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SIGNIFICANCE OF CYCLIC HISTORY DEPENDENT DEFORMATION PHENOMENA OF METALS ON FATIGUE LIFE ESTIMATION

by

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FOREWORD

Recent requirements for increased strength and service life of machines and structures have been met by the use of higher strength materials and new fabrication and joining methods. Simultaneously, failures due to fracture have increased relative to those resulting from excessive deformation. Frequently service conditions are such that low temperature brittle fracture, fatigue fracture, and high temperature creep rupture must be considered in a single system. National concern with increased safety, reliability, and cost has focused attention upon these problems.

Methods are now available to predict both fatigue crack initiation life and crack propagation life. Paradoxically the materials properties required for long fatigue crack initiation life are incompatible with the requirements of high fracture toughness. Thus, the conflicting design approaches and requirements placed on the material are confusing and often impossible to satisfy.

Numerous publications dealing with a variety of fracture problems have led to many new and useful developments. However, the synthesis of the concepts into methods for design, testing and inspection has lagged.

This program of study is intended to contribute to the integration, correlation, and organization of mechanics and materials concepts and research information into a form that will permit enlightened decisions to be made regarding fracture control. Reports are in preparation in three categories:

1. Research reports designed to explore, study and integrate isolated and/or conflicting concepts and methods dealing with life prediction,

2. Reports to introduce and summarize the state-of-the-art concepts and methods in particular areas, and

3. Example problems and solutions intended to illustrate the use of these concepts in decision making.

H. T. Corten
Principal Investigator

ABSTRACT

Simple calculations for determining estimated extremes in fatigue life of metals with or without accounting for cycle dependent hardening, softening and mean stress relaxation in fatigue analysis, are presented. These calculations aid in assessing the importance of cyclic deformation properties in cumulative damage procedures. An SAE 1045 quenched and tempered steel is considered for illustration. In general cycle dependent deformation phenomena may be important only in intermediate and long life situations. The sensitivity of predicted life to cyclic hardening or softening depends on the degree of hardening or softening a material exhibits, whereas in the case of mean stress relaxation it depends on the shape of the cyclic stress-strain curve or the strain hardening exponent. However the need for a proper accounting of cyclic history, recognized as the "cycle counting problem" in fatigue literature, is the most important requisite of a cumulative damage analysis. Two alternative approaches of dealing with the cycle counting problem are discussed.

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NOMENCLATURE

$\frac{\Delta\sigma}{2}$	stress amplitude
$\frac{\Delta\epsilon}{2}$	strain amplitude
σ_o	mean stress
$2N_f$	number of reversals to failure (2 x number of cycles to failure, N_f)
E	elastic modulus
σ_f	monotonic true fracture strength
σ_f', ϵ_f', b and c	fatigue properties (see Table 1)
D	damage per reversal ($1/2N_f$)
σ_{Max}	maximum stress

<u>Subscripts</u>

e	elastic component
p	plastic component

<u>Superscripts</u>

Mon	based on monotonic stress-strain curve
Cyc	based on cyclic stress-strain curve
T	corresponding to maximum mean stress in tension
C	corresponding to maximum mean stress in compression
0	corresponding to zero mean stress

INTRODUCTION

The importance of cyclic plasticity and associated cycle dependent deformation phenomena, such as cyclic hardening, softening, creep and relaxation and cyclic history dependent "memory" behavior, on fatigue life response of materials under uniaxial testing conditions has been emphasized in several recent articles. Computer based models capable of simulating these features of cyclic deformation behavior have been formulated for a number of structural metals including steels and aluminum alloys (1-3).* However, no investigations pertaining to a quantitative evaluation of the sensitivity of predicted fatigue life to these deformation phenomena appear to have been reported so far. Such an attempt is presented in this article. Recognizing that the degree of sensitivity of fatigue life to deformation properties is a strong function of the material in question, an SAE 1045 steel (quenched and tempered) is considered for illustration. Relevant details and properties of this material (4) are listed in Table 1, and Figs. 1 and 2 show the fatigue life and stress-strain curves respectively.

* Numbers in parenthesis denote references.

OVERVIEW OF ANALYSIS PROCEDURE AND PROBLEM DEFINITION

Before outlining the scope of this article, it is necessary to examine current procedures of fatigue data generation, property presentation and cumulative damage analysis and note their implications. We will restrict our discussion to the measurement of fatigue properties using small uniaxial cylindrical or diametral specimens which are subjected to either deformation or load cycling situations. Fatigue life is usually denoted in terms of total number of cycles to complete fracture. No delineation between portions of life spent in crack "initiation" and propagation is made in these tests, mainly because of a lack of good definition of "initiation." However, fatigue properties based on such uniaxial specimen tests have been useful in predicting the crack initiation life at critical locations of notched members (5). For the present investigation our interest is in the fatigue life behavior of uniaxial specimens as influenced by the cycle dependent material properties and, therefore, crack initiation and propagation portions of life will not be delineated.

Basic Fatigue Properties

These properties are usually obtained from constant amplitude (stress or strain) tests and form the basis for establishing a damage criterion for analyzing more general histories. The following forms of characterization are now generally used.

$$\frac{\Delta\epsilon_e}{2} = \frac{\sigma_f'}{E} (2N_f)^b \tag{1}$$

$$\frac{\Delta\epsilon_p}{2} = \epsilon_f' (2N_f)^c \tag{2}$$

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\epsilon_e}{2} + \frac{\Delta\epsilon_p}{2} = \frac{\sigma_f'}{E}(2N_f)^b + \epsilon_f'(2N_f)^c \tag{3}$$

Equations 1 to 3 represent correlation of life expressed as reversals to failure, with elastic strain amplitude after Basquin (6), plastic strain amplitude after Manson-Coffin (7, 8), and total strain amplitude. Equations 1 and 2 result in linear plots on a log-log basis as illustrated for SAE 1045 steel in Fig. 1. The four constants σ_f', ϵ_f', b and c are defined as fatigue properties (9). It may be noted that the above equations do not account for the endurance limit behavior associated with some metals. In such cases the range of applicability of these equations should be defined.

In a strict sense the constants σ_f' and b of Eq. 1 should be determined from constant stress amplitude test data at long lives. Similarly the constants ϵ_f' and c should be calculated from constant plastic strain amplitude test data. It is relatively easier to perform tests in total strain control than in plastic strain control. Therefore, most of the data generated are based on constant total strain amplitude tests.

It should be recognized that due to the cyclic hardening or softening phenomenon, the stress amplitude in a strain control test increases or decreases with cycles. Similarly, the strain amplitude varies in a constant stress amplitude test. Thus the plastic strain does not remain constant in a total strain control or stress control test. However, the observed cyclic hardening or softening phenomenon is transient and a stable state is usually reached by about 20 to 50 percent in life. Therefore, most commonly the plastic and elastic strain components of constant total strain amplitude tests corresponding to the stabilized condition are used in determining the constants σ_f', ϵ_f', b and c. This is justified at high strain levels, where the elastic strain component is a small fraction of the total strain, so that the variation in plastic strain due to cyclic hardening or softening is negligible. Thus at high strains, a constant total strain amplitude test is a good approximation of a constant plastic strain amplitude test. At lower strain levels, where the elastic strains are a significant fraction of the total strains, the variations in either the plastic or the elastic strain due to cyclic hardening or softening cannot be neglected (see Fig. 3 to be discussed later). In other words, the total strain amplitude test at lower strain levels is not a good approximation of either a constant plastic strain or a constant elastic strain (constant stress amplitude) condition. Therefore, it may not be valid to use these data either to verify the Manson-Coffin and the

Basquin equations or to evaluate the constants σ_f', ϵ_f', b and c. Also, other factors, (such as the plastic strain becoming too small to be accurately measurable, the anelastic component of the apparent plastic strain becoming significant and a probable increase in scatter in life) make it difficult to check the validity of the Manson-Coffin law at low strain levels. Therefore, it appears that constant plastic strain amplitude test data are necessary at least at lower strain levels. In case such tests are not feasible it may be more appropriate to derive the plastic strain amplitude-life data from both constant total strain amplitude and constant stress amplitude (at long lives) instead of from constant total strain amplitude tests alone. On this basis, the plastic strain amplitude at a given life at long lives will be the difference between the total strain amplitude at that life obtained from the total strain amplitude tests and the elastic strain amplitude at the same life obtained from the constant stress amplitude tests.

