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# WIRE ROPE WITH COMPLEX CROSS SECTIONS

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## ABSTRACT

A theory has been developed to predict the static response of a wire rope with complex cross sections. The solution of the nonlinear equations of equilibrium are linearized and the results are applied to a 6 x 19 Seale rope with an independent wire rope core. The linearization permits a considerable simplification in the theory so that the results can readily be applied to other types of cross sections. Expressions are presented for the stresses in the rope and the maximum tensile stress is also determined. A load deformation curve of a Seale rope is obtained experimentally and the results are compared with the theory.

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## INTRODUCTION

Many investigations have appeared recently to determine the behavior of thin rods subjected to various loads. The application of the evolved theories to those problems associated with wire rope is discussed in a survey article (2) wherein the interested reader can find many appropriate references. This present work is an extension of the frictionless theory, employed by Costello, Phillips and Miller (1,3,4,6), to wire ropes with complex cross sections.

In the research mentioned above, an exact solution to the nonlinear equations of equilibrium of a thin helical rod is presented. The solution is utilized to determine the true nonlinear behavior of a simple wire strand subjected to an axial force and an axial twisting moment. In this present work, the solution is linearized and a theory is obtained which is valid for most cases of practical interest. Since the linearization results in a considerable simplification, the theory can be extended to ropes with complex cross sections with relative ease.

## ANALYSIS

Several of the equations presented below are derived in (1,3,4,6) and are listed here again for completeness. Figure 1 shows the cross section of a simple strand which consists of a straight center wire, of radius $R_1$, surrounded by $m_2$ helical wires, of radius $R_2$. It is assumed that the center wire is of sufficient size so as to prevent the $m_2$ outer wires from touching each other. This is usually the case in wire rope design since this tends to minimize the effect of friction.

If the $m_2$ wires, with the elliptical cross sections shown in Fig. 1, were just touching each other, the radius $r_2$, of the helix of these $m_2$ wires is given by the relation (3)

$$\frac{r_2}{R_2} = \left[ 1 + \frac{\tan^2\left(\frac{\pi}{2} - \frac{\pi}{m_2}\right)}{\sin^2 \alpha_2} \right]^{\frac{1}{2}} \tag{1}$$

in which $\alpha_2$ = the helix angle of the outer wires. In the usual case $r_2 < R_1 + R_2$ so that the outside wires are not touching each other. The following expression is also valid

$$\tan \alpha_2 = \frac{p_2}{2\pi r_2} \tag{2}$$

where $p_2$ = the pitch of the outer wires. A combination of Eq. 1 and 2 yields the helix radius, $r_2$ , assuming that the outside wires are just touching each other. If $r_2 < R_1 + R_2$ , as determined by Eq. 1 and 2, then the actual helical radius is given by

$$r_2 = R_1 + R_2 \tag{3}$$

Figure 2 shows the undeformed and deformed configuration of an outer wire. An analysis of these configurations (4) yields

$$\xi_1 = \frac{\bar{h} - h}{h} = (1 + \xi_2) \frac{\sin \bar{\alpha}_2}{\sin \alpha_2} - 1 \tag{4}$$

and

$$\beta_2 = \frac{r_2}{h} (\bar{\theta}_2 - \theta_2) = \frac{r_2}{\bar{r}_2} (1 + \xi_1) \left[\frac{1}{\tan \bar{\alpha}_2} - \frac{1}{\tan \alpha_2}\right] \tag{5}$$

in which $\xi_1$ = the axial strain of the strand and of course the center wire; $h$ = the original length of the strand; $\bar{h}$ = the final length of the strand; $\xi_2$ = the axial strain of the outer wires; $\alpha_2$ = the original helix angle of the outer wires; $\bar{\alpha}_2$ = the final helix angle of the outer wires; $\beta_2$ = the rotational strain of the outer wires; $r_2$ = the original radius of the helix of the outer wires; $\bar{r}_2$ = the final radius of the helix of the outer wires; $\theta_2$ = the original total angle the outer wires sweep out in a plane perpendicular to the axis of the strand and $\bar{\theta}_2$ = the final total angle the outer wires sweep out in a plane perpendicular to the axis of the strand. The angle of twist per unit length of the strand is $(\theta_1 - \theta)/h$.

Let

$$\Delta\alpha_2 = \bar{\alpha}_2 - \alpha_2 << 1 \tag{6}$$

which is valid for most metalic strands. Hence, $\sin \bar{\alpha}_2$ can be written as

$$\sin \bar{\alpha}_2 = \sin (\alpha_2 + \Delta\alpha_2) = \sin \alpha_2 + \Delta\alpha_2 \cos \alpha_2 \tag{7}$$

where higher order terms are neglected. Equation 4 can now be expressed as

$$\xi_1 = \xi_2 + \frac{\Delta\alpha_2}{\tan \alpha_2} \tag{8}$$

where terms such as $(\xi_2 \; \Delta\alpha_2)$ are neglected in comparison to the linear terms. Equation 5, after a similar procedure, becomes

$$\beta_2 = \frac{r_2}{\bar{r}_2} \left[ \frac{1 + \xi_2}{\tan \alpha_2} - \Delta\alpha_2 \right] - \frac{1}{\tan \alpha_2} \tag{9}$$

