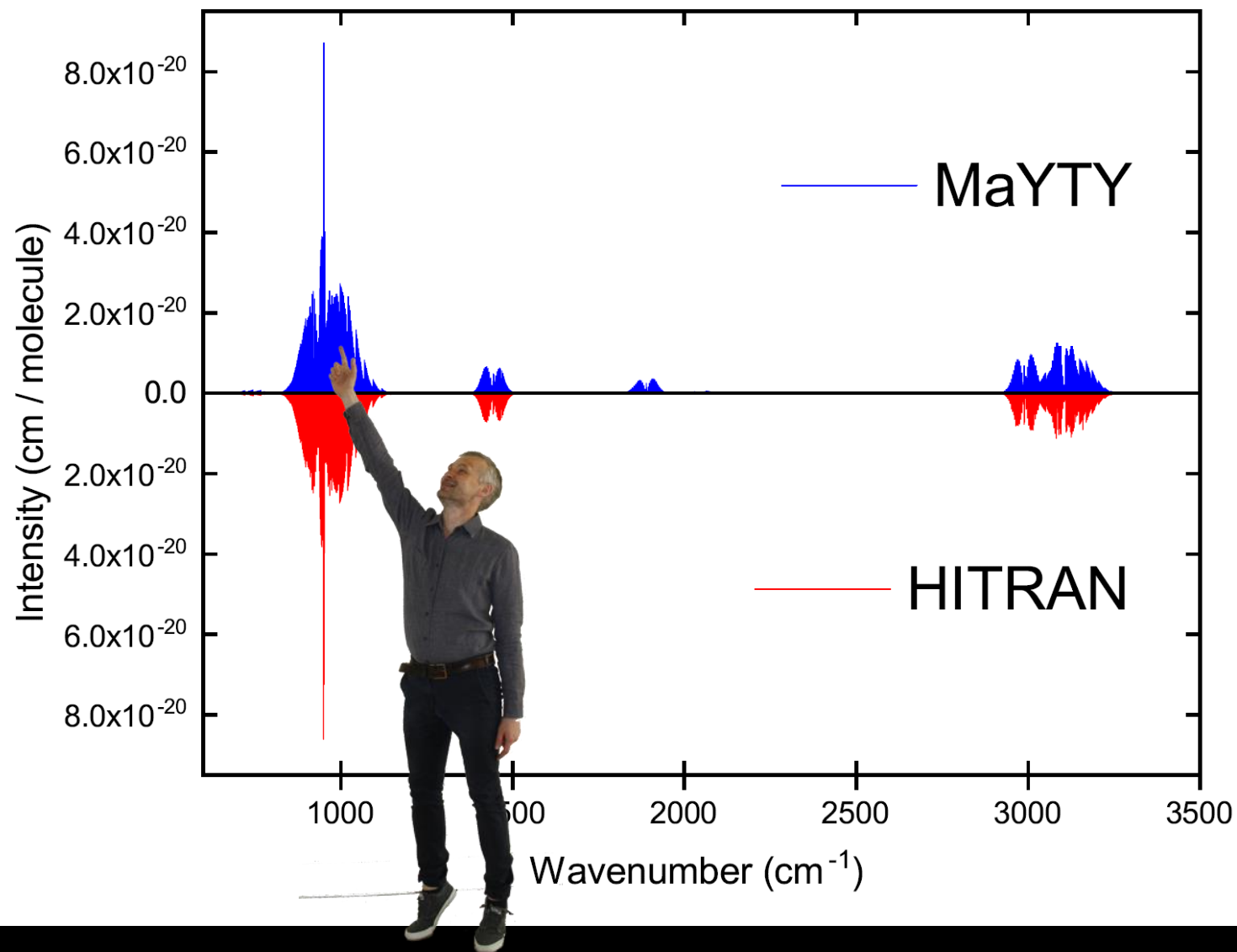


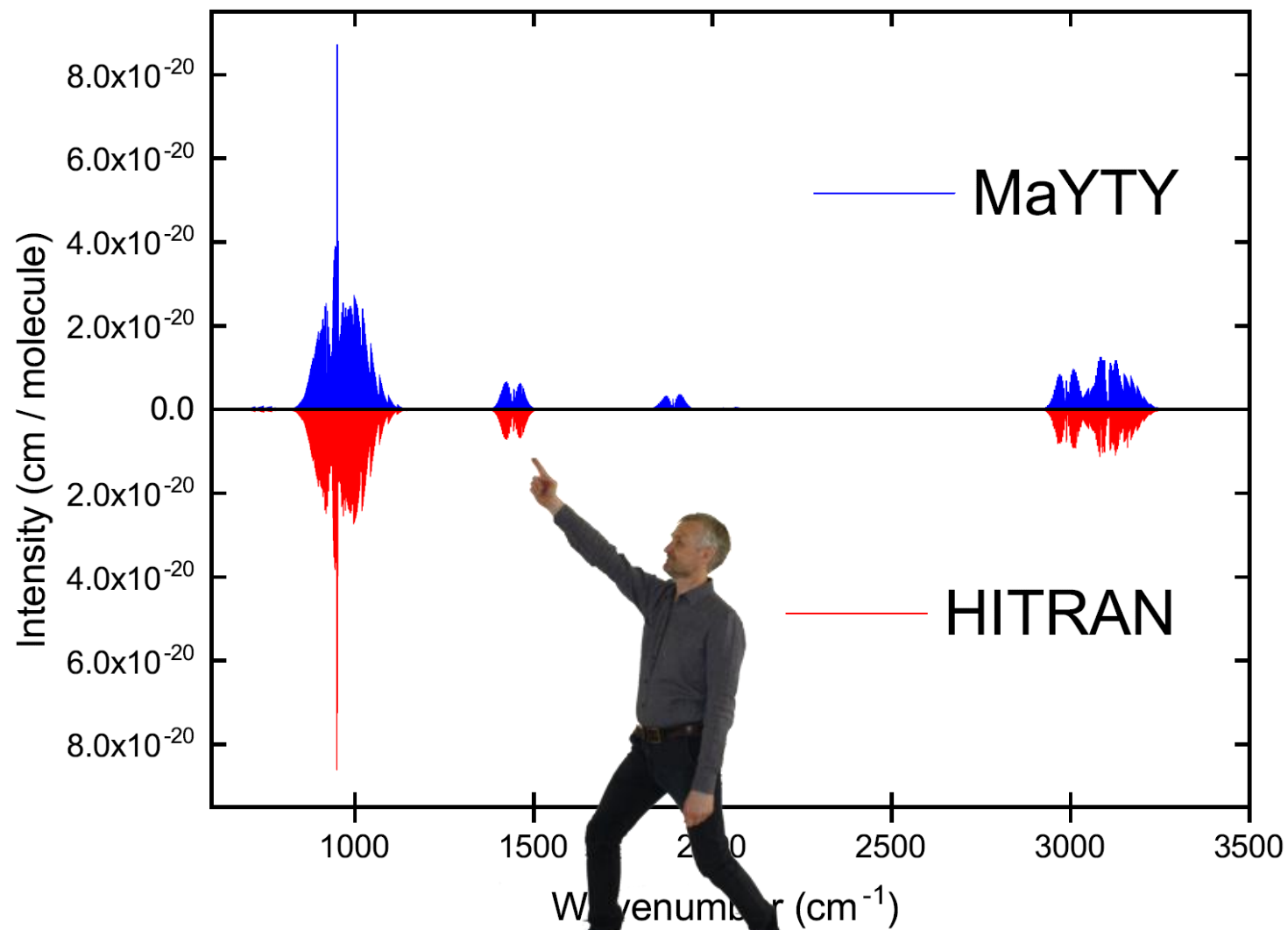


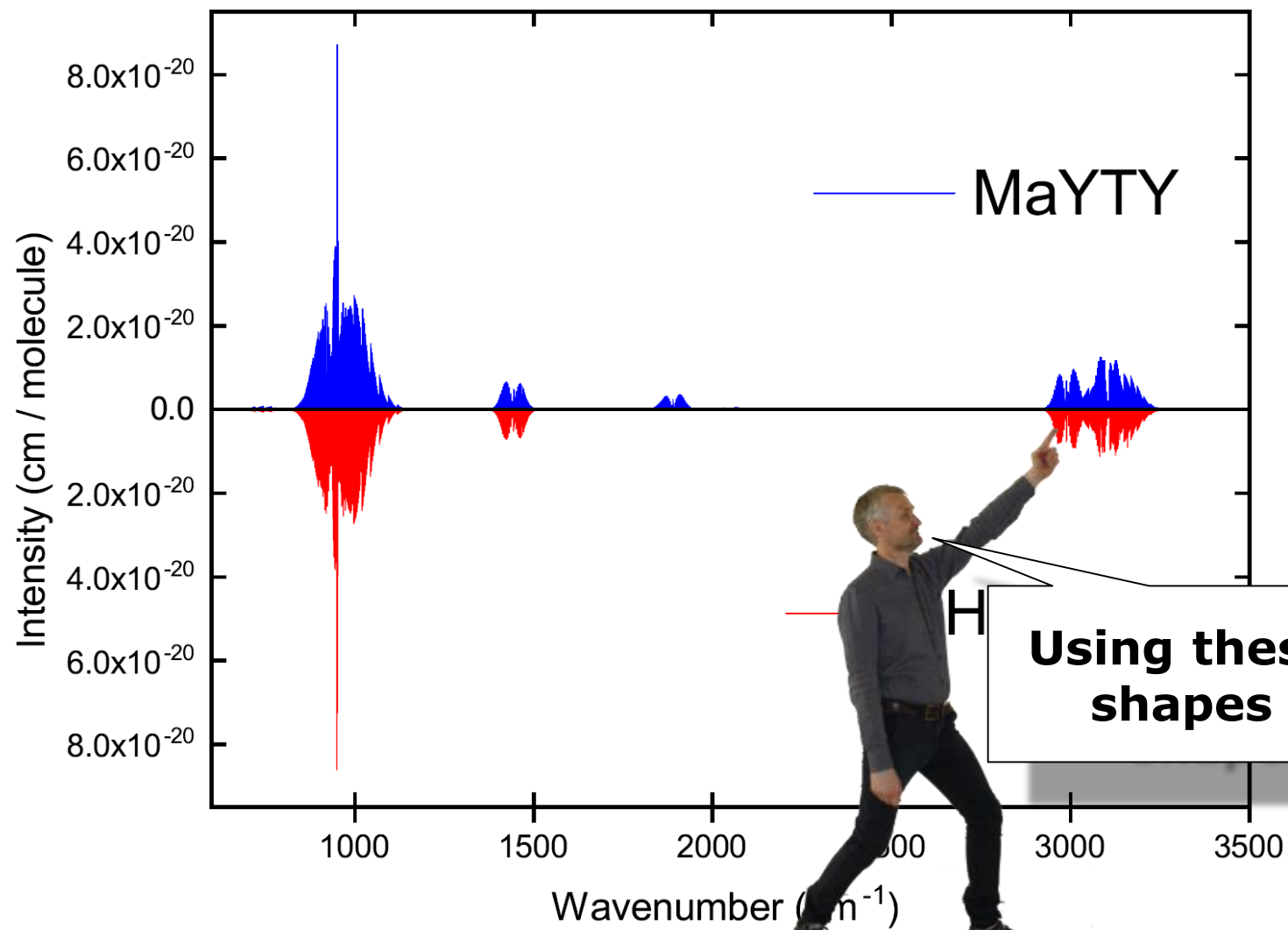


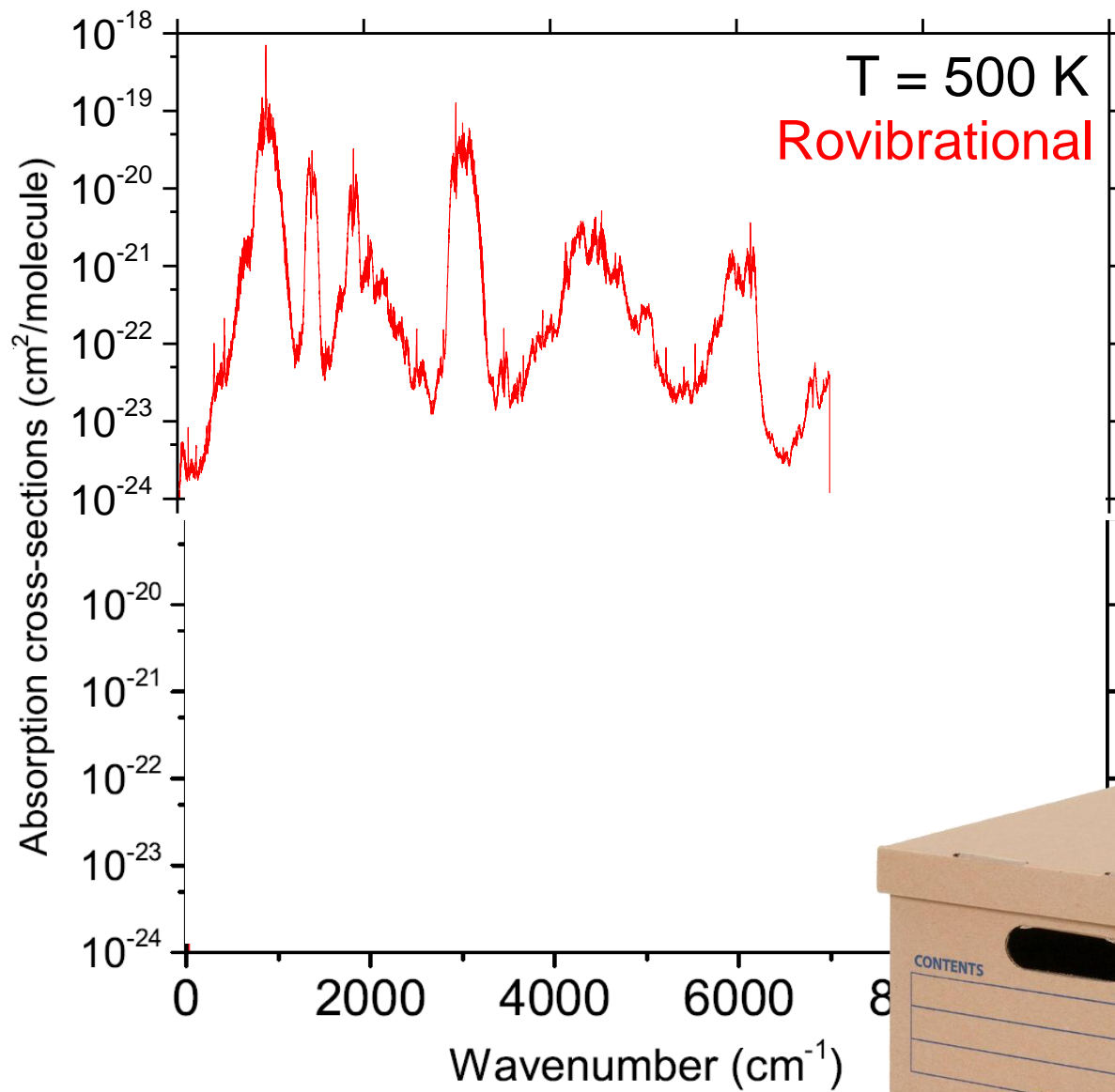




