STATIC AND FATIGUE STRENGTH IN SHEAR OF BEAMS WITH TENSILE REINFORCEMENT

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SYNOPSIS

Dimensional analysis is employed in deriving expressions for the initial diagonal cracking load and the ultimate strength in shear of simply supported reinforced concrete beams with tension reinforcement only. These expressions, which include size effect, are converted into nomographs for ease of application. This study of static strength includes results from tests of 105 beams, 42 of which were tested by the authors.

Fatigue tests were made on 39 reinforced concrete beams with tension reinforcement only. These beams were simply supported on a span of 60 in. and loaded at the third points. Statistical studies of the fatigue behavior with regard to initial diagonal cracking and final failure are included.

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INTRODUCTION

Since the derivation of the familiar expression $v = V/bjd$** by Mörsch (1)*** in 1903, investigations which have been made on reinforced concrete beams failing in shear were generally interpreted by expressing the ultimate nominal unit shearing stress, v_u or V_u/bjd, as a function of certain variables. In 1909, Talbot (2) indicated that the ultimate nominal unit shearing stress depends on many variables, such as the span-depth ratio, the concrete strength, the percentage of tensile reinforcement, and the amount of web reinforcement. Consideration of these variables, with the exception of concrete strength and web reinforcement, have generally been neglected by most subsequent investigators when determining empirical expressions of the ultimate nominal unit shearing stress. Some earlier interpretations even neglected the effect due to the variation of concrete strength. Tests have been made to determine the limiting nominal unit shearing stress, using the expression:

$$v_u = V_u/bjd = Krf'_{yp} + cf'_c, \text{ or}$$

$$v_u = cf'_c \text{ for beams without web reinforcement} \qquad (a)$$

and results thus obtained were varied and inconsistent.

In 1945 Moretto (3) attempted to include other variables which might have an influence on the ultimate nominal unit shearing stress of a reinforced concrete beam failing in shear, and from his data developed the following empirical formula:

**Notations appearing in this paper are summarized in Appendix I.
***Numbers in parentheses refer to the list of references appended to this paper.

$$v_u = V_u/bjd = Krf'_{yp} + 0.10f'_c + 5000p, \text{ or}$$
$$v_u = 0.10f'_c + 5000p \text{ (without web reinforcement)} \quad \text{(b)}$$

A new variable, steel percentage, was included and the expression fitted his experimental data fairly well.

In 1951 Clark (4) fitted the following expression to his experimental data:

$$v_u = V_u/bjd = 2500\sqrt{r} + 0.12f'_c\,(d/a') + 7000p, \text{ or}$$
$$v_u = 0.12f'_c\,(d/a') + 7000p \text{ (without web reinforcement).} \quad \text{(c)}$$

Another variable, the depth-span ratio, has been added to this formula. Although both Moretto's and Clark's formulas fit their own experimental data fairly well, the formulas did not give comparable results.

In April, 1953, Laupa, Siess, and Newmark (5) considered the actual behavior of a reinforced concrete beam without web reinforcement under combined flexure and shear, and found that generally the beam cracked diagonally, and then failed by the destruction of concrete in the compression zone*. They therefore assumed that the final failure of a reinforced concrete beam without web reinforcement due to combined flexure and shear is essentially the same as that due to flexure, the only difference being that the diagonal crack concentrates the angle changes that would normally be distributed along a considerable length of the beam to a relatively short section located at the end of the diagonal crack.

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*Actually the part of beam where final failure occurs is subjected to combined compression and shear; but if the span-depth ratio is large enough, the total force is approximately the same as the compressive force.

Therefore, the maximum moment (shear-moment) the beam can carry with a diagonal crack will be much smaller than in the case for pure flexure. Using this shear-compression theory, an empirical equation expressing the shear-moment for beams without web reinforcement was developed:

$$M_s = kbd^2f'_c \,(0.57 - 4.5f'_c/10^5) \qquad \text{(d)}$$

In June, 1953, Moody, and in 1954 and 1955 Moody, Viest, Elstner and Hognestad (6) presented a semi-empirical formula, based on the same shear-compression failure theory, to express the shear-moment that a concrete beam with tensile reinforcement can carry. They assumed that the moment could be expressed by the same type of equation as for pure flexure, namely:

$$M_s = A_sf_sd \left(1 - k_2 \frac{pf_s}{k_1k_3f'_c}\right) \qquad \text{(e)}$$

Assuming $k_1k_3 = 1.121 - 0.045\, f'_c/1000$, $k_2 = 0.42$, and that the steel stress could be determined from a similar expression as that for over-reinforced concrete beam in pure flexure, the following expirical expression for steel stress at failure in the beam weak in shear was developed:

$$f_s = 15080 \left(-1 + \sqrt{1 + \frac{1450}{E_sp/k_1k_3f'_c}}\right) \qquad \text{(f)}$$

Both Laupa's and Moody's formulas give good predictions of the ultimate shear-moment for beams with relatively large cross sections (d = 8 in.). These formulas did not, however, apply to beams with relatively small cross sections ($d \leq 8$ in.).

Since the development of the shear-compression failure theory, investigations have been made and are now under way to study the effects of size of cross section, web reinforcement, compressive reinforcement, continuous supports, uniform loading and repeated loading. Some of these variables have already been treated in Laupa's and Moody's papers.

The shear-moment will control the ultimate load carrying capacity of a beam weak in shear only if the beam will crack diagonally before the shear-moment of the beam is reached. A beam that cracks diagonally at a load higher than that predicted by the shear-moment theory will therefore take a higher load than that which produces the shear-moment. However, as soon as a diagonal tension crack is fully developed, the beam will fail since the moment on the beam at the critical section is already higher than that of the shear-moment. Moody developed a formula that will predict the load for the initiation of diagonal tension cracking of a beam as determined from load-deflection curves. The predicted load is generally lower than that at which the diagonal tension cracks can be determined with the unaided eye. This formula involves both the concrete strength and the span-depth ratio, a/d or M/Vd, and is as follows:

$$v_c = V_c/(\tfrac{7}{8}\,bd) = 0.12(1 - 0.1M/Vd)\, f'_c\,(1 - f'_c/1{,}000\ \text{psi}) \qquad \text{(g)}$$

$$\text{for } 1{,}000\ \text{psi} < f'_c \leq 5{,}000\ \text{psi}$$

$$v_c = (v_c)_{f'_c = 5{,}000\ \text{psi}}\ ;\ \text{for } f'_c \quad 5{,}000\ \text{psi}$$

Morrow (7) in 1954 considered the cracking of a beam weak in shear as a problem of combined stresses, and developed a formula which predicts the initial diagonal cracking load of knee-frames without web reinforcement and of the beams tested by Moody. This formula is as follows:

$$v_c/f_r = \frac{13.4}{\frac{a_c}{npd} + \sqrt{\left(\frac{a_c}{npd}\right)^2 + 600}} \tag{h}$$

with $f_r = 0.275(1 - \frac{f_c'}{8,250})$ for $f_c' \leq 3,000$ psi

$f_r = 300 + 0.075\, f_c'$ for $f_c' \geq 3,000$ psi

$a_c = a'/2$ for $a/d \geq 2$

$a_c = d$ for $a/d < 2$

and $E_c = 1,800,000 + 460\, f_c'$; $E_s = 28,000,000$ psi

None of the preceeding formulas were found to be applicable to the beams with relatively small cross sections used in this investigation. Nevertheless a satisfactory means of determining the static strength of beams is necessary before the results of repeated load tests made by the authors can be evaluated.

The fatigue strength of reinforced concrete beams is a complicated subject about which little is known. Variables, such as concrete strength, percentage of tensile, compressive, and web reinforcement, maximum value of repeated loading, range and rate of loading, aggregate size and shape, temperature, humidity, and others, will have significant effects on the final test results. If a beam subjected to repeated loading is weaker in shear than flexure, it may fail in any one of the following five ways:

1. Destruction of the compressive zone,
2. Splitting action along the reinforcement,
3. Diagonal cracking,

4. Bond failure, or

5. Fatigue of the reinforcement.

Generally, a beam weak in shear and subjected to repeated load will first crack diagonally and the cracking will then proceed toward the point of maximum moment as additional loads of the same magnitude are applied. The beam will fail when the crack extends so far that the compression zone becomes too small to resist the compressive force acting on it at the maximum value of the cyclic loading. Failure then occurs by destruction of the compression zone*. It has been observed that some splitting action along the reinforcing bars also takes place as while the diagonal crack progresses, but this appears to be only of secondary importance. It has been found that bond failures are unlikely with deformed bars meeting ASTM specification A305 for the l/d ratio tested**. Another interesting phenomenon in a beam under repeated loading is the continual change in stiffness of the beam which is believed to be due to the creep behavior of concrete, the development of the initial vertical tensile cracks, and the formation of the diagonal cracks.

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* Note that unlike the static case, the beam may fail by destruction of the compression zone, even when the beam has a larger predicted cracking load than the shear-moment; because the crack may, under repeated load, occur at a load lower than the load for cracking if loaded statically.

**Minimum Requirements for the Deformations of Deformed Steel Bars for Concrete Reinforcement, A305-53T, Standard, American Society of Testing Materials, Part 1, pp. 1020-2, 1955.

SCOPE OF INVESTIGATION

The purpose of this study was twofold: first, to develop simple and sufficient expressions for the static strength of beams failing in shear; and second, to determine the strength under repeated loading of reinforced concrete beams weak in shear. Emphasis was on small beams with tensile reinforcement only. The variables studied in fatigue are limited to concrete strength, percentage of reinforcement, and magnitude of the repeated load.

OUTLINE OF TESTS

Forty-two simply-supported beams of relatively small cross sections with tensile reinforcement only were tested to failure under static loads. All beams were 4- by 6-in. in cross section, but the values of shear span, steel percentage, and concrete strength were varied as shown on Table 1. The loads at which diagonal tension cracks occurred and the maximum loads were observed. All beams with few exceptions were tested at an age of approximately 28 days, and in all cases the control cylinders were tested on the same dates as the beams.

Data on the shear strength of 64 beams of relatively large cross sections without web or compressive reinforcements were obtained from reports by Moody (6), Richart (8), Richart and Jensen (9), Gaston, Siess, and Newmark (10), Clark (4), and Laupa, Siess and Newmark (5), and are given in Tables 2 and 3.

Thirty-nine beams having 4- by 6-in. cross sections with tensile reinforcement only were subjected to repeated loading. The results of these tests are given in Table 4. The tests were generally continued until final failure occurred. Those that did not fail at a fatigue life of about 10,000,000 cycles were removed from the testing machine and tested to failure statically. Data from this series of beams are given in Table 5.

MATERIALS

The aggregates used in the tests made by the authors were from the Wabash River near Covington, Indiana. These aggregates are part of a glacial outwash, probably from the Wisconsin glaciation. The sand had an average fineness modulus of 3.0. The coarse aggregate was well graded and had a maximum size of one inch. The absorption of the aggregates was about one percent by weight of the surface dry aggregates. Both of these aggregates passed the usual ASTM specification tests. The cement used was a type I portland cement, and was of several brands. The concrete strengths varied from 3150 to 6750 psi, with slumps varying from 1 to 6 in. Both a pan-type mixer of about 2-cu. ft. capacity and a free-falling non-tilting drum mixer of about 6 cu. ft. capacity were used. The concrete was cured in a moist room at 74F and 100 per cent relative humidity for 6-7 days, and then cured in the ordinary laboratory atmosphere until tested.

The reinforcement consisted of deformed bars of intermediate grade billet steel meeting ASTM specification A305. The average yield point was about 47,500 psi, the ultimate strength about 72,500 psi, and the modulus of elasticity about 27×10^6 psi.