The radius $\bar{r}_2$, due to the Poisson's ratio effect, is given by the expression

$$\bar{r}_2 = R_1(1 - \nu\,\xi_1) + R_2(1 - \nu\,\xi_2) \tag{10}$$

where $\nu$ = Poisson's ratio of the wire material. Now $r_2/\bar{r}_2$ can be written as

$$\frac{r_2}{\bar{r}_2} = \frac{R_1 + R_2}{R_1 + R_2 - \nu(R_1\xi_1 + R_2\xi_2)} = 1 + \frac{\nu(R_1\xi_1 + R_2\xi_2)}{R_1 + R_2} \tag{11}$$

since $\xi_1$ and $\xi_2$ are small compared to unity and hence Eq. 9 becomes

$$\beta_2 = \frac{(1 + \nu)}{\tan \alpha_2} \xi_2 + \left( \frac{\nu R_1}{(R_1 + R_2)\tan^2 \alpha_2} - 1 \right) \Delta\alpha_2 \tag{12}$$

The change in curvature, $\Delta\kappa$, of the outer wires is given by (1)

$$R_2 \Delta\kappa = \frac{\cos^2 \bar{\alpha}_2}{\bar{r}_2/R_2} - \frac{\cos^2 \alpha_2}{r_2/R_2} \tag{13}$$

which when linearized becomes

$$R_2 \Delta\kappa = \frac{R_2}{(R_1+R_2)} \left[ \left[ \frac{\nu R_1}{(R_1+R_2)} \frac{\cos^2 \alpha_2}{\tan \alpha_2} - 2 \sin \alpha_2 \cos \alpha_2 \right] \Delta\alpha_2 + \nu \cos^2 \alpha_2 \, \xi_2 \right] \tag{14}$$

The angle of twist, per unit length, $\tau_2$, of the outer wires is given by (1)

$$R_2 \tau_2 = \frac{\sin \bar{\alpha}_2 \cos \bar{\alpha}_2}{\bar{r}_2/R_2} - \frac{\sin \alpha_2 \cos \alpha_2}{r_2/R_2} \tag{15}$$

which when linearized becomes

$$R_2 \tau_2 = \frac{R_2}{(R_1+R_2)} \left[ \left[ 1 - 2 \sin^2 \alpha_2 + \frac{\nu R_1 \cos^2 \alpha_2}{(R_1+R_2)} \right] \Delta\alpha_2 + \nu \sin \alpha_2 \cos \alpha_2 \, \xi_2 \right] \tag{16}$$

The following equations result after a linearization of the expressions in (4)

$$\frac{H}{ER_2^3} = \frac{\pi}{4(1+\nu)} \frac{R_2}{(R_1+R_2)} \left[ \left[ 1 - 2 \sin^2 \alpha_2 + \nu \frac{R_1 \cos^2 \alpha_2}{(R_1+R_2)} \right] \Delta\alpha_2 + \nu \sin \alpha_2 \cos \alpha_2 \, \xi_2 \right] \tag{17}$$

$$\frac{G'}{ER_2^3} = \frac{\pi}{4} \frac{R_2}{(R_1+R_2)} \left[ \left[ \frac{\nu R_1}{(R_1+R_2)} \frac{\cos^2 \alpha_2}{\tan \alpha_2} - 2 \sin \alpha_2 \cos \alpha_2 \right] \Delta\alpha_2 + \nu \cos^2 \alpha_2 \, \xi_2 \right] \tag{18}$$

$$\frac{N'}{ER_2^2} = \frac{H}{ER_2^3} \frac{\cos^2 \alpha_2}{r_2/R_2} - \frac{G'}{ER_2^3} \frac{\sin \alpha_2 \cos \alpha_2}{r_2/R_2} \tag{19}$$

$$\frac{T}{ER_2^2} = \pi \xi_2 \tag{20}$$

$$\frac{X}{ER_2} = \frac{N'}{ER_2^{\,2}} \frac{\sin\alpha_2 \cos\alpha_2}{r_2/R_2} - \frac{T}{ER_2^{\,3}} \frac{\cos^2\alpha_2}{r_2/R_2} \tag{21}$$

$$F = m_2\, ER_2^{\,2} \left[ \frac{T}{ER_2^{\,2}} \sin\alpha_2 + \frac{N'}{ER_2^{\,2}} \cos\alpha_2 \right] + \pi R_1^{\,2} E\, \xi_1 \tag{22}$$

and

$$M = m_2\, ER_2^{\,3} \left[ \frac{H}{ER_2^{\,3}} \sin\alpha_2 + \frac{G'}{ER_2^{\,3}} \cos\alpha_2 + \frac{T}{ER_2^{\,2}} \frac{r_2}{R_2} \cos\alpha_2 - \frac{N'}{ER_2^{\,2}} \frac{r_2}{R_2} \sin\alpha_2 \right] + \frac{\pi ER_1^{\,4}}{4(1+\nu)} \frac{\beta_2}{r_2} \tag{23}$$

where $H$ = the axial twisting moment in an outer wire; $G'$ = the bending moment in an outer wire; $N'$ = the shearing force in an outer wire; $X$ = the contact force per unit length acting on the outer wire; $F$ = the total axial force acting on the strand and $M$ = the total axial twisting moment acting on the strand.

In general, the total axial force and the total axial moment for a rope can be expressed as follows

$$\frac{F}{AE} = K_1\, \varepsilon + K_2\, \beta \tag{24}$$

and

$$\frac{M}{ER^3} = K_3\, \varepsilon + K_4\, \beta \tag{25}$$

where $A = \Sigma\, \pi\, R_i^{\,2}$ = the metalic cross sectional area of the rope; $R$ = the outside radius of the rope; $R_i$ = the radius of an individual wire; $\varepsilon$ = the axial strain of the rope; $K_1$, $K_2$, $K_3$, and $K_4$ are constants; and $\beta$ = the rotational strain of the rope, defined by the equation

$$\beta = R\, \tau \tag{26}$$

where $\tau$ is the angle of twist per unit length of the rope. For example, in the case of a straight wire, $K_1 = 1$, $K_2 = 0$, $K_3 = 0$ and $K_4 = \pi/4(1+\nu)$.