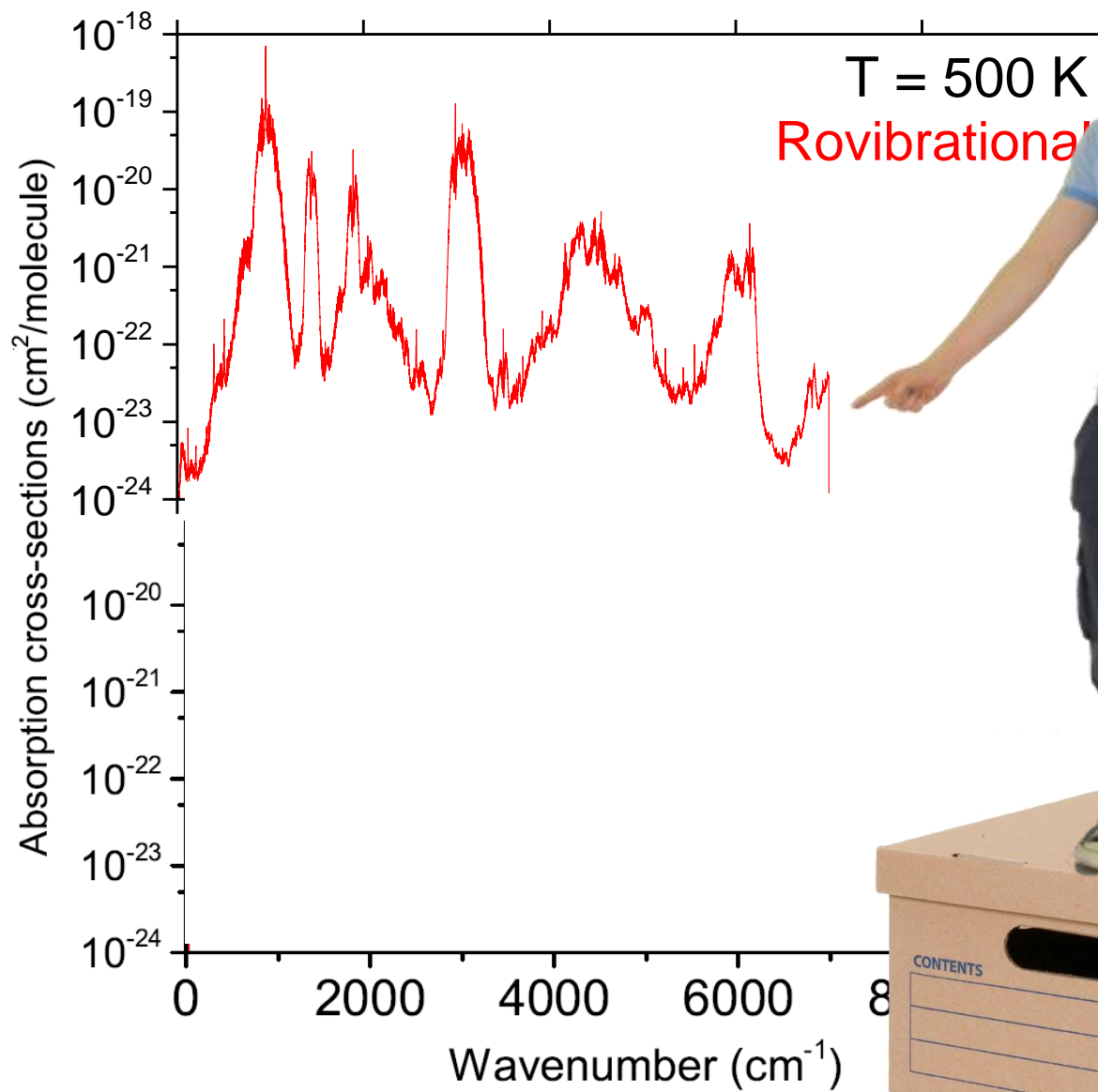


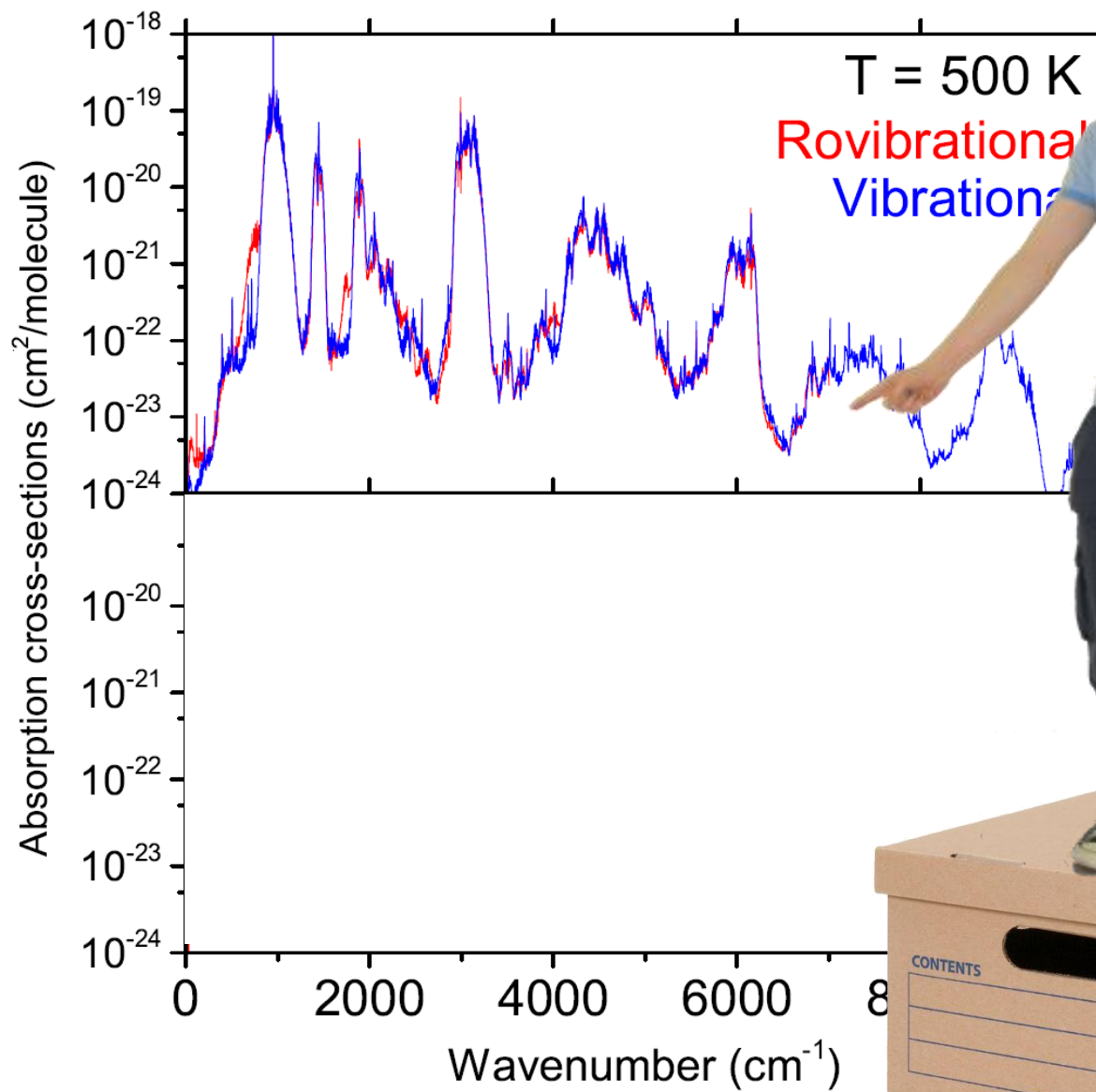


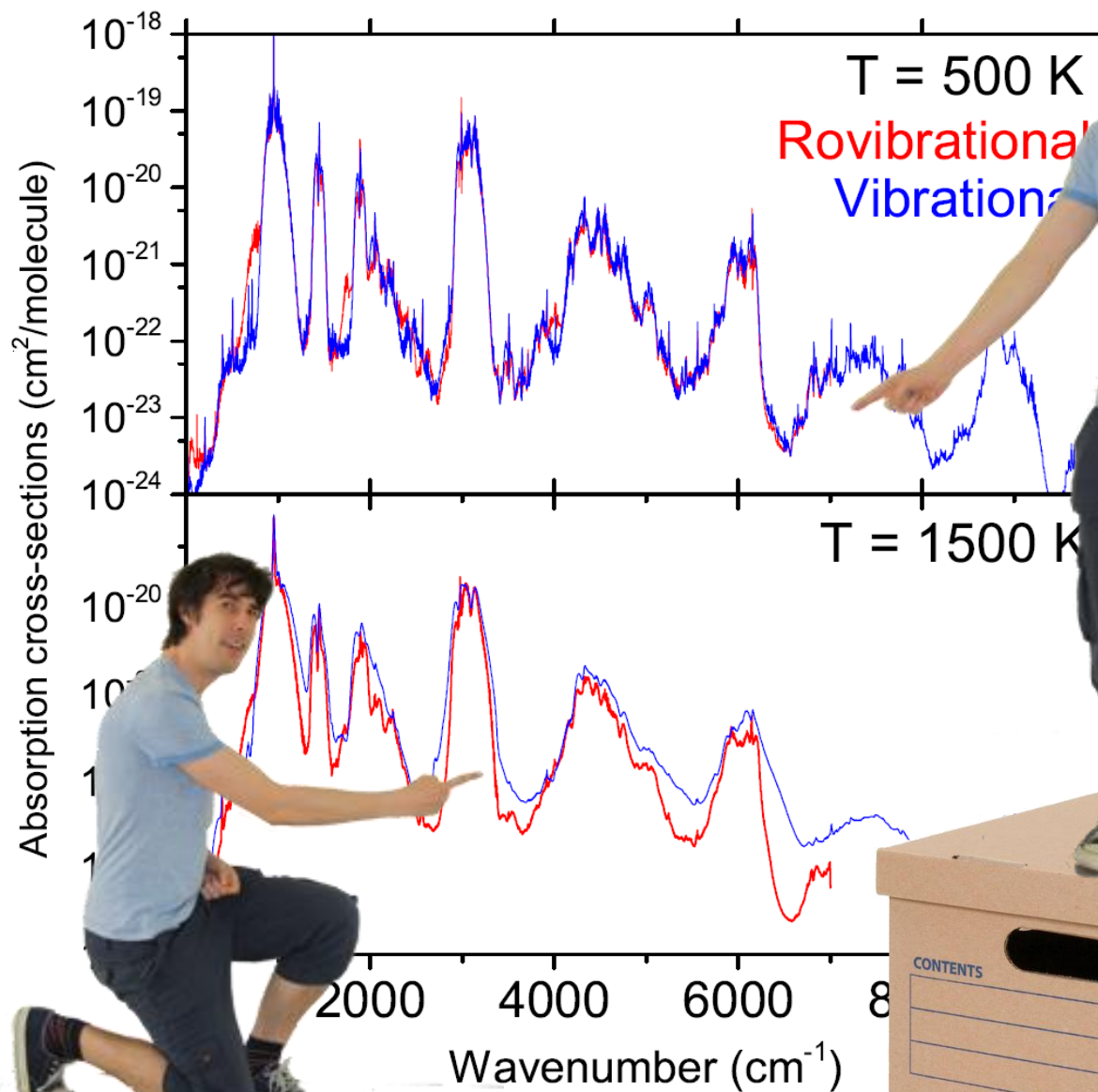