Materials used in tests by other investigators varied considerably and the reader is referred to the original reports for their properties.

TESTING PROCEDURES

Beams designed for the study of size effect were tested statically in a 300,000-lb. testing machine under center- or third-point loading while simply supported. Each test lasted from 15 to 60 minutes.

Beams for the fatigue studies were tested in a specially designed fatigue machine which applied a sinusoidal repeated load at 440 cpm at the third points of a 5-ft. simple span. The maximum value of the repeated loading was maintained by checking the loads at reasonable intervals of time and making adjustments as required. An automatic shut-off stopped the machine when the specimen failed, and the number of cycles to failure was read from a revolution counter. For a detailed description of this fatigue machine, readers are referred to a previous paper (11).

FAILURE OF BEAMS IN FATIGUE

Three modes of failures were observed in the thirty-nine beams tested under repeated loading.

1. <u>Destruction of the Compression Zone: Fig. 1a, b, c, and d.</u> Twenty-three of these beams failed by destruction of the compression zone at the end of the diagonal tension crack. They first cracked diagonally in either one or both of the shear spans. For simply supported beams this shear span is defined as being the length of the specimen between a support and the nearest load point. The diagonal cracks then extended toward the edges of the loading blocks accompanied by some splitting action along the reinforcement. The splitting in some cases

proceeded to the ends of the beams, Fig. 1b, but always after the beams had "failed"; therefore, this splitting behavior was considered as a secondary failure. Final failures occurred when the compression zone at the top end of the diagonal cracks became too small to resist the applied load. The concrete in the compression zone was probably weakened by the repeated loading.

2. Diagonal Cracking Failure: Fig. 1e.

Eleven beams failed by "diagonal cracking". Generally, the diagonal cracks for these beams were fully developed as soon as the cracks were observed. The concrete failed by compression at the top of the loading block as soon as the cracks were formed. This means that the number of cycles for final failure of the beams was the same as that for cracking.

3. Fatigue of Reinforcement: Fig. 1f.

Three beams failed by fatigue failure of the reinforcement in the region of pure flexure between the load points. The beams behaved similarly to those that failed by destruction of the compression zone; but after these beams cracked diagonally, the reinforcement in the beams failed through fatigue.

EXPRESSION FOR THE INITIAL DIAGONAL TENSION CRACKING LOAD

The initial diagonal tension cracking loads obtained from the authors' static beam tests were compared with values given by Morrow's formula Eq. (h); about one-third of the results were found to differ from the formula by about $\pm$ 30 per cent. A new formula for determining the initial diagonal tension cracking load of simply supported concrete beams with tensile reinforcement and weak in shear was developed and found to fit both Moody's and the authors' data for 53 beams with ± 17 per cent with a standard deviation of 0.095. The formula was developed by a combination of dimensional analysis and step-by-step numerical approximations as explained in Appendix II. The expression developed is as follows:

$$V_c/bd = 0.3f'^{0.53}_c \,(1 + 0.00562p^{1.8})(1 + 0.685d^{1.42}/a^{1.42}) \,/\, (1 + 13.4/d^{2.32}) \qquad \text{(i)}$$

The ratio V_c/bd given by Eq. (i) can also be evaluated semi-graphically as the product of the four factors obtained from Fig. 2 for the appropriate values of strength, steel percentage, span-depth ratio, and effective depth of the beam. More conveniently, the nomograph given in Fig. 3 may be used for the solution of Eq. (i)*.

DEVELOPMENT OF THE SHEAR-MOMENT EXPRESSION

The maximum moments for the authors' beams which first cracked diagonally and then failed by destruction of the compression zone were compared with values given by Moody's and Laupa's formulas, and about one-third of the results were found to vary by ± 35 per cent. A new formula for determining the maximum shear-moment of beams with tensile reinforcement was therefore derived, again through dimensional analysis. The detailed procedure is given in Appendix IV.

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*For use of nomograph, Fig. 3, see Appendix III.

The expression developed is:

$$M_s/bd^2 = 7f_c'^{0.35} (1 + 0.188p^{1.62}) (1 + 17.8/d^{3.25})$$

for $p > 2.5$ per cent, and

$$M_s/bd^2 = 4.38f_c'^{0.35} (1 + 25 p^{0.5}) (1 + 17.8d^{3.25})$$

for $p < 2.5$ per cent (j)

This expression predicts ultimate shear-moment for all existing data within $\pm$ 19 per cent with a standard deviation of 0.102.

Equation (j) can also be solved semi-graphically as the product of the three factors given in Fig. 4. The nomograph in Fig. 5 may also be used to solve this equation.*

STUDY OF FATIGUE FAILURES

It has been observed that a reinforced concrete beam will fail at a load considerably less than its static load capacity if the load is repeated a sufficient number of times. One purpose of this investigation was to study statistically the fatigue life for initial diagonal tension cracking and for final failure of a beam which fails in shear when subjected to repeated loading varying from near zero to a maximum load. The span-depth ratio of all beams tested in fatigue was $a/d = 3.53$ or $a'/d = 3.72$**. Two

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*For use of nomograph, Fig. 5, see Appendix V.

**The distance a is measured from the edge of the loading block to the center of the support and is used to determine the shear-moments of the beams, while a' is measured from the center of the loading block to the center of the support and is used for the calculation of the inital diagonal tension cracking load of the beams. In these tests a' = a + 1-in.

values of steel percentages were used in this investigation, 1.86 and 2.89 per cent, and the concrete strength was varied from 2500 to 6500 psi. A minimum load of about two hundred pounds was maintained to prevent impact on the specimen during each cycle of loading. The maximum load varied about 4100 to 9500 lb. which is about 45 to 91 per cent of the initial diagonal tension cracking load and about 51 to 100 per cent of the static shear-moment capacity as predicted by Eqs. (i) and (j).

Statistical studies of fatigue tests on concrete specimens are commonly based on plots of applied load, expressed in per cent of the static load, against the logarithm of the fatigue life of the specimens.

Before any statistical study can be made, however, the criteria for failure of the beams must be determined. Comparison of Eqs. (i) and (j) as plotted in Fig. 6 indicates that the initial diagonal tension cracking load will be higher than the shear-moment load for beams under static loading with steel percentages of 1.86 and 2.89, when the concrete strength is between 2000 and 7000 psi. However, further consideration of the curves in Fig. 6 shows that the maximum variation between the cracking and shear-moment load is only 12 per cent. Hence a variation of 6 per cent of the actual loads from the theoretical loads could give a higher shear-moment load. This means that beams with this particular span-depth ratio and steel percentage when tested statically may fail either by diagonal tension or by destruction of the compression zone after a diagonal tension crack is fully developed. When loaded in fatigue, however, a diagonal tension crack may develop at a load considerably less than the initial diagonal tension cracking load predicted by Eq. (i), and failure may therefore occur in shear-compression. The fatigue life was studied for both initial diagonal cracking and final failure of the beams. The three beams that failed by fatigue of the reinforcing steel were reinforced with two No. 4 bars and had a steel percentage of 1.86. The beams all cracked diagonally before the bars failed in the region of constant moment. The average maximum stresses in the steel as determined from electric resistance strain gages were 31,000 and 32,600 psi for the two beams in which strains were measured. The numbers of cycles of loading sustained by these beams were comparable to those of the beams which did not fail in the steel. All three had a fatigue life greater than one million cycles.

STATISTICAL STUDY OF INITIAL DIAGONAL TENSION CRACKING UNDER REPEATED LOADS

All beams that cracked diagonally in fatigue were included in the analysis. Two values of steel percentages, 1.86 and 2.89 per cent, are available for analysis. The concrete strength of the beams varied from 2150 to 6750 psi. Statistical studies were made by plotting the applied maximum load in terms of per cent of the static initial diagonal tension cracking load, as predicted by Eq. (i) against the logarithm of the number of cycles of loading the beams withstood before cracking diagonally. Separate studies for different steel percentages and concrete strength were made. Curves were drawn for a 50-per cent probability of cracking and the "fatigue factor"* with a probability of 50 per cent was determined as 0.57 for p = 2.89 per cent, 0.57 for p = 1.86 per cent, 0.58 for $f_c' \geq 4300$ psi, and 0.57 for $f_c' \leq 4700$ psi.

Since all the curves seem to have the same characteristics and all the fatigue factors seem to be about the same, a combined plot, Fig. 7, of beams with all steel percentages and concrete strengths was made. The data were found to lie within a limit of ± 15 per cent from the 50 per cent probability curve for cracking with one exception which was 17 per cent off the curve. The fatigue factor for initial diagonal tension cracking with a probability of 50 per cent is about 0.57 and with a probability of near zero per cent is about 0.49.

STATISTICAL STUDY OF FINAL FAILURE UNDER REPEATED LOADS

All data for beams that failed under fatigue loading after a diagonal tension crack formed are included in this study. There are again two values of steel percentages, 1.86 and 2.89. The concrete strengths varied from 2150 to 6750 psi. Separate studies for final failure were again made for different concrete strengths and steel percentages by plotting the applied maximum load in terms of per cent of the static shear moment load, as expressed

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*Fatigue factor is defined as that ratio of the applied maximum repeated load to the static load for cracking or failure below which the beam will crack or fail at a given probability. The most common value used is the average fatigue factor which is the fatigue factor below which 50 per cent of the beams tested will crack or fail.

by Eq. (j), against the logarithm of the number of cycles of repeated load before failure. The failure curve and the fatigue factors with a probability of 50 per cent were again found. The fatigue factors were 0.63 for p = 2.89 per cent, 0.63 for p = 1.86 per cent, 0.63 for $f'_c \geq$ 4300 psi, and 0.63 for $f'_c \leq$ 4700 psi.

Again, all the failure curves and fatigue factors were found to be very close to each other and therefore a combined study, Fig. 8, for all the beams were made. The failure curve with a probability of 50 per cent was capable of predicting the results for all the beams tested within $\pm$ 15 per cent, which is within the range of variation for the prediction of static strength. The fatigue factor for final failure with a probability of 50 per cent was found to be 0.63, and with a probability of near zero per cent was about 0.56.

Since it was believed that the compression zone of a beam was not damaged by repeated loading before diagonal cracking occurred, a statistical study was made on the basis of the fatigue life as represented by the number of cycles of loading required to cause failure after diagonal cracking occurred. The results of this study are shown in Fig 9. Excluding points that have relatively short lives after cracking, the data again seem to form a close band indicating the fatigue factor of 62 per cent with a probability of 50 per cent.

Due to the large flat portion of the curve, the fatigue life of a beam after cracking is more or less interminate, while the fatigue strength of the beam is quite certain. The fatigue factor of the beams can equally well be obtained by making statistical studies of the total fatigue lives of the beams. This study does tend to point out one important point, i.e., not much damage occurs in the shear-compression zone due to fatigue loading for a beam which has no diagonal cracks.

DAMAGE DUE TO REPEATED LOADING

Two beams, 4-20 and 5-1, did not crack diagonally and consequently did not fail even though 10,000,000 cycles of load were applied. They were removed from the fatigue machine and tested to failure under static load. The initial diagonal tension cracking load, Table 5, for both beams were found to be within the range of the formula derived by the authors. Beam 5-1 failed by diagonal tension cracking and destruction of the compression zone simultaneously, and beam 4-20 failed by destruction of the compression zone at a load slightly higher than the diagonal tension cracking load. The final failure load for beam 4-20, Table 5, was found to lie within the range of accuracy of the shear-moment expression derived in this paper.