The value of the constants in Eq. 24 and 25 can be determined for the given simple straight strand discussed above. Let, for example, $\beta = R\,\tau = 0$, and $\xi_2$ = a given value. Since $\beta_2 = (R_1 + R_2)\,\tau = 0$, Eq. 12 can be used to determine $\Delta\alpha_2$. Equations 17, 18, 19, 20, 21, 22 and 23 yield H, G', N', T, X, F and M. The axial strain of the strand $\varepsilon = \xi_1$ is given by Eq. 8. Since F, M, $\varepsilon$ and $\beta$ are now known, Eq. 24 and 25 can be used to determine the constants $K_1$ and $K_3$. A similar procedure, where $\varepsilon = 0$ and $\beta$ = a given value can be used to determine $K_2$ and $K_4$. Let for example, $m_2 = 6$, $R_1 = 0.03155$ in., $R_2 = 0.028925$ in., and $p = 1.30$ in. In this case the 6 outer wires do not touch each other. Hence $r_2 = 0.060475$ in. and $\alpha_2 = 73.7069°$. Letting $E = 30 \times 10^6$ psi and $\nu = 0.29$ and for $\xi_2 = 0.001$ and $\beta = 0$, the following results: $\varepsilon = 0.0011116$, $F = 558.616$ and $M = 7.60$ in. lb. and since $R = 0.0894$ in. and $A = 0.0189$ in.$^2$, $K_1 = 0.886$ and $K_3 = 0.319$. Also when $\varepsilon = 0$ and $\xi_2 = 0.001$, $\beta = 0.00555$, $F = 451.716$ and $M = 14.06$ in. lb. The value of $K_2 = 0.144$ and $K_4 = 0.118$.

It is interesting to note that, for $\beta = 0$, a load of 358.6 lb. produces a stress $E\,\varepsilon = 33{,}350$ psi in the center wire. Since the equations are linear a load of, for example, 1000 lb. would produce a stress of 59,700 psi in the center wire with $\beta = 0$. Also for the axial load of 558.6 lb. and $\beta = 0$, the maximum tensile stress in the outer wires turns out to be 30,000 + 2,600 = 32,600 psi which is the sum of the stresses due to the axial load T and the bending moment G' in the outer wires. This stress is less than the stress in the center wire.

Consider now a rope, as shown in Fig. 3, which consists of a center straight strand (strand 1) and another strand (strand 2) wrapped helically around the center strand. In this case the previous theory can be extended to investigate the response of such a rope.

The bending stiffness of strand 2 will be approximated by a summation of the bending stiffnesses of each wire in the strand, i.e., treating the strand as an assemblage of helical springs. This is felt to be a reasonable assumption since friction is neglected. Recent experimental investigations tend to confirm this (5). Hence the bending stiffness $A_2^*$, for strand 2, is given by the expression (3)

$$A_2^* = m_4 \frac{\pi ER_4^4}{2} \frac{\sin \alpha_4}{(2 + \nu \cos^2 \alpha_4)} + \frac{\pi ER_3^4}{4} \tag{27}$$

where $m_4$ = the number of outer helical wires in strand 2; $R_4$ = the wire radius of the outer wires in strand 2; $R_3$ = the center wire radius in strand 2; and $\alpha_4$ = the helix angle of the outer wires in strand 2.

In the foregoing theory, for the solid wire strands in strand 1, the twisting moment and the axial force in the strand wires are determined from the properties of a straight solid wire with an axial strain $\xi_2$ and an angle of twist per unit length $\tau_2$, i.e., $H = \pi ER_2^4 \tau_2/4(1 + \nu)$ and $T = \pi ER_2^2 \xi_2$. Thus if the cross section of the strand in a rope is like that shown in Fig. 1, the values of $\xi_1$, and $\tau_2$ will be used to determine the axial force and the twisting moment in the curved strand (3).

Let the helix angle of strand 2 in the rope cross section, shown in Fig. 3, be $\alpha_2^*$. As the rope is loaded this helix angle assumes the new value $\bar{\alpha}_2^*$. The angle of twist per unit length for strand 2 becomes

$$\tau_2^* = \frac{\sin \bar{\alpha}_2^* \cos \bar{\alpha}_2^*}{\bar{r}_2^*} - \frac{\sin \alpha_2^* \cos \alpha_2^*}{r_2^*} \tag{28}$$

where

$$r_2^* = R_1 + 2\,R_2 + 2\,R_4 + R_3 \tag{29}$$

and where, due to the Poisson's ratio effect,

$$\overline{r}_2^* = R_1(1 - \nu\,\xi_1) + 2\,R_2(1 - \nu\,\xi_2) + 2\,R_4(1 - \nu\,\xi_4) + R_3(1 - \nu\,\xi_3)$$

$$= r_2^* - \nu(R_1\xi_1 + 2\,R_2\xi_2 + 2\,R_4\xi_4 + R_3\xi_3) \tag{30}$$

where $\xi_3$ and $\xi_4$ are the wire strains in wires 3 and 4. Again Eq. 28 can be linearized.

