FINAL REPORT

on a research project entitled

INVESTIGATION OF REINFORCED CONCRETE

COLUMNS SUBJECT TO HIGH SUSTAINED LOADS

SUSTAINED LOAD STRENGTH OF ECCENTRICALLY LOADED SHORT REINFORCED CONCRETE COLUMNS

I. M. VIEST

R. C. ELSTNER

E. HOGNESTAD

DEPARTMENT OF THEORETICAL AND APPLIED MECHANICS
UNIVERSITY OF ILLINOIS

April 1955

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SUSTAINED LOAD STRENGTH OF ECCENTRICALLY LOADED SHORT REINFORCED CONCRETE COLUMNS

by

I. M. Viest,* R. C. Elstner,+ and E. Hognestad+

SYNOPSIS

An experimental investigation is presented, the principal object of which was to determine which portion of the ultimate strength under fast (short-time) loading an eccentrically loaded reinforced concrete column can sustain indefinitely. Forty-five column tests are reported; 13 tests were made with fast loading, 12 with slow loading, and 19 with sustained loading. In addition to the type of loading, concrete strength and eccentricity of load were the major variables studied.

The test findings indicate that the ultimate strength under sustained loading is only about 10 percent below that for fast loading. It seems satisfactory, therefore, to base safety in ultimate strength design on the ultimate strength equations for fast static loading, which equations are substantiated by numerous previous tests.

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*Member, American Concrete Institute, Research Assistant Professor of Theoretical and Applied Mechanics, University of Illinois, Urbana.

+Members, American Concrete Institute, Development Engineer and Manager Structural Development Section, respectively, Portland Cement Association, Chicago. (Formerly Research Associate and Research Associate Professor, respectively, Department of Theoretical and Applied Mechanics, University of Illinois, Urbana.)

INTRODUCTION

Object of Investigation

In recent years, several theoretical and experimental studies have provided broad and detailed information regarding the short-time static strength of eccentrically loaded reinforced concrete columns. An extensive experimental investigation of such columns was published by the University of Illinois in 1951.[1] As in most other studies involving eccentric loading, the experimental data obtained in this Illinois work were limited to short-time tests, in which column specimens were loaded to failure in about one hour.

A later extension of these 1951 studies was devoted to pilot tests on effects of time on column strength. As reported in the Appendix of this paper, the results indicate that sustained moderate overloads have no significant effect on the ultimate column strength. This confirms previous findings in tests of knee frames,[2] in which a 17-month period under sustained working loads had little effect on ultimate strength. Similarly, it was found in previous tests of concentrically loaded columns[3] that a period of $3\frac{1}{2}$ years under sustained working loads had no significant effect upon ultimate strength. It is clear, therefore, that ultimate strength theories based on short-time tests are applicable to columns loaded to failure after a period of sustained loading somewhat over the working load level.

The investigation reported herein represents a further step in studies of effects of time on the ultimate strength of eccentrically loaded reinforced concrete columns, namely, determination of the <u>maximum load</u> that such columns can <u>sustain</u> <u>indefinitely</u>. Tests were made on 44 rectangular tied columns which were divided into two test series according to the eccentricity of load. One series included 24 columns with moderate eccentricities, and the second series included 20 columns with small eccentricities. Within each series, the principal variables were concrete strength and type of loading. Thirteen columns were tested with fast loading, twelve with slow loading, and nineteen with sustained loading.

Notation

The letter symbols used throughout the paper are defined as follows:

a = length of column capital

A_c = total area of concrete

A_s = area of tension reinforcement

A_s' = area of compression reinforcement

b = width of column

c = distance from neutral axis to compression edge

d = distance from centroid of tension reinforcement to compression edge

d' = distance from centroid of tension to centroid of compression reinforcement

e = eccentricity of load with respect to mid-depth of section

e' = eccentricity of load with respect to centroid of tension reinforcement

E_s = modulus of elasticity of reinforcing steel

f_c' = compressive strength of 6 by 12-in. cylinders

f_s = stress in tension reinforcement

f_y = yield point of reinforcement

k_1 = coefficient defining the magnitude of the internal compressive force in concrete as defined by Fig. 7

k_2 = coefficient defining the position of the internal compressive force in concrete as defined by Fig. 7

k_3 = ratio of flexural compressive strength, f''_c, to cylinder strength, f'_c

ℓ = length of prismatic column shaft

P = load

p = ratio A_s/bd

t = total depth of section

y'_c = lateral deflection of prismatic shaft at mid-height

y_c = lateral deflection at mid-height of column

y''_c = lateral deflection caused by rotation of column capitals

ϵ_o = compressive strain in concrete corresponding to maximum stress

ϵ_u = ultimate concrete strain in flexure

Each column is designated by two numbers and two letters, e.g., 20B2a. The first number expresses the design concrete strength in hundreds of psi; the capital letter refers to the desired mode of failure in a short-time test+ (B = balanced failure, C = compression failure); the second number refers to the type of loading (1-fast, 2-slow, 3 and 4 sustained load); and the lower case letter is used to distinguish between companion specimens.

SPECIMENS AND TESTS

Details of Columns

Details of the test columns are shown in Fig. 1. All were of the same size and were reinforced in an identical manner; they had a

+ A test to failure in a conventional testing machine carried out in about one hour; the corresponding loading will be referred to herein as "fast loading."

prismatic shaft 20 in. long and 5 by 5 in. in cross-section, and a 10-in. capital was provided at each end to accommodate the loads. The reinforcement was made up of two No. 4 bars for tension, two No. 4 bars for compression, and seven 3/16-in. ties spaced at 4 in. The concrete cover of the longitudinal reinforcement was 1/2 in. Sufficient additional capital reinforcement was provided to prevent failure in the capitals.

The eccentricities of load varied to some extent. Using the equations of the 1951 investigation,[1] they were selected on the basis of the desired mode of failure in tests with fast loading. Columns with small eccentricities were designed to fail by compression failures, and columns with moderate eccentricities by balanced failures.

The relatively small size of specimen was chosen to facilitate building of an assembly of ten special sustained loading frames. Because of their size, the specimens were treated as scale models both in selection of the maximum size of aggregates and in tolerances.

Materials

All specimens were made with Type I portland cement purchased in one lot from a local dealer. The aggregates were Wabash River sand and pea gravel. The sieve analyses of the aggregates are given in Table 1. The fineness modulus of the sand was 3.04, and the maximum size of the gravel was 1/2 in.

Three grades of concrete with design strengths of 2000, 3500, and 5000 psi were used. Concrete strength was purposely varied somewhat between companion groups of four specimens each. Average properties of the mixes used are given in Table 2. The slump of all mixes was 1 to 2 in. The strengths of control cylinders are given for each individual column specimen in Table 3.

The longitudinal reinforcement was No. 4 intermediate grade billet steel meeting ASTM Designation A 305-50T for deformations. The steel was purchased in one lot, and mechanical properties were determined by tests of four tensile coupons selected at random. The average properties based on the nominal bar area of 0.20 in. were as follows:

Yield point	43,300 psi
Ultimate strength	71,500 psi
Modulus of elasticity	27.2 million psi

The yield points for individual coupons differed from the average by less than 2 percent.

Fabrication and Curing

All columns were cast in groups of four in four identical steel forms. The casting was done in a horizontal position with the tension side of the column upward. The reinforcing steel was tied into rigid cages and placed in the forms. Then, the concrete required for all four columns and the corresponding control cylinders was mixed in one batch. After a three-minute mixing period, the concrete was placed in a large pan and remixed by shovel. The concrete was then divided into four parts, each for casting one column and three 6 by 12-in. control cylinders. Internal vibrators were used to place the concrete.

The columns and control cylinders were removed from the forms and placed in a moist room after one day, cured in the moist room for six days, and were then stored in the laboratory until tested.

Testing Equipment and Methods

Load was applied to all columns through two 1/2-in. rollers, each placed in a triangular groove in a steel plate seated on the top and bottom of the column with plaster of paris. Tests of the four columns of one group were carried out in a fixed order.

(a) The first column was tested in a 300,000-lb capacity screw-type testing machine. The load was applied in several increments to failure. All measurements were taken at the beginning of the test and after each load increment. Testing of one column took about one hour. This type of loading is referred to herein as fast loading; it is very similar to that used in the 1951 Illinois test series.(1)

(b) The second column was initially loaded to 75 percent of the ultimate load for the first column; this load was applied in three or more increments in the special loading frames shown in Fig. 2. Then the load was increased approximately five percent every 24 hours until failure occurred. All readings were taken before loading and after each increment of load. This type of loading is called slow loading, and was carried out primarily as a guide for selecting the magnitude of the sustained loads.

(c) The third and fourth columns were loaded as shown in Fig. 2 to between 80 and 95 percent of the ultimate load for fast loading. The selected sustained load was usually below the ultimate load for slow loading. The load was first applied in three or more increments and was then maintained on the column until failure took place, or until it became apparent that the column could sustain the load indefinitely. Measurements were taken before loading, after each load increment, and at varying time intervals during the period of sustained loading. The sustained load was adjusted by tightening the loading springs whenever it deviated from the original load level by more than five percent.

(d) The columns which did not fail under sustained loading were removed from the special frame and retested to failure as described for fast loading.

Control cylinders were tested in a 300,000-lb hydraulic testing machine on the day when the corresponding column failed.

Measurements

The measurements made included concrete strains on the compression face of the columns, steel strains on the outside surface of the tension steel, and deflections on the tension face of the columns.

Steel strains were measured on two gage lines and concrete strains on three gage lines with a 6-in. Berry mechanical gage. The locations of the gage lines are shown on Fig. 1. Access to the tension reinforcement was provided through cored holes. For the concrete strain measurements, short cast-in steel plugs were provided.

Deflections were measured by three 0.001-in. dial indicators mounted on a deflection bridge. Details of the deflection bridge and the locations of dials are shown in Fig. 1. The bridge was supported by a pin on the top of the column and by a knife-edge on the tension face above the bottom edge of the column. Because of this arrangement of supports, the deflection bridge did not remain entirely parallel to the load line. All deflection data presented in this report therefore include a small correction for the movement of the bridge.

Records were kept of temperature and relative humidity, which fluctuated between 65 and 84 deg. F, and between 36 and 91 per cent, respectively. The seasonal changes in temperature and humidity had little effect on the column test data.

TEST RESULTS

The principal test data are summarized in Tables 3 and 4. Ultimate load data are given in Table 3, including the maximum loads the columns were able to resist and the corresponding deformations, the strengths of control cylinders, the initial eccentricities measured as the distance from the tension steel to the load, the age of columns at the beginning and conclusion of testing, and the type of loading applied to any particular column. The sustained load test data are given in Table 4; they include the magnitude of the sustained loads, deformations at various stages of the test, and the duration of the test.

Deformations at the ultimate load were usually obtained by extrapolation from measurements taken at loads slightly below the ultimate. All other data included in the two tables were obtained by direct measurements. Steel strains are given as averages of readings on two gage lines located on the outside face of the tension steel. Concrete strains are averages of readings on three gage lines located on the compression surface of the columns. The deflections are given as the total lateral deflections at column mid-height with respect to the line of load.

Except for small variations of concrete strength, the type of loading was the only variable within each group of four columns, such as 20B1a, 20B2a, 20B3a, 20B4a (Table 3). As a rule, column 1 was subjected to fast loading (FA) to failure, column 2 to slow loading (SL) to failure, and columns 3 and 4 to sustained loading (SU) to failure, or, if a column did not fail under sustained loading, to sustained loading followed by fast loading (SU + FA) to failure. Exceptions to this procedure are discussed below.

Column 20B3b, intended for a sustained load test, failed during the initial load application. Column 20C4b was loaded to 31,600 lb; it was intended to maintain this load on the column as a sustained load. Since the column appeared to be in danger of failure, however, the load was allowed to drop and was maintained as a sustained load at 29,000 lb.

Column 35C1a was tested with fast loading at an eccentricity of 3 in. The next column of this group, 35C2a, was initially subjected to slow loading at the same eccentricity. However, at a load of approximately 50,000 lb, it became evident that the sustained load equipment would not permit a sustained test at this load. Consequently, the load was removed and the column was retested with fast loading at an eccentricity of 3.5 in. The remaining columns, 35C3a and 35C4a, were then loaded with this 3.5-in. eccentricity. Column 35C3a was subjected to slow loading, and column 35C4a, which was intended for a sustained load test, failed during the initial load application.

Columns 20C4c and 35C3b were subjected to sustained loading. After it became evident that the columns would support the load indefinitely, the load was raised as shown in Table 4. The increased loads were kept on both columns for 219 days, which period was sufficient time for the deformations to level out under the increased loads.

Behavior under Load

The behavior of the test columns under load is illustrated by typical load-deformation and time-deformation curves in Figs. 3 and 4. The behavior of columns with moderate and with small eccentricities was similar, except for the magnitude of the tension steel strains at failure. In all columns with moderate eccentricities the tension steel yielded before failure, but in several columns with small eccentricities the tension steel stresses remained in the elastic range throughout the test.

It is illustrated by Fig. 3 that deformations were approximately the same for all four columns of one group until the period of sustained loading began. From then on, the deformations were smallest for the column subjected to fast loading and largest for the columns subjected to sustained loading. In all tests the deflections were approximately symmetrical about mid-height (Fig. 3b).