Mean Stress Parameter for Fatigue

Two forms of extending equations 1 to 3 to incorporate the observed effect of mean stress on fatigue life are as follows:

$$\frac{\Delta\epsilon}{2} = (1 - \sigma_o/\sigma_f)\frac{\sigma_f'}{E}(2N_f)^b + \epsilon_f'(2N_f)^c \quad (4)$$

$$\sigma_{Max}\frac{\Delta\epsilon}{2} = \frac{\sigma_f'^2}{E}(2N_f)^{2b} + \sigma_f'\epsilon_f'(2N_f)^{b+c} \quad (5)$$

Equation 4, after Morrow (9), assumes that the mean stress only influences the elastic strain-life relation. Equation 5, after Smith et al (10) is obtained by multiplying equations 1 and 2 and noting that σ_{Max} is equal to $\frac{\Delta\sigma}{2}$ in a fully reversed stress condition. The interesting feature is that for either of these approaches it is not necessary to obtain fatigue data from mean stress tests. All fatigue data with or without mean stress will be represented by either of these equations.

Cumulative Damage Procedure

The first step in a general fatigue analysis procedure is to determine the material deformation response to the given input history. For example, if the strain history is given, the corresponding stress response should be determined. Analytically, this is done by means of a stress-strain model capable of exhibiting the various cyclic deformation phenomena, such as cyclic hardening or softening, cyclic creep or relaxation, and memory of history. Now that both stress and strain histories are known, the plastic strain and mean stress histories can also be determined. Thus with virtually all the deformation parameters at hand, a suitable fatigue life curve can be used as a damage criterion and total damage caused by the given history estimated by using a suitable damage summation rule, such as Palmgren-Miner law (11, 12). In estimating and summing damages, the importance of a proper counting of the cyclic events, such as by "rain flow" technique has been pointed out (13). This topic will be considered later in this paper.

Problem Definition

With this overview of the current fatigue life prediction technique, our problem can be outlined in the following form:

> "What is the maximum possible difference in the estimated life (or cumulative damage) in any given cyclic history between incorporating and not incorporating
>
> (a) the cyclic hardening or softening feature,
>
> (b) the mean stress relaxation or cyclic creep feature in a deformation model, and
>
> (c) a proper cycle counting procedure in estimating and summing damages ?"

Adopting a simple approach, the phenomena of cyclic hardening or softening and cyclic relaxation or creep will be treated independently and "bounds" on estimated lives under constant strain cycling situations will be calculated. The actual "bounds" in a variable amplitude history will depend on the distribution and duration of the various strain levels in that history. The problem of cycle counting will be shown to be directly related to the "Memory" of prior cyclic history (1-3) exhibited by the material.

CYCLIC HARDENING OR SOFTENING AND PREDICTED LIFE

In a cumulative damage analysis procedure which uses a total strain-life criterion to analyze strain-time histories or a stress-life criterion to analyze stress-time histories, the question of accounting for the influence of cyclic hardening or softening does not arise, as a knowledge of the material deformation response is not needed in the analysis. The use of a stable plastic strain-life criterion based on constant total strain amplitude test data, also eliminates the need for incorporating the cyclic hardening or softening feature in a deformation model for cumulative damage analysis. For, implicit in the derivation of stable plastic strain-life criterion from total strain test data, is the assumption that the cyclic hardening or softening is negligible. As the input histories are usually in terms of stress or total strain, it is appropriate to include a cyclic hardening or softening formulation only if plastic strain-life criterion, based on constant plastic strain test data, is used. As pointed out earlier, plastic strain cycling is not commonly used to generate fatigue life curves. Therefore, the use of cyclic hardening or softening formulations for a cumulative damage analysis is not valid in such situations.

For the present analysis, we will assume that we have a plastic strain-life curve generated from constant plastic strain amplitude test data, and that the Manson-Coffin law holds for all plastic strain levels. The cyclic hardening or softening phenomenon is typically characterized by an exponential decay in the hardening and softening rate with cycles, leading to an approximately stable stress-strain response. The stress-strain relationship of the material corresponding to this stable cyclic condition is approximately respresented by the cyclic stress-strain curve defined as the locus of the tips of the stable stress-strain loops of various amplitudes (14). A cyclic incremental step test can also be used to generate a cyclic stress-strain curve. As a first approximation for engineering analysis, the cyclic stress-strain curve can be assumed to be unique to the material, independent of prior history at room temperature.

Therefore, the monotonic (static) and the cyclic stress-strain curves represent two extreme conditions of the material before and after the transient hardening or softening. Consequently, in order to evaluate the maximum influence of the transient hardening or softening phenomenon on the predicted fatigue life, it is sufficient to determine the difference in plastic strain and associated change in the estimated life (based on Eq. 2) between these two states as a function of total strain amplitude.

The monotonic and the cyclic stress-strain curves of SAE 1045 Steel are shown in Fig. 2. For the purpose of our calculations, the upper yield point behavior is neglected and instead, a flat top yield behavior is assumed for the monotonic curve as shown by the dashed line in the figure. The material cyclically softens at all strain levels. The plastic and the elastic strain components corresponding to these two monotonic and cyclic states are estimated and expressed as percent variations with respect to the cyclic state in Fig. 3. The variations in plastic and elastic strain attain significant proportions, reaching a maximum at the region of transition between essentially elastic and predominantly plastic behavior in the monotonic state. The figure clearly demonstrates the invalidity of using total strain control tests to represent plastic or elastic strain cycling conditions at strain levels corresponding to intermediate lives for this material. It may be argued that most of these variations occur in a small fraction of the transient period and the representation of Fig. 3 is somewhat exaggerated. While this is reasonable at high strains and short life regions, as the strain levels decrease the transient period increases and the stabilization becomes less distinct especially in steels thus rendering the representation in Fig. 3 more realistic. However, the sensitivity of fatigue life to these variations in plastic strain is of direct interest to this study.

The stable plastic strain-life relationship of SAE 1045 steel obtained from total strain cycling tests will be used for the purpose of illustration. The results are not expected to be very different from those based on plastic strain-life criterion obtained from plastic strain cycling tests. At a given strain level, the fatigue lives estimated on the basis of the plastic strain at the two extreme conditions namely the monotonic and the cyclic states are compared. This is essentially equivalent to assuming that the material does not exhibit any softening in one case and that it has completely softened from the beginning in the other case.

Figure 4 shows the ratios of estimated fatigue life between these two states (monotonic and cyclic) as a function of the strain level. At strain levels corresponding to intermediate lives (10^4 to 10^5 reversals) the ratios attain significant values and exhibit a peak value of more than two orders of magnitude. At high strain levels corresponding to low cycle fatigue, life ratios close to unity are obtained, because of the negligible difference between plastic strains corresponding to the monotonic and cyclic states. At essentially elastic strain levels, the difference between the monotonic and cyclic stress-strain curves again decreases, thus resulting in a decrease in the variation of predicted life at very long lives. Therefore, it appears that if a plastic strain-life criterion is used in a damage analysis, the consideration of cyclic hardening or softening phenomenon is important at intermediate lives for this material.

