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Static Response of Stranded Wire Helical Springs

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A theory for the large, static, axial response of a helical spring formed of a strand of twisted wires is presented. The wires in the strand are assumed to be in frictionless contact with their neighbors throughout the deformation. The response of a typical tension or compression spring is found to be only weakly nonlinear at large spring strain, and relatively insensitive to the type of end condition.

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Introduction

In an interesting article by Clark [1][1] on the dynamic behavior of stranded wire springs, it is mentioned that helical springs formed of a strand of three wires twisted together were first observed in Russian machine guns used in the Spanish Civil War. Although it has been known for some time that stranded wire springs, when compared with conventional single wire springs, exhibit more damping and longer fatigue life in certain dynamic applications [1], a rigorous analysis of the state of stress in an individual wire in the strand has not appeared, even for the case of static loading.

This paper outlines a procedure for determining the static, large strain, axial (extensional and rotational) response of a helical spring formed from a strand made of any number of smooth wires twisted together. An important assumption made here is that the wires in the strand remain in contact with each other throughout the deformation. Thus, the analysis is restricted to the usual cases where (1) springs designed primarily for tension have the strand and the spring wound in the same sense, as in Fig. 1(a), or where (2) springs designed primarily for compression have the strand and the spring wound in opposite senses [1].

The paper uses previous results of Costello and Phillips [2,3,4] for the axial (extensional and rotational) response of a straight strand, and the recent work of Costello [5] on the bending stiffness of a helical wire,

[1]Numbers in brackets denote References at end of paper.

to determine the torsional, axial and bending response of the twisted wire strand. The axial response of the helical spring formed of this strand can then be determined for arbitrarily chosen values of wire strain and change in wire helix angle, which are the independent variables in this problem. Cross plotting of specified dependent variables, such as the axial spring force and the axial spring strain, for given end conditions, such as zero rotation or zero twisting moment, is then achieved by a Newton or Newton-Raphson procedure.

Analysis

The equations governing the static response of a thin helical wire are presented in Love's treatise [6]. There, the bending and torsional stiffnesses of the wire are required, and for a wire of circular cross section, these stiffnesses are readily determined from elementary analyses of the bending and torsion of a straight wire.

To solve the problem of a helix formed of a strand of twisted wires, one might consider following Love's development for a solid helical wire, making appropriate substitutions for the stiffness terms that arise. As far as the bending stiffness is concerned, this substitution can be effected by employing Costello's results [5] for the bending of a helical wire, as described below. However, the torsional stiffness of a strand depends upon the axial force in the strand, just as the axial stiffness is dependent upon the twisting moment [4]. Perhaps one could circumvent this difficulty by assuming, for example, that the axial force in the strand is negligible; this would in principle allow a unique torsional stiffness to be found. Unfortunately, such an assumption would raise questions regarding the extension of the strand, and might lead to serious errors if springs with large helix angle α^* are considered. See Fig. 1(a).

Instead of making any a priori judgments about the torsional and axial stiffnesses of the strand, the authors adopted a more fundamental approach to the problem. The wire strain ξ and the final wire helix angle α_1 are taken as independent variables. In terms of these variables, all the generalized strains and forces associated with a strand of m wires can be determined [4]. Working with the equilibrium equations for a spring [6] composed of this strand, and using the results of [5], one can then determine the generalized strains and forces associated with the helical spring. Ultimately, the values of ξ and α_1 required to produce a desired type of spring deformation are determined numerically. The procedure follows.

Strand variables. A straight strand of m smooth wires of radius R and initial helix angle α has a wire helix radius r given by the formula [2]

$$\frac{r}{R} = \left[1 + \frac{\tan^2(\pi/2-\pi/m)}{\sin^2\alpha}\right]^{\frac{1}{2}} \tag{1}$$

if the cross sections of the wires, as illustrated in Fig. 1(b), are taken to be elliptical and the wires are in contact in the initial state. Assuming that the wires remain in contact and that the changes in cross sectional dimensions due to contact line loads and Poisson contraction associated with wire extension are negligible, one can also use (1) to determine the final wire helix radius r_1 associated with the final helix angle α_1. The normal and binormal curvatures (κ,κ') and the twist τ of an individual wire are given by [6]

$$\kappa_0 = 0, \qquad \kappa_0' = \cos^2\alpha/r, \qquad \tau_0 = \sin\alpha\cos\alpha/r,$$

and (2)

$$\kappa_1 = 0, \qquad \kappa_1' = \cos^2\alpha_1/r_1, \qquad \tau_1 = \sin\alpha_1\cos\alpha_1/r_1,$$

in the initial and final states (designated by subscripts 0 and 1, respectively). The normal and binormal bending moments (G,G'), twisting moment H, and tension T acting on a wire are given by

$$\frac{G}{ER^3} = \frac{\pi}{4} R\,(\kappa_1 - \kappa_0), \qquad \frac{G'}{ER^3} = \frac{\pi}{4} R\,(\kappa_1' - \kappa_0'),$$

$$\frac{H}{ER^3} = \frac{\pi}{4(1+\nu)} R\,(\tau_1 - \tau_0), \qquad \frac{T}{ER^2} = \pi\,\xi, \tag{3}$$

where E and ν are Young's modulus and Poisson's ratio, respectively, for the wire material, and ξ is the wire strain. Equilibrium equations are used to compute the binormal shear force N' and the normal load per unit length X:

$$\frac{N'}{ER^2} = \frac{H}{ER^3} R\,\kappa_1' - \frac{G'}{ER^3} R\,\tau_1 \tag{4}$$

and

$$\frac{X}{ER} = \frac{N'}{ER^2} R\,\tau_1 - \frac{T}{ER^2} R\,\kappa_1'. \tag{5}$$

In this analysis, it is important that the distributed normal load X be nonpositive, since X is related directly to the (assumed compressive) contact line loads imposed on a given wire by its nearest neighbors [2,3]. The resultant axial force F and axial twisting moment M acting on the m-wire strand are given by

$$\frac{F}{ER^2} = m\left(\frac{T}{ER^2}\sin\alpha_1 + \frac{N'}{ER^2}\cos\alpha_1\right)$$

and (6)

$$\frac{M}{ER^3} = m\left[\frac{H}{ER^3}\sin\alpha_1 + \frac{G'}{ER^3}\cos\alpha_1 + \frac{r_1}{R}\left(\frac{T}{ER^2}\cos\alpha_1 - \frac{N'}{ER^2}\sin\alpha_1\right)\right].$$

Now the associated strains of the strand are computed. The axial strand strain ε is defined as $(h_1-h)/h$, where h and h_1 are the lengths of an arbitrary segment of strand in the original and final configurations, respectively. The rotational strand strain β is defined as $r(\theta_1-\theta)/h$, where θ and θ_1 are the total angles that a given helical wire sweeps out in a plane perpendicular to the axis of the strand, in the original and final configurations, respectively. An analysis of the deformation yields the following expressions for ε and β:

$$\varepsilon = (1+\xi)\frac{\sin\alpha_1}{\sin\alpha} - 1 \qquad (7)$$

and

$$\beta = \frac{r}{r_1}(1+\varepsilon)\cot\alpha_1 - \cot\alpha. \qquad (8)$$

At this point, the generalized strains and forces associated with a straight strand have been found for arbitrarily specified wire strain ξ and change in wire helix angle $(\alpha_1-\alpha)$.