Further static tests were made on some of the beams that failed in fatigue. Failure of a beam under repeated load generally occurred at only one end while the other end remained intact except for a few small tensile cracks or a diagonal tension crack. This end was then loaded at the center while simply supported over a span of 40-in. The results in Table 5 show that for the ends that did not crack diagonally in the fatigue tests, failure occurred either simultaneously by diagonal cracking and final destruction of the compression zone of the beams or by the destruction of the compression zone after the diagonal cracking load had been reached. In either case, the data obtained checked within limits of variation the authors' formulas. For ends that did have diagonal tension cracks as a result of the fatigue tests, failure was by destruction of the compression zone, and the recorded loads were consistent with the authors' formula for predicting the shear-moment capacity of the beams.

These test data seem to indicate two important results. First, if a beam did not crack diagonally under repeated loading, it was not damaged by the repeated loading so far as the initial diagonal tension cracking load and the shear-moment were concerned. Second, if the beam was cracked diagonally under repeated loading, the beam was not damaged so far as the shear-moment carrying capacity of the beam was concerned. It should be noted, however, that if the end of the beam had a diagonal crack, the ultimate load would be predicted by the shear-compression theory, even if theoretically for static loading the beam might have a higher initial diagonal tension cracking load than the shear-moment load.

SUMMARY

The formulas developed by previous investigators for predicting the diagonal tension cracking loads and shear-moments of simply supported reinforced concrete beams with tensile reinforcement only were found to be inapplicable for beams of relatively small cross section. Consequently, equations were developed using dimensional analysis to predict these loads, both for the

relatively small beams tested by the authors and for similar beams of larger cross section for which adequate data could be found. The expression for initial diagonal tension cracking load, Eq. (i), was found to predict Moody's and the authors' test data for 53 beams within $\pm$ 17 per cent with a standard deviation of 0.095. The expression for shear-moment, Eq. (j), was found to predict the existing data within $\pm$ 19 per cent with a standard deviation of 0.102. Because of the complicated form of these two equations, nomographs were developed from which solutions may easily be obtained. Equations (i) and (j) and the nomographs, Figs. 3 and 5, apply exclusively to simply supported reinforced concrete beams with only tensile reinforcement.

The repeated load tests indicate that the fatigue strength of the type of specimens tested is influenced by the percentage of steel and the concrete strength to the same extent as the static strength. When all the data, regardless of percentage of steel or concrete strength, for initial diagonal tension cracking in fatigue were plotted in Fig. 7 with the ordinate being the percentage of the cracking load applied as predicted by Eq. (i), the maximum variation was only $\pm$ 15 per cent from the average curve. The fatigue factor for cracking with a probability of 50 per cent is 0.57 and with a near zero probability is 0.49. A similar plot for final failure in shear, Fig. 8, yielded results with a maximum variation of only $\pm$ 15 per cent from the average failure curve. The fatigue factors for failure with probabilities of 50 per cent and near zero per cent are 0.63 and 0.55, respectively.

Comparison of Figs. 7 and 8 indicates that the cracking load is reduced more than the ultimate failure load at given number of cycles. For example, at 1000 cycles the cracking load has been reduced to 68 per cent of the static value while at the same life the ultimate load has been reduced to only 83 per cent of the static ultimate. The fatigue factors at 10 million cycles and 50 per cent probability are 0.57 and 0.63, respectively.

A few static tests made on beams previously subjected to repeated load indicated that if a beam did not crack diagonally under repeated loading, it was not damaged as regards either the diagonal tension cracking load or the ultimate moment carry capacity. This finding was further strengthened by the statistical studies of fatigue lives of beams after cracking, Fig. 9. Other tests indicated that if a beam was cracked diagonally in the fatigue test but did not fail, its static load-carrying capacity was not affected.

APPENDIX I

NOTATION

a: shear span of a beam measured from edge of loading block to center of support

a': shear span of a beam measured from center of loading block to center of support

a_c: cracking span as defined by J. Morrow

A_s: area of tensile steel of a reinforced concrete beam

b: width of a rectangular reinforced concrete beam

c: constant

d: effective depth of a reinforced concrete beam

d': effective depth of a beam which needs no size correction for computing initial diagonal tension cracking load or shear-moment of the beam

E_c: modulus of elasticity of concrete

E_s: modulus of elasticity of steel

f_r: modulus of rupture of concrete

f_s: stress in tensile reinforcement

f'_c: ultimate strength of concrete as determined from 6 by 12-in. cylinders

f'_{yp}: yield point of web reinforcement

jd: moment arm between the resultant compressive force in the concrete and the tensile stress in the steel as computed by the straight line formula

k: constant relating to the contribution of web reinforcement by truss analogy

k_1k_3: ratio of average concrete stress in the compression zone to the strength of 6 by 12-in. cylinders

k_2 ratio of distance from top fiber to center of compression to distance from top fiber to neutral axis

kd: distance from the top of the beam to the neutral axis of the beam as computed from the straight line theory for flexure

M: moment

M_s: shear-moment

n: ratio of the modulus of elasticity of steel to that of concrete

p: percentage of tensile reinforcement

P: total load on a simply supported beam

P_c: diagonal cracking load

P_s: total load on a simply supported beam causing a maximum moment equal to the shear-moment

r: percentage of web reinforcement

V: shear force

v: nominal unit shear stress

V_c: shear force for initial diagonal tension cracking

V_s: shear force at which the shear-moment of the beam is developed

V_u: shear force at failure of the beam

v_u: nominal unit shear stress at failure

APPENDIX II

DEVELOPMENT OF EQUATION FOR INITIAL DIAGONAL TENSION CRACKING LOAD

The following is the procedure used to develop the expression for initial diagonal tension cracking load.

Experience has indicated that the main variables that influence the initial diagonal tension cracking load of a concrete beam with tensile reinforcement are the concrete strength, the percentage of tensile reinforcement, the effective depth of the beam, the shear span, the width of the beam, the modulus of elasticity of the concrete, the modulus of elasticity of the tensile steel, and the relative size of the beam. Therefore the following expression can be written:

$$V_c = F_1 \; (f_c', \, p, \, d, \, a, \, b, \, E_c, \, E_s, \, \text{relative size})$$

$$\text{or} \quad F_2 \; (V_c, \, f_c', \, p, \, d, \, a, \, b, \, E_c, \, E_s, \, \text{relative size}) = 0 \qquad \text{(k)}$$

If the assumption is made that the modulus of elasticity of concrete is a function of the concrete strength and that the term, relative size, can be expressed by the ratio of the effective depth of the beam to a standard effective depth for which correction of size is not necessary, then the following expression results:

$$F_3 \; (V_c, \, f_c', \, p, \, d, \, a, \, b, \, E_s, \, d/d') = 0$$

$$\text{or} \quad F_4 \; (V_c, \, f_c', \, p, \, d, \, a, \, b, \, E_s, \, d') = 0 \qquad (1)$$

Dimensional Analysis

The purpose of this analysis is to reduce the eight variables that appear in Eq. (1) to a function that contains a minimum number of dimensionless products forming a complete set, or the minimum number still forming a function which describes the phe-omenon completely.

By forming a dimensional matrix for Eq (1) as shown in Table 6a, it is apparent that the determinant formed by the last two columns of the matrix is not equal to zero. This means that the dimensional matrix is of the rank of two, and therefore the number of dimensionless products necessary to form a complete set, which will describe the function $F_4 = 0$, is eight minus two or six.

By observation*, $V_c/E_s d^2$, p, b/d, a/d, d/d', and f'_c/E_s are indicated as the six dimensionless products. To show that they form a complete set, it is necessary to show the solution matrix formed by these six dimensionless products is of the rank of six. This is indeed the case when the matrix is written as shown in Table 6b, since the determinant formed by the last six columns of the matrix is not zero.

The foregoing analysis indicates that the equation of eight variables can be replaced by a function of six dimensionless products as shown in Eq. (m):

$$F_5 \left(V_c/E_s d^2, p, b/d, a/d, d/d', f'_c/E_s\right) = 0$$

$$\text{or} \quad V_c/E_s d^2 = F_6 \left(p, b/d, a/d, d/d', f'_c/E_s\right) \qquad \text{(m)}$$

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*There are standard methods to obtain a complete set of dimensionless products but these are not necessary here.

If it is assumed that V_c is proportional to b, then Eq. (m) becomes:

$$V_c/E_s d^2 = b/d\ F_7\ (f_c', p, a/d, d/d')$$

$$\text{or}\quad V_c/E_s bd = F_7\ (f_c', p, a/d, d/d') \qquad (n)$$

But since the modulus of elasticity of steel can be considered constant, as is also the standard effective depth, the equations further reduces to:

$$V_c/bd = F_8\ (f_c', p, a/d, d) \qquad (o)$$

It can be shown that F_8 can be represented by the product of four functions $f(f_c')$, $g(p)$, $h(a/d)$, and $i(d)$*, thus:

$$V_c/bd = f(f_c') \cdot g(p) \cdot h(a/d) \cdot i(d) \qquad (p)$$

This is a very simple expression. The only step that remains now is to find these functions for the experimental data available to the authors. To do this, a numerical method of successive approximation was developed.

Numerical Analysis

First, the initial diagonal tension cracking load of beams with the same steel percentage, span-depth ratio, and effective depth were plotted against the concrete strength. A tentative function of concrete strength, $f(f_c')$ in Eq. (p), was developed. Then, the initial diagonal tension cracking load of beams with the same span-depth ratio and effective depth were divided by this function according to their corresponding concrete strengths, and

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*See explanation in Appendix VI.

were plotted against the steel percentage. A tentative function of steel percentage, g(p) in Eq. (p) was then determined. All the initial diagonal tension cracking loads for beams with the same effective depth were then divided through by the functions of concrete strength and steel percentage and plotted against the span-depth ratio, and a tentative function of the span-depth ratio resulted. Then, dividing all initial diagonal tension cracking loads available by the functions of concrete strength, steel percentage, and span-depth ratio just determined, and plotting the values obtained against the corresponding effective depth of the beams, a tentative function of the effective depth is obtained. Using these tentative functions of the steel percentage, span-depth ratio, and the effective depth, an improved function of concrete strength was then found. Continuing the cyclic numerical evaluation of these four functions the results converged to:

$$V_c/bd = 0.3f_c'^{0.53}\,(1 + 0.00562\,p^{1.8})\,(1 + 0.685\,d^{1.42}/a^{1.42})\,(1 + 13.4/d^{2.32}) \quad (i)$$

All the final functions of concrete strength, steel percentage, span-depth ratio, and effective depth are shown graphically in Fig. 2.

APPENDIX III

USE OF NOMOGRAPH TO COMPUTE DIAGONAL TENSION CRACKING LOAD

The following is an outline of the procedure for obtaining the initial diagonal tension cracking load of a beam simply supported without web and compressive reinforcement by using the nomograph, Fig. 3.

1. A straight line is passed through the values of concrete strength and steel percentage of the beam on vertical lines (a) and (e) of the nomograph. The intercept of this straight line with (b) is then marked.
2. Connecting this intercept with the value of the span depth ratio of the beam on the left side of the line (c), a value of unit shear force for initial diagonal tension cracking is read on line (d). This unit shear force for diagonal tension cracking is not corrected for size effect.
3. For a more correct value for the unit shear force for initial diagonal tension cracking, connect the uncorrected value with the value of the effective depth of the beam on the right side of line (c), and obtain the corrected value of the unit shear force from line (b), the V_c/bd (corrected) line.
4. The shear force for initial diagonal tension cracking is obtained by multiplying the final value by the width and the effective depth of the beam. For beams loaded at the third points or at the center, the diagonal tension cracking load for the beam is twice the shear force for initial diagonal tension cracking.

APPENDIX IV

DEVELOPMENT OF EQUATION FOR SHEAR-MOMENT

The following method was used to determine the expression for the magnitude of the shear-moment.