In general, the sensitivity of estimated extremes in fatigue life to the phenomenon of cyclic hardening and softening is a strong function of the degree of hardening or softening a material exhibits. However, it may not be significant at extremely long life or short life regions. The SAE 1045 steel studied in the present report significantly softens as illustrated by the difference between its monotonic and cyclic stress-strain curves in Fig. 2 and, consequently, its extreme influence on predicted life is also significant in the intermediate life regions. On the other hand, there are structural metals which are relatively stable thus exhibiting little difference between their monotonic and cyclic stress-strain curves. For such materials, calculations of the type outlined above would show insignificant influence on the estimated extremes in life.

CYCLIC MEAN STRESS RELAXATION AND PREDICTED LIFE

In order to treat this problem independent of the cyclic hardening or softening phenomenon, the deformation response of the material will be assumed to be characterized by the cyclic stress-strain curve as it is a more realistic representation under cyclic conditions. The magnitude of mean stress induced at a given stage in a cyclic history depends not only on the prior history but also on the current strain amplitude. A simple method of inducing a mean stress consists of monotonically straining the material to a reasonably high strain level followed by strain cycling at relatively small strain amplitude as shown in Fig. 5. The initial monotonic loading path will be described by the cyclic stress-strain relationship and the first unloading branch can be approximately described using Masing's (15) hypothesis, namely that the unloading branch is geometrically similar to the initial monotonic curve but magnified by a factor of two. Thus if the maximum monotonic strain and the strain amplitude are known, the initially induced mean stress can be determined. The smaller the strain amplitude, the greater is the initially induced mean stress. The extent to which this initially induced mean stress decreases in magnitude and the cyclic duration of this relaxation depend on the strain amplitude. For the purpose of determining the extreme influence of this phenomenon on predicted life, it is sufficient to compare the initial mean stress and no mean stress situations. In other words, in one case it is assumed that there is no mean stress relaxation and in the other case that the mean stress is assumed to be totally relaxed immediately after induction. While a tensile mean stress is detrimental to life, a compressive mean stress is beneficial but not necessarily to the same extent. Therefore, both tensile and compressive mean stress situations should be studied. Assuming that mean strain is small enough to have negligible influence on life, the problem reduces to comparing lives at a given strain amplitude with a maximum tensile mean stress, with no mean stress and a maximum compressive mean stress.

Figure 5 shows ratios of calculated fatigue lives with and without mean stress for SAE 1045 steel for a maximum monotonic strain of 0.02. Results using both Morrow and Smith et al. criteria described earlier are shown. It is interesting to note that if no mean stress relaxation is assumed, the Smith et al. criterion yields less conservative results than the Morrow criterion at low strain amplitudes. The figure clearly demonstrates that irrespective of either criterion, mean stress has a significant influence on estimated fatigue life for low strain levels corresponding to lives greater than about 5×10^3 cycles for this material.

In general, the significance of mean stress relaxation on the estimated extremes of fatigue life as evaluated in this report, is a strong function of the material's ability to accommodate the initial mean stress. Therefore, the shape of the cyclic stress-strain curve or the strain hardening exponent determines the importance of mean stress relaxation phenomenon in fatigue life estimation. For example, an ideal elastic-plastic material would never be able to sustain an initial mean stress, thus eliminating the need for a mean stress relaxation model in damage calculation.

DAMAGE SUMMATION PROCEDURE AND PREDICTED LIFE

In applying constant amplitude fatigue data to the cumulative damage analysis of variable amplitude histories difficulties arise in determining what constitutes a cycle. This and other practical considerations concerning the recording of cyclic events, have given rise to several cycle counting methods, many of which have been shown to have serious limitations (13). It is not intended to discuss these limitations as applied to specific counting methods here, as a few good treatments of this subject are already available (13, 16). However, it can be shown that the major cause for these limitations is a lack of consideration for the non-linear relationship between the fatigue parameter and the rate of damage. This should be distinguished from the limitations of a linear damage summation rule, which only assumes that the rate of damage per cycle (or per reversal) at a given level of a fatigue parameter such as stress amplitude, strain amplitude or the product of maximum stress times strain amplitude is constant. However, all of the fatigue life formulations known to the author are such that this rate of damage is a non-linear function of the fatigue parameter. Thus, even if a linear damage summation rule is adopted in the cumulative damage analysis, there is still a need to properly account for the non-linear relationship between the fatigue parameter and the rate of damage. This is best illustrated with examples.

Figure 6 shows the non-linear relationships of the three fatigue parameters namely the stress amplitude, the strain amplitude and the product of maximum stress times the strain amplitude with damage per reversal for SAE 1045 steel. It can be seen that the increment in damage per reversal for a given increment in any fatigue parameter very much depends on the parameter level at which it is measured. Consequently, if a reversal (one hysteresis branch) of a cyclic history is split into two, by assuming a fictitious infinitesimal unloading and a simple range counting (magnitude of reversal) method is used, two different values of total damage would result for the unsplit and the split conditions as illustrated in Fig. 7. Extending this argument further, the quantitiative influence of this limitation on predicted life is dramatized in Fig. 8. In this case each reversal of a constant strain cycling history is subjected to a certain number of infinitesimal unloading interruptions and the resulting predicted life based on a simple range counting damage analysis is compared with the fatigue life of the uninterrupted constant amplitude cyclic situation. If a proper cycle counting method is used, the predicted life should not be sensitive to these infinitesimal interruptions. The simple range counting method therefore can result in unrealistic damage prediction. As the parameters used included the mean stress effect, different damages are evaluated for the upper and the lower branches although each cycle has the same overall mean stress and stress and strain ranges. Sensitivity of predicted life increases with decrease in strain amplitude and also with the degree of non-linearity of the fatigue parameter-damage relationship. The absolute need for a proper cycle counting method is thus clearly demonstrated by the representations of Fig. 8.

The cycle counting problem, being a direct consequence of the non-linear relationship between the fatigue parameter and the rate of damage, is inherent to the fatigue analysis procedure irrespective of the material in question. Until a fatigue parameter that is linearly related to the rate of damage is discovered, a proper accounting of cyclic events is indispensible to the cumulative damage analysis.

Rational Forms of Cycle Counting and Their Implications

Most commonly an input history is either in the form of a strain-time or stress-time record as illustrated in Fig. 9. If both stress and strain time records are available, then the input history can also be represented as a stress-strain record. It is clear from Fig. 9 that identification of the cycles is unambiguous in the stress-strain record, (i.e. one cycle of high strain amplitude and two cycles of low strain amplitude) whereas it is not obvious from either the stress-time or the strain-time record alone. Therefore in situations where only one of two time scale histories is known, some method of cycle counting procedure has to be adopted. It has been shown that a "rain flow" or "range pair" counting method (13, 16) properly accounts for the cyclic events and is considered useful in such situations. However, if one is using a deformation response model capable of simulating the stress-strain response for a given input history, the need for a "rain flow" or "range pair" counting procedure does not arise. The "memory" formulation of the deformation model, recognizes the various cyclic events such as for example the infinitesimal unloading referred to in Fig. 7, the proper damage summation in this case being as illustrated in the Figure. Considering the stress-strain record of Fig. 9 the "memory" model recognizes, that portions 2-4 and 5-7 of the paths 3-4 and 6-7 respectively are continuations of 1-2 and 4-5 respectively. These portions are associated with higher rates of damage compared to the portions 3-2 and 6-5 and hence should be distinguished for estimating damage in such events as shown in the figure. The appropriate calculations can be programmed on a digital computer and continously carried out on a reversal-to-reversal basis, thus eliminating the need for the special cycle counting procedures.