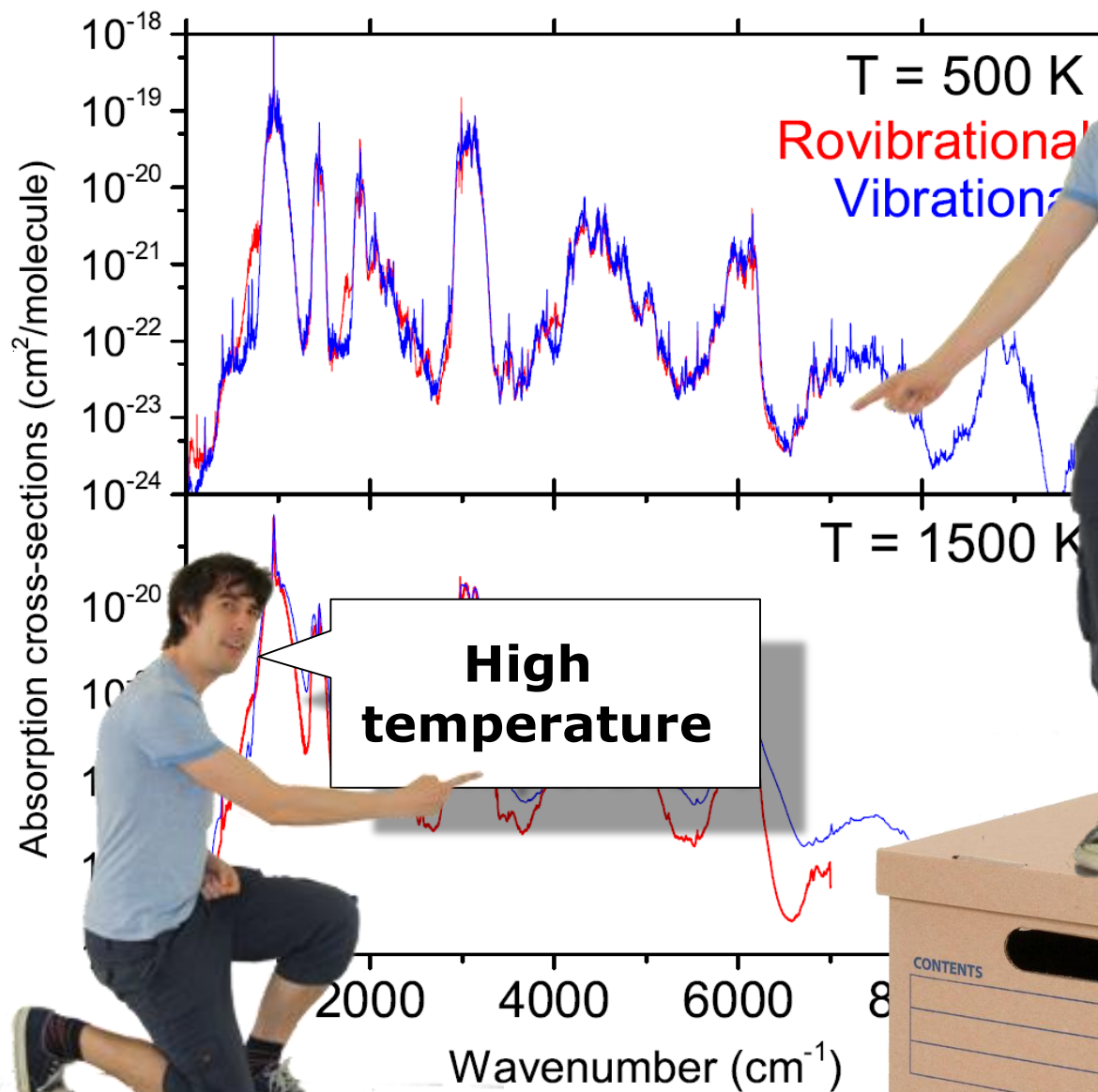






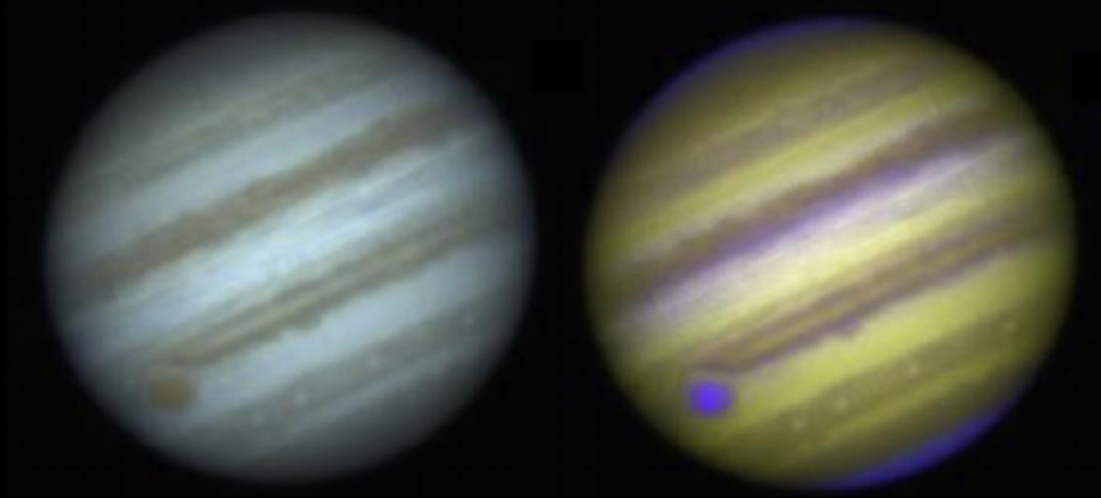




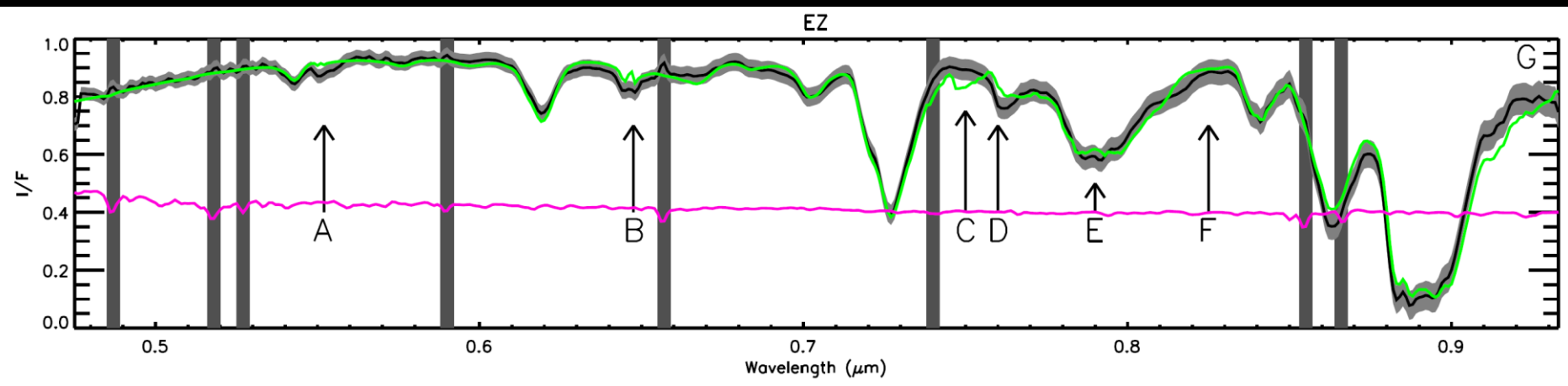


Ammonia

