Theoretical & Applied Mechanics
Report No. 35

AN APPRAISAL OF THE PROT METHOD OF FATIGUE TESTING PART II

By

H. T. Corten, Todor Dimoff
T. J. Dolan and Masaki Sugi

A Research Project of the

Department of Theoretical and applied Mechanics
University of Illinois

Sponsored by

OFFICE OF NAVAL RESEARCH, U.S. NAVY
Contract N6-ori-071(04), Project NR-031-005

Urbana, Illinois
June, 1953

TECHNICAL REPORT NO. 35
on a research project entitled
THE BEHAVIOR OF MATERIALS
UNDER REPEATED STRESS
Project Supervisor, T. J. Dolan

AN APPRAISAL OF THE PROT METHOD OF FATIGUE TESTING - PART II

H. T. Corten
Research Assistant Professor

Todor Dimoff
Research Assistant

T. J. Dolan
Professor

Masaki Sugi
Graduate Research Assistant

DEPARTMENT OF THEORETICAL AND APPLIED MECHANICS
UNIVERSITY OF ILLINOIS

ABSTRACT

This report presents a further study of the method of fatigue testing suggested by Marcel Prot and should be regarded as a continuation of the material in Part I (issued as Technical Report No. 34) of this investigation.

The Prot progressively increasing load method of fatigue testing has been investigated by comparing the experimental results for three ferrous metals with conventional fatigue data. Both notched and unnotched specimens have been studied. Two procedures have been employed in analyzing the data. The first method makes use of the results of conventional fatigue data (obtained with constant stress amplitude) to evaluate an experimental constant required to obtain the optimum value of the endurance limit from the Prot data. The second procedure employs the general method of least squares and statistical analysis to obtain the optimum value of the endurance limit and an estimate of the statistical variation from only the data obtained with progressively increasing loads. The Prot method of fatigue testing appears most promising for rapid estimation of the endurance limit of ferrous metals.

TABLE OF CONTENTS

ACKNOWLEDGMENT

This investigation has been conducted in the research laboratories of the Department of Theoretical and Applied Mechanics as part of the work of the Engineering Experiment Station, University of Illinois, in cooperation with the Office of Naval Research, U. S. Navy, under contract N6-ori-071(04). This work is a continuation of that reported in Technical Report No. 34 of the same title by A. P. Boresi and T. J. Dolan. The advice and assistance of Professor W. G. Madow, Mathematics Department, University of Illinois, on the statistical analysis is greatly appreciated. Acknowledgment is due G. L. Lovestrand for preparing the illustrations.

I. INTRODUCTION

This investigation of the progressive load method of fatigue testing is a continuation of the work reported in Part I (1).* An important group of fatigue problems are concerned with load resisting members subjected to a very large (infinite) number of repeated loads. In this group of problems, the determination of the endurance limit of the material by conventional laboratory methods (S-N curve) is often a long and expensive procedure. This is particularly true when the statistical variation of the endurance limit is desired. Also, after the endurance limit is established, the direct application of this information to the design of complex load resisting members is somewhat uncertain due to such phenomena as the effect of state of stress, notch-sensitivity and size and shape effects. Consequently, testing of expensive complex assemblies is often necessary to determine endurance limit loads. Thus the desirability of a reliable "short time" method for determining the endurance limit and estimating the statistical variation using only a relatively few specimens or complete assemblies has long been evident.

The Prot method (2) of determining the endurance limit appears to incorporate many desirable features from the point of view of reducing the number of specimens required and the length of time for each test. It will be recalled that in the Prot theory it was assumed that conventional fatigue data could be approximately represented by a hyperbola in the region of the endurance limit, that is $(S-E)\,N = K'$, where E is the endurance limit and K is a constant. For load programs where the stress increases linearly with time or number of cycles, the relation between the fracture stress S_R and N was assumed to be another hyperbola with the same horizontal asymptote, E. If the increase in stress per cycle is denoted by α, Prot showed that, based on the above mentioned assumptions, a linear relation should exist between S_R and $\sqrt{\alpha}$, namely

$$S_R = E + K\sqrt{\alpha} \qquad \text{Eq. 1}$$

The endurance limit, E, could be obtained from a diagram of S_R vs. $\sqrt{\alpha}$ by extrapolation to $\alpha = 0$. At this point E could be read directly on the stress scale.

The advantage of this method lies in the fact that every specimen contributes to the determination of the endurance limit. Further, it offers a possibility of obtaining an estimate of the statistical variation of the endur-

---

* Numbers in parentheses refer to references listed in the Bibliography.

ance limit based on the statistical variation of a relatively few specimens by extrapolation* to $\sqrt{\alpha} = 0$.

II. SUMMARY OF PREVIOUS REPORT

The experimental results reported in Part I of this study (1) confirm the findings of another investigator (3), that the theory as originally proposed by Prot in which it was assumed that S-N data could be represented by an hyperbola, was not entirely adequate. Prot's original assumption lead to a value of $n = 0.5$ for the exponent in the more general relation

$$S_R = E + K\alpha^n \qquad \text{Eq. 2}$$

A nonlinear plot of S_R vs. α^n was obtained when a value of $n = 0.5$ was used, particularly for the 75S-T6 aluminum alloy. In the modified theory (3) it was assumed that n is a material constant but may be different for different materials. This theory is based on one of Weibull's (4) approximations to the conventional S-N diagram, namely

$$\log N = \log k - m \log (S-E) \qquad \text{Eq. 3}$$

and Miner's hypothesis (5) which assumes that the cumulative fatigue damage is identical with the summation of the cycle ratios of overstress applied. By employing Eq. 3, the following approximate relation may be obtained for the n of Eq. 2:

$$n = \frac{1}{m+1} \qquad \text{Eq. 4}$$

In this relation m is the slope of the line obtained from a plot of log (S-E) vs. log N (Eq. 3) employing conventional fatigue data. For example, for 75S-T6 aluminum alloy, a value of $n = 0.1786$ was obtained from a plot of log N vs. log (S-E). Using this value of n, it was found that the relation between S_R and α^n was approximately linear (see Fig. 12 of Part I (1)).

Another assumption inherent in the Prot theory is that the relation between S_R and α^n, is independent of the stress at which loading is initiated since it is assumed that cycles of stress below the endurance limit cause

---
* Not necessarily linear extrapolation.

no damage to the material. The experimental results for 75S-T6 aluminum alloy and ingot iron presented in Part I (1) do not completely verify this assumption. The difference is relatively small, however, and may be insignificant in many practical applications.

III. PURPOSE AND SCOPE

It is the purpose of this report to appraise the usefulness and reliability of the Prot method of determining the endurance limit. This report presents the results of an experimental investigation of the Prot method applied to three ferrous metals, an "unaged"* iron, an SAE 2340 steel and a boron steel, 14B50. These steels cover a wide range of hardness and include notched and unnotched specimens using several different levels of stress at the start of the test. Part I contained the results for an ingot iron and 75S-T6 aluminum alloy. The unaged ingot iron (B) was included to study the possible effect of "coaxing" upon the endurance limit as determined by the Prot method. Table I lists all of the materials studied in both Part I and Part II along with the type of specimen and starting stresses used.

Since the exponent n of Eq. 2 varies from one metal to another, appropriate values of n may be determined by the use of Eq. 4 from diagrams of log N vs. log (S-E) but this requires the availability of conventional** fatigue data for the metals studied. To avoid the necessity of using conventional data, the general method of least squares for a nonlinear equation may be applied to the test data obtained by the Prot method. This analysis allows the determination of the constants E, n, and K in Eq. 2 from only the Prot data. The value of n obtained by this method results in a linear relation between S_R and α^n that best fits the data.

The statistical variability of the data are analyzed to obtain an estimate of the standard deviation of the endurance limit as obtained by the Prot method.

---

* The "unaged" ingot iron was heat treated to make it susceptible to "strain aging" during subsequent cyclic loading.

** The term "conventional" is used to indicate tests conducted at a constant stress amplitude.

IV. MATERIALS AND EXPERIMENTAL METHOD

The materials were received as 7/8 in. diameter hot rolled round bars. The blanks for the specimens were prepared from the bars in the "as received" condition. The specimens of ingot iron and boron steel were machined before heat treating. These specimens were left approximately 0.002 in. larger in diameter than the finished specimen to allow for removal of light scale after the heat treatment. The ingot iron, hereafter designated as (B) was quenched in water to retain as much carbon and nitrogen in solid solution as possible. It is known (8) that in this condition the iron is much more susceptible to "coaxing" during the progress of the repeated load test than in the "as rolled" condition (1) designated (A). The SAE 2340 steel was machined after quenching and drawing. The chemical composition, heat treatment and resulting hardness are given in Table II. The specimens were polished using the standard procedure (5).*

The tests were conducted using the equipment and same types of specimens as those described in previous work (1). In testing boron steel 14-B-50, the machine was operated at approximately 7200 rpm and all specimens developed fatigue fractures.

The speed of the machines was reduced to approximately 3600 rpm for SAE 2340 steel and ingot iron (B). The slower speed was employed in an attempt to further reduce the severe shock to the machine caused by the bending failure of some of the specimens; it was also hoped that the slower cyclic frequency might result in more complete fracture of the specimens. However bending of the ingot iron occurred in all specimens for which the value of α was 0.2 or greater. In those cases where α exceeded 0.2, the stress in the specimen had reached or exceeded the yield strength of the metal when plastic bending occurred.

The loading rate α was computed from the initial and final load on the specimen and the number of cycles to failure. The following equation was used

$$\alpha = \frac{(W_f - W_i)\frac{L}{Z}}{N}$$

where W_f is the net final load on the specimen at fracture, W_i is the net initial load, L is the moment arm, Z is the section modulus, and N is the total number of cycles to failure. Since water was added to the load tank to increase the load uniformly, the minor influence of evaporation was automatically compensated by using this method.

---

* After heat treatment, the ingot iron (B) and boron steel 14-B-50 were given a light polish with 2/0 emery polishing paper.

The tensile properties of all materials are included in Table III and representative tensile stress-strain diagrams are given in Figs. 20 through 22.

V. EXPERIMENTAL RESULTS AND DISCUSSION

In the following, the data and results for each material are considered separately. All of the data have been analyzed using the value of the exponent $n = 0.5$ as proposed by Prot, and by using a value of n determined by Eq. 4 from diagrams of log N vs. log (S-E). Some of the data for 14B-50 steel have been analyzed by the general method of least squares and statistical theory. (The mathematical theory on which this method is based is presented in Appendix A along with a sample computation.)

Ingot Iron (B). The results of the experiments for determining the endurance limit under progressively increasing load are presented in Fig. 1 and 2; the starting stresses σ_o, were 20,000 psi and 10,000 psi respectively. Two values of the exponent n were used, $n = 0.5$ and $n = 0.371$. Both of the values appear to give a linear relation between S_R and α^n for the small range of values of α^n covered by the data. The value of the exponent, $n = 0.371$ was determined from a diagram of log N vs. log (S-E)* which is shown in Fig. 3. These latter data (obtained by the conventional method of fatigue testing) are also shown on the S-N diagram in Fig. 4.

The endurance limits determined by the Prot** and conventional methods are summarized in Table IV. Though the absolute differences in values are not large, the Prot method endurance limits are from 16% to 25% higher than that determined by the conventional method. However the use of $n = 0.371$ gives slightly better agreement than $n = 0.5$. It should be noted that both the relative and absolute differences are larger than those obtained for ingot iron (A) in Part I of this report.

---

* Throughout this report the value of E used to obtain log N versus log (S-E) diagrams was obtained from S-N diagrams employing conventional fatigue data.

** In the diagrams of S_R vs. α^n, the intercept with the S axis represents the endurance limit (E) of the material. In determining the equation of the straight line for best fit, the method of least squares was used (7). The equation of this straight line is

$$S_R = E + K\alpha^n \qquad \text{Eq. 2}$$

(Cont'd. on p. 6)

The question of the effect of "coaxing" was raised in Part I (1) and ingot iron (B) was heat treated (see Table II) to a condition that is very susceptible to "coaxing" (8). It is suspected that the coaxing phenomena is largely responsible for the higher values of E determined by the Prot method in these later tests. In support of this, it should also be noted that a lower starting stress resulted in a higher value of E. This is the reverse of the effect noted for different starting stresses for ingot iron (A) (see Table IV) and is consistent with the concept that the "coaxing" phenomena was largely responsible for the increased endurance limit as determined by the Prot method. Thus it appears that for metals that are susceptible to "coaxing" (certain ferrous alloys (8)), the Prot method of determining the endurance limit may lead to estimates of the endurance limit that are too high.

The Prot method of determining the endurance limit was limited to the lower loading rates (low values of α) due to the fact that at higher values of α the specimens failed by plastic bending instead of progressive fracture. Many of the specimens also failed by plastic bending after developing a small crack. However it is thought that the presence of the crack contributed to excessive vibration of the specimen late in the test; therefore, these failures are reported as resulting from progressive fracture.

It is interesting to note from Fig. 4 that when very low loading rates must be employed, the progressive load and conventional methods require comparable numbers of cycles and time to fracture.

<u>SAE 2340 STEEL</u>

The experimental results for the Prot method are presented in Fig. 5 for unnotched specimens tested using a starting stress, σ_o, of 65,000 psi and in Fig. 6 and 7 for notched specimens using starting stresses, σ_o, of 30,000 psi and 15,000 psi respectively. Two values of the exponent n have been used in plotting the data in Fig. 5 and three values in Figs. 6 and 7. In Figs. 5

The constants E and K are determined as follows:

$$E = \frac{\Sigma(\alpha^n)^2 \cdot \Sigma S_R - \Sigma\alpha^n \cdot \Sigma\alpha^n \cdot S_R}{M\Sigma(\alpha^n)^2 - (\Sigma\alpha^n)^2}$$

$$K = \frac{M\Sigma\alpha^n S_R - \Sigma\alpha^n \cdot \Sigma S_R}{M\Sigma(\alpha^n)^2 - (\Sigma\alpha^n)^2}$$

where M is the total number of specimens. A similar procedure was used to determine the exponent m from log N vs log (S-E) diagrams.

through 7 the data are plotted for $n = 0.5$, however it is obvious that the relation between the fracture stress S_R and α^n is not linear. To obtain an estimate of the value of n that would give a linear relation, the diagrams in Fig. 8 and 9 were plotted to include the present conventional fatigue data plus more extensive data available from a previous investigation (9) of the same material. The values of n obtained by applying the method of least squares to only the data from reference (9) gives $n = 0.418$ for unnotched specimens (from Fig. 8) and $n = 0.40$ for notched specimens (from Fig. 9). Using the respective values of n, the data were replotted as shown in Fig. 5 through 7 resulting in a more nearly linear relation between S_R and α^n.

In Fig. 9 the line for notched specimens obtained by the method of least squares appears to be considerably influenced by the wide scatter of points in the range between $N = 5 \times 10^5$ to 10^6 cycles. To further study the use of diagrams such as Fig. 8 and 9, an additional dashed line was drawn by eye in Fig. 9 that appears to better represent the data in the range below 3×10^5 cycles. From the slope of this line, a value of $n = 0.318$ was obtained. The Prot data in Fig. 6 and 7 were also replotted using this value of n, and appear to give the most nearly linear relation between S_R and α^n for this exponent.

The values of the endurance limit, E, found by the Prot method from Fig. 5 through 7 and by the conventional method from Figs. 10 and 11 are tabulated in Table IV. The values of E obtained by the Prot method for unnotched specimens agree very closely with the results of the conventional method. The exponent, $n = 0.418$ appears to give slightly better agreement than $n = 0.5$. For unnotched specimens the values of E obtained by the Prot and conventional method appear to agree best when a value of $n = 0.318$ is used to plot the Prot data. The value of $n = 0.4$ gives good agreement for a starting stress σ_o of 15,000 psi but for $\sigma_o = 30,000$ psi, the difference is larger.

These results should be viewed in the light of the statistical variation inherent in the determination of the endurance limit. The endurance limit by the conventional method was bracketed between one or two specimens that did not fracture at a given stress level and two or three specimens that did fracture at stress levels from 500 to 2000 psi higher. The information that is available (10) on the variability of the endurance limit of other materials indicates that the standard deviation may be several thousand pounds per square inch. Thus when the results of the Prot and conventional methods differ by only several thousand pounds per square inch, it does not appear possible to differentiate accurately between the results obtained by using different values of n.

From the results of the notched and unnotched specimens it is impossible to decide whether or not n is a material constant. Because of the variability of the endurance limit and the relative insensitivity of E to changes in n, it is suggested that for practical purposes the assumption of a constant value of n for a given material is sufficiently reliable.

The variation of the value of E obtained with the Prot method due to different starting stresses for notched specimens is of the same magnitude as was obtained with unnotched specimens. This difference is not large but it does represent a deviation from one of the basic assumptions of the Prot theory. From the practical standpoint, this small difference may not be significant.

