Private Communication
Not for Publication

T. & A. M. Report No. 646

MOISTURE EXCHANGE BEHAVIOR IN CONCRETE AND MORTAR

by

Krishan K. Jain
Clyde E. Kesler

Prepared as a Part of an Investigation
Conducted by

THE ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS

In Cooperation With
THE DIVISION OF HIGHWAYS
STATE OF ILLINOIS
and
BUREAU OF PUBLIC ROADS
U. S. DEPARTMENT OF COMMERCE

Project IHR-72
PREDICTION OF CREEP IN STRUCTURAL CONCRETE
FROM SHORT TIME TESTS

July 1964

ACKNOWLEDGEMENTS

This study was done as a part of the research under the Illinois Cooperative Highway Research Program Project IHR-72, "Prediction of Creep in Structural Concrete from Short-Time Tests." The project has been undertaken by the Engineering Experiment Station of the University of Illinois, in cooperation with the Illinois Division of Highways of the State of Illinois and the Bureau of Public Roads, United States Department of Commerce.

On the part of the University, the work covered by this report was carried out under the general administrative supervision of W. L. Everitt, Dean of the College of Engineering, R. J. Martin, Director of the Engineering Experiment Station, T. J. Dolan, Head of the Department of Theoretical and Applied Mechanics, and Ellis Danner, Director of the Illinois Cooperative Highway Research Program and Professor of Highway Engineering.

On the part of the Highway Division of the State of Illinois, the work was under the administrative direction of Virden Staff, Chief Highway Engineer and J. E. Burke, Engineer of Research and Development.

Technical advice was provided by a Project Advisory Committee consisting of the following personnel:

Representing the Illinois Division of Highways

J. E. Burke, Engineer of Research and Development

C. E. Thunman, Jr., Bureau of Design

Representing the Bureau of Public Roads

J. L. Hirsch, Bridge Engineer

W. W. Hoffman, Assistant Regional Bridge Engineer

Representing the University of Illinois

O. M. Sidebottom, Professor of Theoretical and Applied Mechanics

Mete A. Sozen, Professor of Civil Engineering

Clyde E. Kesler, Professor of Theoretical and Applied Mechanics and Iqbal Ali, Research Associate in the Department of Theoretical and Applied Mechanics, serve as Chairman and Secretary, respectively, of the Project Advisory Committee. Krishan K. Jain, Research Assistant, Department of Theoretical and Applied Mechanics served as the investigator of this study. Mr. Sidney H. L. Kung, Research Associate, assisted in preparing the manuscript.

TABLE OF CONTENTS

1. INTRODUCTION

1.1 Statement of the Problem

Creep in concrete has been indicated by various investigators to depend on the moisture movement relative to the environment. It is considered (1)*to be made of two components: the basic creep, depending on the gel composition of the mix; and the drying creep or stress modified shrinkage, depending on the moisture movement relative to the environment.

A method of estimating the basic creep has been suggested by Ali (1) and therefore the first component of creep does not seem to pose too serious a problem. However, the second component, drying creep or stress modified shrinkage, requires a knowledge of the moisture exchange behavior of concrete with the environment. Very little work has been done in the past (2, 3, 4) on studies of moisture exchange characteristics of concrete and its interdependence on free shrinkage, although extensive work was done earlier by Pickett on the distribution of shrinkage stresses in concrete beams and slabs during the course of drying.

1.2 Purpose of the Investigation

The investigation reported was undertaken with the following aims in view.

(a) To study the phenomena of moisture exchange in concrete specimens of various sizes with its surroundings with time.

(b) To evolve a method of predicting the amount of moisture movement likely to take place at any point, after a given period of time.

(c) To attempt a correlation of the moisture exchange studies in concrete with its free shrinkage behavior.

1.3 Scope of the Experimental Program

The experimental program was of an exploratory nature and only a limited number of specimens were employed. The investigation was restricted to a minimum of variables and to certain idealized conditions, without seriously affecting the significance of the investigation. These restrictions are as follows:

(a) The environment conditions were limited to an initial relative humidity of 100 per cent and a final relative

---

* Numbers in parentheses refer to Bibliography.

humidity of 50 per cent at a temperature of 70 F.

(b) The type of aggregate and cement were kept the same.

(c) To minimize the effect of progressive hydration during moisture exchange with the environment, all concrete specimens were cured to a high degree of maturity before testing.

2. EXPERIMENTAL DETAILS

2.1 General

The following sections describe briefly the materials, mix proportions, type, number, and fabrication of specimens, instrumentation, and the experimental procedure.

2.2 Materials

(a) Cement -- Type I portland cement was used for all specimens.

(b) Aggregate -- Wabash River gravel with a nominal maximum size of 1 in. and a fineness modulus of 7.20 was used as a coarse aggregate. Two varieties of river sand, with fineness moduli of 2.80 and 0.95 were used as fine aggregate. Detailed particulars about the aggregates are given in Table 1.

2.3 Mixes

Concrete and mortar of nominal 28-day strengths of 5000-6000 psi and 3000-4000 psi with water-cement ratios of 0.53 and 0.83 (by weight) respectively were used. The mixes had a slump of 1½ to 2 in. No air entrainment was used. Particulars of the mortar and concrete, designated as mixes A, B and C are given in Table 2.

2.4 Types of Specimens, Fabrication and Number

Three types of specimens were used for studying the moisture exchange behavior. One specimen only of each type was used.

(a) Solid cylinders 12 in. long and with diameters of 3, 6 and 12 inches were cast from mortar mixes A and B, and concrete mix C.

(b) Hollow cylinders 12 in. long with outer and inner diameters of 6 and 4 in. were cast from mortar mixes A and B.

(c) Prismatic bars 1 by 1 by 11 3/4 in. long made from mortar mixes A and B.

Wells were formed in the specimens in order to measure the change of relative humidity in the interior of the drying concrete. These were done by casting in to the specimen 3/16-in. diameter hollow brass tubing, with a close fitting 5/32-in. solid brass rod passing through it and projecting about 1¼ in. below the lower end. While casting, the tube and rod assemblies were held temporarily in a wooden jig at the required spacing and secured at the top of the mold. Concrete or mortar was then carefully placed and vibrated around them. The rods were occasionally rotated, to prevent adhesion at the lower end. After the concrete hardened, the rods could be withdrawn, leaving a well lined with the brass tubing except for the lower 1¼ in. The rods were subsequently used to keep the wells plugged between measurements. Figure 1 shows the details.

2.5 Instrumentation

Variation in the internal humidity of concrete and mortar specimens due to exchange of moisture to the surrounding atmosphere was measured with a probe type gage developed at the Portland Cement Association Laboratories (5). The details of the gage are shown in Fig. 2.

The free shrinkage of concrete and mortar specimens was measured on steel reference points cemented on the surface of the cylinders at a distance of 10 in. on diametrically opposite gage lines. A 10-in. Whittemore gage with a dial indicator reading up to 0.0001 in. was used to measure the deformations.

2.6 Experimental Design

Environments of 100 and 50 per cent relative humidities were used because these were available. All the specimens were first cured to a high degree of maturity at 100 per cent relative humidity to avoid progressive hydration and then transferred to, and kept in an environment of 50 per cent relative humidity. The moisture conditions in the interior of the concrete and mortar were studied in terms of the relative humidity developed in the wells cast in the specimens.

Specimens of a cylindrical shape, with the ends sealed to prevent moisture loss were used so as to have radial two dimensional moisture exchange (in a two dimensional diametrical plane.) Various cylinders of 3, 6, and 12-in. diameter but of the same mix proportions were made to study the effect of size.

Shrinkage measurements were made only on one specimen of each size.

2.7 Experimental Procedure

All specimens were moist cured at 100 per cent relative humidity for 60 days and then brought to an environment maintained at a constant relative humidity of 50 per cent and 70 F. The ends of the specimen were sealed with an epoxy to prevent moisture loss in the longitudinal direction. Internal relative humidity was measured through the preformed well points using the moisture gage, described earlier, in conjunction with a strain indicator. A half bridge circuit was used with a precision 100-ohm wire wound resistor as a compensator. Details of the bridge connections and the general arrangement are shown in Fig. 3.