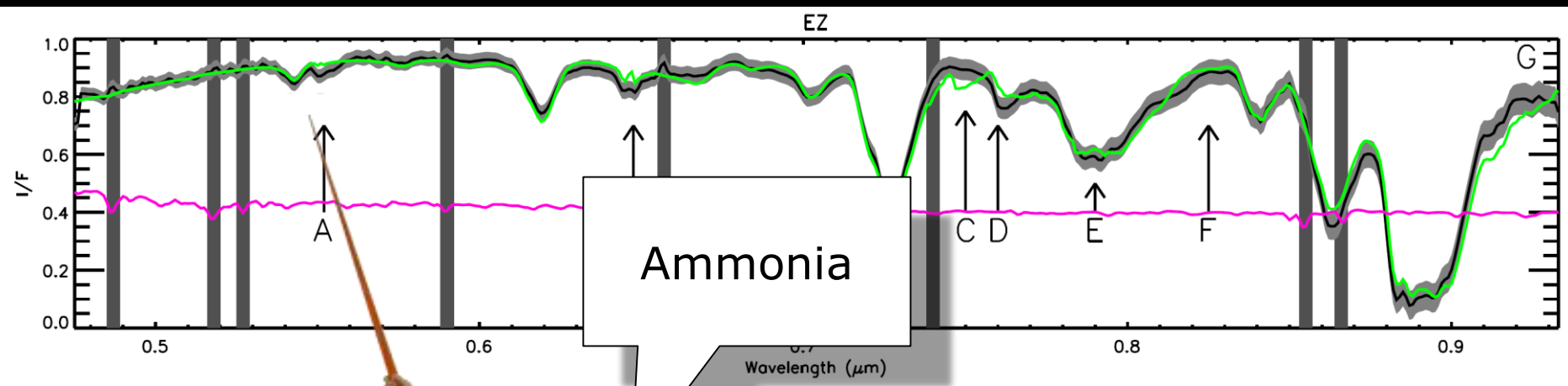
Very Large Telescope (VLT)  
Multi Unit Spectroscopic  
Explorer (MUSE) instrument in  
the spectral range  
