DETERMINATION OF COMPRESSIVE STRENGTH OF CONCRETE USING ITS SONIC PROPERTIES

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DETERMINATION OF COMPRESSIVE STRENGTH OF CONCRETE BY USING ITS SONIC PROPERTIES

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SYNOPSIS

In predicting the quality of concrete by sonic methods, the prediction is usually based on the initial modulus of elasticity; however, considering concrete as viscoelastic, an analysis of a rheological model of concrete clearly shows that a property in addition to the initial modulus of elasticity must be considered. The one property, other than modulus of elasticity, that promises to be of greatest help in predicting the strength of concrete is the coefficient of viscosity. While this property may be an excellent one to use, it cannot be determined readily from sonic tests of concrete; but, logarithmic decrement, which is related to the coefficient of viscosity, can be easily obtained from sonic tests.

To establish whether or not there exists a relationship between the modulus of elasticity and damping capacity of concrete and the compressive strength of concrete some 300 standard

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6 in. by 12 in. cylinders were tested. From the test data a set of curves are obtained from which the strength of concrete, made of aggregates used in this investigation, can be predicted. The accuracy of the prediction is generally within an error of five per cent. This accuracy can be obtained without knowledge of the age, mix, or moisture content of the concrete. The results clearly show why the use of the modulus of elasticity alone is not sufficient, as previously believed, to accurately predict the strength of concrete.

INTRODUCTION

In the study of plain concrete it is often desirable to be able to determine certain physical properties without subjecting the specimen to a destructive test or even to stresses higher than a small percentage of the ultimate stress. The determination of properties such as the dynamic modulus of elasticity and the damping capacity may be of particular importance in the study of fatigue, creep and durability of concrete. It is desirable in the type of studies just mentioned to determine the physical properties of the same specimens upon which the studies are being conducted; and of course, in order to do this, a non-destructive means of testing must be used. Sonic testing of concrete has been recognized for several years as a useful and powerful non-destructive means for studying the quality of concrete. The object of the investigation discussed in this paper was to determine what sonic properties of concrete, if any, would enable one to predict accurately the compressive strength of the concrete.

Engineers have believed for many years that a relationship exists between the modulus of elasticity and compressive strength of concrete. Several experimental formulas have been presented giving such a relationship; however, it is possible to be more than a hundred per cent in error if these formulas are used for material or conditions for which they were not derived. Thus, it becomes clear that if the compressive strength of concrete is to be predicted with any degree of accuracy, some physical property in addition to the modulus of elasticity must be used. Furthermore this additional property must be one which can be obtained in a non-destructive test.

If concrete is assumed to be a viscoelastic material, it immediately becomes evident that some property of the concrete in addition to the initial modulus of elasticity must be considered. If an analysis is made of the rheological model shown in Fig. 1, which is the simplest model which may be assumed to represent concrete, it becomes clear that damping properties should be taken into consideration as well as the elastic properties. Thus, it would seem clear that knowledge of the coefficient of viscosity would be extremely useful in predicting the behavior of concrete. Although this property cannot be determined readily from sonic tests of concrete, the logarithmic decrement, which is a measure of the specific damping capacity, and is related to the coefficient of viscosity can easily be obtained from sonic tests.

The dynamic modulus of elasticity may be determined from the resonant frequency of the specimen and may be computed by the following equation(3):

$$E_D = C\ w\ f_o^{\ 2} \qquad (1)$$

where

E_D = dynamic modulus of elasticity

f_o = resonant frequency

w = weight of specimen

C = a factor which depends upon the shape and size of specimen, the mode of vibration, and Poisson's ratio.

Graphs are given by Pickett(3) for determining the size and shape factor C.

While the logarithmic decrement may be obtained in several ways, it may be obtained most easily by measuring the sharpness of the resonsnce curve and may be computed from the following approximate equation(4):

$$\delta = \pi \frac{(f_2 - f_1)}{f_o} \qquad (2)$$

where

δ = logarithmic decrement

f_o = resonant frequency

f_1, f_2 =frequencies on either side of resonance at which the amplitude of vibration is 0.707 of the maximum

This method of obtaining the logarithmic decrement was chosen because of its simplicity.