As the "range pair" or "rain flow" method of cycle counting requires only a time scale input history, it eliminates the need for a deformation response model. Consequently, it would not be possible to incorporate the effect of cyclic hardening, softening, relaxation and creep on damage estimations. Also, the method requires the entire input history before starting the analysis. The alternative approach of using a deformation response model allows the incorporation of the transient features, takes care of the cycle counting without resort to a special counting procedure such as the "range pair" or "rain flow" method and is capable of calculating and summing damages continuously. It is not therefore required to scan the entire history before starting the analysis. However, if it is decided to neglect the effects of transient phenomena, the computing expenses of a deformation model approach may be considerably reduced by resorting to the "range pair" or "rain flow" type of analysis. Depending on the type of history, it may even be possible to further reduce the computational expenses of the "range pair" or "rain flow" method by using the "shortcut" approach suggested by Fuchs et al. (17).

DISCUSSION

Whereas a proper accounting of cyclic events is imperative to the present format of cumulative damage analysis independent of the material in question, the importance of cyclic hardening, softening and relaxation on predicted life depends not only on material properties but also on the type of input load or deformation history. The procedure outlined in this article considers only the material's aspect, and determines the maximum possible difference in life that one predicts between accounting and not accounting for these cyclic phenomena. These maximum differences in predicted life are expressed as functions of strain amplitude. The actual differences in predicted life, however, depend on the type of input history also, being governed by the distribution and duration of the various strain amplitudes in that history. Even if the cyclic phenomena appear to be significant at the material level of investigation as observed in the case of SAE 1045 steel at strain levels corresponding to intermediate and long lives, the durations and distributions of strains in a given input history may be such that these cyclic phenomena may not be significant for that history. On the other hand, if these cyclic phenomena are found to be insignificant at the material level itself, as would be expected in relatively stable (cyclically) materials with low strain hardening exponents, the decision not to account for the cyclic phenomena can be made at an early stage, using the approach suggested in this article, without having to examine the input history.

The calculations involved are simple and require only the four following material property data, all of which are being specified in current fatigue literature.

1. The monotonic stress-strain curve
2. The cyclic stress-strain curve
3. Plastic strain-life curve, and
4. Mean stress incorporated fatigue life curve

The calculations required for evaluating the significance of cyclic phenomena as affected by both material and input history are relatively more involved and require in addition to the above material data, a stress-strain model. Using the same ideas adopted in the foregoing analysis, lives are predicted by considering the monotonic stress-strain curve in one case and the cyclic stress-strain curve in the other and also considering full mean stress and zero mean stress conditions in both the cases. These predicted lives establish bounds and thus aid in evaluating the significance of cyclic phenomena for the given history. The involved calculations amount to four repetitions of the cumulative damage analysis using a relatively simple deformation model with no provision for cyclic hardening, softening and relaxation. On the other hand, a single calculation using a sophisticated deformation model incorporating all the cyclic phenomena, may sometimes be more economical in terms of computational expenses. In such cases the need for evaluating the significance of cyclic phenomena as affected by a given history does not arise.

CONCLUSIONS

The presented procedure for determining the maximum possible influence of cyclic hardening, softening and relaxation phenomena of metals on predicted fatigue life requires monotonic and cyclic stress-strain curves and constant amplitude fatigue life data using plastic strain and mean stress criteria.

The formulation of transient hardening or softening phenomenon in a cumulative damage model is unnecessary when fatigue criteria based on steady state parameters are adopted. It is consistent to use this formulation only if constant plastic strain-life criterion is adopted. The sensitivity of predicted life to cyclic hardening or softening depends on the degree of hardening or softening a material exhibits, whereas in the case of mean stress relaxation, it depends on the shape of the stress-strain curve or the strain hardening exponent. For the SAE 1045 steel studied in this article, consideration of cyclic hardening, softening, and relaxation can significantly increase the accuracy of predicted life in intermediate and long life regions.

The need for a proper accounting of cyclic events is consequential to the non-linear relationship between the currently used fatigue parameters and rate of damage defined as the inverse of the number of cycles to failure. Therefore, it is the most important and necessary requisite of a cumulative damage analysis independent of the material in question. The "rain flow" or the "range pair" counting method meets this requirement in the analysis of strain-time or stress-time history. In the case of computer based models capable of simulating stress-strain response, the "memory" formulation is sufficient to properly account for the cyclic events in a damage analysis.

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TABLE 1

Material: SAE 1045 Steel

Heat Treatment: Austenitized $1500^{o}F/30$ min;
Brine Quenched and Tempered $1200^{o}F/1$ hr.

Data Source: Reference 4

Monotonic Properties:

Modulus of Elasticity, $E = 29 \times 10^{3}$ ksi

Yield Strength, $S_y = 92$ ksi

Ultimate Strength, $S_u = 105$ ksi

Monotonic True Fracture Strength, $\sigma_f = 178$ ksi

Brinnel Hardness Number: 225

Fatigue Properties:

