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EFFECT OF RANGE OF STRESS ON FATIGUE STRENGTH OF PLAIN CONCRETE BEAMS

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SUMMARY

From the results of tests of 175 plain concrete beams subjected to repeated flexural loading, it was found that plain concrete exhibits no fatigue limit when subjected to loads which produce no reversals of stress. Fatigue strengths at ten million repetitions of stress were determined for each of the several ranges of stress investigated, and these strengths were found to be dependent on the range of stress to which the specimens were subjected.

The test results are in good agreement with previous investigations insofar as valid comparisons may be made. The need for additional research is, however, most apparent; the results obtained from this investigation do not permit the definition of the fatigue behavior of plain concrete subjected to reversals of stress, nor do they permit a valid description of the process by which fatigue failures occur.

INTRODUCTION

The increased application of concrete as an engineering material demands an additional knowledge of its behavior under the several types of loading to which it is subjected. Such knowledge is necessary not only to provide safe, efficient, and economical designs for the present, but to serve as a rational basis for the conception of improved and extended applications for the future.

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The behavior of plain concrete when subjected to repeated or fatigue loading can only be approximated. The existence of a fatigue limit has not been established, nor have fatigue strengths been defined for any finite life of the material. The effects of service-encountered variables which may alter the behavior may only be inferred, and the mechanism of failure is not clearly defined.

The controlled laboratory investigation affords an effective tool for the determination of this information.

Scope

The discussion of the investigation reported in this paper will be confined essentially to the following points:

(a) The establishment of a fatigue limit for repeated loading, or, if no such limit exists, the establishment of fatigue strengths for probable finite lives.

(b) The effect of the range of stress on the behavior of plain concrete beams subjected to repeated flexural loading.

PREVIOUS INVESTIGATIONS

Although the previous investigations which have been made to determine the fatigue behavior of plain concrete are relatively few in number and limited in extent, they do indicate a pattern of behavior to which the results of more recent investigations may be compared.

Repeated load tests made on tension briquets by DeJoly and reviewed by Mills and Dawson (1)[2], together with repeated load tests on compression specimens by Van Ornum (2, 3), indicate the possible existence of a fatigue limit of the material.

Repeated load tests on compression specimens by Probst and Mehmel as reported by Moore and Kommers (4) indicate a fatigue limit for repeated compressive

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[2] The numbers in parentheses refer to the list of references appended to this paper.

loading which lies between 47 and 60 per cent of the static ultimate compressive strength. In addition they indicate a change in the fatigue limit with an alteration of the range of stress.

Investigations which indicate the behavior of plain concrete subjected to repeated flexural loading have been reported by Clemmer (5), Hatt and Crepps (6, 7, 8), Williams (9), and Kesler (10). These investigations more closely approximate that reported in this paper and will therefore be summarized in somewhat greater detail.

The tests reported by Clemmer were conducted by the Illinois Division of Highways to determine the extent to which fatigue loads affect concrete pavements. In these tests an effort was made to duplicate the manner in which the fatigue loads are applied to the pavement slabs.

The test specimens were plain concrete cantilever beams subjected to wheel loads applied at a frequency of 40 cpm which produced stresses which varied from zero to a maximum. Four series of tests were made and both quality of the concrete and the loading pattern were included as variables in the tests. Although an attempt was made to minimize their effects, it is probable that impact and torsion affected the behavior to some degree. Seven specimens were tested simultaneously, new specimens replacing their predecessors as failures occurred. The magnitude of the load was varied in some of the tests and it was noted that specimens which had sustained a number of repetitions of load at a particular magnitude withstood a greater number of repetitions of load at an increased magnitude than did specimens which were initially loaded at the greater load.

Subject to additional tests, Clemmer concluded that the fatigue phenomenon does exist in concrete and he tentatively assigned a fatigue limit between 51 and 54 per cent of the static modulus of rupture, though it is to be noted that this limit was based on an arithmetic plot of the data obtained. Clemmer also concluded that fatigue resistance is increased by repeated applications of load at a level somewhat below the fatigue limit. He noted decreased resistance in lean mixes and a reduction in the number of applications of load required for failure when the applied stress exceeded the fatigue limit.

The fatigue investigation by Hatt and Crepps at Purdue University was concurrent with that by Clemmer in Illinois. The Purdue tests, like those at Illinois, were conducted to provide data which might be utilized in the design of concrete pavements. The significant differences between the investigations lay in the type of loading, the frequency of application, the duration of loading, and the time interval between periods of loading.

In reviewing the Illinois tests, Hatt considered the high frequency of application, 40 cpm, and the absence of rest periods to be a significant departure from service conditions. He also noted that pavements are subjected to loads which produce reversals of stress. As a consequence, the Purdue tests were devised to place primary emphasis on the type and frequency of loading rather than the mode of application.

The test specimens were mortar beams tested as vertical cantilevers loaded by an alternating couple which produced reversed stresses of equal magnitude throughout the length of the test section. The frequency of application was 10 cpm, and the loading was generally halted overnight and throughout the weekends although some rest periods were of a greater duration.

It was concluded that for specimens which were at least four months old, the fatigue limit was between 50 and 55 per cent of the static ultimate strength. For specimens tested at 28 days the fatigue limit could only be said to lie between 40 and 60 per cent of the static ultimate strength. It is again noted that the fatigue limits were based on an arithmetic plot of the data obtained, however, it is also of interest to note that the results of these tests are in agreement with the Illinois tests although the range of stress was twice as great.

As in the Illinois tests, it was found that the fatigue limit may be raised by applications of stress of an intensity less than the fatigue limit of specimens which have had no previous loading history. On the whole, the Purdue tests tend to corroborate the findings reported by Clemmer.

A limited series of tests was made by Williams at Stanford University to determine the fatigue resistance of concrete made with light-weight aggregates. His test specimens were similar to those used in the Purdue tests and the manner in which the loads were applied was the same, however, the frequency was increased to 15 cpm.

Williams' conducted two series of tests. In the first series the specimens were subjected only to stresses which varied from zero to a maximum, while in the second series the specimens were subjected to complete reversals of stress. Based on a semi-logarithmic plot, no fatigue limit was found in either series, however, a fatigue strength of approximately 50 per cent of the static ultimate strength at 1,000,000 repetitions was found for the first series, while a fatigue strength of 40 per cent of static ultimate was determined in the second.

Williams' results for the first series of tests is in reasonably close agreement with the Illinois tests, while that for reversed loading is substantially less than the fatigue limit for similar loading determined in the Purdue tests. Two factors which may account for a substantial part of the difference are the type of concrete employed and the use of the semi-logarithmic plot rather than an arithmetic plot to interpret the test results. The latter factor may be particularly significant.

The tests by Kesler were made to determine the effect of the speed of testing on the flexural fatigue strength of plain concrete, and were the initial investigation of a comprehensive program now in progress at the University of Illinois. The test specimens employed were plain concrete beams subjected to third-point loading which produced repeated stresses from near zero to a maximum. Specimens were tested at loading frequencies of 70, 230, and 440 cpm.

In none of the series of tests was a fatigue limit indicated; however, fatigue strengths at 10,000,000 repetitions of stress were determined. The average fatigue strengths at loadings of 70, 230, and 440 cpm were, respectively, 65, 62, and 64 per cent of the static ultimate strength. It was concluded that the speed of testing, within the limits of the investigation, had negligible effect on the fatigue strength of plain concrete made with sound aggregate and of a strength normally used in construction.