<u>Spring variables</u>. Consider now the strains and forces associated with a helical spring formed of the strand just considered. Let variables associated with the spring be denoted by an asterisk (*); for example, r^* and α^* will denote the original strand helix radius and helix angle, respectively. See Fig. 1(a). If r_1^* and α_1^* are the final values of strand helix radius and helix angle, respectively, then the initial and final curvatures and twists of the strand will be given by equations analogous to (2):

$$\kappa_0^* = 0, \qquad \kappa_0'^* = \cos^2\alpha^*/r^*, \qquad \tau_0^* = \sin\alpha^*\cos\alpha^*/r^*,$$

and (9)

$$\kappa_1^* = 0, \qquad \kappa_1'^* = \cos^2\alpha_1^*/r_1^*, \qquad \tau_1^* = \sin\alpha_1^*\cos\alpha_1^*/r_1^*.$$

The strand strain ξ^* has the same meaning as the ε given in (7), and the change in twist, $\tau_1^*-\tau_0^*$, is directly related to the β given in (8):

$$\tau_1^* - \tau_0^* = \frac{1}{r}\beta. \tag{10}$$

As for the strand forces, if the symbol T^* is used to denote the axial tension in the strand, it will be seen that T^* denotes the quantity previously called F; and similarly, H^* is the same as M. The new dependent variable for which an expression must be derived is the bending moment G'^*. It is here that Costello's results [5] for the bending of a helical wire are employed, by assuming that the bending stiffness A^* of an m-wire strand is m times the bending stiffness of a single helical wire. From [5], it may be inferred that

$$G'^* = A^*\,(\kappa_1'^* - \kappa_0'^*) \tag{11}$$

where

$$A^* = m\,\frac{\pi E R^4}{4}\cdot\frac{\sin\alpha}{1+\frac{1}{2}\nu\cos^2\alpha}\,. \tag{12}$$

Although (11) is a linear expression strictly appropriate only for small changes in curvature, the approximation is found to be excellent even for large changes in curvature [5], and will be used without modification in the present work.

Equilibrium equations analogous to (4) and (5) may now be written for the spring:

$$\frac{N'^*}{ER^2} = \frac{H^*}{ER^3}\,R\,\kappa_1'^* - \frac{G'^*}{ER^3}\,R\,\tau_1^* \tag{13}$$

and

$$\frac{X^*}{ER} = \frac{N'^*}{ER^2} R \tau_1^* - \frac{T^*}{ER^2} R \kappa_1'^*. \tag{14}$$

The distributed normal load per unit length X^* in (14) vanishes identically since the spring is formed of a single strand. All the quantities in (13) and (14) are known except for the final spring curvature $\kappa_1'^*$ and G'^*, which depends upon $\kappa_1'^*$ from (11). Setting $X^* = 0$ yields the following equation for $\kappa_1'^*$:

$$\kappa_1'^* = \frac{A^* \tau_1^* \kappa_0'^*}{[(T^*/\tau_1^*) - H^* + A^*\tau_1^*]} \cdot \tag{15}$$

With $\kappa_1'^*$ known, the quantities G'^* and N'^* can be calculated, and the final strand helix angle α_1^* can be computed by noting from (9) that

$$\tan\alpha_1^* = \tau_1^*/\kappa_1'^* . \tag{16}$$

The final strand helix radius r_1^* is thus known:

$$r_1^* = \cos^2\alpha_1^*/\kappa_1'^* . \tag{17}$$

Finally, the extensional and rotational strains of the spring, and the generalized forces associated with these strains are calculated. Equations similar to (7) and (8) yield the strains:

$$\varepsilon^* = (1+\xi^*) \frac{\sin\alpha_1^*}{\sin\alpha^*} - 1 \tag{18}$$

and

$$\beta^* = \frac{r^*}{r_1^*} (1+\varepsilon^*) \cot\alpha_1^* - \cot\alpha^*, \tag{19}$$

while equations analogous to (6) yield the axial spring force F^* and the axial spring twisting moment M^*:

$$\frac{F^*}{ER^2} = \frac{T^*}{ER^2}\sin\alpha_1^* + \frac{N'^*}{ER^2}\cos\alpha_1^*$$

and (20)

$$\frac{M^*}{ER^3} = \frac{H^*}{ER^3}\sin\alpha_1^* + \frac{G'^*}{ER^3}\cos\alpha_1^* + \frac{r_1^*}{R}\left(\frac{T^*}{ER^2}\cos\alpha_1^* - \frac{N'^*}{ER^2}\sin\alpha_1^*\right) .$$

Numerical Results

General response. For a given spring the wire strain ξ and the final wire helix angle α_1 determine the complete response of the spring, including the spring extensional and rotational strains ε^* and β^* and their associated generalized forces F^* and M^*. It is remarkable that even for very small values of ξ and $(\alpha_1-\alpha)$, the response of the spring can be large and strongly nonlinear, as illustrated in Figs. 2-9 for two typical springs. Indeed, in constructing these figures, many extreme behaviors have already been suppressed by omitting results for which the compressive line load condition $(X \leq 0)$ is violated, or for which the extensional spring strain ε^* falls outside the "reasonable" range $-1 \leq \varepsilon^* \leq +2$. Under these conditions, it is found that the wire strain ξ will generally be positive for either type of spring (tension or compression).

Figs. 2-5 illustrate the dependence of ε^*, β^*, F^* and M^*, respectively, on ξ and $(\alpha_1-\alpha)$ for a 3-wire stranded spring having the parameters $\alpha = 60°$, $\alpha^* = 10°$, $r^* = 10$ R, $\nu = 0.3$. This spring, illustrated in Fig. 1(a), would be suitable for tensile loading since the strand and the spring are wound in the same sense. Note that the permissible changes in wire helix angle $(\alpha_1-\alpha)$ are negative in this case.

Figs. 6-9 illustrate the response of a spring identical to the previous one except for the sense of the strand, *i.e.* $\alpha = 120°$ instead of 60°. This spring would be suitable for compressive loading; note that the permissible changes in wire helix angle $(\alpha_1 - \alpha)$ are positive.

Effect of end conditions. In some applications, springs are loaded with an axial force in such a way that the ends cannot rotate (*i.e.* $\beta^* = 0$). In other applications, the ends may rotate freely (*i.e.* $M^* = 0$). There are other types of end condition which can arise. Whatever the end condition is, it can in principle be written in the form

$$f(\xi, \alpha_1) = 0, \tag{21}$$

where f is a known function of the independent variables ξ and α_1. In view of (21), α_1 then becomes a function of ξ and *vice versa*; choosing ξ as "the" independent variable, one can solve (21) by a graphical or numerical procedure to yield the functional dependence

$$\alpha_1 = g(\xi). \tag{22}$$

All dependent variables are then a function of ξ alone.