Variables

If the theory of shear-compression failure is correct*, then the variables that will influence the shear-moment of a beam with tensile reinforcement are k, f_c', p, n, b, d, and E_s or k_1k_3, f_c', p, f_s, b, d, and E_s as suggested by Laupa's and Moody's formulas, respectively. A closer examination of either set of these variables shows that the actual independent variables are M_s, f_c', p, n, b, d, and E_s, or

$$F_1' (M_s, f_c', p, n, b, d, E_s) = 0 \qquad (g)$$

By introducing the size effect through a ratio of the effective depth of the beams to a standard effective depth, the function becomes:

$$F_2' (M_s, f_c', p, b, d, d', E_s) = 0 \qquad (r)$$

Dimensional Analysis

The function of seven variables as appeared in Eq. (r) was further reduced to a function of a complete set of five dimensionless products through dimensional analysis. The analysis is

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*With deformed bars meeting ASTM A305 or end anchorages, bond failures are rare and diagonal cracking will occur before reaching the critical shear-moment if the span-depth ratio of the beam is reasonably small, but not so small that the shear force will have a large effect on the compressive force in the compression zone.

essentially the same as that for the formula for initial diagonal tension cracking and will not be shown here. The resulting function after the analysis is:

$$F_3' \, (M_s/E_s d^3, \, p, \, b/d \; f_c'/E_s, \, d/d') = 0, \text{ or}$$

$$M_s/E_s d^3 = F_4' \, (p, \, b/d, \, f_c'/E_s, \, d/d') \qquad \text{(s)}$$

If M_s is assumed to be directly proportional to b, then,

$$M_s/E_s d^3 = b/d \cdot F_5' \, (p, \, f_c'/E_s, \, d/d'), \text{ or}$$

$$M_s/E_s bd^2 = F_5' \, (p, \, f_c'/E_s, \, d/d') \qquad \text{(t)}$$

But E_s can be considered constant and d' is a constant, therefore Eq. (t) further reduces to:

$$M_s/bd^2 = F_6' \, (f_c', \, p, \, d), \text{ or}$$

$$M_s/bd^2 = f'(f_c') \cdot g'(p) \cdot h'(d) \qquad \text{(u)}$$

The functions $f'(f_c')$, $g'(p)$ and $h'(d)$, are again found through numerical approximation. The functions are shown graphically in Fig. 4. The final expression for the shear-moment of a concrete beam of any size with tensile reinforcements is:

$$M_s/bd^2 = 7 \, f_c'^{0.35} \, (1 + 0.188 \, p^{1.62}) \, (1 + 17.8/d^{3.25})$$

for p smaller than 2.5 per cent.

$$M_s/bd^2 = 4.28 \, f_c'^{0.35} \, (1 + 1.25 \, p^{0.5}) \, (1 + 17.8/d^{3.25})$$

for p smaller than 2.5 per cent (j)

APPENDIX V

USE OF NOMOGRAPH TO COMPUTE SHEAR-MOMENT

The following is an outline of the procedure for obtaining the shear-moment of a concrete beam without web and compressive reinforcement by using the nomograph in Fig. 5.

1. A straight line is passed through the values of concrete strength and steel percentage of the beam on vertical lines (a) and (c) on the nomograph. An intercept of this straight line on (b) gives an unit shear-moment for the beam, but which is not corrected for size effect.
2. A correction for size effect of this unit shear-moment can be made by connecting the uncorrected value and the value of the effective depth of the beam on the right side of vertical line (c), and obtaining the corrected value of the unit shear-moment of the beam on the vertical line (d).
3. The shear-moment of the beam is obtained by multiplying the unit shear-moment by the width, and the square of the effective depth of the beam.

APPENDIX VI

TRANSFORMATION OF FUNCTION OF DIMENSIONLESS PRODUCTS INTO A MULTIPLE OF FUNCTIONS

A function of a complete set of n dimensionless products can generally be expressed by a multiple of n functions, each of which varies with one of the n dimensionless products. This can be proved by noting that there are infinite numbers of sets of dimensionless products which can describe the physical phenomenon completely, or

$$F(\pi_1, \pi_2 ------\pi_n) = F^1(\pi_1', \pi_2' ------\pi_n') ---- =$$

$$F^i(\pi_1^i, \pi_2^i, ----\pi_n^i) = ---------$$

In particular, the case of n = 2, then the problem reduces to the following:

$$F(\pi_1, \pi_2) = f_1(\pi_1) \cdot F_2(\pi_2),$$

$$F^1(\pi_1', \pi_2') = f_1'(\pi_1') \cdot f_2'(\pi_2'),$$

-----------------------------, or

$$F^i(\pi_1^i, \pi_2^i) = f_1^i(\pi_1^i) \cdot f_2^i(\pi_2^i)$$

It can be shown that the function $F(\pi_1, \pi_2)$ can be expressed as a sum of the multiple of functions which are functions of π_1 and π_2.

$$F(\pi_1, \pi_2) = g_1(\pi_1) \cdot h_1(\pi_2) + g_2(\pi_1) \cdot h_2(\pi_2) + -----+$$

$$g_j(\pi_1) \cdot h_j(\pi_2) + -------$$

where $g_j(\pi_1)$ and $h_j(\pi_2)$ are of the following type: $a_j\pi_1^k$ and $b_j\pi_2^m$.

But $g_1(\pi_1)h_1(\pi_2) + g_2(\pi_1)h_2(\pi_2)$ can be expressed by a function of multiple of two new dimensionless products:

$$g_1(\pi_1)h_1(\pi_2) + g_2(\pi_1)h_2(\pi_2)$$

$$= g_1(\pi_1)h_1(\pi_2) \left[\frac{g_2(\pi_1)h_2(\pi_2)}{g_1(\pi_1)h_1(\pi_2)}\right] + 1$$

$$= g'(\pi_1')\ h'(\pi_2')$$

or by analogy,

$$F(\pi_1, \pi_2) = g_1(\pi_1)h_1(\pi_2) + \text{----------} + g_n(\pi_1)h_n(\pi_2)$$

$$= f_1'(\pi_1')f_2'(\pi_2')$$

where π_1' and π_2' can be proved to still form a complete set which describes the physical phenomenon completely. Since $F(\pi_1, \pi_2) = F^1(\pi_1{}^1, \pi_2{}^1)$, therefore,

$$F^1(\pi_1^1, \pi_2 1) = f_1^1(\pi_1^1)\ f_2^1(\pi_2^1)$$

This proof can be extended to the case of n dimensionless products so that the following is true:

$$F(\pi_1, \pi_2 \text{--------} \pi_n) = f_1(\pi_1)\ f_2(\pi_2) \text{------} f_n(\pi_n).$$

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TABLE I

DATA AND RESULTS OF ANALYSIS OF STATIC TESTS OF AUTHORS' BEAMS

(For all beams: b = 4 in. and d = 5.38 in.)

Beam No.	f'_c psi	p per cent	a in.	V_c(Test) lb.	V_c(Test)* / V_c(Calc.)	M_s(Test) lb. in.	M_s(Test) / M_s(Calc)
IA1	4,000	2.89	21	4,470	0.88	104,500	1.10
IA2	4,000	2.89	10	——	——	101,500	1.08
IB1	4,000	1.86	13	4,400	0.97	62,700	0.86
IB2	4,000	1.86	13	——	——	71,900	0.98
IC1	4,000	2.37	17	4,410	0.92	75,200	0.88
IC2	4,000	2.37	17	4,000	0.84	80,100	0.93
IIA1	2,560	1.86	17	3,830	1.02	65,300	1.06
IIA2	2,560	1.86	17	3,880	1.03	66,300	1.07
IIB1	2,560	2.37	21	3,730	0.98	83,600	1.14
IIB2	2,560	2,37	10	——	——	80,400	1.09
IIC1	2,560	2.89	13	4,000	0.85	82,200	1.01
IIIA1	2,160	2.37	21	3,830	1.13	76,000	1.09
IIIA2	2,160	2.37	10	4,650	1.16	74,300	1.02
IIIB1	2,160	1.86	17	3,450	1.07	58,600	1.03
IIIB2	2,160	1.86	17	3,480	1.08	59,200	0.95
IIIC1	2,160	2.89	13	4,130	0.86	82,600	1.06
IIIC2	2,160	2.89	13	——	——	77,200	0.99
4-1a	5,860	1.86	19	——	——	95,000	1.12
4-2a	5,210	1.86	19	——	——	85,500	1.06
4-3a	5,540	1.86	19	——	——	85,500	1.03
4-4a	5,940	1.86	19	——	——	95,000	1.11
4-5a	5,730	1.86	19	——	——	85,500	1.02
4-13a	4,930	1.86	19	——	——	72,000	0.93
4-13b	4,930	1.86	19	——	——	66,500	0.85
4-14a	5,410	1.86	19	——	——	85,500	1.04
4-14b	5,410	1.86	19	——	——	91,200	1.11
4-21a	5,600	1.86	19	4,740	0.96	——	——

cont.

TABLE I (CONTINUED)

Beam No.	f'_c psi	p per cent	a in.	V_c(Test) lb.	$\frac{V_c(\text{Test})^*}{V_c(\text{Calc.})}$	M_s(Test) lb. in.	$\frac{M_s(\text{Test})}{M_s(\text{Calc.})}$
4-21b	5,600	1.86	19	5,540	1.11	———	———
4-22a	4,620	1.86	19	4,820	1.04	———	———
4-22b	4,600	1.86	19	5,270	1.14	———	———
4-23a	4,670	1.86	19	4,860	1.05	———	———
4-23b	4,670	1.86	19	5,060	1.09	———	———
5-7a	4,680	2.89	19	———	———	89,400	0.90
5-7b	4,680	2.89	19	———	———	89,000	0.90
5-9a	4,470	2.89	19	———	———	87,900	0.90
5-9b	4,470	2.89	19	———	———	89,800	0.92
5-21a	4,670	2.89	19	6,490	1.15	———	———
5-21b	4,670	2.89	19	6,180	1.10	———	———
5-22a	4,520	2.89	19	5,040	0.91	———	———
5-22b	4,520	2.89	19	5,830	1.04	———	———
5-23a	4,650	2.89	19	5,510	0.98	———	———
5-23b	4,650	2.89	19	5,250	0.94	———	———

*In computing V_c, $a' = (a + 1)$ in. is used instead of a.