It required 5-10 minutes to stabilize the humidity and the gage after insertion into the well. The response was quicker when the gage was transferred from a lower humidity to a higher one. To take advantage of this effect and avoid any hysteresis, the readings on various specimens were taken in increasing order of humidity wherever possible. Calibration of the gage was checked with each set of readings by first taking a reading at zero per cent relative humidity over "Drierite-$CaSo_4$" desiccant and closing the set of measurements at 100 per cent relative humidity over distilled water. The gage was allowed to remain in each well for 3-5 minutes to stabilize before switching on the bridge circuit.

3. EXPERIMENTAL RESULTS

3.1 Moisture Exchange

The variation of internal relative humidity in concrete and mortar specimens with time was measured for specimens of various sizes and shapes. All the specimens were kept in an environment at 50 per cent relative humidity while drying from an initially saturated state. The specimens include 12-in. long hollow cylinders with an outside diameter 6 in. and an inside diameter 4 in., 1 by 1 by 11-3/4 in. prisms, and 12-in. long solid cylinders of 3, 6 and 12-in. diameter. Mortar mixes of A and B and concrete mix C are represented.

Figure 4 shows the time variation of internal relative humidity at the center of the 1-in. wall thickness of a hollow cylinder and at the center of the 1 by 1 in. cross section of a prismatic bar. The hollow cylinders lost moisture from the inside and outside faces, the prismatic bar from all four faces.

Figures 5 through 9 show the variation of internal relative humidity with time at various radial distances from the center of solid cylinders of different mixes and sizes.

Figures 10 and 11 show the distribution of internal relative humidity across the plane of moisture exchange for specimens of different sizes and mixes.

3.2 Free Shrinkage

The free shrinkage of mortar and concrete was measured as surface deformations in a direction perpendicular to the plane of moisture exchange. All the specimens were kept in the same environmental conditions as the specimens for moisture exchange measurements. The data is presented as axial strain. Figure 12 shows the variation of free drying shrinkage with time for mixes A and C.

4. DISCUSSION AND ANALYSIS OF TEST DATA

4.1 Discussion of Test Data

From Fig. 4 it can be seen that the internal humidity at the center of the prismatic bars dropped at a much faster rate than the humidity at the center of the 1 in. thick hollow cylinders. This can be explained by the fact that the square bar provides four faces and therefore a larger surface area than the two faces of the hollow cylinder. Mortar mix A having a larger cement water ratio than B, has a larger gel volume with smaller interstices to resist the moisture movement, and therefore shows a slower moisture loss.

Figures 5 through 7 show the variation of internal humidity with time in cement mortar (mix A) specimens of different sizes at various locations in the plane of moisture exchange. The internal relative humidity at any location in a specimen dropped from the initial 100 per cent relative humidity at a decreasing rate to an asymptotic value of 50 per cent relative humidity--the humidity of the environment. While at locations further distant from the center, the decrease of internal relative humidity was at much faster rate initially and later at the same rate as the center locations. It is observed that smaller specimens dried faster than the larger ones. At the center of the 6-in. diameter specimen an internal humidity of 80 per cent is reached after 140 days, while at the center of 3-in. diameter specimen the same internal humidity is reached at about 36 days. Thus at the center of each specimen the times required to reach the same internal humidity seemed to be in proportion to the square of the radius. Figure 10 affords a convenient comparison of the internal relative humidities developed at similar locations in specimens of the same mix A and diameters 3, 6 and 12 in.

Variation of internal humidity for specimens of the same size but different mix proportions can be seen from Figures 6, 8 and 9. Distribution across the plane of moisture exchange can be seen from Fig. 11. Mix B seemed to lose moisture much faster than mix A, possibly because mix B developed a smaller volume of cement gel than mix A and thus allowing less restrictions to moisture exchange with the environment.

Measurement of the internal relative humidity was subject to certain restrictions and experimental errors. These are listed as follows:

(a) Temperature compensation was not attained completely.

(b) Slight bending of the gage stem in the preformed well points due to imperfect alignment seemed to cause considerable variance in the readings.

(c) Internal humidity is subject to variations in the inside surface texture of the humidity wells.

4.2 Analysis

4.2.1 Effect of Size

It can be shown by dimensional analysis and the principle of similitude (6) that effect of size on the moisture exchange phenomena with time can be reduced on a comparable basis using a non dimensional radial position parameter and normalized time based on scale factor of the specimens.

Selecting as a common base a cylinder of unit radius, a scale factor K for different size specimens which is equal to the radius r_o of the cylinder, is obtained. Internal relative humidity at any radial distance r is referred to a non dimensional position coordinate Φ, where $\Phi = \frac{r}{r_o}$. The time required for a cylinder of unit radius to reach the same internal humidity as a cylinder of radius r_o is as follows by the model laws, (6):

$$t = \frac{T}{K^2} = \frac{T}{r_o^2} \text{ (normalized time)} \qquad (1)$$

where r_o = radius of the cylinder

K = scale factor referred to unit radius cylinder, $K = r_o$

T = time required for a cylinder of radius r_o to reach a specified relative humidity H at any radial distance r.

Φ = non dimensional position coordinate such as r/r_o

t = time required for a unit radius cylinder to reach the above relative humidity H at a similar radial position Φ.

Figure 13 shows moisture exchange phenomena referred in a non dimensional plot for specimens of 3, 6, and 12 in. diameter. The abscissa is normalized time t, and the ordinate, degree of drying, as fraction of full desiccation. Experimental data for a particular value of Φ for different specimen sizes can be grouped and plotted in a single curve.

4.2.2 Distribution of Internal Humidity

(a) Analytical

An analytical solution was attempted for radial two dimensional moisture exchange in concrete by considering it as similar to heat flow in a long cylinder whose surface is kept at a constant temperature 0 and whose initial temperature in the interior is unity. Internal humidity is considered to be similar to internal temperature and concrete is idealized to have a constant diffusivity. The following expression involving Bessel functions of order zero and one was obtained in Appendix A. (7)

$$H = \sum_{n=1}^{\infty} \frac{2}{x_n J_1(x_n)} e^{-x_n^2(aT/r_o^2)} J_o(x_n \frac{r}{r_o}) \qquad (2)$$

where:

H = Internal humidity at any time T and a radius r.

T = time

r = radial distance between the point in consideration and the center of cylinder

r_o = radius of the cylinder

J_o = Bessel function of zero order

J_1 = Bessel function of order one

x_n = n^{th} root of the Bessel function $J_o(x)$

a = diffusivity of the material

The above solution did not wholly agree with the experimental data, possibly because concrete can not be assumed to be a homogeneous isotropic material with a constant diffusivity. It might be more reasonable to assume two values of diffusivities, one for the capillary pores and another value for the gel pores. An analytical solution considering two different diffusivities of concrete is beyond the scope of this investigation. However, an empirical approach was made recognizing the existence of two separate diffusivities.

(b) Empirical

To represent the desiccation or dryness with normalized time an empirical expression of the following form was obtained after several trials.

$$h = C_1(1 - e^{-t/\tau_1}) + C_2(1 - e^{-t/\tau_2}) \qquad (3)$$

where:

h = desiccation = $\dfrac{\text{Initial Humidity } H_o - \text{Humidity at any instant}}{\text{Initial Humidity} - \text{Final humidity}}$

t = normalized time (time of drying for a cylinder of unit radius)

τ_1 and τ_2 = constants, representing reciprocal of diffusivities.

C_1 and C_2 = empirical constants such that $C_1 + C_2 = 1$

Values of the constants τ_1, τ_2, C_1 and C_2 were determined by a procedure similar to estimation of rheological model parameters (1). The following expressions were found to give a reasonably close fit to the experimental data.

$$\phi = \frac{r}{r_o} = 0; \quad h = 0.1(1-e^{-t/3.5}) + 0.9(1-e^{-t/45}) \qquad (4)$$

$$\phi = \frac{r}{r_o} = 0.25 \quad h = 0.15(1-e^{-t/4}) + 0.85(1-e^{-t/40}) \qquad (5)$$

$$\phi = \frac{r}{r_o} = 0.5 \quad h = 0.3(1-e^{-t/5}) + 0.7(1-e^{-t/40}) \qquad (6)$$

$$\phi = \frac{r}{r_o} = 0.75 \quad h = 0.5(1-e^{-t/5}) + 0.5(1-e^{-t/40}) \qquad (7)$$

The values of the constants τ_1 and τ_2 do not vary appreciably with the parameter, ϕ, and thus may be assumed constant. C_1 and C_2 are seen to vary parabolically with the parameter ϕ and can be represented respectively as

$$C_1 = 0.1 + 0.9\phi^2 \qquad (8)$$

$$C_2 = 0.9(1 - \phi^2) \qquad (9)$$

where $\phi = \frac{r}{r_o}$; ranges from 0 to 1.