A simple graphical procedure was used to generate the dashed curves labeled "$\beta^* = 0$" and "$M^* = 0$" in Figs. 2, 4, 6 and 8, as well as the dashed curves labeled "$\varepsilon^* = 0$" and "$F^* = 0$" in Figs. 3, 5, 7 and 9. Note in Fig. 2, for example, that for a given wire strain ξ, the spring axial strain ε^* is somewhat larger under the $M^* = 0$ condition than it is under the $\beta^* = 0$ condition. Referring to Fig. 4, one finds that the axial force F^* has a similar, but less pronounced, dependence upon the type of end condition. An analysis of the relative dependences of ε^* and F^* upon the type of end condition suggests that the spring would be stiffer under the zero-rotation condition than it would be under the zero-moment condition.

Verification of this finding for a tension spring is provided in Fig. 10, where F^* has been cross-plotted with ε^* for the two end conditions under consideration. All results in Fig. 10 were obtained numerically by a Newton procedure employing (21) with ξ treated as the independent variable. For the tension spring, ξ varied from zero to +0.0010, and for the compression spring, ξ varied from zero to +0.0006. Note that the axial stiffness of the compression spring is virtually independent of the type of end condition. This result could have been forseen by observing that the "$\beta^* = 0$" and "$M^* = 0$" curves practically coincide in Figs. 6 and 8.

Although helical springs are usually designed for tensile or compressive loading, one might also be interested in the torsional stiffness in some applications. For a tension spring, it will be seen in Figs. 3 and 5 that the manner in which β^* and M^* depend upon the wire strain ξ is strongly influenced by the types of end condition considered here, namely zero axial strain ($\varepsilon^* = 0$) and zero axial force ($F^* = 0$). However, when M^* is cross-plotted with β^*, it is found that the torsional stiffness is virtually independent of the type of end condition and that it varies insignificantly with the amount of rotational strain β^*. The same is true for compression springs (Figs. 7 and 9).

Summary and Conclusions

A large-deflection analysis of the static behavior of stranded wire helical springs has been developed with a minimum of assumptions, perhaps the most notable of them being that friction can be neglected and that each wire in the strand maintains contact with neighboring wires throughout the deformation. It was found, as Clark [1] argues, that the contact condition requires the strand to be wound in the same sense as that of the spring if the spring is to be loaded in tension, but in the opposite sense if the spring is to be loaded in compression.

It has also been found that the spring strains and forces are nonlinear functions of the extensional wire strain and the change in wire helix angle, even though these independent variables are assumed to be small. However, the overall response of the spring, as measured conventionally in tension, compression or torsion, is only weakly nonlinear and is relatively insensitive to ordinary types of end condition, even though the spring strains may be large.

The results in Fig. 10 indicate that for small spring strains, the axial stiffness of a tension spring is approximately equal to that of a compression spring of comparable construction, and it is plausible that an expression could be derived for the axial stiffness of either type of spring under the assumption of small spring strains. It is unlikely that such an expression could be written concisely. This matter is under investigation.

Another matter being considered is the effect of wire separation, which might occur, for example, if a compression spring were loaded in tension. It is anticipated that a distinct change in stiffness would occur when wire contact within the strand is either made or broken.

Acknowledgment

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Figure Captions

Fig. 1 Initial configuration of a stranded wire helical spring: (a) Three-dimensional drawing of spring; (b) Cross-sectional view of strand.

Fig. 2 Axial spring strain ε^* as a function of wire strain ξ for various values of change in wire helix angle $(\alpha_1-\alpha)$. Results are for a typical tension spring with $\alpha = 60°$.

Fig. 3 Rotational spring strain β^* as a function of wire strain ξ for various values of change in wire helix angle $(\alpha_1-\alpha)$. Results are for a typical tension spring with $\alpha = 60°$.

Fig. 4 Dimensionless axial spring force F^*/ER^2 as a function of wire strain ξ for various values of change in wire helix angle $(\alpha_1-\alpha)$. Results are for a typical tension spring with $\alpha = 60°$.

Fig. 5 Dimensionless axial twisting moment M^*/ER^3 as a function of wire strain ξ for various values of change in wire helix angle $(\alpha_1-\alpha)$. Results are for a typical tension spring with $\alpha = 60°$.

Fig. 6 Axial spring strain ε^* as a function of wire strain ξ for various values of change in wire helix angle $(\alpha_1-\alpha)$. Results are for a typical compression spring with $\alpha = 120°$.

Fig. 7 Rotational spring strain β^* as a function of wire strain ξ for various values of change in wire helix angle $(\alpha_1-\alpha)$. Results are for a typical compression spring with $\alpha = 120°$.

Fig. 8 Dimensionless axial spring force F^*/ER^2 as a function of wire strain ξ for various values of change in wire helix angle $(\alpha_1-\alpha)$. Results are for a typical compression spring with $\alpha = 120°$.

Fig. 9 Dimensionless axial twisting moment M^*/ER^3 as a function of wire strain ξ for various values of change in wire helix angle $(\alpha_1-\alpha)$. Results are for a typical compression spring with $\alpha = 120°$.

Fig. 10 Conventional response curves for typical tension and compression springs of comparable construction. For each type of spring the dimensionless axial spring force F^*/ER^2 has been plotted as a function of the axial spring strain ε^* for the zero-rotation ($\beta^* = 0$) condition and the zero-moment ($M^* = 0$) condition. Negative values of F^* and ε^* are associated with the compression spring.