The ultimate loads were, as a rule, highest for columns subjected to fast loading and smallest for columns which failed under sustained load. The load for all columns that failed under sustained loading was as an average 89 percent of the corresponding ultimate loads for fast loading. The slow loading produced failures at loads averaging 93.5 percent of fast loading, and the fast loading after a period of sustained loading gave failure loads averaging 94.6 percent of fast loading.

The location of the failure zone varied from column to column. Crushing of concrete occurred at random above, below, or at mid-height of the columns.

Fast loading - The yield point strain of the reinforcing bars was 0.00159. It can be seen from Table 3 that under fast loading the tension steel strains in columns with moderate eccentricities exceeded the yield point value only by a small margin. Therefore, the specimens failed by nearly balanced failures. The columns with small eccentricities, however, failed by compression failures while the tension steel strains were still in the elastic range. Thus, the modes of failure of all columns subjected to fast loading were as planned.

In general, the behavior of columns subjected to fast loading was the same as that observed in the 1951 Illinois series.[1] The principal difference was in the distribution of deflections and in the location of the failure zone. The 1951 columns deflected more above than below mid-height, and failure consistently took place in the upper half, as contrasted by the symmetrical deflection pattern and random location of the zone of failure in the present investigation. The difference is caused by the casting procedures used. The 1951 columns were cast in a vertical position; the columns of this investigation, in a horizontal position. Columns cast vertically are weaker near the top as a result of water gain, whereas columns cast horizontally are equally strong along their full height.

Sustained loading - Nineteen columns were subjected to sustained loading. Typical effects of time on strains and deflections are illustrated in Fig. 4. For columns which did not fail under sustained loading, deformations increased rapidly for only about 50 days, and at 500 days the deformations had leveled out (Column 50B3a). On the other hand, for columns which failed under sustained loading, deformation increased rapidly to failure (35B3a). Typical effects of increasing the magnitude of the sustained load after a long period of sustained loading are illustrated by the curves for Column 35C3b in Fig. 4.

The steel strains at the end of the sustained load period exceeded the yield point value for all columns except 20C3a, 20C3b, 20C3c, and 20C4c (Table 4). For columns 35C3b and 35C4b, the steel strains exceeded the yield point value in spite of the fact that the strains at first loading were in the elastic range and the corresponding fast loading specimens failed by compression. This illustrates that creep can change the mode of column failure.

Concrete Strain

The compression-face concrete strains at various stages of loading are given in Tables 3 and 4. Effects of concrete strength and of type of loading on the magnitude of ultimate strain are shown in

Fig. 5. An effect of the initial strain at first loading on the magnitude of concrete strains at various stages of sustained loading is shown in Fig. 6.

Effect of concrete strength - It can be seen in Fig. 5 that the concrete strength had no significant effect on the magnitude of ultimate concrete strain for any one type of loading. This is in agreement with previous findings.[1]

Effect of type of loading - The ultimate strain in columns subjected to fast loading (Fig. 5a) varied from 0.0023 to 0.0040 with an average of 0.0032. Since the plotted values represent average strains measured over three 6-in. gage lengths, it is most probable that local ultimate strains in the failure region were somewhat higher. This is in general agreement with the 1951 Illinois tests[1] which gave an average ultimate concrete strain in the failure region of 0.0038.

The ultimate strains for columns failing under sustained loading averaged 0.0061. This is approximately twice the ultimate strains obtained from tests with fast loading. Again, since the plotted values represent average strains, it is reasonable to assume that the ultimate strains in the zone of failure were about twice the value found in the 1951 tests for fast loading.

Ultimate strains for slow loading are not shown in Fig. 5. They ranged between those for fast and sustained loading.

The ultimate strains for columns subjected to fast loading to failure after a period of sustained loading were comparable to those of columns failing under sustained load. However, if the creep strains are subtracted from the ultimate strains, the resulting values (triangles in Fig. 5b) are in good agreement with ultimate strains for fast loading.

Thus, the increment of concrete strains caused by fast loading from zero load to failure is roughly independent of the previous history of loading.

Effect of initial strains - Quantitative studies of the effects of creep on concrete strains revealed a correlation between the concrete strains at first loading and the corresponding concrete strains at later ages. This correlation is illustrated in Fig. 6a for strains after 10 days of sustained loading and in Fig. 6b for strains measured at the conclusion of the sustained load period. It can be seen from Fig. 6 that columns subjected to high initial strains underwent larger total deformations than the columns subjected to smaller strains at the outset of the sustained load period.

The columns represented by dots and crosses in Fig. 6b could probably sustain the applied loads indefinitely without any appreciable further increase of strain. It may be noted that without exception these columns were first strained to 0.00231 or less. On the other hand, columns which failed under sustained load (circles) were strained at first loading to 0.00231 or more. Only one exception to this rule was observed. Thus it appears possible that initial fast loading beyond a compression-face concrete strain of 0.00231 leads to localized damage to the concrete which spreads and causes the column to fail if the loading is sustained for a sufficiently long period of time.

If the suggested hypothesis is correct, then the load causing an initial strain of 0.0023 represents the sustained load strength of a column. It should be pointed out, however, that to the authors' knowledge the test data shown in Fig. 6b represent the only direct experimental indication of the existence of a critical strain as applied to the sustained load strength of concrete.

ANALYTICAL STUDIES

Ultimate Load Equations

Fast loading - The Illinois 1951 report[1] has shown that the ultimate strength of eccentrically loaded reinforced concrete columns subjected to fast loading may be computed with a satisfactory accuracy on the basis of the following assumptions:

1. The compressive stresses in the concrete are distributed as shown in Fig. 7.
2. No tensile stresses exist in concrete.
3. Bernoulli's hypothesis of linear distribution of strains is valid.
4. No general slip takes place between concrete and reinforcing steel.
5. The stress-strain relation for reinforcing steel is trapezoidal with the flat portion corresponding to the yield point stress. For tension failures both tension and compression steel stresses are equal to the yield point value; for compression failures the compression steel stress is equal to the yield point value and the tension steel stress is in the elastic range.

For tension failures of symmetrically reinforced sections, assumptions 1 to 5 combined with the equations of statical equilibrium result in the following expression for the ultimate load:

$$P = f_c'bd \frac{k_1k_3}{2k_2} \left[1 - \frac{e'}{d} + \sqrt{\left(\frac{e'}{d} - 1\right)^2 + 4 \frac{k_2}{k_1k_3} \frac{pf_y}{f_c'} \frac{d'}{d}} \right] \quad (1)$$

For compression failures, the following two equations of equilibrium and one equation of compatibility of strains may be written on the basis of assumptions 1 to 5:

$$P\,e' = k_1 k_3 f'_c\, bc(d - k_2 c) + A'_s\, d'\, f_y \tag{2a}$$

$$P = k_1 k_3 f'_c\, bc + A'_s\, f_y - A_s f_s \tag{2b}$$

$$f_s = \varepsilon_u E_s \frac{d - c}{c} \leqq f_y \tag{2c}$$

The ultimate load P for columns failing in compression may be evaluated by a simultaneous solution of Eq. 2a to c for P, tension steel stress f_s, and depth of neutral axis c.

All symbols used in Eq. 1 and 2 are defined in the section on notations. The symbols for the column dimensions and for most of the properties of materials are those in common use. The eccentricity of the load, e', is the maximum eccentricity at failure, including both the initial eccentricity and the deflection of the column at failure. It was found in the 1951 Illinois tests[1] that the ultimate load for fast loading is reached when the maximum strain in concrete ε_u is about 0.0038. Since the test data from this investigation were essentially in agreement with this finding, this value was adopted herein. Parameters k_1, k_2, and k_3 characterize the stress-strain relationship for concrete. With the aid of Fig. 7, the following equation may be written for parameters k_1 and k_2 when $\varepsilon_u = 0.0038$:

$$k_1 = \frac{3620 + 0.63\, k_3 f'_c}{3910 + k_3 f'_c} \tag{3a}$$

$$k_2 \cong 0.55\, k_1 \tag{3b}$$

Thus k_1 and k_2 may be evaluated if k_3 and f'_c are known.

The parameter k_3 designates the ratio of the flexural compressive strength of concrete in the column to the corresponding cylinder strength. In the 1951 report(1) this ratio was assumed equal to 0.85. Correlation of test data with Eq. 1 and 2 indicates, however, that $k_3 = 1.0$ gives better agreement with the tests reported herein than $k_3 = 0.85$. This difference between the results of the two investigations may be expected as a result of the differences in casting procedures mentioned before. The concrete in the failure zone of the vertically-cast 1951 columns was weaker than the concrete in the corresponding cylinders. There is, however, no reason to expect a similar effect of water gain in the horizontally cast columns of this investigation. Therefore $k_3 = 1.0$ was adopted for this study.

Sustained loading - A procedure for predicting the maximum load that an eccentrically loaded column can sustain indefinitely may be based either on the conditions at failure under sustained loading or on the initial strains at first loading.

In the discussion of the test data it was indicated that the load on columns which failed under sustained loading produced initial concrete strains of 0.00231 or more at first loading. Accordingly, the critical strain $\varepsilon_{cr} = 0.0023$ could also be adopted as a criterion for evaluating the ultimate sustained load. However, the experimental evidence regarding the existence of an initial critical strain is limited to only six failures under sustained loading of this investigation. It was decided, therefore, to base the analytical studies of this report on the ultimate rather than on the critical strain.

It is well known that the compressive strains in concrete increase as a result of creep. In previous tests with sustained overloads on concentrically loaded columns[4] and cylinders,[5] compressive strains substantially in excess of 0.0038 were observed before failure. Similar findings are reported in this paper. It has been shown in the section on ultimate strains that, at failure of eccentrically loaded columns under sustained loading, the maximum concrete strains are roughly twice as large as those at failure under fast loading. It can be shown that the ultimate load is not very sensitive to the magnitude of the ultimate strain, and a value of $\varepsilon_u = 0.0076$ may therefore be adopted as an approximate criterion for the ultimate sustained load.

The ultimate column strength under sustained loading may then be computed on the basis of assumptions similar to those made for fast loading.

1. The distribution of compressive stresses in the concrete is undoubtedly affected by creep to some extent. Since information regarding such quantitative effects of creep is very limited, it is assumed for the purpose of computing the ultimate load that the shape of the concrete stress-block is not affected by creep. The parameters k_1 and k_2 may then be computed from Eq. 3 for sustained as well as for fast loading.

2. Since the difference between the ultimate loads for sustained and fast loading is relatively small, the assumption of the absence of tension stresses in concrete should be applicable also to sustained loading.

3. No test data are available on the effect of high overloads on the strain distribution across a column. Studies of the test data from this investigation have shown, however, that ultimate sustained loads and the corresponding steel stresses computed with the aid of Bernoulli's hypothesis agree reasonably well with the test data.

4. The assumption of no general slip seems to be applicable equally well to sustained loading as to fast loading.

5. The stress-strain relationship for steel is not affected by time.

Since the assumptions 1 through 5 are similar to those for fast loading, the ultimate sustained load may be computed from Eq. 1 and 2. The principal difference between the two types of loading lies in the values of the ultimate strain ε_u and the value of the parameter k_3. In accord with the preceding paragraphs, an ultimate strain $\varepsilon_u = 0.0076$ was chosen for sustained loading. The ratio k_2/k_1 is given by Eq. 3b as approximately 0.55, so that the value of k_3 may be computed from Eq. 1 for any column which failed by a tension failure. An average value of $k_3 = 0.9$ was found in this manner from the test data.

For fast loading, $k_3 = 1.0$ was found to give good agreement with the test data. Since both values of k_3 are based on the conventional cylinder strength f'_c, the flexural compressive strength for sustained loading may have been equal to about 0.9 times the value for fast loading. A similar conclusion for cylinders was reached by Shank, who reported that the ultimate sustained load strength of concrete cylinders is equal to $0.9\ f'_c$.(5) The mode of failure can be different for sustained than for fast loading, since the stress in the tension steel is a direct function of ε_u (see Eq. 2c), and ε_u is greater for sustained loading than for fast loading. Therefore, columns failing in compression under fast loading can in some cases fail in tension under sustained loading.

<u>Fast loading after sustained loading</u> - The ultimate load of a column subjected to sustained loading for a period of time and subsequently

loaded to failure with fast loading may also be computed from Eq. 1, 2, and 3, if the ultimate strain ε_u and the parameter k_3 are properly chosen.

It has been shown in the discussion of the test data that the concrete strain caused by fast loading is roughly independent of the previous history of loading. Thus the ultimate strain ε_u for columns subjected to fast loading after a period of sustained loading may be computed as the sum of the creep strain and 0.0038. For the purposes of this study, the creep strain actually observed in tests was used in evaluating the ultimate strain ε_u.

A value of $k_3 = 1.0$ was found to give good computed values of the ultimate load, thus indicating that the flexural compressive strength for fast loading is not affected by preceding creep deformations.

Slow loading - The magnitude of the ultimate load for a column subjected to slow loading depends on the length of the time intervals between load increments. Its upper and lower limits are given by fast and sustained loading.

Ultimate Deflections

Equations 1 and 2 contain the eccentricity of load at failure, e', which includes both the original eccentricity and the ultimate deflection. Thus, the deflection at failure is needed for computing the ultimate load of an eccentrically loaded column.

For a prismatic column loaded with equal eccentricities at both ends, the relationship between the deflection at mid-height and the ultimate strain ε_u may be expressed approximately as follows:

$$y_c' = \frac{\varepsilon_u}{c}\,\frac{\ell^2}{8} \qquad (4a)$$

where ℓ is the free length of the column and c the depth of the neutral axis.