The following equations can be written down

$$\xi_1 = \xi_3 + \frac{\Delta\,\alpha_2^*}{\tan\,\alpha_2^*} \tag{31}$$

$$\beta_2^* = \tau\,r_2^* = \frac{r_2^*}{\overline{r}_2^*}\left[\frac{1 + \xi_3}{\tan\,\alpha_2^*} - \Delta\,\alpha_2^*\right] - \frac{1}{\tan\,\alpha_2^*} =$$

$$= \frac{\xi_3}{\tan\,\alpha_2^*} - \Delta\,\alpha_2^* + \frac{\nu}{r_2^*}\;\frac{(R_1\xi_1 + 2\,R_2\xi_2 + 2\,R_4\xi_4 + R_3\xi_3)}{\tan\,\alpha_2^*} \tag{32}$$

$$\xi_3 = \xi_4 + \frac{\Delta\,\alpha_4}{\tan\,\alpha_4} \tag{33}$$

$$\beta_4 = (R_3 + R_4)\,\tau_2^* = \frac{(1 + \nu)}{\tan\,\alpha_4}\,\xi_4 + \left(\frac{\nu\,R_3}{(R_3 + R_4)\tan^2\,\alpha_4} - 1\right)\Delta\,\alpha_4 =$$

$$\frac{(R_3 + R_4)}{r_2^*}\left[(1 - 2\,\sin^2\,\alpha_2^*)\Delta\,\alpha_2^* + \frac{\nu\,\sin\,\alpha_2^*\,\cos\,\alpha_2^*}{r_2^*}(R_1\xi_1 + 2\,R_2\xi_2 + 2\,R_4\xi_4 + R_3\xi_3)\right] \tag{34}$$

The following procedure can now be used to determine the response of a rope with the cross section shown in Fig. 3. Choose a value of $\varepsilon$ and $\beta$, which are the axial and rotational strain of the rope. Since $\varepsilon = \xi_1$ and $\beta = R\,\tau = (R_1 + 2\,R_2 + 2\,R_3 + 4\,R_4)\,\beta_2/(R_1 + R_2)$, Eq. 8 and 12 can be used to determine $\xi_2$ and $\Delta\,\alpha_2$. Equations 31, 32, 33 and 34 can now be solved for $\xi_3$, $\xi_4$, $\Delta\,\alpha_2^*$ and $\Delta\,\alpha_4$. Once $\xi_3$ and $\beta_4$ (Eq. 34) are known, the total axial force $T^*$ and the total axial twisting moment $H^*$ in strand 2 can be determined. The bending moment in strand 2, $G'^*$, is given by the expression

$$G'^* = A_2^*\,\Delta\,\kappa_2^* = A^* \left( \frac{\sin\bar{\alpha}_2^*\,\cos\bar{\alpha}_2^*}{\bar{r}_2^*} - \frac{\sin\alpha_2^*\,\cos\alpha_2^*}{r_2^*} \right) \tag{35}$$

which again can be linearized. Equations similar to Eq. 22 and 23 can be written down (3) for the total axial force and the total axial twisting moment acting on the cross section shown in Fig. 3.

The preceeding theory can now be applied to a rope which for example consists of three strands. Figure 4 shows such a rope which is a 6 x 19 Seale wire rope with an independent wire rope core.

## RESULTS

Consider a 6 x 19 Seale rope, with an IWRC (independent wire rope core), having the following parameters: $R_1$ = 0.03155 in., $R_2$ = 0.028925 in., $R_3$ = 0.027725 in., $R_4$ = 0.025815 in., $R_5$ = 0.05731 in., $R_6$ = 0.02805 in., $R_7$ = 0.049928 in., $\alpha_2$ = 73.7069°, $\alpha_4$ = 81.0664°, $\alpha_6$ = 77.7330°, $\alpha_7$ = 68.7688°, $\alpha_2^*$ = 70.8302° and $\alpha_3^*$ = 70.2389°. The following results from an application of the preceeding theory:

$$\frac{F}{AE} = 0.8864\ \varepsilon + 0.1436\ \beta$$
$$\frac{M}{ER^3} = 0.3189\ \varepsilon + 0.1182\ \beta \qquad \text{strand 1} \qquad (36)$$

$$\frac{F}{AE} = 0.9642\ \varepsilon + 0.0853\ \beta$$
$$\frac{M}{ER^3} = 0.1928\ \varepsilon + 0.0721\ \beta \qquad \text{strand 2} \qquad (37)$$

$$\frac{F}{AE} = 0.8295\ \varepsilon - 0.1765\ \beta$$
$$\frac{M}{ER^3} = -0.4240\ \varepsilon + 0.1480\ \beta \qquad \text{strand 3} \qquad (38)$$

The minus signs occur in Eq. 38 since strand 3 is left lay. When strand 1 and 2 are placed together to form the independent wire rope core the follow results:

$$\frac{F}{AE} = 0.7984\ \varepsilon + 0.1799\ \beta$$
$$\frac{M}{ER^3} = 0.3092\ \varepsilon + 0.0840\ \beta \qquad \text{strands 1 and 2} \qquad (39)$$

It is instructive at this point to express the effective modulus of elasticity $E_e$ as follows.

$$E_e = K_1\ E \qquad (40)$$

which is the modulus of elasticity of the rope when $\beta = 0$. Strand 1 has an effective modulus of 0.8864 E and has a helix angle $\alpha_2 = 73.7069°$, while strand 2 has an effective modulus of 0.9642 E with a helix angle of $\alpha_4 = 81.0664°$. When strand 1 and 2 are placed together to form a rope the effective modulus drops to 0.7984 E. When strands 1, 2 and 3 are placed together to form the Seale rope the effective modulus becomes 0.704 E.

If $\beta = 0$ and $\varepsilon = 0.0015 = \xi_1$, the values of the wire strands are

$\xi_1 = 0.0015$, $\xi_2 = 0.001349$, $\xi_3 = 0.001297$, $\xi_4 = 0.001240$,

$\xi_5 = 0.001286$, $\xi_6 = 0.001196$ and $\xi_7 = 0.00106$

which is an indication of how the wires strains vary in the Seale rope. The total axial force on the Seale rope assuming modules of elasticity of $30 \times 10^6$ with $\beta = 0$ and $\varepsilon = 0.0015$ is 23,040 lb. and the total axial twisting moment is 2,603 in. lb. The axial stress in the center wire under this loading is 45,000 psi. Again the results are linear. The axial rope loads, acting on strands 1, 2 and 3, are $F_1 = 754$ lb., $F_2 = 3153$ lb. and $F_3 = 19133$ lb., which means that strands 1, 2 and 3 take about 3.27, 13.68 and 83.04 per cent respectively of the total axial load. Again this is for a rope which is not allowed to rotate.