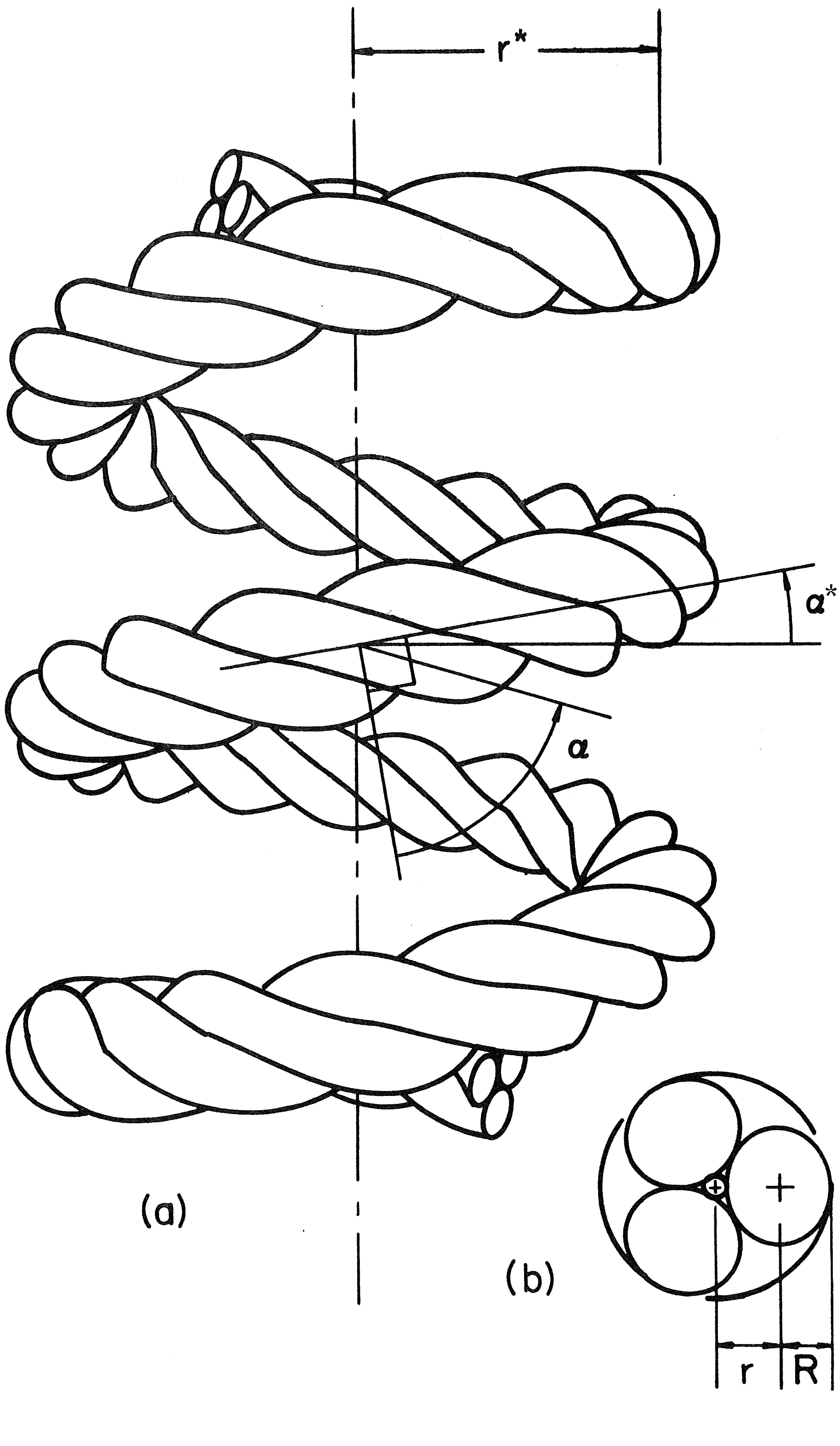
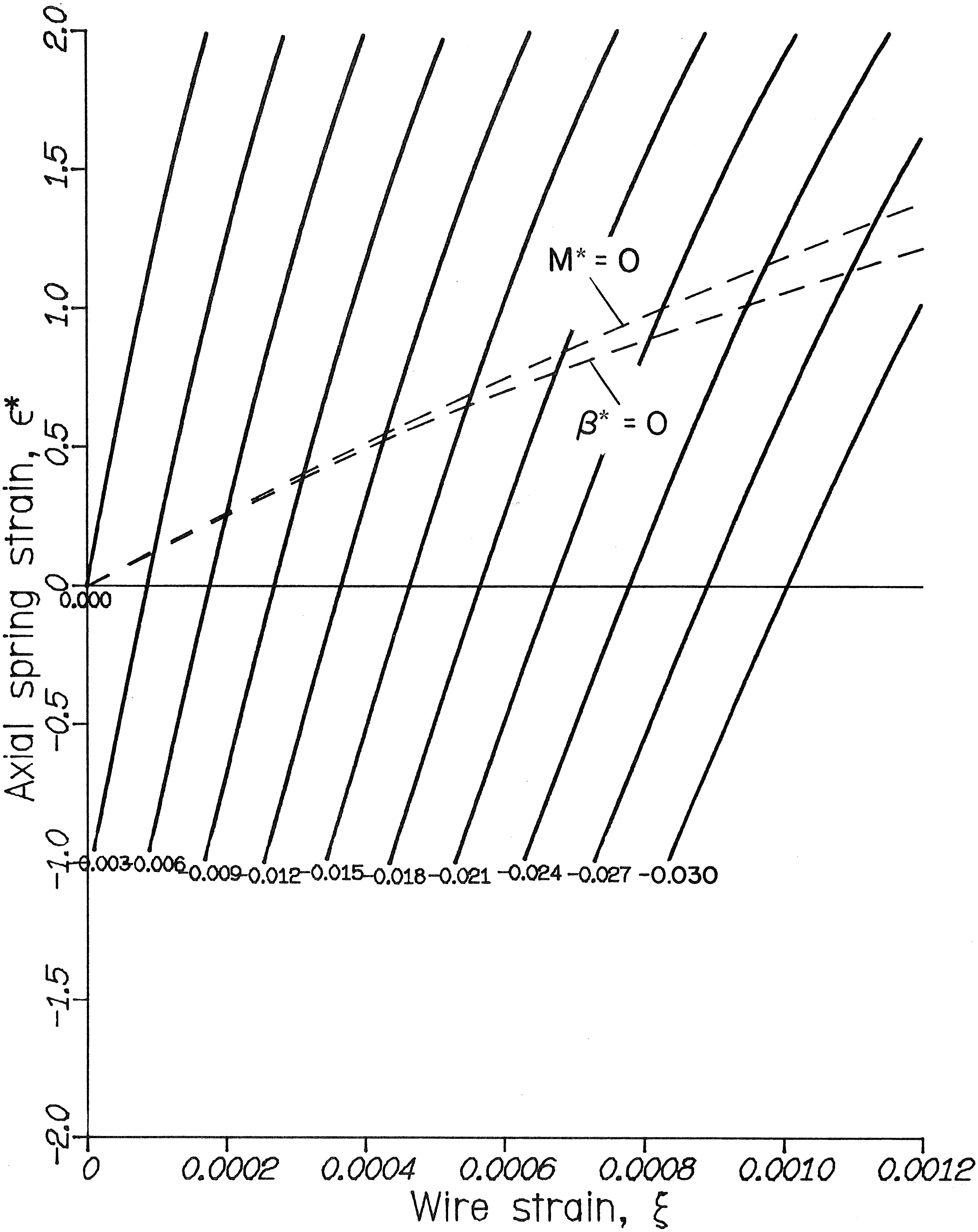