Variation of desiccation at any point on the cross section of a cylinder can therefore be expressed by a relation of the form

$$h = \left\{0.1 + 0.9\left(\frac{r}{r_o}\right)^2\right\}\left(1-e^{-T/4r_o^2}\right) + 0.9\left\{1-\left(\frac{r}{r_o}\right)^2\right\}\left(1-e^{-T/40r_o^2}\right) \qquad (10)$$

or

$$h(r,T) = \left\{D_1 + D_2\left(\frac{r}{r_o}\right)^2\right\}\left(1-e^{-T/\tau_1 r_o^2}\right) + D_2\left\{1-\left(\frac{r}{r_o}\right)^2\right\}\left(1-e^{-T/\tau_2 r_o^2}\right) \qquad (11)$$

where D_1 and D_2 are another set of empirical constants. Thus with the evaluated values of the constants D_1, D_2, τ_1 and τ_2, the desiccation pattern for a cylindrical specimen can be predicted at any time.

4.2.3 Interrelation of Moisture Exchange and Free Shrinkage

Free shrinkage strains are known to remain almost constant over sections parallel to the plane of moisture movement to the environments. As movement of moisture is accompanied by shrinkage deformations, the variation of desiccation averaged over the section of moisture movement is believed to be of the same general form as variation of free shrinkage strains with time.

The average desiccation over any plane parallel to the plane of moisture exchange can be found by integrating the point variation of desiccation over the whole section of moisture exchange. In the present case, for symmetrical radial moisture exchange, the average desiccation is as follows.

$$\bar{h}(t) = \frac{1}{\pi \cdot 1^2}\int_0^{\phi=1} H(\phi,t)2\pi\phi d\phi \qquad (12)$$

where:

ϕ = a non dimensional parameter, $\frac{r}{r_o}$

t = normalized time = $\frac{T}{r_o^2}$ where T = desiccation time at a specified point for a cylinder of radius r_o.

For cement mortar of mix A, the average variation of desiccation, $\bar{h}$, with time was obtained as below by integrating equation 10.

$$\bar{h} = 0.7(1-e^{-t/4}) + 0.3(1-e^{-t/40}) \qquad (13)$$

Equation 13 has been plotted as a solid curve in Fig. 14. Corresponding free shrinkage strains for the same mortar mix are shown as traction of ultimate shrinkage with normalized time. Experimental data on free shrinkage strains were reduced to the non dimensional plot by assuming that the average of variation desiccation with time is of the same form as the variation of the fraction of ultimate shrinkage. The value of ultimate shrinkage is estimated by extrapolating the shrinkage curve of Fig. 12.

The shrinkage and moisture exchange phenomena seems therefore to be closely interlinked. Knowing the variation of one, the other can be predicted.

5. SUMMARY AND CONCLUSIONS

5.1 Summary

Behavior of moisture movement in cement mortar and concrete specimens of various size and shape was studied. Only symmetrical two-dimensional radial moisture movement was studied and analyzed in any detail. Tests were restricted to specimens having moisture movement to an environment at 50 per cent relative humidity from an initial state of 100 per cent relative humidity at all interior points. Time variation of desiccation at any position has been found to be represented by a semi-empirical expression as

$$h = \left\{D_1 + D_2\left(\frac{r}{r_o}\right)^2\right\}\left(1-e^{-T/\tau_1 r_o^2}\right) + D_2\left\{1-\left(\frac{r}{r_o}\right)^2\right\}\left(1-e^{-T/\tau_2 r_o^2}\right)$$

where:

h = desiccation at any radial distance r at time T.

r_o = radius of the specimen

D_1, D_2 and τ_1, τ_2 = constants depending on the internal structure of the mix and the difference of the initial and subsequent environmental humidities.

The constants can be evaluated experimentally for a given mix by running moisture exchange tests on a small specimen, which provides all the necessary information in a comparatively shorter period of time than the prototype. Variation with time of the average desiccation on a section parallel to the plane of moisture exchange seems to have the same general trends as the accompanying free shrinkage strains normal to the plane of moisture loss.

5.2 Conclusions

As the investigation employed only a limited number of specimens and considerable scatter was observed in the experimental data, no definite quantitative conclusions can be drawn except some qualitative remarks.

A theoretical solution, though conceivably possible, is difficult to arrive at, unless moisture exchange is considered to involve at least two diffusivities, one for the pore and the other for the gel structure of concrete. A semi-empirical approach appears to give a fairly satisfactory fit to the experimental data by which distribution and variation of desiccation with time can be estimated across the section of moisture movement. Variation of average desiccation with time appears to have the same general pattern as variation of free drying shrinkage.

5.3 Suggestions for Further Work

As in the present investigation of moisture exchange, only a two dimensional radially symmetrical case was studied, it seems more logical to extend the investigation by employing rectangular shapes and also considering moisture movement from two opposite long faces. Such a study would be useful from the point of view of shrinkage and creep in slabs losing moisture from their opposite faces.

Measurement of free drying shrinkage should be included together with the investigation of moisture movement so that the interdependence of the two can be studied.

6. BIBLIOGRAPHY

1. Ali, Iqbal and Kesler, C. E., "Creep in Concrete With and Without Exchange of Moisture with the Environment, T. & A. M. Report No. 641, University of Illinois, Urbana, Illinois, December 1963.

2. Carlson, Roy, "Drying Shrinkage of Large Concrete Members," Proceedings of the American Concrete Institute, Vol. 33, Jan. 1937, pp. 327.

3. Newman, A. B., "The Drying of Porous Solids," Interim Publication, American Institute of Chemical Engineering, August 1, 1931.

4. Pickett, Gerald, "Shrinkage Stresses in Concrete," Proceedings of the American Concrete Institute, Vol. 17, No. 3, January 1946, pp. 165-195.

5. Monfore, G. E., "A Small Probe Type Gage for Measuring Relative Humidity," Journal of the Portland Cement Association Research and Development Laboratories, Vol. 5, No. 2, May 1963, pp. 41-47.

6. Langhaar, H. L., "Dimensional Analysis and Theory of Models," John Wiley & Sons.

7. Sokolnikoff, I. S. and Redheffer, R. M., "Mathematics of Physics and Modern Engineering," McGraw Hill Book Company, Inc. pp. 479-482.

APPENDIX A

DISTRIBUTION OF RELATIVE HUMIDITY IN A LONG CYLINDER WHOSE SURFACE IS KEPT AT A CONSTANT HUMIDITY, ZERO, AND WHOSE INITIAL HUMIDITY IN THE INTERIOR IS UNITY.

1. Governing Differential Equation

The relative humidity, as well as shrinkage, of a concrete body may be treated mathematically just as heat is so treated. The partial differential equation governing this phenomenon is

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = \frac{1}{a}\frac{\partial H}{\partial t} \quad (1)$$

In the case of a concrete cylinder, the transformation of Eq (1) is

$$\frac{\partial^2 H}{\partial r^2} + \frac{1}{r}\frac{\partial H}{\partial r} = \frac{1}{a}\frac{\partial H}{\partial t} \quad (2)$$

where $H(r,t)$ is the relative humidity at any radial position r and at any given instant, and 'a' is the diffusivity constant of concrete.