Although there appear to be certain inconsistencies among the results of the several investigations, they do afford a basis for further tests and provide a guide for the interpretation of additional data. In order that the information may be used effectively it is necessary that the apparent inconsistencies be reconciled or that erroneous information be corrected.

SPECIMENS AND TEST PROCEDURE

Type I portland cement was used in all specimens and the aggregates were well graded Wabash River sand and gravel which passed the usual specification tests. The coarse aggregate had a maximum nominal size of 1 in. and the fineness modulus of the sand was approximately 3.0. Each of the mixes was designed by trial batching to yield concrete with an ultimate compressive strength of approximately 4500 psi. Mixes of dry consistency had average slumps of one inch; those of wet consistency had average slumps of 6 in.

One test specimen and four control cylinders or two control beams were cast from a single batch and the concrete was compacted with mechanical vibrators. Test specimens, cylinders, and control beams remained in the forms for 24 hours; they were then moist cured for a period of 6 days and removed to storage in normal laboratory atmosphere until tested.

All test specimens were plain concrete beams 6 in. square and 64 in. long. The companion specimens were either standard 6 by 12 in. control cylinders or control beams 6 in. square and 21 in. long.

Designation

The specimens used in the preliminary investigations were designated by a numeral preceded by either a letter D or a letter W. The letter denoted either a dry or wet consistency concrete, while the numeral indicated casting sequence: Specimens cast specifically for the investigation reported in this paper were designated by a sequence of letters followed by a numeral. The letter sequences were DH, DHF, and

WHF. The D or W had the same connotation as before, while the H indicated specimens cast specifically for this investigation. The letter F denoted the use of control beams in lieu of control cylinders and numerals again indicated casting sequence.

Loading

All specimens were simply supported on a 60 in. span and subjected to repeated loads of varying range and magnitude applied at the one-third points. The central third of the test specimen was thus subjected to pure bending, the shearing stresses due to the weight of the specimen being negligible.

The range of the repeated loads was systematically varied throughout the investigation. The variation within each series, with one exception, was such as to maintain a constant ratio, R, between the minimum and maximum applied stresses.

Series of tests were made with concrete of both wet and dry consistency in which R had values of 0.25, 0.50, and 0.75. One series of tests was made in which the minimum applied load and stress was constant, R then being a variable throughout the series. This modification was made because the manner in which the loads were applied precluded the reduction of the minimum applied load to zero without the simultaneous introduction of impact loads. An arbitrary minimum load of 100 pounds was used throughout this series.

Equipment

Three testing machines were used in this investigation, one of which is shown schematically in Fig. 1.

The specimens were supported and loaded through a system of rollers and balls to prevent the accidental introduction of torsional forces. The repeated loads were applied by a loading lever through an adjustable loading bar and loading plate. The lever was actuated by a connecting rod attached to a motor-driven variable eccentric.

The range of load was determined by the set in the variable eccentric while the intensity of the load was controlled by the adjustable loading bar and measured by the loading plate which served as a dynamometer. Necessary correction or readjustment of the load during the course of the test was accomplished by varying the length of the connecting rod.

The machines operated continuously, applying between 400 and 440 repetitions of load per minute. In all tests the machines were stopped automatically whenever a specimen failed. The repetitions of load applied to each specimen were recorded by a counter.

Calibration

The loading plates were initially calibrated for each specimen tested. The plate was simply supported and subjected to a static center point load equal to that which was to be applied to the test specimen. The center deflection of the plate was measured with an Ames dial which read to 0.001 in. In the testing machine the variable eccentric and the adjustable loading bar were set to reproduce the static deflection of the plate.

The procedure was inherently tedious and subject to error. The Ames dial was rather easily damaged and the zero setting was subject to constant readjustment. To increase both the ease and accuracy of measurement, two Type A5 SR-4 electric strain gages were applied to the tension and compression surfaces of the loading plate. Shielded leads were attached and both gages were waterproofed with wax which, in addition, afforded some protection from mechanical damage. Strains were measured with a portable strain indicator.

The plates were calibrated statically to a total load in excess of the flexural capacity of the test specimens and calibration curves were drawn. In loading the specimens the eccentric and loading bar were set to reproduce predetermined strain differentials taken from the calibration curves.

Estimation of Ultimate Flexural Strength

An accurate estimate of the ultimate flexural strength of the test specimen was a desirable prerequisite for the selection of a fatigue load; for interpretation of the test results it was essential. The inability to duplicate concrete specimens invalidates the measurement of fatigue resistance solely in terms of the intensity of the applied stress; a more satisfactory measure is made in terms of the static ultimate flexural strength. Once a specimen fails under repeated loading it is obviously impossible to determine the static strength at the point of failure; however, a series of statistical studies by Kesler (11) affords a means of approximating this strength.

For specimens of D, W, and DH designation, the initial estimate of the flexural strength was made from Kesler's statistical relationship between cylinder strength and beam strength. Although statistically sound, when applied to single specimens these estimates were frequently quite erroneous and loads unreasonably high or low were then applied in the tests. In an attempt to reduce the error, companion flexure specimens were substituted for control cylinders in specimens of both DHF and WHF designation. The average flexural strength of the two companion specimens was taken as the estimate of the flexural strength of the test specimen; this procedure reduced, but did not eliminate, the error.

The statistical study also indicated a one-to-one correlation between the modulus of rupture of a specimen tested on a 60 in. span and the same property determined from tests of the broken portions of the specimen tested on an 18 in. span when both types of specimens were subjected to third-point loading. The "true" flexural strength of the test specimens was therefore assumed to be the strength determined from the broken portions of the specimen upon completion of the fatigue test.

For specimens with which no control cylinders were cast, the quality of the concrete was determined from compression tests of the modified cubes obtained from the flexural tests of the specimens. The correlation with cylinder strength was again based on Kesler's statistical studies. It is considered that the error which may have been introduced by the application of statistical correlation was offset by use of the actual specimen in measuring the quality of the concrete.

TEST RESULTS

Notation

The following notation is used throughout the presentation and analysis of the test results:

f_1 = flexural stress at minimum load

f_2 = flexural stress at maximum load

f_r = modulus of rupture as determined from static tests of portions of test specimen after failure under repeated loading

f'_c = ultimate compressive strength as determined from standard 6 by 12 in. control cylinders

$R = f_1/f_2$ = a measure of the range of loading

$F = f_2/f_r$ = fatigue factor

$M = f_1/f_r$ = a measure of the minimum stress

F_{10} = F at ten million repetitions of stress - fatigue strength

All stresses are computed by the usual flexure formula and so represent the theoretical rather than the actual intensities of stress which existed in the specimen.

Results

The data obtained from the tests of the specimens are given in Tables I to IV.

Concrete of both dry and wet consistency was used in these tests. Whenever possible, specimens which had aged a minimum of 90 days were used so that the concrete would not continue to gain strength during the course of the test and so influence the results.

In Fig. 2 to 5 are plotted the data obtained from the four series of tests. These data have been plotted in a manner similar to a conventional S-N diagram, the ordinates to the curve being the fatigue factors, F, and the abscissas the logarithms of the cycles or repetitions of stress sustained prior to failure.

In none of the test series did the data indicate the existence of an endurance limit, although many of the specimens sustained more than ten million repetitions of stress without failure.

It was concluded that a straight line would best represent the curve determined by each set of data, and these lines were established by the method of least squares. In making the computations the cycles to failure were assumed to be the independent variable. The straight lines would, of course, have different slopes were the fatigue factors assumed as the independent variables; however, since there is a greater probability of error in the fatigue factors it was felt that the method employed was the more likely to give a proper analysis of the data. The following groups of specimens were excluded from the computations:

(1) Specimens which did not fail during the course of the test. No accurate estimate could be made of the true position of these specimens on the plot.