 $0.48\text{--}0.93\ \mu\text{m}$

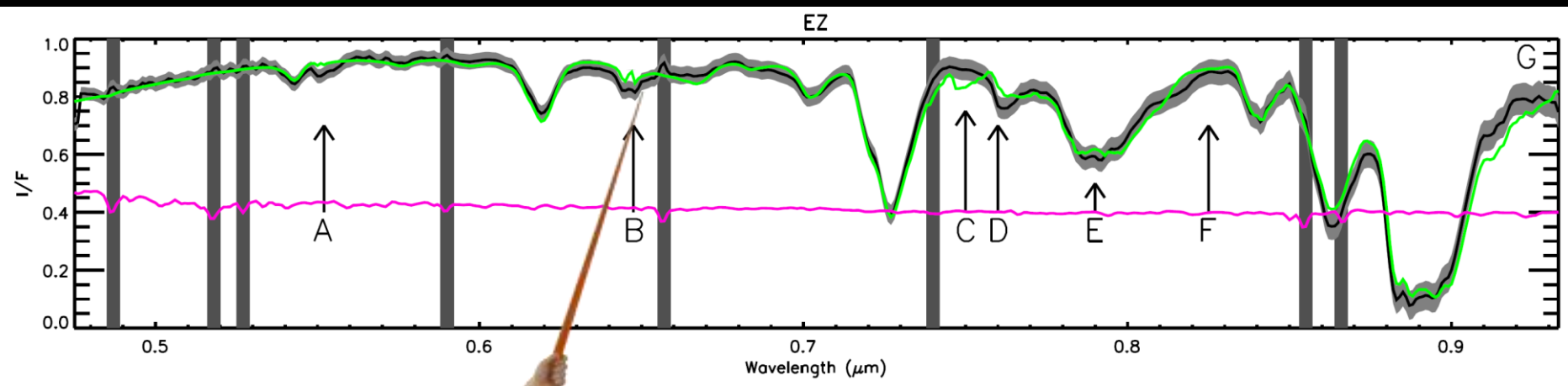


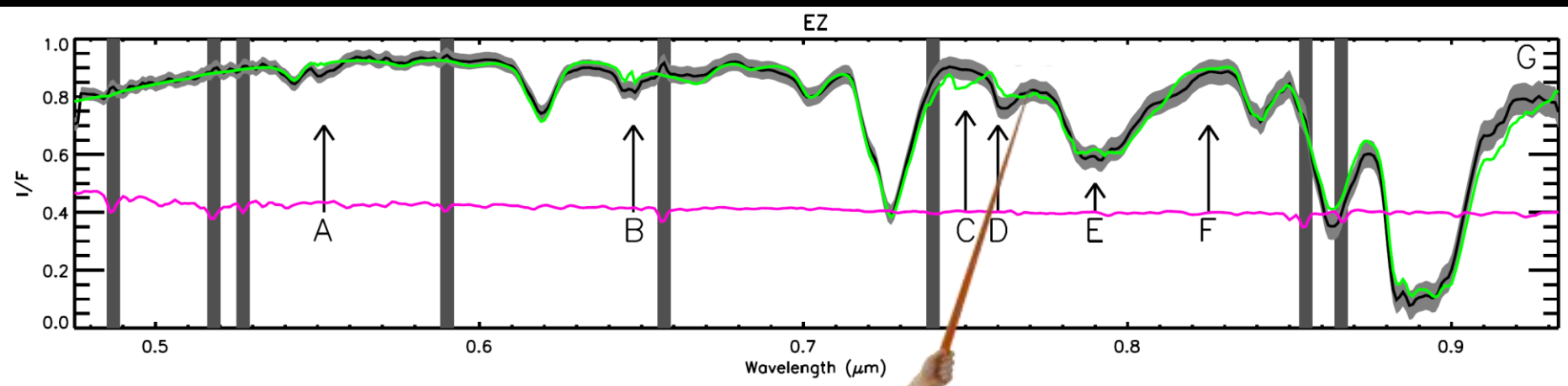
Visible spectrum of Jupiter:  
Ammonia bands

