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RELAXATION OF RESIDUAL STRESSES DUE TO FATIGUE LOADING

by

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Abstract

Macro-residual stresses are usually accidentally or purposely induced in parts during fabrication. These stresses may change if the part is subjected to fatigue loading in service. To make an estimate of the effect of residual stresses on the life of the part, one must know the magnitude and importance of this change.

Axial fatigue results can be used to determine the mechanism by which residual stresses at the surface of a member relax due to fatigue loading.

Such an approach leads to the following conclusions:

1. Little relaxation of residual stresses should be expected for alternating stresses near the fatigue limit except for "soft" materials.

2. Because the fatigue limit of "soft" metals is, in general, relatively insensitive to mean stress, partial relaxation of residual stress will affect the fatigue limit only slightly.

3. For the purpose of estimating the fatigue limit of a member, the <u>initial</u> residual stress at the surface should be considered to remain unchanged during fatigue loading. Treating this stress as part of the service stresses, proceed as in any other fatigue problem in which mean stresses are present.

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Introduction

To manufacture parts, metal is heated, deformed, abraded, hacked, or in some way forced to assume a useful shape. As a result there exists in the part, internal misfits. Some regions simply do not fit into the space available. Internal stresses arise.

These residual stresses are usually greatest at the surface where the largest plastic deformation occurs during fabrication. Below the surface, complex tri-axial states of stress may exist.

There are other complications. The surface hardness and roughness of the part may be altered. Such parts often experience repeated loads in service which cause fatigue failures to initiate at the surface where the residual stresses are greatest.

Residual stresses have the effect of shifting the range of fatigue stressing much the same as mean stresses do in an axial fatigue test. Thus, compressive residual stresses are considered beneficial. Tensile residual stresses are detrimental.

The designer must insure satisfactory fatigue performance in the presence of still another complicating factor. Residual stresses in a member do not necessarily remain unchanged during its life. Possible sources of relief and redistribution of residual stresses in metals have been outlined by Richards.(1) One source is the fatigue loading itself. Yielding will occur at points where the sum of the residual stress and the applied stress exceeds the flow stress of the material. The material at

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* Numbers in parentheses refer to the list of references at the end of this paper.

these points will then "fit" better. Residual stresses change. The change will be largest early in the life of the part. Later in the life, the change for a single cycle may be so small as to be imperceptible. However, millions of such cycles may cause an accumulative effect which is large.

It would seem that before the significance of residual stresses in fatigue can be appreciated, it is first necessary to know the influence of fatigue on residual stresses.

1.0 OBJECT AND SCOPE

Experimental investigations pertaining to the change of macro-residual stresses due to fatigue loading are summarized and interpreted. The mechanism by which residual stress is believed to change is formulated. An attempt is made to answer the following questions:

1. What determines the amount of change?
2. How should the designer account for this change?

2.0 SOME EXPERIMENTAL EVIDENCE

Residual stresses may be decreased, increased, or left unchanged by fatigue loading. For example, Fig. 1 shows the decrease of tri-axial residual stresses measured by Bühler and Buchholtz.(2) Residual stresses were measured before and after fatigue testing (8.36×10^6 cycles). No attempt was made to determine the cyclic rate of change.

Change in residual stress is largest for the first few cycles. In fact, Wallace and Frankel(3) found that the residual stress in a structural steel member was removed by a single cycle of fatigue loading.

Moore(4) measured the percent change of residual stresses as a function of the number of fatigue cycles. These results are shown in Fig. 2. Here the fraction of the initial residual stress remaining after a given number of cycles is plotted versus the log of the number of cycles. Notice that no relaxation occurred in the surface hardened steel. In the soft structural steel there was a progressive decrease in the residual stress as the number of cycles increased.

The progressive nature of the relaxation of residual stresses due to fatigue loading received considerable attention by a Japanese investigator. Taira(5) induced surface compressive residual stresses of about 25,000 psi. by prestraining unnotched soft steel specimens. These were subjected to repeated bending loads. The residual stresses were determined at the surface periodically by X-ray. Taira's results are shown in Fig. 3. Notice that the rate of relaxation is greatly dependent on the amplitude of alternating stress. After 10^4 cycles the plot of the fraction of remaining residual stress versus log of the number of cycles is practically linear.

Elsesser and Corten(6) report, "---an increase in the residual stresses in the outermost fibers of the beams with increasing number of load repetitions." These tests were performed on SAE 1035, 1045, and 4340 steels. Initially stress free beams were loaded from zero to maximum. Rosenthal and Sines(7) also found an increase in the residual stress at the root of a notch in aluminum specimens subjected to zero to maximum loading.

For hard steel (Rockwell hardness C 59), Tarasov, Hyler, and Letner(8) reported, "Cyclic stressing did not cause any measurable reduction in the magnitude of the residual grinding stresses." This comment was based on cyclic tests with applied stresses near the fatigue limit.

The experimental evidence indicates that the factors which determine the change of residual stress during a fatigue test are:

1. Composition and hardness of the material.
2. Magnitude and sense of the initial stresses present.

3. Magnitude and direction (whether reversed or unidirectional) of the applied fatigue stress.

4. Number of cycles of fatigue loading.

There have been no comprehensive experiments to directly determine the relative importance of these factors. Many experimental difficulties make it impractical to carry out such a program. For example, methods of inducing residual stresses such as shot peening or quenching may alter the surface material drastically. Techniques for determining residual stresses are tedious and time consuming, making extensive testing prohibitive.

In the next section an experimental approach is described which overcomes some of these difficulties.

3.0 EXPERIMENTAL APPROACH

In Fig. 4(a) a member is shown with longitudinal residual stresses present. To simplify the problem, only the surface element shown in Fig. 4(b) will be considered. It is held in place by the surface residual stress, σ_o. This means that the surface element would be longer by the amount, ϵ_o, if it were removed from the surrounding material.

Presume the material has a stress-strain curve as shown in the right portion of Fig. 4(b). The amount of stress and strain in the element is shown as a point on the stress-strain diagram.

Apply an axial load or bending moment to the member. Choose the direction of loading such that the element shown in Fig. 4(c) is subjected to the compressive strain, ϵ_a. Provided $\epsilon_o + \epsilon_a$ is large enough, yielding will occur in the surface element. The stress on the surface element will then by σ_y, the flow stress of the material.

Unload the member. Provided only a small region of the total cross-section experiences yielding the portion which remains elastic will restore the strain in the surface element near the original value, ϵ_o. The stress will decrease to the value σ_1 as shown in Fig. 4(d).

Repeated loading of the member would cause additional relaxation of the surface residual stresses.

Why not consider the surface element of Fig. 4 to be analogous to a uniaxially loaded fatigue specimen? Initial stresses and strains in such an axially loaded filament are analogous to the initial stresses and strain present in the surface elements of members with residual stresses. The amplitude of alternating strain imposed on the axial fatigue specimen is analogous to the repeated strains caused in the surface element of the member due to fatigue loading. The change of stress from the original value due to the fatigue loading is analogous to the change of residual stress in the member.

There is justification for using this simple approach to study the change of residual stresses in members subjected to fatigue loading. The justifications are:

1. Fatigue failure usually nucleates at the surface. Thus, a knowledge of what happens at the surface is sufficient to determine the fatigue behavior of the part.

2. Service stresses are normally not large enough to cause excessive yielding of the part as a whole. Yielding takes place at discrete regions where the stresses are highest. This feature makes it possible to use the simplifying assumption that surface

elements experience repeated strain when the member is subjected to repeated loads.

3. Inducing stress in this manner does not alter the surface finish, or the properties of the material being tested. Basic information is obtained for separating the effect of work hardening, surface finish, and residual stresses.

4. No complicated measurement of residual stress is needed. Large amounts of data are obtained relatively easy. The separate effect of the factors which influence the relaxation of residual stress can be exhaustively studied.

Such an experimental program and the resulting analysis are reported in the next two sections.

4.0 CYCLE-DEPENDENT STRESS RELAXATION TESTS

There follows a discussion of axial fatigue tests on SAE 4340[9] and the alloy A-286.[10] The mean strain and amplitude of alternating strain were held constant throughout the fatigue tests, as shown in Fig. 5. These conditions were intended to approximate the conditions of loading of the surface element of Fig. 4 discussed in the preceeding section.

Under these conditions, any accumulated inelastic action would cause the mean stress to decrease. This is also shown in Fig. 5. The relaxation or change of the mean stress is considered to be similar to the change of residual stresses in members subjected to fatigue loading. Since the stress depends on the number of cycles, this phenomenon has been called cycle-dependent stress relaxation.

Three levels of hardness were investigated for the SAE 4340 steel (Rockwell hardness C 30--Soft, 40--Medium, 50--Hard) and the hardness of the A-286 was C 25. Initial mean stresses ranging from 60,000 psi. in compression to 180,000 psi. in tension were investigated. Table I summarizes the experimental program and gives some of the pertinent data. For further details the reader is referred to References 9 and 10. Figures 6 through 11 are samples of cycle-dependent stress relaxation data, which are discussed below.

4.1 Discussion of Sample Relaxation Data

In Fig. 6 the fraction of the initial mean stress remaining at N cycles is plotted against the log of N. The material is SAE 4340 in the soft condition with an initial tensile mean stress of 60,000 psi. Each line represents the results of one specimen tested at a particular value of alternating stress.* The curves shown in this figure are simply to guide the eye and do not represent theoretical or expected behavior. On the semi-log plot used, these curves are nearly linear below N equal to approximately 10^6 cycles.

For alternating stresses which cause fatigue fracture to occur, there is a decrease of mean stress during the first cycle. The decrease continues as the number of cycles increases. For large amplitudes of alternating stress, the first cycle decrease of mean stress is large and the slope of the relaxation line on the semi-log plot used is large.

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* For convenience in this paper, the term alternating stress is used for the product of the alternating strain and elastic modulus, $\epsilon_a E$. The <u>actual</u> alternating stress is less than $\epsilon_a E$. early in the test but approaches this value after the first few cycles.

For smaller amplitudes there is little relaxation of mean stress in less than one million cycles. After a million cycles there is sometimes a decrease of the mean stress even for specimens which endure 10^7 cycles.

The test conditions for the data displayed in Fig. 7 are identical to those in Fig. 6 except for the initial mean stress which was in compression. Comparison of the two figures reveals that the behavior is nominally the same.

Figures 8, 9, and 10 give cycle-dependent relaxation data for the soft, medium, and hard conditions of SAE 4340. The initial mean stresses for these three figures are approximately 80 percent of the 0.2 percent offset yield strength for the three conditions of hardness. The influence of hardness is evident from these figures. For the soft condition, the relaxation occurs more readily than for the hard condition. For the medium condition the behavior is intermediate. As in Figs. 6 and 7, very little relaxation of mean stress took place before 10^6 cycles for specimens which did not fail.

Note the similarities between Figs. 6, 7, 8, 9, and 10 for cycle-dependent stress relaxation and Figs. 2 and 3 for change of residual stresses due to fatigue loading.

4.2 Alloy A-286

A sample of the relaxation of mean stress obtained for alloy A-286 is given in Fig. 11. While these results agree qualitatively with the results for SAE 4340 steel, a quantitative comparison is a bit disappointing. The A-286 used in this investigation was softer than the soft SAE 4340. On this basis, one would expect the relaxation of mean stress to be larger in this material for the same amplitude of alternating stress. Yet, a comparison of similar tests from Fig. 11 for A-286 and Fig. 8 for the soft SAE 4340, reveals that the SAE 4340 relaxed more than A-286 for the same alternating stress. In fact, for A-286 there is virtually no relaxation of mean stress after the first ten cycles.

Little relaxation of the mean stress occurred below 10^6 cycles for A-286 specimens which did not fail. For both SAE 4340 and A-286 at least 75 percent of the initial mean stress was still present after a million cycles for those specimens which did not fail in fatigue.

5.0 ANALYSIS OF CYCLE-DEPENDENT STRESS RELAXATION

Cycle-dependent relaxation is a result of material yielding. On the first cycle, yielding may be quite large. On ensuing cycles, yielding is small. It is, therefore, convenient to break the problem of analysis into two parts--first cycle relaxation, and relaxation for cycles subsequent to the first.

5.1 Mean Stress After One Cycle

In Fig. 12 the stress-strain response to the first cycle of alternating strain about a fixed mean strain is schematically shown. Assume the material to have a "flat top" stress-strain curve with a yield point, σ_y. Assume elastic unloading along a line parallel to the elastic slope, E. The following expression for the mean stress at the end of the first cycle, σ_1, can be obtained from inspection of the figure:

$$\sigma_1 = \sigma_y - \epsilon_a E = \sigma_y - \sigma_a \tag{1}$$

In this expression, ϵ_a is the amplitude of the alternating strain and $\sigma_a = \epsilon_a E$.

