EFFECT OF ROTATION ON THE STRUCTURE OF A CONVECTING MUSHY LAYER

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Summary

Nonlinear natural convection in a mushy layer during solidification of a binary alloy is investigated under a high gravity environment where the rotation axis is inclined to the high gravity vector. Asymptotic and scaling analyses are applied to convective flow within the mushy layer and in vertical chimneys. The main result is that, for some particular moderate rotation range, vertical velocity in the chimneys decreases rapidly with increasing rotation rate and appears to have opposite signs across some rotation dependent vertical level.

1. Introduction

Recently Worster [1] analysed governing equations for a mushy layer in the limit of sufficiently large mush solutal Rayleigh number R_m. He proposed a model in which there is upward convective flow in localized chimneys within the mushy layer. These chimneys are characterized by having zero solid fraction, and they are assumed to be in vertical direction. These channels are locations of severe compositional nonhomegeneity which are called freckles. Freckles are imperfections that interrupt the uniformity of the solidified material causing areas of mechanical weakness. Sample and Hellawell [2,3] did NH_4Cl alloy experiment in a cylindrical mold with a chilled bottom surface where solidification was induced. They employed a rotation and tilting technique to change the orientation of gravity relative to the bottom surface of the cylinder. They found that for slow and steady rotation of the mold about the vertical axis, the channel formation and development was about the same as in the case without rotation. But, for slow and steady rotation of a tilted mold, the number of channels was reduced substantially and, under some conditions, completely eliminated. Their conclusion was that the decrease in freckles was due mainly to bulk liquid translation across the liquidus front. The main purpose of the present study was to investigate the effects of inclined rotation on the chimneys within the mushy layer using asymptotic and scaling analyses of the type carried out by Worster in the zero-rotation case in order to determine results which can be compared with the experimental results [2,3] as well as can be used, in a beneficial way, to eliminate or at least reduce the undesirable freckles effects.

The motivations for the present study under high gravity condition have been those discussed above as well as the recent experimental results due to Rodot et al. [4] in a centrifuge (high gravity environment), which indicated that convective transport can be suppressed at a well-defined acceleration level $Ng (N>1)$ (g is the acceleration due to gravity) during solidification in a centrifuge. Suppression of convection is useful because it can lead to uniform production of crystal which is desirable for crystal growers. Our goal has been to carry out the present theoretical investigation in order to gain some fundamental understanding of physical processes and driving forces in a high gravity environment which can lead to useful and beneficial results especially due to the fact that the present experimental data, taken during solidification in a centrifuge, are mostly ill-understood.

2. Formulation

We consider a thin layer of binary alloy melt of some constant composition C_o and temperature T_∞ which is solidified at a constant rate V_o, with the eutectic temperature T_e at the position $z=o$ held fixed in a frame moving with the solidification speed in z-direction, where the z-axis is assumed to be anti-parallel with the high gravity vector to be described below. The physical model at normal gravity conditions is based on the assumptions of the type considered by Worster [1], and we refer the reader to this reference for details regarding such assumptions. The extension of such model in high gravity environment is done based on assumptions of the type considered by Arnold et al.[5] for solidification in a centrifuge, and we refer the reader to this reference for details regarding such high gravity considerations. Within the layer of melt, there is a very thin mushy layer adjacent to the solidifying surface and of thickness *h*. In general *h* is a function of the variables *x* and *y*, but we shall follow [1] and make the simplifying assumption that *h* is a constant. Here *x* and *y* axes are in a plane perpendicular to *z*-axis. We assume that the solidifying system is placed in a centrifuge rotating at some constant angular velocity Ω about the centrifuge axis which makes an angle γ with respect to *z*-axis. The centrifuge axis is anti-parallel to the earth gravity vector.