(2) Specimens which failed at less than 50 repetitions of stress. It was concluded that many of these failures may have been influenced by other variables and were not representative fatigue failures.

(3) Specimens which were retested under loads of increased intensity. The degree to which the behavior of these specimens was influenced by the previous loading history could not be accurately assessed.

Notations of these exclusions are made in the Tables.

Test Series I

The minimum stress, f_1, was constant for all specimens and equal to 70 psi. The ratio, R, ranged between 0.13 and 0.18.

It will be noted that several of the specimens used in Series I had aged only 40 to 64 days, but, with one exception, the tests in which these specimens were used were completed in less than a day. The test results could not have been influenced by an increase in concrete strength.

A fatigue strength, at 10,000,000 repetitions of stress, of 61 per cent of the static ultimate strength is indicated by the plot in Fig. 2.

Test Series II

In all tests in this series, R was constant and equal to 0.25. The range of loading was thus decreased somewhat from that in Series I.

Specimen DHF-19 was excluded from the computations because it was believed that its value of f_r, as determined from the static tests after failure, was not indicative of its true flexural strength. The inclusion of this specimen would have significantly distorted the trend established by all of the other specimens in this series. A notation of the exclusion is made in Table II.

The fatigue strength, at 10,000,000 repetitions of stress is 63 per cent of the static ultimate strength, only slightly higher than the strength determined from the tests of Series I.

Test Series III

In this series of tests, R was held constant and equal to 0.50, a further reduction in the range of loading.

A fatigue strength, at 10,000,000 repetitions of stress, of 73 per cent of the static ultimate strength is indicated in the plot in Fig. 4. This limit is significantly greater than either of those which was determined in Series I or II.

Test Series IV

For all specimens in this series the ratio R was held constant at 0.75, the minimum range of loading for any of the series of tests.

Specimen WHF-53 has been omitted from the computations. There is an obvious error in the determination of the modulus of rupture, f_r, and the inclusion of this specimen in the data would distort the pattern of behavior determined by the remaining specimens.

It will be observed that the scatter of the data in this series of tests seems somewhat more pronounced than that observed in either Series II or III. This may be due, in part at least, to the fact that all of the specimens were tested at a rather high percentage of their static ultimate strength; no specimen failed at less than 73 per cent of the static ultimate strength. The curve, however, seems to be a reasonable statistical average.

The plot in Fig. 5 indicates a fatigue strength, again at 10,000,000 repetitions of stress, of 85 per cent of the static ultimate strength, a significant increase with a further reduction in the range of stress.

Comparison of Test Results

The four curves are shown for comparison purposes in Fig. 6. The influence of the range of stress, as measured by the ratio R, may be seen; successive fatigue strengths, for R equal to 0.75, 0.50, 0.25, and 0.13 to 0.18, are 85, 73, 63, and 61 per cent, respectively, of the static ultimate strength. The differences between successive fatigue strengths is constantly diminishing, suggesting the possibility of a lower limiting value.

Although concretes of both wet and dry consistency are consolidated in these series, the results are not influenced thereby. Separate computations were made for both series and the differences were negligible.

The value of R is necessarily approximate in Series I. Considering all specimens in the series, the average value of M was 0.11 so as F varied from 0.86 to 0.61, R ranged between the approximate limits of 0.13 to 0.18.

In Fig. 7 the effect of the range of stress on the fatigue limit is shown by the use of a modified Goodman diagram. In this diagram the ordinates to the 45 degree line passing through the origin represent the values of M, the ratio of the minimum stress to the modulus of rupture. Ordinates to the curve at the same abscissas, represent the fatigue strength, F_{10}, taken at 10,000,000 repetitions of stress, which may be obtained with the corresponding value of M. As the minimum stress is made a greater percentage of the ultimate strength, greater fatigue strengths may be obtained for specimens in which no reversals of stress occur.

The data from the tests by Hatt and Crepps afford the only approximation of the fatigue strength when concrete is subjected to reversals of stress. However, the tests summarized at the beginning of the paper indicate that the fatigue limit for concrete subjected to repeated compressive stresses is approximately 50 per cent of f'_c; compressive flexural stresses are but a small percentage of this limit when the flexural tensile stresses reach the critical values described in this paper. It seems reasonable to assume that tensile stresses remain critical as the stresses are reversed and the fatigue strength is therefore approximated as remaining constant at 56 per cent of the static ultimate flexural strength.

The tests by Clemmer and Hatt and Crepps are included in Fig. 7 to show the correlation among the Illinois and Purdue tests and those reported in this paper. Dotted portions of the curve represent regions in which there is limited substantiating data.

ANALYSIS AND CONCLUSIONS

Fatigue Strengths

Fatigue strengths, at ten million repetitions of stress, have been determined as follows:

Ratio of Minimum to Maximum Stress	Fatigue Strength as a Percentage of f_r
0.75	85
0.50	73
0.25	63
0.00	56*
-1.00	56*

* Extrapolated from Fig. 7.

In none of the series of tests was there evidence of a fatigue limit; it must be concluded that no fatigue limit may be assigned for plain concrete of the type tested and within the range of the tests reported in this investigation.

Effect of Range of Stress

In the series of tests reported the range of stress was found to have a significant influence on the fatigue strength. This effect is shown in Figs. 6 and 7.

From the plot in Fig. 7, the following equations, which may be used to approximate the fatigue strength, were derived:

(a) for M and R between 0 and 1,

$$F_{10} = 0.56 + 0.44\ M, \text{ or}$$

$$F_{10} = 1.3/(2.3 - R)$$

(b) for M between -0.56 and 0, and R between -1 and 0

$$F_{10} = 0.56$$

The fatigue limit for reversal of stress is based on the assumption that failure in fatigue under reversed loading is conrolled by tensile failure in the concrete. This assumption gains some measure of support from the tests reported by Hatt and Crepps; however, there is a need for additional tests in which the range of stress varies from a partial reversal to a complete reversal.

Limitations of the Test Procedure

The accuracy with which the fatigue strengths have been established is determined by the accuracy with which the ultimate static flexural strength of the specimens was approximated. In a few of the static tests there were significant differences between the indicated flexural strengths of the two broken portions of the test specimen, and the average of these strengths may not have been indicative of the true flexural capacity of the unbroken specimen. The modulus of rupture, even when determined from static tests which were in close agreement, may have been somewhat different from that of the original specimen. These limitations must have influenced the amount of scatter in the data.

Despite these limitations, the trend of the data in each of the tests is well defined and it is considered that the curves established from these data are representative of the fatigue behavior.

Conclusions

The data obtained from investigation give support to the following conclusions:

(a) Plain concrete, made with sand and gravel aggregates, subjected to repeated flexural loading, exhibits no fatigue limit, at least through ten million repetitions of stress.

(b) The repeated loads which plain concrete may sustain for a finite number of repetitions without failure is a critical percentage of the static ultimate flexural strength. Furthermore, this percentage is a function of the range of stress to which the concrete is subjected.

(c) The data obtained in this investigation, taken in conjunction with that of the Illinois and Purdue tests, tentatively defines the behavior of plain concrete subjected to loadings which produce reversals of stress. Although the agreement among the investigations is generally quite good, it cannot be construed as conclusive. Additional research is therefore needed to define the fatigue behavior in the region where the concrete is subjected to reversals of stress, both partial and complete.