In Fig. 13 the mean stress at the end of the first cycle is plotted versus the alternating stress for all the SAE 4340 specimens in which large scale yielding occurred on the first cycle. Three lines are placed through the data for the soft, medium, and hard specimens. The lines are of the form of Eq. 1. The following values of σ_y were used:

Soft	123,000 psi.
Medium	172,000 psi.
Hard	222,000 psi.

These values are slightly less than the 0.2 percent offset yield strength for the three hardness levels given in Table I.

Good correlation of the mean stress at the end of the first cycle on the basis of Eq. 1 is possible for SAE 4340 as Fig. 13 demonstrates.

For A-286 no simple equation for correlating the mean stress at the end of the first cycle was found. This material demonstrated stress-strain properties which were more complex than the simple model shown in Fig. 12 from which Eq. 1 was derived. Stress-strain loops for A-286 at 1, 2, 5, and 10 cycles are shown in Fig. 14. Notice that two of the assumptions which were made in the derivation of Eq. 1 are violated during the first cycle. The stress-strain curve is <u>not</u> "flat top" and the material does <u>not</u> unload linearly along a line parallel to the elastic modulus line. Due to these factors, the mean stress at the end of the first cycle was always found to be above the value predicted from Eq. 1. The larger the strain amplitude and the initial mean stress, the larger the difference. An analysis of the first cycle relaxation taking these factors into account is given in Reference 10.

No appreciable change in the mean stress was found to occur during the first cycle for either the SAE 4340 or the A-286 if the sum of the alternating stress and the initial mean stress was below a value roughly equal to the 0.2 percent offset yield strength.

5.2 Relaxation of Mean Stress for Cycles Subsequent to the First

After the first cycle there should be no large scale plastic yielding for the condition of controlled strain used in this investigation. There might be, however, micro-inelastic action which could only be observed by accumulating these small strains over a large number of repeated strain cycles. A helpful model for considering this type of micro-plasticity is shown in Fig. 15. Here the deviation from linearity is exaggerated for clarity.

For the Nth cycle (shown in Fig. 15), the material is presumed to be strained along some nonlinear path to the maximum strain. It is then unloaded elastically to zero stress. It is presumed to follow the same shape path when loaded in compression as was followed in tension. Returning the strain to the initial mean value completes one strain cycle. The material experiences the small plastic strain $d\epsilon/dN$ which causes the stress-strain loop <u>to fail too close</u> by the amount $d\sigma/dN$. From the enlarged view at the bottom right of the figure, $d\epsilon/dN$ can be related to $d\sigma/dN$ as follows:

$$E \frac{d\epsilon}{dN} = - \frac{d\sigma}{dN} \tag{2}$$

Further study of this assumed stress-strain loop reveals that $d\epsilon/dN$ is the difference between the plastic strain at the maximum stress and the plastic strain at the minimum stress.

Assume the existance of a function of stress, $f_p(\sigma) = \epsilon_p$, which determines the amount of plastic strain for a given stress level.* The following equation can be written for the mean stress at N cycles:

$$-\frac{1}{E}\frac{d\sigma_N}{dN} = f_p(\sigma_{max}) - f_p(|\sigma_{min}|) \tag{3}$$

To solve Eq. 3 a function must be chosen which realistically represents the cyclic stress-plastic strain behavior of the material.

Such a solution has been made in Reference 9 and the resulting equations can be found there. In the same reference it is shown that the following approximate equation can be obtained by making certain simplifying approximations:

$$\frac{\sigma_N}{\sigma_o} = \frac{\sigma_1}{\sigma_o} - \left(\frac{\sigma_a}{\sigma_y}\right)^b \log N \tag{4}$$

In this equation, σ_N is the mean stress at N cycles, σ_o is the initial mean stress, b is a material constant dependent on the cyclic stress-strain properties, and the other parameters have

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* This function is assumed to be the same in tension as in compression and to remain unchanged by repeated straining of the material.

been defined previously.

Equation 4 is not applicable for values of σ_N/σ_o, less than about 0.20 nor for N greater than about 10^6 cycles.

To compare Eq. 4 to experimental results, Fig. 16 has been prepared. The bold lines are reproductions of representative experimental relaxation curves for SAE 4340 taken from Figs. 8, 9, and 10, and the dashed lines are theoretical curves obtained with the aid of Eqs. 1 and 4 with b = 5.7 and σ_y defined as before. The agreement can be seen to be good. Space does not permit graphical comparison of more data. However, it was found that the experimental values of σ_N/σ_o was within about ± 0.10 of the value from Eq. 4 for all SAE 4340 data reported in Reference 9 except for $N > 10^6$ and for values of $\sigma_N/\sigma_o < 0.20$.

As mentioned before, the cycle-dependent relaxation behavior of A-286 was substantially different from that of SAE 4340. Most of the relaxation occurred in the first ten cycles. Reasons for the difference in the behavior of SAE 4340 and A-286 can be found by looking at Fig. 14. After a few cycles of fatigue loading, the cyclic stress-strain behavior of A-286 is such that the hysteresis loop tends to join into a continuous loop (see the 10th cycle, Fig. 14). Under these conditions the model used for relaxation (Fig. 15) is no longer applicable since $d\sigma/dN$ approaches 0. The hysteresis loop reaches a stable shape which does not permit further relaxation of mean stress under conditions of controlled strain.

Despite the absence of cycle-dependent relaxation in A-286, it was evident that a broad hysteresis loop was present because specimens tended to heat excessively during testing.

Further details concerning cycle-dependent stress relaxation are not appropriate to the purpose of this paper. The question is--how can this information be used to better understand the change of residual stresses in parts which are subjected to fatigue loading? This question is taken up in the next section.

6.0 CHANGE OF RESIDUAL STRESSES DUE TO FATIGUE LOADING

The surface element of a member containing residual stresses has been considered to be analogous to an axially loaded fatigue specimen. On the basis of this analogy, the factors which are considered to determine the amount of change of residual stresses due to fatigue loading will be discussed.

6.1 Magnitude and Sense of Initial Residual Stresses and Applied Stresses

As Dolan(11) suggested, residual stresses should be considered "---a portion of the imposed loading pattern that the material must resist." For all types of loading the surface element is subjected to an alternating stress about a mean stress.* The mean stress will always tend to relax toward zero when the member is subjected to fatigue loading. That is, yielding or cycle-dependent relaxation will cause the cyclic stress at the surface to approach conditions of completely reversed stressing (mean stress = zero).

Surface residual stress in a member such as shown in Fig. 4 is analogous to mean stress if the member is subjected to

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* The amplitude of alternating load is considered to remain constant.

completely reversed loading. For the case of one directional loading of the member the residual stress will be the maximum or minimum stress in the cycle depending on its sense and the direction of the applied load. If the member is subjected to a mean load and an alternating load, the residual stress will not be the mean nor will it be the maximum or minimum stress.

A few examples will be used to demonstrate why residual stresses can decrease, increase, or remain unchanged.

6.11 Example I (Completely reversed loading)

Under these conditions the residual stress is the mean stress. Should the sum of the residual stress and the alternating stress at the surface be large enough, the residual stress will relax toward zero. Thus, the residual stress decreases. Examples have been cited from the literature and from the experimental program reported here in which such a decrease was observed.

6.12 Example II (Zero initial residual stresses-one directional loading)

The stress-strain response of the surface element is shown in Fig. 17. While the mean stress relaxes toward zero, the residual stress at the surface is changed from zero to a compressive value. Should the mean stress eventually reach zero, the residual stress at the surface of the member would be equal to the amplitude of the alternating stress. A residual stress induced by cycling an initially stress free bar has been reported by Elsesser and Corten(6) and is discussed earlier in this paper.

It is also possible for an initial residual tensile stress to be changed to a residual compressive stress in the same manner as shown in Fig. 17. Or compressive stresses can be changed to tension.

6.13 Example III (High residual stress present, but loaded such that the mean stress is zero)

These conditions are shown in Fig. 18. There is a large residual stress present, and the range of applied stress is large. The mean stress is zero. The stress-strain behavior of the surface material is such that no change occurs in the residual stress. Strain peening[12,13] is based on this principle. It insures dependable performance of leaf springs in service by inducing residual stresses of opposite sense to the applied stresses.

6.2 Hardness

For the same material the residual stress will be changed by fatigue loading more easily in the soft state than in the hard state, other conditions being equal. However, hardness alone is not a sufficient index. More significant is the cyclic stress-strain characteristics of the material. Such properties may be determined by tests similar to those reported above.

It is important to remember that a change in hardness at the surface of a member may be accomplished by work hardening or thermal softening. Therefore, a process such as shot peening would not only induce desirable compressive residual stresses but would also tend to insure that the residual stresses do not change as easily because of the increase in the hardness of the surface material.

6.3 Numbers of Load Repetitions

Most of the change in the residual stress will occur during the first few cycles. For sufficiently large ranges of applied alternating loading there will be a progressive change in the residual stress as the number of cycles increases.

For alternating stresses near the fatigue limit, the residual stress probably does not change much after the first cycle. There is some evidence in the case of SAE 4340 (see Fig. 6) that after about 10^6 cycles rather marked changes can occur. The data is so scant in this region that no definite statements can be made concerning this behavior at present.

With the data and interpretation offered above, we are now ready to proceed to the question, "How should the designer account for changes of residual stresses."

7.0 HOW SHOULD THE DESIGNER ACCOUNT FOR THE CHANGE OF RESIDUAL STRESSES?

This question can only be partially answered at present.

7.1 Estimate of the Fatigue Limit of a Part

Residual stresses should be looked upon as a portion of the stresses which the surface material must resist. These stresses will not change in hard materials with moderately low residual stresses for amplitudes of alternating stresses near the fatigue limit. Even in a soft material there will be little change when the member is stressed near the fatigue limit. Hard materials are more sensitive to mean stress than are soft materials. This can be shown by Fig. 19 which is data on SAE 4340 from Reference 9. A change of residual stress would only occur in materials which are relatively insensitive to the change.

A reasonable estimate can be made of the fatigue limit of a member or part which has moderately low initial residual stresses, in the following manner: (1) Consider the initial surface residual stresses to be superimposed on the service stresses; (2) Consider these stresses to remain unchanged during fatigue loading; and (3) Treat the problem as any other in which a mean stress is present.

For high initial residual stresses or a load pattern which produces a stress greater than the yield strength of the material at the surface, use Eq. 1 to determine the residual stress present at the end of the first cycle. Consider that the residual stresses do not change after the first cycle and proceed to estimate the fatigue limit as before. An example of this proceedure will be found in a paper by Felgar.[14]

7.2 Fatigue Life

For stresses above the fatigue limit there is almost certain to be a change of the residual stresses. With present knowledge, it would be difficult to take this into account. One thing can be said, however. For sufficient high alternating stresses or for soft metals, the surface material experiences completely reversed stressing after the first few cycles. Initial residual stresses therefore have very little effect. An estimate of the life can be made using a completely reversed (zero mean stress) S-N curve for the material. Figure 20 shows the futility of initial residual stresses for large amplitudes in soft materials.

This same figure shows diagramatically how the life would be affected by change in residual stress due to fatigue loading in the region between the fatigue limit and very high stress amplitudes. The fatigue life for this region will be between the life for zero mean stress and the life for a mean stress which is maintained throughout the life of the part and is equal to the residual stress. For stresses near the fatigue limit, the fatigue life will be near to the life for the S-N curve in which a mean stress was maintained. For very high stresses the life will be near to the S-N curve for completely reversed conditions. This is still a zone of uncertainty in which the designer must exercise his famous judgment.