Next, we consider the equations for momentum, continuity, heat and solute for both liquid and mushy layers in the moving frame *oxyz* whose origin *o* is centered on the solid-mush interface. The governing system of these equations for the solidifying system rotating with the centrifuge basket [5] and translating with the solidification front at speed V_O is non-dimensionalized using $V_0, K/V_0, K/V_0^2, \beta\Delta CP_o g K/V_0$, ΔC and ΔT as scales for velocity, length, time, pressure, solute and temperature, respectively. Here K is the thermal diffusivity, P_0 is a reference (constant) density, $\beta = \beta^* - \Gamma a^*$, where α^* and β^* are the expansion coefficients for heat and solate respectively and Γ is the slope of the liquidus curve [1], which is assumed to be constant, $\Delta C \equiv C_O - C_e$, C_e is the eutectic concentration of the alloy, $\Delta T = T_L(C_o) - T_e$, and T_L is the local liquidus temperature. Due to the variations of density with respect to solute concentration and temperature, the centrifugal acceleration terms in the momentum equations for both liquid and mushy layers can not be converted into a passive gradient terms and become important at significant rotation rate. Some recent results due to Riahi [6] indicate that there can be unusual and unexpected effects due to centrifugal acceleration term for a solidifying system in the absence of mushy layer. The centrifugal acceleration term, for either liquid or mushy layer, is then splitted into an average term, which is superimposed on the gravity term, and a so-called gradient acceleration term [5]. The non-dimensional parameters representing the modified gravity term can then become significantly larger than the corresponding one due to earth's gravity alone for significant rotation rate. Following [1,7,8], we treat the mushy layer as a porous medium where Darcy 's law is adopted. In general, a constitutive equation for the permeability $\Pi = \Pi(1-\phi)$ as a function of the local solid fraction ϕ of the mushy layer is needed to relate Π to ϕ. However, we do not need such relation in the present study, since we will be dealing with asymptotics states in which the term involving Π does not require specific form to leading terms and will not enter our analysis.

Since we will be concerned mainly with the convective flow in vertical and cylindrical chimneys [1] within the mushy layer, we consider cylindrical coordinate system. In order to make further simplifications, which can lead to theoretical progress in the present investigation, we shall consider axisymmetric steady state asymptotic for strong buoyancy force, due to solute concentration, and in the limits of $\beta \equiv \beta *$, sufficiently large Prandtl number $P_r = \mu/K$, sufficiently large Lewis number K/D and zero thermal bouyancy [1]. Here μ is the Kinematic viscosity and D is the solute diffusivity. The non-dimensional form of the governing equations and the boundary conditions for liquid and mushy layers are given below based on the above assumptions and the limiting conditions. The non-dimensional form of the equations for the momentum, continuity, temperature and solute concentration in the liquid layer are

$$\nabla^2 \underset{\sim}{u} = R_l\left(\nabla P + S\hat{K}\right) + T_l \hat{\Omega} \times \underset{\sim}{u} + A_l S \underset{\sim}{R}, \tag{1.1}$$

$$\Delta \cdot \underset{\sim}{u} = o, \tag{1.2}$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial z} + \underset{\sim}{u} \cdot \nabla\right)\theta = \nabla^2 \theta, \tag{1.3}$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial z} + \underset{\sim}{u} \cdot \nabla\right)S = o, \tag{1.4}$$

where $\underset{\sim}{u}$ is the velocity vector, P is the pressure, S is the solute concentration, θ is the temperature, $\hat{\Omega}$ is a unit vector along the rotation axis defined by

$$\hat{\Omega} = \cos\gamma\hat{K} + \sin\gamma\left(\cos\xi\hat{r} - \sin\xi\hat{\xi}\right), \tag{2.1}$$

$\hat{K}$ is a unit vector along axial (z) axis, $\hat{r}$ is a unit vector along radial axis, $\hat{\xi}$ is a unit vector along azimuthal axis, ξ is the azimuthal variable, r is the radial variable, $\underset{\sim}{R}$ is a radial position vector from the rotation axis defined by

$$\begin{aligned}\underset{\sim}{R} = &\left(-r\cos^2\xi\cos^2\gamma + z\sin\gamma\cos\gamma\cos\xi - r\sin^2\xi\right)\hat{r} + \\ &\left(-z\sin\gamma\cos\gamma\sin\xi + r\cos\xi\sin\xi\cos^2\gamma - r\sin\xi\cos\xi\right)\hat{\xi} + \\ &\left(r\sin\gamma\cos\gamma\cos\xi - z\sin^2\gamma\right)\hat{K},\end{aligned} \tag{2.2}$$