The columns of this investigation had a prismatic shaft with capitals at both ends. Thus, in addition to the deflection of the shaft (Eq. 4a), the total column deflection includes the contribution of the capitals, which may be approximated by the following expression:

$$y_c'' = \frac{1}{4} \frac{\varepsilon_u}{c} \ell a + \frac{\varepsilon_u}{0.0038} 0.05 \qquad (4b)$$

where a is the length of the column capital. The first term in Eq. 4b was derived assuming infinite rigidity of the capitals. The second term is empirical and accounts for cracking of the capitals. The total deflection of the column at mid-height may be computed by adding Eq. 4a and 4b. Since ℓ = 20 in. and a = 10 in., the ultimate mid-height deflections of the columns of this investigation with respect to the line of load may be computed from:

$$y_c = \frac{\varepsilon_u}{0.0038} \left(\frac{0.38}{c} + 0.05 \right). \quad \text{(c in inches)} \qquad (4c)$$

Comparison between Computed Values and Test Data

Ultimate loads, deflections, and steel stress were computed from Eq. 1 to 4 for all columns subjected to fast loading, to sustained loading, and to fast loading following a period of sustained loading. These computed values are listed in Table 5, together with the corresponding test data and the ratios of measured to calculated values.

The arithmetic average of the ratios of measured to computed ultimate loads for all columns listed in Table 5 is 1.008 and the standard deviation is 0.075. The correlation is equally good for all three types of loading.

Two quantities are listed in Table 5 for deflections at mid-height: the deflections of the prismatic shaft and those of the column. For the prismatic shaft, test values are given only for those columns for which complete measurements were taken at the instant of failure, since errors in extrapolation were judged prohibitively large. The agreement between the computed and test values for deflections is not as good as for ultimate loads, but the computed values are sufficiently accurate for calculating the eccentricity of load at failure.

The stresses in the tension steel at failure are given in the last column. The test values were computed from average bar strains obtained by correcting the strains measured on the surface of the bar; the average strains were then multiplied by the modulus of elasticity determined from tests of coupons. The calculated stresses are in reasonably good agreement with the test values.

Application in Ultimate Strength Design

The principal object of this investigation was to determine which portion of the ultimate strength under fast loading an eccentrically loaded column can sustain indefinitely. This is of considerable fundamental importance in ultimate strength design. If the ultimate strength under sustained loading were considerably lower than the strength under fast loading, then it would not be entirely satisfactory to base safety on fast-load strength. Most of the thousands of tests to failure available for reinforced concrete members were, of course, made with fast loading.

Since the yield strength of mild steel reinforcement is negligibly affected by time, there is little reason for concern regarding the sustained-load strength of columns with large eccentricities. In this case, ultimate strength is controlled by yielding of the reinforcement, and changes in the internal moment arm resulting from creep even at high loads is unlikely to cause any decrease of importance in the ultimate load. The columns discussed herein were therefore designed for balanced and compression failures only.

Test data given in Table 4 for six columns that failed under sustained loading indicate a sustained-load strength equal to an average of 89 percent of the strength of companion columns tested under fast loading. Similarly, 13 columns sustained without failure loads equal to an average of 88 percent of the fast-load strength of companion columns. For these latter 13 columns, the ultimate strength as compared to fast loading was little affected when they were loaded fast to failure after the period of sustained loading. This extends the previous findings[2,3] that sustained working loads do not appreciably affect ultimate strength.

The significance of sustained loading to ultimate strength design may also be studied in terms of ultimate strength equations suggested for practical application, such as those presented by C. S. Whitney:[6]

For compression failures

$$P = \frac{2A'_s f_y}{\frac{2e}{d'} + 1} + \frac{bt f'_c}{\frac{3te}{d^2} + 1.18} \qquad (5)$$

For tension failures with symmetrical reinforcement

$$P = 0.85\, tb\, f'_c \left[\sqrt{\left(\frac{e}{t} - 0.5\right) + \frac{d'}{t}\,\frac{2A'_s}{bt}\,\frac{f_y}{0.85\, f'_c}} - \left(\frac{e}{t} - 0.5\right) \right] \qquad (6)$$

which is Eq. 1 with $k_2/k_1 = 1/2$ and $k_3 = 0.85$. It is very important to note that Eq. 5 and 6 were derived from vertically cast columns. Thus, the small horizontally cast columns of this investigation should, for fast loading, be somewhat stronger than predicted by these equations.

Table 6 gives sustained loads, measured ultimate loads, and ultimate loads computed by Whitney's Eqs. 5 and 6. The initially applied eccentricity plus the measured mid-height deflection at failure were used as e in the computations. The table shows that the average ratio of sustained load to the Whitney ultimate load was 0.99 for the six columns that failed under sustained loading, and 0.98 for the 13 columns that carried the sustained loads without failure. The average ratio of measured ultimate strength to the Whitney ultimate was 1.07 for the 13 fast-load columns, and 1.04 for all 44 columns tested.

Fig. 8 gives the ratio of sustained load to the Whitney ultimate load as a function of the duration of sustained loading. It should be noted that the open circles represent sustained loadings that were maintained <u>without</u> failure.

The test data reported herein indicate, therefore, that an eccentrically loaded column can usually be expected to sustain indefinitely about 90 percent of its fast-load ultimate strength.

In ultimate strength design of columns an overload factor of about two may be expected to be used. If a high sustained overload should occur, this overload factor would reduce to about 1.8 so that ample safety would still be provided. Furthermore, in structures where high sustained overloads may be of future concern, this should be provided for simply by choosing a sufficiently high design loading, not by infringing knowingly upon the margin of safety.

By this reasoning, it seems entirely satisfactory to base ultimate strength design on fast-load ultimate strength equations substantiated by numerous tests to failure.

SUMMARY

Forty-four eccentrically loaded reinforced concrete columns were tested to determine how large a percentage of the ultimate loads obtained with fast loading can be sustained indefinitely. In order to determine this ratio between the strength for sustained and for fast loading, 13 columns were subjected to fast loading, 12 columns to slow loading, and 19 columns to sustained loading. The sustained loads were equal to 82-95 percent of the ultimate load for fast loading. Among the 19 columns subjected to sustained loading 6 failed under load, and the behavior of the remaining 13 columns indicated that the sustained load could be carried indefinitely.

The concrete strength, the initial eccentricity, and the manner of loading were the primary variables. The duration of the sustained load test varied from column to column. The failures caused by sustained load occurred in the period of $1\frac{1}{2}$ hours to 151 days. For the columns which did not fail under sustained load, the shortest period of loading was 217 days and the longest period was 933 days.

On the average, the columns of this investigation were able to carry a sustained load of about 90 percent of the ultimate load for fast loading, regardless of concrete strength and eccentricity. The smallest failure load under sustained loading was equal to 82.6 percent, and the highest load sustained without failure was 94.0 percent of the ultimate load of companion specimens tested to failure with fast loading.

The results of tests with fast loading were in good agreement with earlier findings. The results of tests with sustained loading led to an extension of the Illinois 1951 analysis for eccentrically loaded columns.(1) This extension permits prediction of the ultimate sustained load of an eccentrically loaded reinforced concrete column and the ultimate load for columns subjected to fast loading after a period of sustained loading.

By comparison with C. S. Whitney's practical ultimate strength equations, it seems satisfactory to base safety in ultimate strength design on the ultimate strength equations of fast (short-time) loading, which loading has been used in most tests of reinforced concrete members.

ACKNOWLEDGMENTS

This investigation was carried out in the Department of Theoretical and Applied Mechanics of the University of Illinois under the auspices of the Engineering Foundation through the Reinforced Concrete Research Council.

A Task Committee, consisting of Messrs. D. McHenry, Chairman, N. M. Newmark, and E. J. Ruble, was appointed by the Council to provide close supervision of the investigation. The contributions of the Task Committee and of other members of the Council are gratefully acknowledged.

Part of the laboratory work was carried out by Mr. Bengt Broms, Research Assistant in Theoretical and Applied Mechanics, whose able assistance is gratefully acknowledged.

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Table 1--SIEVE ANALYSES

Aggregate	Percentage Retained on Sieve No.								
	1/2 in.	3/8 in.	4	8	16	30	50	100	F.M.
Sand	-	0	3.6	13.7	33.1	63.9	91.1	98.2	3.04
Gravel	0	35.7	94.8	99.8	-	-	-	-	-

Table 2--CONCRETE MIXES

Design f'_c psi	Mix Ratio* (by wt.)	w/c (by wt.)	Average 28-day Strength, f'_c psi
2000	1:4.3:6.7	0.91	2280
3500	1:3.3:5.2	0.63	4150
5000	1:2.9:4.8	0.55	4870

*Saturated surface dry basis.

Table 3--ULTIMATE LOAD TEST DATA

Column No.	Concrete Strength+ f'_c, psi	Initial Eccentricity x e', in.	Age at Loading days	Age at Failure days	At Failure: Load kips	At Failure: Deflection y_c, in.	At Failure: Steel Strain x 10^6	At Failure: Concrete Strain x 10^6	Type of Loading++
Columns with Moderate Eccentricities									
20B1a	2290	5.38	29	29	27.0	0.191	2160	2780	FA
2a	2550	5.38	30	32	25.5	0.227	2030	3210	SL
3a	2350	5.38	35	540	25.0	0.355	2850	6300	SU+FA
4a	2660	5.38	34	547	24.3	0.396	3130	7500	SU+FA
20B1b	2120	5.56	31	31	22.1	0.189	1940	2760	FA
2b	2530	5.56	37	45	21.4	0.555	3870	7450	SL
3b	2300	5.56	83	83	22.0	0.260	2350	4000	FA
4b	1860	5.56	83	541	22.0	0.345	2700	4500	SU+FA
35B1a	4240	4.25	29	29	49.3	0.206	2150	3340	FA
2a	4410	4.25	41	43	44.5	0.240	2180	4070	SL
3a	4610	4.25	47	75	43.7	0.385	4000	6250	SU
4a	4640	4.25	48	199	43.4	0.534	3100	6500	SU
35B1b	4760	4.00	258	258	48.0	0.180	1620	2800	FA
2b	4770	4.00	260	264	45.6	0.170	1530	2950	SL
3b	4710	4.00	274	274	43.1				SU
4b	4760	4.00	271	936	47.3	0.246	2080	4200	SU+FA
50B1a	5370	4.00	28	28	59.0	0.232	1990	3550	FA
2a	5670	4.00	29	37	58.5	0.294	2700	5000	SL
3a	5420	4.00	40	592	57.0	0.302	2800	5350	SU+FA
4a	5680	4.00	41	589	51.0	0.440	5070	6780	SU+FA
50B1b	4360	4.25	46	46	50.0	0.159	1610	2540	FA
2b	4540	4.25	48	48	41.2	0.181	2000	3090	SL
3b	4740	4.25	52	490	49.5	0.356	3140	5880	SU+FA
4b	5320	4.25	52	124	41.3	0.346	3800	5760	SU
Columns with Small Eccentricities									
20C1a	2980	3.00	28	28	57.6	0.150	720	3450	FA
2a	3130	3.00	31	35	53.0	0.250	1300	5600	SL
3a	3470	3.00	40	98	52.5	0.300	1100	6300	SU
4a	2900	3.00	34	602	53.0	0.402	1960	8720	SU+FA
20C1b	1560	3.50	33	33	34.0	0.181	1060	3610	FA
2b	1810	3.50	34	39	31.5				SL
3b	1820	3.50	43	48	32.4	0.321	1540	5420	SU
4b	1770	3.50	43	479	32.0	0.560	2850		SU+FA

Table 3 (concluded)

Column No.	Concrete Strength+ f'_c, psi	Initial Eccen-tricity^x e', in.	Age at Loading days	Age at Failure days	At Failure: Load kips	At Failure: Deflec-tion y_c, in.	At Failure: Steel Strain x 10^6	At Failure: Con-crete Strain x 10^6	Type of Loading++
Columns with Small Eccentricities (continued)									
20C1c	2440	3.00	30	30	50.0	0.144	730	2910	FA
2c	2500	3.00	38	44	48.2	0.301	1060	4540	SL
3c	2380	3.00	50	948	51.2	0.262	1330	5940	SU+FA
4c	2810	3.00	82	1052	48.4	0.278	1600	6600	SU+FA
35C1a	3940	3.00	28	28	70.0	0.153	850	3450	FA
2a	4460	3.50	355	533	57.0	0.190	1810	3110	SL+FA
3a	4240	3.50	544	551	55.4	0.150	1450	2900	SL
4a	4230	3.50	561	561	50.9	0.150	1360	2980	FA
35C1b	4280	3.50	78	78	54.0	0.201	1250	2850	FA
2b	4670	3.50	80	85	49.4	0.256	2290	5050	SL
3b	4330	3.50	90	604	47.4	0.368	2400	7250	SU+FA
4b	4120	3.50	87	545	57.0	0.348	2450	6970	SU+FA

+Control cylinders were tested the day the corresponding column failed.

xDistance from the tension steel to the line of load.

++FA = fast loading; SL = slow loading; SU = sustained loading; SU+FA = fast loading following sustained loading.