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TABLE I

TEST PROGRAM (9)(10)

Material	Rockwell Hardness C Scale	Number of Specimens in Set	Initial Mean Stress psi.	Approximate Fatigue Limit, psi.
SAE 4340	29	15	0	57,000
SAE 4340	29	13	60,000	45,000
SAE 4340	29	13	100,000	36,000
SAE 4340	29	9	-60,000	66,000
SAE 4340	41	6	0	60,000
SAE 4340	41	10	60,000	42,000
SAE 4340	41	12	140,000	30,000
SAE 4340	50	18	0	80,000
SAE 4340	50	14	60,000	60,000
SAE 4340	50	21	120,000	20,000
SAE 4340	50	7	180,000	15,000
Alloy A-286	25	7	0	58,000
Alloy A-286	25	11	30,000	55,000
Alloy A-286	25	9	60,000	46,000
Alloy A-286	25	8	80,000	41,000
Alloy A-286	25	9	-60,000	65,000

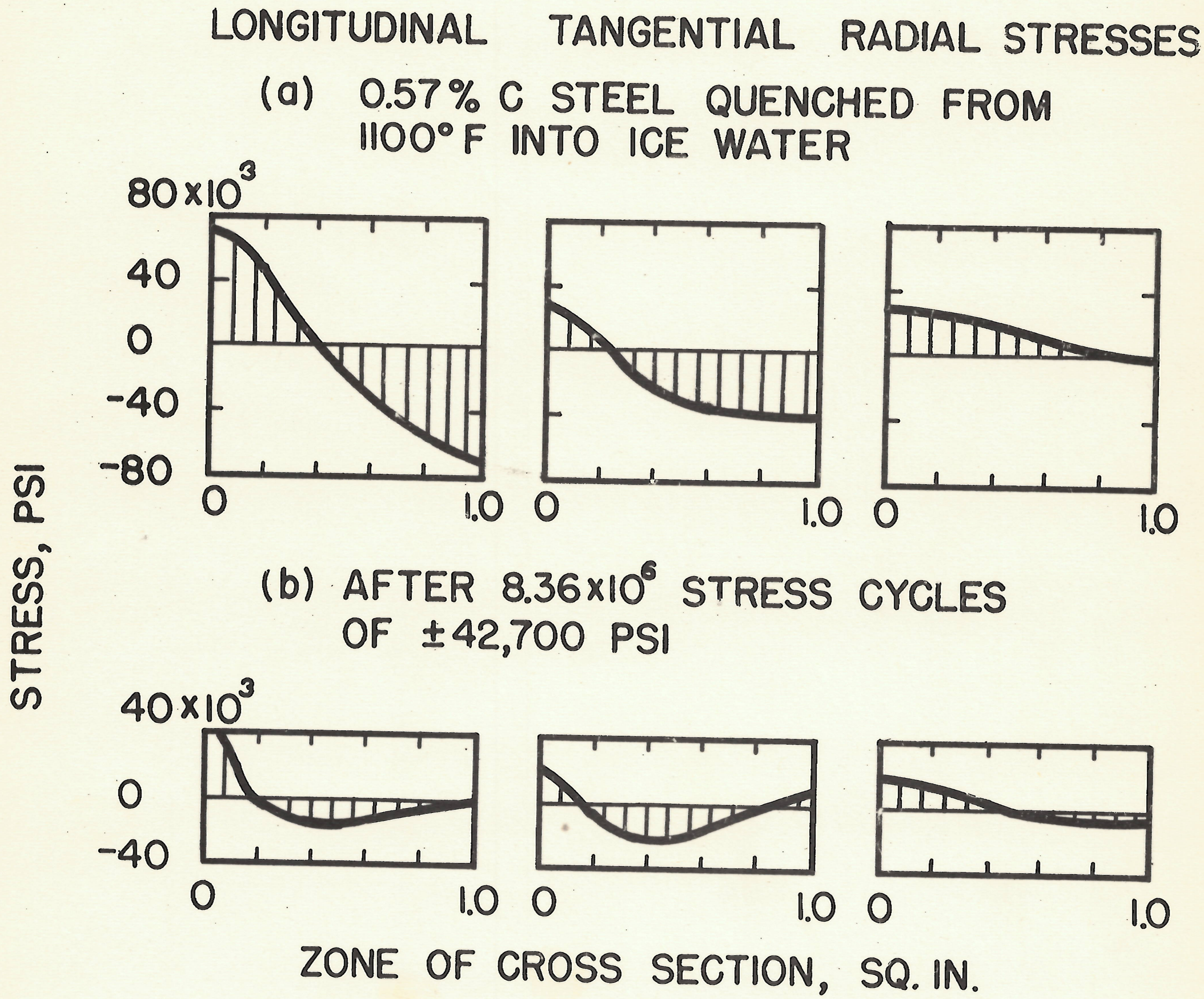

FIG. 1 CHANGE OF RESIDUAL STRESSES DUE TO FATIGUE LOADING, (2)

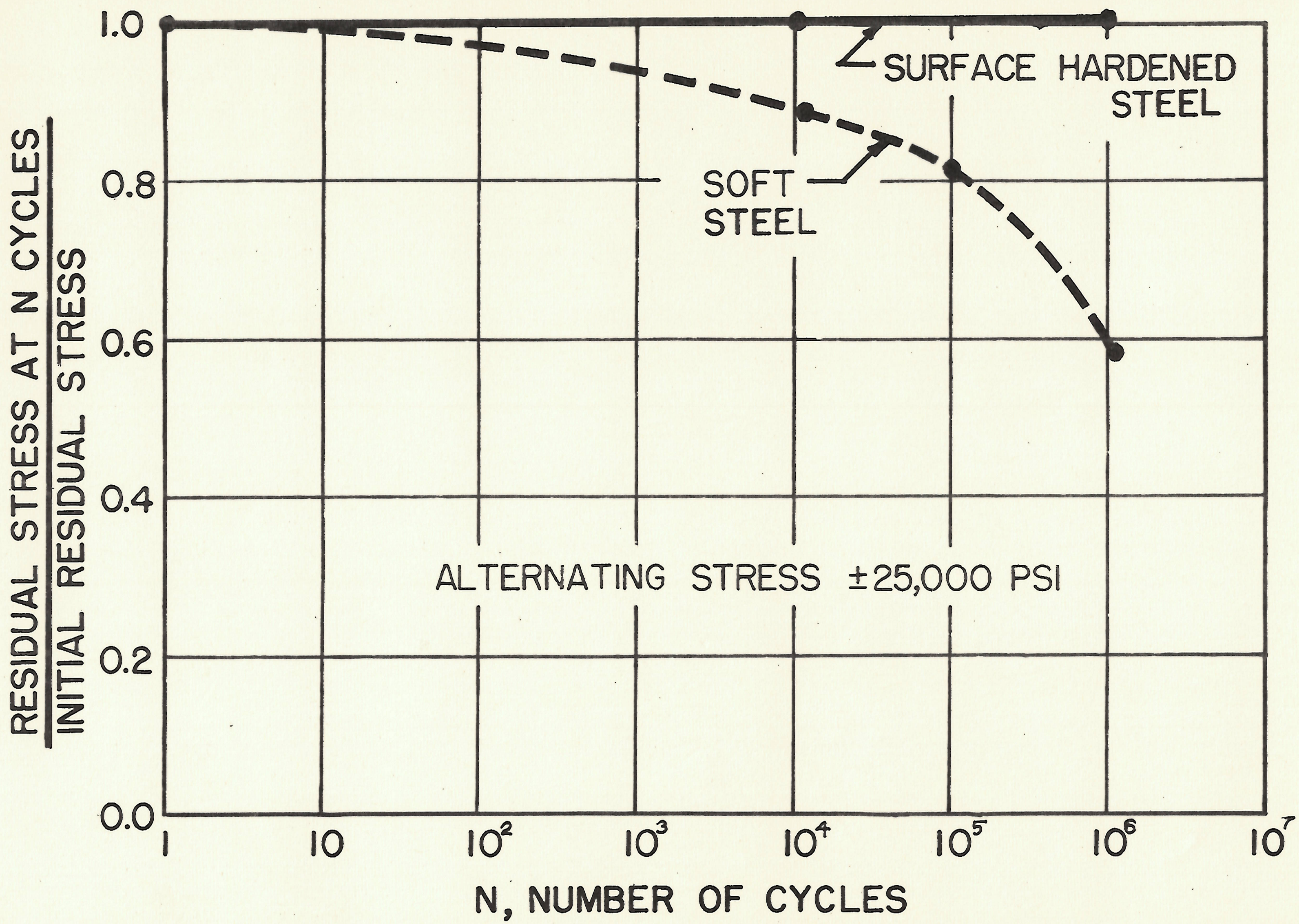

FIG. 2 CYCLE-DEPENDENT CHANGE OF RESIDUAL STRESSES IN SOFT AND HARD STEEL, (4)

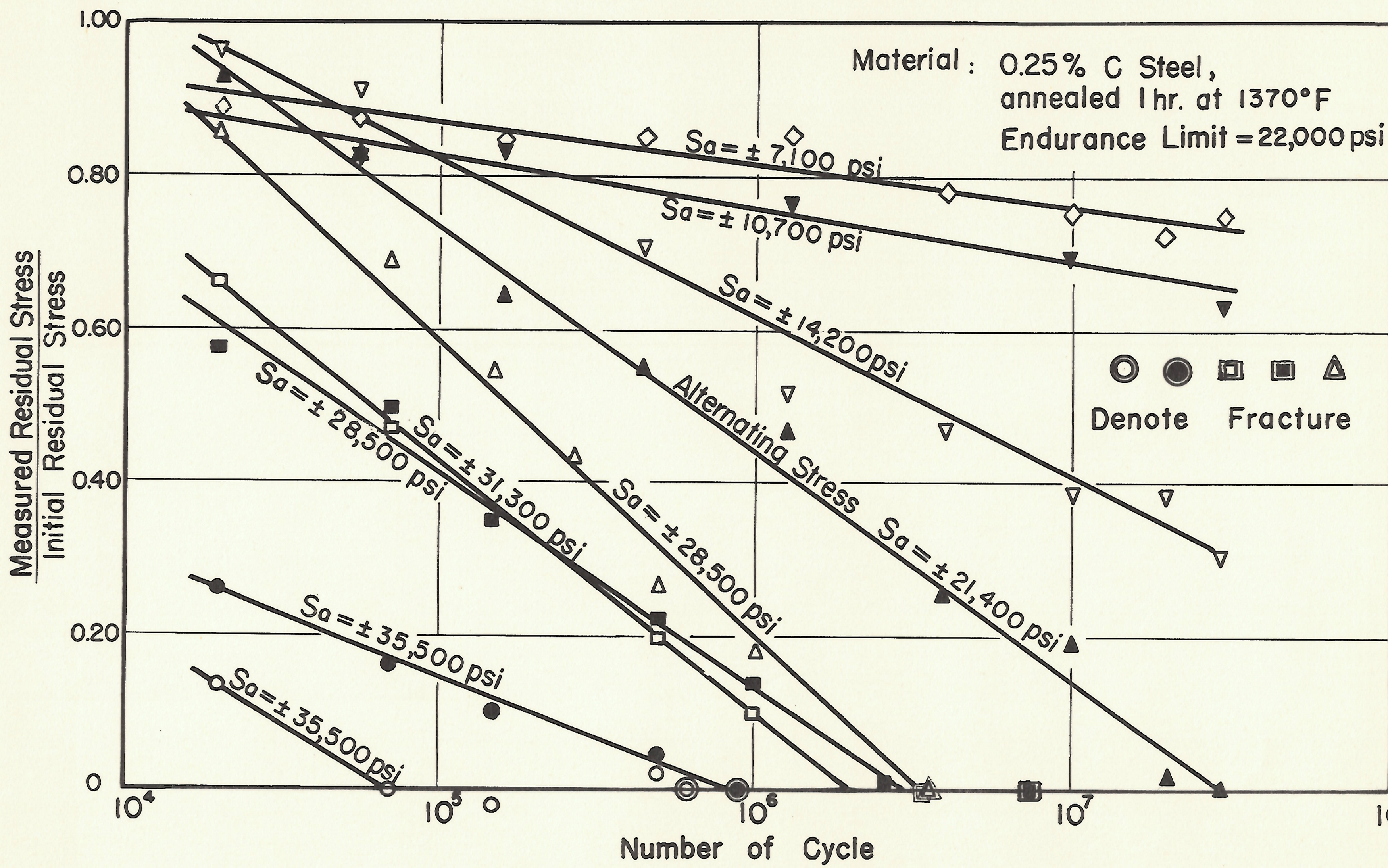

FIG. 3 DECREASE OF RESIDUAL STRESS DUE TO REPEATED LOADING, (5)