TABLE 2

DATA AND RESULTS OF ANALYSIS OF MOODY'S BEAMS

Beam No.	f'_c psi	p per cent	b in.	d in.	a' in.	V_c(Test) lb.	$\frac{V_c\text{(Test)}}{V_c\text{(Calc.)}}$	M_s(Test) lb. in.	$\frac{M_s\text{(Test)}}{M_s\text{(Calc.)}}$
A1	4,400	2.17	7	10.30	25	13,000	0.89	425,000	0.81
A2	4,500	2.15	7	10.50	25	15,000	1.03	473,000	0.88
A3	4,500	2.22	7	10.55	25	14,000	0.96	535,000	0.95
A4	4,570	2.37	7	10.63	25	15,000	0.98	505,000	0.82
B1	3,070	1.62	7	10.50	25	11,450	0.99	399,000	1.15
B2	3,130	1.63	7	10.55	25	13,500	1.13	426,000	1.19
B3	2,790	1.60	7	10.63	25	12,000	1.07	394,000	1.15
B4	2,430	1.66	7	10.69	25	11,900	1.13	394,000	1.18
C1	920	0.81	7	10.55	25	4,500	0.87	142,000	0.97
C2	880	0.83	7	10.70	25	5,500	1.07	174,000	1.16
C3	1,000	0.80	7	10.75	25	5,700	0.99	179,000	1.16
C4	980	0.82	7	10.80	25	5,650	1.01	178,000	1.14
1	5,320	1.89	6	10.56	40	11,500	0.97	468,000	1.05
2	2,420	1.89	6	10.56	40	7,500	0.87	288,000	0.88
3	3,740	1.89	6	10.56	40	11,500	1.09	423,000	1.11
4	2,230	1.89	6	10.56	40	8,500	1.03	356,000	1.06
5	4,450	1.89	6	10.56	40	10,500	0.95	422,000	1.02
6	2,290	1.89	6	10.56	40	7,500	0.92	284,000	0.92
7	4,480	1.89	6	10.56	40	10,000	0.90	414,000	1.00
8	1,770	1.89	6	10.56	40	7,000	0.97	252,000	0.86
9	5,970	1.89	6	10.56	40	11,500	0.93	432,000	0.91
10	3,470	1.89	6	10.56	40	9,500	0.94	316,000	1.05
11	5,530	1.89	6	10.56	40	11,000	0.89	486,000	1.07
12	2,930	1.89	6	10.56	40	10,500	1.12	382,000	1.09
13	5,480	1.89	6	10.56	40	10,000	0.82	450,000	1.00
14	3,270	1.89	6	10.56	40	9,700	0.98	349,000	0.95
15	5,420	1.89	6	10.56	40	11,500	0.95	414,000	0.90
16	2,370	1.89	6	10.56	40	8,000	0.94	306,000	0.95

TABLE 3

DATA AND RESULTS OF ANALYSIS OF CLARK'S, LAUPA'S, RICHART'S, AND GASTON'S BEAMS

Beam No.	f'_c psi	p per cent	b in.	d in.	M_s(Test) lb. in.	$\frac{M_s\text{(Test)}}{M_s\text{(Calc.)}}$
Reference 4 - Clark						
AO-1	3,120	0.98	8	15.37	468,000	0.96
AO-2	3,770	0.98	8	15.37	288,000	1.06
BO-1	3,420	0.98	8	15.37	423,000	1.04
BO-2	3,470	0.98	8	15.37	356,000	0.81
BO-3	3,410	0.98	8	15.37	422,000	1.09
CO-1	3,580	0.98	8	15.37	284,000	1.18
CO-3	3,420	0.98	8	15.37	414,000	1.16
DO-1	3,750	0.98	8	15.37	252,000	1.06
DO-3	3,760	0.98	8	15.37	432,000	1.07
Reference 5 - Laupa, Siess and Newmark						
S-2	3,900	2.08	6	10.58	458,000	1.05
S-3	4,690	2.52	6	10.44	574,000	1.14
S-4	4,470	3.21	6	10.37	600,000	1.14
S-5	4,330	4.11	6	10.31	538,000	0.97
S-11	2,140	1.90	6	10.51	365,000	1.18
S-13	3,800	4.11	6	10.31	538,000	1.03
Reference 8 - Richart						
291.1	1,690	1.65	8	10	304,000	0.98
291.2	1,690	1.65	8	10	270,000	0.97
291.3	1,690	1.65	8	10	333,000	1.07
294.1	1,490	1.65	8	10	300,000	1.00
294.2	1,490	1.65	8	10	243,000	0.81
293.4	2,350	1.65	8	10	329,000	0.93
293.5	2,350	1.65	8	10	417,000	1.17
221.1	4,076	2.33	8	21	2,690,000	1.14

TABLE 3 (CONTINUED)

Beam No.	f'_c psi	p per cent	b in.	d in.	M_s(Test) lb. in.	$\frac{M_s(\text{Test})}{M_s(\text{Calc.})}$
221.2	3,696	2.33	8	21	2,670,000	1.18
222.1	4,522	2.33	8	21	2,980,000	1.19
222.2	4,337	2.33	8	21	2,270,000	0.93
Reference 9 - Richard and Jensen						
1	4,760	2.80	8	21	2,290,000	0.83
2	4,620	2.80	8	21	2,550,000	0.93
3	4,290	2.80	8	21	2,430,000	0.91
4	3,860	2.80	8	21	2,150,000	0.84
5	2,230	2.80	8	21	1,690,000	0.83
6	2,630	2.80	8	21	1,865,000	0.84
Reference 10 - Gaston, Siess and Newmark						
T2Ma	4,320	1.38	6	10.58	332,000	0.96
T2Mb	4,020	1.38	6	10.58	352,000	1.05
T2Mc	4,470	1.90	6	10.58	450,000	1.08

TABLE 4

RESULTS OF FATIGUE TESTS

Beam No.	f'_c psi	p per cent	P lb.	P/P_c	Cracking Life Cycles	P/P_s	Failure Life Cycles
4-3*	4,580	1.86	5,000	0.54	519,200	0.61	2,114,500
4-6	4,250	1.86	7,000	0.78	400	0.89	800
4-7	4,650	1.86	8,000	0.87	200	0.98	200
4-8	4,640	1.86	6,000	0.65	425,000	0.74	557,000
4-9	3,910	1.86	6,000	0.69	1,500	0.79	16,800
4-11	3,950	1.86	4,270	0.49	292,800	0.56	1,900,200
4-12	4,290	1.86	5,000	0.56	5,700	0.63	1,142,100
4-14	5,410	1.86	5,500	0.56	8,000	0.63	19,300
4-15	2,150	1.86	5,500	0.87	90	0.91	1,940
4-16	4,240	1.86	6,750	0.76	280	0.86	440
4-17	4,300	1.86	7,130	0.79	80	0.90	360
4-19	5,200	1.86	4,920	0.53	6,690,200	0.57	6,690,200
4-20	5,630	1.86	4,520	0.45	---------	0.49	---------
4-24	4,600	1.86	5,500	0.60	135,100	0.68	4,822,400
4-25	4,280	1.86	5,700	0.63	1,000	0.72	1,097,300
4-26*	4,680	1.86	5,500	0.59	97,500	0.67	1,493,600
4-27	5,120	1.86	5,750	0.61	700	0.67	1,250,400
4-28	5,430	1.86	6,250	0.65	500	0.70	578,800
4-29	5,360	1.86	6,000	0.62	1,207,600	0.69	1,207,600
1-5	2,890	2.89	5,500	0.61	63,400	0.63	1,027,200
2-5	2,860	2.89	5,000	0.56	976,100	0.58	976,100
3-5	3,630	2.89	6,000	0.60	418,600	0.64	467,200
5-1	6,750	2.89	6,500	0.53	---------	0.54	---------
5-3	4,730	2.89	7,500	0.68	1,500	0.72	23,200
5-4	5,360	2.89	8,000	0.70	1,700	0.76	1,700
5-5	4,670	2.89	7,400	0.68	1,300	0.72	402,900
5-6	5,040	2.89	6,500	0.58	71,200	0.61	15,871,700

TABLE 4 (CONTINUED)

Beam No.	f'_c psi	p per cent	P lb.	P/P_c	Cracking Life Cycles	P/P_s	Failure Life Cycles
5-3	5,000	2.89	7,000	0.62	202,800	0.66	11,217,700
5-10	2,160	2.89	7,000	0.90	100	0.91	100
5-11	3,590	2.89	6,000	0.61	39,800	0.64	39,800
5-12	4,060	2.89	8,000	0.77	530	0.81	530
5-13	4,440	2.89	6,000	0.56	1,133,100	0.59	3,666,500
5-14	5,340	2.89	7,000	0.61	13,000	0.64	13,000
5-15	4,810	2.89	6,000	0.54	239,000	0.57	239,000
5-16	4,000	2.89	6,500	0.63	2,100	0.67	4,300
5-17	5,190	2.89	6,500	0.57	87,800	0.60	87,800
5-18	Broke on First Loading						
5-19	Broke on First Loading						
5-20	4,740	2.89	8,880	0.81	170	0.85	900

*Reinforcing bar broke in pure moment region.

TABLE 5

DAMAGE TESTS AND COMPARISON OF RESULTS OF BROKEN BEAMS

Beam No.	f'_c psi	p per cent	P_c(Meas.) lb.	P_s(Meas.) lb.	P_c(Meas.)/P_c(Calc.)	P_s(Meas.)/P_s(Calc.)
4-3	4,580	1.86	———	9,070	——	1.12
4-6	4,250	1.86	———	8,170	——	1.04
4-7	4,650	1.86	———	9,800	——	1.19
4-8	4,640	1.86	———	6,620	——	0.81
4-9	3,910	1.86	———	8,600	——	1.13
4-11	3,950	1.86	———	8,010	——	1.05
4-12	4,290	1.86	———	8,200	——	1.04
4-15	2,150	1.86	———	6,890	——	1.14
4-17	4,300	1.86	———	7,960	——	1.01
4-19	5,200	1.86	9,300	———	1.00	——
4-20	5,630	1.86	9,720	10,300	0.99	1.15
4-24	4,600	1.86	10,470	———	1.14	——
4-25	4,230	1.86	———	9,160	——	1.16
4-28	5,430	1.86	———	9,580	——	1.08
4-29	5,360	1.86	10,970	———	1.13	——
1-5	2,890	2.89	———	7,420	——	0.85
2-5	2,860	2.89	8,000	8,320	0.89	0.97
3-5	3,630	2.89	———	9,700	——	1.03
5-1	6,750	2.89	13,290	———	1.09	——
5-3	4,730	2.89	———	12,250	——	1.18
5-4	5,360	2.89	———	9,250	——	0.83
5-5	4,670	2.89	———	9,980	——	0.97
5-6	5,040	2.89	———	10,300	——	0.96
5-8	5,000	2.89	———	8,600	——	0.81
5-11	3,590	2.89	9,100	9,150	0.93	0.98
5-12	4,060	2.89	9,170	9,310	0.88	0.95
5-13	4,440	2.89	———	10,250	——	1.01
5-14	5,340	2.89	———	8,930	——	0.81

TABLE 5 (CONTINUED)

Beam No.	f'_c psi	p per cent	P_c(Meas.) lb.	P_s(Meas.) lb.	$\frac{P_c(\text{Meas.})}{P_c(\text{Calc.})}$	$\frac{P_s(\text{Meas.})}{P_s(\text{Calc.})}$
5-15	4.810	2.89	10,030	10,610	0.90	1.02
5-17	5,190	2.89	11,770	———	1.03	———
5-19	3,950	2.89	8,770	19,000	0.87	0.92
5-20	4,740	2.89	———	11,750	———	0.91

TABLE 6

(a) DIMENSIONAL MATRIX FOR THE FUNCTION OF THE SHEAR FORCE FOR INITIAL DIAGONAL TENSION CRACKING

	V_c	f_c'	p	d	a	b	E_s	d'
F	1	1	0	0	0	0	1	0
L	0	-2	0	1	1	1	-2	1

$$\begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix} = 1 \neq 0$$

(b) SOLUTION MATRIX FOR THE FUNCTION OF THE SHEAR FORCE FOR INITIAL DIAGONAL TENSION CRACKING

	V_c	f_c'	p	d	a	b	E_s	d'
$V_c/E_s d^2$	1	0	0	-2	0	0	-1	0
p	0	0	1	0	0	0	0	0
b/d	0	0	0	-1	0	1	0	0
a/d	0	0	0	-1	1	0	0	0
d/d'	0	0	0	1	0	0	0	-1
f_c'/E_s	0	1	0	0	0	0	-1	0

$$\begin{vmatrix} 0 & -2 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{vmatrix} = 2 \neq 0$$