$R_l = \beta * \Delta CNg\,K^2/\left(V_0^3\mu\right)$ is the liquid solutal Rayleigh number, $T_l = 2\Omega K^2/\left(V_0^2\mu\right)$ is the liquid Coriolis Parameter (square root of Taylor number), $A_l = \beta\Delta C\Omega^2 K^3/\left(V_0^4\mu\right)$ is the liquid gradient acceleration parameter, $Ng = \left(g^2 + \Omega^4 R_0^2\right)^{\frac{1}{2}}$ is the acceleration due to high gravity, $N = 1$ corresponds to normal gravity case while $N > 1$ indicates level of high gravity, R_0 is the perpendicular distance from the center of gravity of the centrifuge basket to the rotation axis and t is the time variable. The boundary conditions for the liquid layer are

$$\theta - S = \frac{\partial}{\partial z}(\theta - S) = \left[\hat{n}.\underset{\sim}{u}\right] = \underset{\sim}{u} - \hat{n}.\underset{\sim}{u} = o \text{ at } z = h, \tag{3.1}$$

$$\theta \to \theta_\infty, S \to o, \underset{\sim}{u} \to o \text{ as } z \to \infty, \tag{3.2}$$

where θ_∞ is the non-dimensional form of T_∞ the square brackets denote the jump in the enclosed quantity across the interface and $\hat{n}$ is a unit vector normal to the interface. The non-dimensional form of the equations for momentum, continuity, temperature and solute concentration in the mushy layer are

$$\frac{\underset{\sim}{u}}{\Pi} = R_m\left(\nabla P + S\hat{K}\right) + T_m\hat{\Omega} \times \underset{\sim}{u} + A_m S\,\underset{\sim}{R}, \tag{4.1}$$

$$\nabla.\underset{\sim}{u} = o, \tag{4.2}$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial z}+\underset{\sim}{u}.\nabla\right)\theta=\nabla^2\theta-S_t\frac{\partial\phi}{\partial z},\qquad(4.3)$$

$$\left(-\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right)\frac{\partial S}{\partial t}+\frac{\partial}{\partial z}\left[(1-\phi)(C_r-S)\right]+\underset{\sim}{u}\cdot\nabla S=o,\qquad(4.4)$$

where $S_t = L/(C\Delta T)$, is the Stefan number, C is the specific heat per unit volume, L is the latent heat of solidification per unit volume, $C_r=(C_s-C_0)/\Delta C$ is a concentration ratio, C_s is the composition of the solid phase forming the dendrites, Π is the permeability of the mushy layer, $R_m=\beta\Delta CNg\Pi_0/(V_0\mu)$ is the mush solutal Rayleigh number, Π_0 is a reference value of the permeability, $T_m=2\Omega\Pi_0/\mu$ is the mush Coriolis Parameter and $A_m=\beta\Delta C\Pi_0K\Omega^2/(V_0^2\mu)$ is the mush gradient acceleration parameter. The boundary conditions for the mushy layer are

$$\theta=-1, \underset{\sim}{u}.\hat{K}=o \text{ at } z=o,\qquad(5.1)$$

$$[\theta]=[\hat{n}.\nabla\theta]=[P]=o \text{ at } z=h\qquad(5.2)$$

Analysis in Worster [1] indicates that $\theta=S$ in the mushy layer. This result is also valid here. For details regarding (1.1)-(5.2) and the assumption involved the reader is referred to [1] on solidification aspects and to [5] on high gravity aspects.

In the following three sections, we shall apply scaling analysis for (1.1)-(5.2) to determine the strongly nonlinear steady state axisymmetric behaviour and structure of the convective mushy layer and mainly the flow features in chimneys once they are fully developed within the mushy layer in the asymptotic limit of sufficiently large R_m [1].

3. Analysis for weak rotation

Let us designate a to be the radius of a chimney under consideration whose axis coincides with $z-$axis. It is assumed that a is small $(a<<1)$. Define a stream function $\psi(r,z)$ for the flow in the chimney

$$\underset{\sim}{u} = (u, w) = \left(-\frac{1}{r}\frac{\partial \psi}{\partial z}, \frac{1}{r}\frac{\partial \psi}{\partial r} \right), \tag{6}$$

where u and w are the radial and axial components of the velocity vector. We assume that the orders of magnitude of r and z are a and 1, respectively. Following Worster [9] that R_l is large generally in comparison to R_m, we find in the present problem that the following relations hold

$$\frac{R_l}{R_m} = \frac{A_l}{A_m} = \frac{T_l}{T_m} >> 1 \cdot \tag{7}$$

Let us consider the asymptotic regime of large $R_m(R_m >> 1)$ [1] and assume that $|\underset{\sim}{u}| \sim 1$ in the mushy layer, $A_l << R_l$ and $aT_l << R_l$. Then, (4.1) implies that to the leading terms pressure field is hydrostatic and $\theta = S$ is independent of r. Using these results in (4.3)-(4.4) implies that $w \equiv w_0(z)$ and $\phi \equiv \phi_0(z)$ at most. Assuming that $C_r >> \theta$ in the mushy layer [1], (4.3) – (4.4) then yield

$$(w_0 - 1)\theta_0' = \theta_0'' - S_l\phi_0', \tag{8.1}$$

$$(1 - \phi_0)\theta_0' + C_r\phi_0' = w_0\theta_0', \tag{8.2}$$

where $\theta_o(z)$ is the leading order term for θ and prime indicates derivative with respect to z.