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3. Gerald Pickett, "Equations for Computing Elastic Constants from Flexural and Torsional Resonant Frequencies of Vibration of Prisms and Cylinders," Proceedings, Am. Soc. Testing Mats., Vol. 45, p. 846 (1945).

4. W. T. Thomson, "Measuring Changes in Physical Properties of Concrete by the Dynamic Method," Proceedings, Am. Soc. Testing Mats., Vol. 40, p. 1113 (1945).

APPARATUS

A schematic diagram of the apparatus used in this investigation is shown in Fig. 2. All equipment was selected and used so as to meet the requirements of the method of test ASTM designation C215. The oscillator is a Hewlett-Packard Model 200-I tuned circuit type. It covers a frequency range of 6 to 6000 cps and delivers approximately 10 volts. The oscillator feeds a Knight 10 watt power amplifier which in turn controls the driver.

The driver consists of a standard Jensen eight-inch speaker modified as follows: the protecting felt covering over the voice coil was removed and a small aluminum rod was cemented to the voice coil. A length of 1/4 in. round aluminum rod was then attached to the first rod to act as a driving rod. The driving rod was initially supported at the face of the speaker by a thin leather diaphragm. However, in the belief that excessive damping in the speaker may have been produced by this leather support, it was removed after the tests had begun and was replaced by two very thin brass straps. There was no noticeable change of damping in the speaker. As an aid in eliminating damping and also in reducing noise, a major part of the cone was removed. However, sufficient cone was left to adequately support the voice coil.

The specimen is vibrated by placing the 1/4 in. aluminum rod in firm contact with it. The vibration of the specimen excites a small crystal pickup in a Brush model KN-1 "Vibromike", which is a contact microphone of the inertia type with a frequency range from 30 to 6000 cps. Below 1000 cps the pickup delivers a voltage proportional to the acceleration it receives from the vibrating specimen. Above 1000 cps the voltage delivered is proportional to the velocity of the vibrating body.

Amplification for the pickup and a meter for indicating the strength of the signal picked up are combined in a single unit, a Hewlett-Packard Model 400-C, vacuum tube voltmeter. It has a frequency range from 20 to 2 million cps. The amplifier provides a gain of approximately 50db. A DuMont 274-A cathode ray oscilloscope with A5BP1A cathode ray tube is used with this equipment more as a matter of convenience than necessity. The oscilloscope may be used as an aid in checking the other equipment and is most frequently used to indicate the mode of vibration of the specimen.

Because of the accuracy required in determining the quantity $(f_2 - f_1)$ of Eq. 2, the frequencies are determined by use of a Berkaley Eput Meter Model 554B. With the use of this frequency counter it is possible to measure the frequencies from the oscillator to approximately one-tenth cycle per second. However, this accuracy was not needed from a practical standpoint, and the frequencies were determined only to the nearest cycle.

Equipment by other manufacturers may be used with equal facility. The particular components of this apparatus were used because of their availability.

The relative error in determining the modulus of elasticity, $\underline{E_D}$, caused by using incorrect frequency readings is, from equation1,

$$\frac{dE_D}{E_D} = \frac{dC}{C} + \frac{dw}{w} + \frac{2df_o}{f_o} .$$

This equation shows that for an error of 0.5 per cent in reading f_o, the error in E_D will be 1 per cent. The scales of most oscillators are marked in such a manner that it is not difficult to read them within 0.5 per cent error and a 1 per cent error is generally permissable. However, to assure that the error in reading the frequency is within 0.5 per cent, the oscillator must be correctly calibrated and the scale must be correct. This possible difficulty is avoided by the use of the frequency counter.

The relative error caused in the logarithmic decrement, δ by incorrect frequency readings is, from equation 2

$$\frac{d\delta}{\delta} = \frac{d(f_2 - f_1)}{(f_2 - f_1)} - \frac{df_o}{f_o}$$

Here the difficulty is in the determination of the quantity $(f_2 - f_1)$. The magnitude of this quantity is only about 0.01 of the resonant frequency. Hence an error of 9.5 per cent in reading f_1 and f_2 could cause an error of 100 per cent in the term $(f_2 - f_1)$. By using the frequency counter and taking the average of three readings to determine f_1 and f_2, the maximum error should be less than 2 per cent in logarithmic decrement, δ.