A load deformation curve was obtained experimentally for a 1.306 in. diameter 6 x 19 Seale IWRC wire rope on a 600,000 lb. testing machine. The plot of the load as a function of the deformation is shown in Fig. 5. A 45 in. gage length was used and the total cross sectional area of the rope $(\Sigma \pi R_i^2)$ is 0.727 $in^2$. The experimental effective modulus of the rope is about 18,400,000 psi. while the preceeding theory predicts a effective modulus, based on E = 29,000,000 psi., of about $0.7 \times 29{,}000{,}000 = 20{,}300{,}000$ psi.

## SUMMARY AND CONCLUSIONS

A theory is developed that will predict the static response of a wire rope with complex cross sections such as a 6 x 19 Seale IWRC. The theory consists of linearizing a previous theory so that the results can be applied, with relative ease, to ropes with complex cross sections. The results show that, if the rope is not allowed to rotate, the maximum tensile stress occurs in the center wire. In a crude sense, the center wire is surrounded by helical springs. The actual stress of course depends upon both the axial load and the axial twisting moment. If the rope, for instance, were subjected only to an axial twisting moment the maximum tensile stress would shift to another wire; presumably the center wire of one of the outer strands.

The theory indicates how the effective modulus of a rope decreases as additional strands are added. In the present case, $E_e = 0.886\ E$ for the center strand, $E_e = 0.798\ E$ for the independent wire rope and $E_e = 0.704\ E$ for a Seale IWRC.

In the case of no rotation, the center wire of the Seale rope experiences the greatest tensile stress. If the rope however is bent over a sheare, the maximum tensile stress, depending upon the radius of curvature of the sheave, may shift to the center wire in one of the outer strands. This is because of the larger wire diameter of the center wire in the outer strand as compared with the center wire in the rope.

The theory predicts an effective modulus larger than the experimentally determined value (20,300,000 psi. as compared to 18,400,000 psi.). The theoretically determined modulus is usually higher than the experimentally determined one since contact deformation between the wires is neglected. Also the theory assumes that plane sections in the rope remain plane which is not the case if the ends of the ropes are held in zinc sockets. This effect, however, would decrease with an increase in the length of the rope tested.

The coefficient $K_1$, $K_2$, $K_3$ and $K_4$ were determined for a 1.306 in. diameter Seale IWRC. These values are however felt to be valid for any diameter Seale IWRC since they are dimensionless and if the diameter of the rope were, for example reduced, a similar reduction would occur in all the wire diameters and the various pitches of the wires in the rope.

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APPENDIX 1 - REFERENCES


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## APPENDIX II - NOTATION

The following symbols are used in this paper:

$A$ = metalic cross sectional area of rope;

$A_2^*$ = bending stiffness of initially curved strand 2;

$E$ = modulus of elasticity;

$E_e$ = effective modulus of elasticity;

$F$ = resultant axial force acting on strand;

$G'$ = bending moment in outer wire;

$G'^*$ = bending moment in strand 2;

$H$ = axial twisting moment in outer wire;

$h$, $\bar{h}$ = initial and final length, respectively, of strand;

$K_1$, $K_2$, $K_3$, $K_4$ = rope constants;

$M$ = axial twisting moment acting on strand;

$m_2$ = number of outer wires

$N'$ = shearing force in outer wire;

$p_2$ = pitch of outer wires;

$R$ = radius of rope

$R_1$ = radius of center wire;

$R_2$ = wire radius of outer wires;

$r_2$, $\bar{r}_2$ = initial and final helix radius, respectively, of outer wires;

$T$ = axial force in outer wire;

$X$ = contact force per unit length in outer wire;

$\alpha_2$, $\bar{\alpha}_2$ = initial and final helix angle, respectively, of outer wires;

$\alpha_2^*$, $\bar{\alpha}_2^*$ = initial and final helix angle, repsectively, of strand 2;

$\beta$ = rotational rope strain;

$\beta_2$ = rotational strain of outer wires;

$\beta_2^*$ = rotational strain of strand 2;

$\Delta\alpha_2$ = change in helix angle of outer wires;

$\Delta K$ = change in curvature of outer wires;

$\varepsilon$ = axial rope strain;

$\theta_2$, $\overline{\theta}_2$ = initial and final angles, respectively, that the outer wires sweep out in a plane perpendicular to the axis of the strand;

$\nu$ = Poission's ratio

$\tau$ = angle of twist per unit length of rope;

$\tau_2$ = angle of twist per unit length of outer wires;