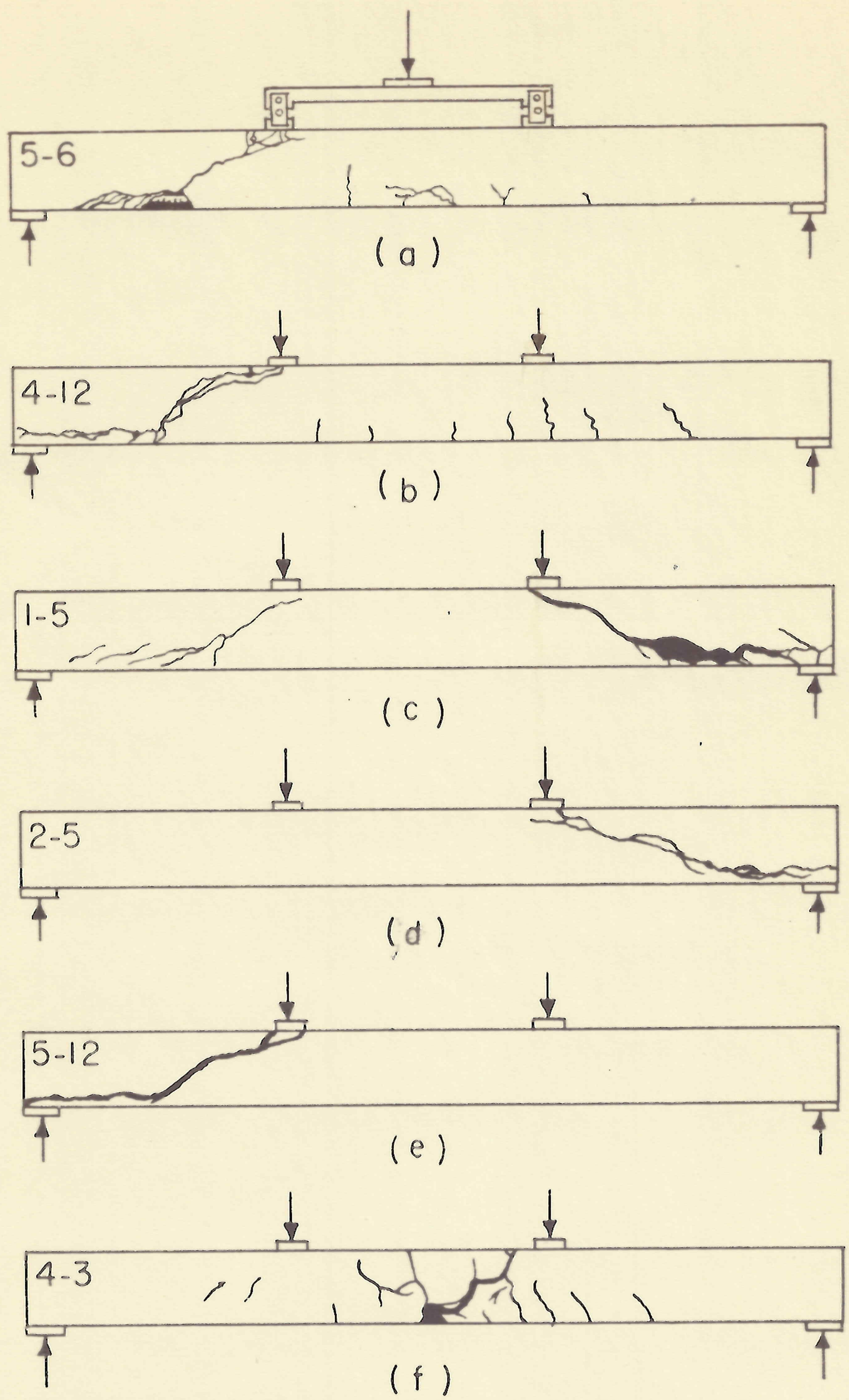

Fig. 1 Typical Fatigue Failures

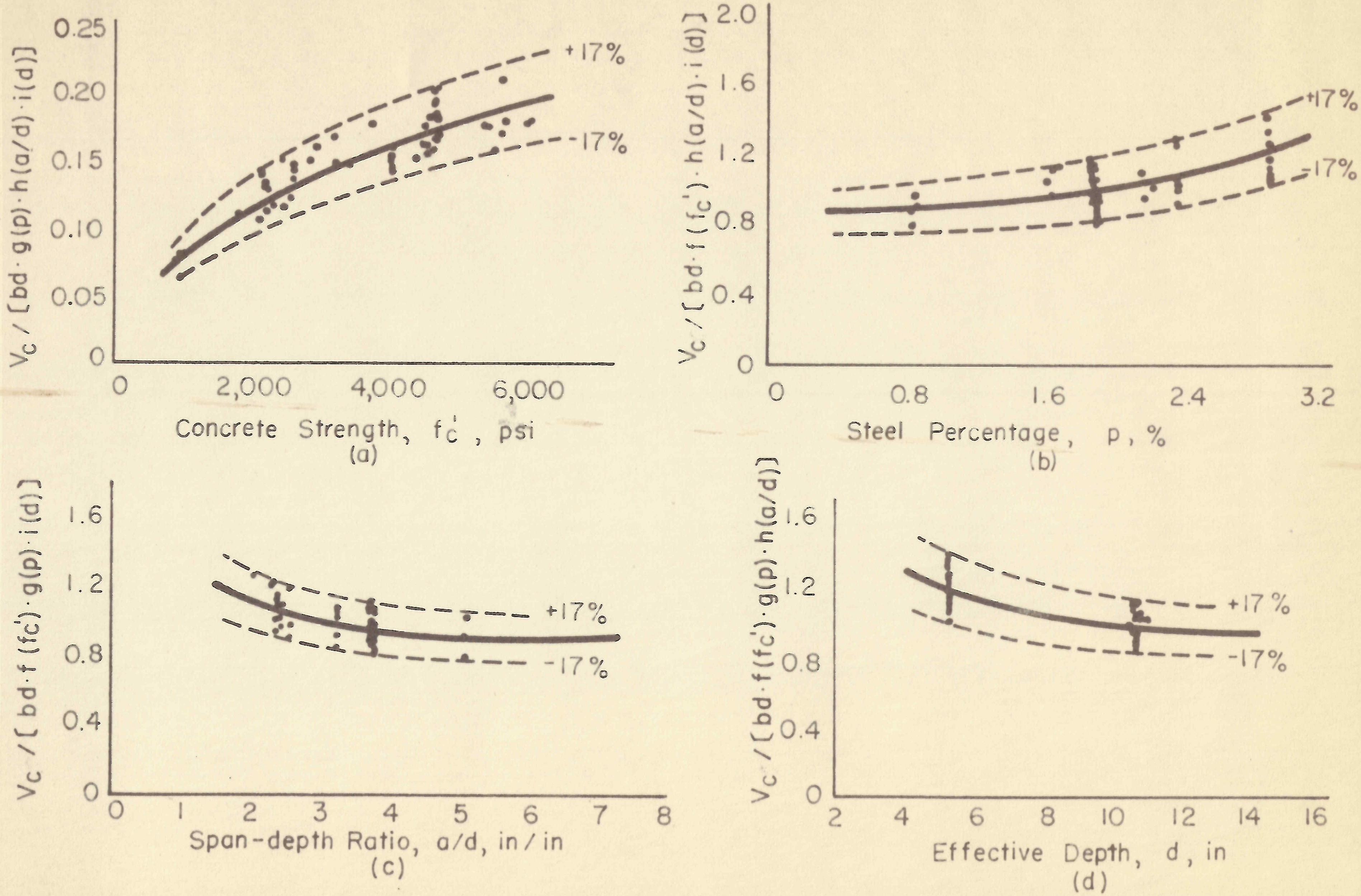

Fig. 2 Functional Dependence of the Cracking Load with Concrete Strength, Steel Percentage, Span-Depth Ratio and Effective Depth.

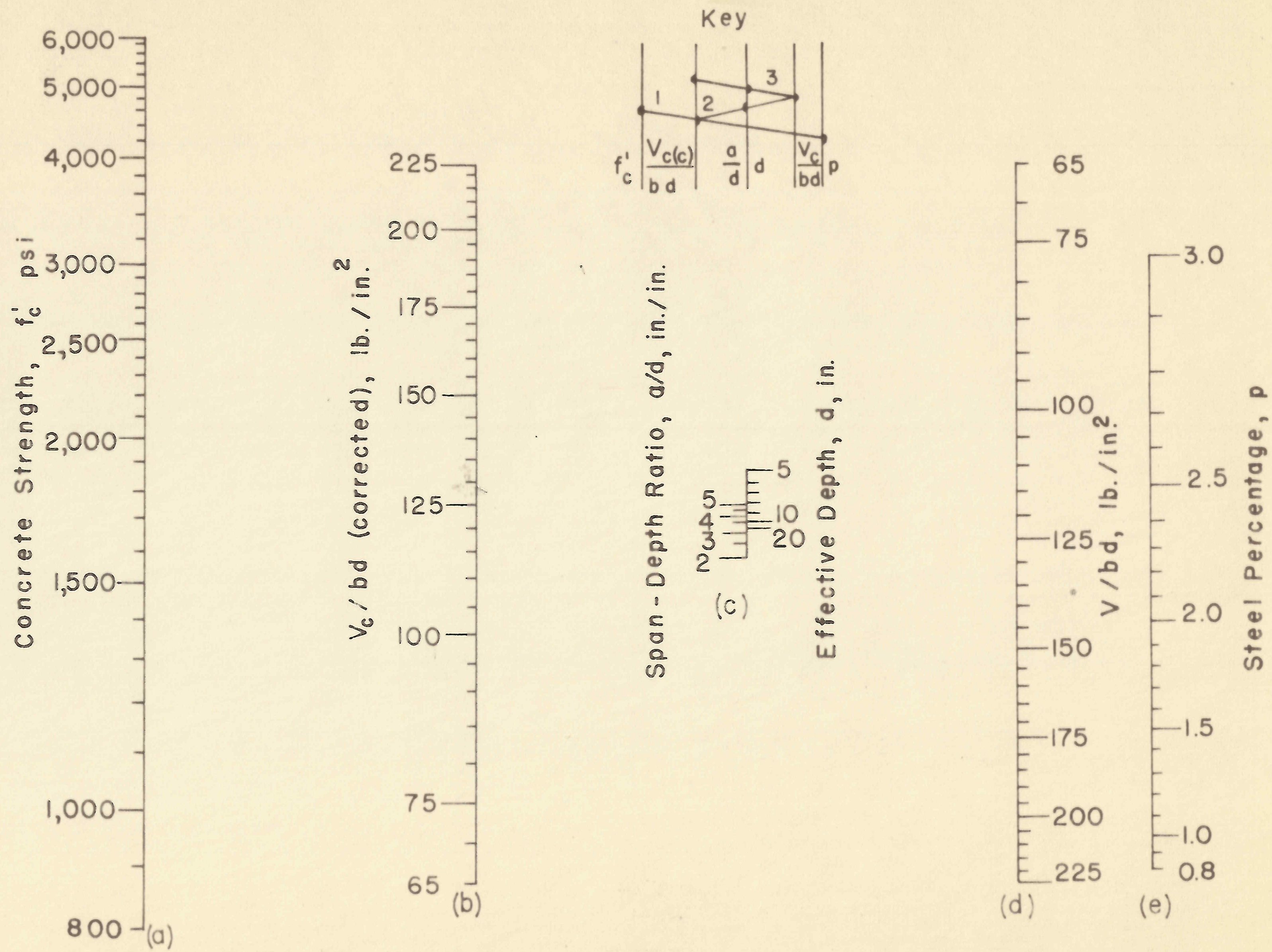

Fig. 3 Nomograph for Predicting the Cracking Load of Simply Supported Reinforced Concrete Beams without Web and Compressed Reinforcement.