Fatigue Strength Coefficient, $\sigma_f' = 178$ ksi

Fatigue Strength Exponent, $b = -0.095$

Fatigue Ductility Coefficient, $\epsilon_f' = 1.0$

Fatigue Ductility Exponent, $c = -0.66$

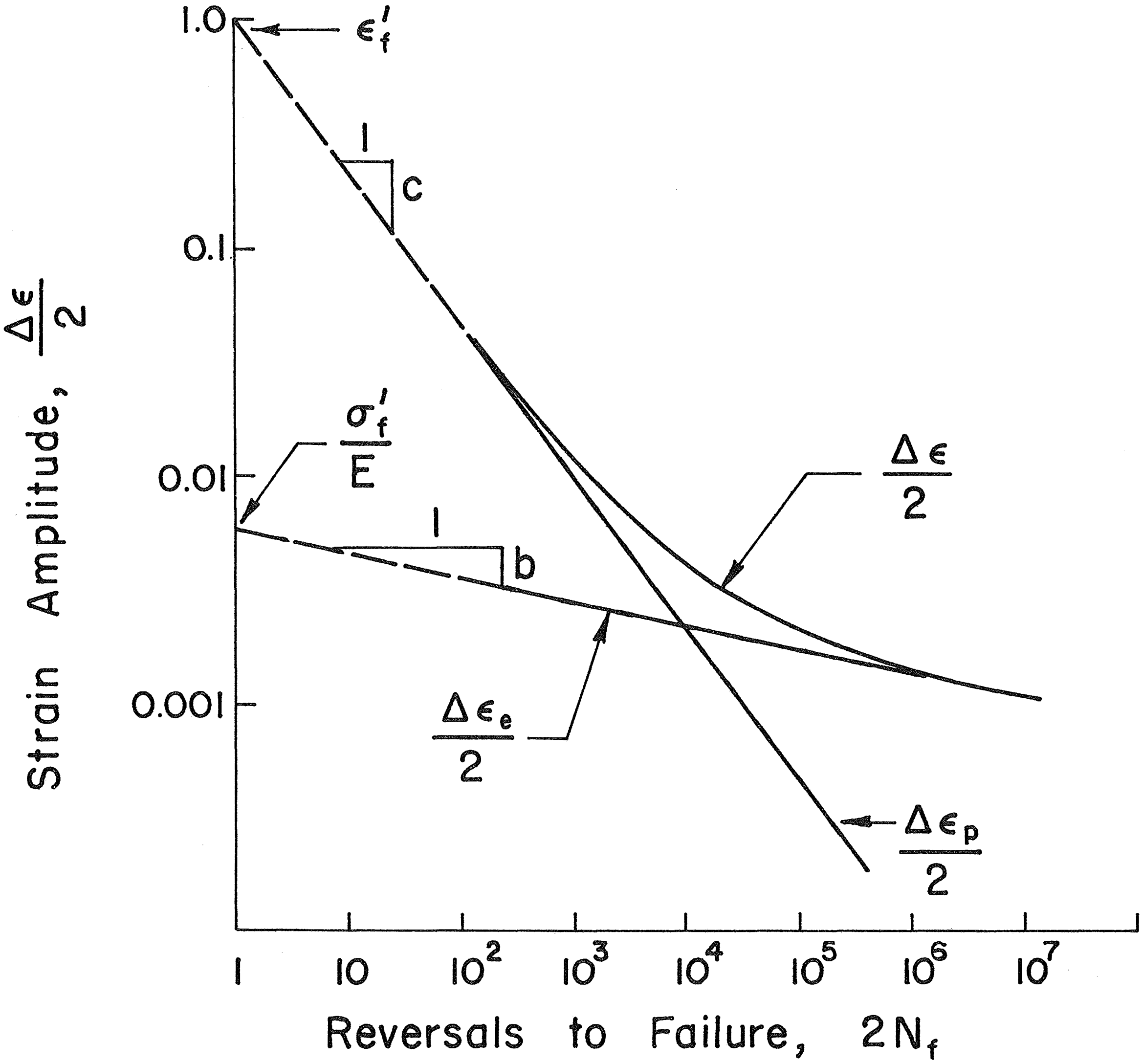


Fig. 1 Strain-Life Plot for SAE 1045 Steel (4)

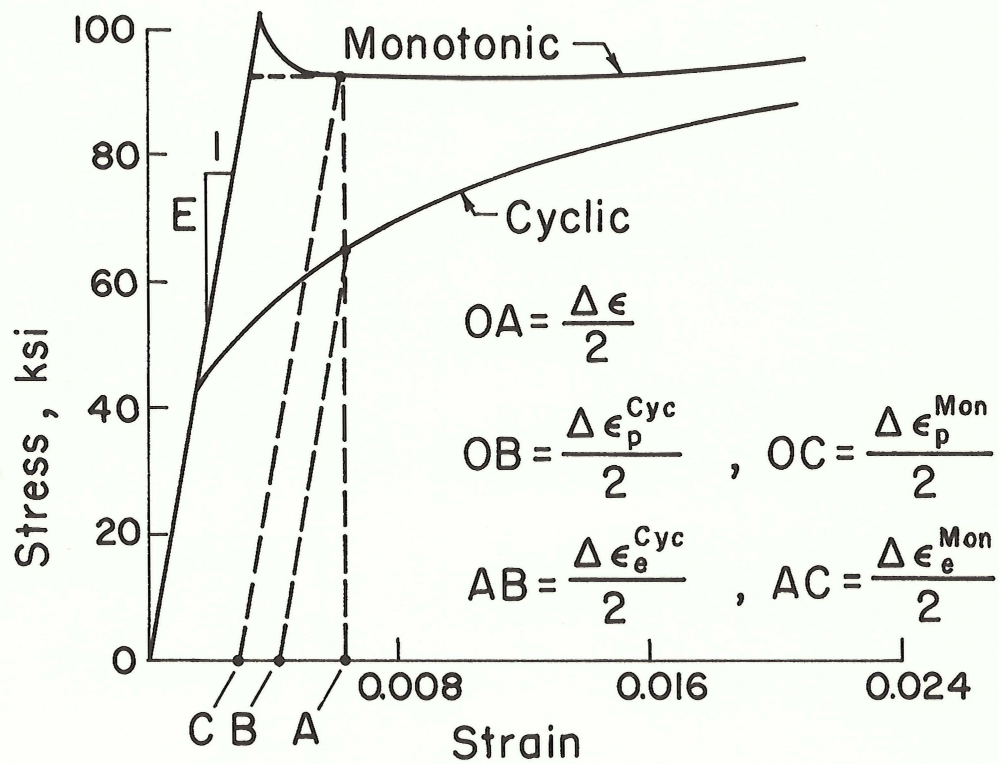


Fig. 2 Monotonic and Cyclic Stress-Strain Curves of SAE 1045 Steel

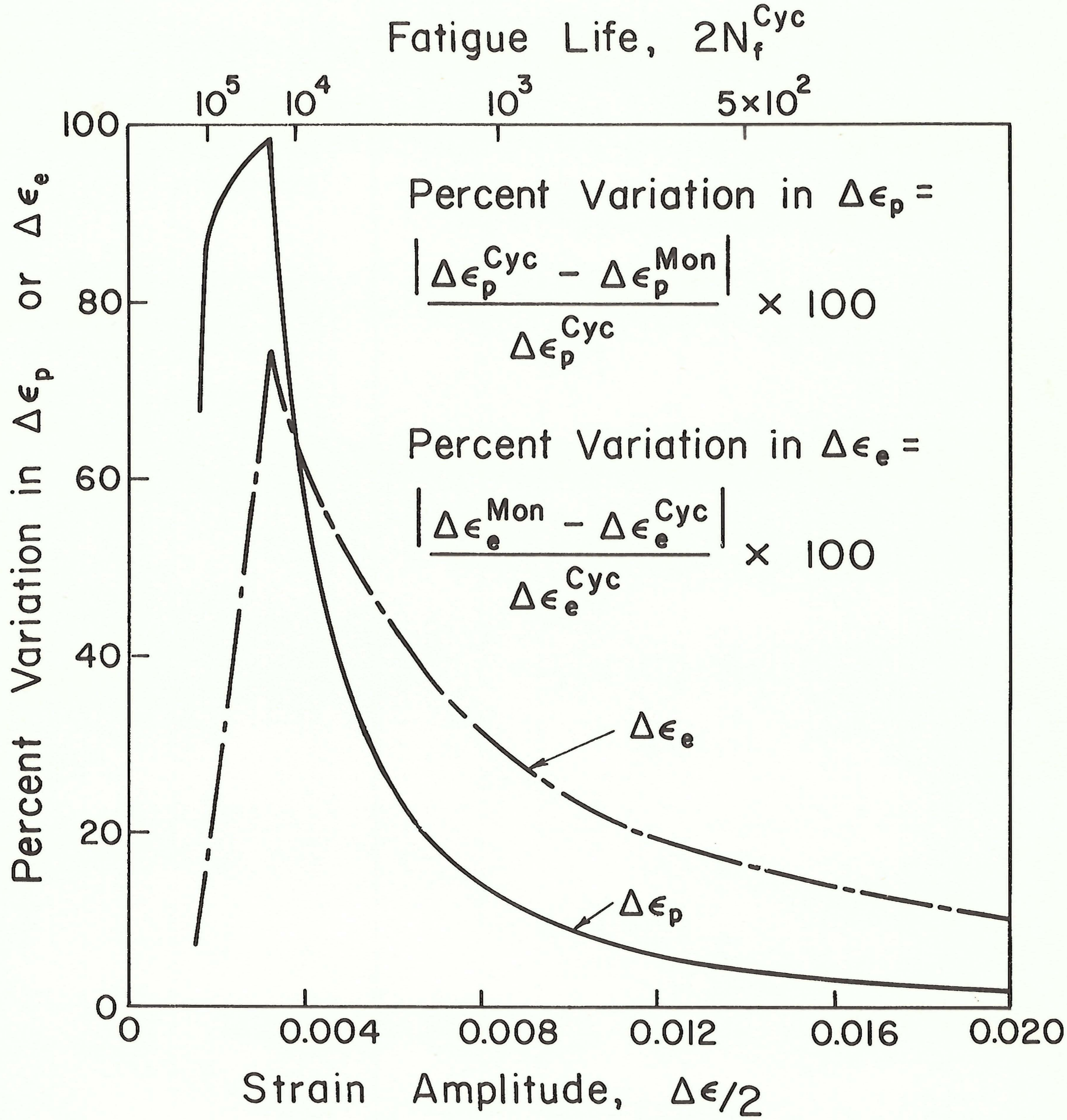


Fig. 3 Maximum Percentage Variations in Elastic and Plastic Strains due to Cyclic Softening of SAE 1045 Steel