The boundary conditions associated with Eq (2) are

$$H(r,o) = 1 \quad (3a)$$

$$H(r_o,t) = 0 \quad (3b)$$

$$\left.\frac{\partial H}{\partial T}\right|_{r=0} = 0 \quad (3c)$$

and assuming a solution of the form

$$H(r,t) = R(r)T(t), \quad (4)$$

the following two ordinary differential equations are obtained

$$T' + a\lambda^2 T = 0 \quad (5)$$

$$R'' + \frac{1}{r}R' + \lambda^2 R = 0 \quad (6)$$

By means of the substitution $x = \lambda r$, Eq (6) reduces to the standard form of Bessel's equation of order zero:

$$x^2 \frac{d^2R}{dx^2} + x\frac{dR}{dx} + x^2 R = 0,$$

whose solutions are $J_o(x)$ and $Y_o(x)$. Hence the solutions of Equation (2) are $J_o(\lambda r)$ and $Y_o(\lambda r)$, and the function

$$H(r,t) = e^{-a\lambda^2 t}\left[AJ_o(\lambda r) + BY_o(\lambda r)\right] \quad (7)$$

satisfies Eq (2) whatever the values of the constant λ, A, B. Since

$\lim_{n \to 0} Y_o(x) = \infty$ and H(o,t) is certainly finite, the function Y_o must be dropped from Eq (7). Hence

$$H(r,t) = Ae^{-a\lambda^2 t} J_o(\lambda r)$$

From the second boundary condition of Eq (3b),

$$H(r_o t) = Ae^{-a\lambda^2 t} J_o(\lambda r_o) = 0$$

This implies $J_o(\lambda r_o) = 0$ and that λr_o must be a root of $J_o(x) = 0$.

$$\lambda r_o = X_n, \quad \lambda = \frac{x_n}{r_o}, \quad n = 1, 2, -----$$

Due to the linearity of the equation, it is concluded that the function

$$H(r,t) = \sum_{n=1}^{\infty} A_n e^{-a(\frac{x_n}{r_o})^2 t} J_o(X_n \frac{r}{r_o})$$

where the A_n are arbitrary constants, is a solution of Eq (2) which satisfies the second of the boundary conditions of Eq (3).

If U(r,t) is to equal f(r) when t = 0, then

$$f(r) = \sum_{n=1}^{\infty} A_n e^{-(o)} J_o(X_n \frac{r}{r_o}) \tag{8}$$

The A_n will be obtained by means of the orthogonality properties of the Bessel functions. Multiplying Eq (8) by

$$r J_o(X_m \frac{r}{r_o})$$

and integrating from 0 to r_o with respect to r

$$\int_o^{r_o} rf(r) J_o(X_m \frac{r}{r_o}) dr = \sum_{n=1}^{\infty} A_n \int_o^{r_o} rJ_o(X_n \frac{r}{r_o})J_o(X_m \frac{r}{r_o})dr = A_m \frac{r_o^2}{2} J_1^2(X_m)$$

from which

$$A_m = \frac{2}{r_o^2 J_1^2(X_m)} \int_o^{r_o} rf(r)(X_m \frac{r}{r_o})dr$$

The solution to the humidity problem thus becomes

$$H(r,t) = \frac{2}{r_o^2} \sum_{n=1}^{\infty} \frac{1}{J_1^2(X_n)} \left[\int_o^{r_o} r f(r) J_o(X_n \frac{r}{r_o}) dr \right] J_o(X_n \frac{r}{r_o}) e^{-a(\frac{x_n}{r_o})^2 t} \qquad (9)$$

For the present case, Eq (3a), H(r,o) = 1, then

$$A_n = \frac{2}{r_o^2 J_1^2 \left[X_n\right]} \int_o^{r_o} r J_o(X_n \frac{r}{r_o}) dr \qquad (10)$$

To integrate Eq (10) the change of dummy variable is made.

$$Z = X_n \frac{r}{r_o}$$

$$A_n = \frac{2}{J_1^2(X_n)} \frac{1}{x_n^2} \int_o^{X_n} X J_o(z) dz = \frac{2}{X_n J_1(X_n)}$$

The numerical result is therefore

$$H(r,t) = 2 \sum_{n=1}^{\infty} \frac{1}{X_n J_1(X_n)} e^{-a(\frac{1}{r_o^2}) X_n^2 t} J_o(X_n \frac{r}{r_o}) \qquad (11)$$

2. <u>Approximate Solutions</u>

If the first three terms are taken for the right side of Eq (11)

$$H = A_1 e^{-x_1^2(\frac{a}{r_o^2})t} J_o(X_1 \frac{r}{r_o}) + A_2 e^{-x_2^2(\frac{a}{r_o^2})t} J_o(x_2 \frac{r}{r_o}) + A_3 e^{-x_1^3(\frac{a}{r_o^2})t} J_o(x_3 \frac{r}{r_o})$$

$$= 2\left[\frac{1}{x_1 J_1(x)} e^{-x_1^2(\frac{a}{r_o^2})t} J_o(x_1 \frac{r}{r_o}) + \frac{1}{x_2 J_1(x_2)} e^{-x_2^2(\frac{a}{r_o^2})t} J_o(x_2 \frac{r}{r_o})\right.$$

$$\left. + \frac{1}{x_3 J_1(x_3)} e^{-x_3^2(\frac{a}{r_o^2})t} J_o(x_3 \frac{r}{r_o})\right]$$

$$= 2 \frac{1}{2.405} \frac{1}{0.52} e^{-(+2.4)^2 (\frac{a}{r_o^2}) t} J_o(2.405 \frac{r}{r_o})$$

$$+ \frac{1}{5.52} \frac{1}{(-0.34)} e^{-(5.52)^2 (\frac{a}{r_o^2}) t} J_o(5.52 \frac{r}{r_o})$$

$$+ \frac{1}{8.654} \frac{1}{0.27} e^{-(8.654)^2 (\frac{a}{r_o^2}) t} J_o(8.654 \frac{r}{r_o})$$

$$H(r,t) = 1.6e^{-5.8 \frac{a}{r_o^2} t} J_o(2.405 \frac{r}{r_o}) - 1.1e^{-30.5 \frac{a}{r_o^2} t} J_o(5.52 \frac{r}{r_o})$$

$$+ 0.85e^{-75 \frac{a}{r_o^2} t} J_o(8.654 \frac{r}{r_o})$$

TABLE 1

CHARACTERISTICS OF AGGREGATES USED

Properties	Aggregates		
	Gravel Max. size 1-in.	Coarse Sand	Fine Sand
1. Fineness modulus	7.20	2.80	0.95
2. Specific Gravity (water = 1.00)	2.71	2.62	2.62
3. Absorption (per cent)	1.50	1.00	0.20

TABLE 2

PARTICULARS OF MIX PROPORTIONS

Designation	Cement		Fine Sand		Coarse Sand		Gravel		Water		Air
	Wt.	* Vol %	Wt.	* Vol %	Wt.	* Vol %	Wt.	* Vol %	W/C by Wt.	* Vol %	Vol %
A	1	13.86	0.50	8.31	3.00	49.98	----	-----	0.55	24.12	3.73
B	1	10.28	0.78	9.61	4.22	52.14	----	-----	0.75	24.30	3.67
C	1	10.21	----	----	2.77	24.05	3.13	37.15	0.53	16.95	1.64