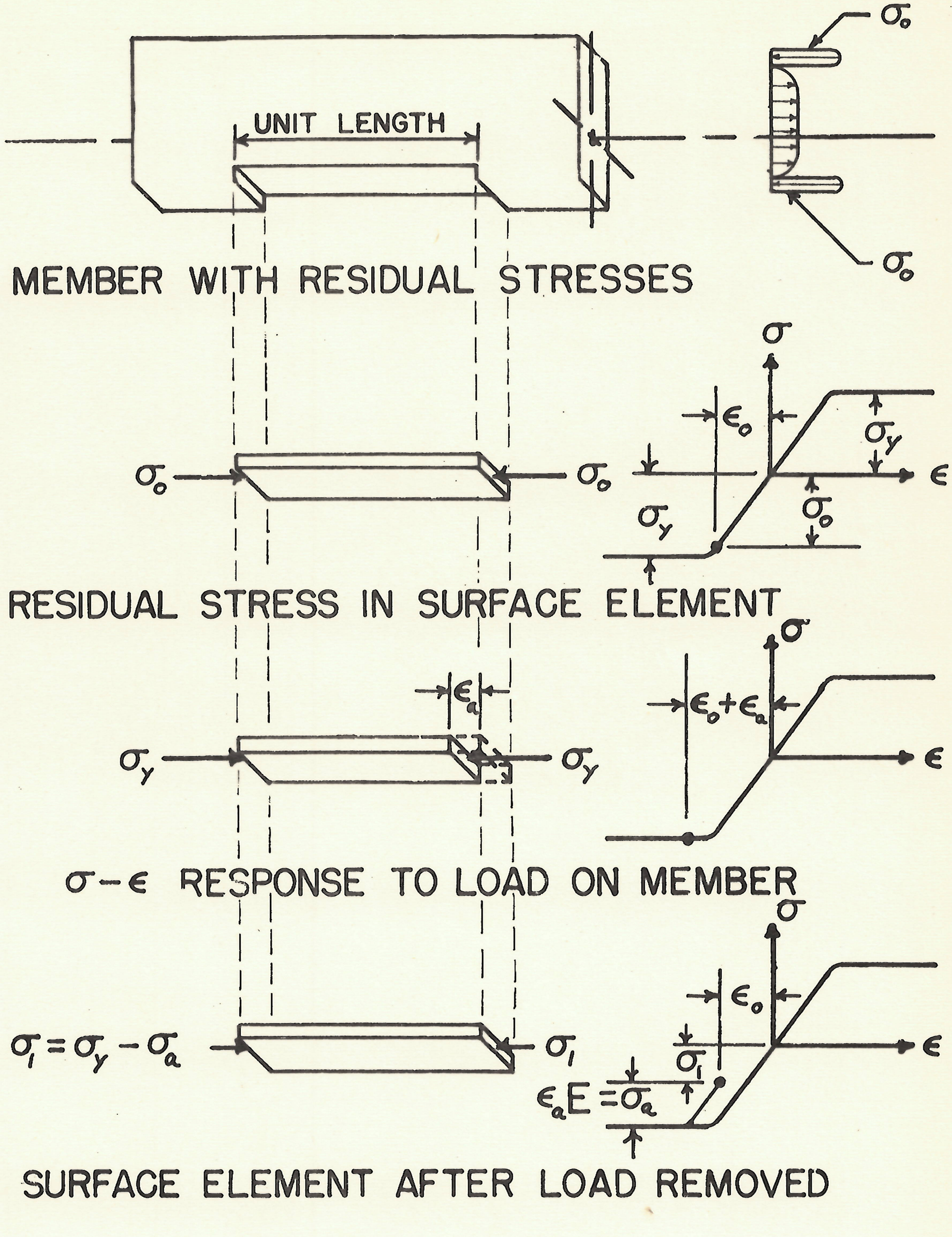

FIG. 4 ILLUSTRATION OF RESIDUAL STRESS CHANGE

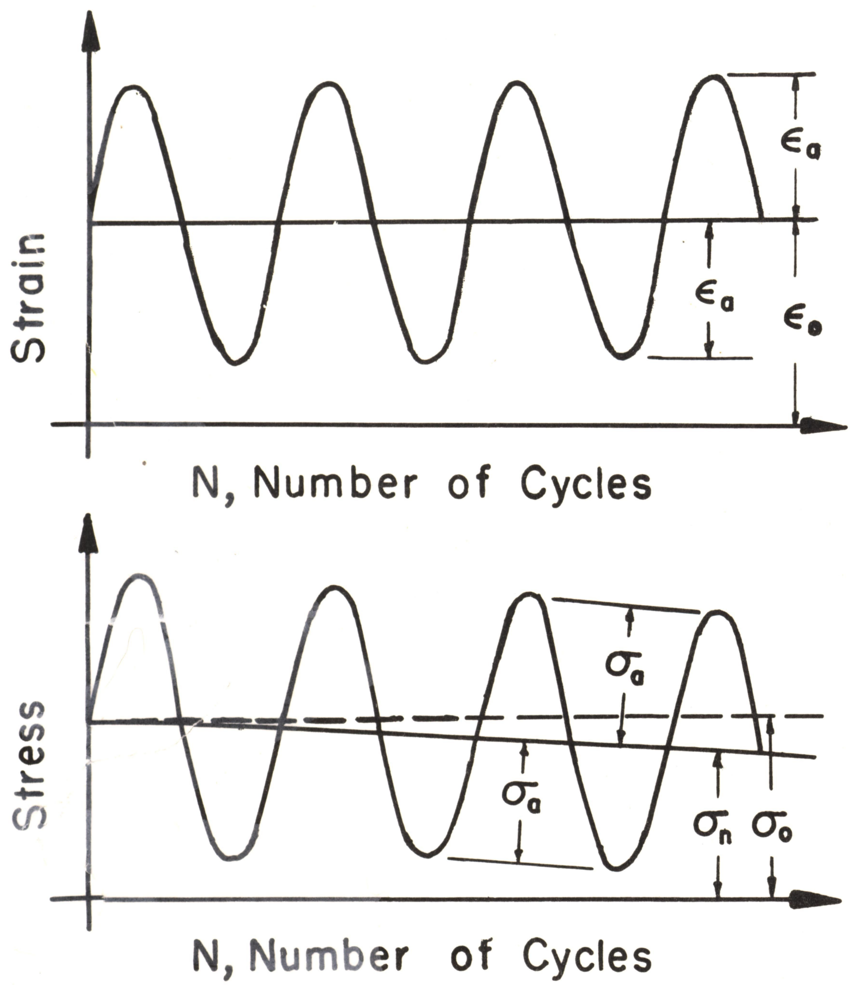

FIG. 5 SCHEMATIC OF TEST CONDITIONS

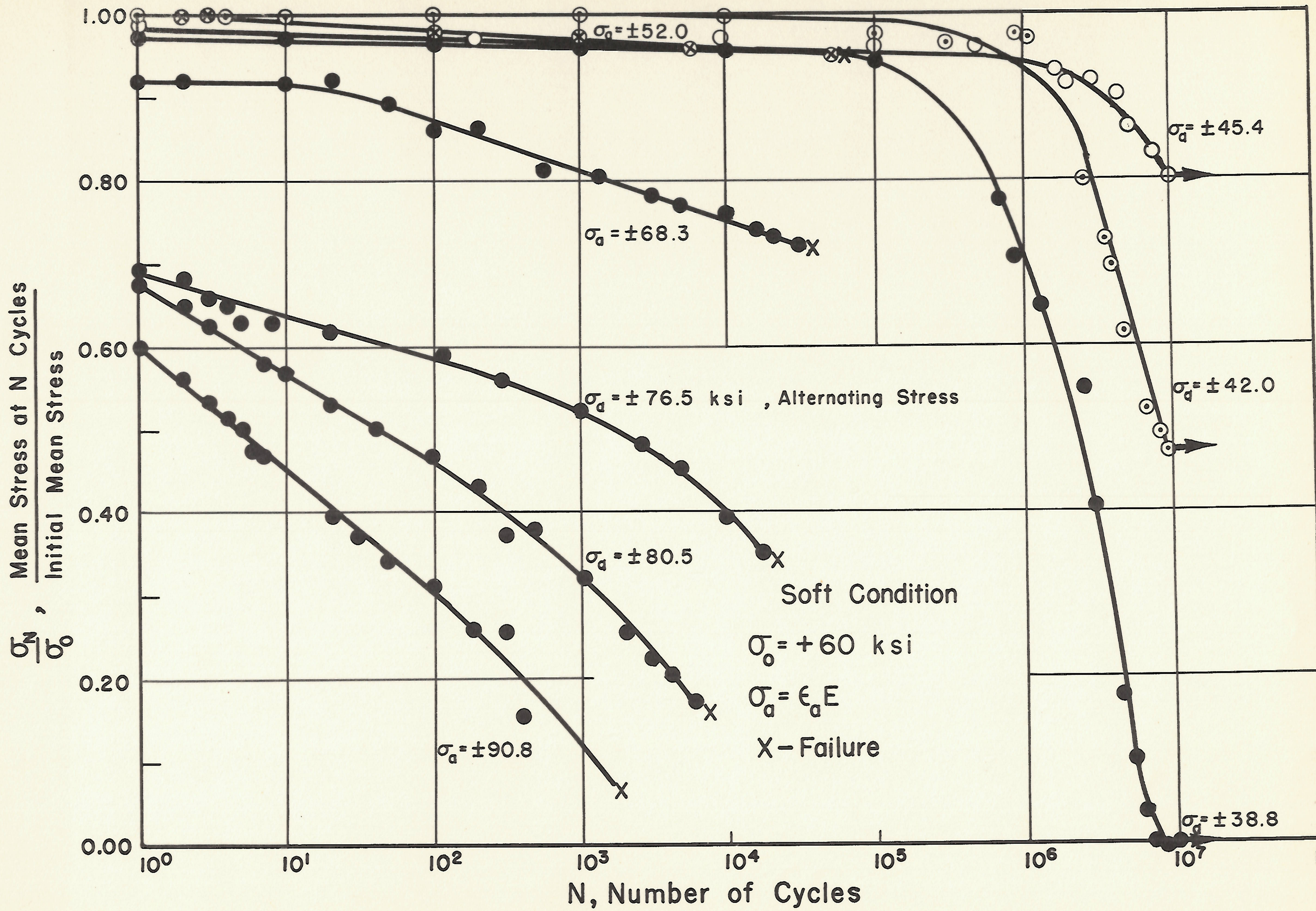

FIG. 6 RELAXATION OF MEAN STRESS, SAE 4340, (9)

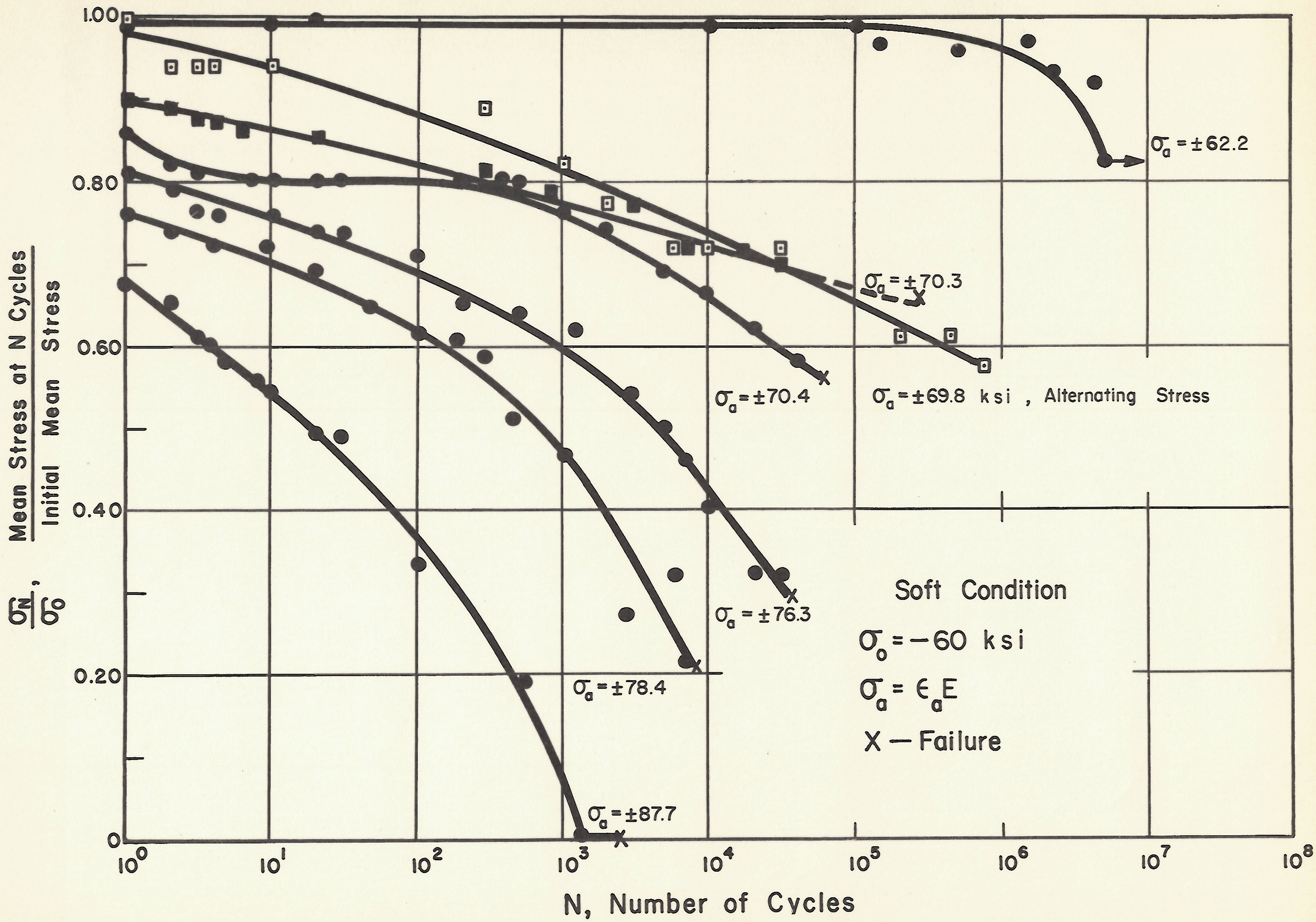

FIG. 7 RELAXATION OF MEAN STRESS, SAE 4340, (9)