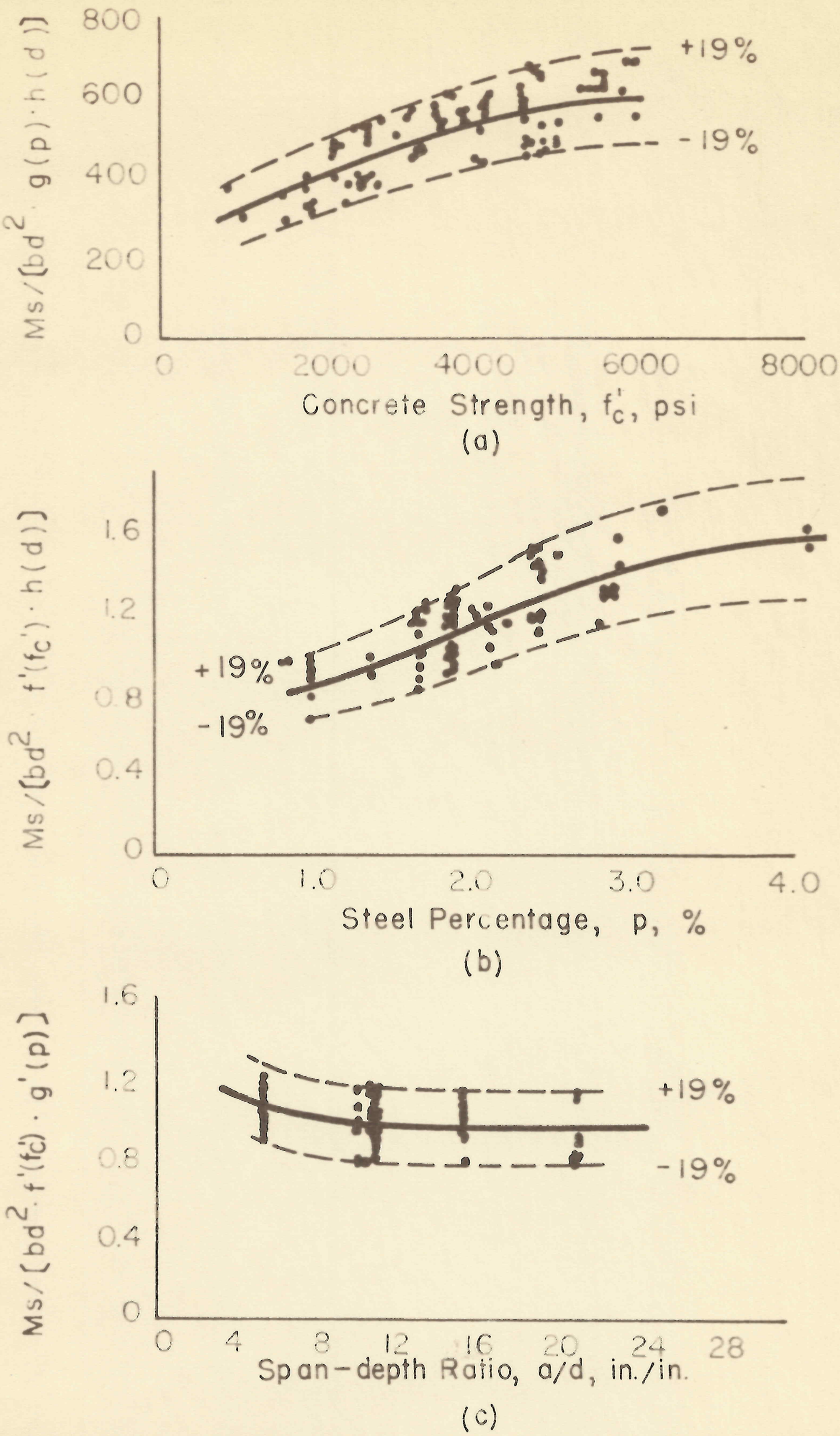

Fig. 4 Functional Dependence of the Shear-Moment with Concrete Strength, Steel Percentage and Effective Depth.

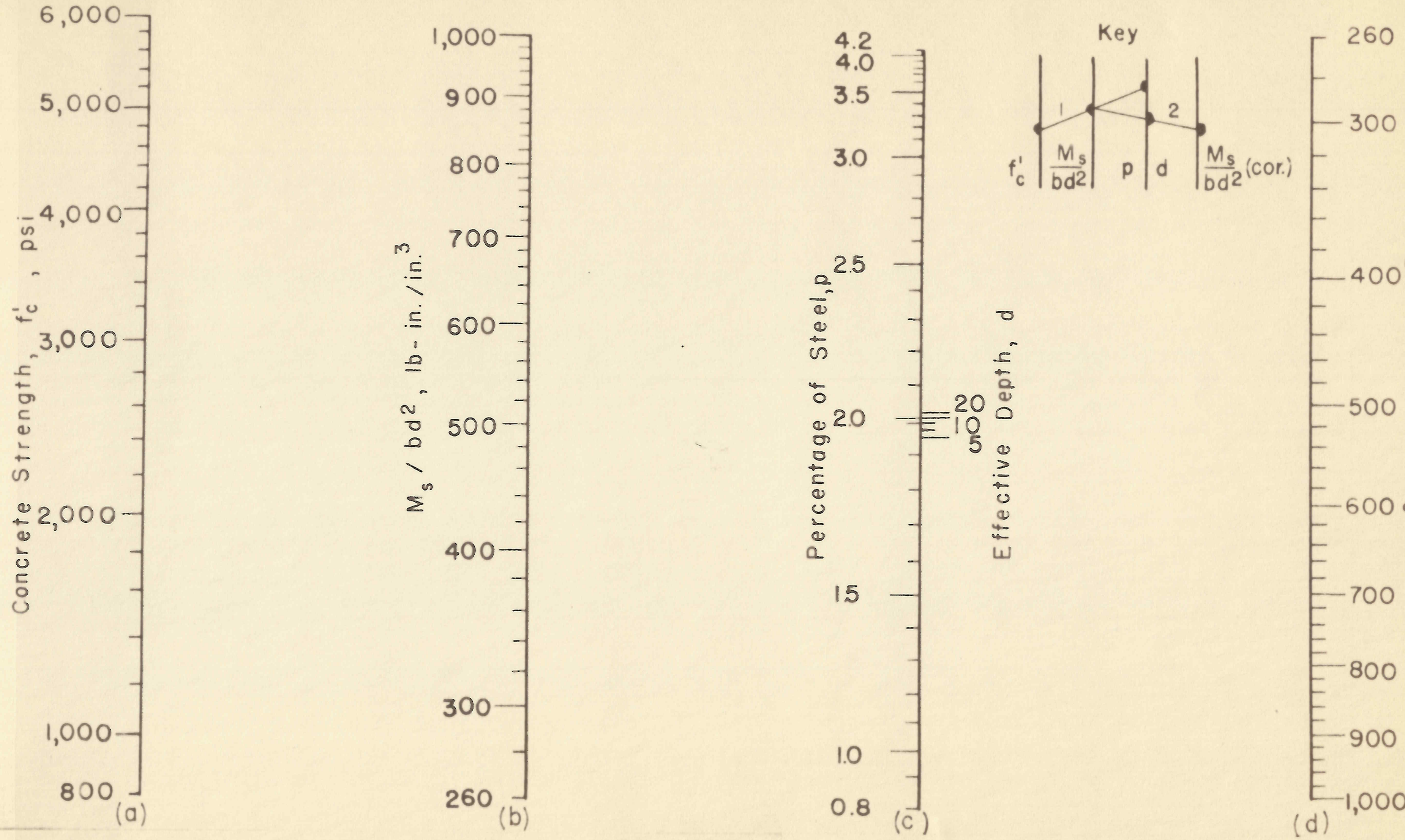

Fig. 5 Nomograph for Predicting the Shear Moment of Simply Supported Reinforced Concrete Beams without Web and Compressive Reinforcement.

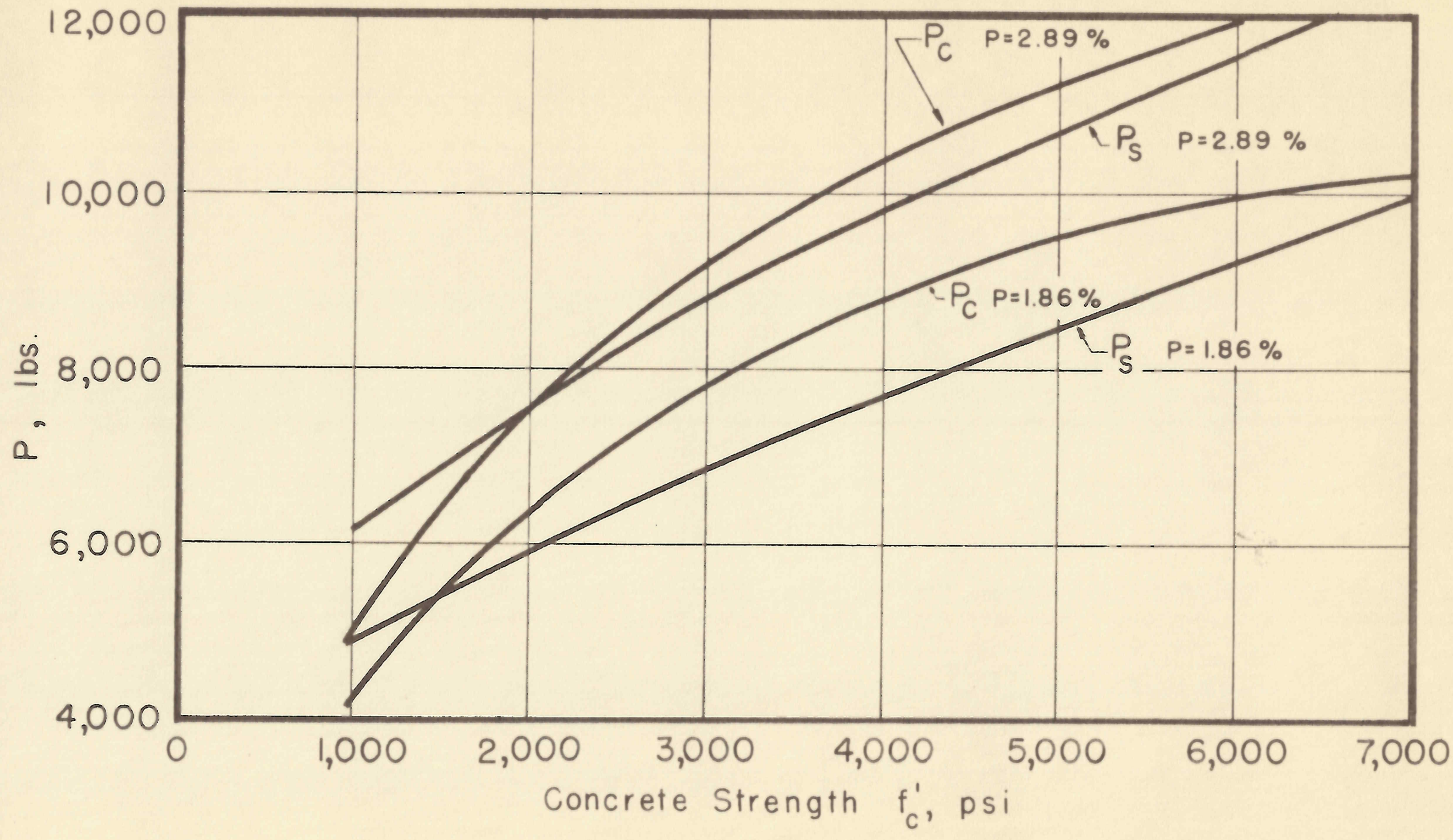

Fig. 6 Comparison of Load Carrying Capacities of Beams with Respect to Concrete Strength and Steel Percentage.