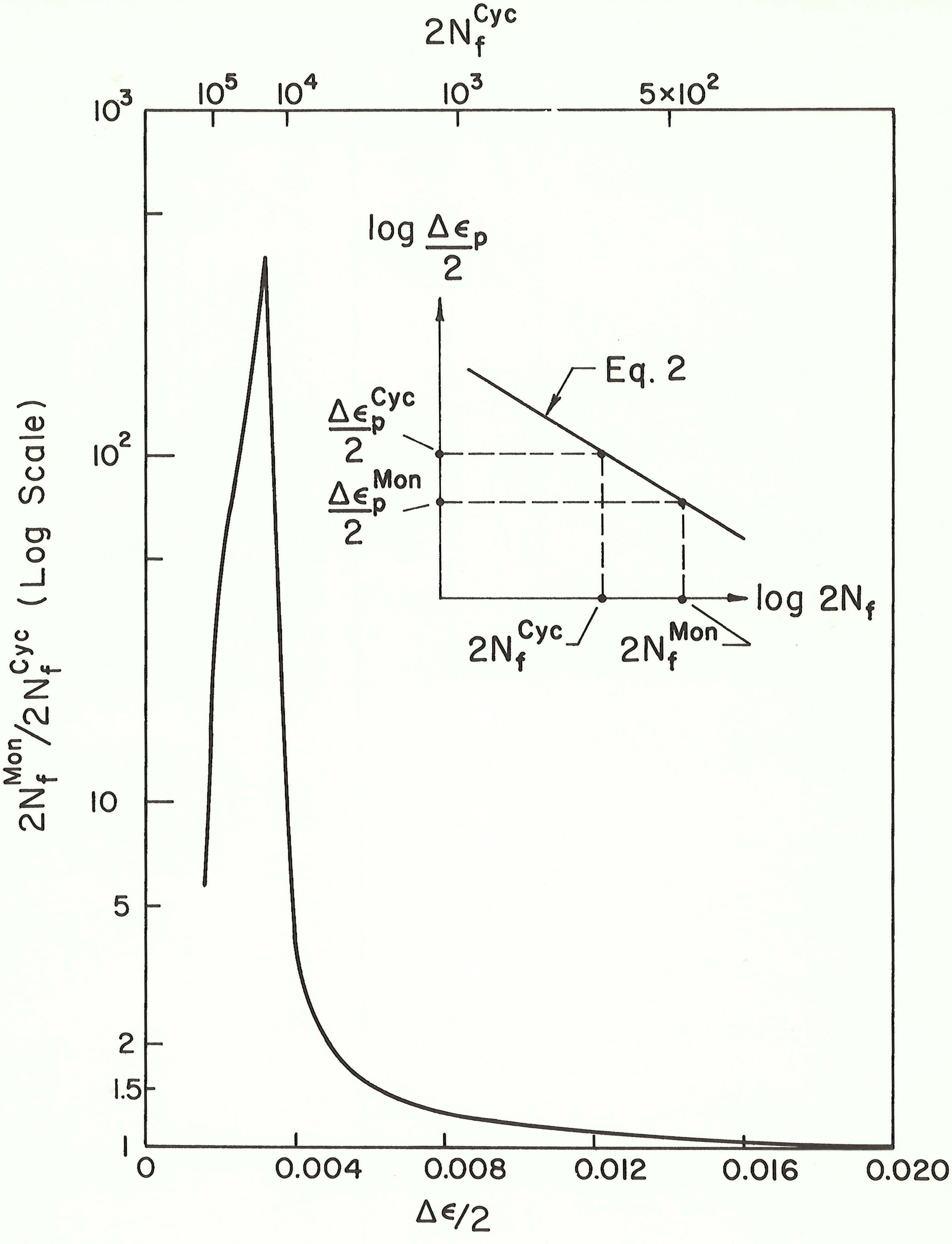


Fig. 4 Maximum Possible Influence of Cyclic Softening on Predicted Fatigue Life for SAE 1045 Steel

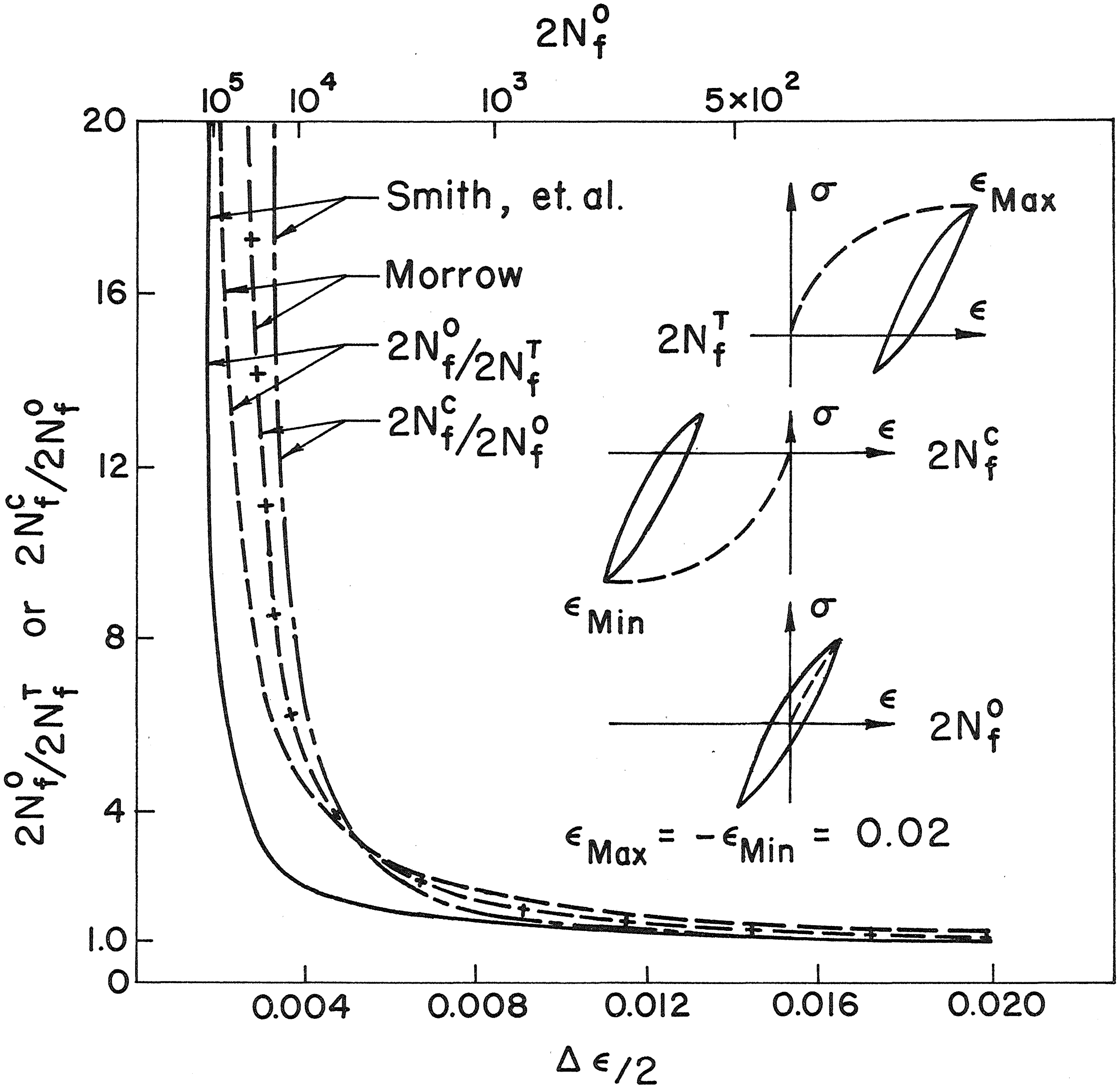


Fig. 5 Influence of Cyclic Mean Stress Relaxation on Predicted Life for SAE 1045 Steel