TEST PROCEDURE AND RESULTS

Because of the fundamental nature of this study only one size specimen, the standard 6-by 12-in. cylinder, was used. These were cast in steel molds. The concrete mixes were designed so as to cover almost any strength concrete that might be used on the majority of jobs. The water-cement ratio varied in such a manner that the slump varied from approximately one to seven inches.

Different curing procedures were used and some cylinders were tested wet and others dry. The mixes, curing conditions and results of both the dynamic and static tests are given in Table I. All experimental values represent the average of five cylinders, except in a veryfew cases where only four were tested.

The fine aggregate was a Wabash River sand with a fineness modulus of about 3.0. The coarse aggregate was a well-rounded Wabash River gravel of 1" maximum size. Both of these aggregates passed the usual specification tests. The cement used was a Type I portland cement.

The "double hump" characteristics of the frequency-response curves for certain concrete specimens, as noted by Thomson, were also observed by the authors in only a few of the specimens tested. The following observation was made in testing 6 by 12 in. concrete cylinders. If a double hump was noted, either hump could be made the predominating one by merely changing the plane of vibration. This phenomenon may be caused either by the inherent heterogeneity of concrete or by a slight lack of symmetry in the shape of the specimen. As a result, all concrete cylinders tested in this investigation were vibrated in three planes at angles of approximately 120° to each other. These readings were averaged and the result was considered to be the reading for the specimen.

A graphical presentation showing the influence of water-cement ratio, age and curing on the logarithmic decrement is given in Fig. 3. Four conclusions may be drawn from the figure:

1. If the curing conditions are the same the logarithmic decrement decreases with an increase in age.

2. The logarithmic decrement decreases as the moisture content of the specimen decreases.
3. The logarithmic decrement becomes less dependent on water-cement ratio as the moisture content decreases.
4. Logarithmic decrement, if used alone, is not a measure of concrete strength.

The influence of strength, age and curing on the dynamic modulus of elasticity is shown in Fig. 4. The following conclusions may be drawn from this figure:

1. For the same curing conditions the modulus of elasticity increases as the strength increases.
2. If the concrete is kept moist the modulus of elasticity increases with age, and if the concrete is allowed to dry the modulus of elasticity decreases with age.
3. Modulus of elasticity, if used alone, is not a measure of concrete strength.

The conclusion that neither logarithmic decrement nor modulus of elasticity alone is sufficient to estimate the compressive strength was predicted in the introduction. It was further suggested that the combination of logarithmic decrement and modulus of elasticity should be useful in predicting the compressive strength. The inner relation of these three variables taken from Table I is shown in Fig. 5.

In general the lines of equal modulus of elasticity which have positive slope indicate concretes which have been moist cured until tested in a wet condition; those with negative slopes indicate concrete which has been allowed to dry, after an initial moist curing period, and tested dry.

With the aid of Fig. 5 it is a simple matter to predict the compressive strength of concrete. The diagram should be entered on the horizontal axis with the logarithmic decrement and this point projected upward to the proper value of the modulus of elasticity; the compressive strength can then be read on the vertical axis. The concrete strength determined in this manner from Fig. 5 should agree with the values given in Table I within ten per cent regardless of age, mix, or curing. The concrete was in all cases a workable mix. Additional experience at the University of Illinois with cylinders whose mix, age, and curing were unknown to the authors indicated that compressive strength could be predicted with an average error of about three per cent.

To estimate the strength of concrete using other aggregates such as light-weight aggregates, other types of cement and mixes containing admixtures, this diagram may have to be modified.