From Fig. 9 and 10 it is clear that there is some question about the proper interpretation of these diagrams. For small values of N, approximately 100,000 cycles and less, a second line of different slope often appears to better fit this data (3,4). For larger values of N, the scatter is often large and the trend of the data is difficult to determine. Thus this method of estimating n may be misleading when conventional data from only a few specimens are available. In terms of the modified Prot theory this may also mean that it would be desirable to use a variable value of n in plotting the S_R vs. α^n diagrams. However considering the insensitivity of E to changes of n and the empirical assumptions upon which the modified Prot theory is based, it appears doubtful that such a refinement is justified.

BORON STEEL 14-B-50

The experimental results of the tests under progressively increasing load are presented in Fig. 12 for unnotched specimens using a starting stress of 50,000 psi and in Fig. 13 and 14 for notched specimens using starting stresses of 35,000 and 25,000 psi respectively. In Fig. 12 for unnotched specimens the data are plotted using n = 0.5 and n = 0.456. The value of n of 0.456 was obtained from Fig. 15, a diagram of log N vs. log (S-E), using conventional fatigue data for unnotched specimens and Eq. 4. As sufficient conventional data was not available, a value of n for notched specimens was not determined by this procedure.*

* In the next section another method of obtaining a value of n is discussed. The value of n = 0.55 was obtained for notched specimens by this method and the results will be discussed in the next section.

The values of the endurance limit as obtained by the Prot and conventional methods are also summarized in Table IV. For the unnotched specimens, the values of the endurance limit are both (for n = 0.5 and n = 0.456) in close agreement with the value obtained by the conventional method. The conventional data are presented in S-N diagrams in Fig. 16 and 17 for unnotched and notched specimens respectively. The endurance limits obtained by the Prot method for notched specimens using n = 0.5 were slightly lower than the value obtained by conventional methods for both starting stresses. However considering the statistical variability of the endurance limit (10), it is not possible to determine accurately whether the difference between the Prot and conventional results is really significant.

The results for Boron steel indicate that, like the SAE 2340 steel, the Prot method appears to be as reliable for notched as for unnotched specimens. The implication is that the Prot method is probably as applicable for testing odd-shaped complex machine members as for laboratory specimens.

For the notched specimens, the two starting stresses resulted in slightly different values of E; however, for the Boron steel the difference is so slight that it must be considered insignificant. It is interesting to note, by comparison of Fig. 13 and 14, that different starting stresses result in larger differences at higher values of α but the lines representing the data tend to converge as α approaches zero. This tendency was found to exist for all materials; however, from a practical standpoint in determination of the endurance limit, it has no apparent significance.

From Tables II and III and Fig. 22 it will be noted that the Boron steel is a very hard, high static strength material. However the fatigue strength does not reflect this tendency. Photomicrographs of the cross section of the specimen at the surface showed that the surface layer was slightly decarburized. Presumably fatigue cracks develop in the decarburized surface layer and then spread to the higher strength interior with the aid of the stress concentration at the root of the sharp crack. This softer surface layer may also account for the relatively low notch-sensitivity exhibited by the notched specimens of Boron steel. In the presence of a high stress gradient, the stress on the hard interior material was sufficiently reduced that the softer surface determined the effective notch-sensitivity.

VI. INTERPRETATION OF PROT DATA BY THE GENERAL METHOD OF LEAST SQUARES

In order to avoid the necessity of employing conventional data to obtain a suitable value of n to be used in plotting the Prot data, it is desirable to consider the methods that require only the Prot data for a complete analysis. One such method is the "General Method of Least Squares Applied to a Nonlinear Formula".* This method utilizes estimates of the constants E, K, and n in Eq. 2 to obtain corrections to these constants by the ordinary method of least squares. The corrections are obtained from an approximate formula which becomes more exact the smaller the values of the corrections. The successive application of this procedure results in corrections which rapidly become small compared to the constants E, K and n; thus the optimum values are obtained employing only the Prot data. Development of the mathematical theory for application to this problem and a specific numerical example of the computations are included in Appendix A.

This method was applied to the Prot data for steel 14-B-50. The data in Fig. 12 for unnotched specimens and in Fig. 13 for notched specimens using a starting stress of 35,000 psi were chosen for this study. The values of n obtained by this method were 0.717 and 0.55 for unnotched and notched specimens respectively. Diagrams of S_R vs. α^n using these values of n are shown in Figs. 18 and 19. The solid straight lines represent the optimum fit of a straight line to the test data. The value of E obtained for the unnotched specimens (see Table IV) is somewhat higher than the value obtained by the conventional method. It appears that the two unusually low points ($\alpha^n = 0.17$ in Fig. 18) influenced the value of n and E considerably in this case. The value of E computed for the notched specimens by this method is in excellent agreement with the value obtained by the conventional method. This procedure was not applied to the notched specimens tested using a starting stress of 25,000 psi. However the value of $n = 0.55$ (obtained from notched specimens tested with a starting stress of 35,000 psi) was also used to plot the data for notched specimens that were started with $\sigma_o = 25,000$ psi. The results are shown in Fig. 14 and the value of E is listed in Table IV. The values of E for both sets of notched Boron steel specimens appear to be in closer agreement with the results of the conventional method when plotted using $n = 0.55$ than for $n = 0.5$.

---

* Often referred to as Regression Analysis. See ref. (15).

The advantage of this method is that only the data obtained with progressively increasing loads are required; however, the method inherently assumes that the relation between S_R and α^n is linear and that the distribution of the observed values of S_R about its mean is normal. As the assumptions made in the theory to arrive at a linear relation are empirical and not always borne out by experimental data (11), the usefulness of this procedure is restricted to those cases where S_R and α^n can reasonably be assumed to be linearly related. From all of the data available (1, 2, 12, 13, 14), it appears that for small values of α, the assumption of a linear relation is reasonable. The assumption of a normal distribution of S_R about the mean also appears reasonable, at least as a first approximation (12).

DETERMINATION OF THE STANDARD DEVIATION FROM PROT DATA

The statistical variability of the results of fatigue experiments makes knowledge of the expected variation of the endurance limit as important as knowledge of the mean value.* The Prot method allows measurement of the fracture stress, S_R, which is related to the endurance limit by Eq. 2. Consequently the variation of the endurance limit is obtainable only indirectly from the variability of S_R. Prot originally suggested that the variation of S_R at different values of α may be constant and equal to the variation of E. There is some data to indicate that the variation of S_R may be constant and approximately equal to that for the endurance limit obtained by the "step up" method of fatigue testing (12). However the various methods of determining the scatter of the endurance limit do not appear to give entirely consistent results. (10)

In view of the lack of knowledge of the relation between the variation of E and S_R, the standard deviation of S_R at $\alpha = 0$ will be determined purely on the basis of statistics, assuming only that the two distributions are normal. The theory is an extension of the theory for obtaining the optimum values of E, K, and n by the method of least squares. The standard deviation of values of S_R is very closely related to the sum of the squares of the difference between the observed and estimated (optimum) values of S_R. A brief discussion of the theory with application to the Prot data is presented in Appendix A.

This method applied to the data for Boron steel gave results that are presented in Figs. 18 and 19. In Fig. 18 the short dashes on the stress axis

---

* By definition, the endurance limit stress is that stress at or below which fatigue fracture does not occur. However when considering the scatter of experimental data, it is convenient to deal with the mean value and the standard deviation about the mean. The term "endurance limit" is used in this report to indicate the mean value.

represent $\pm$ one standard deviation of E, that is for S_R at $\alpha = 0$. In Fig. 19 the dashed lines on either side of the solid line represent upper and lower bounds for one standard deviation from the optimum curve over a range of values of α. The values obtained for the magnitude of the standard deviation of E are reasonable (10, 12); however, it should be noted that the standard deviation increases markedly (diverging dashed lines in Fig. 19) outside of the range covered by the experimental data. It appears that the value selected for the smallest α employed in testing has a pronounced influence on the value of standard deviation obtained for $\alpha = 0$. That is, the standard deviation of E will be increased if only larger values of α are used in the experiment. Thus the value of standard deviation obtained by this method depends upon the experimental design (choice of α) and may be larger than the true value. For the purpose of estimating the scatter, this result may be useful as it leads to an estimate that is on the safe side.

VII. CONCLUSIONS

1. The Prot method of fatigue testing gives information about the endurance limit of the metal only. Consequently the method appears to be more promising for ferrous metals with a well defined endurance limit than for nonferrous metals where the fatigue strength at a given life must be considered.

2. The modified Prot theory in which the exponent n (in Eq. 2) may be different for different metals, produces better agreement with conventional fatigue data than does the original Prot interpretation based on $n = 0.5$.

3. The use of progressively increasing loads to obtain a reliable estimate of the endurance limit depends upon a knowledge of the appropriate value of n; its value may be approximately constant for a given material regardless of the shape of the specimen. Approximate values of n may be obtained from previous conventional fatigue data for similar metal using log N vs log (S-E) diagrams and Eq. 4. If such data are not available, the method presented in Appendix A may be used which employs only the data obtained from progressively increasing loads.

4. For ferrous metals the exponent n was found to be reasonably close to 0.5 ($0.37 < n < 0.71$) in all cases. For rapid estimation of the endurance limit, $n = 0.5$ appears to give satisfactory results if a smooth curve is drawn through the data. However such a procedure was unsatifactory for 75S-T6 aluminum alloy.

5. Various levels of starting stresses that are below the endurance limit lead to approximately the same value for the endurance limit. In general, lower starting stresses resulted in a slightly lower value of E. This would indicate than an influence of repeated loading at stress levels below the endurance limit was present; however, the variations were relatively small except when the metal was susceptible to coaxing.

6. For ferrous metals that are susceptible to coaxing, the Prot procedure affords an opportunity for coaxing to occur which appreciably raises the estimated value of E as compared to the value obtained by conventional method.

7. Endurance limits determined by progressive loading of notched specimens were in close agreement with the values obtained by the conventional (constant stress amplitude) method. This indicates that the Prot method may be applicable to members of any shape including full-sized structures. However it is doubtful that the Prot method will give reliable results in instances where environmental conditions such as corrosion, erosion, or elevated temperatures contribute to the initiation of fatigue cracks. The short time nature of the test precludes the complete development of detrimental conditions that are functions of time.

8. A method of estimating the standard deviation of the fracture stress at $\alpha = 0$ (and hence of E) is presented. This method gives reasonable estimates of the variability in the cases investigated. However due to lack of comparable data obtained by other methods, the reliability of this method is not known. The extimate of variability depends upon the smallest value of α used in the experiments and may overestimate the statistical variation in endurance limit if this value of α is reasonably large.

APPENDIX A

GENERAL METHOD OF LEAST SQUARES FOR A NONLINEAR FORMULA (16)

by Masaki Sugi

The general relation between two variables, x and y is defined as

$$y = f(x; a, b, c) \qquad \text{Eq. 5}$$

where x and y are the observed variables and a, b, and c are unknown constants involved in the relation between x and y. It is assumed that the form of Eq. 5 is known. The problem consists of determining values of a, b, and c that best fit the experimental data, that is, simultaneously observed values of x and y. The observed values of y are assumed to exhibit statistical variation.

From Eq. 2 it may be observed that Eq. 5 is nonlinear with respect to the unknown constants and is not of a form that can be modified to be linear with respect to the unknowns a, b, and c (15). However by considering small changes of the unknown constant in Eq. 5, a new equation may be formed that is linear with respect to the small changes of a, b, and c. This procedure consists of estimating the values of the constants a, b, and c that will require correction. Let the estimated values be a_o, b_o, and c_o. Then the second approximation of the constants will be

$$a_1 = a_o + \delta a$$

$$b_1 = b_o + \delta b \qquad \text{Eqs. 6}$$

$$c_1 = c_o + \delta c$$

The corrected values of a, b, and c allow computation of a corrected value of y. From Eq. 5 the second approximation of y is

$$y_1 = f(x; a_o + \delta a, b_o + \delta b, c_o + \delta c) \qquad \text{Eq. 7}$$

The value of y based on the estimated parameters is

$$y' = f(x; a_o, b_o, c_o) \qquad \text{Eq. 8}$$

Equation 7 may be expanded using Taylor's expansion to give

$$y_1 = f(x;\, a_o, b_o, c_o) + \delta a\left(\frac{\partial f}{\partial a}\right)_o + \delta b\left(\frac{\partial f}{\partial b}\right)_o + \delta c\left(\frac{\partial f}{\partial c}\right)_o + O(\delta a^2, \delta b^2, \delta c^2) \qquad \text{Eq. 9}$$

The difference, ν, between the corrected value, y_1, given by Eq. 9 and an observed value of y may be written as

$$\nu = y_1 - y = f(x;\, a_o, b_o, c_o) + \delta a\left(\frac{\partial f}{\partial a}\right)_o + \delta b\left(\frac{\partial f}{\partial b}\right)_o + \delta c\left(\frac{\partial f}{\partial c}\right)_o$$

$$- f(x;\, a, b, c)$$

and from Eq. 8,

$$\nu = \delta a\left(\frac{\partial f}{\partial a}\right)_o + \delta b\left(\frac{\partial f}{\partial b}\right)_o + \delta c\left(\frac{\partial f}{\partial c}\right)_o + (y' - y) \qquad \text{Eq. 10}$$

where the higher orders of δa, δb, and δc are neglected.

A group of r samples of observations leads to r equations of the type of Eq. 10, namely

$$\left\{\begin{array}{l}
\delta a\left(\frac{\partial f_1}{\partial a}\right)_o + \delta b\left(\frac{\partial f_1}{\partial b}\right)_o + \delta c\left(\frac{\partial f_1}{\partial c}\right)_o + (y'_1 - y_1) = \nu_1 \\
\delta a\left(\frac{\partial f_2}{\partial a}\right)_o + \delta b\left(\frac{\partial f_2}{\partial b}\right)_o + \delta c\left(\frac{\partial f_2}{\partial c}\right)_o + (y'_2 - y_2) = \nu_2 \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\delta a\left(\frac{\partial f_r}{\partial a}\right)_o + \delta b\left(\frac{\partial f_r}{\partial b}\right)_o + \delta c\left(\frac{\partial f_r}{\partial c}\right)_o + (y'_r - y_r) = \nu_r
\end{array}\right\} \qquad \text{Eqs. 11}$$

It should be noted that Eqs. 11 are linear in the corrections to the constants, δa, δb, and δc, and δa, δb, and δc may be found by the method of least squares.

The condition of least squares may be obtained by minimizing the sum of the squares of ν_i with respect to δa, δb, and δc. This condition may be expressed as

$$\left\{\begin{aligned} &\frac{\partial}{\partial(\delta a)}\left(\sum_{i=1}^{r} v_i^2\right) = 0 \\ &\frac{\partial}{\partial(\delta b)}\left(\sum_{i=1}^{r} v_i^2\right) = 0 \\ &\frac{\partial}{\partial(\delta c)}\left(\sum_{i=1}^{r} v_i^2\right) = 0 \end{aligned}\right\} \qquad \text{Eqs. 12}$$

Performing the operation indicated in Eq. 12* on Eq. 11 results in the following three equations known as the normal equations,

$$\left\{\begin{aligned} &\delta a\sum\left(\frac{\partial f_i}{\partial a}\right)_0^2 + \delta b\sum\left(\frac{\partial f_i}{\partial a}\right)_0\left(\frac{\partial f_i}{\partial b}\right)_0 + \delta c\sum\left(\frac{\partial f_i}{\partial a}\right)_0\left(\frac{\partial f_i}{\partial c}\right)_0 = -\sum\left(\frac{\partial f_i}{\partial a}\right)_0(y_i'-y_i) \\ &\delta a\sum\left(\frac{\partial f_i}{\partial a}\right)_0\left(\frac{\partial f_i}{\partial b}\right)_0 + \delta b\sum\left(\frac{\partial f_i}{\partial b}\right)_0^2 + \delta c\sum\left(\frac{\partial f_i}{\partial b}\right)_0\left(\frac{\partial f_i}{\partial c}\right)_0 = -\sum\left(\frac{\partial f_i}{\partial b}\right)_0(y_i'-y_i) \\ &\delta a\sum\left(\frac{\partial f_i}{\partial a}\right)_0\left(\frac{\partial f_i}{\partial c}\right)_0 + \delta b\sum\left(\frac{\partial f_i}{\partial b}\right)_0\left(\frac{\partial f_i}{\partial c}\right)_0 + \delta c\sum\left(\frac{\partial f_i}{\partial c}\right)_0^2 = -\sum\left(\frac{\partial f_i}{\partial c}\right)_0(y_i'-y_i) \end{aligned}\right\}$$

Eqs. 13

Solving Eq. 13 for the corrections δa, δb, and δc, the corrected constant s are given by Eq. 6. Taking a, b, and c, thus obtained as the second approximations, successive applications of this method lead to values of δa, δb, and δc that rapidly become small compared to the constants a, b, and c. Thus the successively corrected constants a, b, and c rapidly approach an optimum value.