* Based on solid volumes

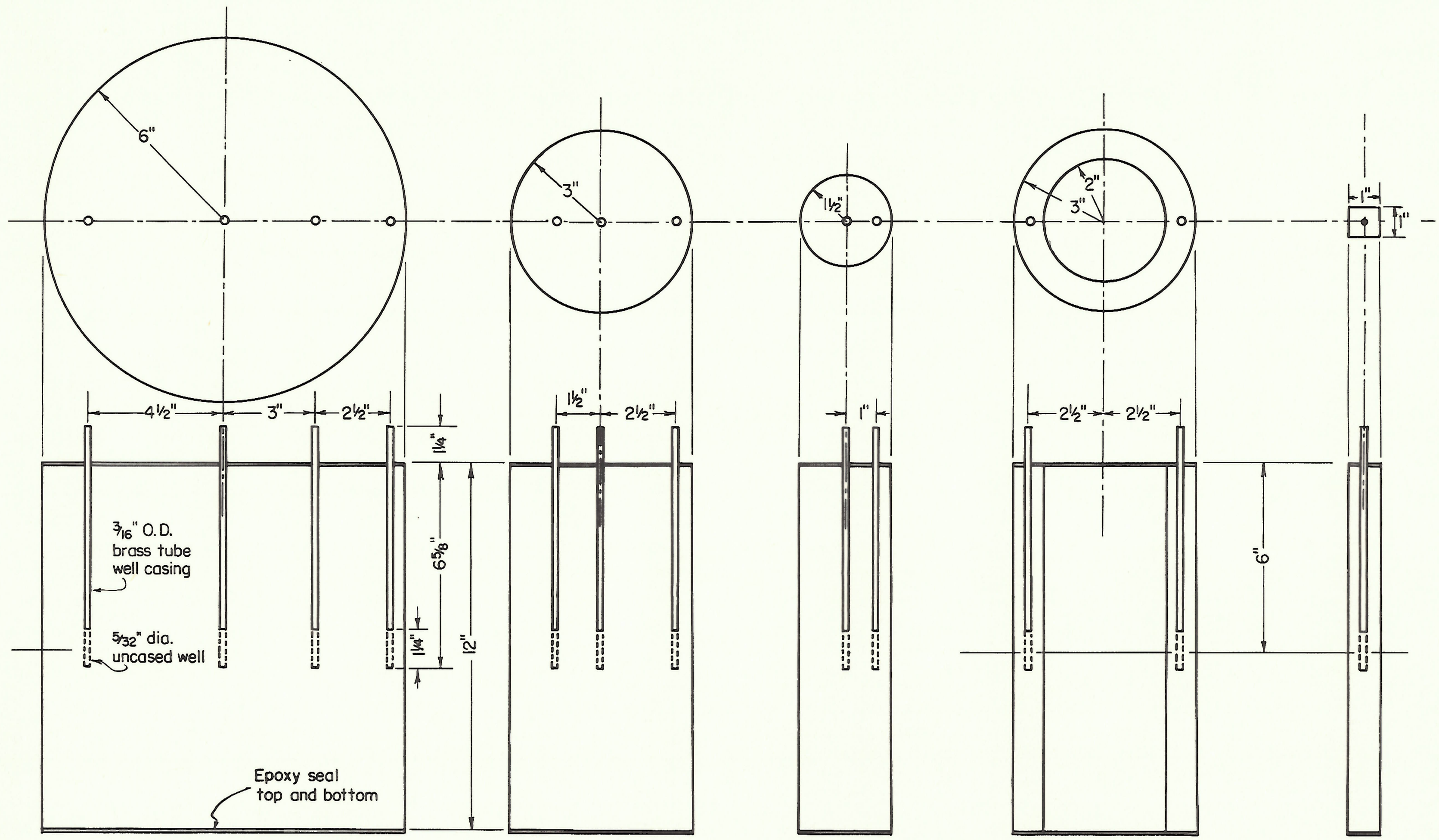

Fig. I Specimens Used for Moisture Exchange Studies

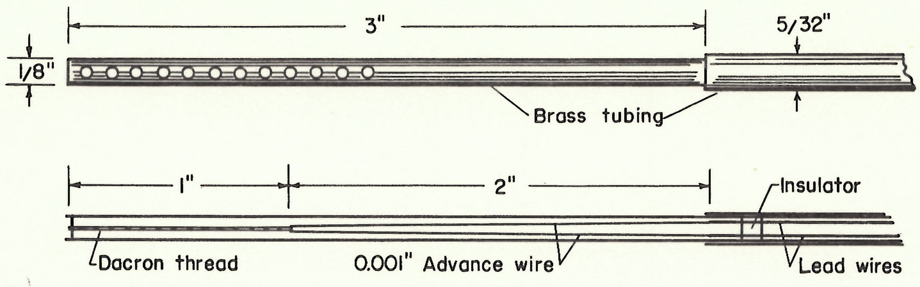

Fig. 2 Details of the Relative Humidity Gage

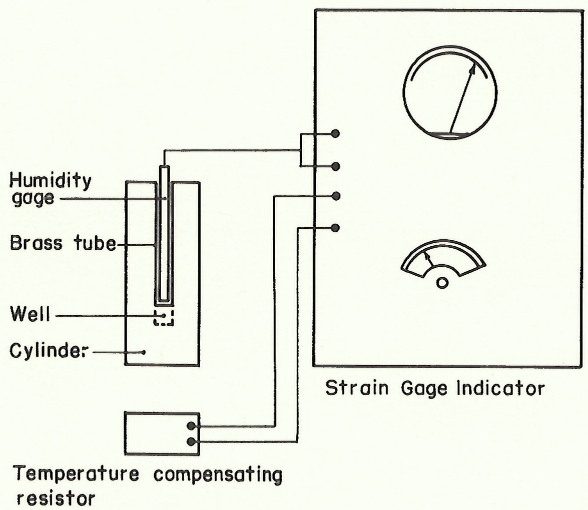

Fig. 3 Measurement of Internal Humidity

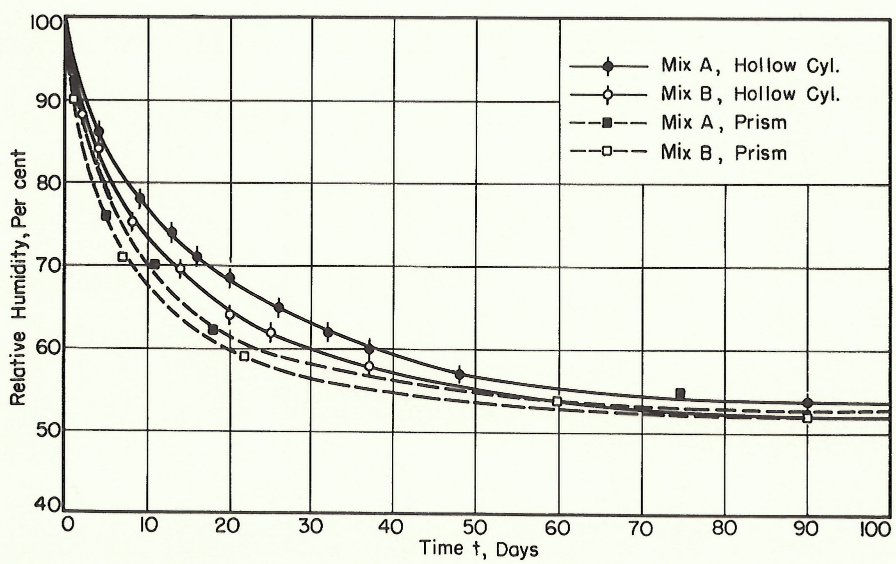

Fig. 4 Variation of Relative Humidity with Time in Hollow Cylinders and Prisms

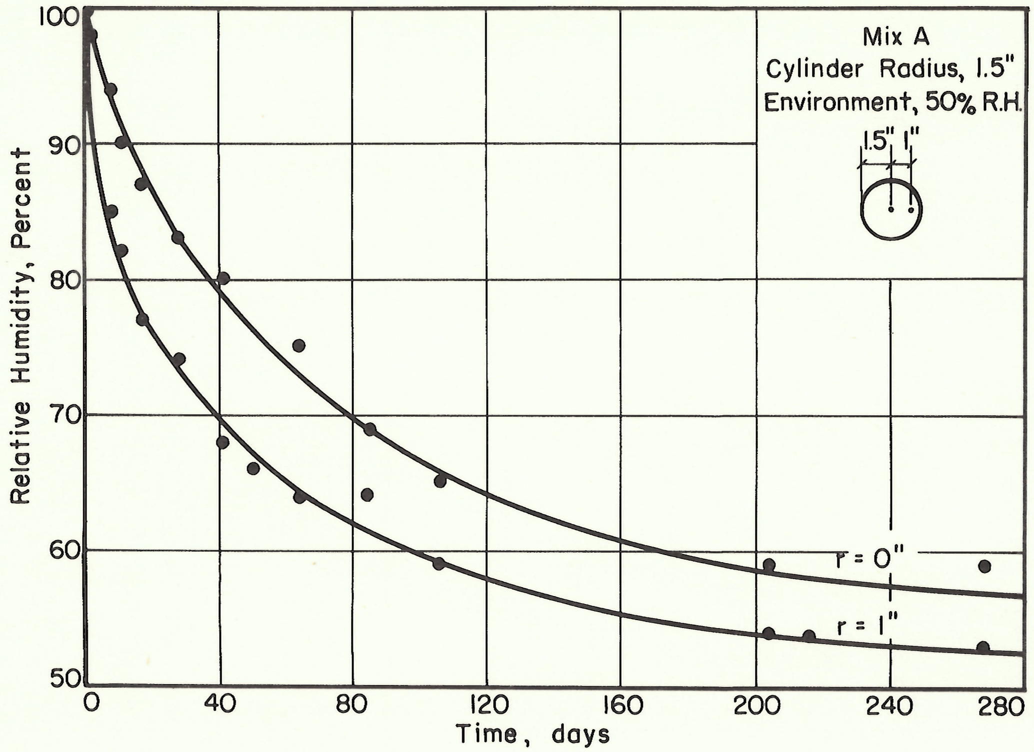

Fig. 5 Variation of Internal Relative Humidity with Time (3" dia.)