Table 4--SUSTAINED LOAD TEST DATA

Column No.	Duration of Loading, days	Sustained Load		End of Sustained Load Period			Concrete Strains, x 10^6				
		kips	per cent[+]	Deflection y_c, in.	Steel Strain x 10^6	Concrete Strain x 10^6	at First Loading	at 10 Days	at 100 Days	at 1 Year	Recovery
Columns with Moderate Eccentricities											
20B3a	505	24.2	89.7	0.345	2640	6080	1990	3200	4580	5900	2060
4a	513	22.0	81.5	0.344	2620	5760	1860	2960	4500	5580	2150
20B4b	457	20.6	93.2	0.286	2380	4030	1960	2740	3490	3970	2030
35B3a	28	43.7[x]	88.7	0.385	4000	6250	2310	4400	--	--	--
4a	151	43.4[x]	85.4	0.534	3100	6500	2470	4270	5750	--	--
35B3b	1.5hr.	43.1[x]	89.8	0.167	1610	--	2910	--	--	--	--
4b	664	42.1	87.7	0.217	1910	3490	2130	2390	2880	3350	1900
50B3a	552	55.5	94.0	0.264	2080	4740	1960	2800	4000	4820	1910
4a	548	52.0	88.2	0.402	3930	6460	2310	3070	5150	6220	2100
50B3b	437	41.9	83.8	0.311	2350	5220	1760	2870	4300	4990	1960
4b	72	41.3[x]	82.6	0.346	3800	5760	2960	5220	--	--	--
Columns with Small Eccentricities											
20C3a	58	52.5[x]	91.2	0.300	1000	6300	1780	3860	--	--	--
4a	567	48.1	83.5	0.375	1600	8290	1810	3350	5860	7770	2190
20C3b	5	32.4[x]	95.4	0.321	1540	5420	2690	--	--	--	--
4b	436	29.0[++]	85.3	0.504	2300	--	1790	--	--	--	--
20C3c	933	44.1	88.2	0.226	1080	4600	1410	2190	3720	4490	1680
4c[xx]	(750	40.1	80.2	0.205	930	4230	1510	1860	3300	4070	1850
	(219	43.1	86.2	0.225	1070	4800					
35C3b[xx]	(295	46.9	86.9	0.294	1690	5840	2170	3050	4980	6360	2250
	(219	49.0	90.8	0.231	2110	6940					
4b	457	48.0	88.9	0.309	2230	6270	1820	2900	4350	5900	2140

+ Per cent of the ultimate load of the companion specimen subjected to fast loading.

x Failed under this load. All other specimens sustained the load without failure.

++ Loaded to 31.6 kips, but load was maintained at 29.0 kips.

xx After it became evident that the original load could be sustained indefinitely, the load was raised to a higher level.

Table 4 (concluded)

Column No.	Steel Strain, x 10^6 at First Loading	at 10 Days	at 100 Days	at 1 Year	Recovery	Deflection, y_c, 0.001 in. at First Loading	at 10 Days	at 100 Days	at 1 Year	Recovery
Columns with Moderate Eccentricities										
20B3a	1650	1830	2120	2700	1500	143	198	275	340	145
4a	1600	1940	2150	2570	1560	141	199	270	336	152
20B4b	1610	1790	2140	2370	1440	156	196	247	284	158
35B3a	1700	2400	--	--	--	160	268	--	--	--
4a	1590	2100	2530	--	--	160	247	429	--	--
35B3b	1610	--	--	--	--	167	--	--	--	--
4b	1180	1300	1270	1280	1200	134	149	170	204	119
50B3a	1180	1510	1770	1800	1270	120	171	226	256	129
4a	1590	1880	2940	3610	1270	165	207	313	384	149
50B3b	1150	1300	1950	2250	1490	123	173	257	311	127
4b	2110	3400	--	--	--	194	300	--	--	--
Columns with Small Eccentricities										
20C3a	300	690	--	--	--	82	178	--	--	--
4a	440	750	1310	1610	700	90	162	273	360	117
20C3b	900	--	--	--	--	143	--	--	--	--
4b	980	1650	1810	2110	980	141	327	453	497	136
20C3c	460	410	620	570	310	70	108	173	203	84
4c[xx]	390	450	680	630	470	73	89	161	187	91
35C3b[xx]	770	1010	1450	1920	1020	121	165	250	324	127
4b	880	1110	1570	2130	1090	101	156	228	300	113

[xx]After it became evident that the original load could be sustained indefinitely, the load was raised to a higher level.

Table 5 -- COMPARISON OF CALCULATED VALUES WITH TEST DATA

Column No.	Type of Loading	Ultimate Loads Calc. kips	Test kips	Test/Calc.	Deflection of Shaft y'_c Calc. in.	Test+ in.	Test/Calc.	Deflection of Column y_c Calc. in.	Test in.	Test/Calc.	Tension Steel Stress Calc. ksi	Test[x] ksi	Test/Calc
Columns with Moderate Eccentricities													
20B1a	FA	24.3	27.0	1.11	0.073	--	--	0.196	0.191	0.98	43.3	43.3	1.00
3a	SU+FA	23.0	25.0	1.09	0.164	--	--	0.429	0.355	0.83	43.3	43.3	1.00
4a	SU+FA	23.7	24.3	1.03	0.173	--	--	0.449	0.396	0.88	43.3	43.3	1.00
20B1b	FA	22.6	22.1	0.98	0.073	--	--	0.196	0.189	0.97	43.3	43.3	1.00
3b	FA	23.2	22.0	0.95	0.077	--	--	0.203	0.260	1.28	43.3	43.3	1.00
4b	SU+FA	21.2	22.0	1.04	0.107	0.094	0.88	0.291	0.345	1.18	43.3	43.3	1.00
35B1a	FA	44.9	49.3	1.10	0.069	--	--	0.189	0.206	1.09	43.3	43.3	1.00
3a	SU	40.9	43.7	1.07	0.149	--	--	0.398	0.385	0.97	43.3	43.3	1.00
4a	SU	41.0	43.4	1.06	0.150	0.186	1.24	0.399	0.534	1.34	43.3	43.3	1.00
35B1b	FA	52.7	48.0	0.91	0.065	--	--	0.181	0.180	0.99	43.3	37.2	0.86
3b	SU	46.1	43.1	0.93	0.135	--	--	0.370	--	--	43.3	--	---
4b	SU+FA	51.2	47.3	0.92	0.092	0.065	0.71	0.252	0.246	0.98	43.3	43.3	1.00
50B1a	FA	55.9	59.0	1.06	0.069	--	--	0.188	0.232	1.23	43.3	43.3	1.00
3a	SU+FA	52.5	57.0	1.09	0.128	--	--	0.342	0.302	0.88	43.3	43.3	1.00
4a	SU+FA	51.2	51.0	0.99	0.165	0.131	0.80	0.437	0.440	1.01	43.3	43.3	1.00
50B1b	FA	45.3	50.0	1.10	0.071	--	--	0.191	0.159	0.83	43.3	37.5	0.87
3b	SU+FA	43.3	49.5	1.14	0.151	0.109	0.72	0.397	0.356	0.90	43.3	43.3	1.00
4b	SU	42.9	41.3	0.96	0.167	--	--	0.424	0.346	0.82	43.3	43.3	1.00
Average FA =				1.03						1.05			0.96
Average SU =				1.00						1.04			1.00
Average SU+FA =				1.04						0.95			1.00

+ Measured values only, no extrapolation involved.

x Computed from measured strains corrected to the centerline of bars; 43.3 ksi yield point stress.

Table 5 (concluded)

Column No.	Type of Loading	Ultimate Loads Calc. kips	Test kips	Test/Calc.	Deflection of Shaft y_c' Calc. in.	Test+ in.	Test/Calc.	Deflection of Column y_c Calc. in.	Test in.	Test/Calc.	Tension Steel Stress Calc. ksi	Test[x] ksi	Test/Calc.
Columns with Small Eccentricities													
20C1a	FA	56.0	57.6	1.03	0.051	--	--	0.152	0.150	0.99	14.1	13.2	0.94
3a	SU	55.4	52.5	0.95	0.100	--	--	0.299	0.300	1.00	23.0	21.1	0.92
4a	SU+FA	51.5	53.0	1.03	0.133	--	--	0.402	0.402	1.00	26.5	37.0	1.40
20C1b	FA	33.2	34.0	1.02	0.053	0.056	1.06	0.156	0.181	1.16	19.0	21.6	1.14
3b	SU	33.0	32.4	0.98	0.101	0.104	1.03	0.302	0.321	1.06	25.4	31.4	1.24
4b	SU+FA	--	32.0	--	--	0.159	--	--	0.560	--	--	--	--
20C1c	FA	49.6	50.0	1.01	0.051	0.055	1.08	0.151	0.144	0.96	12.8	14.3	1.12
3c	SU+FA	47.5	51.2	1.08	0.090	--	--	0.273	0.262	0.96	18.6	25.2	1.35
4c	SU+FA	52.3	48.4	0.93	0.093	--	--	0.278	0.278	1.00	20.7	36.2	1.75
35C1a	FA	67.1	70.0	1.04	0.052	0.058	1.11	0.153	0.153	1.00	16.3	16.6	1.02
4a	FA	58.0	50.9	0.88	0.058	--	--	0.166	0.150	0.91	31.4	30.4	0.97
35C1b	FA	58.5	54.0	0.92	0.059	--	--	0.167	0.201	1.20	31.4	--	--
3b	SU+FA	57.0	47.4	0.83	0.123	--	--	0.363	0.368	1.01	43.3	43.3	1.00
4b	SU+FA	55.9	57.0	1.02	0.118	0.107	0.91	0.344	0.348	1.01	43.3	43.3	1.00
Average FA =				0.98						1.04			1.04
Average SU =				0.97						1.03			1.08
Average SU+FA =				0.98						1.00			1.30
Overall Average FA =				1.01						1.05			0.99
Overall Average SU =				0.99						1.04			1.03
Overall Average SU+FA =				1.01						0.97			1.13

+ Measured values only, no extrapolation involved.

x Computed from measured strains corrected to the centerline of bars; 43.3 ksi yield point stress.

Table 6--COMPARISON WITH C. S. WHITNEY'S EQUATIONS

Column No.	Type of Loading	Sustained Load kips	Ultimate Load Test kips	Ultimate Load Whitney kips	Sustained Whitney	Test Whitney
20B1a	FA	--	27.0	23.7T	--	1.14
2a	SL	--	25.5	24.4T	--	1.05
3a	SU+FA	24.2	25.0	23.0T	1.05	1.09
4a	SU+FA	22.0	24.3	23.6T	0.93	1.03
20B1b	FA	--	22.1	22.2T	--	1.00
2b	SL	--	21.4	21.2T	--	1.01
3b	FA	--	22.0	22.3T	--	0.99
4b	SU+FA	20.6	22.0	20.3C	1.01	1.08
35B1a	FA	--	49.3	43.1T	--	1.14
2a	SL	--	44.5	43.3T	--	1.03
3a	SU	43.7*	43.7	42.2T	1.04*	1.04
4a	SU	43.4*	43.4	39.4T	1.10*	1.10
35B1b	FA	--	48.0	51.0T	--	0.94
2b	SL	--	45.6	51.1T	--	0.89
3b	SU	43.1*	43.1	50.2T	0.86*	0.86
4b	SU+FA	42.1	47.3	49.6T	0.85	0.95
50B1a	FA	--	59.0	52.9T	--	1.12
2a	SL	--	58.5	53.0T	--	1.10
3a	SU+FA	55.5	57.0	51.7T	1.07	1.10
4a	SU+FA	52.0	51.0	49.5T	1.05	1.03
50B1b	FA	--	50.0	44.5T	--	1.12
2b	SL	--	41.2	45.0T	--	0.92
3b	SU+FA	41.9	49.5	42.7T	0.98	1.16
4b	SU	41.3*	41.3	45.0T	0.92*	0.92

Column No.	Type of Loading	Sustained Load kips	Ultimate Load Test kips	Ultimate Load Whitney kips	Sustained Whitney	Test Whit'y
20C1a	FA	--	57.6	51.1C	--	1.13
2a	SL	--	53.0	50.9C	--	1.04
3a	SU	52.5*	52.5	53.6C	0.98*	0.98
4a	SU+FA	48.1	53.0	46.3C	1.04	1.04
20C1b	FA	--	34.0	30.5C	--	1.11
2b	SL	--	31.5	32.1C	--	0.98
3b	SU	32.4*	32.4	31.6C	1.03*	1.03
4b	SU+FA	29.0	32.0	29.2C	0.99	1.10
20C1c	FA	--	50.0	45.4C	--	1.10
2c	SL	--	48.2	43.7C	--	1.10
3c	SU+FA	44.1	51.2	43.1C	1.02	1.19
4c	SU+FA	43.1	48.4	47.2C	0.91	1.03
35C1a	FA	--	70.0	61.3C	--	1.14
2a	SL+FA	--	57.0	56.4C	--	1.01
3a	SL	--	55.4	55.1C	--	1.01
4a	FA	--	50.9	55.0C	--	0.93
35C1b	FA	--	54.0	54.6C	--	0.99
2b	SL	--	49.4	57.1C	--	0.87
3b	SU+FA	49.0	47.4	52.5C	0.93	0.90
4b	SU+FA	48.0	57.0	51.0C	0.94	1.12

Average for sustained load without failure	0.982	
" " " " with "	0.988	
Average for all ultimate loads		1.039
Average for fast-load ultimate		1.065

* Failed under sustained loading.