Application to Prot Data

According to the modified Prot theory, the relation between S_R and α is given by

$$S_R = E + K\alpha^n \qquad \text{Eq. 2.}$$

---

* In the following it will be understood that the summation runs from i = 1 to r.

where it will be recalled that S_R is the fracture stress, α is the increase of stress per cycle, E is the endurance limit, and K and n are constants.

In this case α and S_R correspond to the variables x, and y, and E, K, and n to the unknown constants, a, b, and c in Eq. 5. Thus

$$S_R = f(\alpha; E, K, n) = E + K\alpha^n \qquad \text{Eq. 14}$$

From Eq. 13 it will be noted that the derivatives of $f(\alpha; E, K, n)$ with respect to E, K, and n evaluated at E_o, K_o and n_o are required. These are as follows:

$$\left\{\begin{aligned} &\left(\frac{\partial f}{\partial a}\right)_o = \left(\frac{\partial f}{\partial E}\right)_o = 1 \\ &\left(\frac{\partial f}{\partial b}\right)_o = \left(\frac{\partial f}{\partial K}\right)_o = \alpha^{n_o} \\ &\left(\frac{\partial f}{\partial c}\right)_o = \left(\frac{\partial f}{\partial n}\right)_o = K_o \alpha^{n_o} \ln \alpha = 2.3026\, K_o\, \alpha^{n_o} \log_{10} \alpha \end{aligned}\right. \qquad \text{Eqs. 15}$$

Making these substitutions, the three linear equations designated as Eqs. 13 become

$$\left\{\begin{aligned} &+\delta E + \Sigma \alpha_i^{n_o} \delta K + (2.3026 K_o \Sigma \alpha_i^{n_o} \log \alpha_i)\delta n = \Sigma(S_i' - S_i) \\ &\Sigma \alpha_i^{n_o} \delta E + \Sigma \alpha_i^{2n_o} \delta K + 2.3026 K_o \Sigma \alpha_i^{2n_o} \log \alpha_i \delta n = -\Sigma \alpha_i^{n_o}(S_i' - S_i) \\ &\Sigma \alpha_i^{n_o} \log \alpha_i \delta E + \Sigma \alpha_i^{2n_o} \log \alpha_i \delta K + 2.3026 K_o \Sigma \alpha_i^{2n_o} (\log \alpha_i)^2 \delta n = \\ &\qquad -\Sigma \alpha_i^{n_o} \log \alpha_i (S_i' - S_i) \end{aligned}\right\}$$

Eqs. 16.

where the subscript R of S'_{Ri} and S_{Ri} is omitted for convenience of notation. The solution for δE, δK, and δn in the form of determinants is as follows:

$$\delta E = -\frac{\begin{vmatrix} \Sigma(S_i'-S_i) & \Sigma\alpha_i^{n_o} & \Sigma\alpha_i^{n_o}\log\alpha_i \\ \Sigma\alpha_i^{n_o}(S_i'-S_i) & \Sigma\alpha_i^{2n_o} & \Sigma\alpha_i^{2n_o}\log\alpha_i \\ \Sigma\alpha_i^{n_o}\log\alpha_i(S_i'-S_i) & \Sigma\alpha_i^{2n_o}\log\alpha_i & \Sigma\alpha_i^{2n_o}(\log\alpha_i)^2 \end{vmatrix}}{\Delta}$$

Eq. 17a

$$\delta K = -\frac{\begin{vmatrix} \tau & \Sigma(S_i'-S_i) & \Sigma\alpha_i^{n_o}\log\alpha_i \\ \Sigma\alpha_i^{n_o} & \Sigma\alpha_i^{n_o}(S_i'-S_i) & \Sigma\alpha_i^{2n_o}\log\alpha_i \\ \Sigma\alpha_i^{n_o}\log\alpha_i & \Sigma\alpha_i^{n_o}\log\alpha_i(S_i'-S_i) & \Sigma\alpha_i^{2n_o}(\log\alpha_i)^2 \end{vmatrix}}{\Delta}$$

Eq. 17b

$$\delta n = -\frac{\begin{vmatrix} \tau & \Sigma\alpha_i^{n_o} & \Sigma(S_i'-S_i) \\ \Sigma\alpha_i^{n_o} & \Sigma\alpha_i^{2n_o} & \Sigma\alpha_i^{n_o}(S_i'-S_i) \\ \Sigma\alpha_i^{n_o}\log\alpha_i & \Sigma\alpha_i^{2n_o}\log\alpha_i & \Sigma\alpha_i^{n_o}\log\alpha_i(S_i'-S_i) \end{vmatrix}}{2.3026\,K_o\,\Delta}$$

Eq. 17c

where Δ is given by

$$\Delta = \begin{vmatrix} \tau & \Sigma\alpha_i^{n_o} & \Sigma\alpha_i^{n_o}\log\alpha_i \\ \Sigma\alpha_i^{n_o} & \Sigma\alpha_i^{2n_o} & \Sigma\alpha_i^{2n_o}\log\alpha_i \\ \Sigma\alpha_i^{n_o}\log\alpha_i & \Sigma\alpha_i^{2n_o}\log\alpha_i & \Sigma\alpha_i^{2n_o}(\log\alpha_i)^2 \end{vmatrix}$$

Eq. 17d

Effect of Neglecting Higher Order Terms in Eq. 4

The last term of Eq. 9, namely $O(\delta a^2, \delta b^2, \delta c^2)$ may be written as

$$O(\delta a^2, \delta b^2, \delta c^2) = \frac{1}{2}(\delta a)^2 \left(\frac{\partial^2 f}{\partial a^2}\right)_o + \frac{1}{2}(\delta b)^2 \left(\frac{\partial^2 f}{\partial b^2}\right)_o + \frac{1}{2}(\delta c)^2 \left(\frac{\partial^2 f}{\partial c^2}\right)_o$$

$$+ O(\delta a^3, \delta b^3, \delta c^3)$$

Applied to the Prot analysis

$$O(\delta E^2, \delta K^2, \delta n^2) = \frac{1}{2}(\delta E)^2 \left(\frac{\partial^2 f}{\partial E^2}\right)_o + \frac{1}{2}(\delta K)^2 \left(\frac{\partial^2 f}{\partial K^2}\right)_o + \frac{1}{2}(\delta n)^2 \left(\frac{\partial^2 f}{\partial n^2}\right)_o$$

$$+ O(\delta E^3, \delta K^3, \delta n^3)$$

and

$$\left(\frac{\partial^i f}{\partial E^i}\right)_o = \left(\frac{\partial^i f}{\partial K^i}\right)_o = 0 \quad \text{for } i = 2, 3, 4, \ldots$$

$$\left(\frac{\partial^i f}{\partial n^i}\right)_o = K_o (\ln \alpha)^i \alpha^{n_o} \quad \text{for } i = 1, 2, 3, \ldots$$

therefore

$$O(\delta E^2, \delta K^2, \delta n^2) = K_o \alpha^{n_o} \left\{ \frac{1}{2!}(\delta n \ln \alpha)^2 + \frac{1}{3!}(\delta n \ln \alpha)^3 + \ldots \right\}$$

The expected range of values of n is between 0 and 1 and the estimated value, n_o is 0.5. Therefore the correction δn is smaller than 0.5, often approximately 0.1 or less. Values of α varied from approximately 10^{-4} to 1 (lb/in^2/cycle). If $\delta n \leq 0.1$ and $\alpha \geqslant 10^{-4}$, the absolute value of the term $\delta n \ln \alpha$ is smaller than unity and the higher order terms may be neglected. Successive approximations lead to very small values of δn and the remainder terms $O(\delta E^i, \delta K^i, \delta n^i)$approach zero.

STANDARD DEVIATION

The expansion of Eq. 5 into Taylor's series considering only the linear terms of δa, δb, and δc gives

$$\delta y = \left(\frac{\partial f}{\partial a}\right)\delta a + \left(\frac{\partial f}{\partial b}\right)\delta b + \left(\frac{\partial f}{\partial c}\right)\delta c \qquad \text{Eq. 18}$$

in which the partial derivatives have been evaluated at the optimum values of a, b, and c determined previously. If the higher order terms in δa, δb, and δc are small compared to the linear terms (17), δy may be considered to be a linear function of the variables δa, δb, and δc.* The general variance law (17) for a function, δy, which is the sum of three linear variables δa, δb, and δc may be written as

$$\sigma_y^2 = \left(\frac{\partial f}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial f}{\partial b}\right)^2 \sigma_b^2 + \left(\frac{\partial f}{\partial c}\right)^2 \sigma_c^2 + 2\left(\frac{\partial f}{\partial a}\right)\left(\frac{\partial f}{\partial b}\right)\sigma_a \sigma_b \rho_{ab}$$

$$+ 2\left(\frac{\partial f}{\partial a}\right)\left(\frac{\partial f}{\partial c}\right)\sigma_a \sigma_c \rho_{ac} + 2\left(\frac{\partial f}{\partial b}\right)\left(\frac{\partial f}{\partial c}\right)\sigma_b \sigma_c \rho_{bc} \qquad \text{Eq. 19}$$

It may be shown (18) that the variance of δa, δb, and δc are given by

$$\sigma_a^2 = (\Delta^{-1})_{aa}\sigma^2;\ \sigma_b^2 = (\Delta^{-1})_{bb}\sigma^2;\ \sigma_c^2 = (\Delta^{-1})_{cc}\sigma^2$$

and the co-variance by

$$\sigma_a \sigma_b \rho_{ab} = (\Delta^{-1})_{ab}\sigma^2;\ \sigma_{ac}\rho_{ac} = (\Delta^{-1})_{ac}\sigma^2;$$

$$\sigma_b \sigma_c \rho_{bc} = (\Delta^{-1})_{bc}\sigma^2$$

where $(\Delta^{-1})_{aa}$, $(\Delta^{-1})_{ab}$ are the terms of the reciprocal matrix Δ^{-1} of the matrix Δ (Eq. 17d) and given by

$$(\Delta^{-1})_{aa} = \frac{\text{cofactor}^{**} \text{ of } \left(\frac{\partial f}{\partial a}\right)^2}{\Delta} = \frac{\Delta_{aa}}{\Delta}$$

etc., and

$$(\Delta^{-1})_{ab} = \frac{\text{cofactor of } \left(\frac{\partial f}{\partial a}\right)\left(\frac{\partial f}{\partial b}\right)}{\Delta} = \frac{\Delta_{ab}}{\Delta}$$

etc., and

$$\sigma^2 = \frac{\sum(\Delta y)^2}{r - m}$$ where m is the number of parameters.

---

* The independent variable x in Eq. 5 enters Eq. 18 in the evaluation of the partial derivatives only. Since the partial derivatives are evaluated at specific values of x, they may be considered to be constant.

** Cofactor of $\left(\frac{\partial f}{\partial a}\right)^2$ is the minor of this term in Eq. 17d multiplied by the appropriate sign, $(-1)^{i+j}$.

Application to the Prot Data

Writing Eq. 19 in terms of the Prot symbols (see Eq. 15) gives

$$\sigma_{S_R}^{\ 2} = \frac{\sigma^2}{\Delta}\left[\Delta_{EE} + \Delta_{KK}\,\alpha^{2n} + \Delta_{nn}\,\alpha^{2n}(\log\alpha)^2 + 2\,\Delta_{EK}\,\alpha^{n} + 2\Delta_{En}\,\alpha^{n}\log\alpha + 2\Delta_{Kn}\,\alpha^{2n}\log\alpha\right] \qquad \text{Eq. 20}$$

where σ^2 is given by

$$\sigma^2 = \frac{\sum_{i=1}^{r}(S_R - S'_R)^2}{r-3}$$

Δ is given by Eq. 17d and Δ_{EE}, Δ_{EK}, etc. are cofactors of Δ.

The standard deviation of S_R may be computed from Eq. 20 for any desired value of α. For the special case of $\alpha = 0$, $S_R = E$ and Eq. 20 reduces to

$$\sigma_{S_R}^{\ 2} = \sigma_E^{\ 2} = \frac{\sigma^2}{\Delta}\,\Delta_{EE} \qquad \text{Eq. 21}$$

(Note in Eq. 21 that although $\log\alpha \to \infty$ as $\alpha \to 0$, $\alpha^n \log\alpha \to 0$ as $\alpha \to 0$.) where

$$\Delta_{EE} = \begin{vmatrix} \sum \alpha_i^{2n} & \sum \alpha_i^{2n}\log\alpha \\ \sum \alpha_i^{2n}\log\alpha_i & \sum \alpha_i^{2n}(\log\alpha_i)^2 \end{vmatrix}$$

A sample computation is given in Table V to illustrate the procedure.

BIBLIOGRAPHY

1. Boresi, A. P. and T. J. Dolan, "An Appraisal of the Prot Method of Fatigue Testing, Part I," Tech. Report 34, ONR Project NR-031-005, University of Illinois, January 1953.

2. Prot, E. Marcel, "Fatigue Testing Under Progressive Loading; A New Technique for Testing Materials," Rèvue de Metallurgie, Vol. XLV, No. 12, 1948, p. 481. English Translation by E. J. Ward, WADC Tech. Report 52-148, Wright Air Development Center, September 1952.

3. Henry, D. L., "Prediction of Endurance Limits Using Linearly Increasing Loads," Unpublished Report, 10 March 1951.

4. Weibull, W., "Statistical Representation of Fatigue Failures in Solids," KTH Handl. No. 27, 1949.

5. Miner, M. A., "Cumulative Damage in Fatigue," Proceedings ASME, Vol. 67, 1945, pp. A-159-164.

6. Manual on Fatigue Testing, ASTM Special Technical Publication No. 91, 1949, p. 35.

7. Worthing, A. G. and J. Geffner, Treatment of Experimental Data, John Wiley and Sons, Inc., N. Y., pp. 239-243.

8. Sinclair, G. M., "An Investigation of the Coaxing Effect in Fatigue of Metals," Tech. Report No. 28, ONR Project NR-031-005, University of Illinois, March 1952; also ASTM Preprint No. 92, 1952.

9. Dolan, T. J., F. E. Richart, and C. E. Work, "The Influence of Fluctuations in Stress Amplitude on the Fatigue of Metals; Part I," Seventh Progress Report, and Part II, Ninth Progress Report, ONR, Project NR-031-005, University of Illinois, 1948. Also Trans. ASTM, Vol. 49, 1949, pp. 646-682.

10. Symposium on Statistical Aspects of Fatigue. Special Technical Publication No. 121, ASTM 1951. Symposium on Fatigue with Emphasis on the Statistical Approach, II, Special Technical Publication No. 137, ASTM 1952.

11. Dolan, T. J. and H. F. Brown, "Effect of Prior Repeated Stressing on the Fatigue Life of 75ST Aluminum," Tech. Report 29, ONR Project No. NR-031-005, University of Illinois, April 1952. Also ASTM Preprint No. 91, 1952.

12. Ward, E. J., and D. C. Schwartz, "Investigation of Prot Accelerated Fatigue Test," WADC Tech. Report 52-234, Materials Laboratory, Wright Air Development Center, November 1952.

13. Prot, E. Marcel, "Essais de Fatigue sous charge progressive, Etude de la dispersion des rèsultats," Rèvue Gènèrale De Mècanique, Janvier 1953.

14. Stulen, F. B., W. D. Lamson, "Preliminary Report of the Progressive-Load Method of Fatigue Testing," Unpublished report, Curtiss-Wright Corp., Propeller Div., April 1951.

15. Hald, A., Statistical Theory with Engineering Applications, John Wiley and Sons, Inc., 1952, Chap. 18 and 20.

16. Scarborough, J. B., Numerical Mathematical Analysis, Second Ed., Johns Hopkins Press, 1950, pp. 463-465.

17. Arley, N. and K. R. Bush, Introduction to the Theory of Probability and Statistics, John Wiley and Sons, Inc., 1950, Art. 6.5.

18. Arley, N. and K. R. Bush, Introduction to the Theory of Probability and Statistics, John Wiley and Sons, Inc., 1950, Chapter 12.

TABLE I

OUTLINE OF EXPERIMENTAL PROGRAM

Material	Type of Specimen	Starting Stress
Aluminum Alloy 75S-T6*	Unnotched Unnotched	10,000 psi 20,000 psi
Ingot Iron (B)	Unnotched Unnotched	20,000 psi 10,000 psi
Ingot Iron (A)*	Unnotched Unnotched	10,000 psi 30,000 psi
SAE 2340 Steel	Unnotched Notched Notched	65,000 psi 30,000 psi 15,000 psi
14-B-50 Steel	Unnotched Notched Notched	50,000 psi 35,000 psi 25,000 psi

* Experimental results reported in Part I (1).