ACKNOWLEDGMENTS

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TABLE I

MIXES, CURING, AND TEST RESULTS

Series	Mix by Weight C: S : G	w/c by Weight	Cured		Tested*	Age Days	E_D psi.	δ	f'_c psi.
			Moist Days	Dry Days					
IA	1:1.72:2.85	0.467	7		W	7	5.16×10^6	0.049	3920
			7	7	D	14	5.17	0.033	4760
			7	21	D	28	4.84	0.029	4720
			7	84	D	91	4.63	0.023	5000
IB	1:1.74:2.87	0.446	7		W	7	5.31	0.050	4390
			14		W	14	5.52	0.044	4590
			28		W	28	5.92	0.041	5430
			91		W	91	6.25	0.038	6080
			7	7	D	14	5.30	0.031	4890
			7	21	D	28	5.32	0.026	5930
			7	84	D	91	5.20	0.023	5870
			7	1	D	8	5.20	0.040	4340
IC	1:1.77:2.92	0.420	7		W	7	5.54	0.046	5270
			7	7	D	14	5.48	0.031	5520
			7	21	D	28	5.35	0.027	6970
			7	84	D	91	5.05	0.023	6790
IIA	1:2.28:3.31	0.555	7		W	7	4.89	0.053	3410
			7	7	D	14	5.18	0.033	4530
			7	21	D	28	4.98	0.028	5070
			7	84	D	91	4.94	0.023	5370
			7	1	D	8	5.05	0.042	3640
			7	1	D	**9	5.07	0.054	3690
IIB	1:2.31:3.34	0.530	7		W	7	5.05	0.052	3610
			14		W	14	5.40	0.047	4520
			28		W	28	5.71	0.044	4830
			91		W	91	6.01	0.040	5390
			7	7	D	14	5.21	0.035	4380
			7	21	D	28	5.38	0.027	5530
			7	84	D	91	5.08	0.022	5720
IIC	1:2.33:3.39	0.498	7		W	7	5.50	0.050	4240
			7	7	D	14	5.42	0.032	5200
			7	21	D	28	5.27	0.028	5460
			7	84	D	91	5.08	0.024	5710
			7	1	D	8	5.16	0.039	3700
			7	1	W	**9	5.27	0.047	3700

(continued)

TABLE I

(Continued)

Series	Mix by Weight C: S : G	w/c by Weight	Cured		Tested*	Age Days	E_D psi.	δ	f'_c psi.
			Moist Days	Dry Days					
IIIA	1:3.14:3.93	0.681	7		W	7	$4.69\text{x}10^6$	0.055	2980
			7	7	D	14	4.71	0.038	3550
			7	21	D	28	4.81	0.030	4110
			7	84	D	91	4.64	0.025	4040
IIIB	1:3.17:3.96	0.651	7		W	7	4.83	0.054	2860
			14		W	14	5.22	0.048	3920
			28		W	28	5.28	0.046	4040
			91		W	91	5.59	0.043	4370
			7	7	D	14	4.83	0.034	3580
			7	21	D	28	4.77	0.028	4120
			7	84	D	91	4.55	0.023	4240
			7	1	D	8	4.82	0.049	2730
IIIC	1:3.22:4.03	0.612	7		W	7	5.03	0.053	3350
			7	7	D	14	5.10	0.034	4160
			7	21	D	28	5.03	0.029	4260
			7	84	D	91	4.94	0.023	4980
IVA	1:3.79:5.28	0.860	7		W	7	4.02	0.059	1390
			7	7	D	14	4.14	0.042	1710
			7	21	D	28	4.13	0.034	2070
			7	84	D	91	3.97	0.021	2680
VC	1:4.36:6.19	0.860	7		W	7	4.65	0.053	1890
			14		W	14	4.92	0.050	2300
			28		W	14	4.92	0.050	2300
			91		W	91	5.43	0.046	2940
			7	21	D	28	4.18	0.035	2200
			7	84	D	91	4.24	0.025	2470