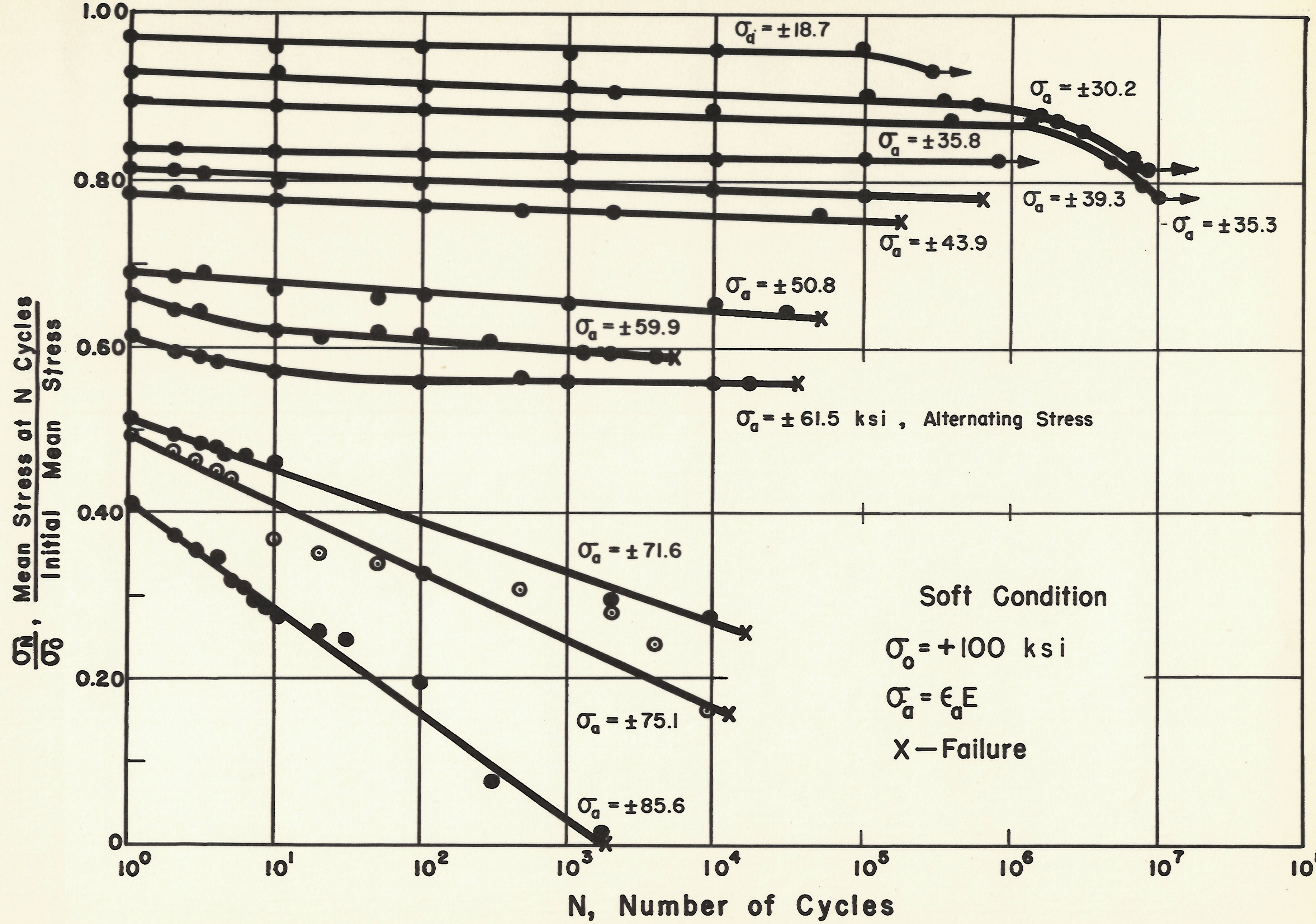

FIG. 8 RELAXATION OF MEAN STRESS, SAE 4340, (9)

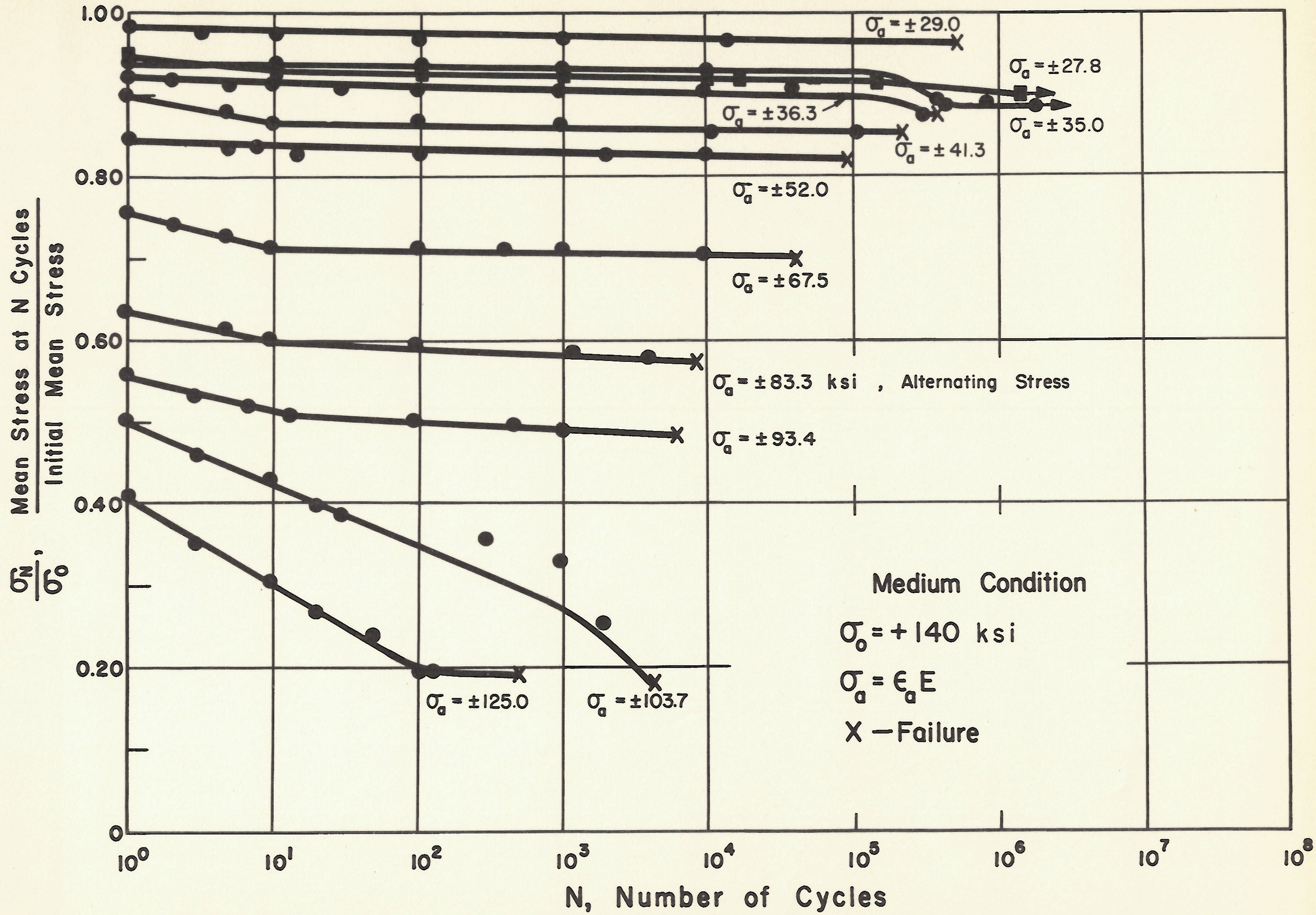

FIG. 9 RELAXATION OF MEAN STRESS, SAE 4340, (9)

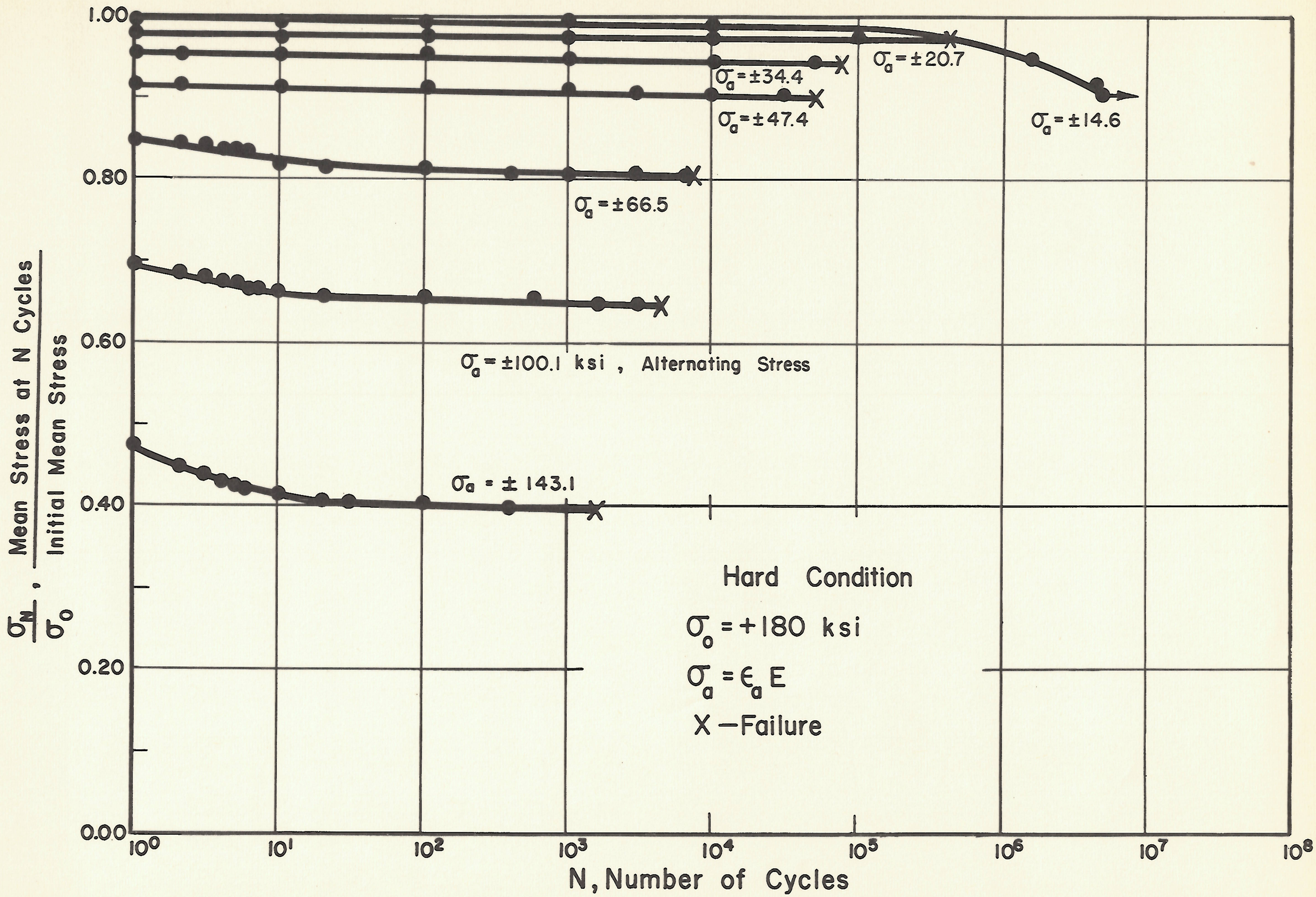

FIG. 10 RELAXATION OF MEAN STRESS, SAE 4340, (9)