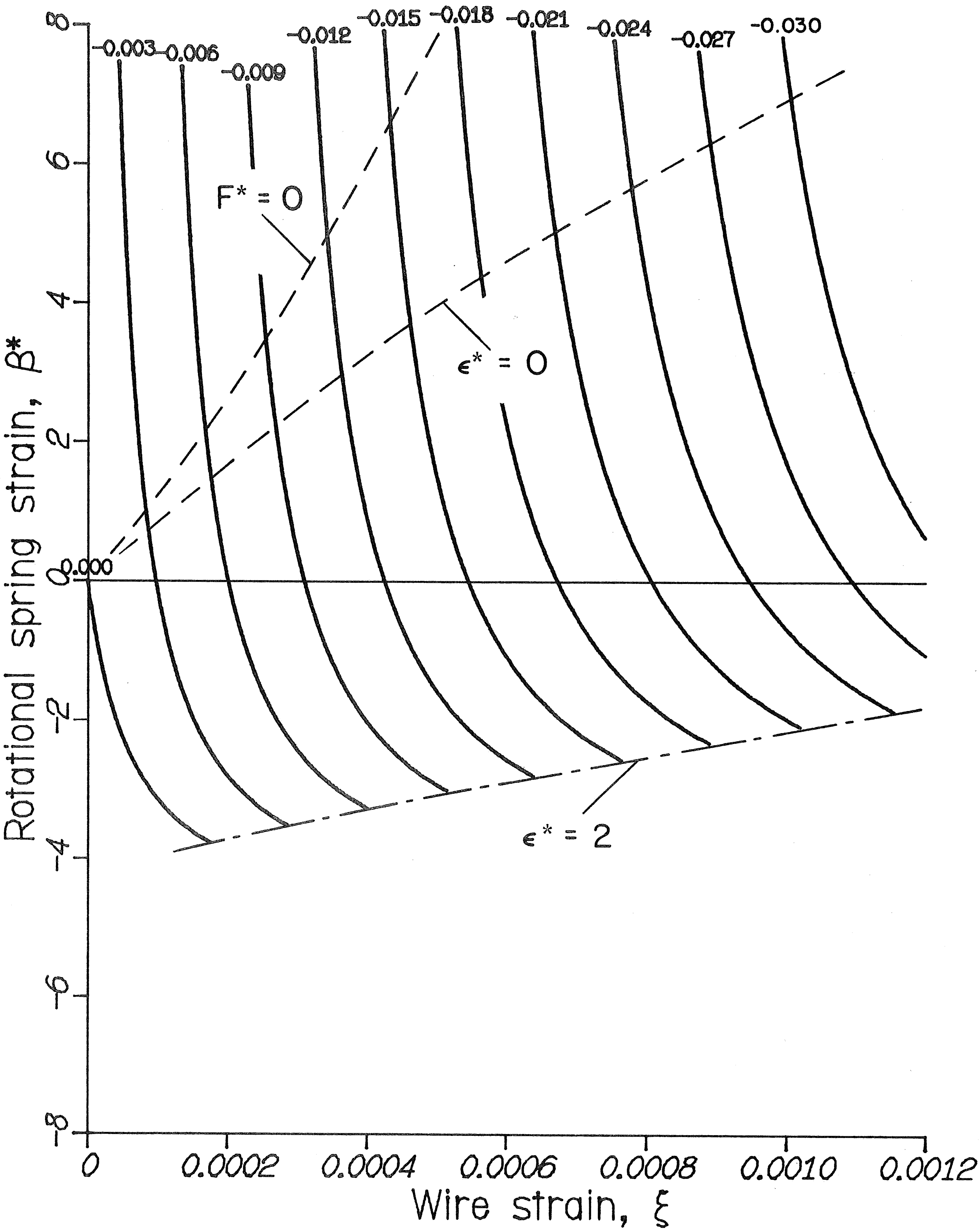