* W - Tested Wet
D - Tested Dry

** Soaked in water one day after being cured seven days moist and one day dry.

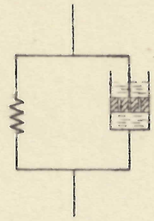

FIG. 1 Simplest Mechanical Model which can be considered to represent Concrete

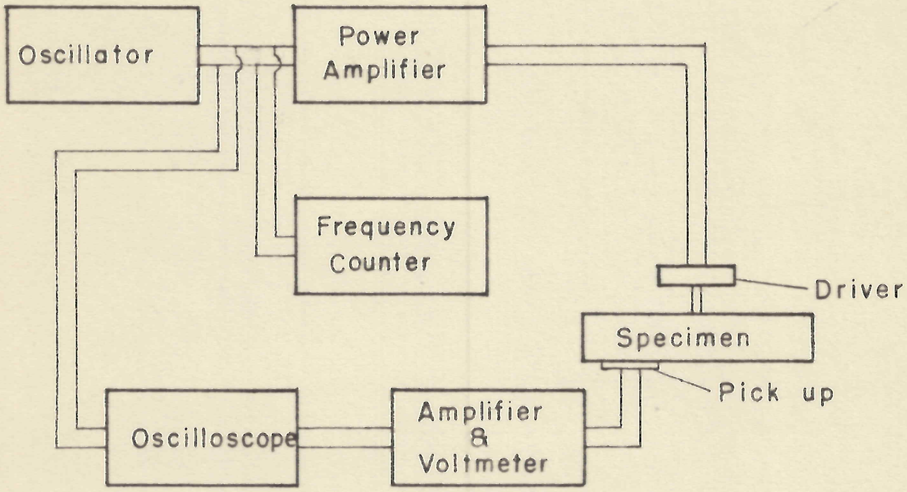

FIG. 2 Schematic Diagram of Apparatus

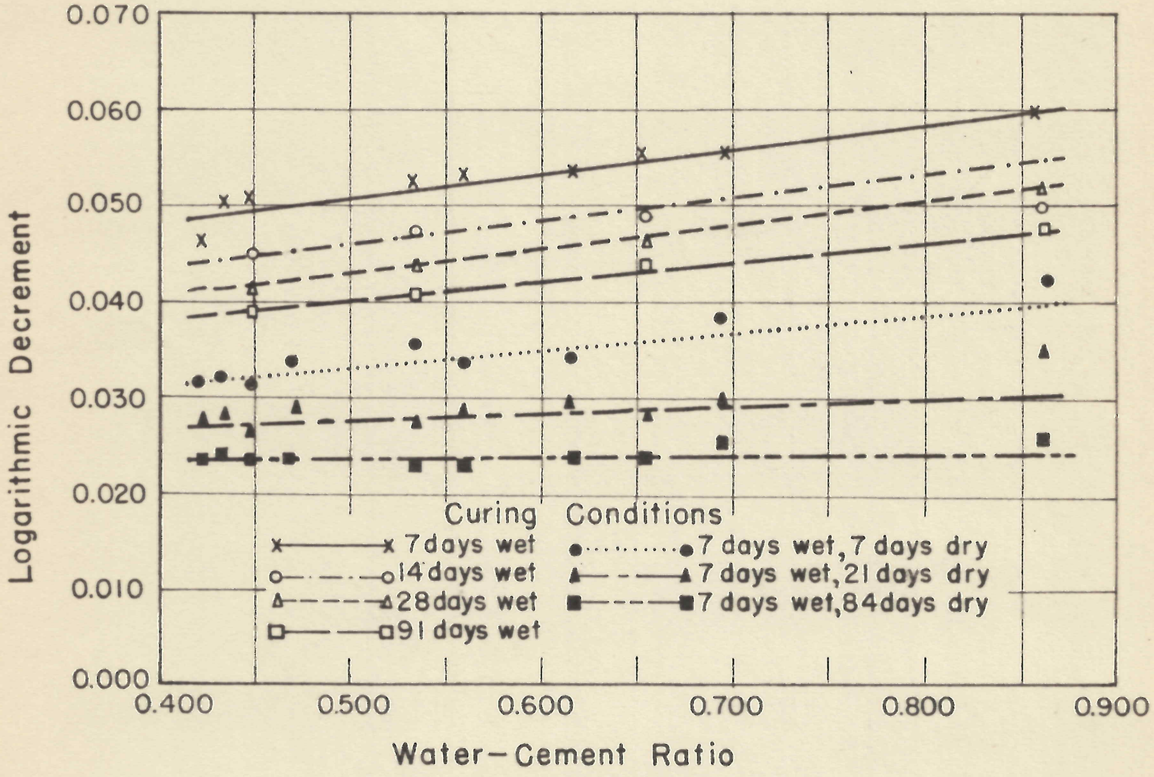

FIG. 3 Influence of Curing on Logarithmic Decrement

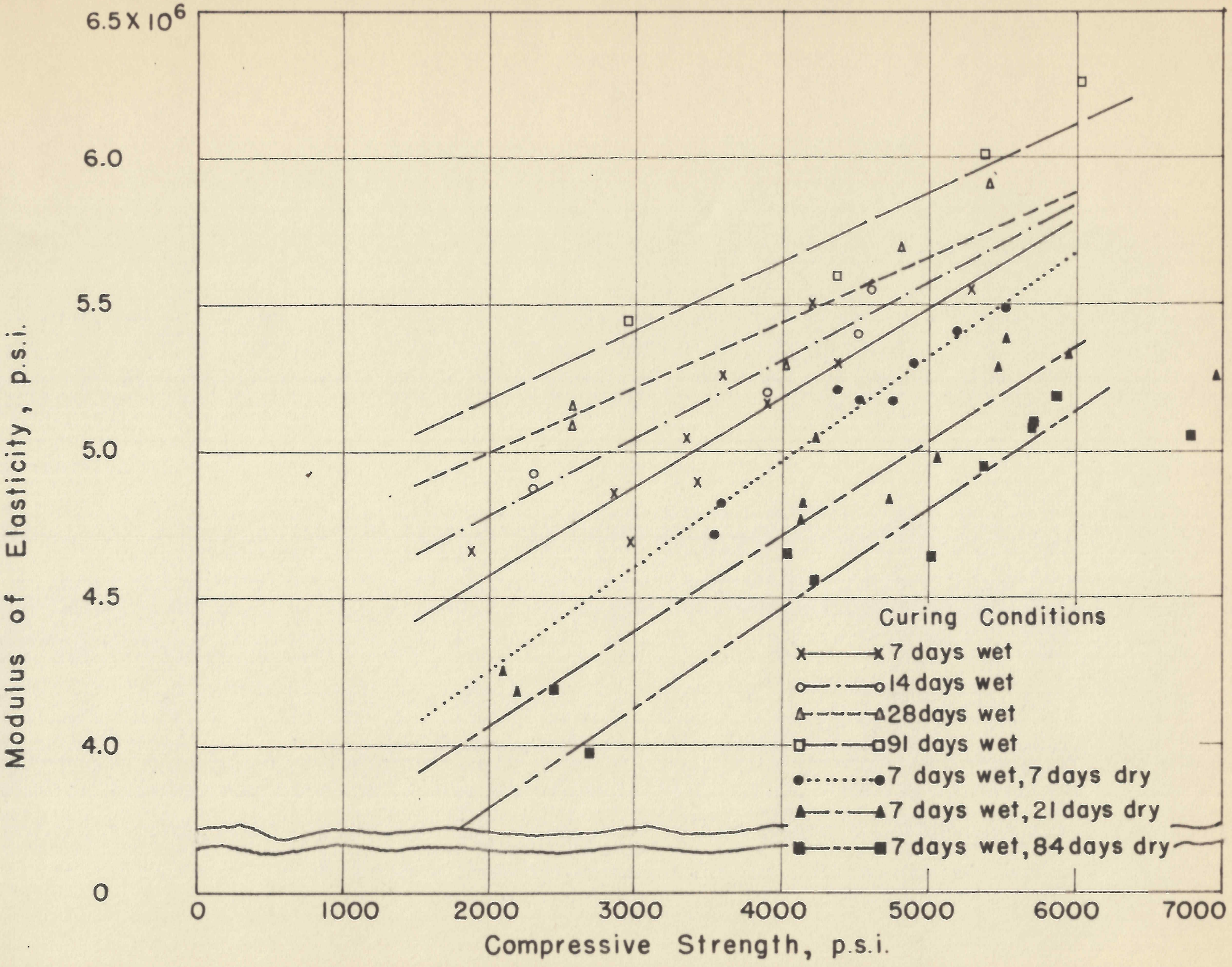