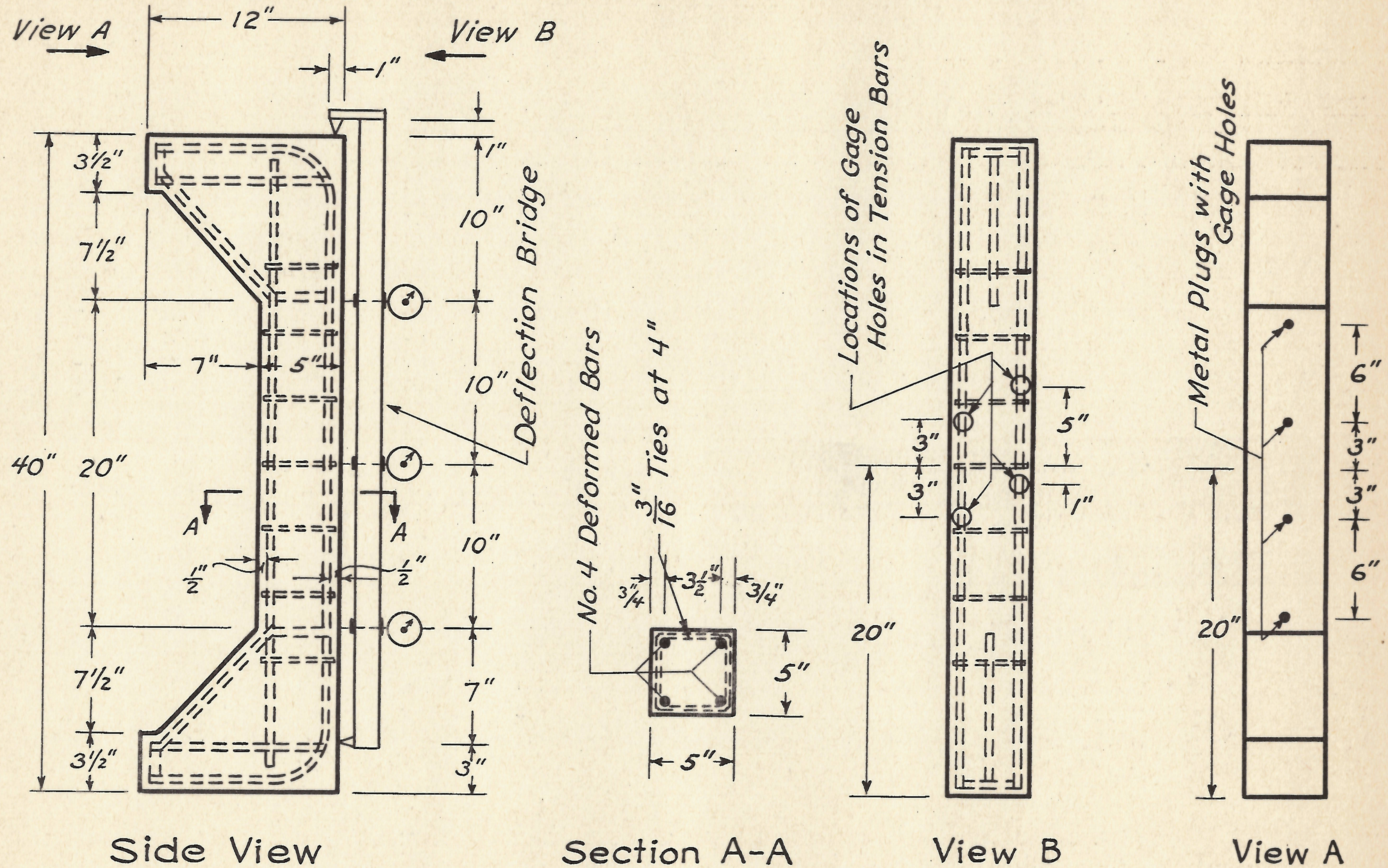


Fig. 1 - Details of Specimens

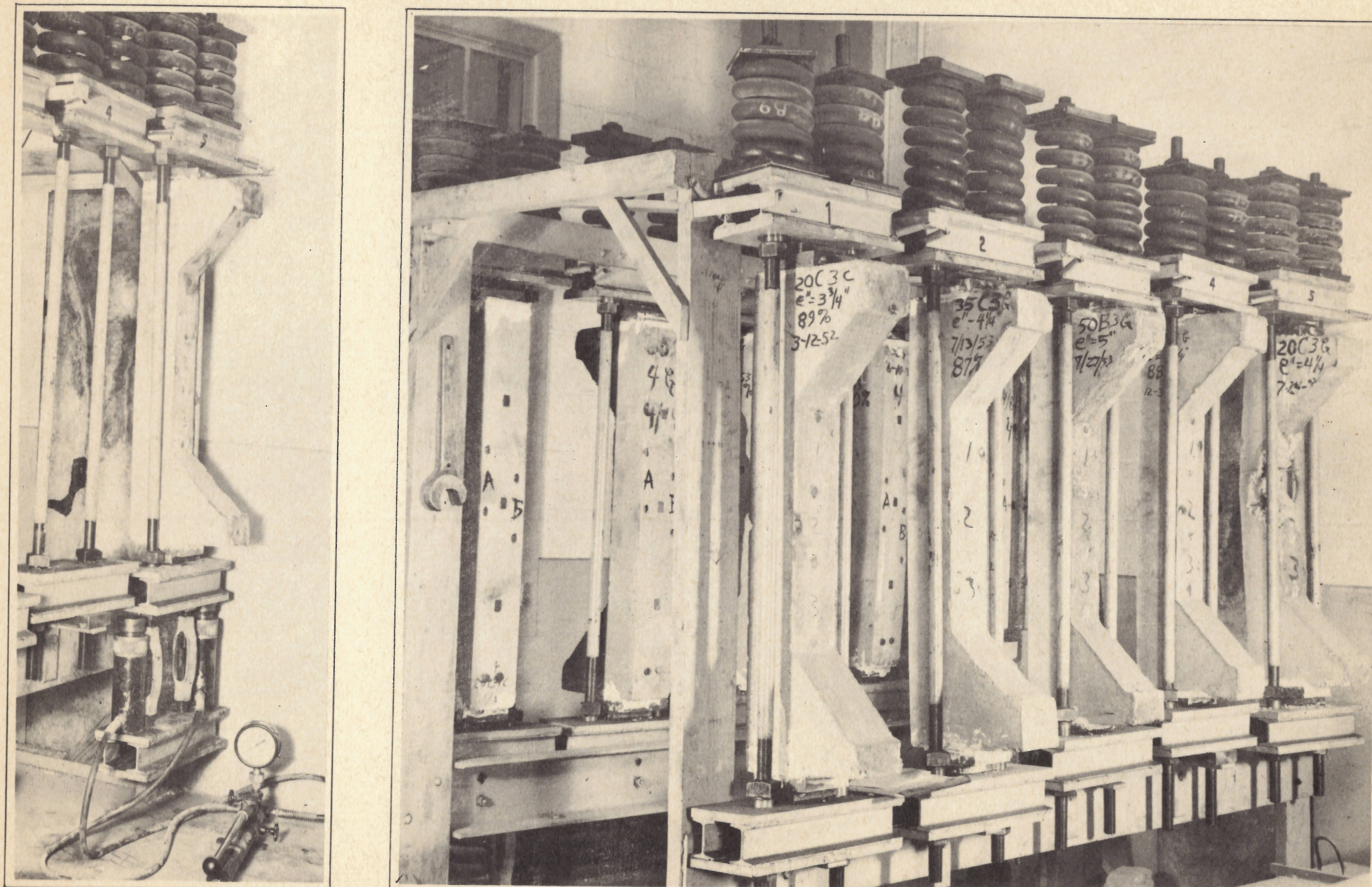


Fig. 2 - Loading Frames for Sustained Load Tests

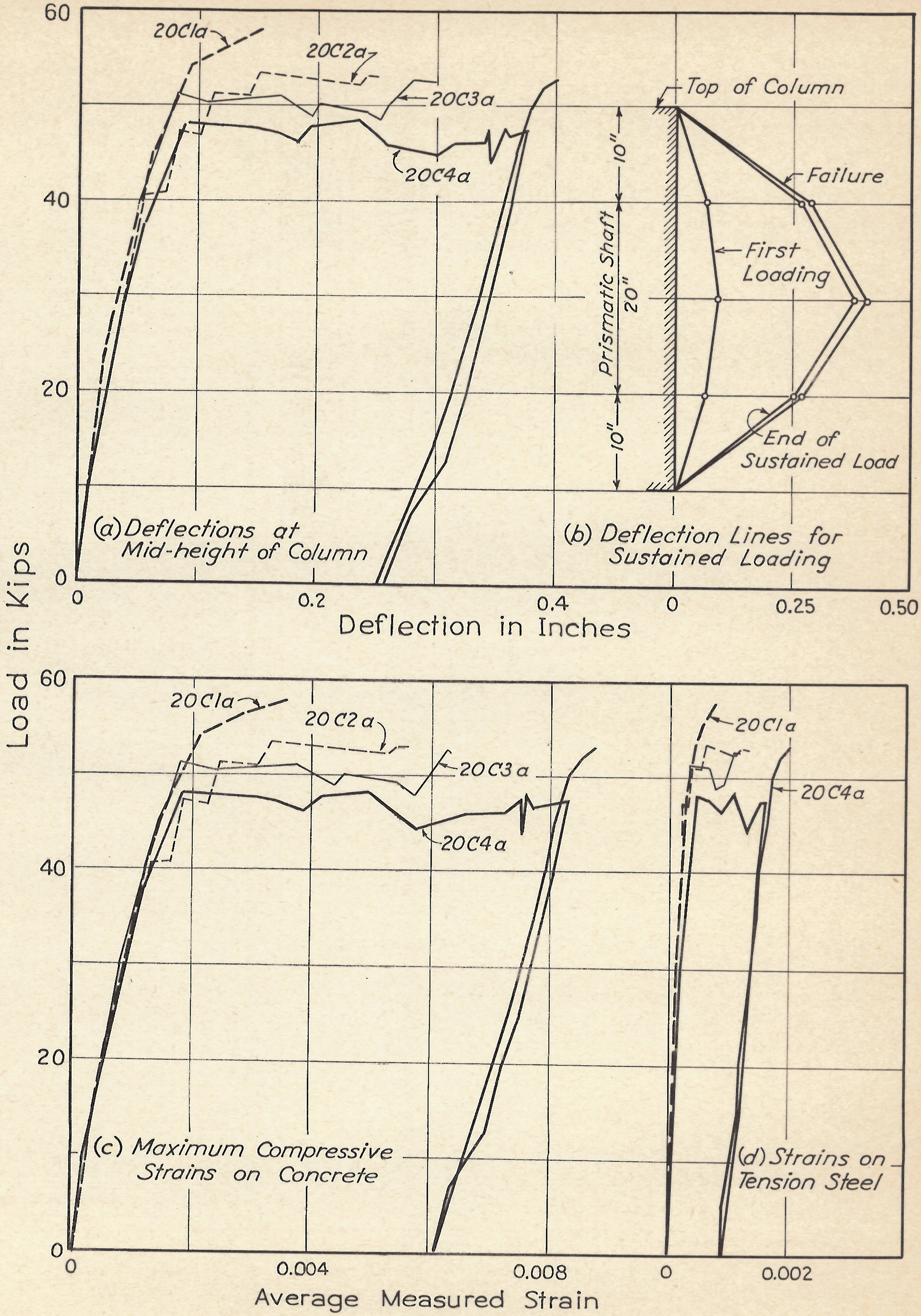
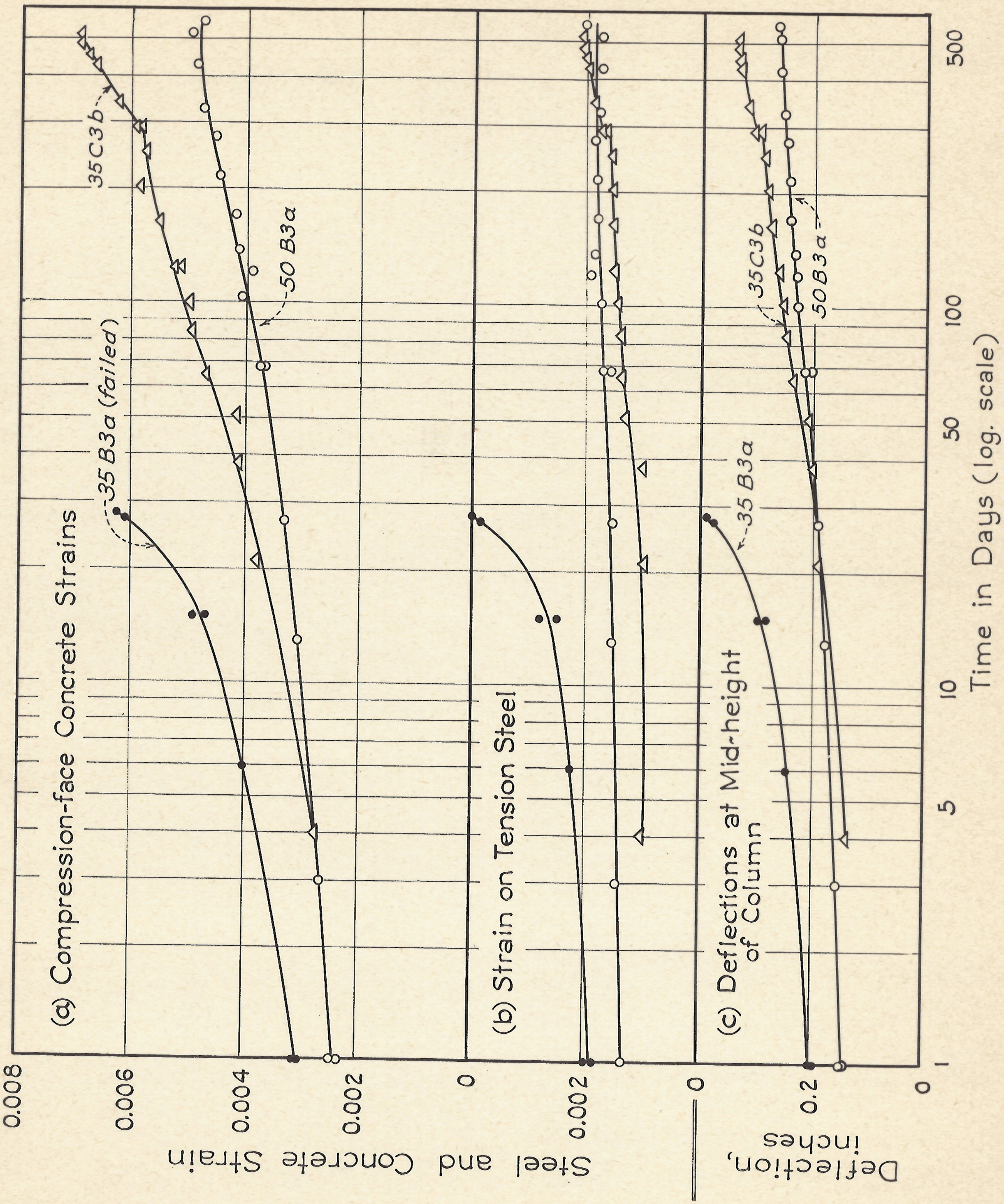