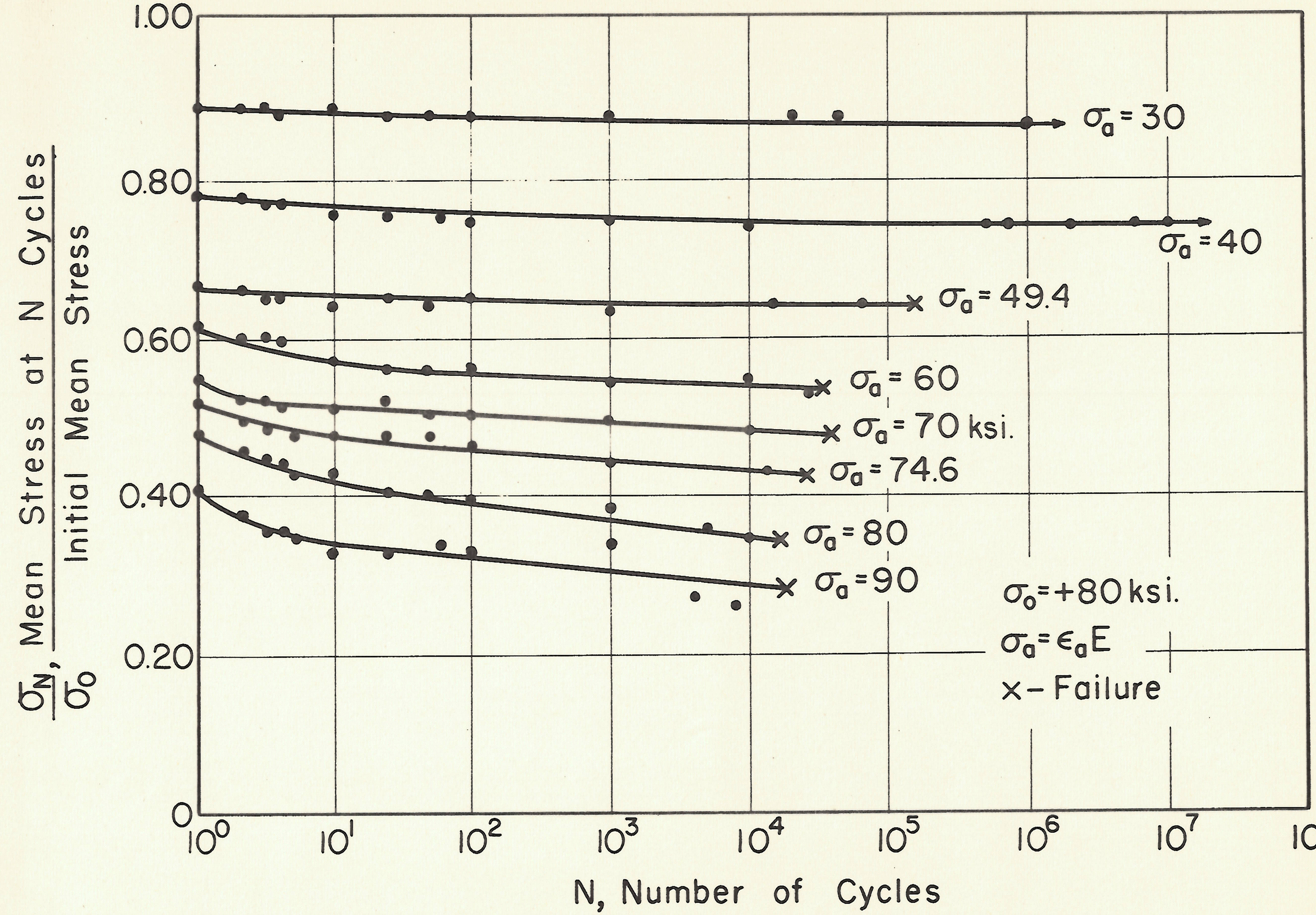

Fig. 11 Relaxation of Mean Stress, A-286, (10)

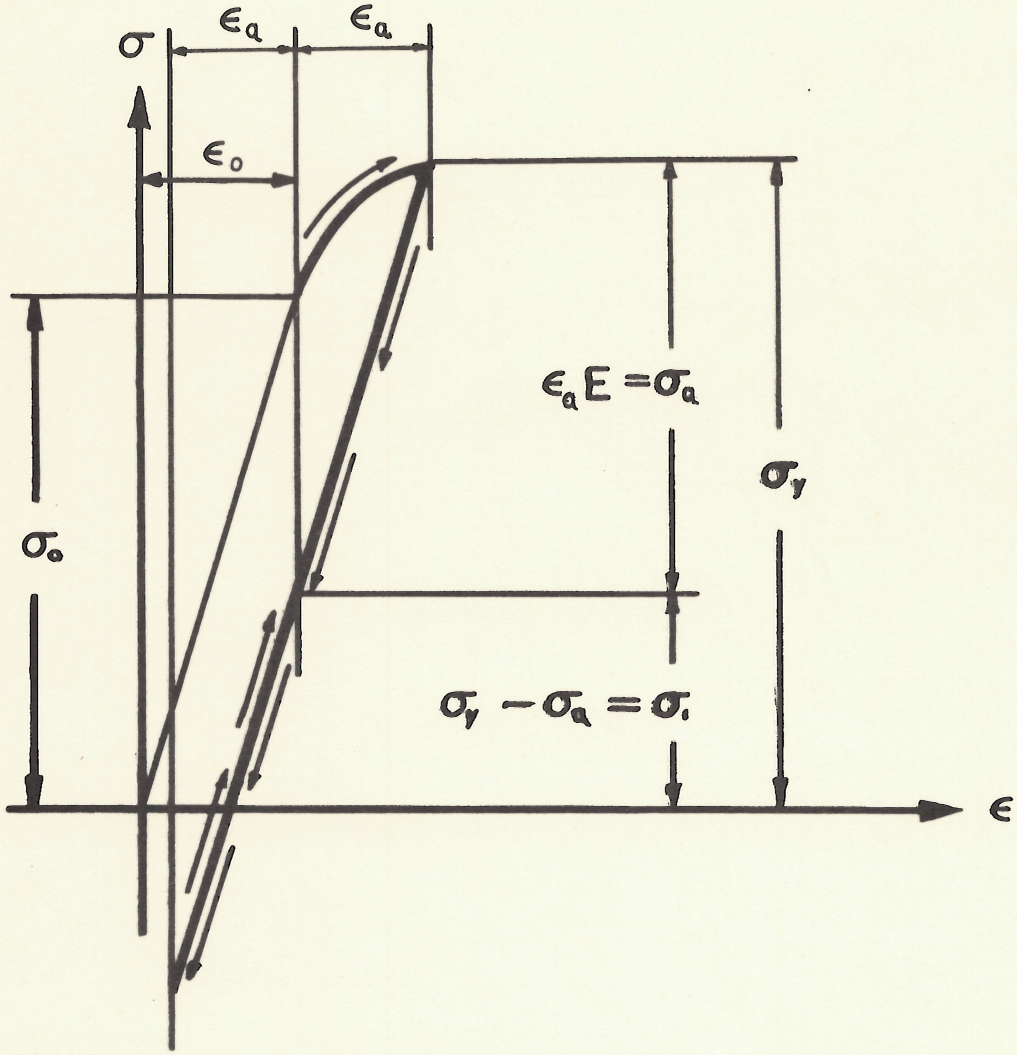

FIG. 12 FIRST CYCLE CHANGE OF MEAN STRESS

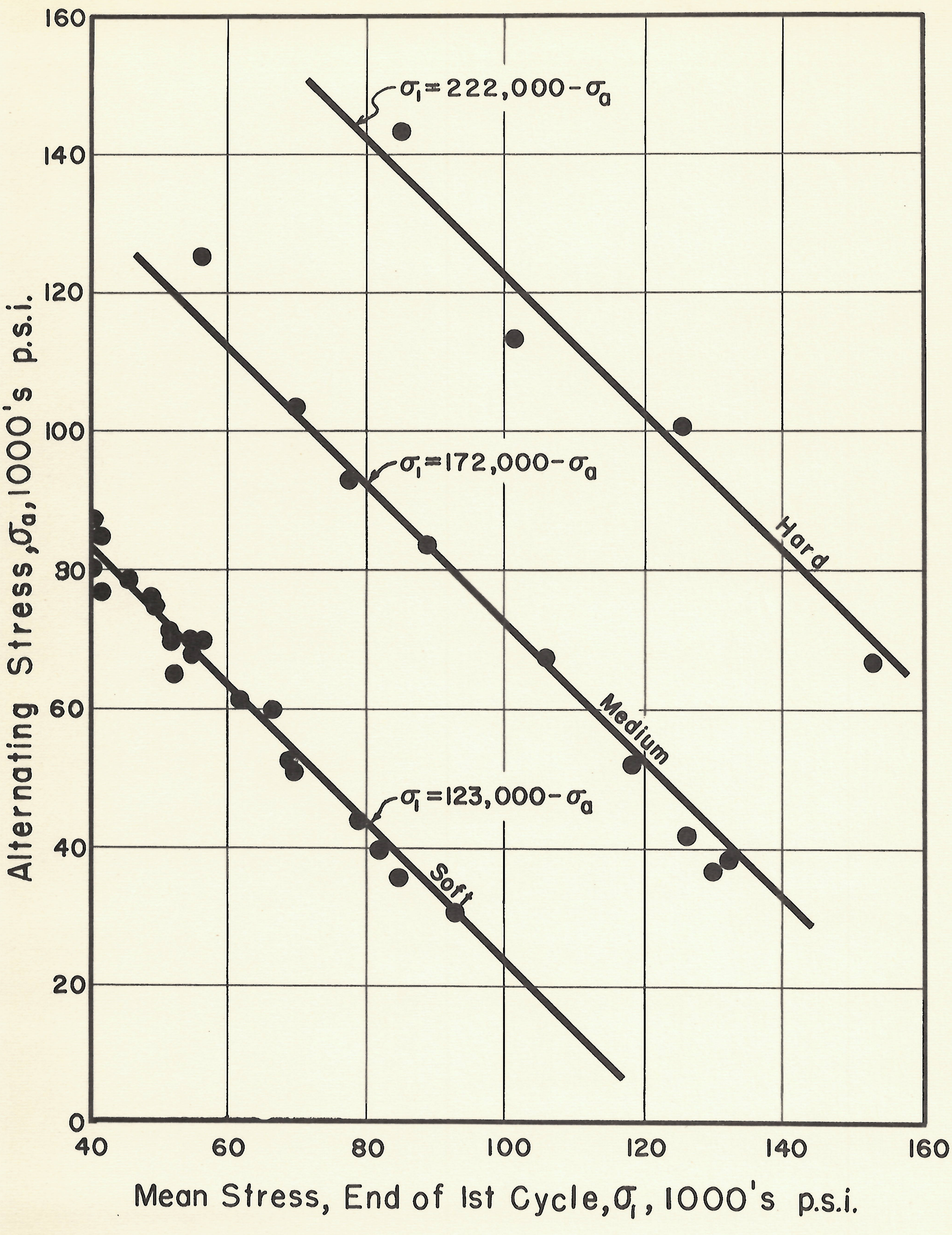

Fig. 13 Mean Stress after One Cycle, (9)

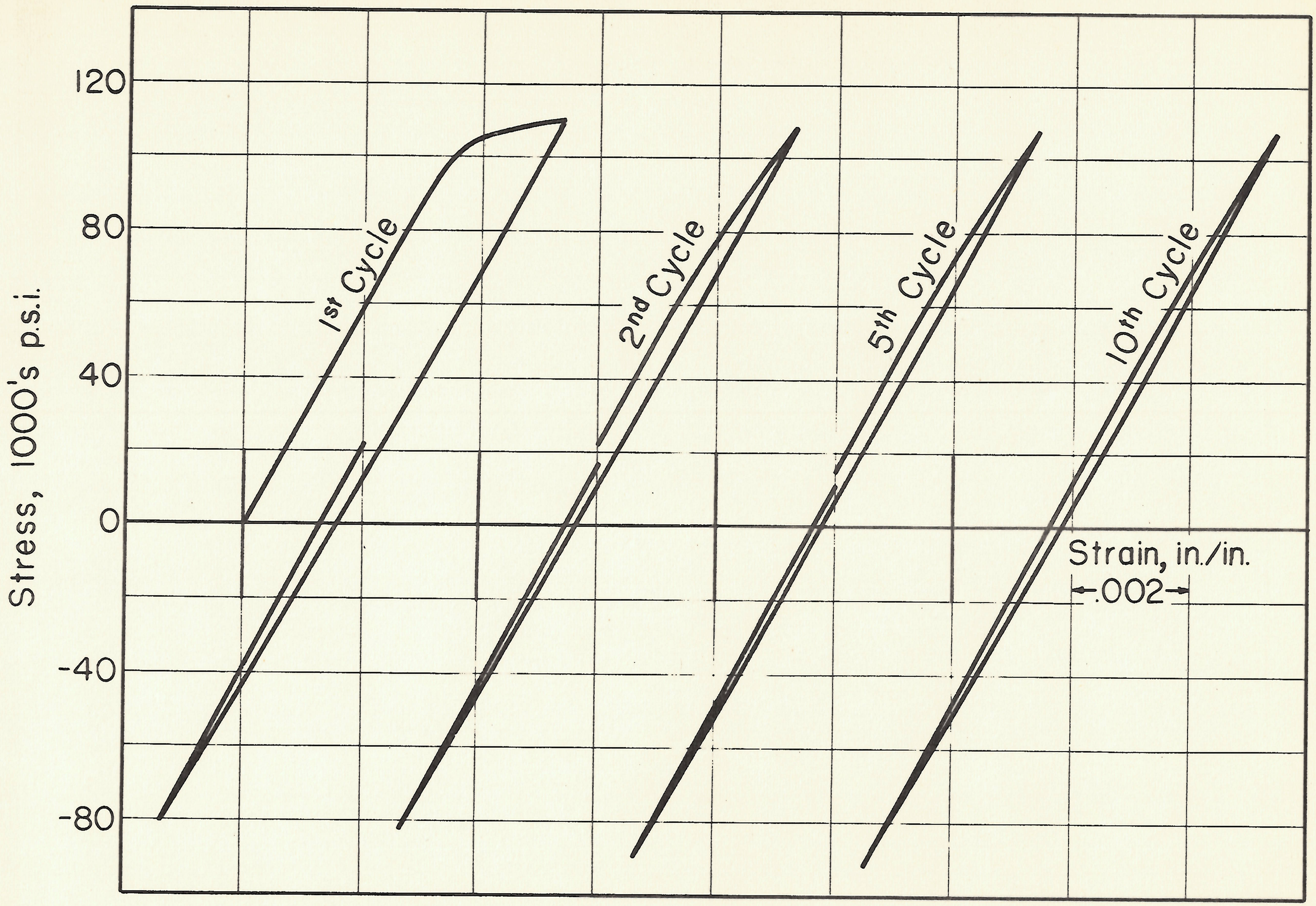

Fig. 14 Cyclic Stress-Strain Behavior, A-286, (10)