$\xi_1$ = strain in center wire

$\xi_2$ = axial strain in outer wire

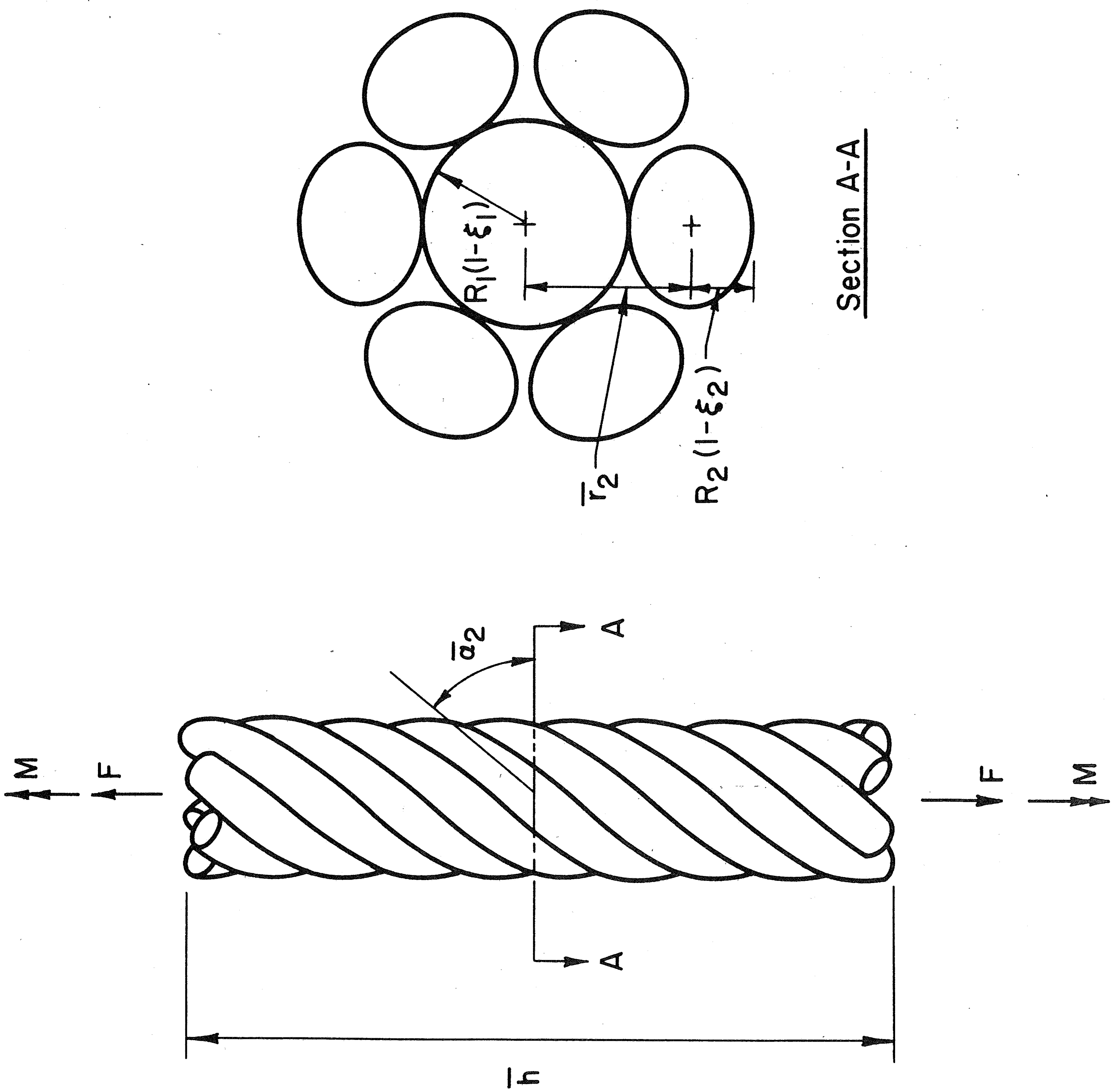


FIGURE 1

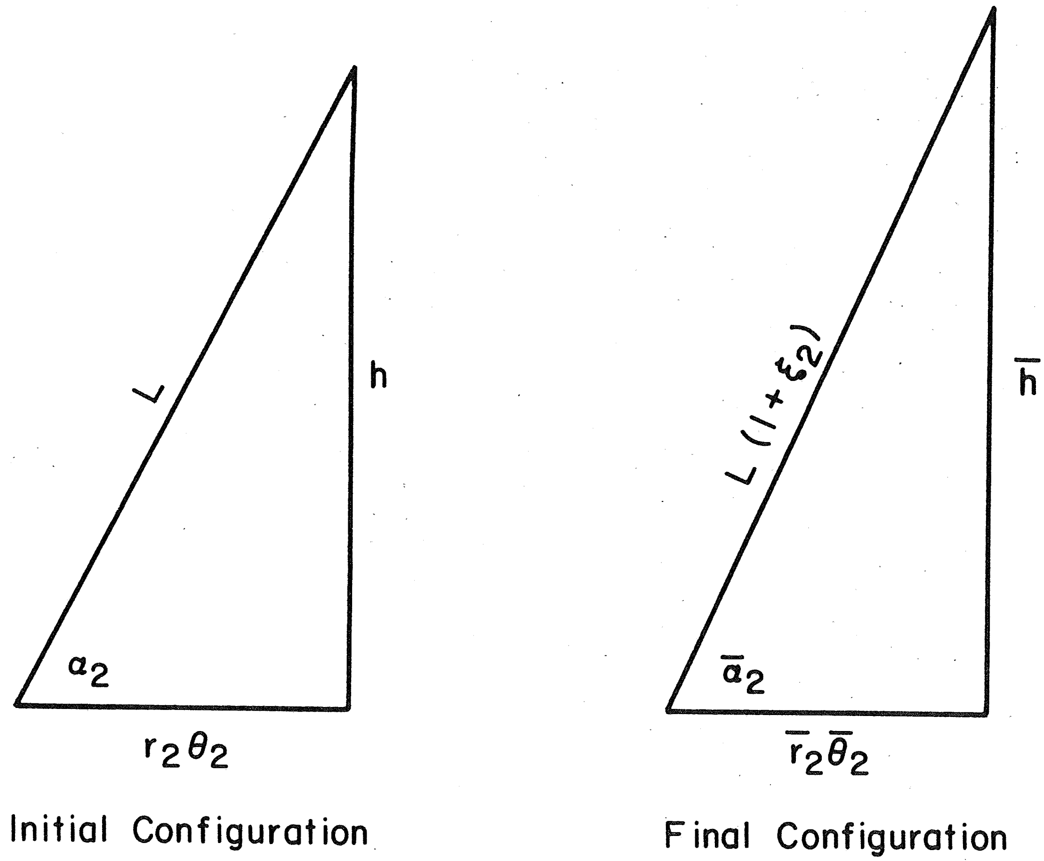
L
h
$\alpha_2$
$r_2\theta_2$
Initial Configuration
$L(1+\varepsilon_2)$
$\bar{h}$
$\bar{\alpha}_2$
$\bar{r}_2\bar{\theta}_2$
Final Configuration