Fig. 3 - Typical Column Deformations



<u>Fig. 4 – Effect of Time on Column Deformations</u>

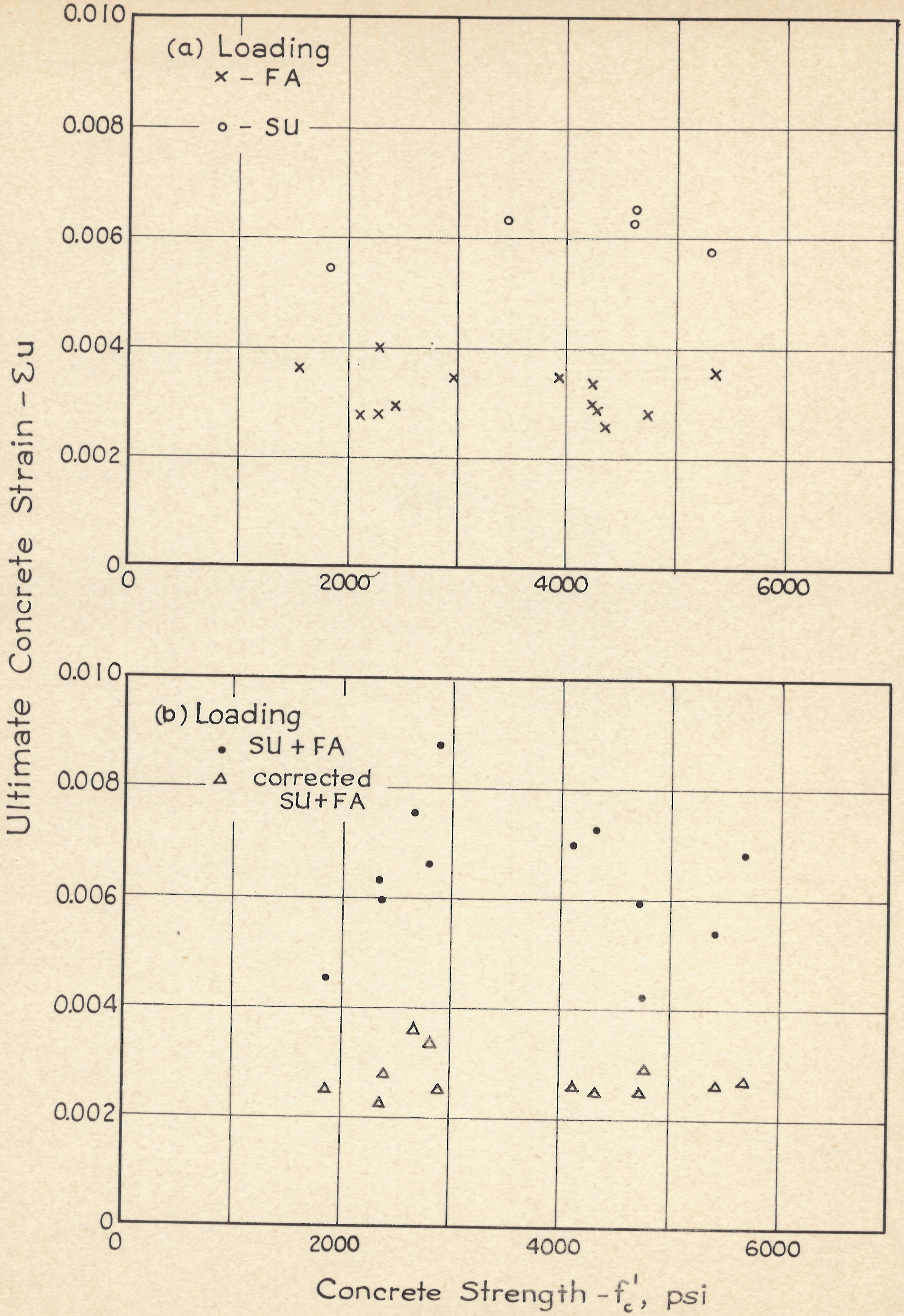


Fig.5 – Ultimate Concrete Strains

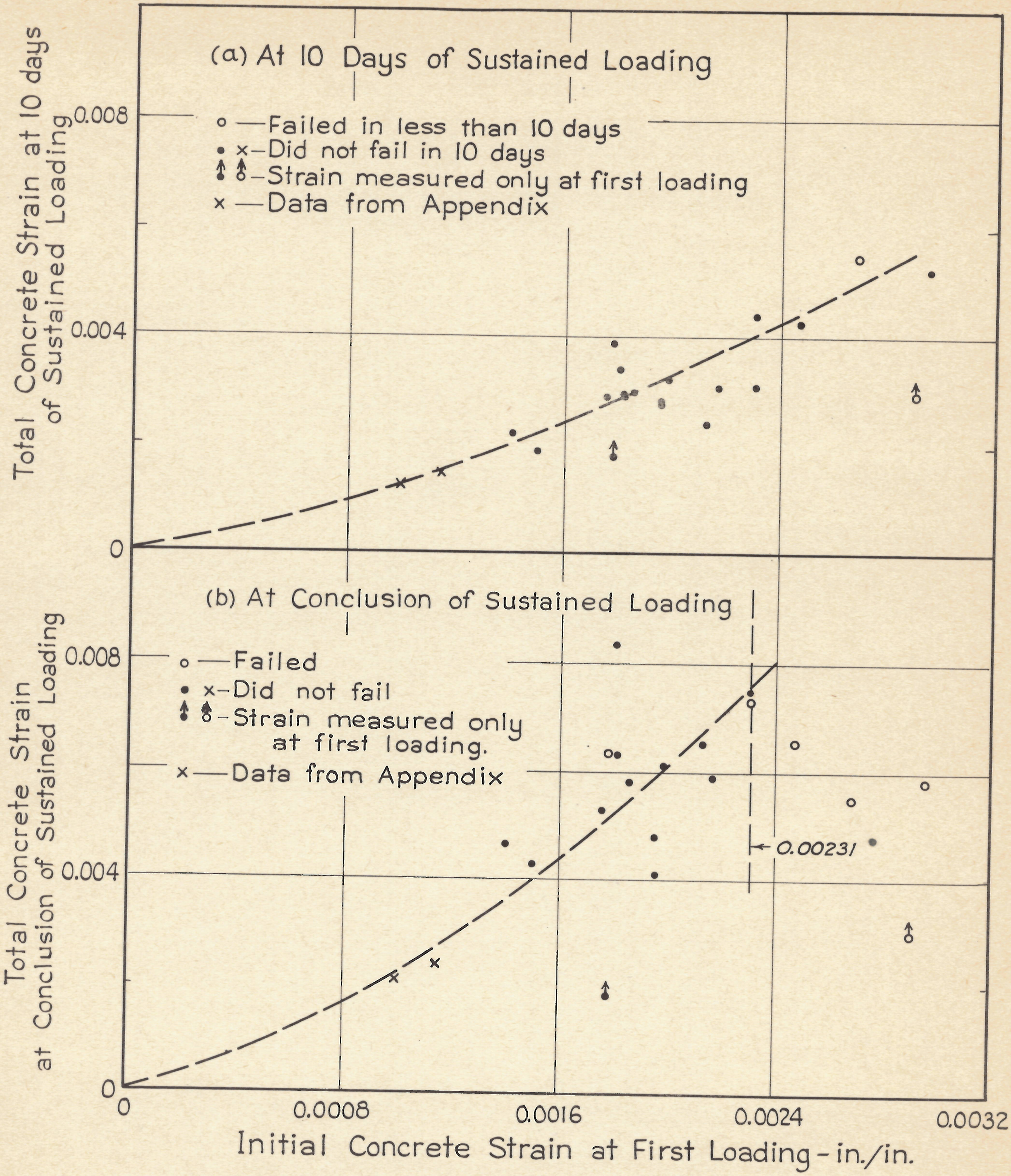


Fig. 6 – Total Concrete Strains After Sustained Loading Related to Initial Value

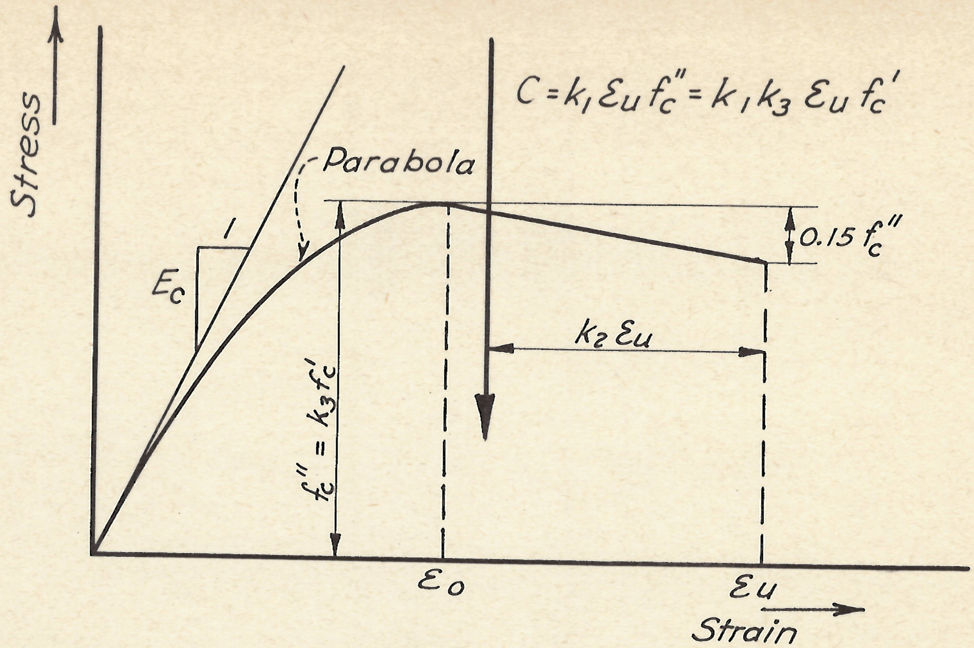


Fig. 7- Assumed Stress-Strain Diagram for Concrete in Flexure

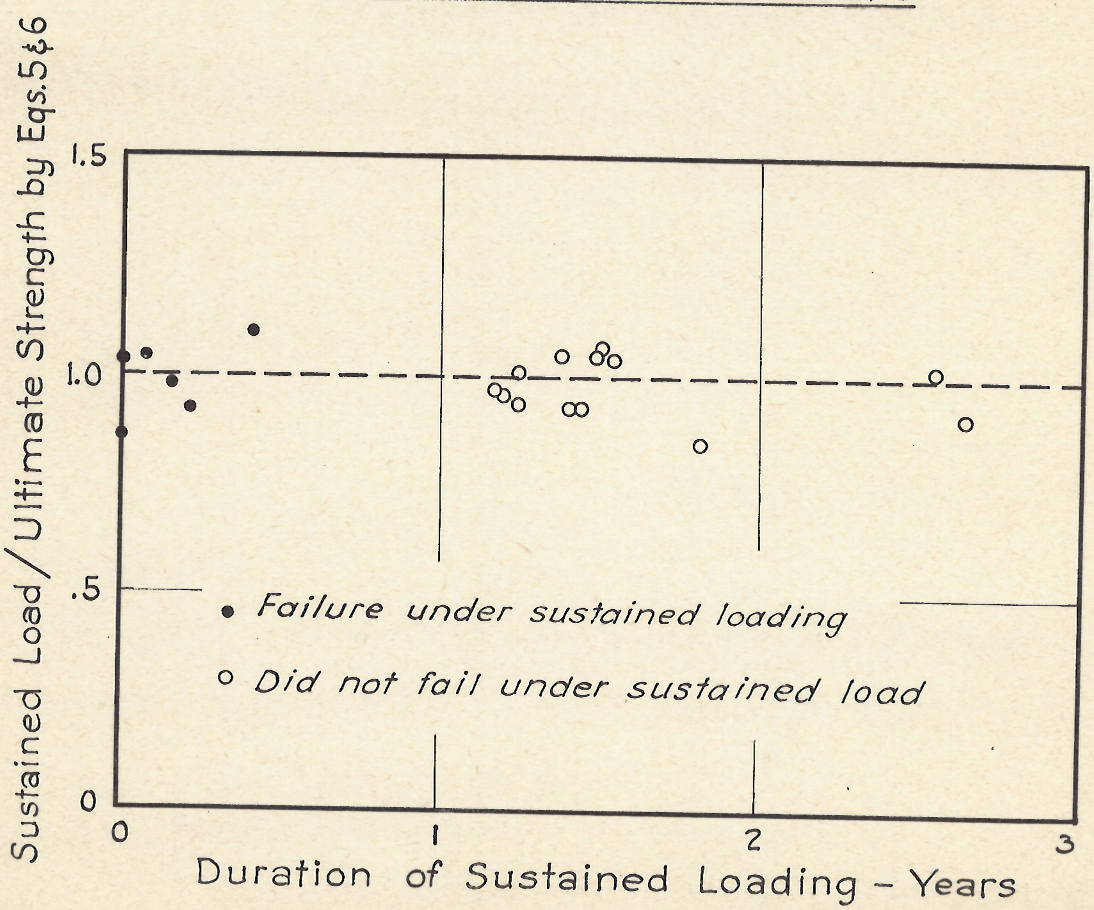


Fig. 8- Comparison of Sustained Loads to Ultimate Loads
Computed by C.S. Whitney's Equations

APPENDIX - PILOT TESTS OF COLUMNS WITH SUSTAINED ECCENTRIC MODERATE OVERLOADS

Summary

The tests of four eccentrically and one concentrically loaded tied columns reported in this appendix were undertaken to study two major phenomena: (a) the behavior of such columns under sustained moderate overloads, and (b) the effect of a period of sustained loading on the ultimate strength.

Within the specific limitations of these tests, the following conclusions may be made:

(1) During the period of sustained loading the strains in the tension steel changed very little, while the strains in the compression steel and on the compression face of the concrete as well as the lateral deflection increased considerably.

(2) Evidence of a variation in concrete strength over the column height was observed.

(3) The strain distribution across the column cross-section remained fairly linear during the creep deformations.

(4) The creep deformations came practically to rest after about one year.

(5) No signs of distress appeared during the period of sustained loading.

(6) During unloading after a period of sustained loading, cracks appeared on the "compression" face of the columns. These cracks closed again during reloading and appeared to be as harmless as the cracks normally found near tension reinforcement.

(7) In the tests to failure, the columns which had been subject to a period of sustained loading were at least equally strong as the corresponding companion columns stored without load.

(8) The strength of a tied column with unsymmetrical reinforcement tested to failure with flat ends was in general agreement with the equation for concentrically loaded columns.

Object and Scope

The object of the pilot tests reported herein was to investigate the effect of sustained moderate overloads on the behavior and strength of eccentrically loaded columns. Hence, two major phenomena were involved: (a) the behavior of columns subject to sustained loads, and (b) the effect of a period of sustained loading on the ultimate strength as measured in a subsequent fast-load test to failure. Two columns designated B9c and B10c, were subjected to sustained loads 62.5 and 65 percent, respectively, of the short-time strength computed on the basis of the 28-day cylinder strength. The sustained loads were maintained for about 500 days. After this period, the loads were removed and both columns tested to failure with fast loading. Two companion columns, designated B9d and B10d, were stored the same 500 days without load and were tested to failure with fast loading the same day as the corresponding sustained load columns.

The tests were an extension of the 1951 Illinois investigation of eccentrically loaded, reinforced concrete columns tested to failure with fast loading.[1] Accordingly, columns B9c and B9d corresponded to columns B9a and B9d of the 1951 tests, and columns B10c and B10d corresponded to columns B10a and B10b. All four columns were 10 in. square, 75 in. long, reinforced with eight 5/8-in. bars, made with a 3000-psi concrete, and loaded at eccentricities from the center line of the column to the load line of 7.5 and 12.5 in., respectively.*

In addition, one column designated B1c, reinforced with four 5/8-in. and two 1/4-in. bars was loaded to failure in a short-time test with "flat ends" in order to study the effect of end conditions on concentrically loaded columns.

Materials, Fabrication, and Test Methods

The same materials as reported for the 1951 tests were used.[1] The 5/8-in. reinforcing bars had a yield point of 43,600 psi. The concrete strengths at 28 days and at test to failure are given in Table 7 as an average of about nine 6- by 12-in. cylinders.

The reinforcement was prepared and the columns were cast in pairs in steel forms in the same manner as described in the 1951 report.[1] One electric SR-4 gage of type A-11 with 1-in. gage length was mounted on each 5/8-in. longitudinal bar prior to casting. In addition, plugs for a 6-in. mechanical Berry gage were prepared in the columns subjected to sustained loads. All columns were cured wet for 7 days, and were then stored in the laboratory.

The columns B9c and B10c were subjected to sustained loads at an age of 28 days by means of tie-rods and springs as shown in Fig. 9. The following quantities were observed as a function of time: (1) strains on the concrete surface, (2) strains in the tension and compression steel, (3) lateral deflections in the plane of eccentricity and (4) deflections of the loading springs.

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*For other details see Fig. 4 and Fig. 5 in reference (1).

The loading springs were tightened at intervals so as to maintain the sustained load within $\pm$ 3 percent. As the springs were calibrated prior to the test, all measured quantities were corrected to the exact sustained load by linear interpolation of the corresponding quantities for the last increment of loading before the sustained load was reached. The springs returned to their original length after completion of the test. The strains in the reinforcement measured by electric gages were corrected for changes in the zero-point of the SR-4 indicator used as well as for the effect of the local removal of bar deformations indicated in Fig. 3b of reference (1).