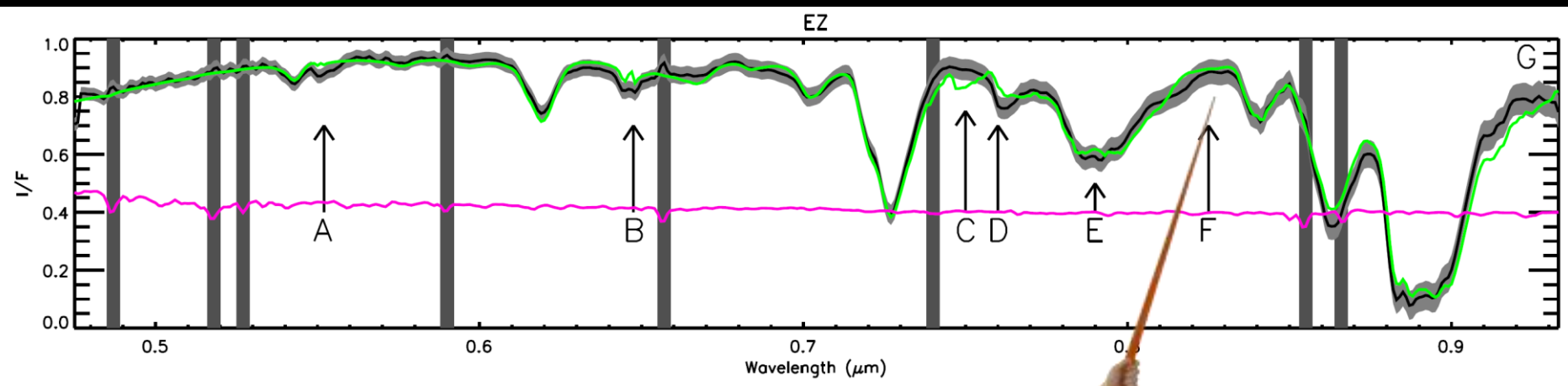


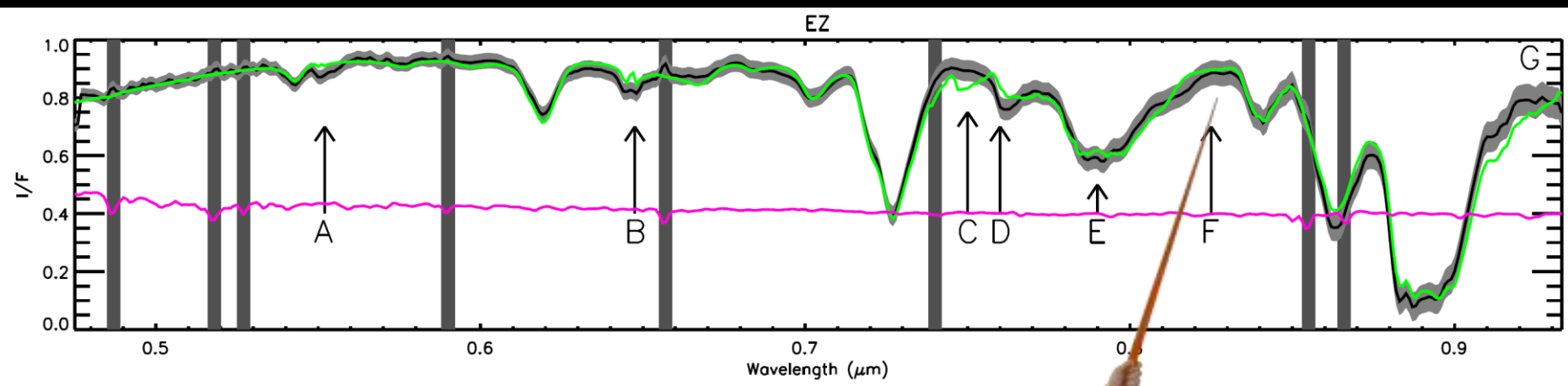










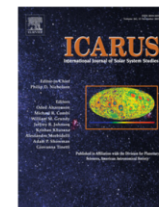


Icarus 302 (2018) 426–436

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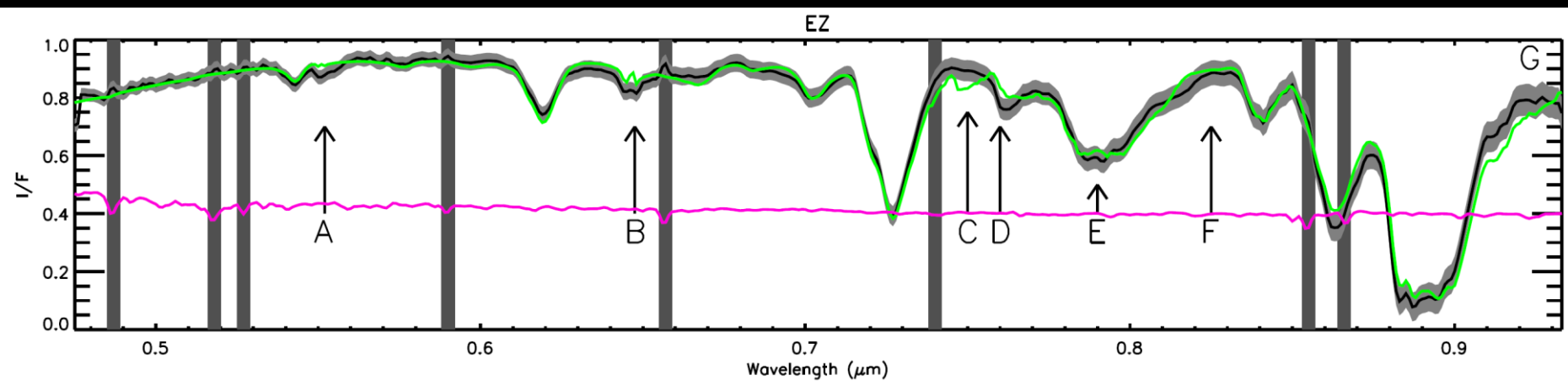


# Analysis of gaseous ammonia ( $\text{NH}_3$ ) absorption in the visible spectrum of Jupiter

Patrick G.J. Irwin\*, Neil Bowles, Ashwin S. Braude, Ryan Garland, Simon C.ott

Department of Physics, University of Oxford, Parks Rd, Oxford OX1 3PU, UK



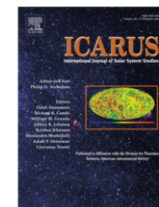


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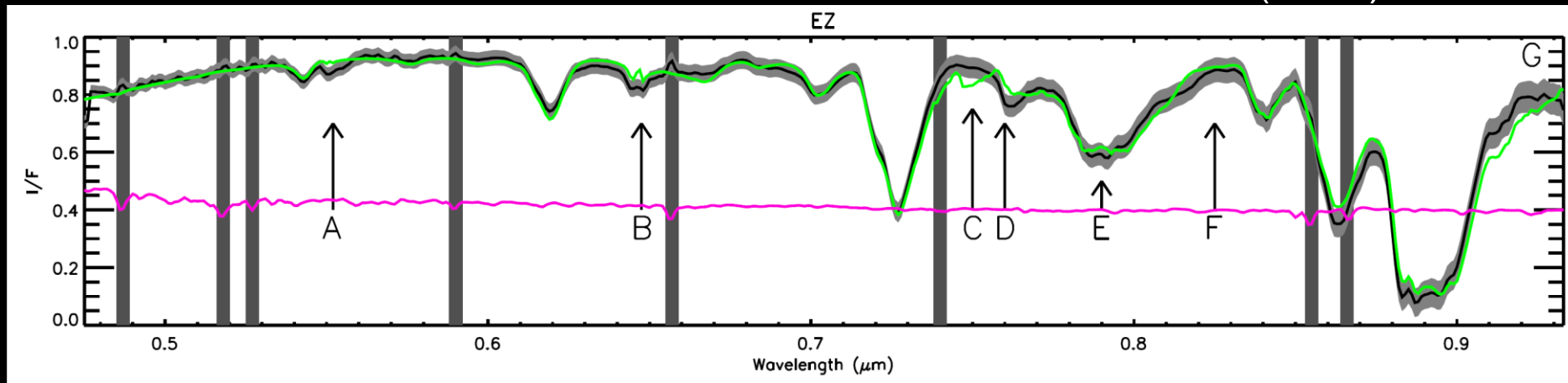