Next, consider the flow in the chimney which is described by (1.1)-(1.4) since $\phi = o$ there. Using (6), we have $u \sim \psi / a, w \sim \psi / a^2$, and thus $u \sim aw$. It is also reasonable to assume that $S \sim 1$ and $w >> 1$ [1]. Then (1.1) implies that gradient acceleration term is negligible compared to the viscous term for $A_l << w / a^2$, which we assume here for a weak rotation case. It then follows from (1.1) that $w \sim R_l a^2, u \sim R_l a^3, \psi \sim R_l a^4$. Now θ_0 is the leading order temperature solution in the mushy zone outside the chimney and its surrounding vertical boundary layer [1]. Designate $\theta_1(r, z)$ to be the deviation of θ from

θ_0. From (1.2) and the condition $1/a^2 << R_l << 1/a^4$, we find $\theta_1 << 1$ and consequently

$$A_l << R_l \tag{9}$$

Also assuming $a^2 T_l u << w$, which together with (9) make rotation negligible in (1.1), it follows that

$$T_l << 1/a^3 \cdot \tag{10}$$

Using these results, (1.1)-(1.4) in the chimney are reduced to the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = R_l\left(S + \frac{\partial \mathrm{P}}{\partial z}\right), \tag{11.1}$$

$$\nabla \cdot \underset{\sim}{u} = o, \tag{11.2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta_1}{\partial r}\right) = \frac{1}{r}\frac{\partial \psi}{\partial r}\theta_0', \tag{11.3}$$

$$\underset{\sim}{u} \cdot \nabla S = 0 \cdot \tag{11.4}$$

Now, the results in the later analysis and (7) indicate that it is reasonable to assume that

$$T_m << 1/a \cdot \tag{12}$$

Using (9), (12) and (4.1) in the mushy zone outside the chimney, we find that

$$\frac{\partial \rho}{\partial z} = -\theta_0 \tag{13}$$

to the reading terms. Using (13) in (11.1), we find

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = R_l(S - \theta_0) \cdot \tag{14}$$

Integrate (11.3) in r from $r = o$ to $r = a$ and follow [1], we find the following result

$$\theta_1 \sim \psi_a \theta_o' 1nr, \tag{15.1}$$

where $$2\Pi \psi a = \int_o^a 2\Pi r w dr \tag{15.2}$$

is the vertical volume flux in the chimney. Using (6), we have

$$u \sim -\psi_a' / r \text{ as } r \to a \cdot \tag{16}$$

From (4.1), we have

$$\frac{u}{\Pi} \sim -R_m \frac{\partial \mathrm{P}}{\partial r} \cdot \tag{17}$$

Use (16) in (17), we find

$$\Delta P \sim \frac{\psi_a'}{R_m} \ln a, \tag{18}$$

where it is assumed that $\Pi = 0(1)$ and ΔP represents the pressure near the chimney. Using (15.1) and (4.1) we find that

$$w \sim R_m \psi_a \ln a \tag{19}$$

holds near the wall of the chimney. Using (4.4) and the condition $C_r >> 1$, we find

$$C_r \frac{\partial \phi}{\partial z} \sim \underset{\sim}{u} \cdot \nabla S \cdot \tag{20}$$

Within the mushy zone outside the chimney, we have

$$C_r \frac{\partial \phi}{\partial z} \sim u \frac{\partial \theta_1}{\partial r} + w\theta_0' \cdot \tag{21}$$

For $u << 1$, (21) reduces to

$$C_r \frac{\partial \phi}{\partial z} \sim w\theta_0' \cdot \tag{22}$$

Near the wall of the chimney, $\theta_0' \sim 1, C_r >> 1$ and $\phi C_r \sim 1$ [1], which implies that $w \sim 1 \cdot$

From (19), (16) and earlier result on the order of magnitude of u, we find

$$R_m R_l a^4 \ln a \sim 1 \cdot \tag{23}$$

Using these results, (23) and (18), we find that

$$\Delta P \sim 1/R_m^2 \cdot \tag{24}$$

Using (15.1) and the above results, we find that

$$\theta - \theta_0 \sim 1/R_m \cdot \tag{25}$$

From (16) and (23)-(25), we find

$$u \frac{\partial \theta_1}{\partial r} \sim \left[R_l / (R_m \ln a)^3 \right]^{1/2} \cdot \tag{26}$$