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TABLE I

DATA FOR SERIES I

$R = 0.13$ to 0.18, $f_1 = 70$ psi

Specimen	Age at Test days	f'_c psi	Cycles to Failure	f_2 psi	f_r psi	F
D-2	47	4990	7500	428	572	.748
D-3	51	4110	11,900	390	516	.756
D-4	55	4730	2800	451	549	.821
D-5	91	4040	31,700	401	635	.631
D-6	54	4800	7000	463	536	.864
D-7	89	5740	2,274,300	522	731	.714
D-8	90	5440	1900	567	644	.880
D-9	86	5280	600	569	737	.772
D-10	89	5440	2500	582	697	.835
D-12	90	5330	56	632	718	.883
D-17	100	4910	61,900	402	666	.604
D-23	56	5330	50,400	450	696	.646
D-25*	64	4850	10,622,600	402	663	.606
D-25°	81	4850	8100	516	663	.778
D-34	40	4490	2300	450	520	.865
D-35	40	4200	4300	451	561	.804
D-37	41	4770	1000	390	545	.716
D-38	43	4720	5500	362	514	.704
D-39	43	4680	11,600	390	531	.734
D-58	113	5450	220,900	463	595	.778
D-59	118	5180	1300	571	621	.919
D-60*	119	5300	600	590	693	.851
D-61	120	5300	11,667,900	437	681	.641
D-61°	139	5300	4,724,600	477	681	.700
D-62	119	4580	3,411,300	399	645	.612
D-63*	126	4980	20,666,700	412	659	.625
D-63°	163	4980	8,902,400	435	659	.660
D-64	155	4470	1,759,400	385	518	.743
D-65	169	5680	132,700	509	695	.733
D-66	174	5430	110,400	467	658	.710
D-67	190	5030	31,800	417	655	.637
D-68*	189	4800	16,034,500	381	604	.631
D-68°	215	4800	1,581,200	415	604	.687
D-69*	203	5170	11,331,900	431	672	.641
D-69°	218	5170	203,500	470	672	.699
D-70	252	4690	15,000	378	550	.687
D-72	259	4470	1800	450	637	.706
D-73	262	5140	370	554	667	.831
D-74	265	5550	252	614	764	.804
D-75	281	3950	3,989,000	415	624	.665
D-76	348	4630	2175	489	640	.764
D-77	341	4200	5870	430	663	.649

* Excluded from computations. Did not fail

° Excluded from computations. Previous loading history

(continued)

TABLE I (concluded)

R = 0.13 to 0.18, f_1 = 70 psi

Specimen	Age at Test days	f_c' psi	Cycles to Failure	f_2 psi	f_r psi	F
W-3	49	4070	77,100	390	513	.760
W-5	54	3770	800	401	476	.842
W-6	93	4950	8000	517	626	.826
W-7	86	5240	200	549	670	.819
W-8	89	4880	11,200	504	745	.676
W-9	90	5180	7600	548	680	.806
W-10*	89	4460	14,590,300	467	790	.591
W-10°	116	4460	7,863,900	490	790	.620
W-11	91	5240	600	616	755	.816
W-12	89	5510	67	632	695	.909
W-16	102	4610	3300	451	595	.758
W-17	98	4560	9100	451	641	.704
W-18	104	4420	36,200	379	579	.655
W-19	55	5060	1,817,100	443	607	.730
W-20	56	4740	90,600	379	602	.630
W-21*	54	4580	12,000,000	380	640	.594
W-21[1]	74	4580	8,710,300	443	640	.693
W-21°	88	4580	20,500	517	640	.808
W-23*	76	4650	11,797,200	431	766	.563
W-23°	114	4650	13,355,400	461	766	.602
W-25*	85	4420	21,621,100	403	694	.581
W-27	141	4780	7,042,700	431	675	.639
W-28*	295	3390	10,088,900	362	701	.516
W-28°	353	3390	309,600	434	701	.619
W-29*	295	3600	10,040,700	320	549	.583
W-29°	343	3600	220,900	434	549	.790
W-30	258	4040	30,200	417	659	.633
W-31	258	4080	23,000	445	600	.742
WHF-48	233	4610+	1000	548	706	.776
WHF-49	233	3420+	75	530	572	.927
WHF-57	234	4840+	400	515	572	.900
WHF-58	234	4490+	200	553	701	.789
WHF-64	231	4410+	200	490	559	.877

* Excluded from computations. Did not fail.

° Excluded from computations. Previous loading history.

[1] Excluded from computations. Previous loading history. Did not fail.

+ f_c' determined from statistical correlation. f_c' = 0.85 x modified cube strength.

TABLE II

DATA FOR SERIES II

R = 0.25

Specimen	Age at Test days	f'_c+ psi	Cycles to Failure	f_2 psi	f_r psi	F
DHF-12	217	4830	1600	463	630	.735
DHF-14	217	5210	27,300	468	610	.768
DHF-15*	197	4700	16,041,500	404	595	.680
DHF-16	190	5350	4,837,800	454	699	.698
DHF-17	306	5250	1,013,300	475	760	.625
DHF-18[1]	194	5660	1200	515	601	.856
DHF-19[o]	140	4810	563,200	518	555	.934
DHF-20[2]	154	5090	2	525	637	.824
DHF-21	167	4800	100	455	578	.787
DHF-22*	194	5220	17,239,200	399	713	.560
DHF-23	194	5050	10,500	399	576	.693
DHF-24*	264	5530	12,826,700	408	695	.587
DHF-26*	261	5480	11,586,100	403	708	.569
DHF-27	286	5380	1,519,900	473	710	.666
DHF-28	139	4780	97,600	398	572	.697
DHF-29	138	4490	654,300	364	532	.685
DHF-30	134	5230	12,700	441	553	.797
DHF-31	257	4450	5100	507	610	.831
DHF-32	262	4560	50	548	610	.898
DHF-33	262	4830	100	525	593	.885

* Excluded from computations. Did not fail.

o Excluded from computations. Error in f_r.

1 Excluded from computations. Reloaded at higher stress level after approximately 400,000 repetitions.

2 Excluded from computations. Not true fatigue.

\+ f'_c determined from statistical correlation. $f'_c = 0.85$ x modified cube strength.

TABLE III

DATA FOR SERIES III

R = 0.50

Specimen	Age at Test days	f'_c psi	Cycles to Failure	f_2 psi	f_r psi	F
DH-15*	120	5240	11,198,400	425	668	.636
DH-16*	113	5050	10,462,400	390	618	.631
DH-17*	112	5130	10,163,400	438	660	.664
DH-18*	118	5160	12,430,900	446	635	.702
DH-19*	162	5130	10,678,400	540	703	.768
DH-20*	146	5240	10,015,700	512	720	.709
DH-21*	118	5970	12,477,200	525	768	.684
DH-22	68	5170	20,600	460	557	.823
DH-23	141	5440	5000	630	701	.900
DH-24	139	5330	1,126,000	594	745	.797
DH-26*	125	5420	10,352,000	536	752	.714
DH-27°	97	5930	1	640	641	1.000
DH-28	94	5540	175	639	706	.905
DH-35	111	4400	1250	540	587	.920
DH-36°	112	5370	15	583	643	.907
DH-37	114	5540	67,400	578	676	.855
DHF-1	105	5030+	4400	612	647	.946
DHF-2°	110	3740+	2	600	647	.927
DHF-3	126	4780+	100	600	751	.799
DHF-4	133	3960+	54,300	612	735	.833
DHF-5	121	4870+	25,900	580	731	.793
DHF-6°	128	4900+	1	675	768	.879
W-32*	377	5100	11,894,800	484	660	.733
W-34*	381	4480	10,600,000	490	710	.690
W-35	396	4230	34,960	492	640	.769
W-36	394	4930	2680	570	804	.709
W-37	401	5360	2,880,000	510	716	.712
W-38	395	4560	269,700	487	725	.672
W-39°	392	4890	2	607	610	.995

* Excluded from computations. Did not fail.