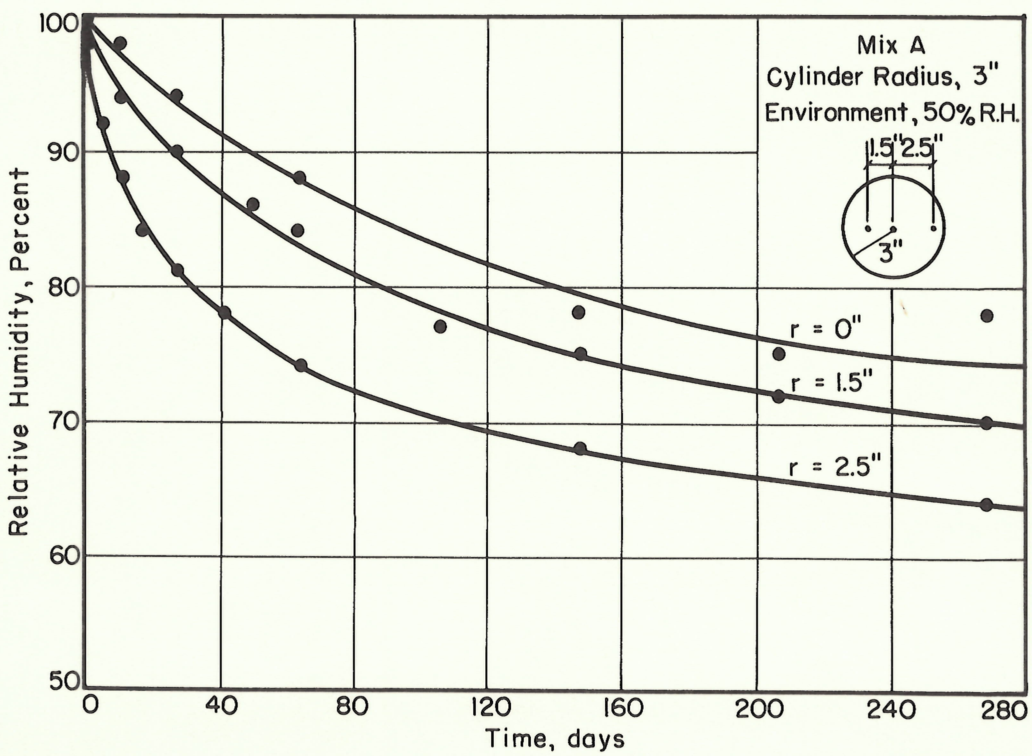

Fig. 6 Variation of Internal Relative Humidity with Time (6" dia.)

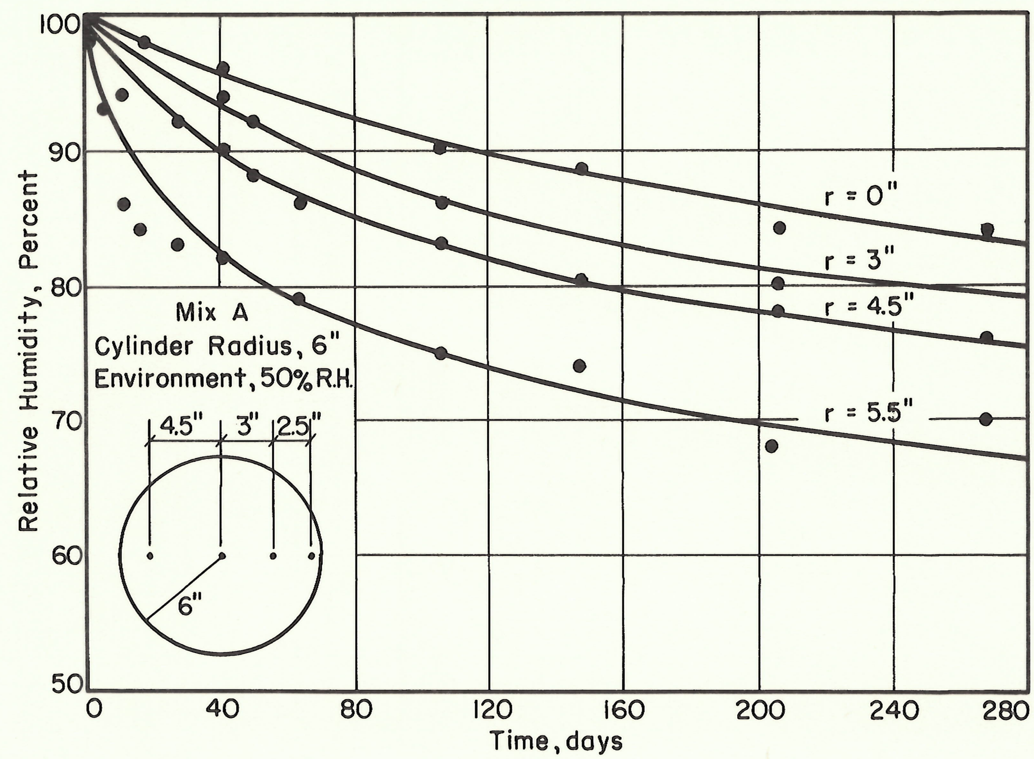

Fig. 7 Variation of Internal Relative Humidity with Time (12" dia.)

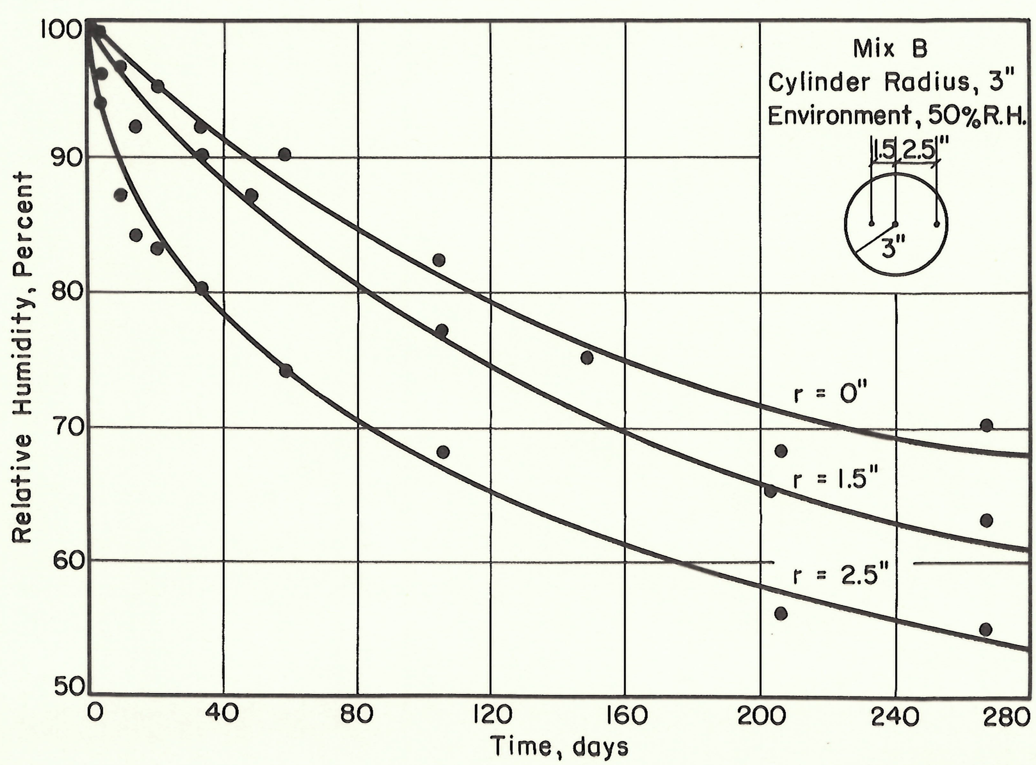

Fig. 8 Variation of Internal Relative Humidity with Time (6" dia.)