Fig. 1. Costello & Phillips

α = 1.047000 Fig. 2. Costello & Phillips



α = 1.047000 Fig. 3. Costello & Phillips



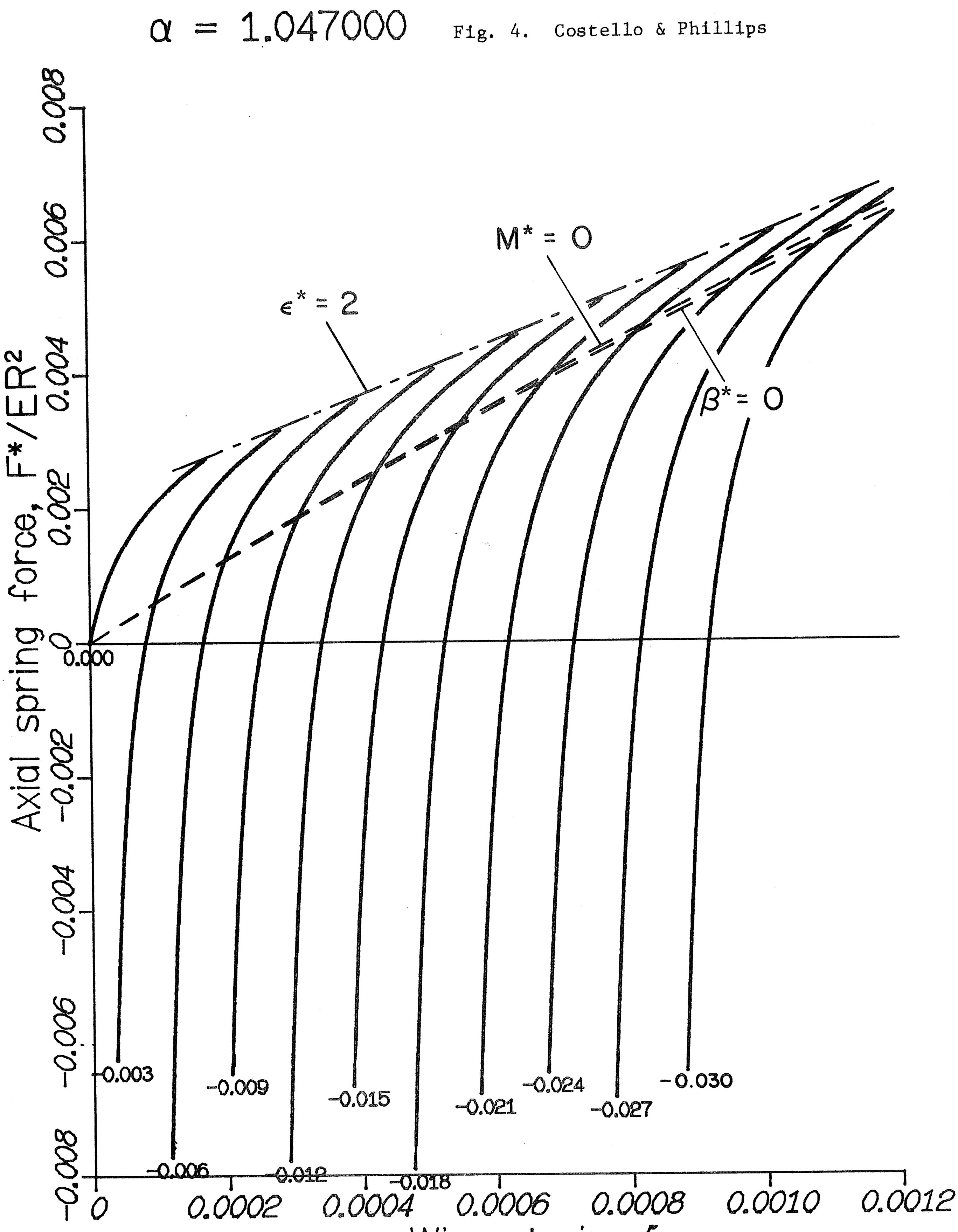

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Fig. 4. Costello & Phillips
Axial spring force, F*/ER²
Wire strain, ξ
ε* = 2
M* = 0