TABLE II

CHEMICAL COMPOSITION, HEAT TREATMENT AND HARDNESS OF METALS TESTED

Material	Chemical Comp.	Time	°F	Quench	Temper Time	Temper °F	Hardness Rockwell Scale B	Hardness Rockwell Scale C
75S-T6 Aluminum Alloy*	Zn 5.6 Mg 2.5 Cu 1.6 Cr 0.3	---------As received------------------ (Rolled)					91.2	
Ingot Iron (B)	C .012 Mn .017 P .005 S .025	1 hr.	1400	Water			54.	
Ingot Iron (A)*	C .012 Mn .017 P .005 S .025	----------As received------------------ (Hot Rolled)						
SAE 2340 Steel	C .40 Mn .74 P .019 S .020 Si .28 Ni 3.48	1/2 hr.	1450	Oil	1 hr.	1200	99	
14-B-50 Steel	C .52 Mn .84 P .011 S .030 Si .27 Brn. .0005	10 min.	1550	Oil	1 hr.	550		51

* Experimental results reported in Part I (1).

TABLE III

TENSILE PROPERTIES OF METALS TESTED*

Material	Yield Strength 0.2% offset psi	Tensile Strength psi	Elongation % in 2 in.	Reduction of Area %
75S-T6** Aluminum Alloy	73,100	83,000	16	31.6
Ingot Iron (B)	33,000	55,125	21.5	66.5
Ingot Iron (A)**	54,000	62,900	18	65.6
SAE 2340 Steel	92,100	112,900	26.5	64.8
14-B-50 Steel	242,000	271,500	9.25	45.8

* All values represent the average of at least two tests. Specimens were 0.505 in. dia. and gage length was 2 in.

** Experimental results reported in Part I (1).

TABLE IV

COMPARISON OF ENDURANCE LIMITS OBTAINED BY THE CONVENTIONAL METHOD AND THE PROT METHOD

Material	Type of Spec.	Endurance Limit by Conventional Test lb/in^2	Starting Stress lb/in^2	Exponent n	Endurance Limit by Prot method lb/in^2
75S-T6 Aluminum Alloy*	Unnotch	25,000 at 10^8 cycles	20,000	0.5	------
			20,000	0.1786	20,500
			10,000	0.5	------
			10,000	0.1786	17,000
Ingot Iron (B)	Unnotch	23,000	20,000	0.5	28,100
			20,000	0.371	26,800
			10,000	0.5	28,800
			10,000	0.371	26,800
Ingot Iron (A)*	Unnotch	34,000	30,000	0.5	38,500
			30,000	0.371	36,200
			10,000	0.5	36,700
			10,000	0.371	35,400
SAE 2340 Steel	Unnotch	69,000	65,000	0.5	71,700
			65,000	0.418	69,500
	Notch	38,000	30,000	0.5	42,500
			30,000	0.4	40,700
			30,000	0.318	38,800
			15,000	0.5	39,200
			15,000	0.4	38,300
			15,000	0.318	36,900
14-B-50 Steel	Unnotch	61,000	50,000	0.5	60,000
			50,000	0.456	61,100
			50,000	0.717	68,000
	Notch	39,000	35,000	0.50	37,500
			35,000	0.55	38,400
			25,000	0.5	37,000
			25,000	0.55	38,300

* Experimental results reported in Part I (1).

TABLE V

SAMPLE COMPUTATION OF OPTIMUM VALUES OF E, K AND n AND STANDARD DEVIATION BY THE METHOD OF APPENDIX A.

DATA FOR NOTCHED 14-B-50 STEEL, σ_o = 35,000 psi

To evaluate Eq. 17 the following quantities are required:
E_o = 37,500; K_o = 70,000; n_o = 0.500

Specimen Number	S_R	α	$\log \alpha$	$n_o \log \alpha$	α^{n_o}
1	41,400	3.42612×10^{-3}	-2.4651975	-1.2325987	5.85330×10^{-2}
2	42,400	4.41791×10^{-3}	-2.3547821	-1.1773911	6.64674×10^{-2}
3	43,900	5.02541×10^{-3}	-2.2988285	-1.1494143	7.08901×10^{-2}
4	50,000	4.36047×10^{-2}	-1.3604667	-0.6802334	2.08817×10^{-1}
5	52,900	4.60154×10^{-2}	-1.3370968	-0.6685484	2.14512×10^{-1}
6	48,750	4.84155×10^{-2}	-1.3150156	-0.6575068	2.20035×10^{-1}
7	62,750	1.42747×10^{-1}	-0.8454330	-0.4227165	3.77819×10^{-1}
8	59,000	1.44578×10^{-1}	-0.8398978	-0.4199489	3.80234×10^{-1}
9	70,100	1.52875×10^{-1}	-0.8156635	-0.4078318	3.90992×10^{-1}
10	71,800	1.55734×10^{-1}	-0.8076165	-0.4038083	3.94632×10^{-1}
11	76,900	3.20336×10^{-1}	-0.4943942	-0.2471971	5.65982×10^{-1}
12	84,100	3.21546×10^{-1}	-0.4927569	-0.2463785	5.67050×10^{-1}
13	72,400	3.22971×10^{-1}	-0.4908365	-0.2454183	5.68305×10^{-1}
Σ					4.084268

Sample Number	$A_o \alpha^n$	S_R'	$S_R' - S_R$	α^{2n_o}	$\alpha^{n_o} \log \alpha$
1	4,097	41,597	197	34.26112×10^{-4}	-14.42954×10^{-2}
2	4,653	42,153	- 247	44.1795×10^{-4}	-15.65152×10^{-2}
3	4,962	42,462	-1438	50.25406×10^{-4}	-16.29642×10^{-2}
4	14,617	52,117	2117	4.36045×10^{-2}	-2.84089×10^{-1}
5	15,016	52,516	- 384	4.60154×10^{-2}	-2.86823×10^{-1}
6	15,402	52,906	4152	4.84154×10^{-2}	-2.89349×10^{-1}
7	26,447	63,947	1197	14.27472×10^{-2}	-3.19421×10^{-1}
8	26,616	64,116	5115	14.45779×10^{-2}	-3.19358×10^{-1}
9	27,369	64,869	-5231	15.28747×10^{-2}	-3.18918×10^{-1}
10	27,624	65,124	-6676	15.57344×10^{-2}	-3.18711×10^{-1}
11	39,618	77,118	218	32.03356×10^{-2}	-2.79818×10^{-1}
12	39,694	77,194	-6906	32.15457×10^{-2}	-2.79418×10^{-1}
13	39,781	77,281	4881	32.29705×10^{-2}	-2.78945×10^{-1}
Σ			-3004	1.711691	- 3.438625

TABLE V
(Cont'd.)

Sample Number	$\alpha^{2n_o} \log \alpha$	$\alpha^{2n_o} (\log \alpha)^2$	$\alpha^{n_o} (S'_R - S_R)$	$\alpha^{n_o} \log \alpha (S'_R - S_R)$
1	-84.46043×10^{-4}	2.08212×10^{-2}	11.531	- 28.426
2	$-104.03225 \times 10^{-4}$	2.44973×10^{-2}	- 16.417	38.660
3	$-115.52548 \times 10^{-4}$	2.65573×10^{-2}	- 101.940	234.343
4	-5.93226×10^{-2}	8.07066×10^{-2}	442.060	- 601.416
5	-6.15270×10^{-2}	8.22674×10^{-2}	- 82.373	111.140
6	-6.33669×10^{-2}	8.37228×10^{-2}	913.585	-1201.377
7	-12.06833×10^{-2}	10.20298×10^{-2}	452.249	- 382.347
8	-12.14308×10^{-2}	10.19895×10^{-2}	1945.277	-1633.836
9	-12.46944×10^{-2}	10.17087×10^{-2}	-2045.279	1668.260
10	-12.57736×10^{-2}	10.15767×10^{-2}	-2634.563	2127.715
11	-15.83720×10^{-2}	7.82981×10^{-2}	123.384	- 61.000
12	-15.84440×10^{-2}	7.80744×10^{-2}	3196.047	1929.661
13	-15.85258×10^{-2}	7.78103×10^{-2}	2773.897	-1361.531
Σ	- 1.182842	0.960060	-2134.630	838.846

The numerators of Eq. 17a, b, and c become

$$\Delta_E = \begin{vmatrix} -3004. & 4.084268 & -3.438625 \\ -2134.630 & 1.711691 & -1.182842 \\ +838.846 & -1.182842 & +0.960060 \end{vmatrix} = -160.877$$

$$\Delta_K = \begin{vmatrix} 13 & -3004 & -3.438625 \\ 4.084268 & -2134.630 & -1.182842 \\ -3.438625 & +838.846 & +0.960060 \end{vmatrix} = -722.973$$

$$\Delta_n = \begin{vmatrix} 13 & 4.084268 & -3004. \\ 4.084268 & 1.711691 & -2134.630 \\ -3.438625 & -1.182842 & +838.846 \end{vmatrix} = -1340.466$$

and

$$\Delta = \begin{vmatrix} 13 & 4.084268 & -3.438625 \\ 4.084268 & 1.711691 & -1.182842 \\ -3.438625 & -1.182842 & +0.960060 \end{vmatrix} = 0.144764$$

TABLE V
(Cont'd.)

From Eqs. 17, δE, δK, and δn become

$$\delta E = -\frac{\Delta_E}{\Delta} = 1111.31$$

$$K = -\frac{\Delta_K}{\Delta} = 4994.15$$

$$n = -\frac{\Delta_n}{2.3026\, K_o \Delta} = 0.05745$$

Therefore from Eq. 6, E_1, K_1, and n_1 become

$$E_1 = E_o + \delta E = 38,611.31; \; K_1 = K_o + \delta K = 74,994.15$$

$$n_1 = n_o + \delta n = 0.55745$$

Successive applications of this method give

$$E_2 = 38,315.72; \; E_3 = 38,330$$

$$K_2 = 75,012.08; \; K_3 = 75,138$$

$$n_2 = 0.5508237; \; n_3 = 0.55149$$

From Eq. 21, for the standard deviation of S_R at $\alpha = 0$, the following are required:

$$\sigma^2 = \frac{\sum (S_R - S'_R)^2}{r - 3} = \frac{189,113,175}{10} = 18,911,318$$

$$\Delta_{EE} = \begin{vmatrix} 1.459245 & -0.981910 \\ -0.981910 & 0.766014 \end{vmatrix} = 0.1536549$$

$$\Delta = 0.120650 \qquad \text{(after correction)}$$

$$\sigma_{S_R}{}^2 = \sigma_E{}^2 = \sigma^2 \frac{\Delta_{EE}}{\Delta} = 24,084,680$$

and the standard deviation is

$$\sigma_E = \pm 4,908 \ \text{lb/in}^2$$

For 14-B-50 steel, unnotched specimens, the standard deviation at $\alpha = 0$ was found to be

$$\sigma_E = \pm 3,730 \ \text{lb/in}^2$$

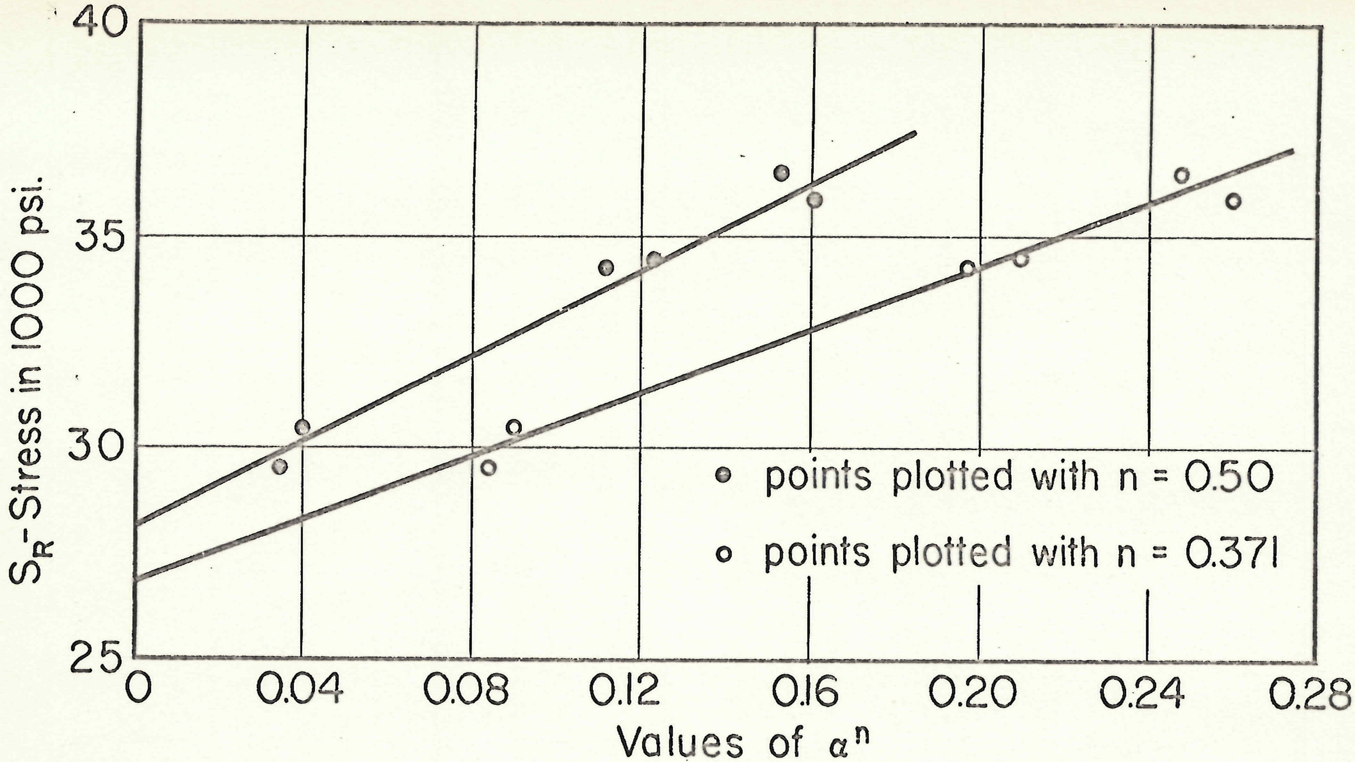

Fig. 1. Appraisal of Prot Method for Ingot Iron (B) (σ_o = 20,000 psi.).

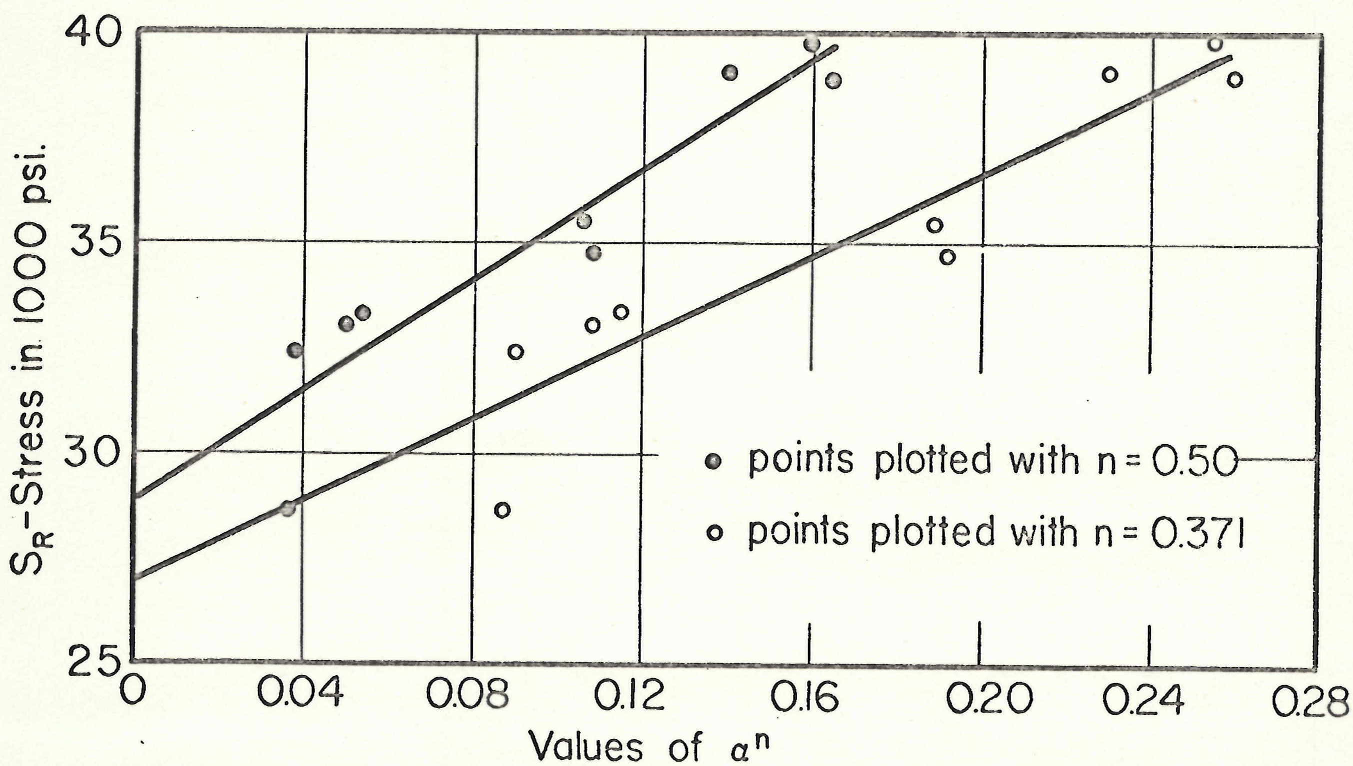

Fig. 2. Appraisal of Prot Method for Ingot Iron (B) (σ_o = 10,000 psi.).