FIG. 4 Effect of Curing on the Dynamic Modulus of Elasticity

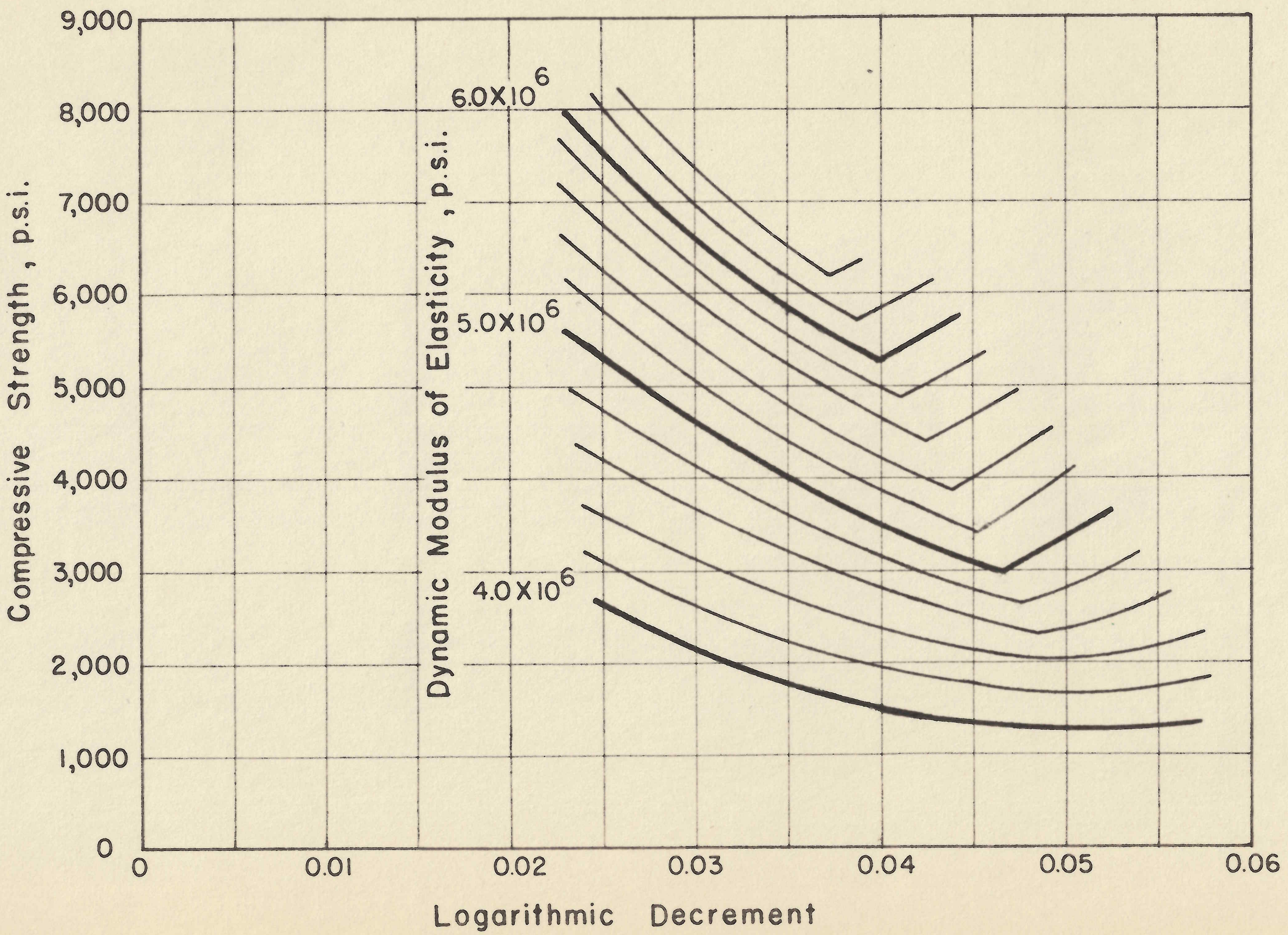

FIG. 5 Relation between Dynamic Modulus of Elasticity, Logarithmic Decrement, and Compressive Strength