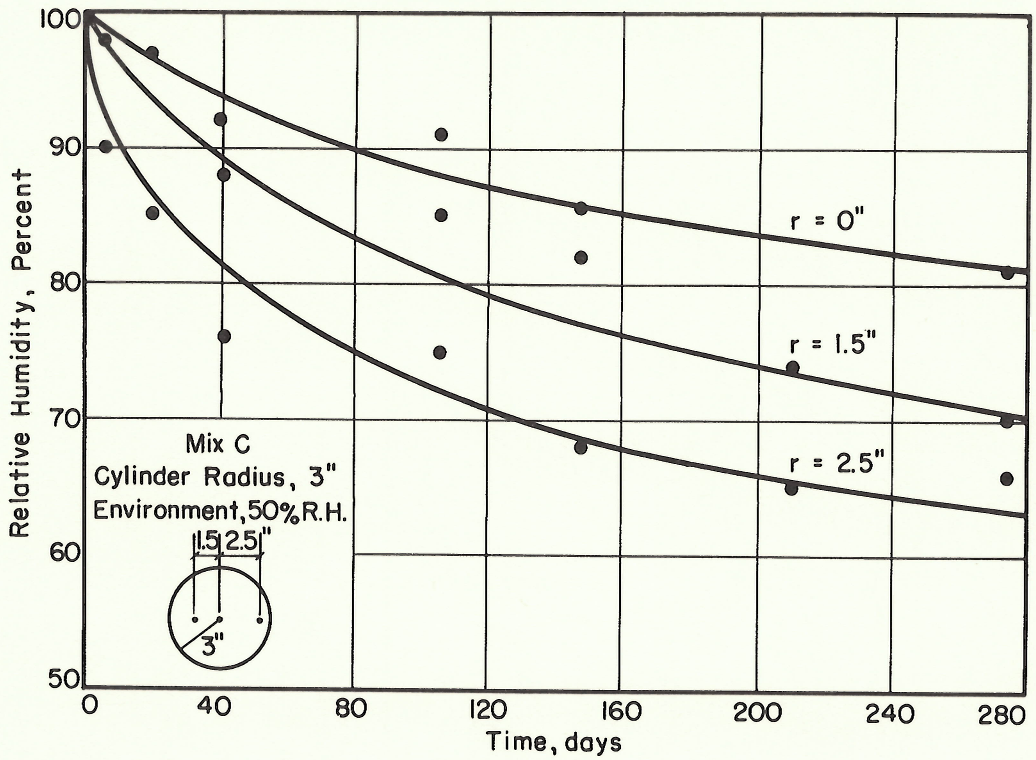

Fig. 9 Variation of Internal Relative Humidity with Time, (6" dia.)