° Excluded from computations. Not true fatigue.

\+ f'_c determined from statistical correlation. $f'_c = 0.85 \times$ modified cube strength.

(continued)

TABLE III (concluded)

R = 0.50

Specimen	Age at Test days	f'_c psi	Cycles to Failure	f_2 psi	f_r psi	F
WHF-2	251	4280+	818,100	491	668	.735
WHF-4*	250	4320+	10,384,600	508	639	.795
WHF-5	250	4560+	2000	508	622	.817
WHF-6	259	4560+	15,100	574	722	.795
WHF-7	264	3780+	9950	518	616	.841
WHF-8	261	3940+	90	553	643	.860
WHF-10	265	5290+	14,550	595	689	.864
WHF-11	265	4610+	1000	500	572	.874
WHF-12	265	5370+	1,075,700	540	706	.765

* Excluded from computations. Did not fail.

+ f'_c determined from statistical correlation. f'_c = 0.85 x modified cube strength.

TABLE IV

DATA FOR SERIES IV

R = 0.75

Specimen	Age at Test days	f'_c psi	Cycles to Failure	f_2 psi	f_r psi	F
D-95*	220	5040	19,984,300	506	601	.842
D-96°	197	4240	5	460	550	.836
D-97*	132	5030	10,947,000	459	555	.827
D-98	156	5570	20,450	450	593	.759
D-99	167	4970	1,272,000	421	572	.736
D-100	173	5310	200	524	559	.937
D-101*	169	5520	11,566,100	441	714	.618
D-102*	175	5750	10,488,700	522	622	.839
D-103*	183	5590	14,174,400	540	685	.788
DH-1	99	5220	501	580	589	.985
DH-2	99	4540	3,417,400	523	635	.824
DH-3	100	5630	206,700	576	616	.935
DH-4	100	4120	50,200	495	505	.980
DH-5	99	5150	180,650	540	612	.882
DH-6°	99	4970	3	540	647	.835
DH-7	99	4580	1500	518	526	.985
DH-8°	101	5160	1	554	601	.922
DH-9	93	5240	3,558,700	575	645	.891
DH-10°	93	5290	1	588	597	.985
DH-11	100	4950	64,925	540	576	.938
DH-12	100	5160	9650	558	589	.947
DH-13	93	5570	100	544	672	.810
DHF-9	82	4730+	1580	650	781	.832
DHF-10	78	4810+	1625	672	735	.914
WHF-13*	204	4890+	10,356,200	480	620	.774
WHF-14*	204	5210+	10,377,600	488	706	.691
WHF-17*	229	4980+	11,155,900	575	651	.883
WHF-18	246	4530+	2400	608	693	.877
WHF-19	246	4570+	2000	624	701	.890
WHF-20	245	4150+	100	660	660	1.000

* Excluded from computations. Did not fail.

° Excluded from computations. Not true fatigue.

\+ f'_c determined from statistical correlation. f'_c = 0.85 x modified cube strength.

(continued)

TABLE IV (concluded)

R = 0.75

Specimen	Age at Test days	f'_c psi	Cycles to Failure	f_2 psi	f_r psi	F
WHF-24	215	5120+	3, 409, 600	589	664	.887
WHF-26*	215	5130+	10, 916, 400	574	700	.820
WHF-27	248	4840+	2, 303, 900	585	660	.886
WHF-28	248	4110+	44, 900	550	635	.866
WHF-29	247	5340+	6000	590	601	.982
WHF-30	245	4950+	5000	610	747	.817
WHF-32	239	4850+	2000	606	622	.974
WHF-33°	234	4950+	1	618	660	.936
WHF-34	236	4470+	4000	560	626	.895
WHF-35	236	5250+	51, 400	591	772	.766
WHF-51	228	5000+	200	600	730	.822
WHF-53[1]	229	4980+	5, 266, 800	587	589	.966

* Excluded from computations. Did not fail.
° Excluded from computations. Not true fatigue.
[1] Excluded from computations. Error in f_r.
\+ f'_c determined from statistical correlation. f'_c = 0.85 x modified cube strength.