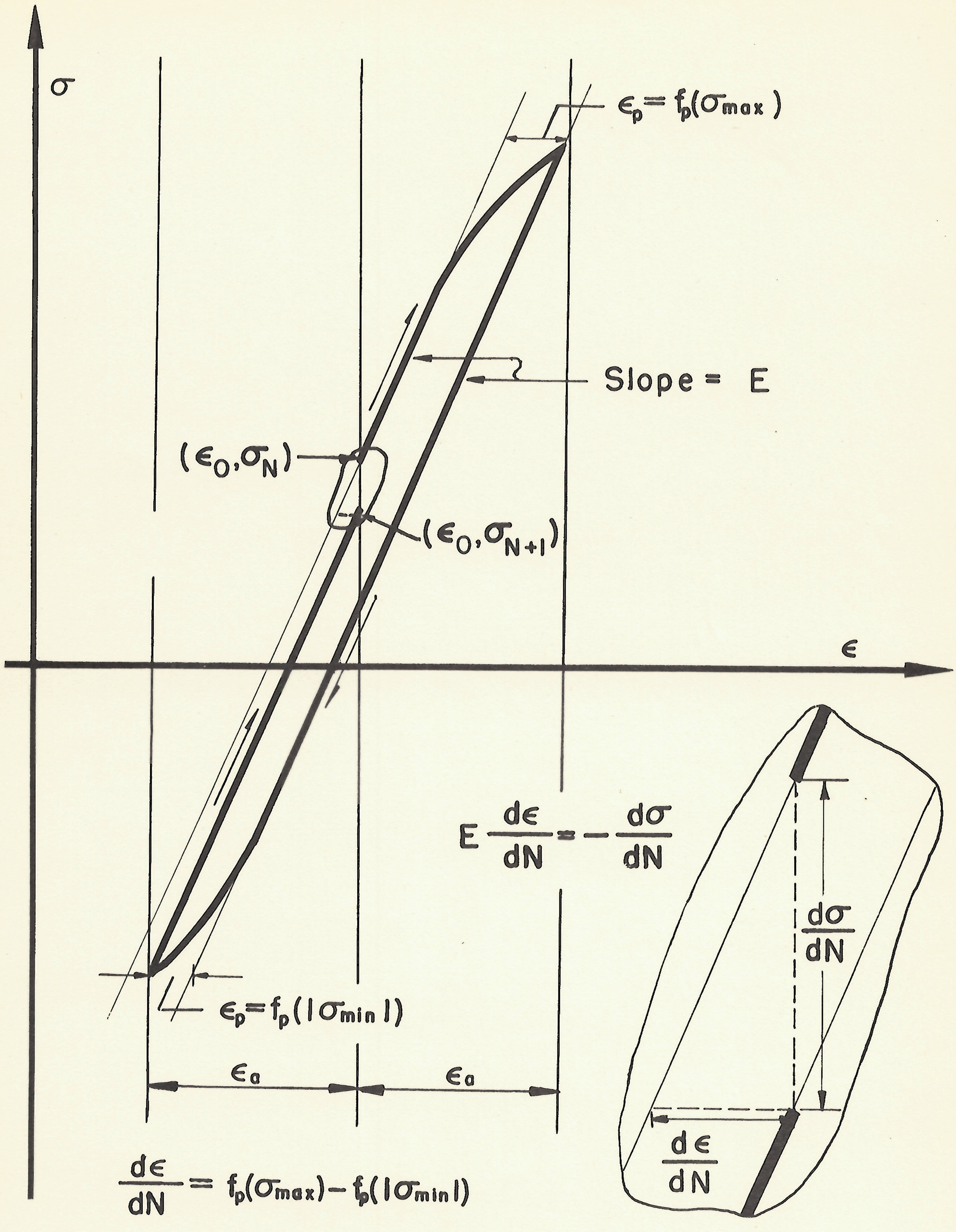

FIG. 15 ASSUMED CHANGE OF MEAN STRESS ON THE N^{th} CYCLE

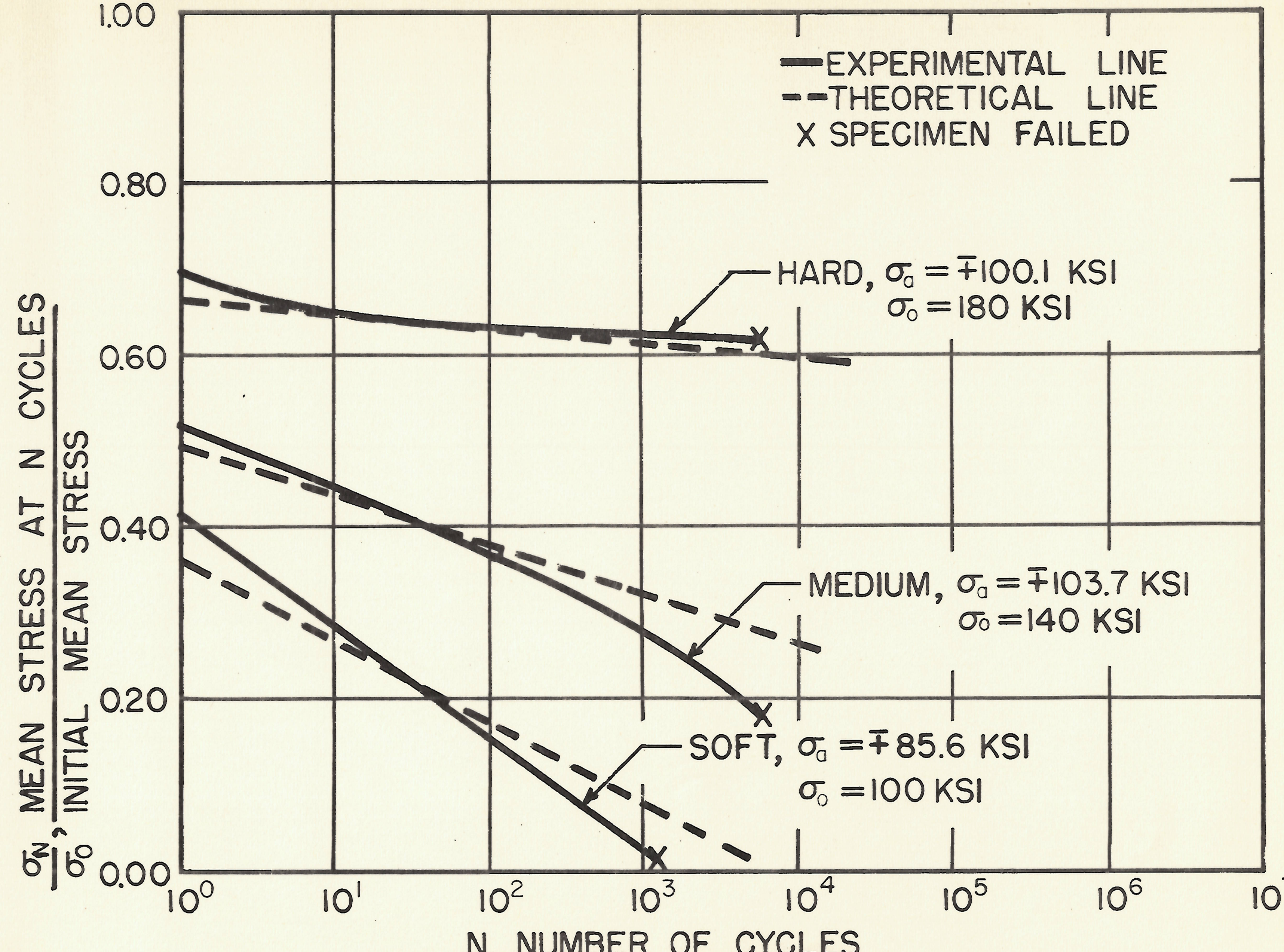

FIG. 16 COMPARISON OF COMPUTED AND OBSERVED BEHAVIOR OF SAE 4340, (9)