# Analysis of gaseous ammonia ( $\text{NH}_3$ ) absorption in the visible spectrum of Jupiter

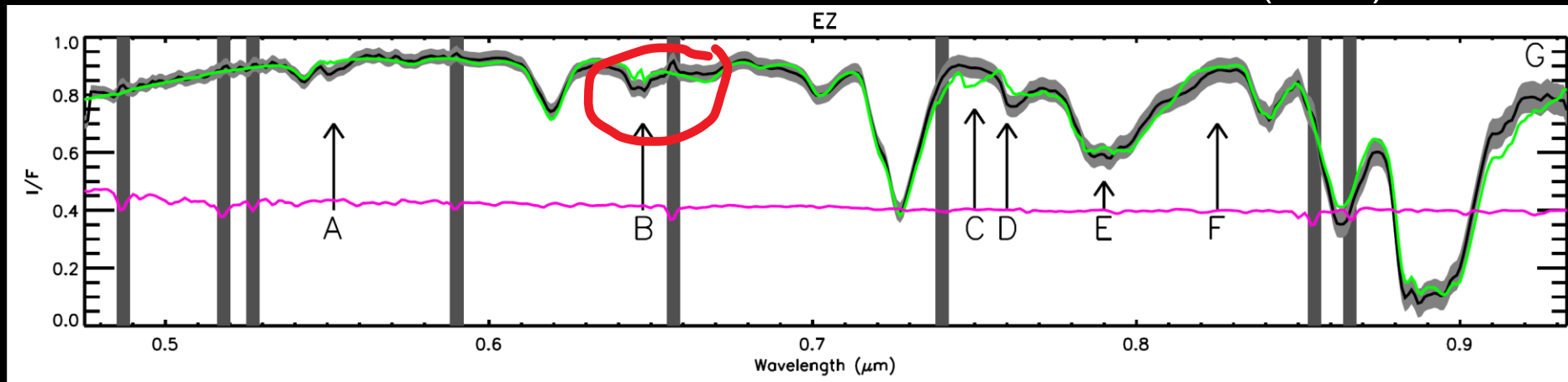
Patrick G.J. Irwin\*, Neil Bowles, Ashwin S. Braude, Ryan Garland, Simon J. Peacock

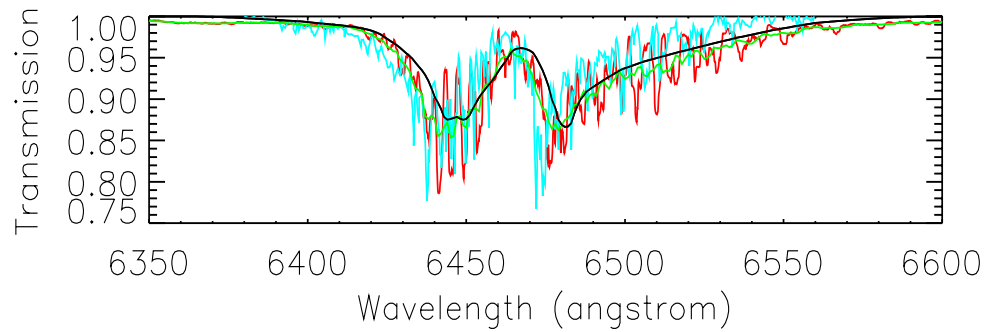
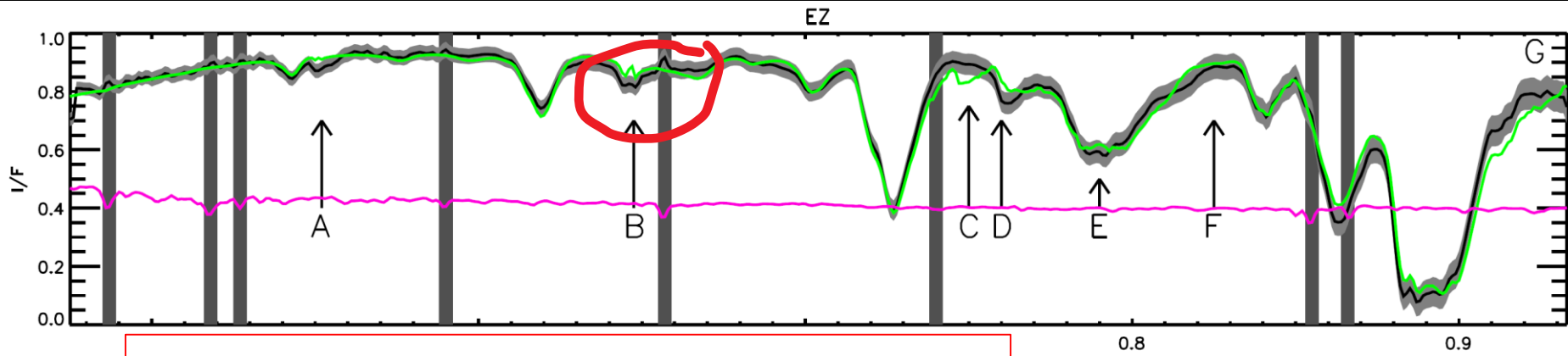
Department of Physics, University of Oxford, Parks Rd, Oxford OX1 3PU, UK

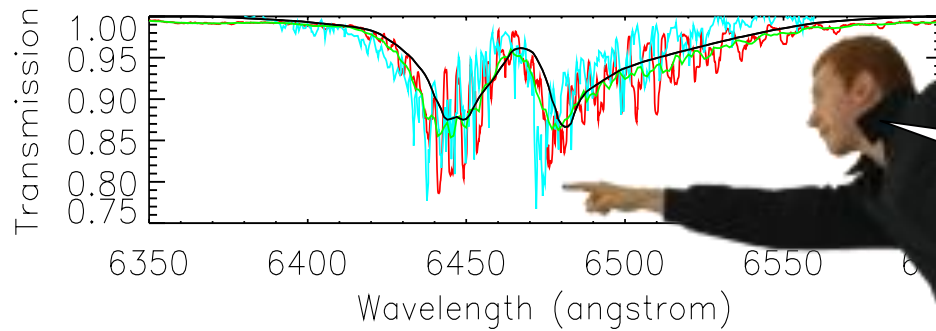
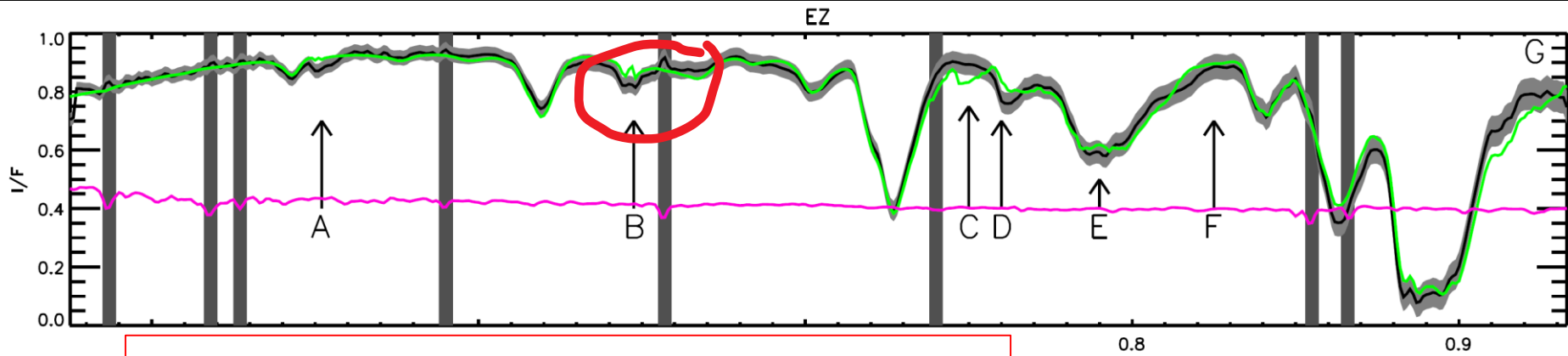
They tried our new empirical  
line list for  $\text{NH}_3$