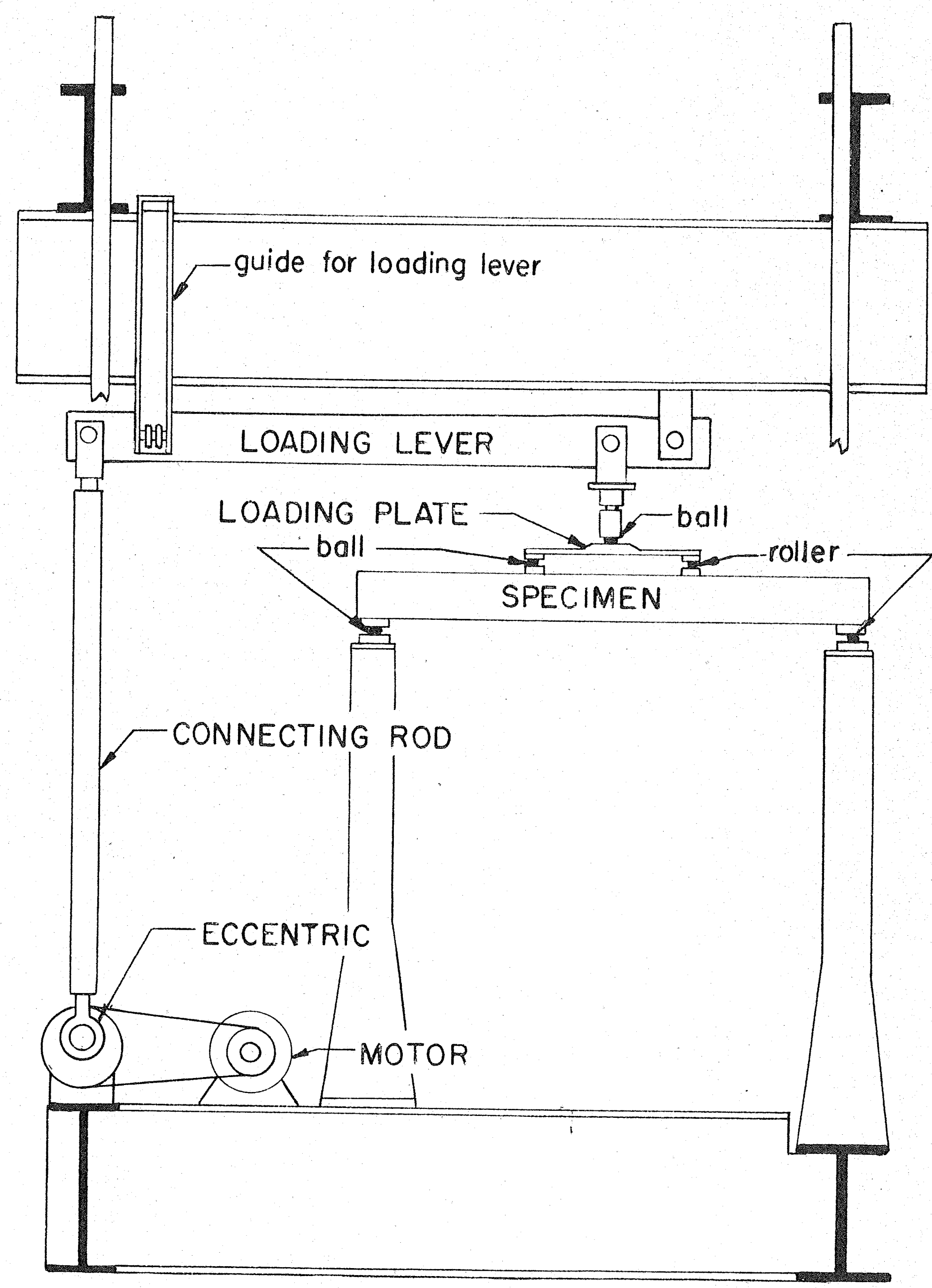

Figure l. SCHEMATIC DIAGRAM OF FATIGUE TESTING MACHINE

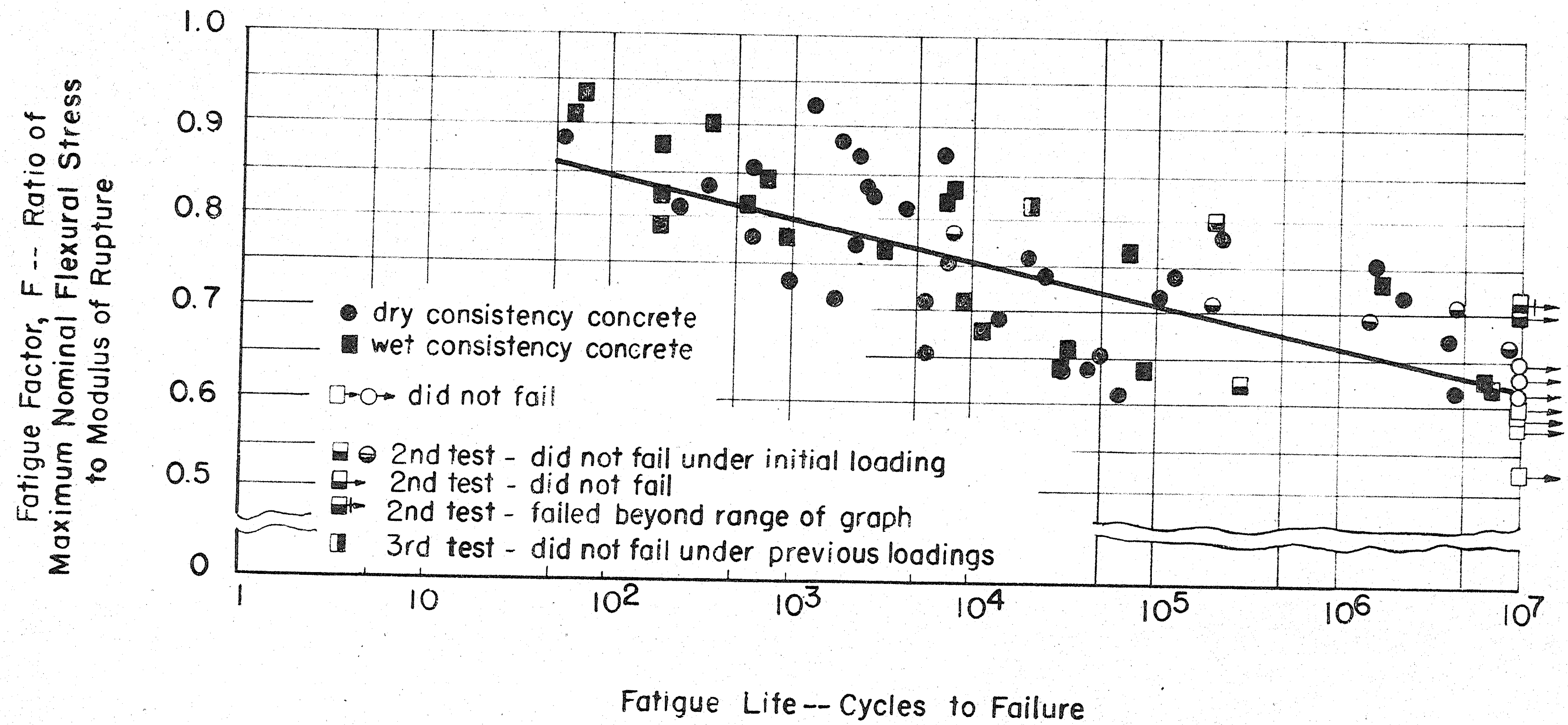

Figure 2. BEHAVIOR OF PLAIN CONCRETE UNDER FATIGUE LOADING -- R = 0.13 to 0.18

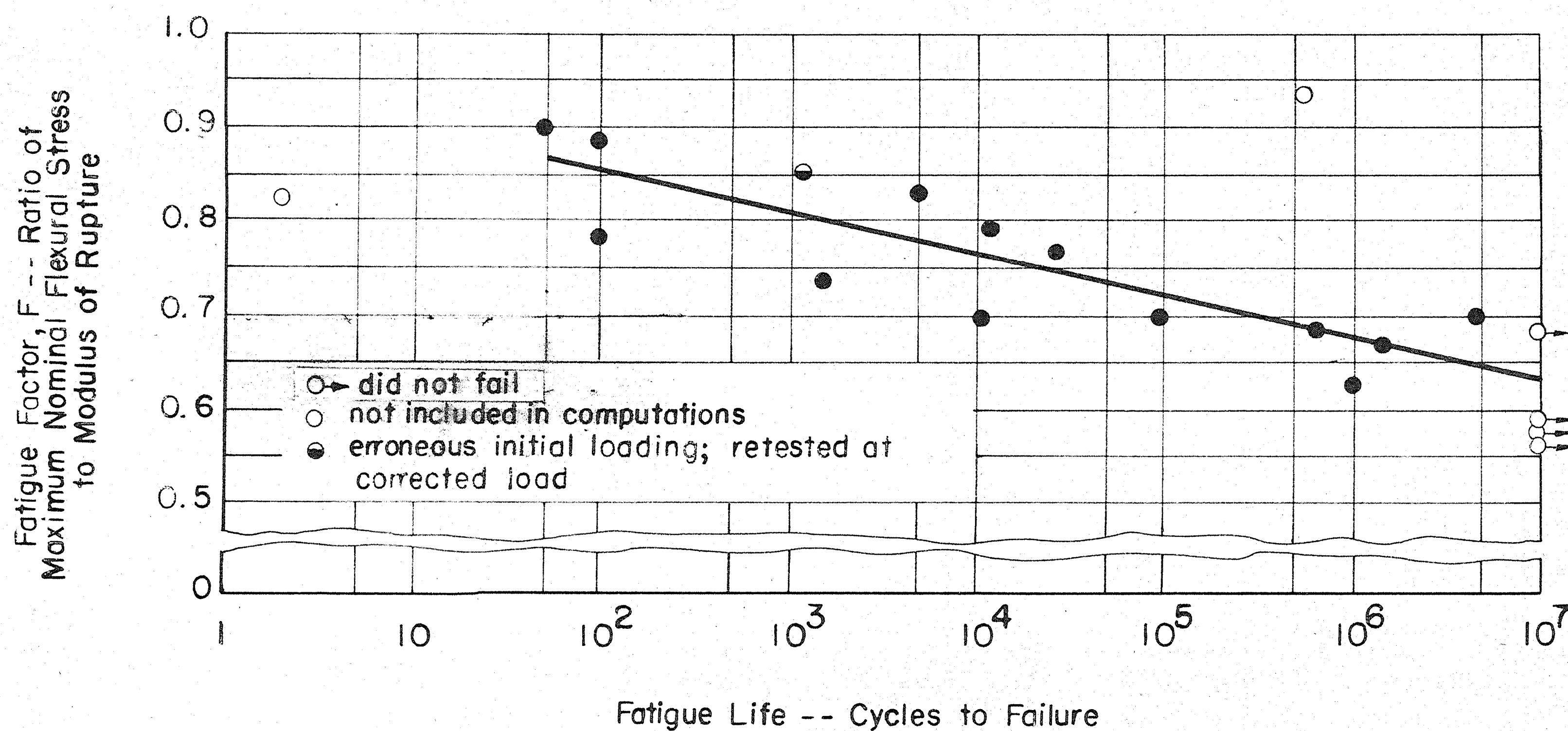