FIGURE 2

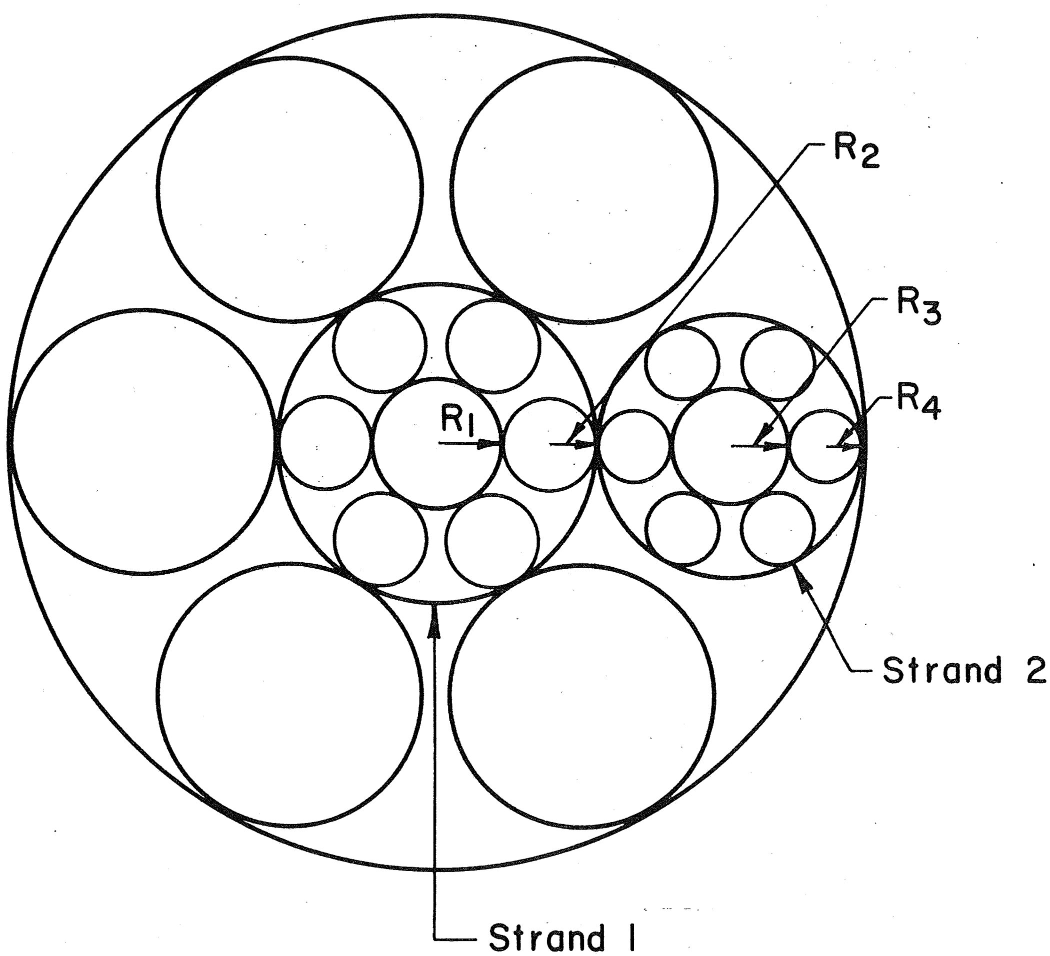


FIGURE 3

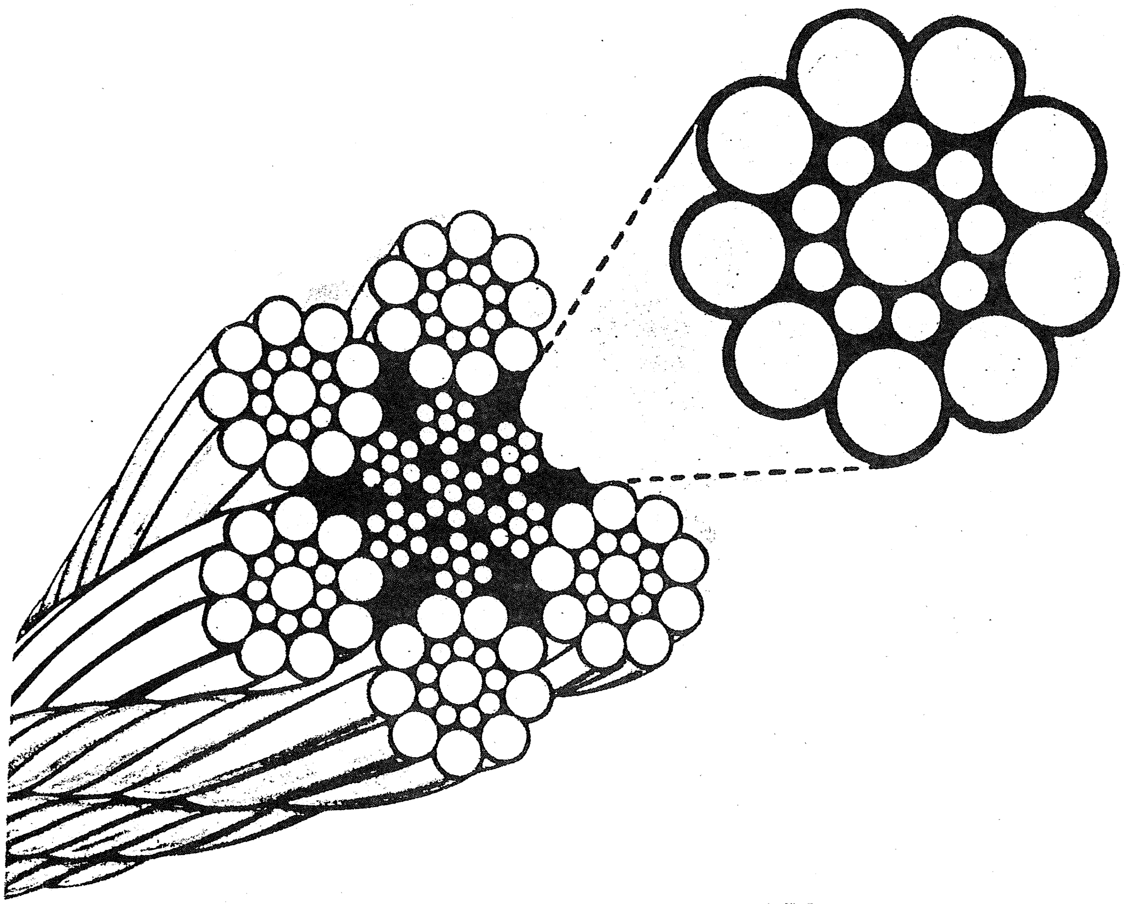

FIGURE 4