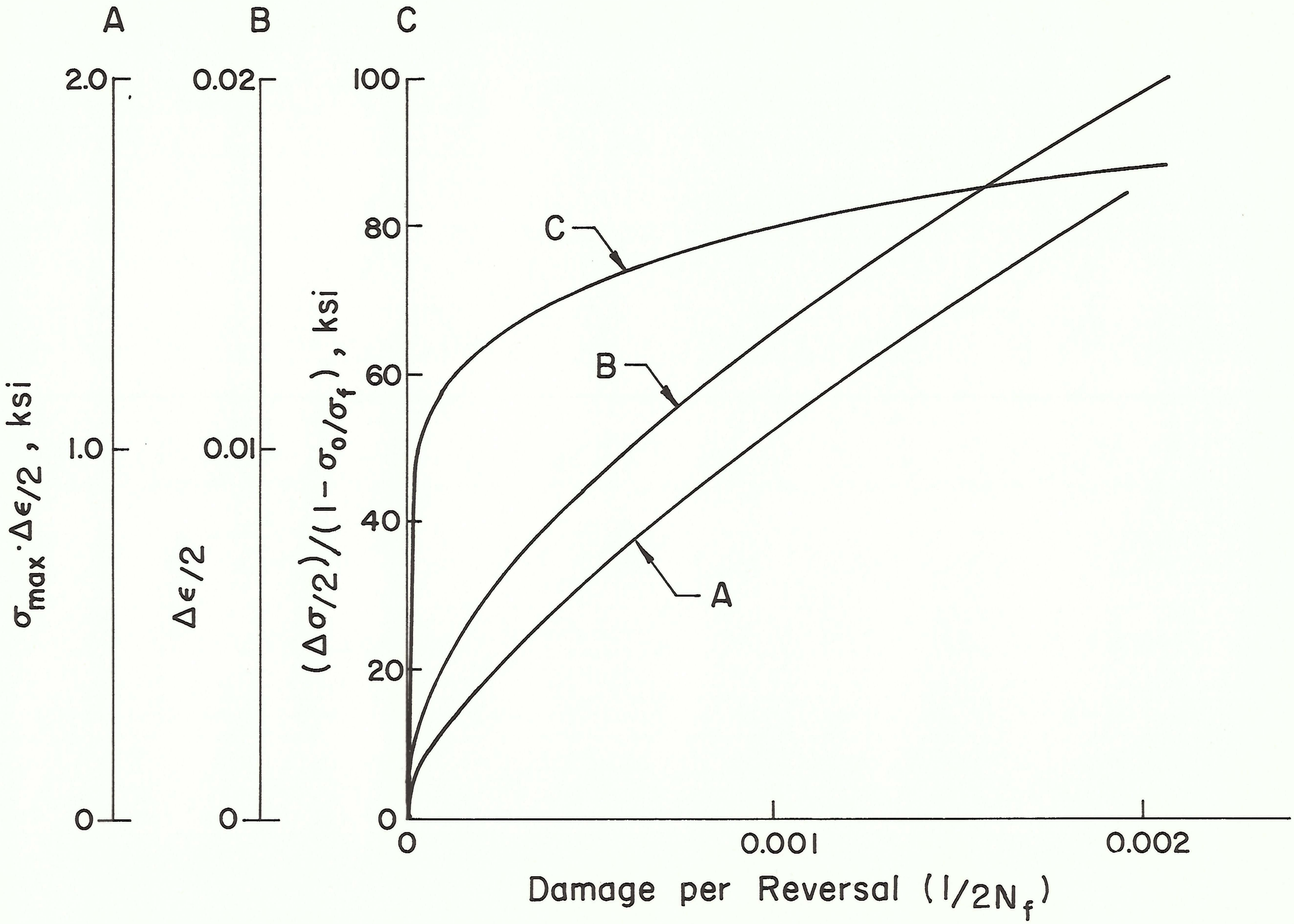
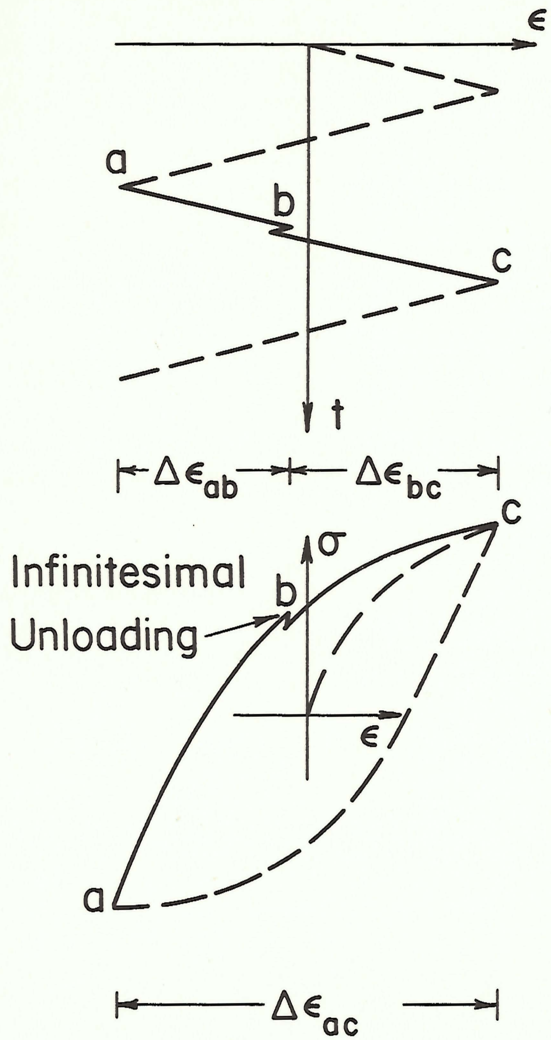
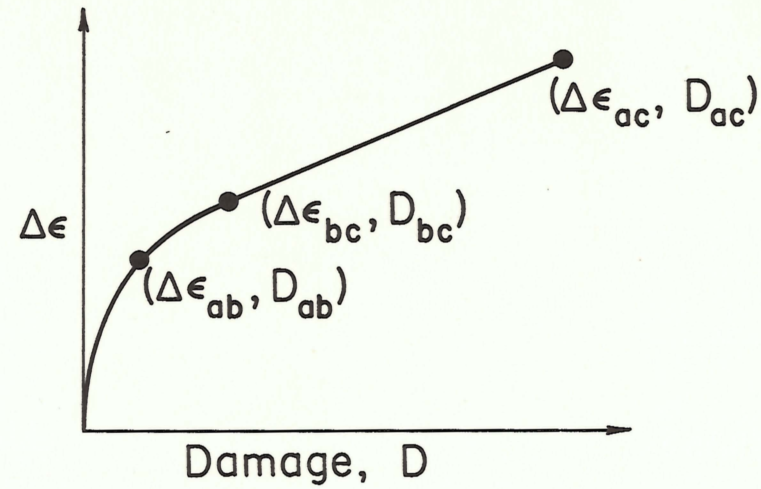