Figure 3. BEHAVIOR OF PLAIN CONCRETE UNDER FATIGUE LOADING
R= 0.25

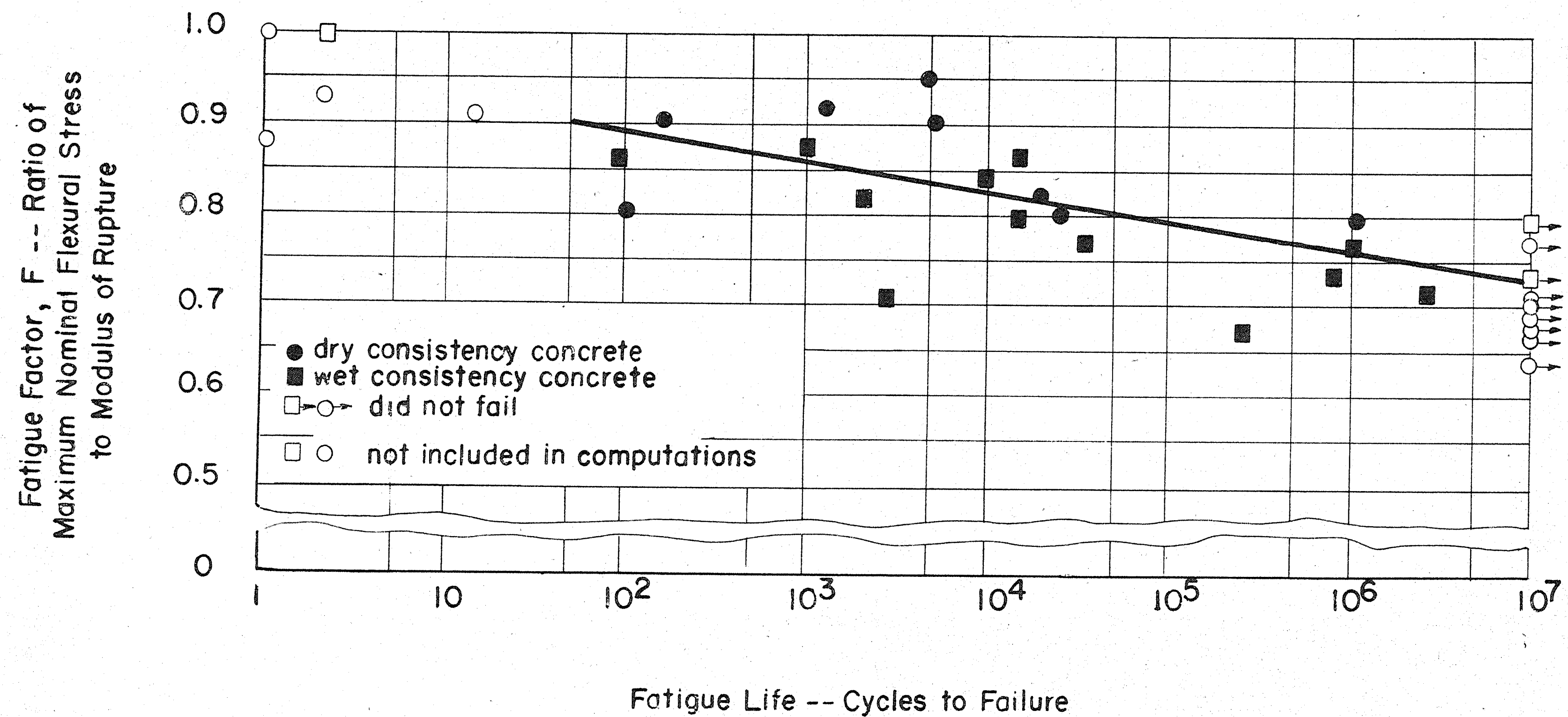

Figure 4. BEHAVIOR OF PLAIN CONCRETE UNDER FATIGUE LOADING -- R= 0.50

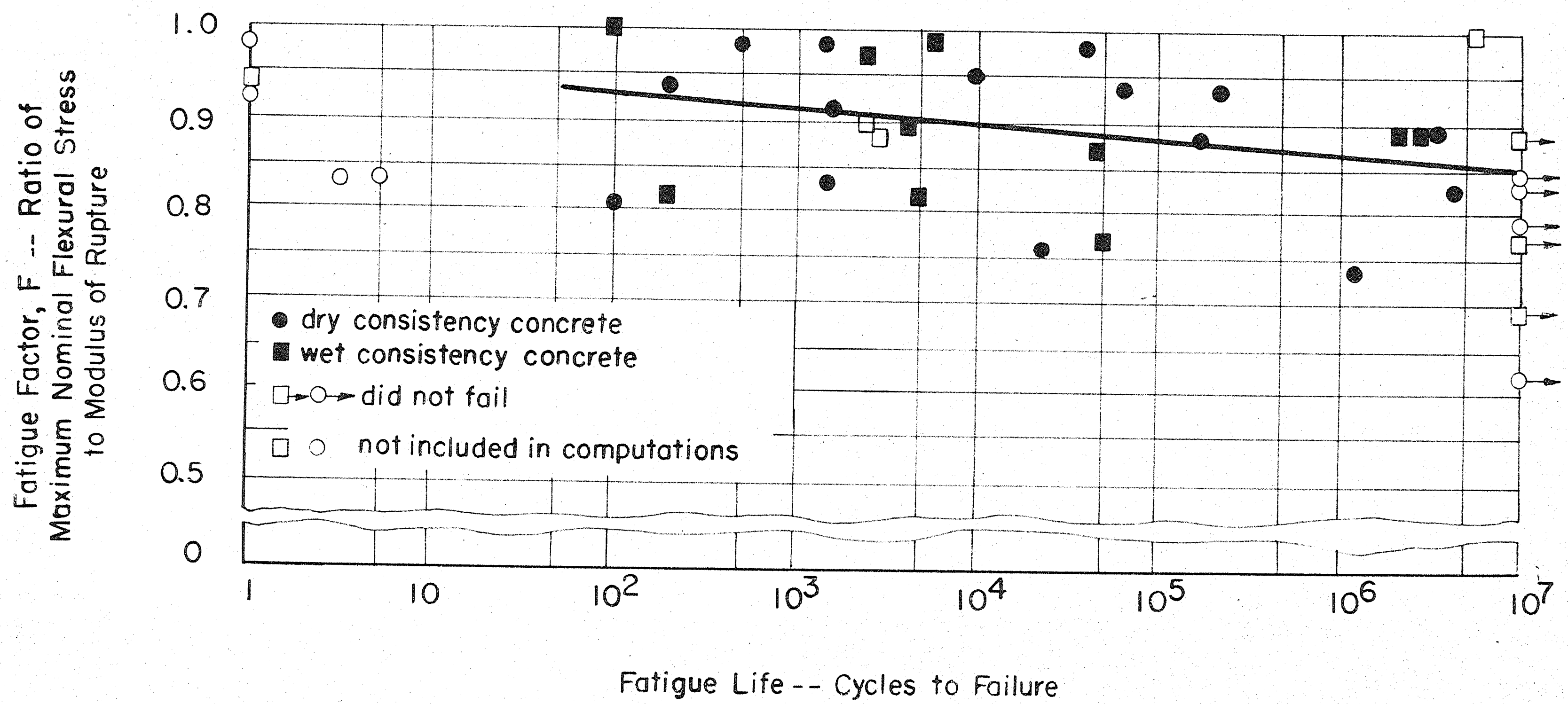

Figure 5. BEHAVIOR OF PLAIN CONCRETE UNDER FATIGUE LOADING
R = 0. 75

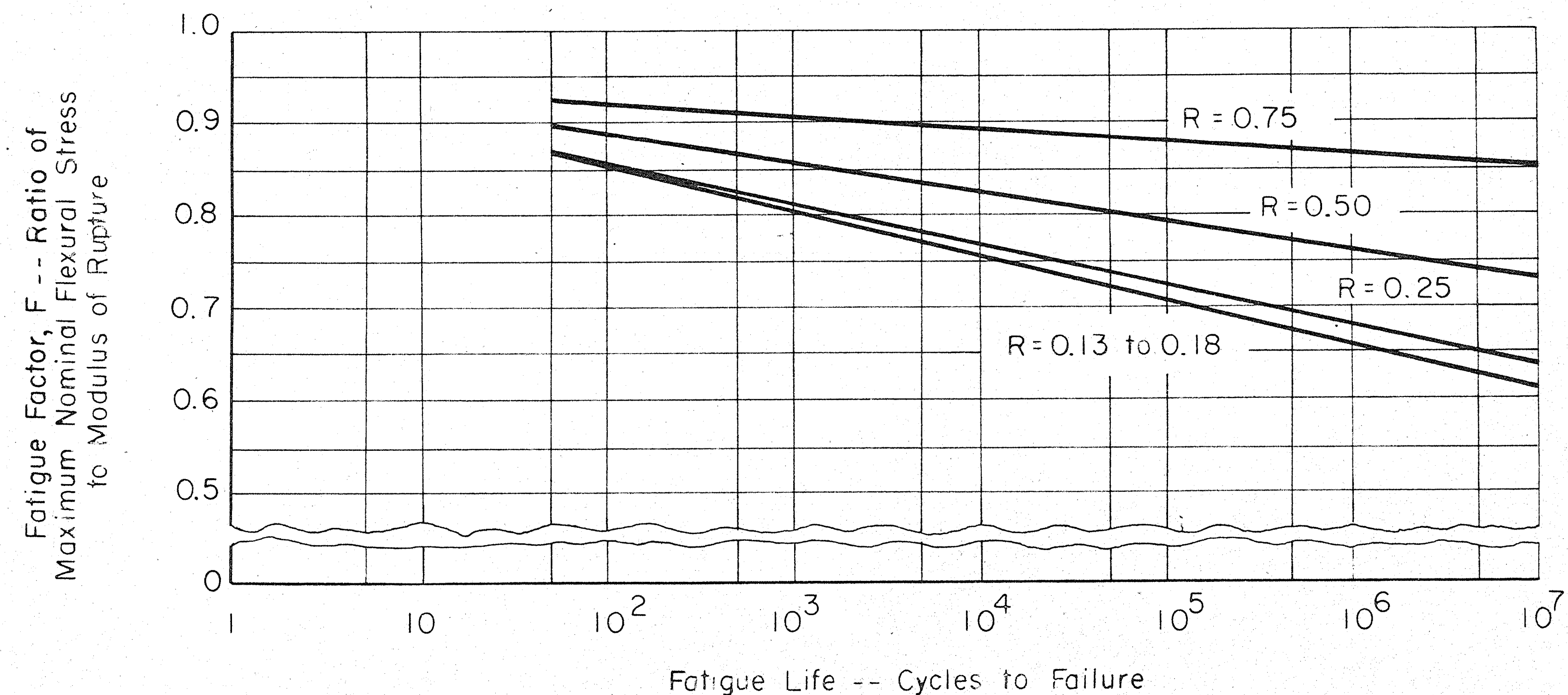