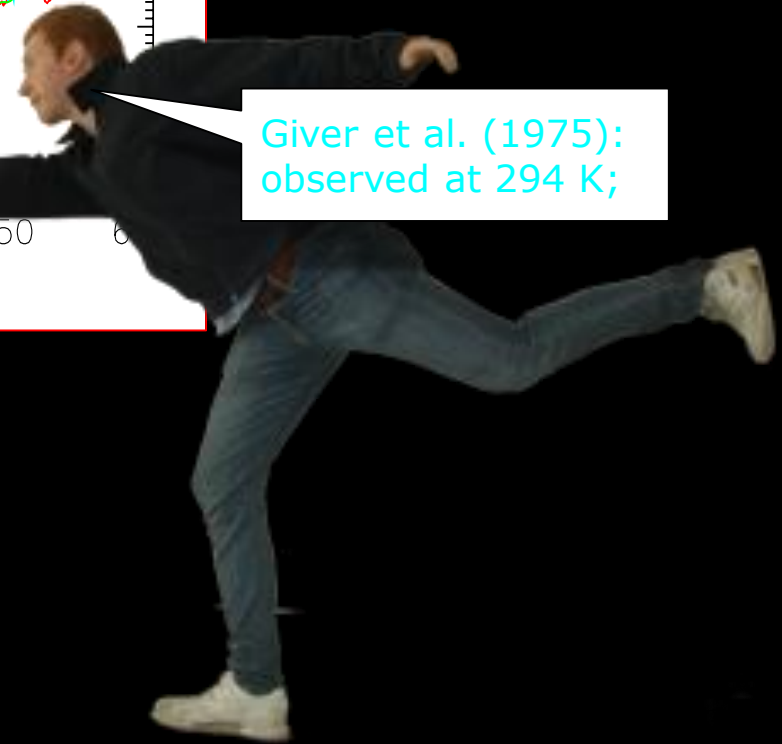


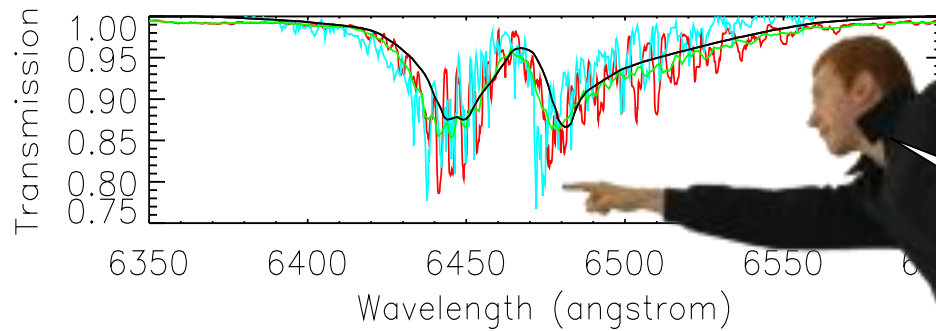
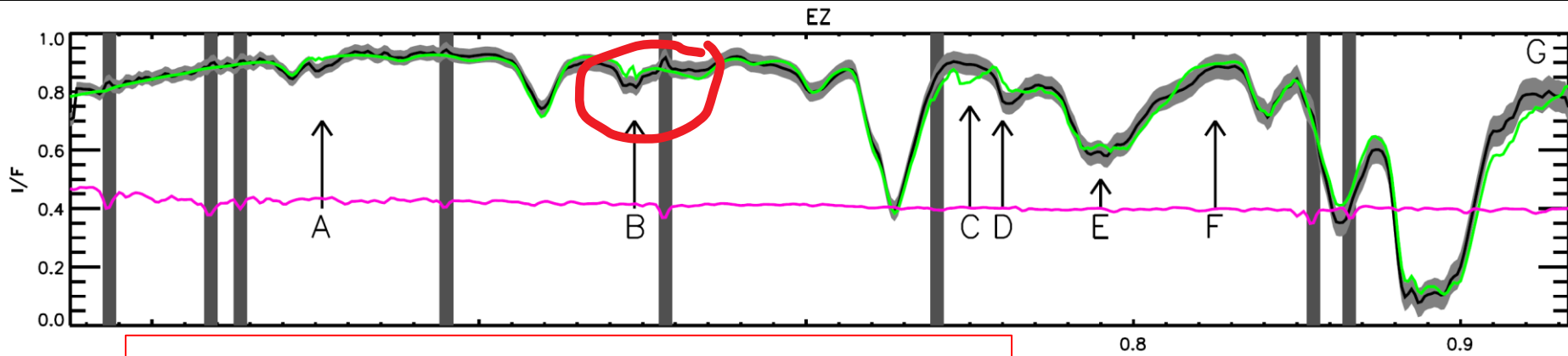




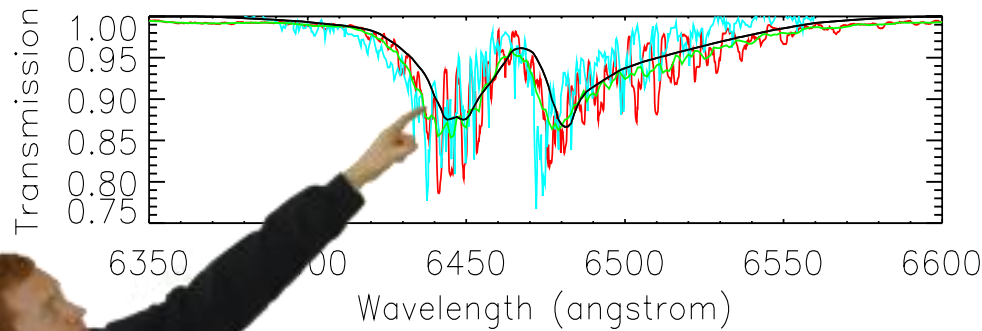
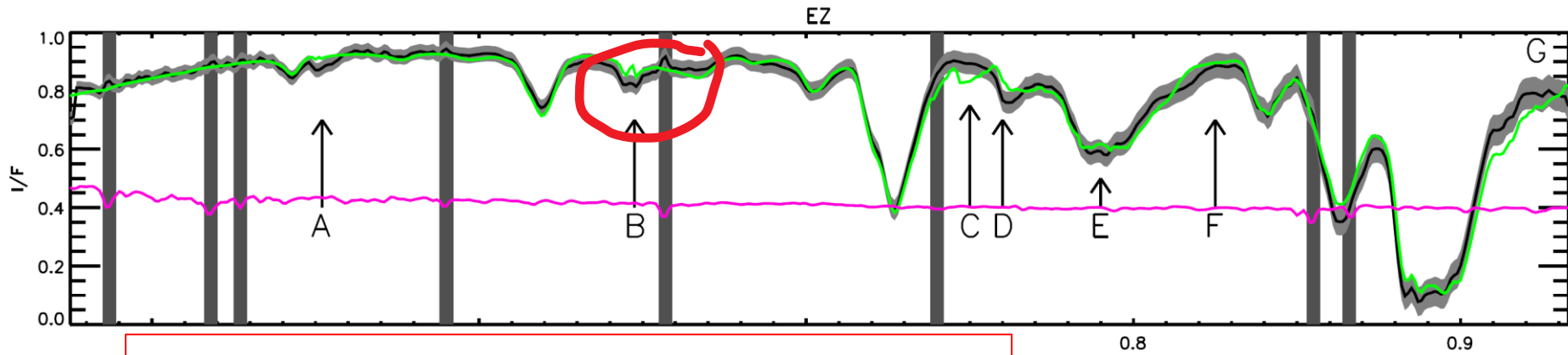


Giver et al. (1975):  
observed at 294 K;





Lutz and Owen (1980):  
low resolution  
absorption coefficients.



Our theoretical  
spectrum is  
represented by the  
green and red lines

Quasi-continuum

Methane line list contains 34  
billion transitions

*The End*