Columns B9d and B10d were stored without any load. Strains in the reinforcement due to shrinkage were observed as a function of time.

A few hours before the final test to failure, the sustained loads were removed, and all columns were loaded to failure through knife-edges in a 3,000,000-lb hydraulic testing machine as in the 1951 tests.[1]

Behavior of Columns under Sustained Loads

At the age of 28 days, columns B9c and B10c were subjected to sustained loads 62.5 and 65 percent, respectively, of the theoretical ultimate loads at that age (Table 7).[+] At these loads, the stresses in the tension reinforcement of both columns were about 30,000 psi.

The various measured determinations are presented in Fig. 10 as a function of time. It is seen that the strains in the tension steel changed very little, while the strains in the compression reinforcement and on the compression face of the concrete as well as lateral deflections increased

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+ The loads were computed from Eq. 1 with k_1/k_2 = 0.55, k_3 = 0.85 and f'_c as given in Table 7 for 28 days; the deflection at failure was assumed as 0.50 in.

considerably. These movements came practically to rest after about one year.

It was noted in the 1951 tests that a differential in strength existed over the column height due to the vertical casting position. A similar phenomenon was observed in the present tests as shown in Fig. 11a and b, indicating larger strains in the concrete near the top of the columns and relatively larger deflections in the upper half.

The distributions of strains across the column sections are given in Fig. 11c and d. A fairly linear distribution of strains was found at all times. The relatively low strains in the compression steel are probably due to the fact that the strains in the reinforcement were measured near mid-depth of the columns, while the concrete strains are given as the average of several gage lines at the top, center, and bottom of the prismatic shaft.

No signs of distress were observed throughout the period of sustained loading.

Tests to Failure

Columns B9c and B10c were unloaded a few hours before the tests to failure. At loads about one-third of the sustained load, cracks appeared on the "compression" faces of both columns at about the same 4-in. spacing as that which appeared on the tension face during loading. This crack formation was by no means serious. During reloading in the testing machine, the cracks closed up at low loads.

During the final test to failure, both the columns stored with sustained loads and those stored unloaded appeared to the eye to behave in a manner very similar to that of the corresponding columns B9 and B10 in the 1951 tests.[1]

The test and theoretical values of ultimate loads are given in Table 7. The theoretical values were computed from Eq. 1 with the concrete strength at the time of test to failure, the measured eccentricity at failure (Fig. 12a and b), and with $k_2/k_1 = 0.55$ and $k_3 = 0.85$; this method of calculating the ultimate load is identical with that of the 1951 report. The test results for columns B10c and B10d were very nearly the same as the computed values. The results for columns B9c and B9d seem to indicate that the period of sustained loading may have strengthened column B9c; however, the difference in the computed and measured ultimate strengths is within the range of scatter to be expected in this type of tests.

The lateral deflections of columns B9 and B10 during the test to failure are given in Fig. 12a and b. All columns failed initially by yielding of the tension steel, and the maximum load was reached when the concrete failed in compression. For columns B9, buckling of the compression steel took place at a deflection of about 1.0 in., while the tests of columns B10 were discontinued before buckling took place.

Strains measured during the tests to failure are presented in Fig. 12c and d. For columns B9d and B10d, which were stored without load, the initial strains were caused by shrinkage. During the unloading and reloading of columns B9c and B10c, which had been stored with sustained loads, hysteresis loops appeared. These loops were not parallel to the initial loading curves because of the tension cracking of the concrete on the "compression" face during unloading and the closing of these cracks during reloading. Thus, during reloading, columns B9c and B10c were more flexible than the corresponding companion specimens. When loaded beyond the sustained load level, however, these columns deformed in a manner very similar to that of the companion specimens.

Column with Flat Ends

It was found in the 1951 tests[1] that tied columns loaded concentrically through knife edges were 10 to 15 per cent weaker than computed from the formula

$$P = 0.85\,A_c\,f'_c + A_{st}\,f'_y \qquad (7)$$

This was explained as being due to the fact that it is practically impossible to obtain a true concentric loading through knife edges.

In the tests reported herein, one column, B1c, was loaded to failure with flat ends. This column had an unsymmetrical reinforcement consisting of four 5/8-in. bars on one side, two 1/4-in. bars on the other side. Nevertheless, the column appeared to have been concentrically loaded as the ultimate load was 6 percent over that predicted by Eq. 7 (Table 7).

Table 7--TEST RESULTS

Column No.	Cylinder Strength f'_c, psi at		e in.	P_{calc} 28 days+ kips	Sustained Load			Test to Failure			
	28 days	Failure			days	kips	Percent of P_{calc}, 28d	Age days	Test kips	Calc kips	Test/Calc.
B9c	3650	3450	7.5	88.0	453	54.8	62.5	481	99.8	85.1	1.17
B9d	"	"	"	"	0	0	0	"	91.2	86.7	1.05
B10c	2940	3175	12.5	44.5	558	28.9	65.0	586	44.9	43.4	1.03
B10d	"	"	"	"	0	0	0	"	45.4	"	1.04
B1c	4220	3800	0	----	0	0	0	479	403	379	1.06