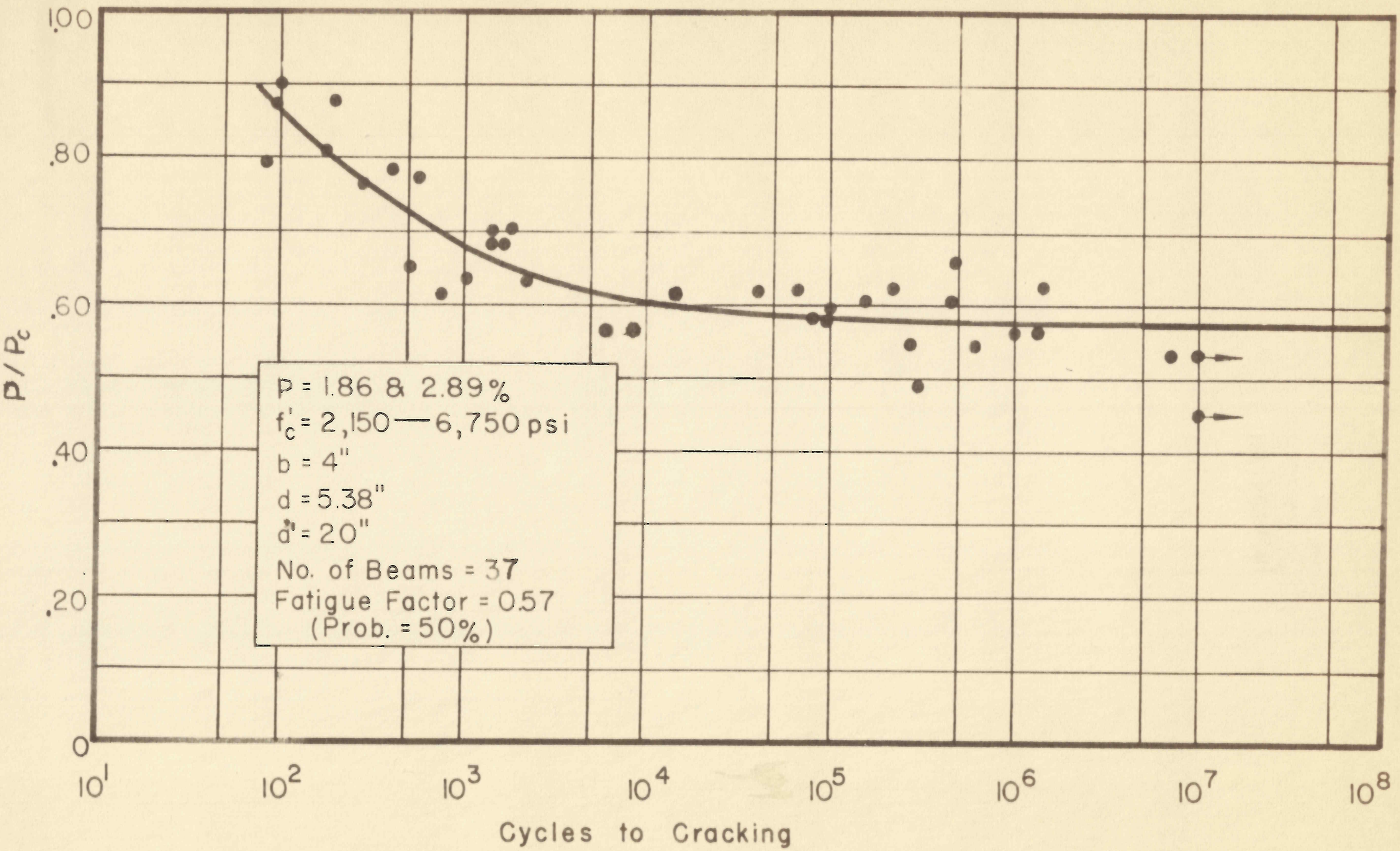

Fig. 7 Statistical Study of the Fatigue Strength of Beams against Diagonal Cracking

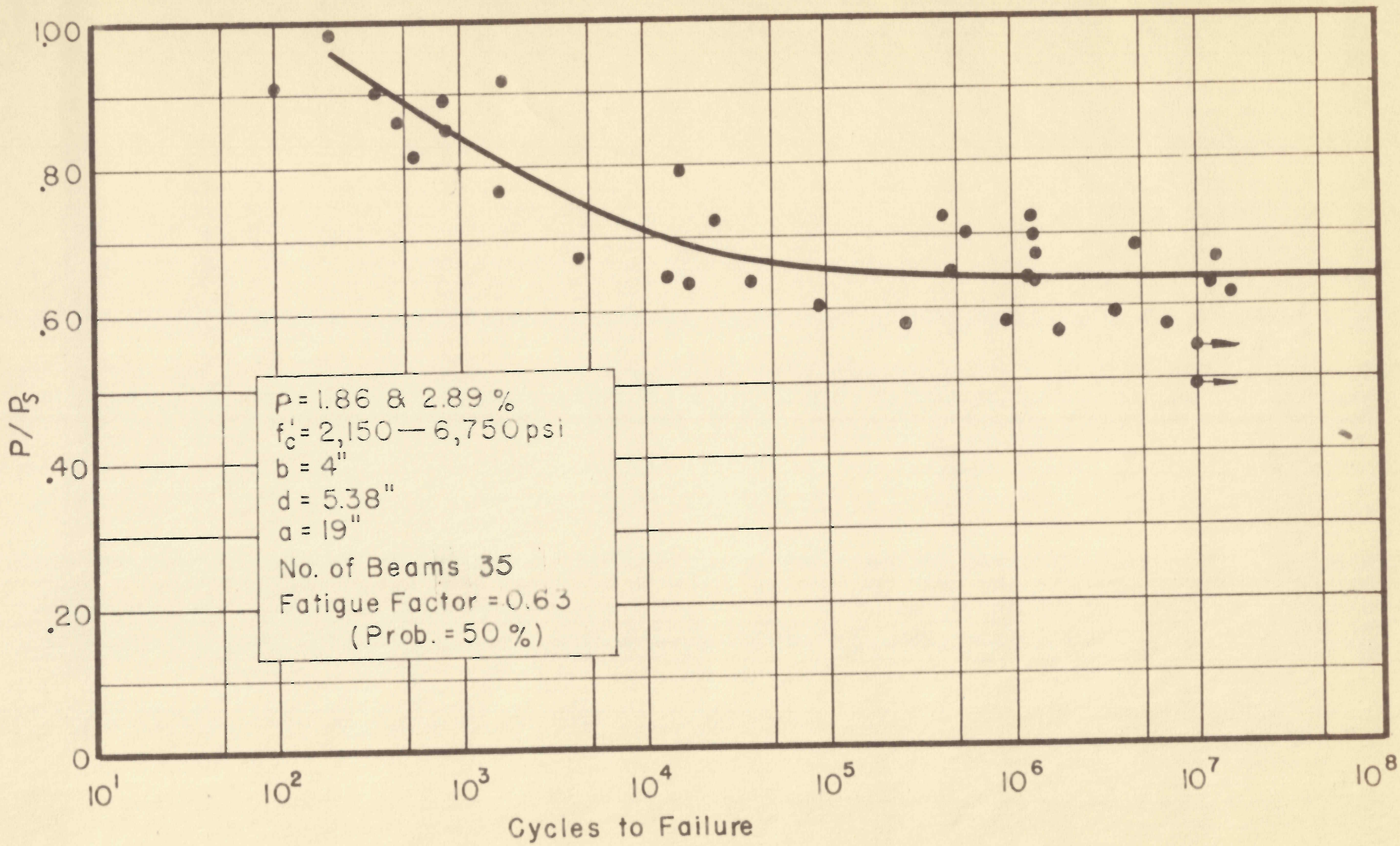

Fig. 8 Statistical Study of the Fatigue Strength of Beams against Failure.

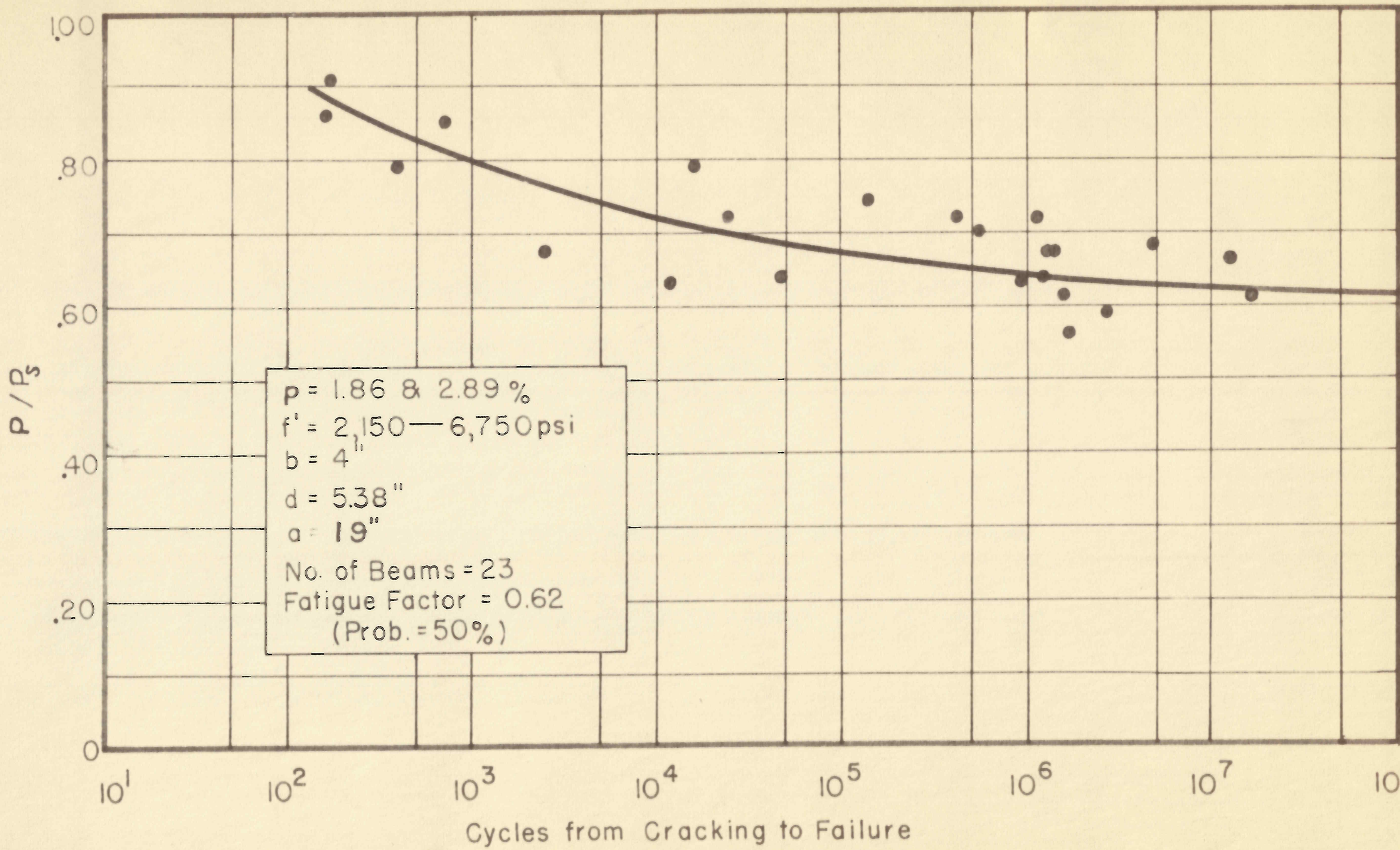

Fig. 9 Statistical Study of the Fatigue Lives of Beams against Failure after Cracking