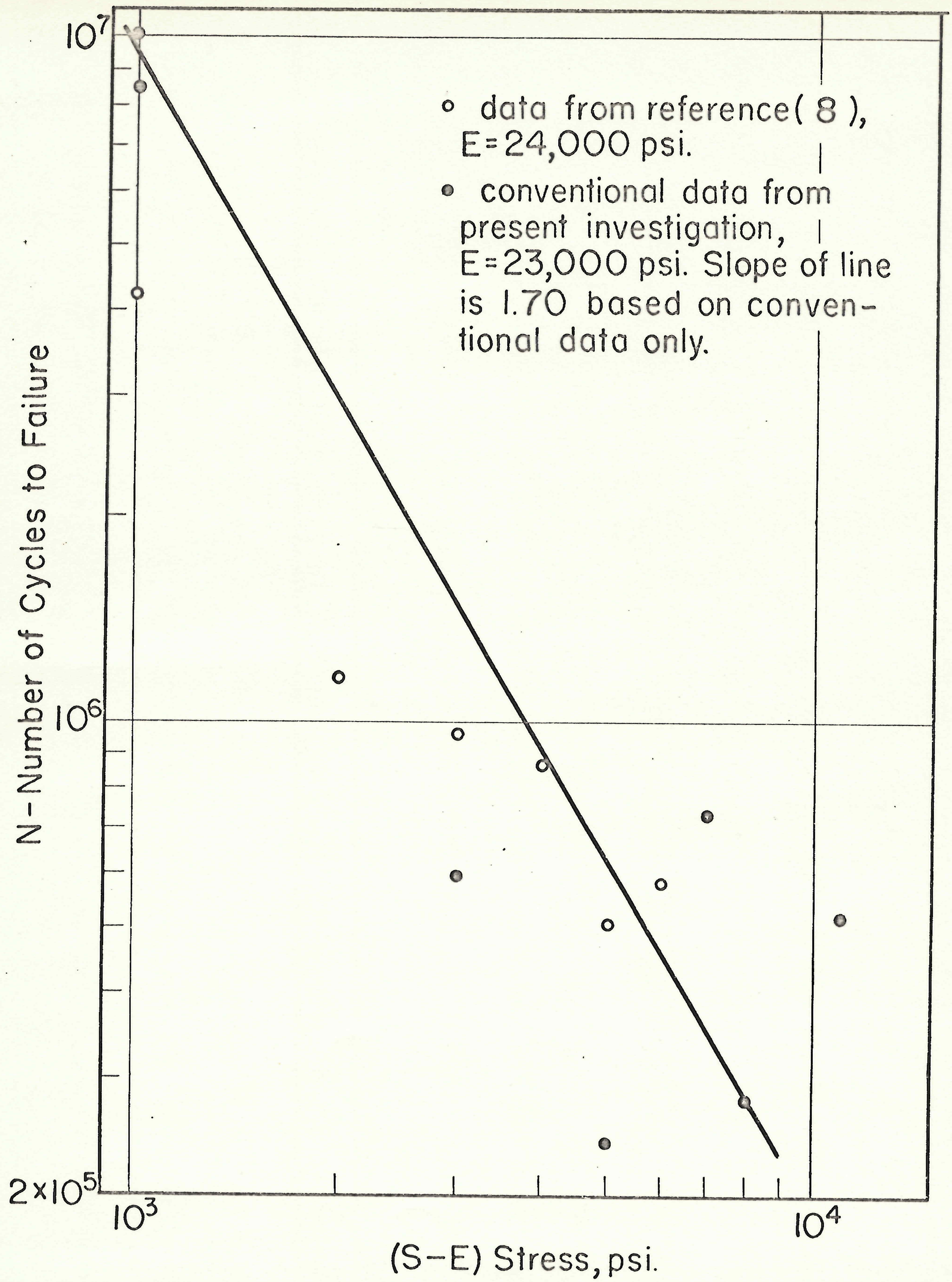

Fig. 3. N vs. (S-E) Diagram for Ingot Iron Specimens.

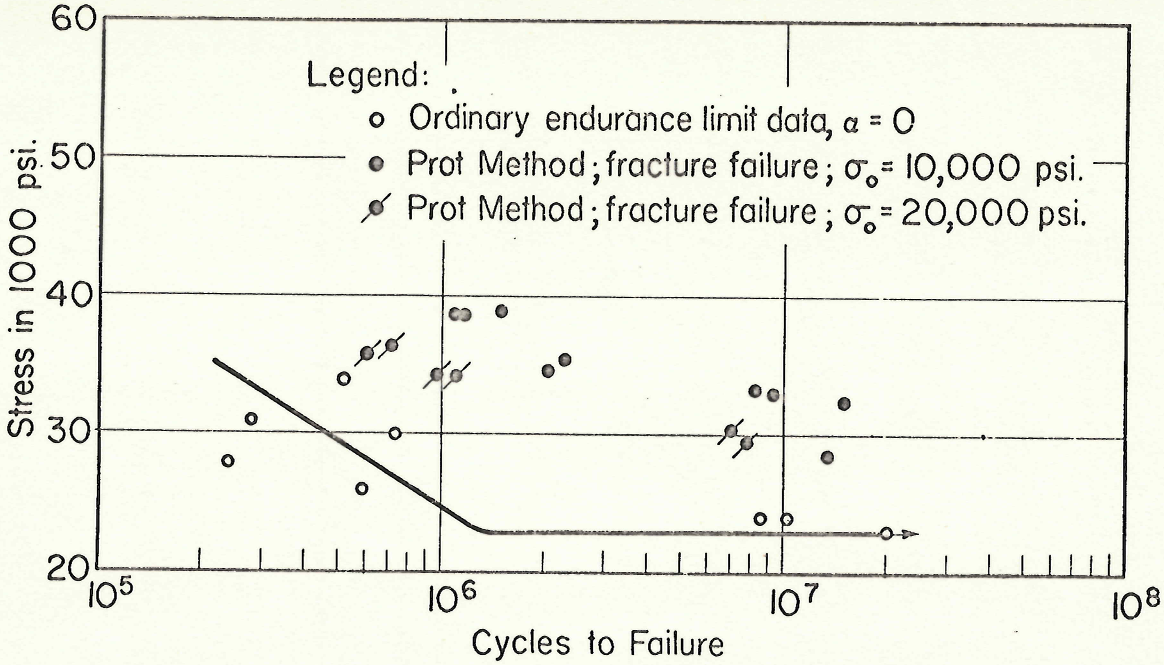

Fig. 4. S-N Curve for Complete Stress Reversals, Ingot Iron Specimens.

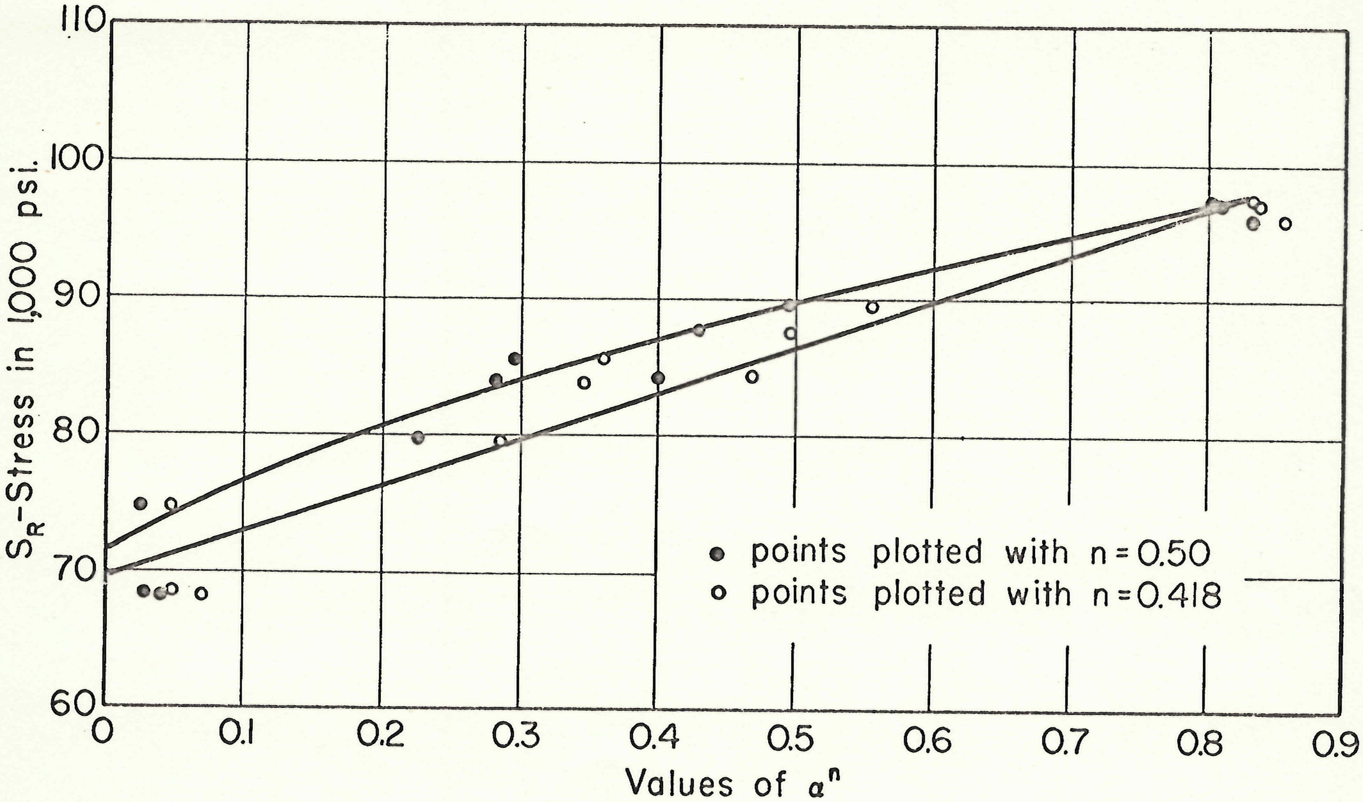

Fig. 5. Appraisal of Prot Method for SAE 2340 Steel Unnotched (σ_o = 65,000 psi.)

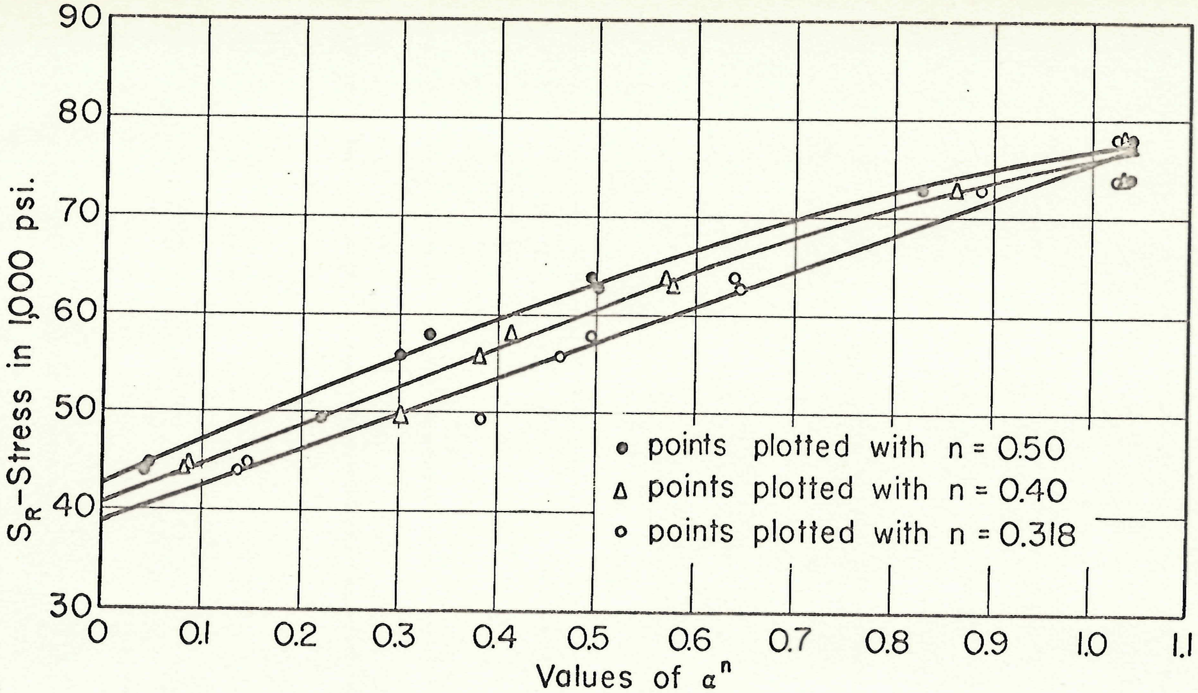

Fig. 6. Appraisal of Prot Method for SAE 2340 Steel Notched (σ_o = 30,000 psi.)

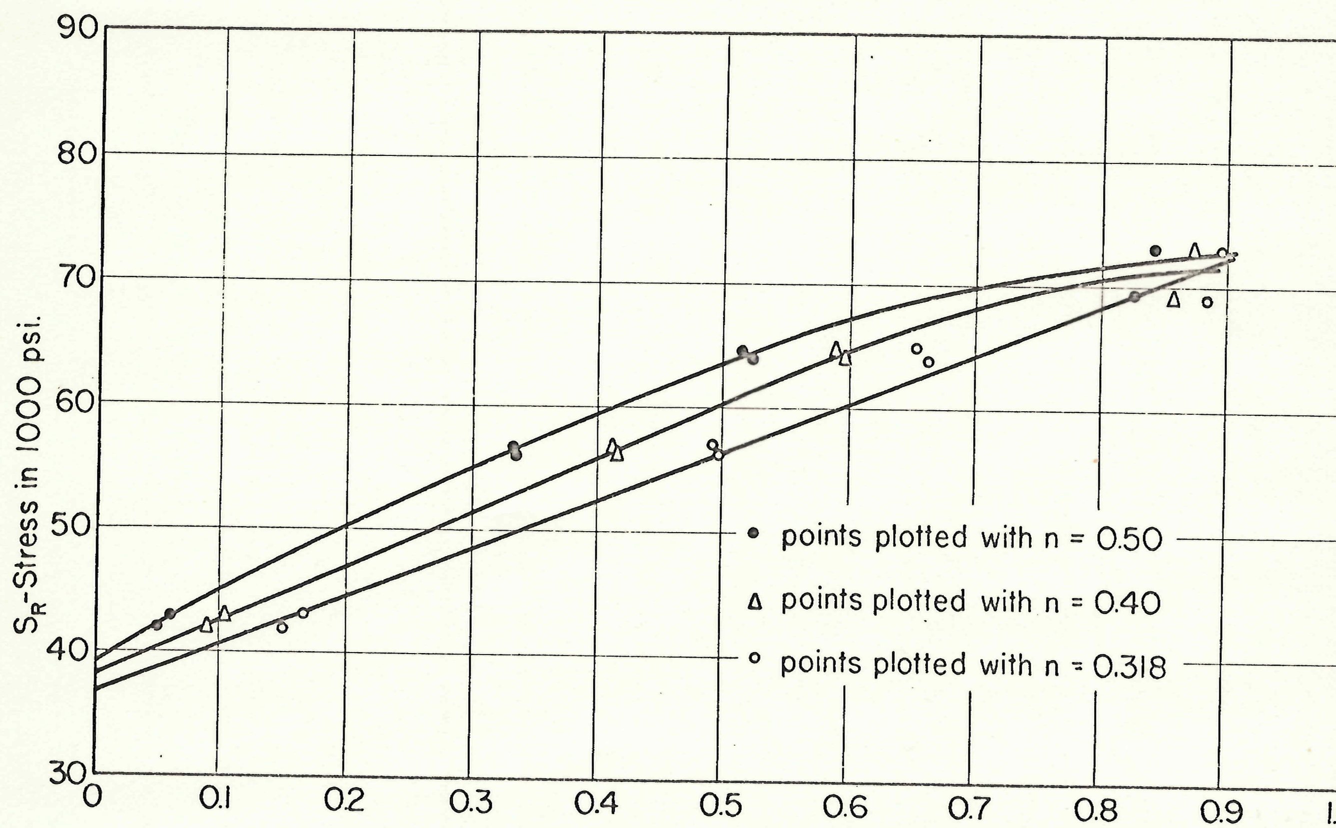

Fig. 7. Appraisal of Prot Method for SAE 2340 Steel Notched (σ_o=15,000 psi.)