+ Calculated short-time ultimate load at 28 days

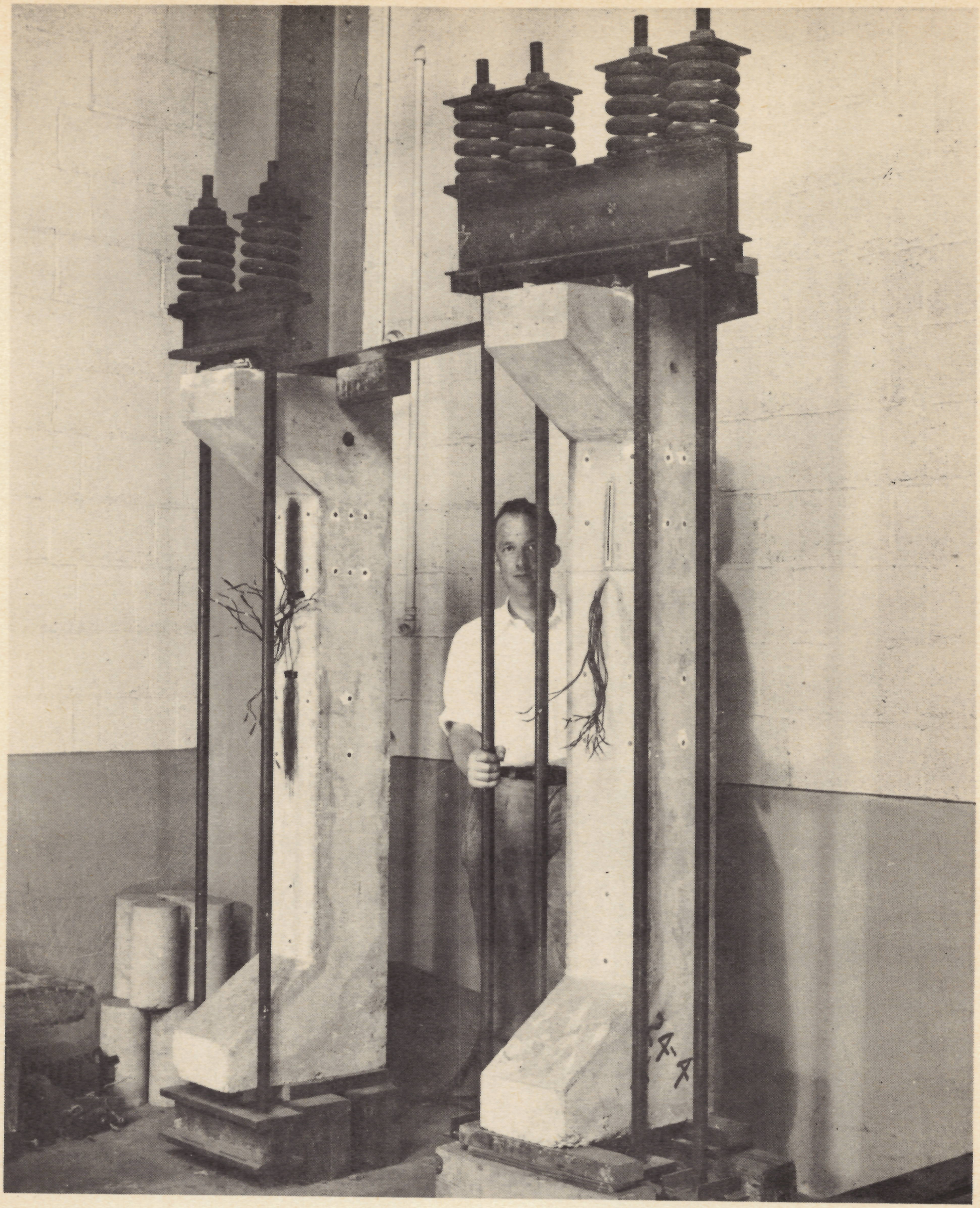

Fig. 9 - Columns Under Sustained Loads

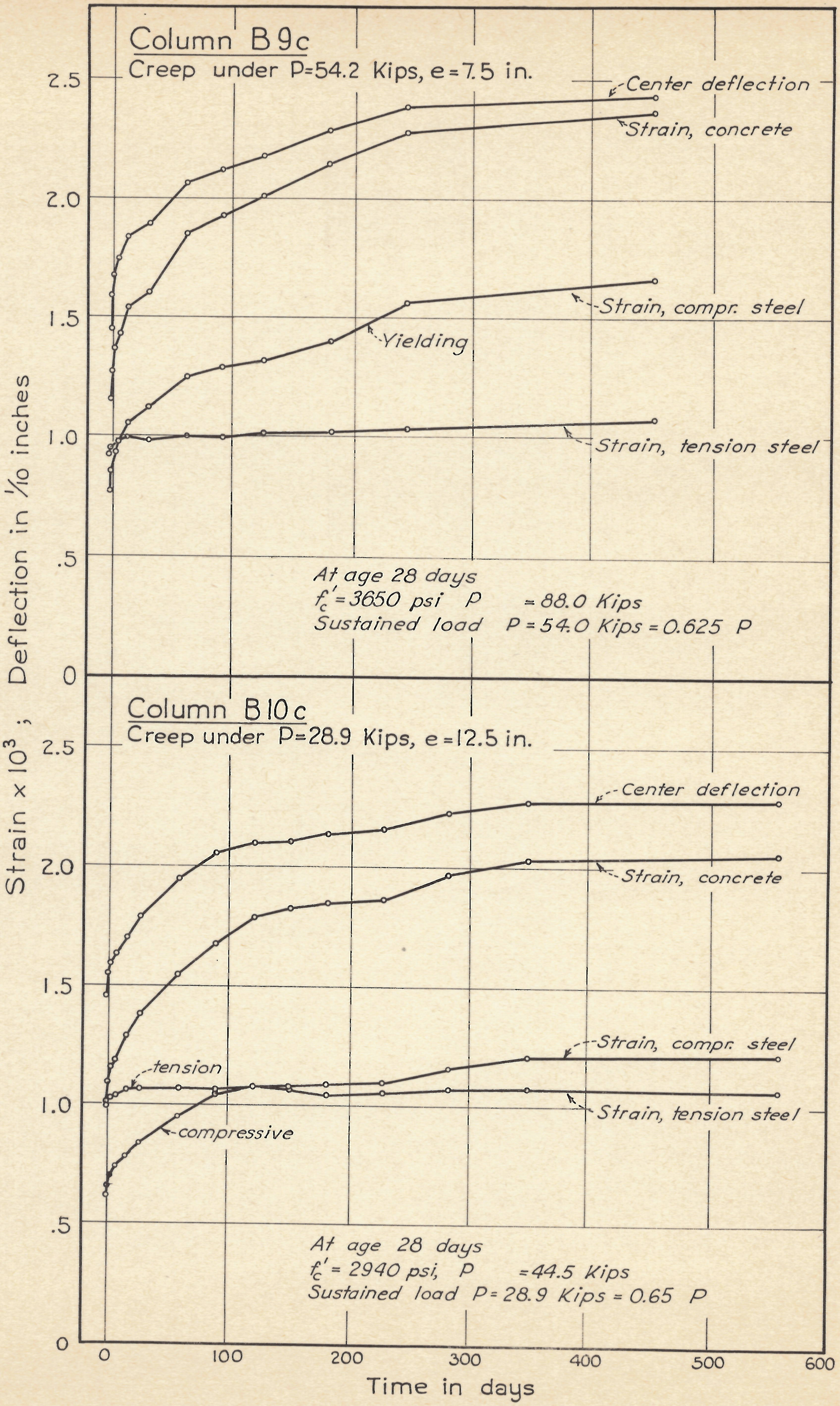

Fig. 10 - Deformations Under Sustained Loading

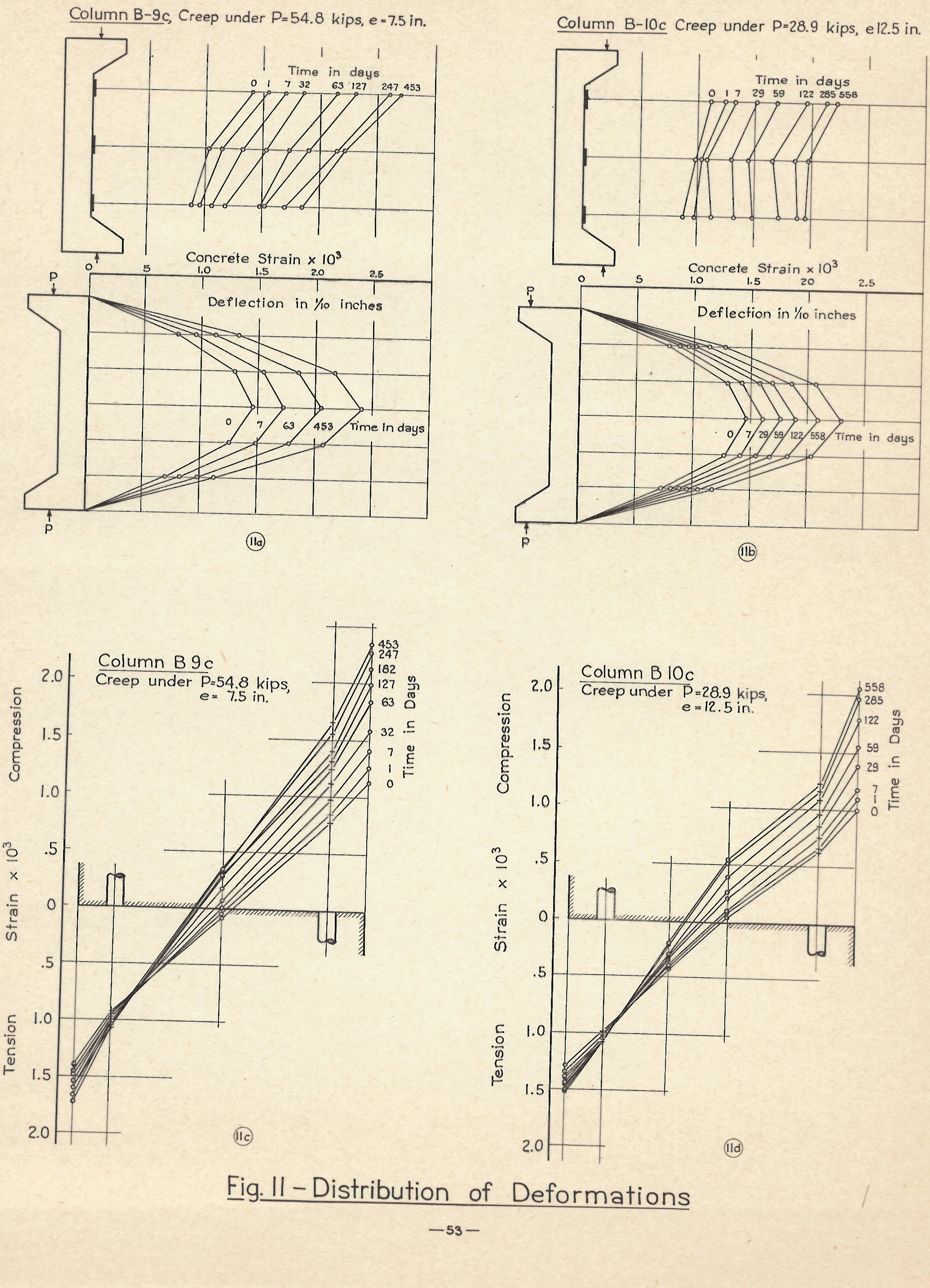

Fig. 11 – Distribution of Deformations

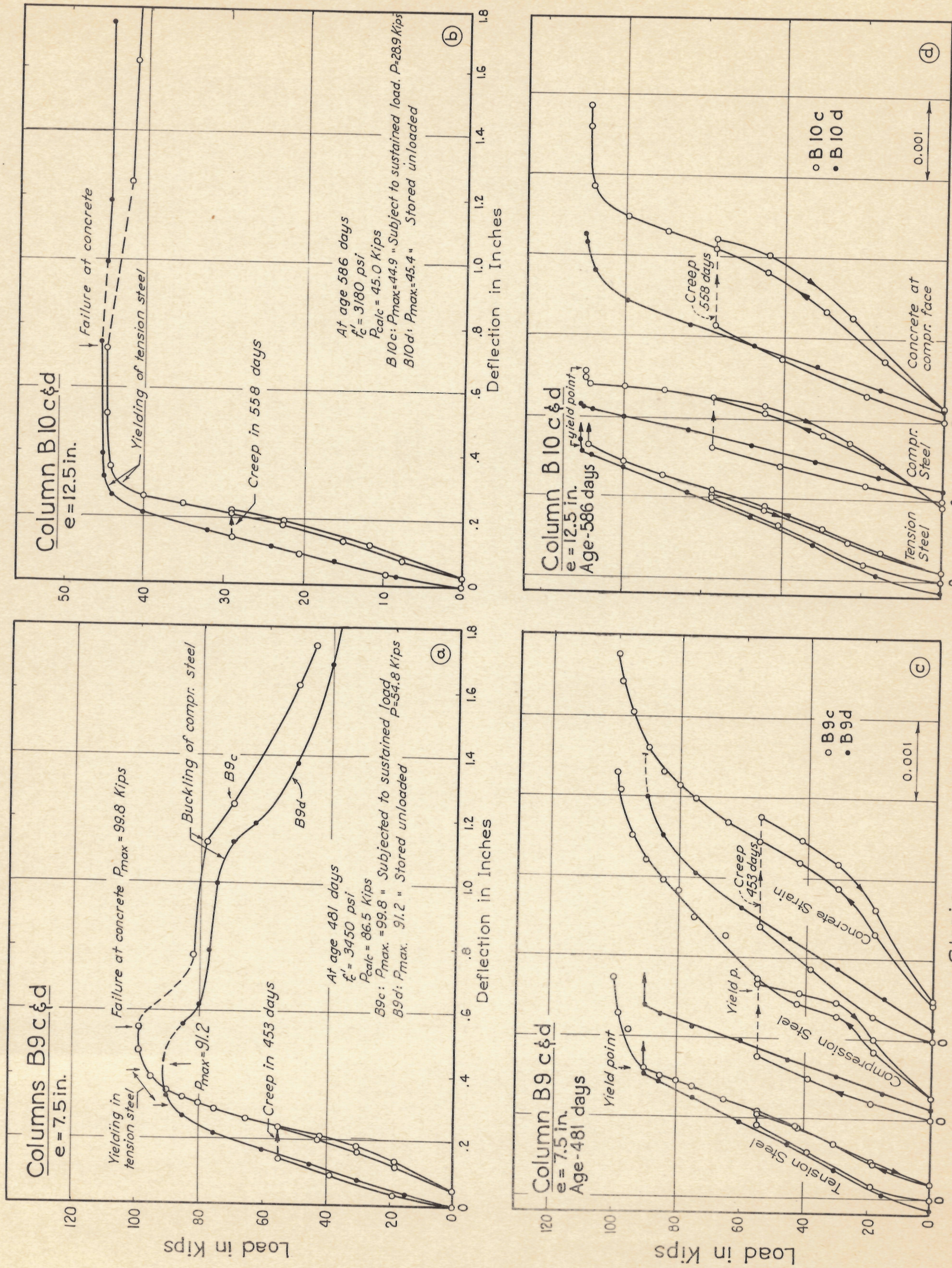

Fig. 12 - Test to Failure