Fig. 6 Relationships between Several Fatigue Parameters and Damage per Reversal for SAE 1045 Steel





<u>Damage Calculation for the Branch ac</u>

Actual Damage $= D_{ac}$

Damage by Range Count $= D_{ab} + D_{bc} < D_{ac}$

Damage by "Memory" of Unloading $= D_{ab} + (D_{ac} - D_{ab}) = D_{ac}$

Fig. 7 Influence of a Ficticious Infinitesimal Unloading on Damage Summation

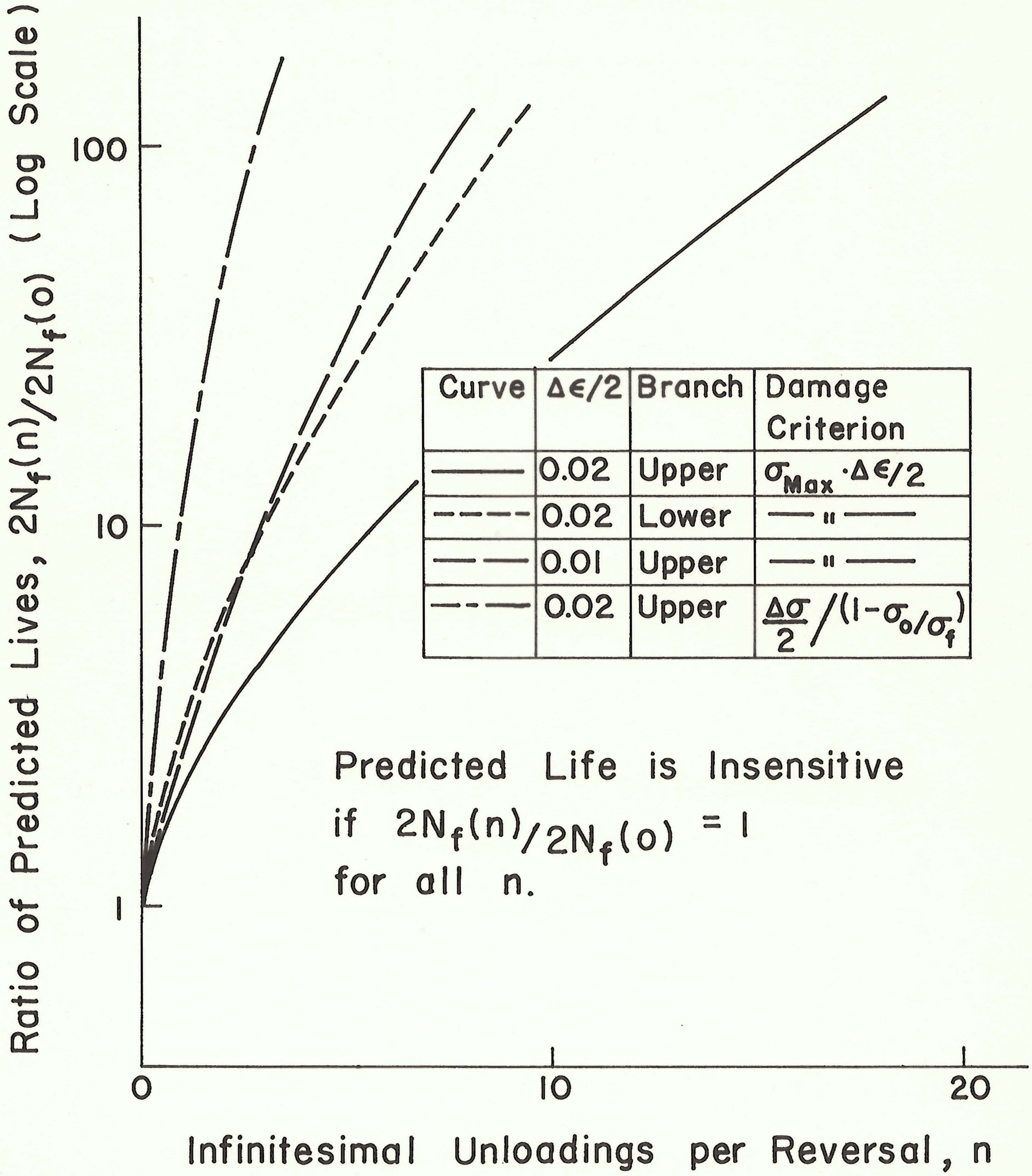


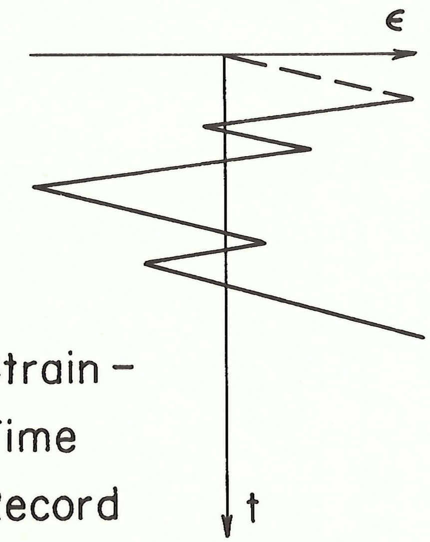
Fig. 8 Sensitivity of Predicted Life to Infinitesimal Unloadings Using Range Count Method for SAE1045 Steel

Incorrect Damage Summation By Simple Range Count:

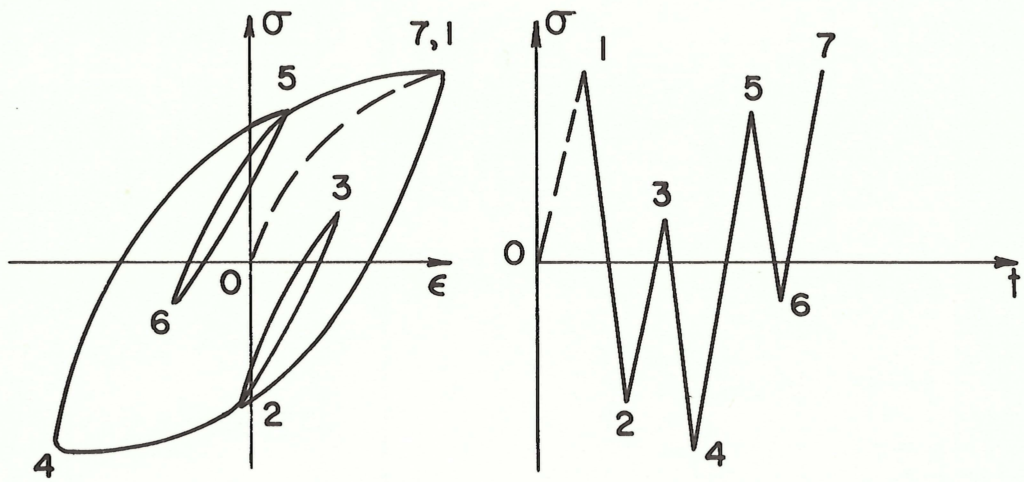
$$D_{07} = D_{01} + D_{12} + D_{23} + D_{34} + D_{45} + D_{56} + D_{67}$$

Correct Damage Summation Using Memory Formulation:

$$D_{07} = D_{01} + \cancel{D_{12}} + D_{23} + (D_{32} + D_{14} - \cancel{D_{12}}) + \cancel{D_{45}} + D_{56} + (D_{65} + D_{47} - \cancel{D_{45}})$$



Strain-Time Record



Stress-Strain Record   Stress-Time Record

Fig. 9 Illustration of Proper Accounting of Cyclic Events in Damage Summation