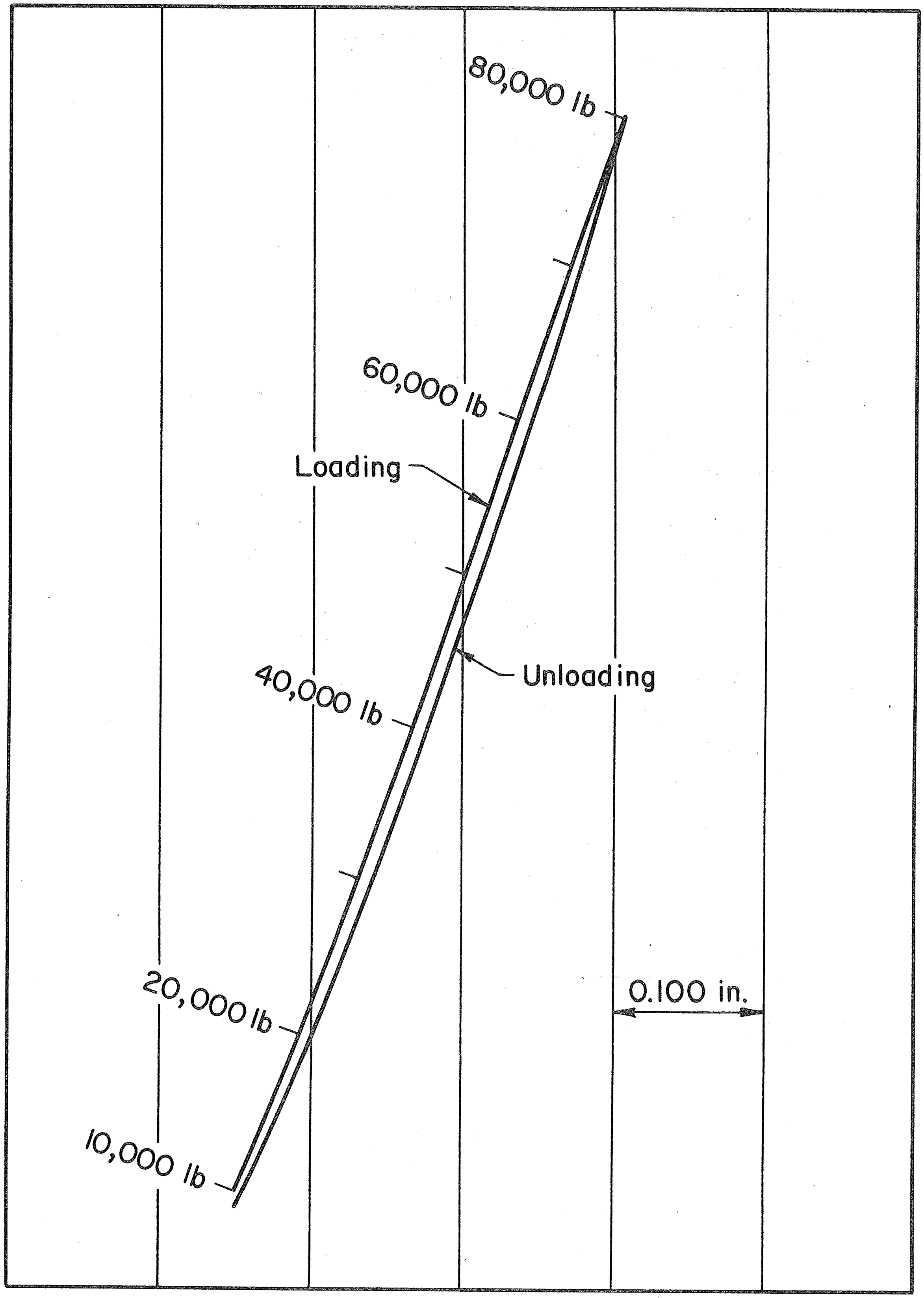
80,000 lb
60,000 lb
Loading
40,000 lb
Unloading
20,000 lb
10,000 lb
0.100 in.

FIGURE 5