Figure 6. EFFECT OF THE RANGE OF STRESS ON THE BEHAVIOR OF PLAIN CONCRETE UNDER FATIGUE LOADING

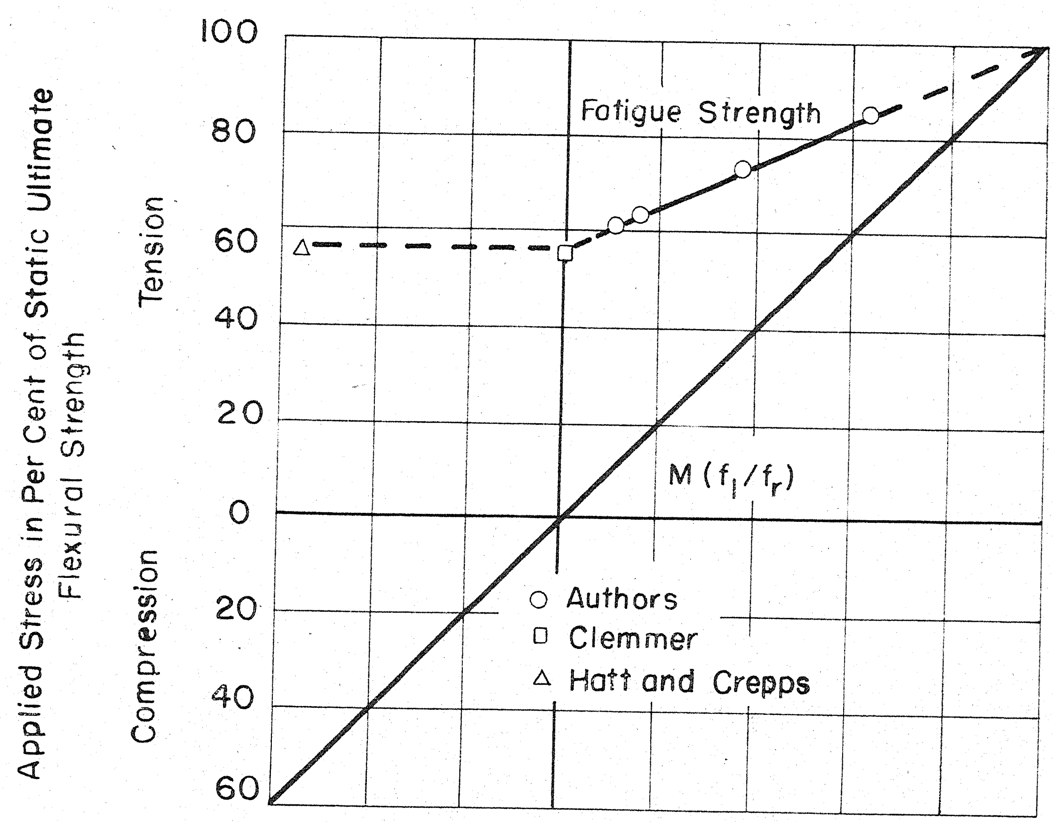

Figure 7. MODIFIED GOODMAN DIAGRAM SHOWING THE EFFECT OF THE RANGE OF STRESS ON THE FATIGUE LIMIT OF PLAIN CONCRETE UNDER REPEATED LOADING