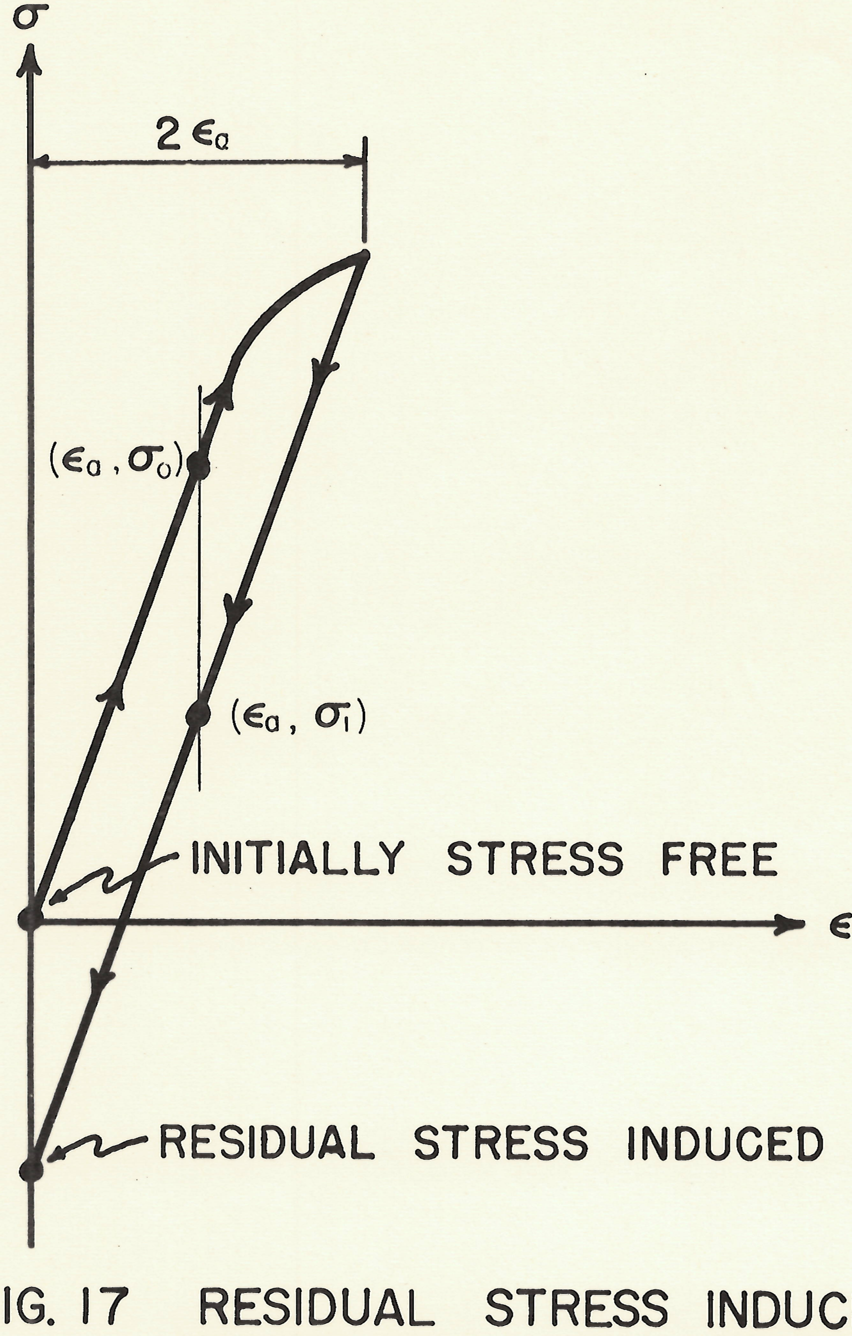

FIG. 17 RESIDUAL STRESS INDUCED INTO A STRESS FREE MEMBER BY FATIGUE LOADING

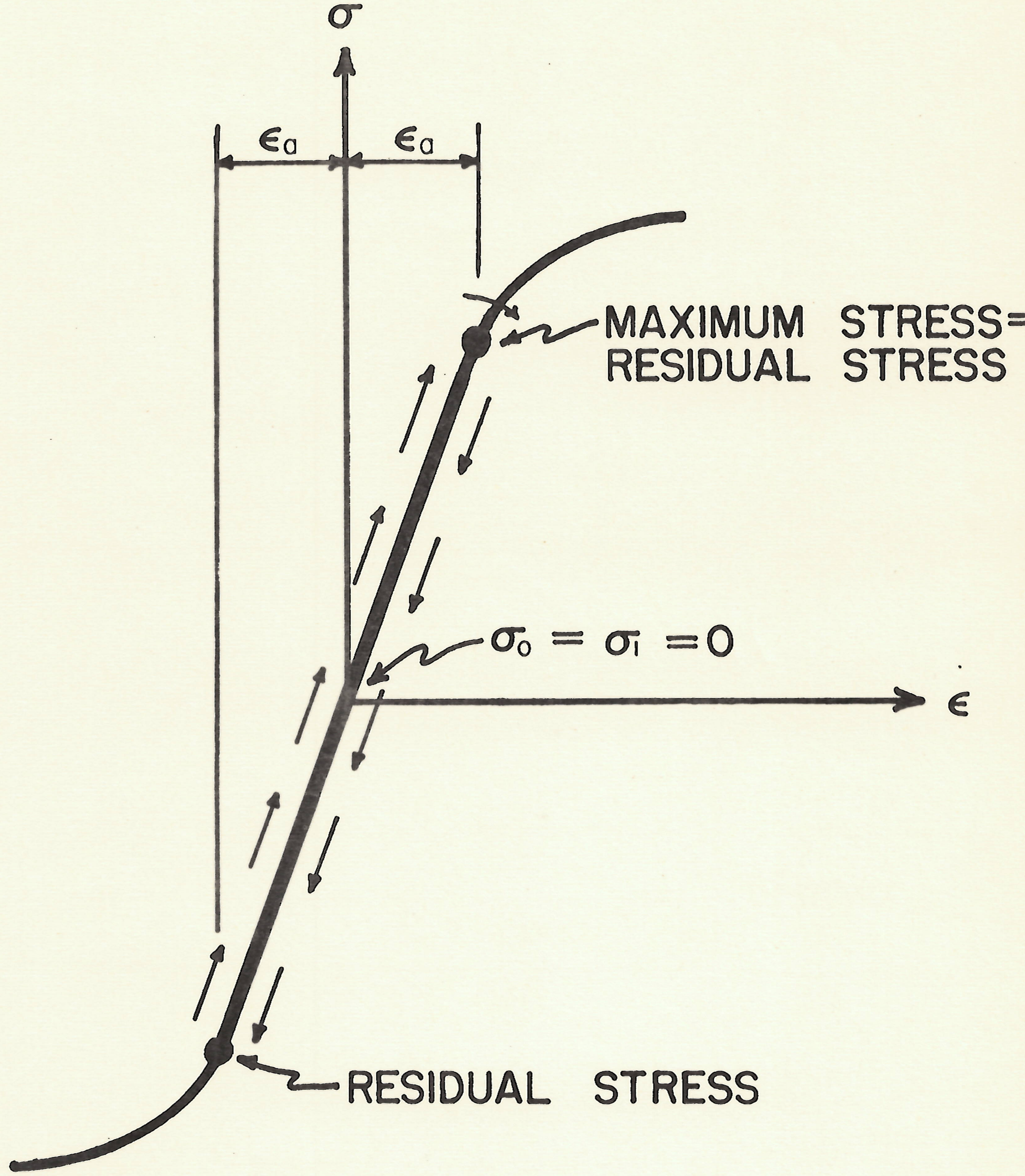

FIG. 18 RESIDUAL STRESS DOESN'T CHANGE IF MEAN STRESS IS ZERO

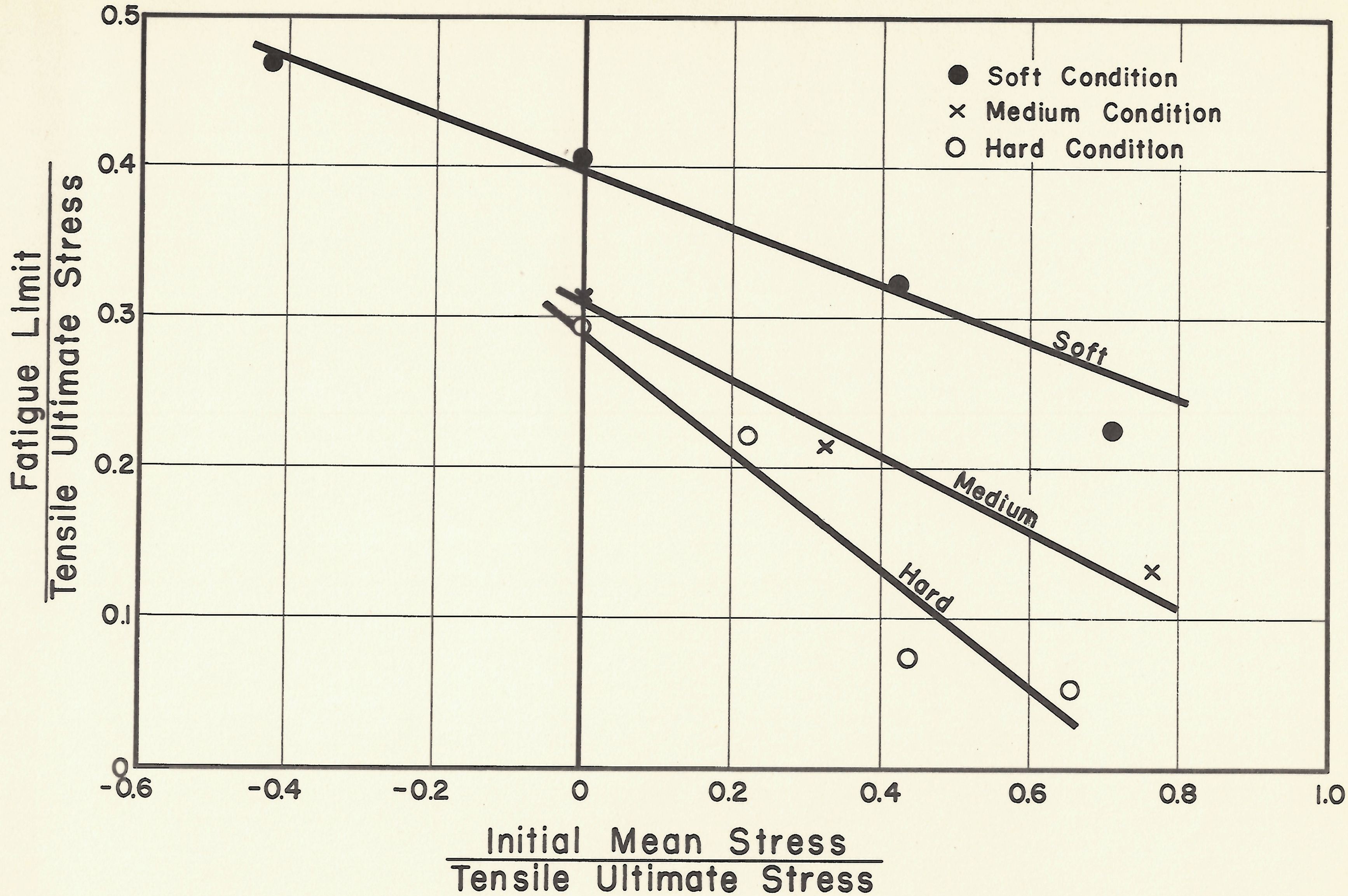

FIG. 19 EFFECT OF INITIAL MEAN STRESS ON FATIGUE LIMIT FOR SAE 4340, (9)

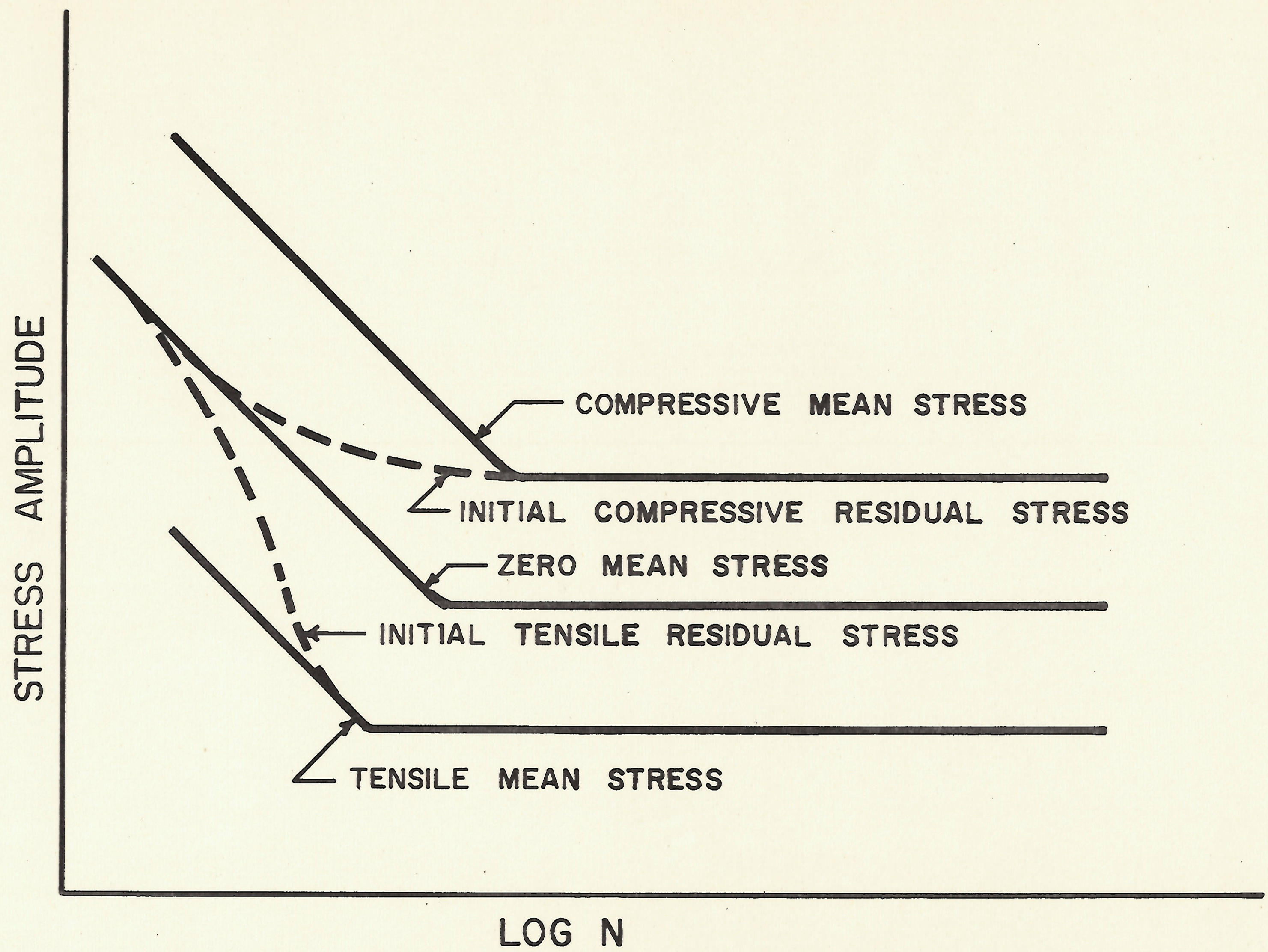

FIG. 20 SCHEMATIC S-N CURVES FOR MEAN STRESS HELD CONSTANT AND RESIDUAL STRESSES ALLOWED TO CHANGE