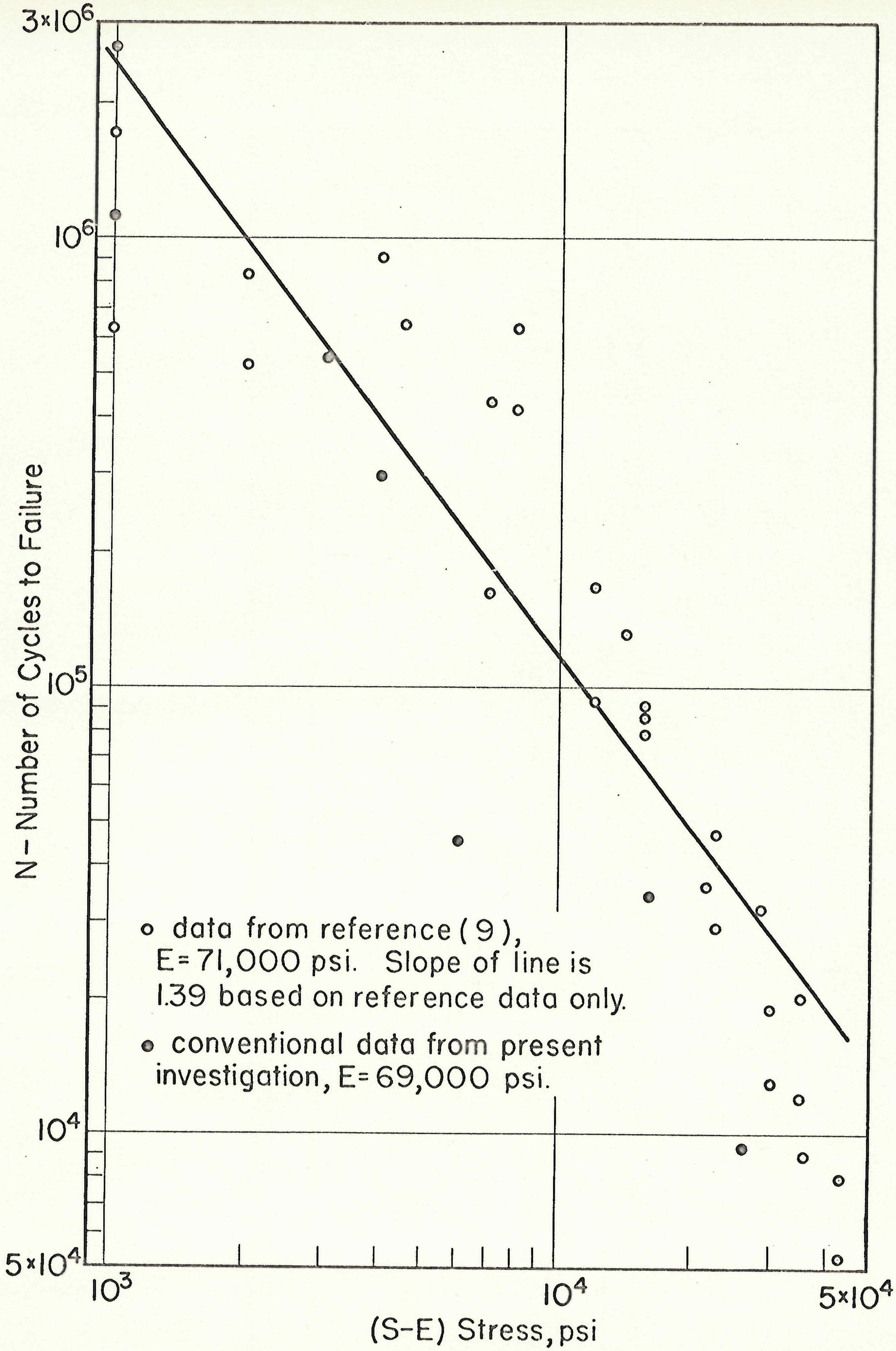

Fig. 8. N vs. (S-E) Diagram for SAE 2340 Steel Unnotched Specimens.

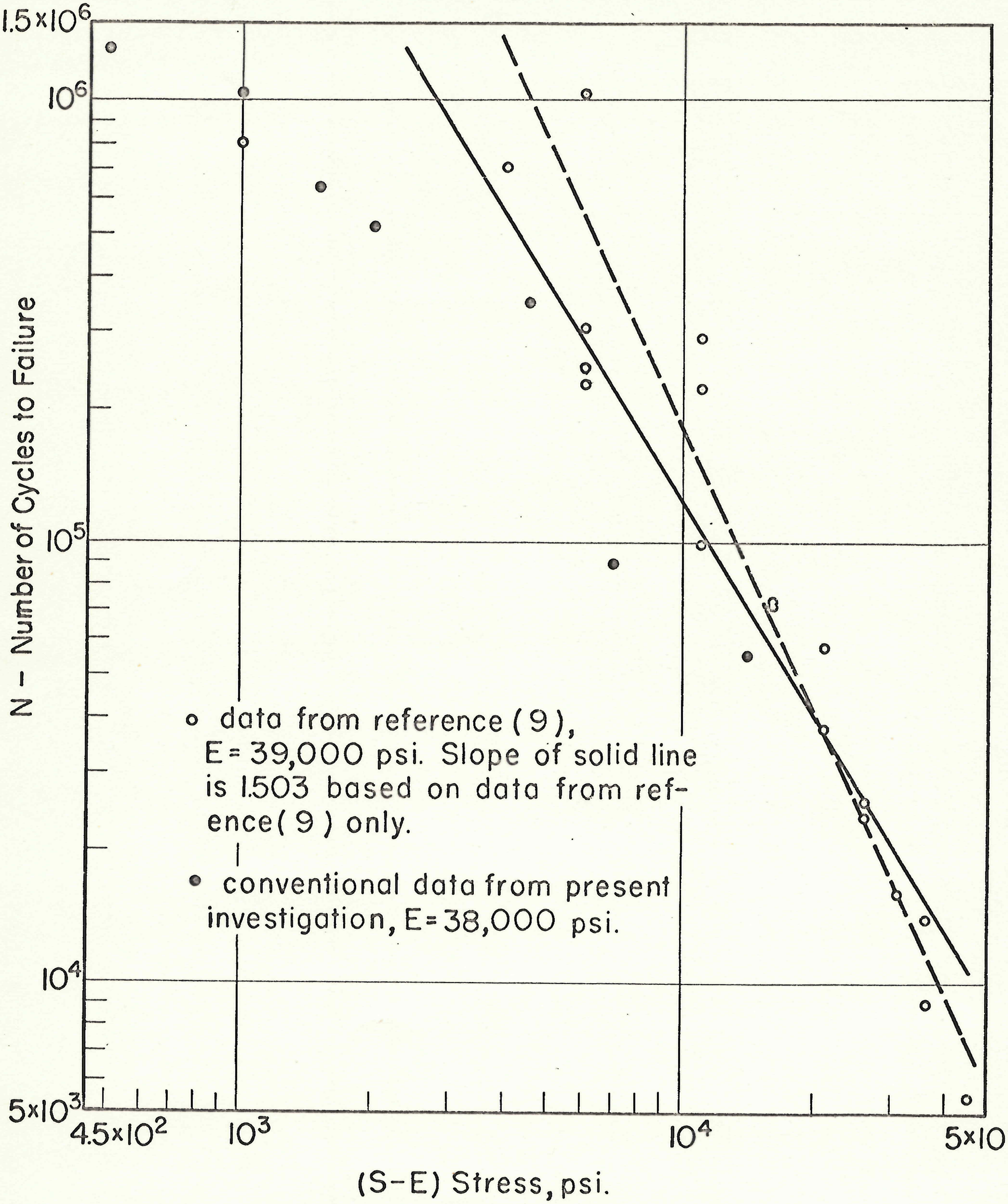

Fig. 9. N vs. (S−E) Diagram for SAE 2340 Steel Notched Specimens.

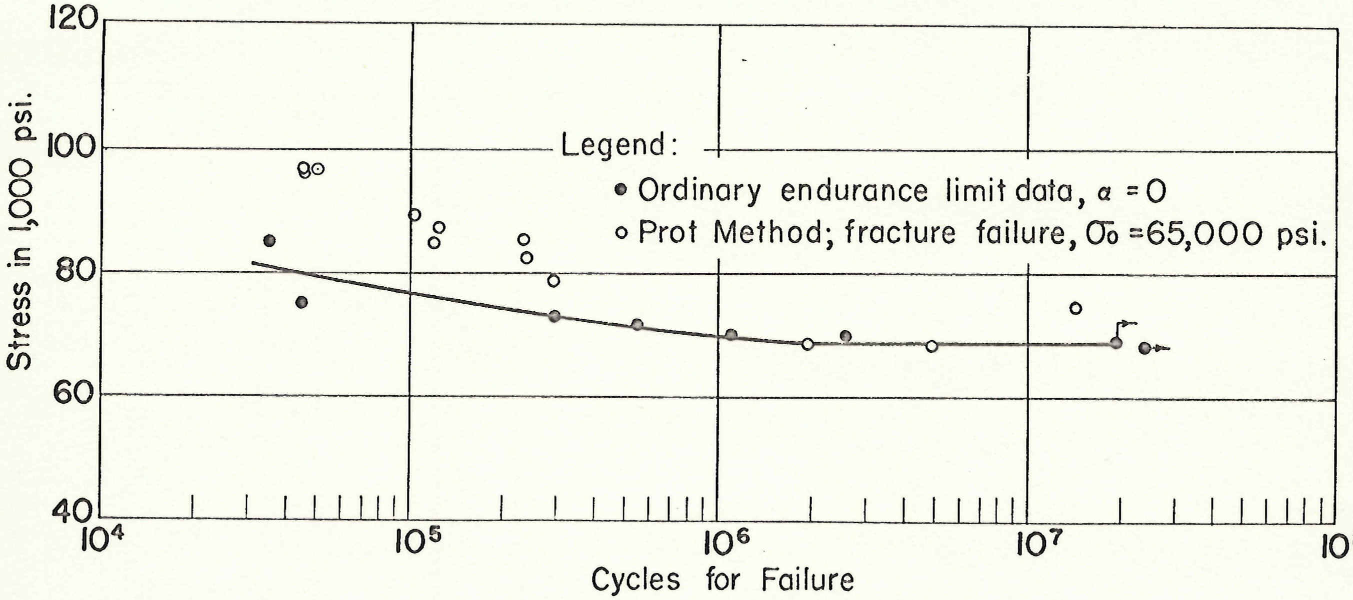

Fig.10. S-N Curve for Complete Stress Reversals; SAE 2340 Steel Unnotched Specimens.

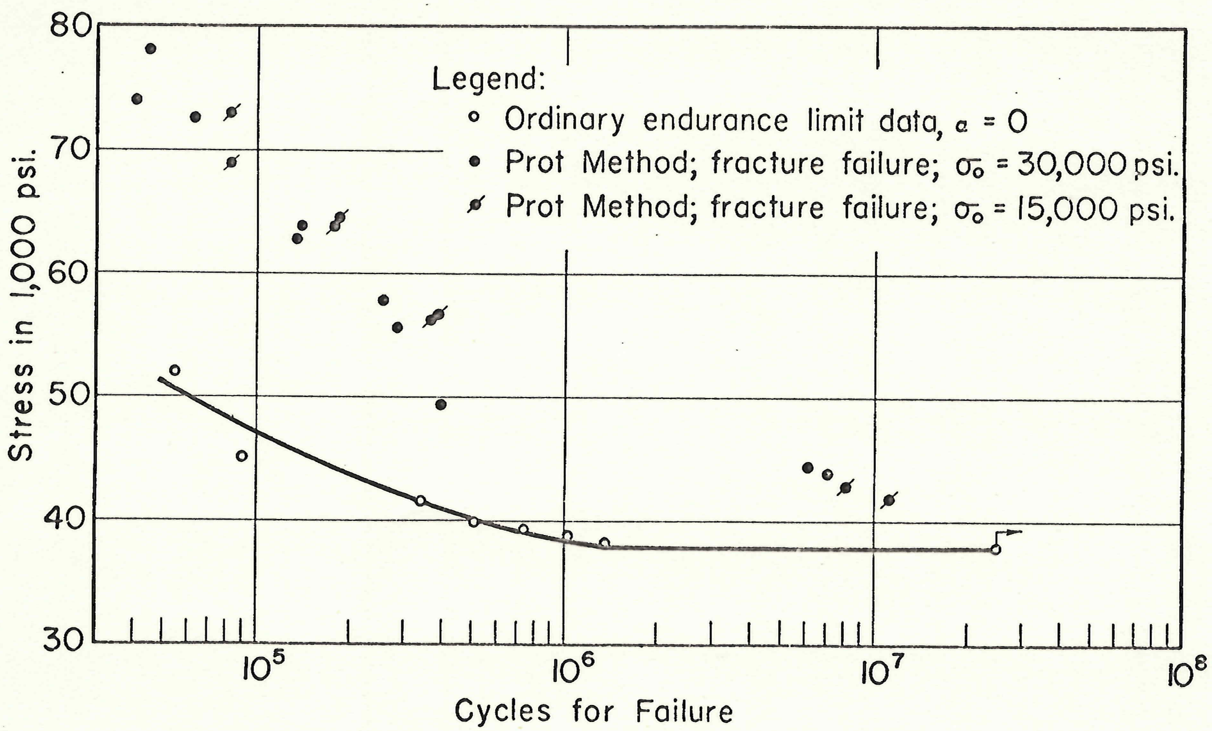

Fig.11. S-N Curve for Complete Stress Reversals; SAE 2340 Steel Notched Specimens.

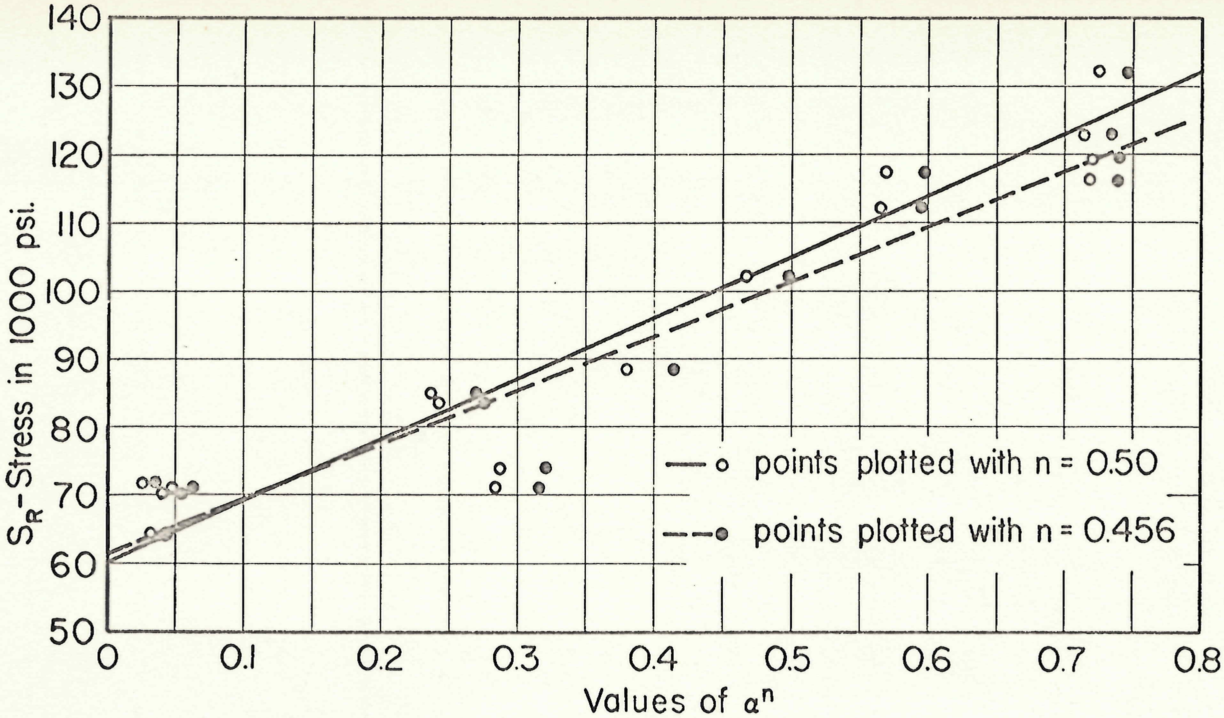

Fig. 12. Appraisal of Prot Method for 14 B50 Steel Unnotched (σ_o = 50,000 psi.).