β* = 0
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
0
0.0002
0.0004
0.0006
0.0008
0.0010
0.0012
0.000
-0.003
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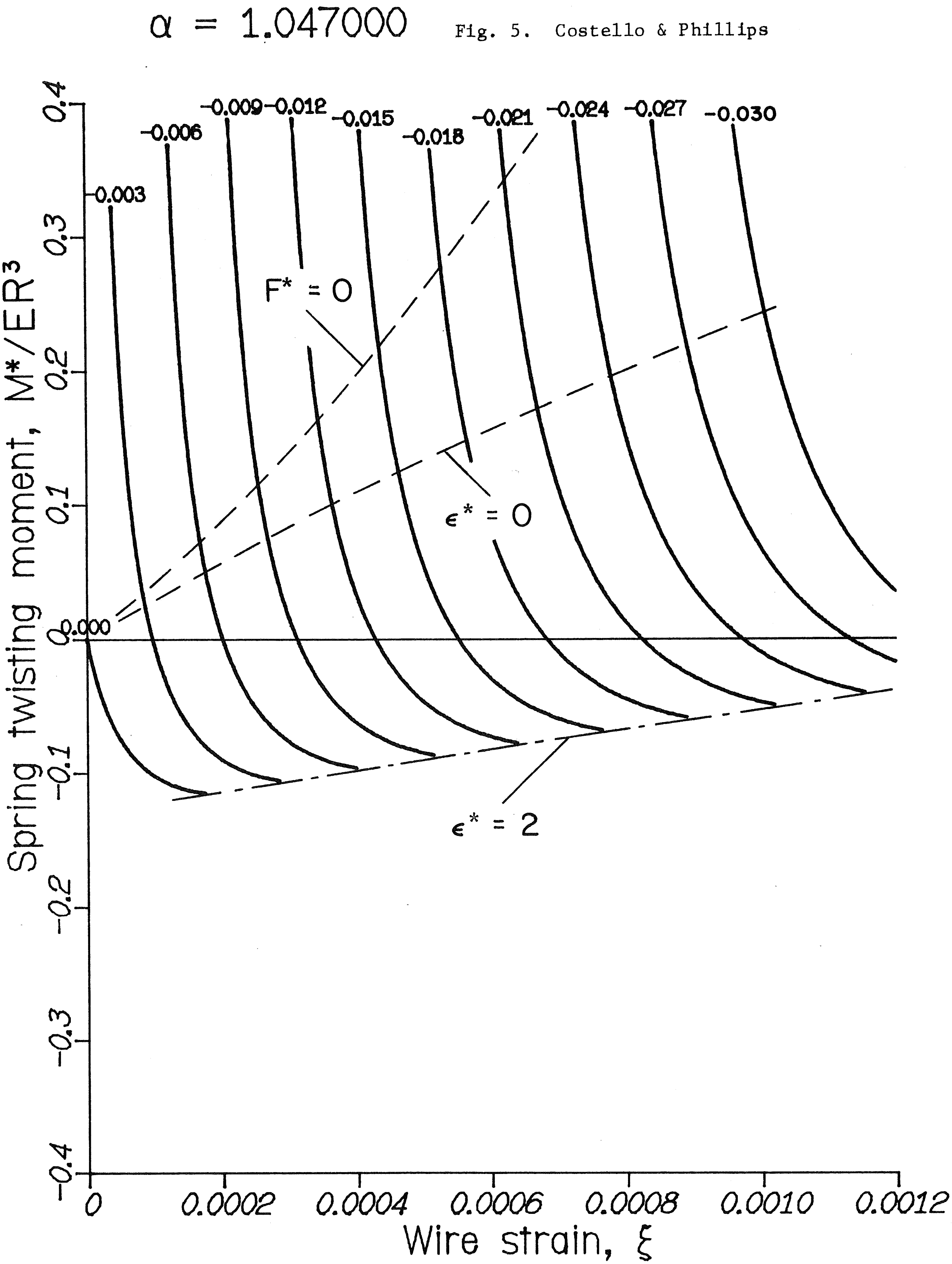
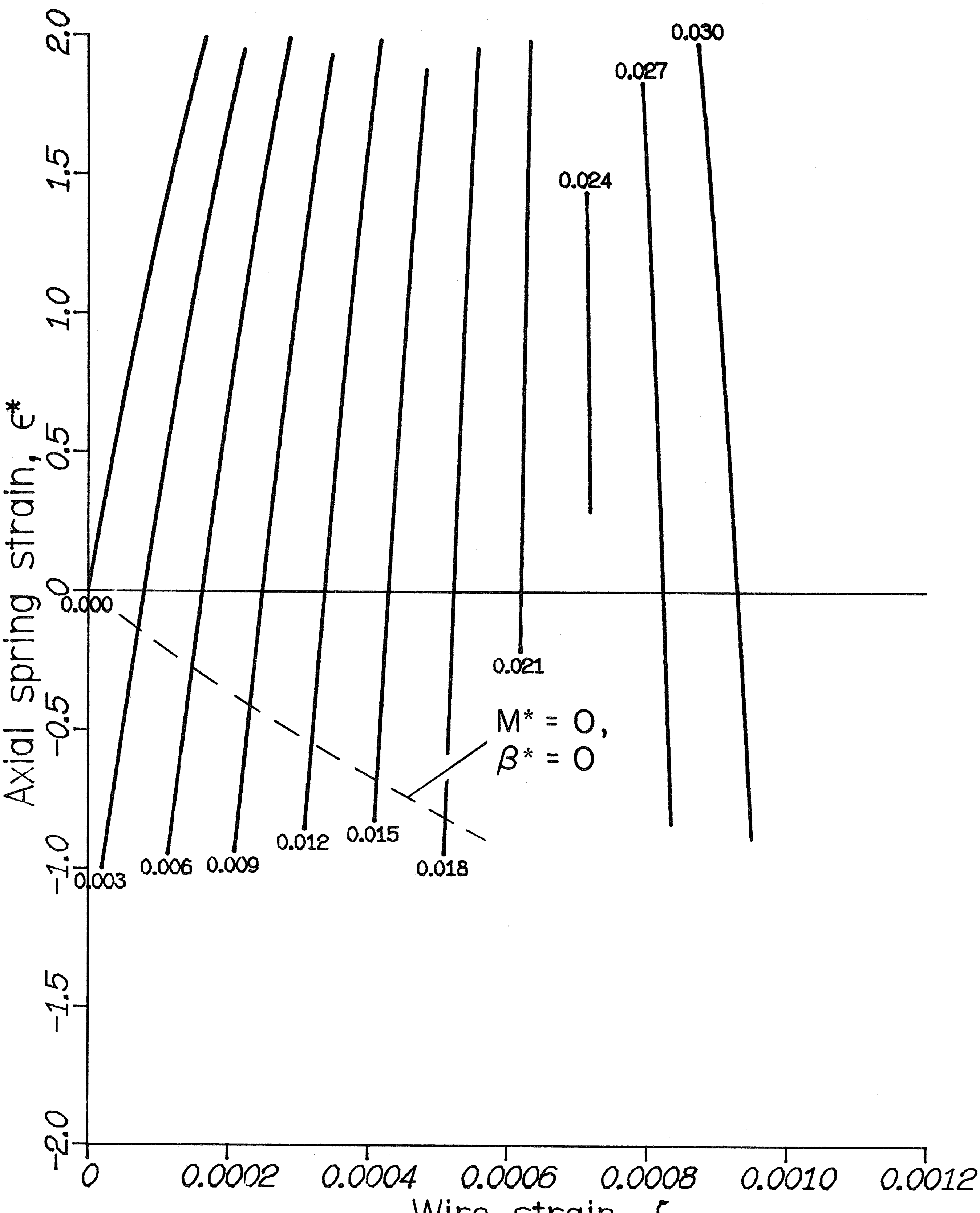
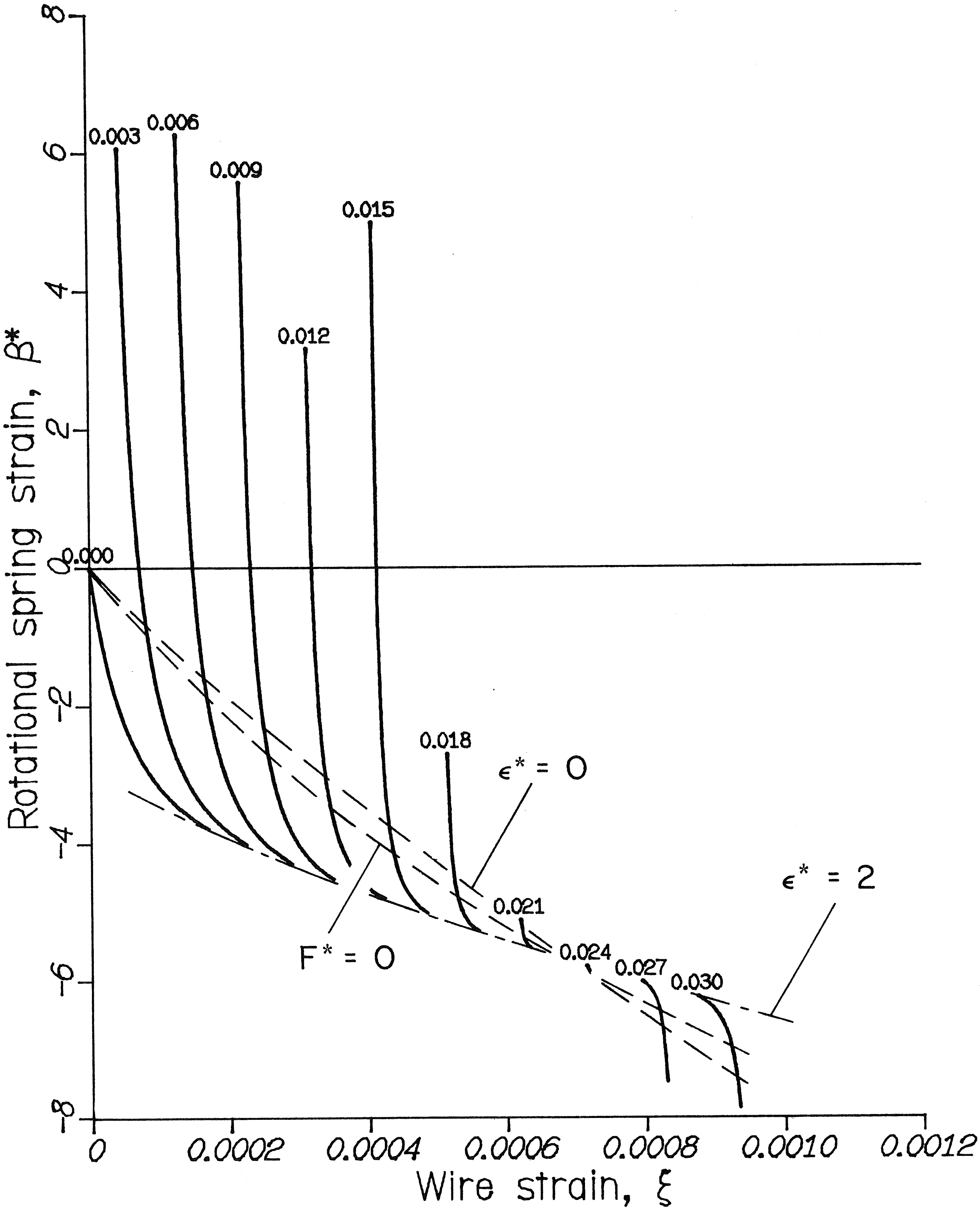