Thus $u \frac{\partial \theta_1}{\partial r}$ is negligible if $R_l << (R_m \ln a)^3$ with a given by (23). For

$$R_l >> (R_m \ln a)^3, \tag{27}$$

all the three terms in (21) must balance and using (23) yield

$$w \sim R_l^2 a^6 \cdot \tag{28}$$

Using (19) and (28), we find

$$\frac{R_m}{R_l} \sim a^2/\ln a \cdot \qquad (29)$$

Using (18) and (29), we find

$$\Delta P \sim (\ln a)^3 R_m/R_l \cdot \qquad (30)$$

From (26), we have

$$\theta - \theta_0 \sim R_m^2(\ln a)^3 R_l \cdot \qquad (31)$$

Vertical and horizontal advection of solute balance here in this regime. Using (4.3) and (28)-(29), we find

$$\frac{\partial \phi}{\partial z} \sim (Rm \ln a)^3 / R_l \qquad (32)$$

which is small based on the conditon (27). Now, the wall of the chimney can be defined by

$$\phi[a(z), z] = o \text{ at } r = a(z) \cdot \qquad (33)$$

Taking derivative with respect to z of (33) implies

$$\frac{a'(z)}{a} \sim \frac{\partial \phi}{\partial z} << 1, \qquad (34)$$

which indicates that, based on the scalings of the type (29)-(31), the walls of the chimney are vertical to leading order terms. Applying the scalings (29)-(31), using a Polhausen type method suggested by Lighthill [10] and following Worster [1], we find that the total volume flux in the chimney, due to upward flow [1], is given by

$$2\pi\psi_a = 2\pi\lambda a^4 R_l[1 + \theta_0(z)], \qquad (35)$$

where λ is a constant [1].

To satisfy the mass conservation, downward flow through the mushy zone $w_0(z)$ must be equal to the total upflow through all the chimneys per unit horizontal area. Thus

$$w \cdot = 2\pi\psi_a N, \qquad (36)$$

where N is the number density of chimneys.

For the case of weak rotation, say for Ω of order of $0.1 rad/\sec\cdot$, it can be seen from the definition of Ng given in section two that, for realistic centrifuge cases [5], Ng is hardly different from $1g$ so that the results are essentially those for normal gravity conditions.

4. Analysis for moderate rotation

With the same basic assumptions as in the case of weak rotation, the results (6)-(8.2) are found to be valid again for the moderate rotation case, where

$$A_l \sim R_l \cdot \tag{37}$$

The results (10) and (11.2)-(11.4) are found to be valid again here. However, (11.1) is now replaced by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = R_l\left(S + \frac{\partial \mathrm{P}}{\partial z}\right) - A_l S z \sin^2 \gamma \cdot \tag{38}$$

The result (12) is valid again here. However, (13)-(14) are replaced, respectively, now by the following two equations

$$\frac{\partial \mathrm{P}}{\partial z} = -\theta_0\left[1 - (A_m / R_m) z \sin^2 \gamma\right], \tag{39}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = R_l(S - \theta_0) + R_l A_m \theta_0 z \sin^2 \gamma / R_m + A_l S z \sin^2 \gamma \tag{40}$$

The results (15.1)-(15.2) and (16) are valid again here. However, (17)-(19) are now replaced by $(\xi = 90^\circ\, case)$

$$\frac{u}{\Pi} \sim -R_m \frac{\partial \mathrm{P}}{\partial r} + r A_m S, \tag{41}$$

$$\Delta \mathrm{P} \sim A_m a^2 \theta_0 / R_m, \tag{42}$$

$$w \sim R_m \psi_a \ln a\left(1 - A_m z \sin^2 \gamma / R_m\right) \cdot \tag{43}$$

The results (20)-(22) are valid again. However, (23)-(24) are now replaced by

$$R_m \psi_a \ln a\left(1 - A_m z \sin^2 \gamma / R_m\right) \sim 1, \tag{44}$$

$$\Delta \mathrm{P} \sim a^2 \cdot \tag{45}$$

The results (25) -(29) are valid again. However, (30) is now replaced by

$$\Delta \mathrm{P} \sim R_m \ln a / R_l \cdot \tag{46}$$

The results (31)-(34) and (36) are valid again. However, (35) is now replaced by

$$2\pi\psi_a = 2\lambda(z) a^4 R_l\left[1 + \theta_0(z)\right]\left(1 - A_m z \sin^2 \gamma / R_m\right), \tag{47}$$

where λ is now a function of z.