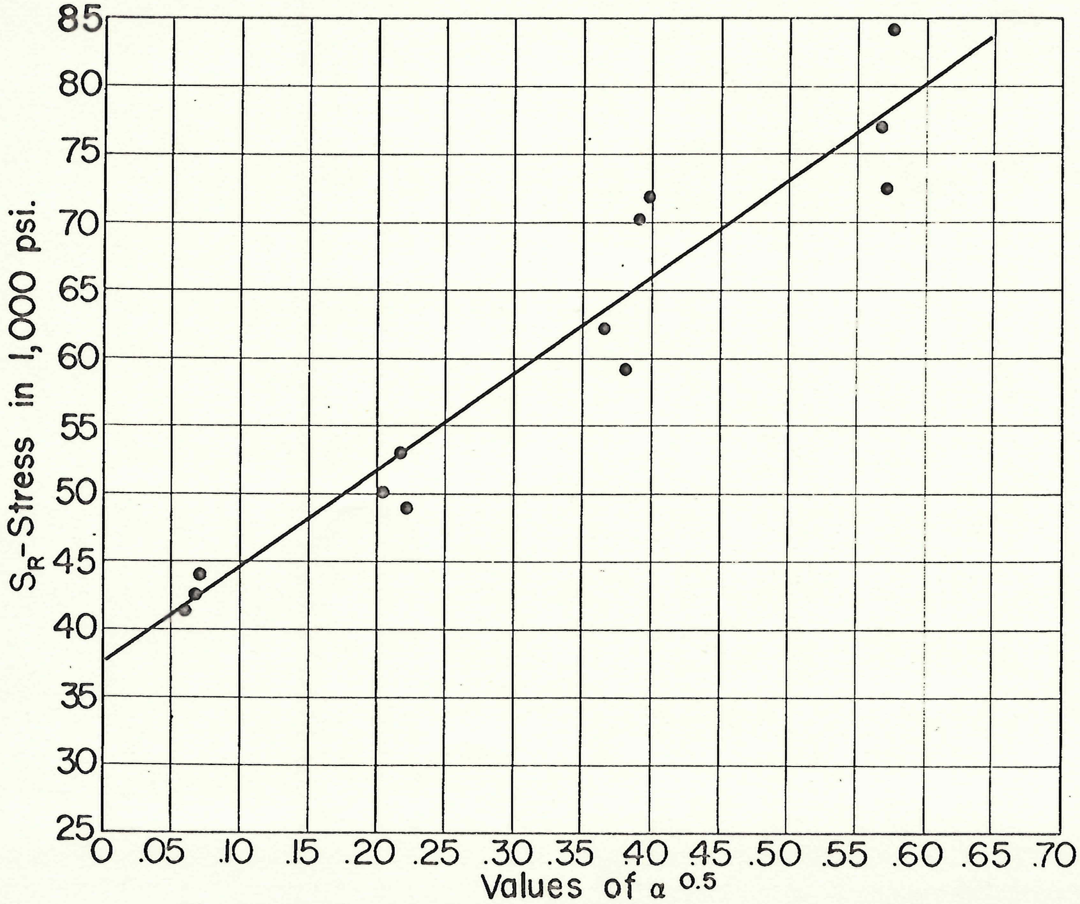

Fig. 13. Fatigue Data for Notched Specimens of 14 B 50 Steel (σ_o= 35,000 psi.)

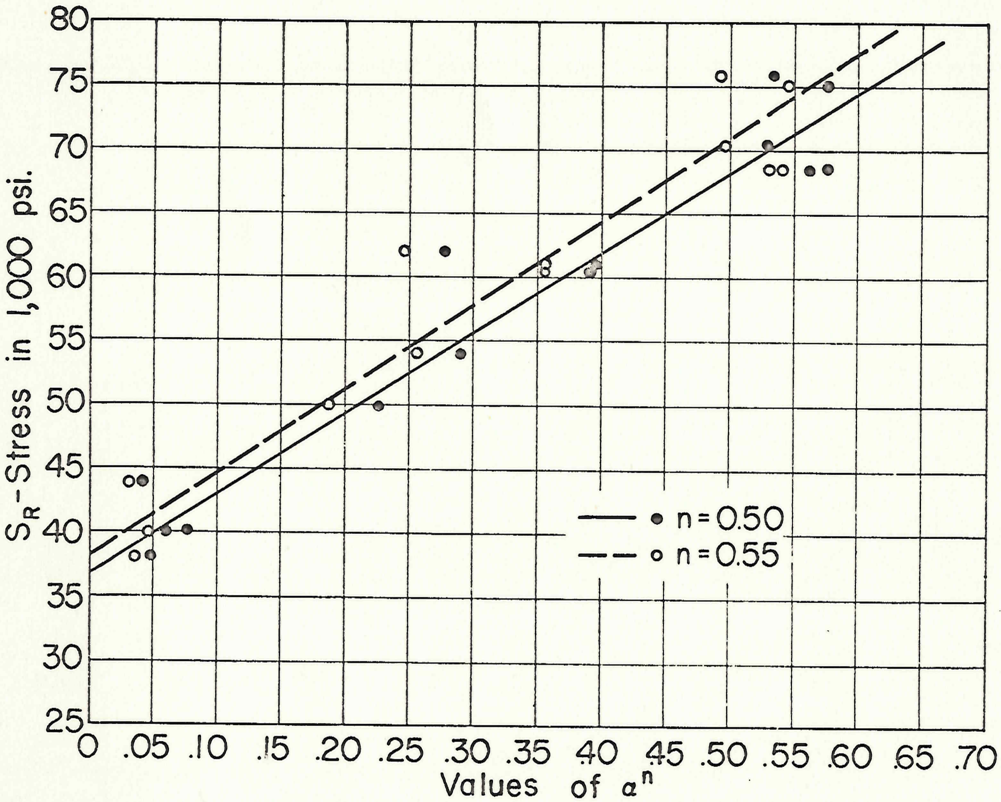

Fig. 14. Fatigue Data for Notched Specimens of 14 B 50 Steel (σ_o=25,000 psi.)

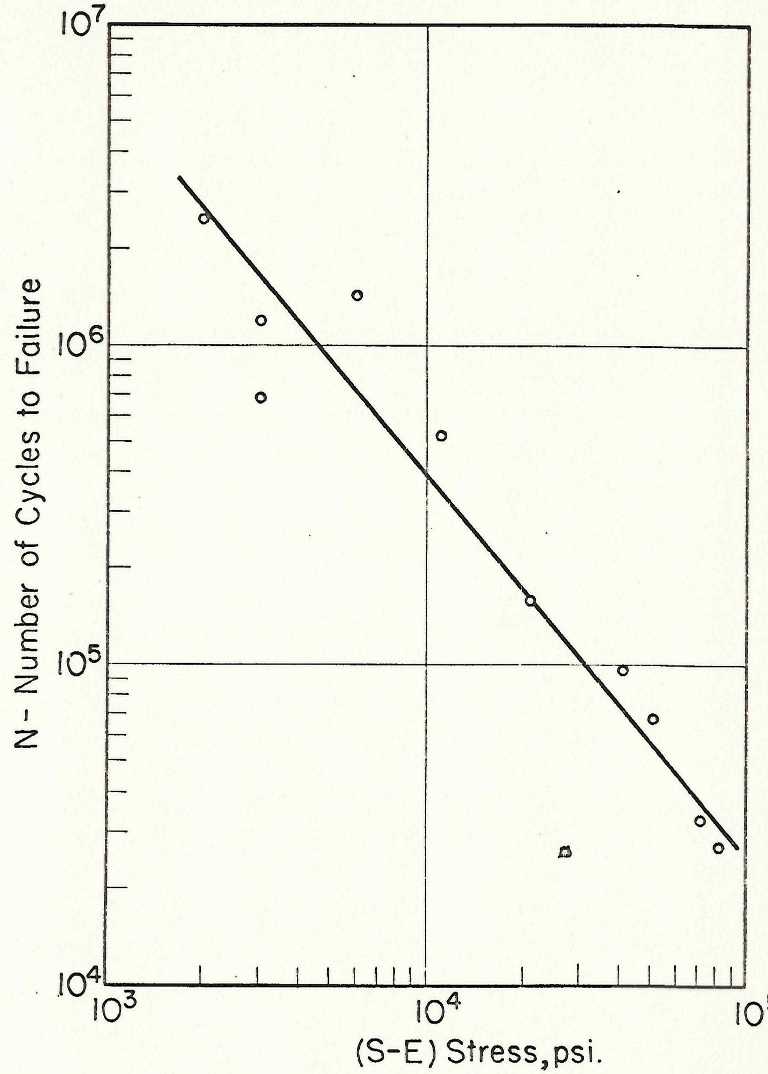

Fig. 15. N vs. (S-E) Diagram for 14 B 50 Steel Unnotched Specimens (E = 59,000 psi.). Slope of line is 1.193.

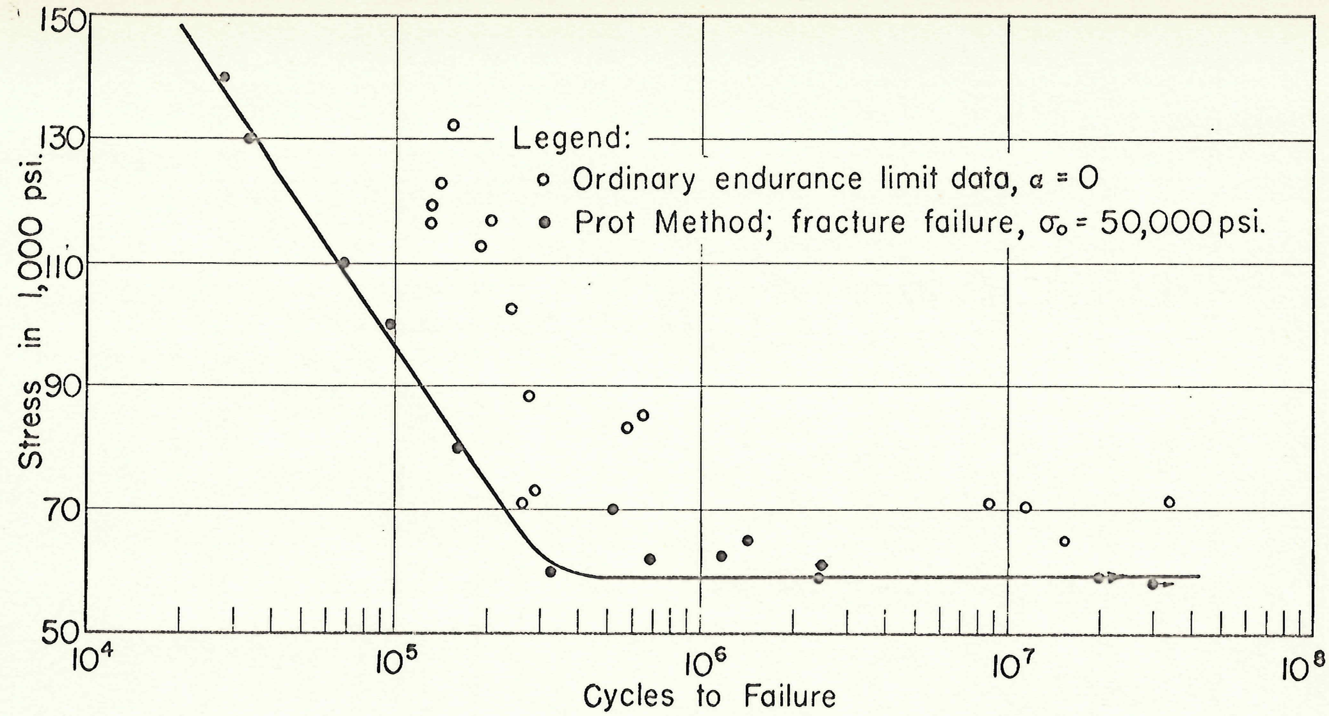

Fig. 16. S-N Curve for Complete Stress Reversals; 14 B 50 Steel Specimens.

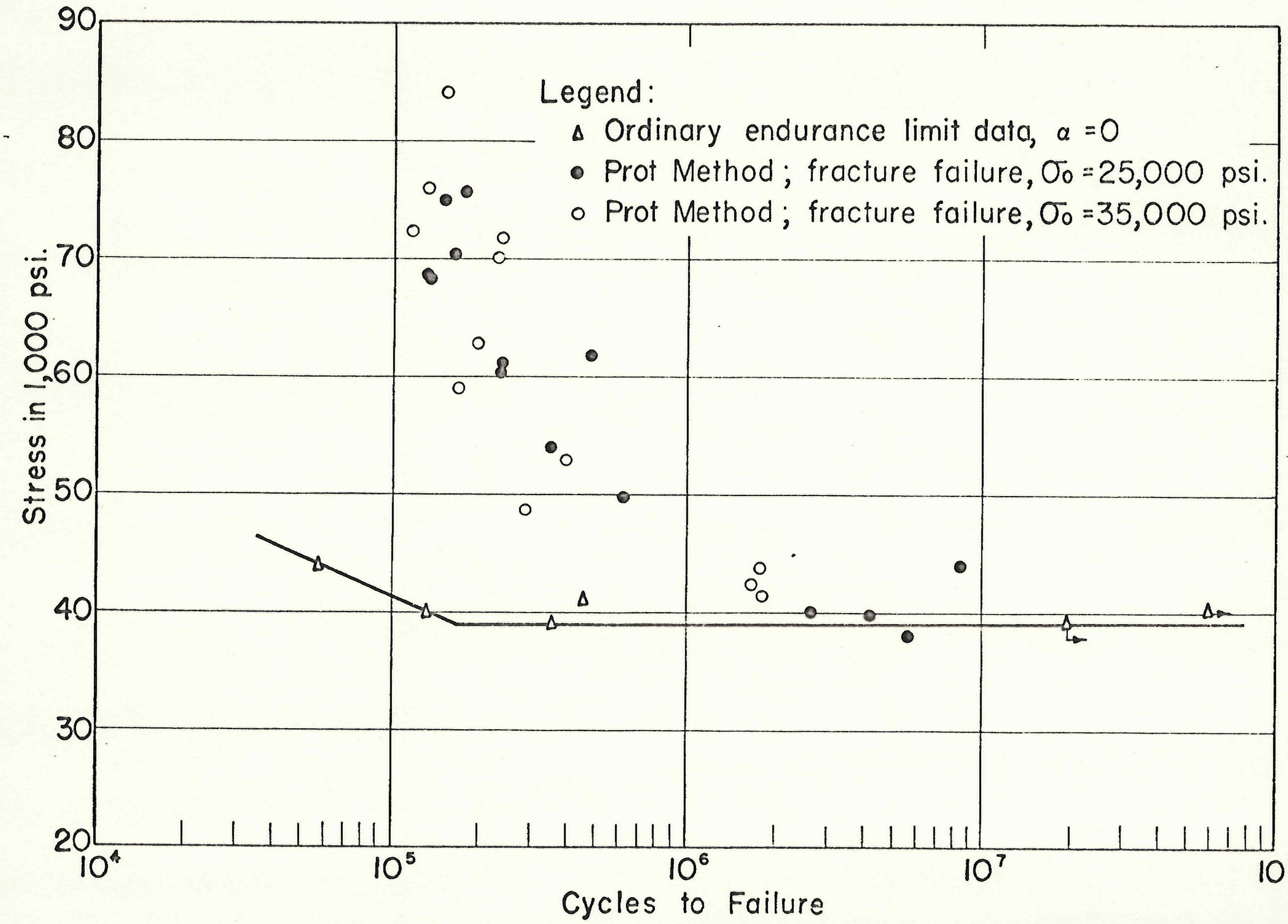

Fig. 17. S-N Curve for Complete Stress Reversals; 14 B 50 Steel Notched Specimens.