α = 1.047000
Fig. 5. Costello & Phillips
Spring twisting moment, M*/ER³
Wire strain, ξ
-0.003
-0.006
-0.009
-0.012
-0.015
-0.018
-0.021
-0.024
-0.027
-0.030
0.000
F* = 0
ϵ* = 0
ϵ* = 2
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
0.0002
0.0004
0.0006
0.0008
0.0010
0.0012

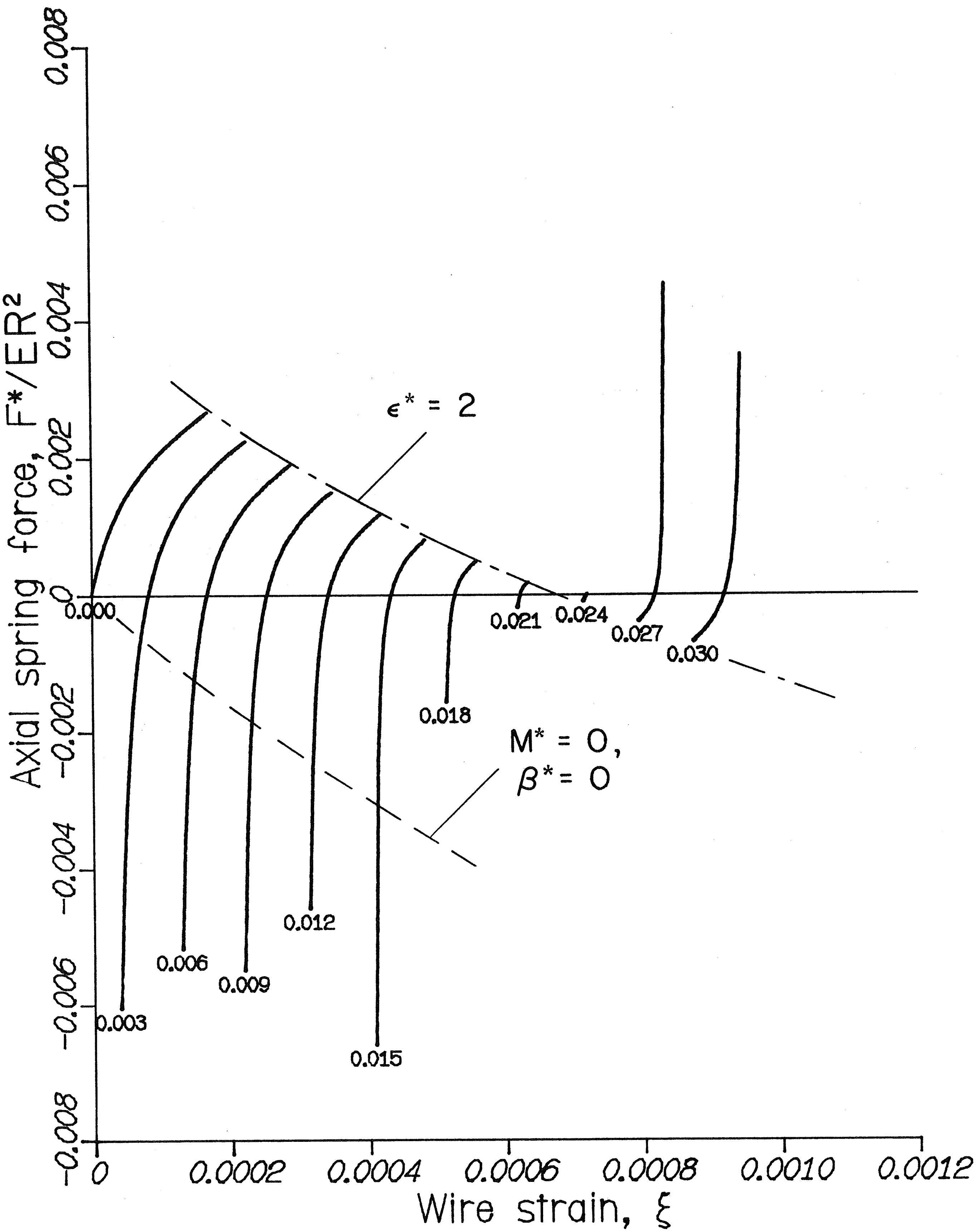
$\alpha = 2.094593$ Fig. 6. Costello & Phillips



$\alpha = 2.094593$ Fig. 7. Costello & Phillips



$\alpha = 2.094593$ Fig. 8. Costello & Phillips



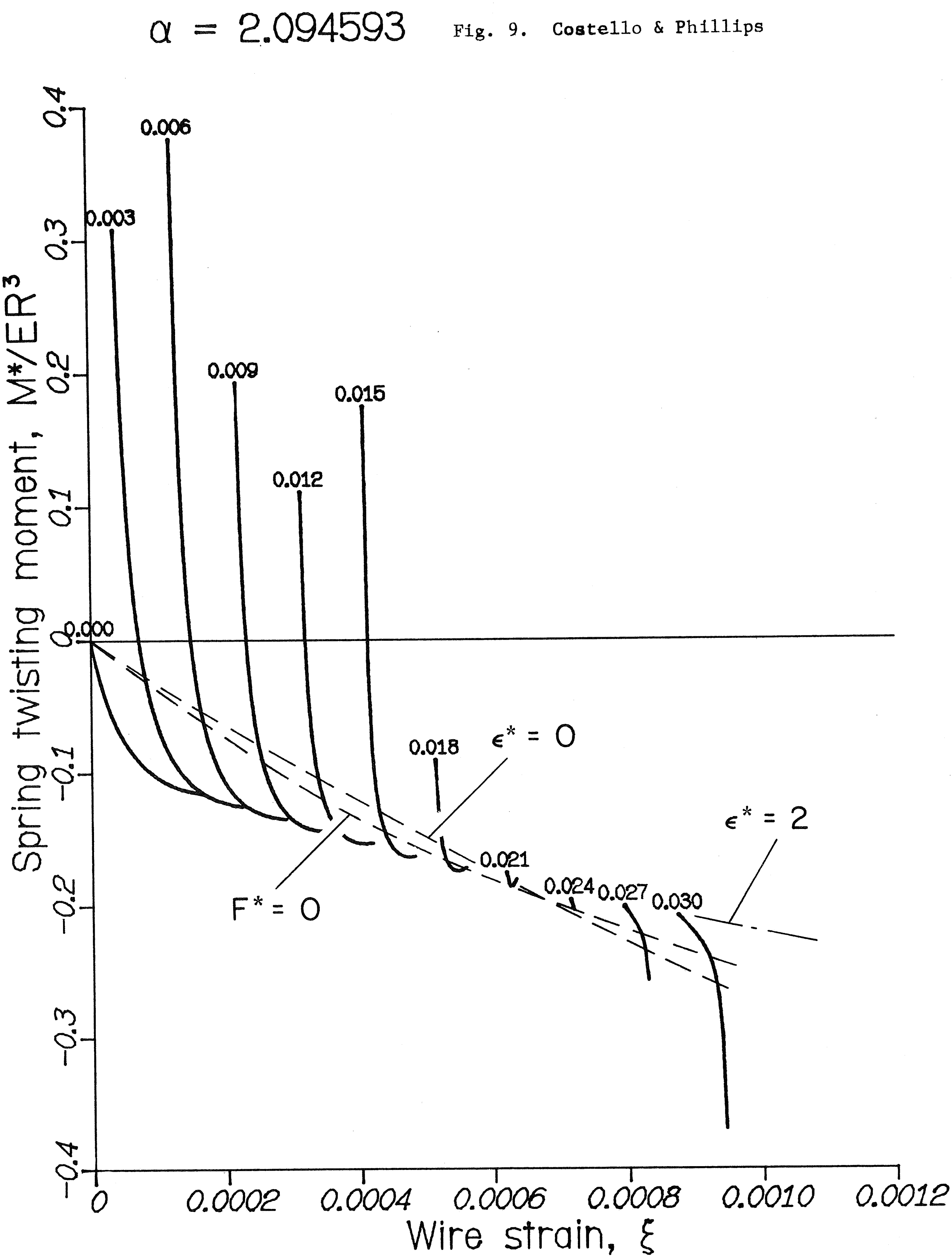


Fig. 9. Costello & Phillips

Fig. 10. Costello & Phillips