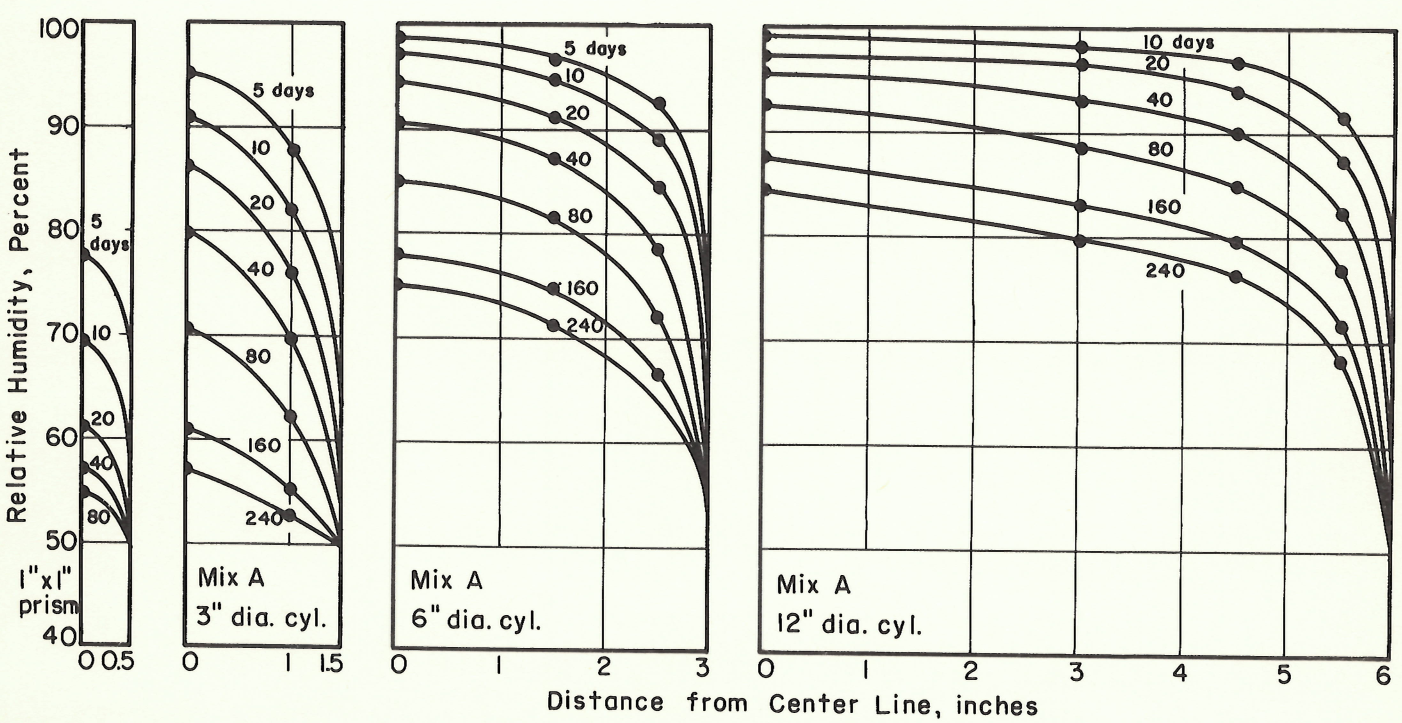

Fig. 10 Distribution of Relative Humidity in Specimens of Various Sizes

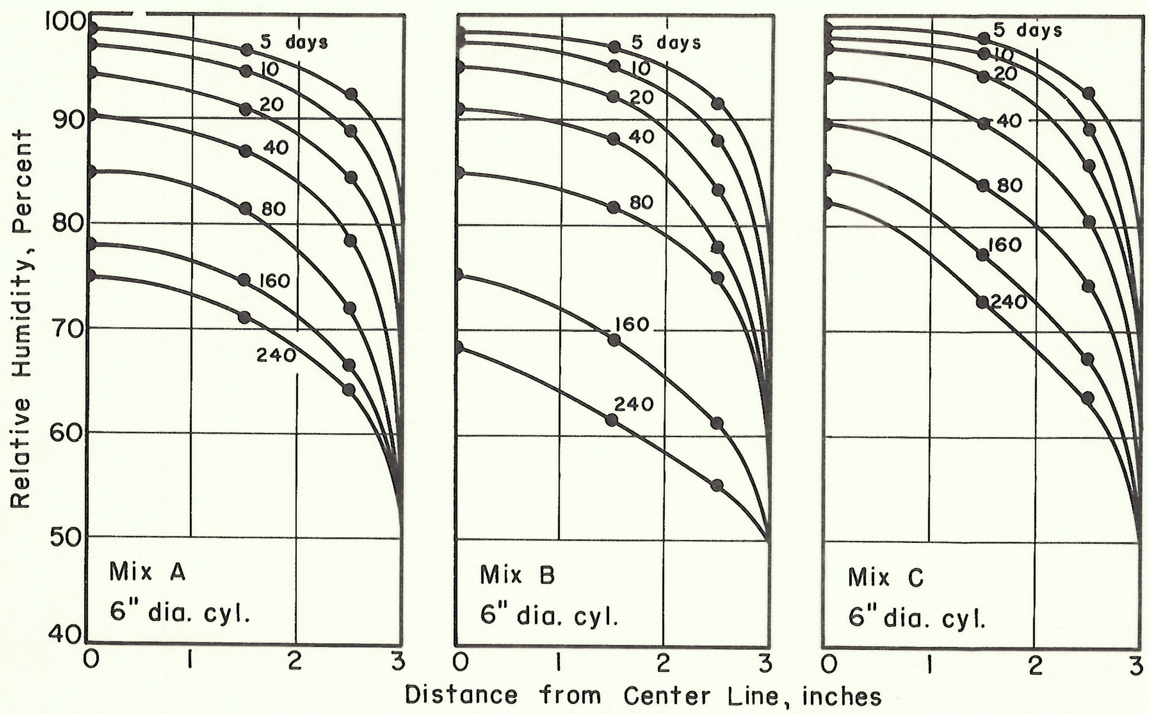

Fig. 11 Distribution of Relative Humidity in Specimens of Various Mixes

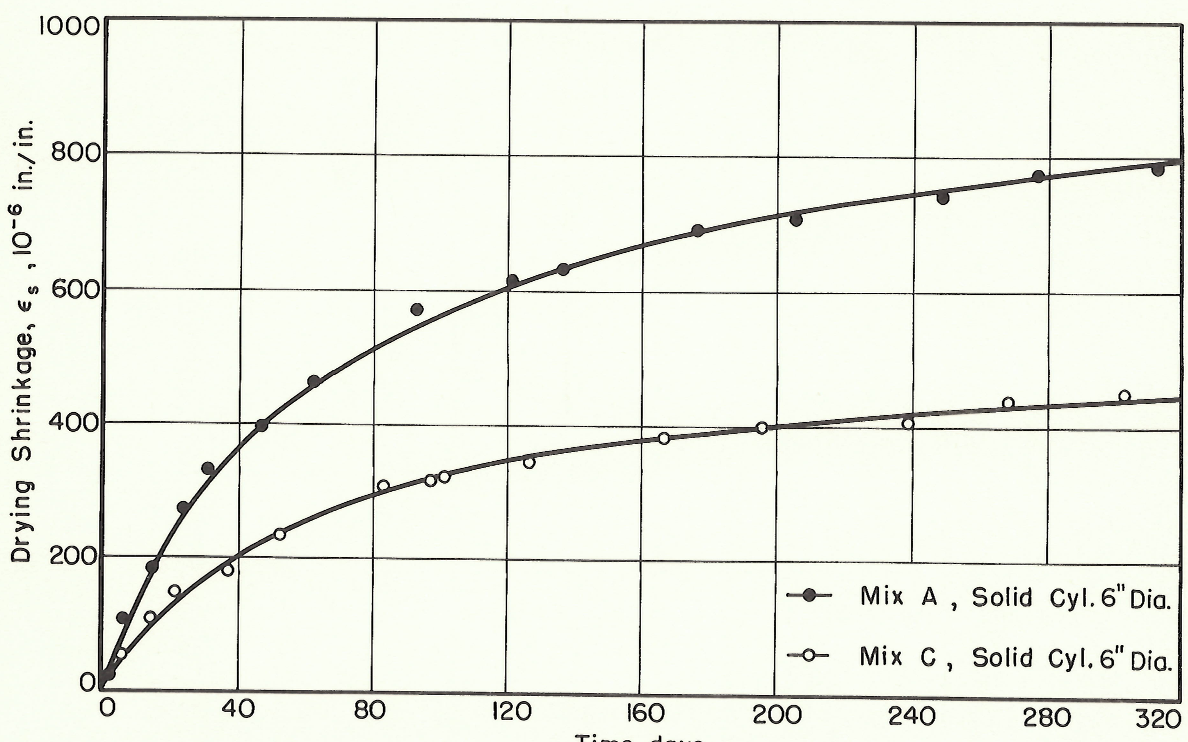

Fig. 12 Free Drying Shrinkage at 50 Percent Relative Humidity

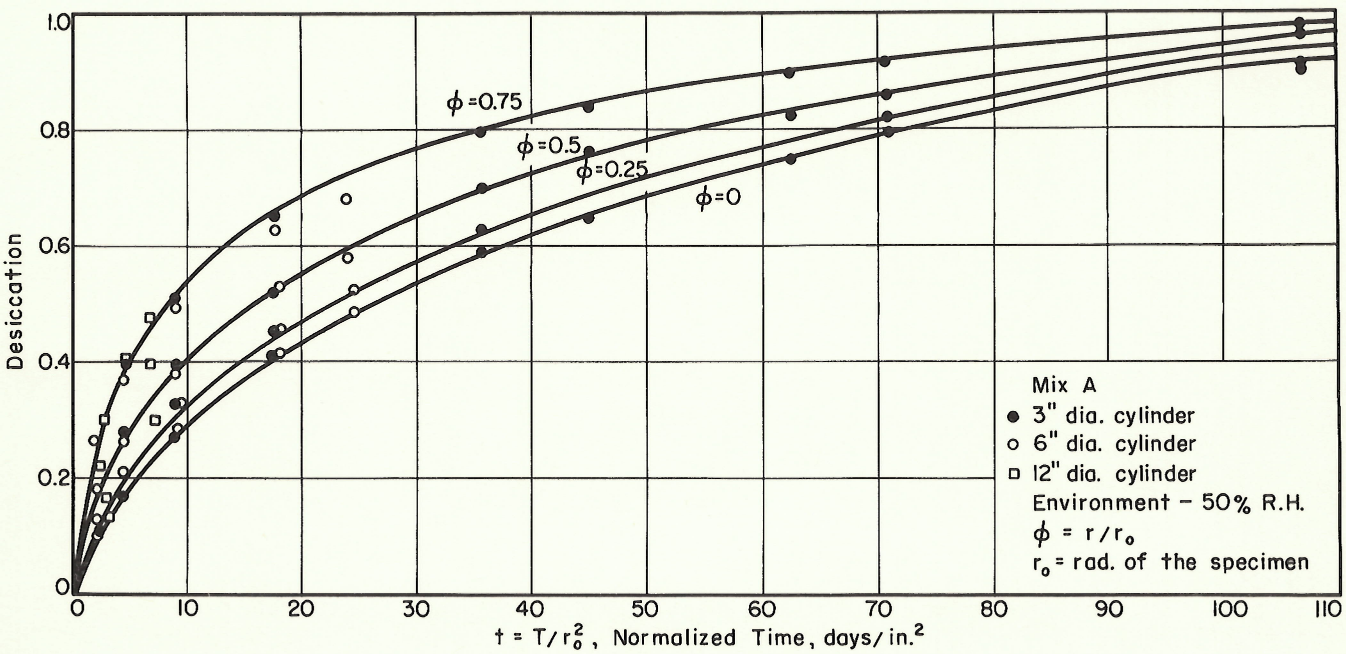

Fig. 13 Variation of Desiccation with Normalized Time, at 50 Percent Relative Humidity

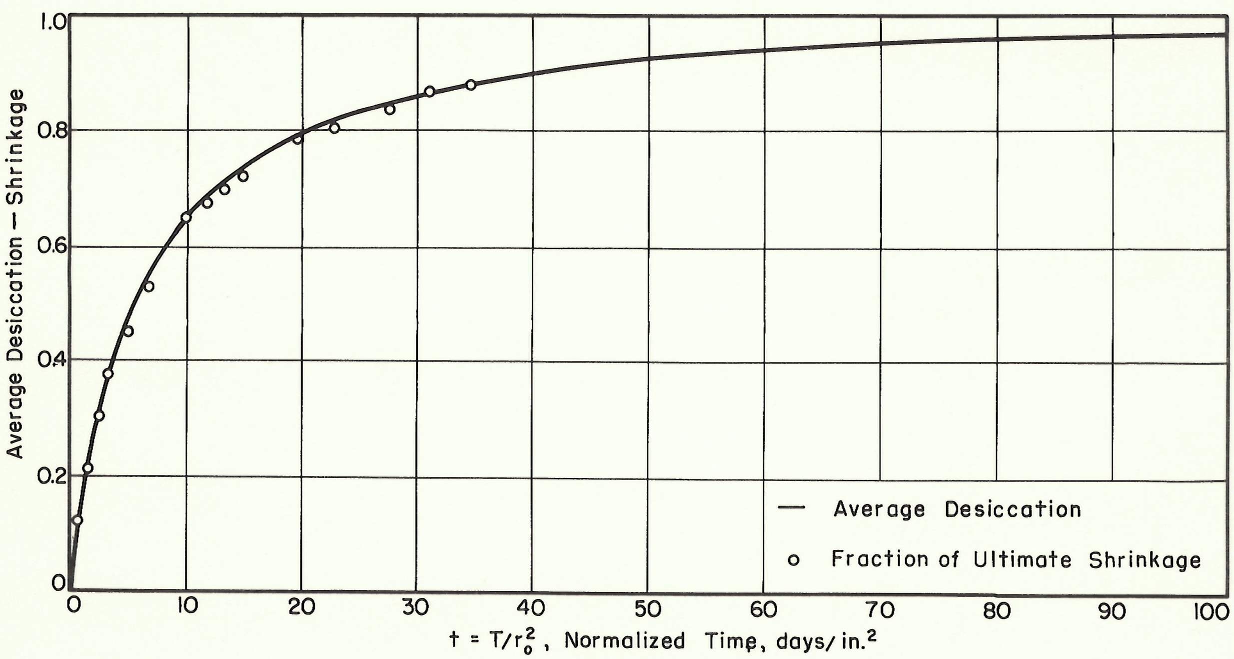

Fig. 14 Average Desiccation — Shrinkage at 50 Percent Relative Humidity