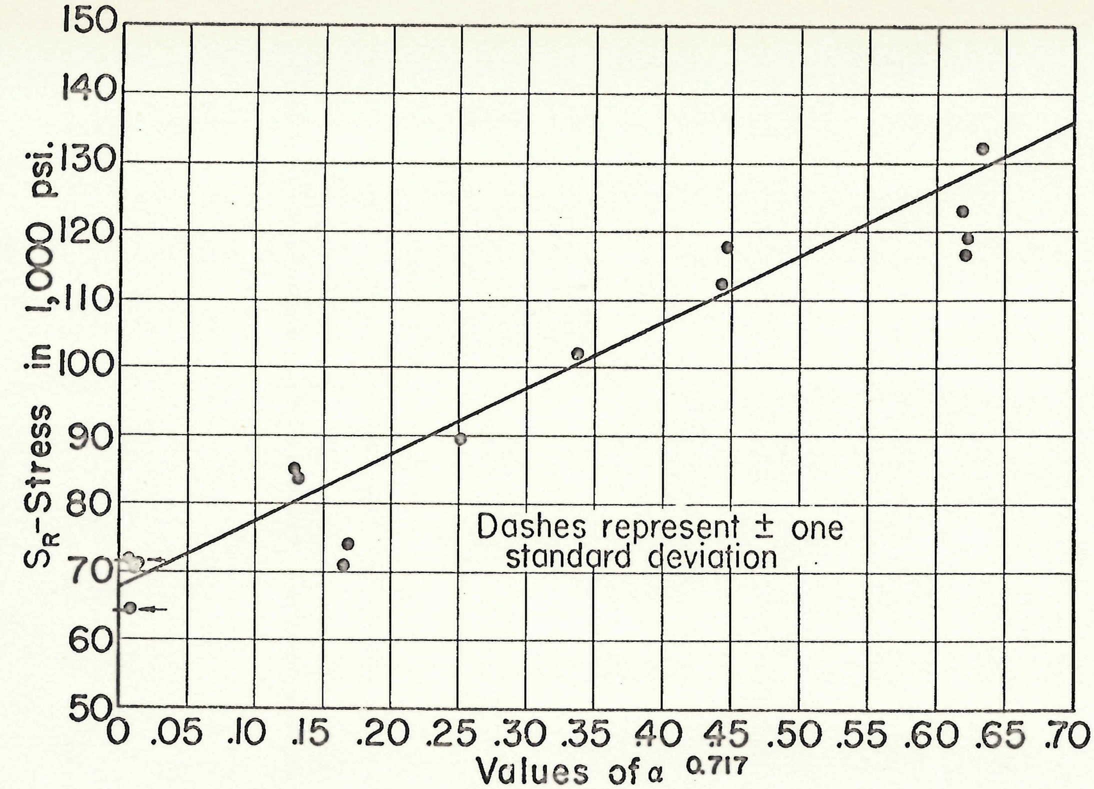

Fig. 18. Fatigue Data for Unnotched Specimens of 14 50 B Steel (σ_0 = 50,000 psi.)

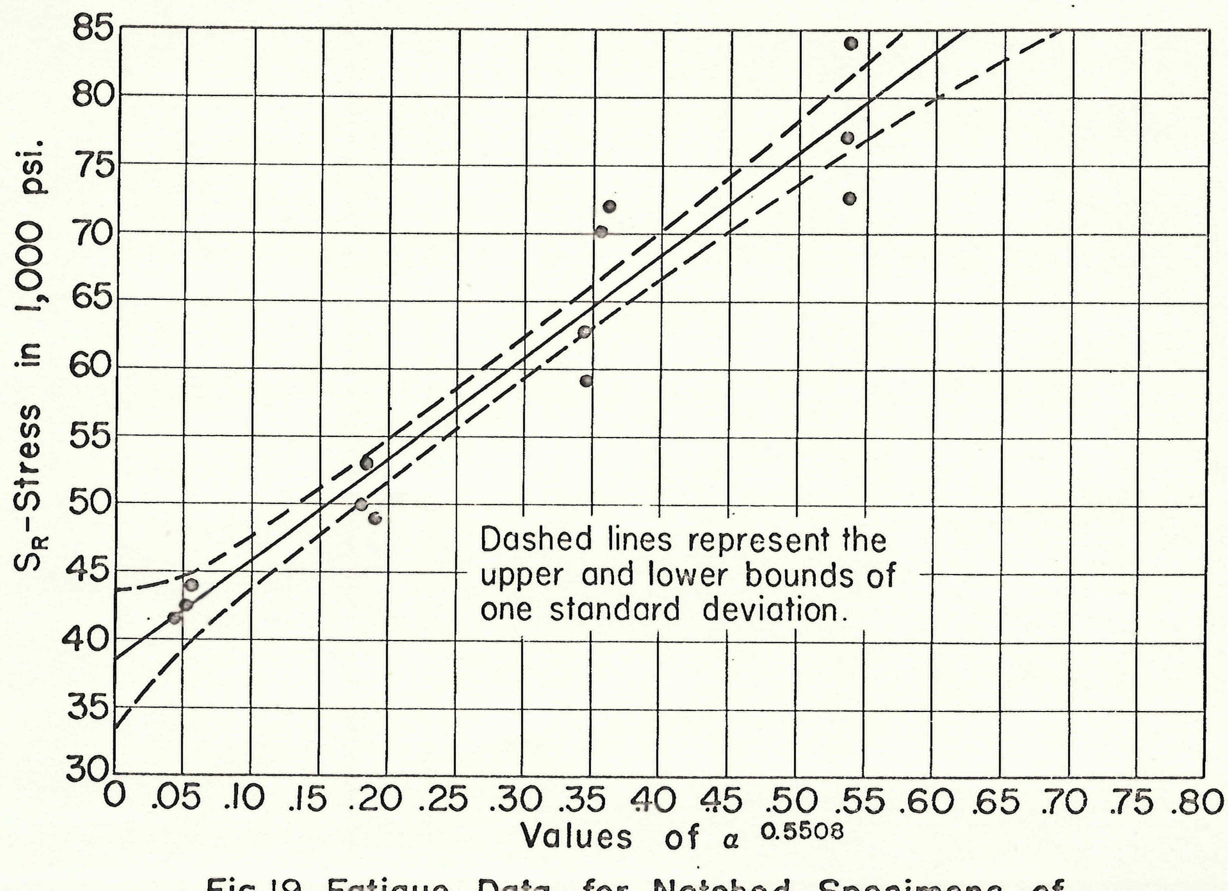

Fig. 19. Fatigue Data for Notched Specimens of 14 B 50 Steel (σ_0 = 35,000 psi.)

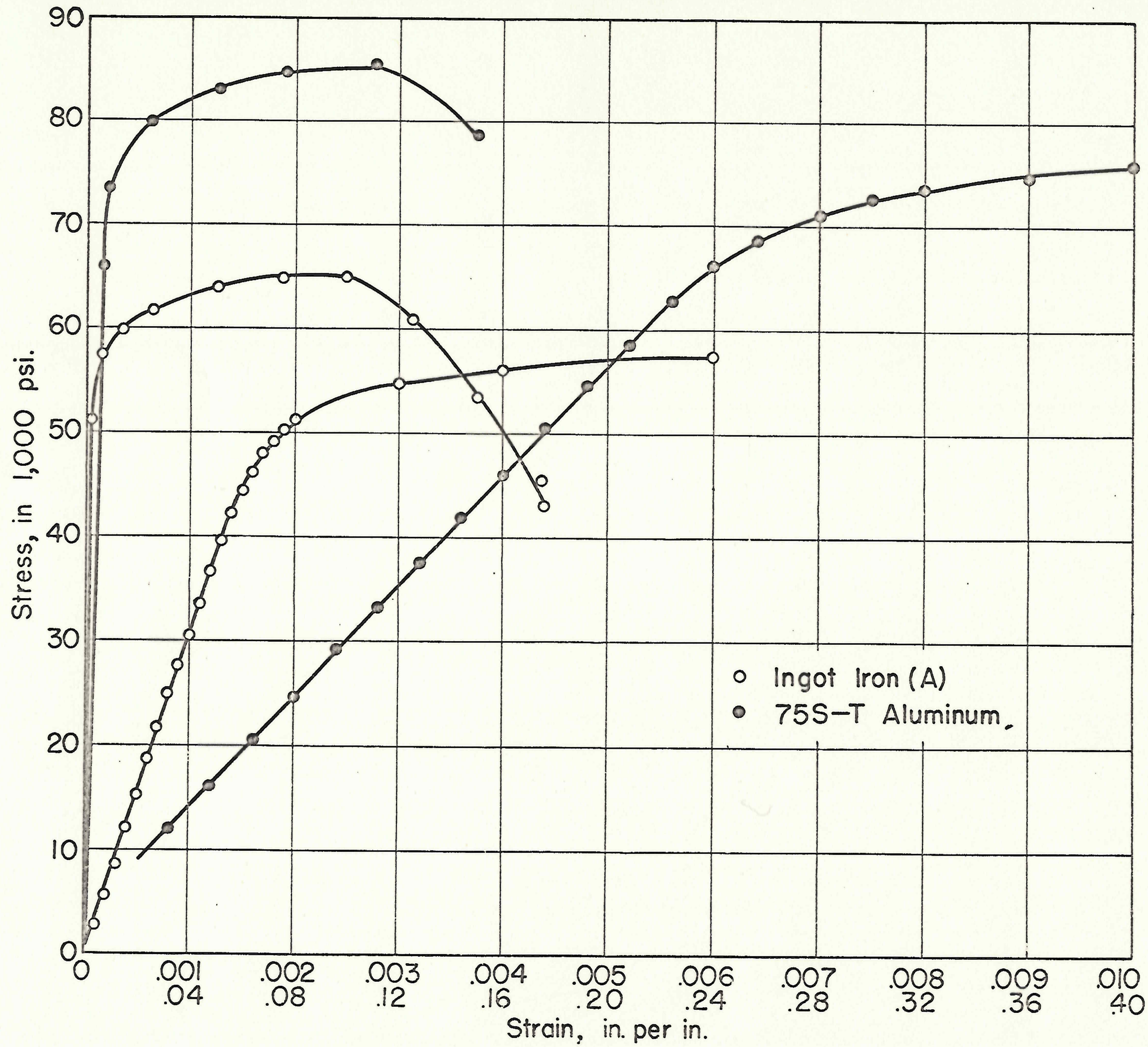

Fig. 20. Tensile Stress–Strain Curves.

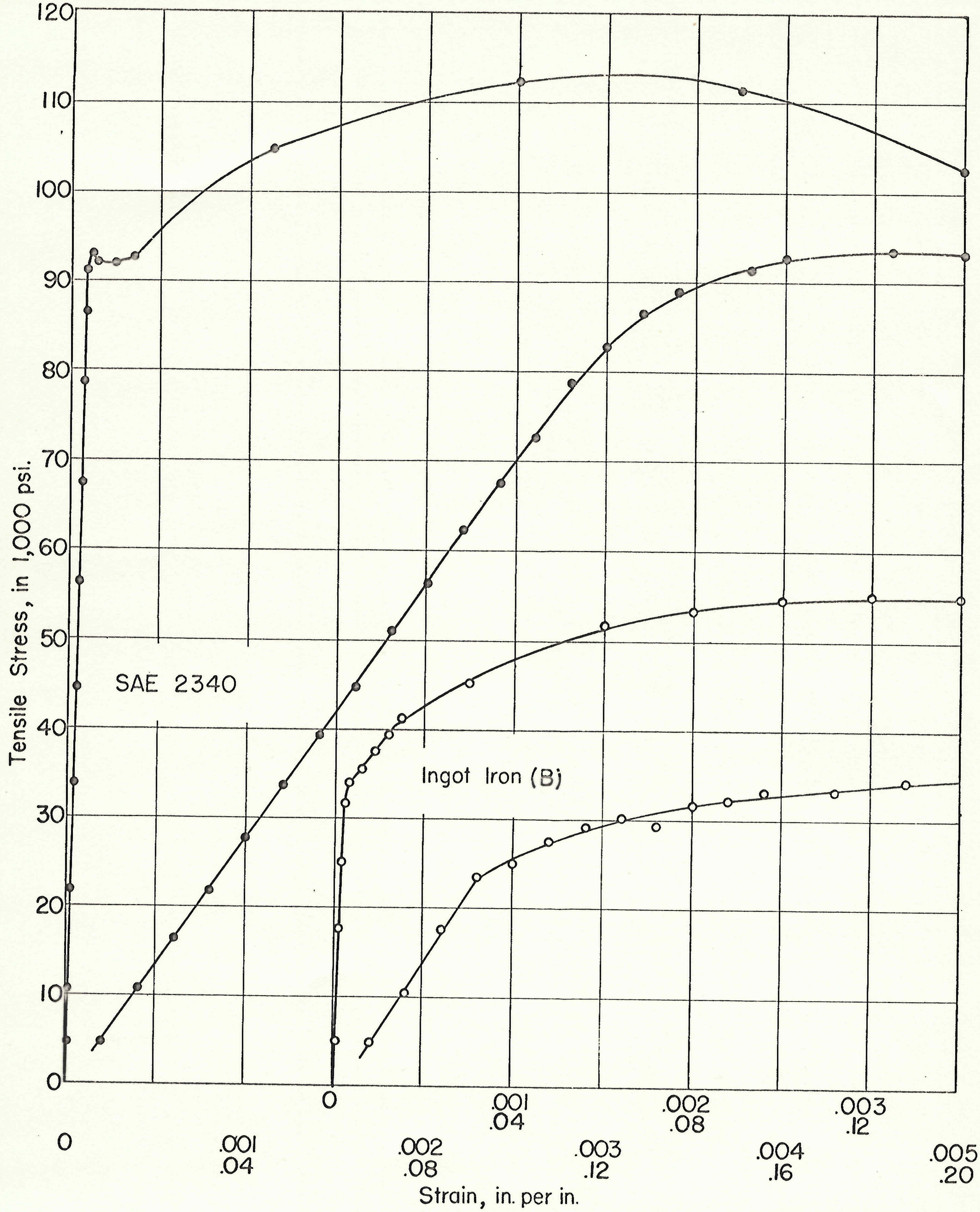

Fig. 21. Tensile Stress-Strain Curves

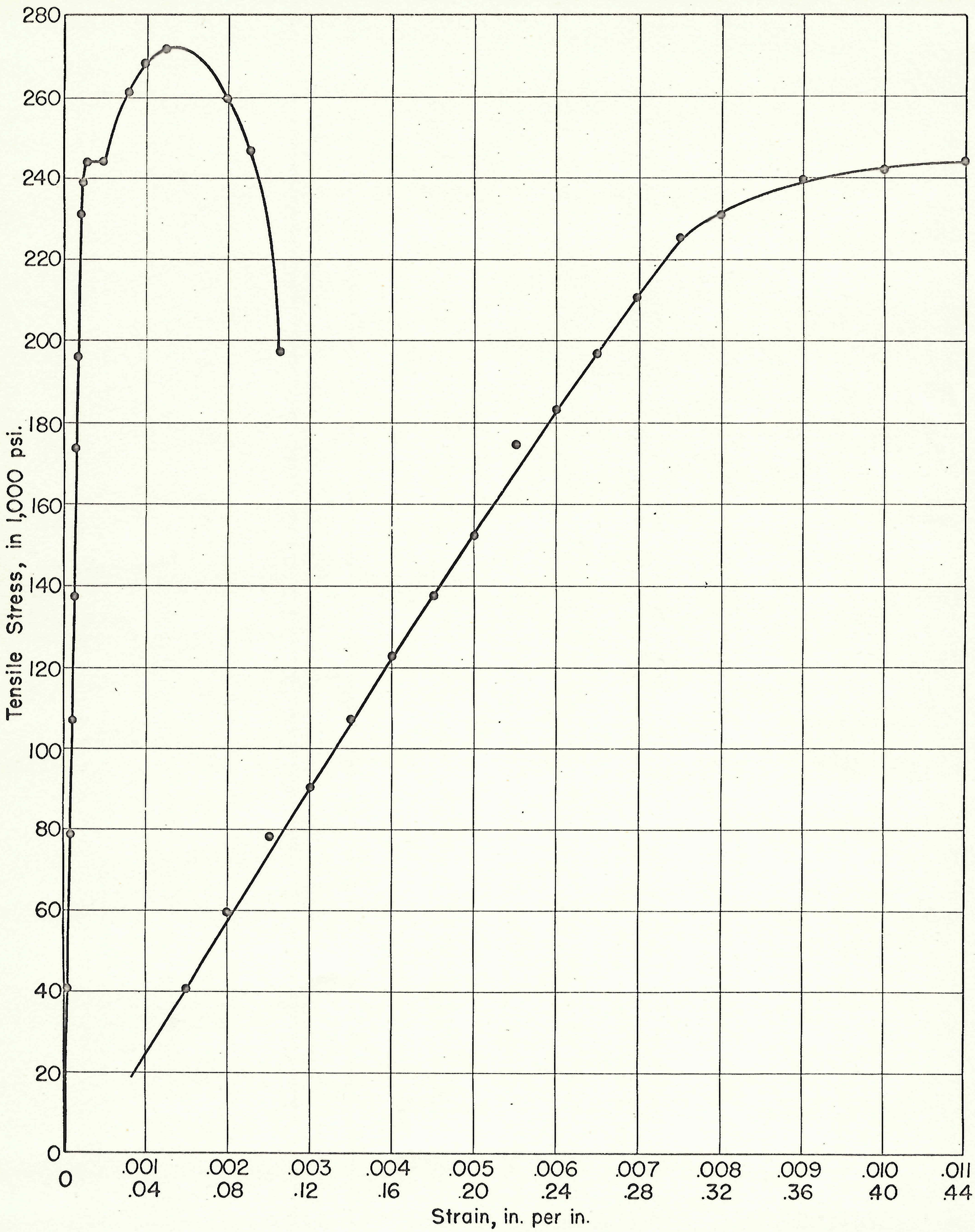

Fig. 22. Tensile Stress–Strain Curve; 14 B 50 Steel