The most interesting results for the case of moderate rotation, where (37) holds, are that due to (43) and (47). These results indicate that the vertical velocity as well as the vertical volume flux in the chimney decrease with increasing rotation rate. For some particular rotation rate limit, where $A_m z \sin^2 \gamma \to R_m$ for some critical value $z = z_c, w \to o$ based on (43) and the next leading order terms will determine the order of magnitude of w, which subsequently will be much less than that for weak rotation case. However, for $z > z_c$, (47) indicates that ψ_a has opposite sign to that for $z < z_c$. Furthermore, in such limiting circumstances, a smaller value of z_c corresponds to a larger rotation rate and visce versa a larger value of z_c corresponds to a smaller rotation rate. Clearly these results indicate the stabilization of the convective flow within the chimney by the external constraint of the inclined rotation. Also, there is some indication of the possible double-cell formation in the vertical direction within the chimney due to the effect of a moderate rotation rate. The important cell in the immediate vicinity of the mush-solid interface is shorter for higher rotation rate.

For the case of moderate rotation, say for Ω of order of $6\,rad/\sec$, it can be seen from the definition of Ng given in section two that, for realistic centrifuge cases [5], Ng is of order of about $10g(\Pr = 100)$ so that the results given in this section are applicable in high gravity conditions.

5. Analysis for strong rotation

With the same basic assumptions as in the case of weak or moderate rotation, the results (6)-(18.2) are found to be valid again here for the strong rotation case, where

$$A_l >> R_l \cdot \tag{48}$$

The results (10) and (11.2)-(11.4) are found to be valid again here. However, (11.1) and (38) are now replaced by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = R_l \frac{\partial P}{\partial z} + A_l S z \sin^2 \gamma \cdot \tag{49}$$

The result (12) is valid again here. However, (13)-(14) and (39)-(40) are now replaced by

$$\frac{\partial \mathrm{P}}{\partial z} = A_m S z \sin^2 \gamma / R_m, \tag{50}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial w}{\partial r} \right) = R_l A_m \theta_0 z \sin^2 \gamma / R_m - A_l S z \sin^2 \gamma \cdot \tag{51}$$

The results (15.1)-(15.2) and (16) are valid again here. However, (17)-(19) and are now replaced by (41)-(42) and by

$$w \sim A_m \psi_a \ln a z \sin^2 \gamma \cdot \tag{55}$$

The results (20)-(22) are valid again here. However, (23)-(24) and (44)-(45) are now replaced by

$$A_m \psi \ln a z \sin^2 \gamma \sim 1, \tag{56}$$

$$\Delta \mathrm{P} \sim A_m a^2 / R_m \cdot \tag{57}$$

The results (25)-(29) are valid again here. However, (30) and (46) are now replaced by

$$\Delta \mathrm{P} \sim A_m \ln a / R_l \cdot \tag{58}$$

The results (31)-(34) and (36) are valid again here. However, (35) and (47) are now replaced by

$$2\pi \psi_a = -2\pi \lambda(z) a^4 R_l A_m z \sin^2 \gamma (1 + \theta_0)/R_m \cdot \tag{59}$$

It is seen from the above results in this section for the case of strong rotation, where (48) holds, that the order of magnitudes of pressure and vertical velocity in the chimney increases with rotation rate. Hence, rotational effects are destabilizing in this case and can lead quickly to channel formation in the mushy layer. These results are in agreement with the experimental results reported by Kou et al. [11] which indicated that if rotational speeds became too large, then segregates formed along a ring between the axis and outer edge of the ingot.

6. Some conclusions

(i) For $A_l << R_l$, the volume flux and the vertical velocity in the chimney increase with R_l but are un-affected by the rotational effects.

(ii) For $A_l \sim R_l$, the volume flux and the vertical velocity in the chimney increase with R_l, but they decrease with increasing the rotational effects. Rotation is stabilizing in this case.

(iii) For $A_l >> R_l$, the volume flux and the vertical velocity in the chimney increase with R_l, and they also increase with increasing the rotational effects. Rotation is destabilizing in this case.

(iv) The results of the present study concludes that only some moderate gravity level $(\sim 10g)$ in the high gravity environment appears to have a beneficial effect on macrosegregation.

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