On the Structure of an Unsteady Convecting Mushy Layer

PACS numbers: 47.25QV, 47.20KY, 47.20BP, 47.20HW

D.N. Riahi
Department of Theoretical and Applied Mechanics,
216 Talbot Laboratory, 104 S. Wright Street
University of Illinois, Urbana, IL 61801

Abstract

Nonlinear time dependent natural convection in a mushy layer during solidification of a binary alloy is investigated. Asymptotic and scaling analyses are applied to convective flow within the mushy layer and in cylindrical chimneys with small radius a. Various results in four distinctly identified regimes, corresponding to high or low Prandtl number melt and strongly or weakly time dependent flow, are determined. In particular, it is found that strongly time dependent flow can provide non-vertical chimneys, and, for weakly time dependent flow of a low Prandtl number melt vertical chimneys are possible only for sufficiently small a.

I. INTRODUCTION

Recently Worster[1] analyzed governing equations for a mushy layer in the limit of infinite Prandtl number and asymptotically large mush solutal Rayleigh number R_m. He proposed a steady state model in which there is upward steady convection in localized chimneys within the mushy layer. These chimneys are characterized by having zero solid fraction, and they were assumed to be in vertical direction. These channels are locations of severe compositional nonhomogeneities which are called freckles. Freckles are imperfections that interrupt the uniformity of the solidified material causing areas of mechanical weakness. The steady model described above is consistent with experimental observation in aqueous systems[2] where Prandtl number is high. However, such steady state model may not be preferred for low Prandtl number melt cases[3], such as metallic alloys which are of commercial values. Sayre and Riahi[3] investigated linear flow instabilities of the liquid and mushy regions during solidification of binary alloys in the absence and presence of external constraint of rotation. For the case without rotation, they found, in particular, that an oscillatory mode of convection is preferred for low Prandtl number melt cases, such as those of metallic alloys. Although such results were obtained for low values of the mush solutal Rayleigh number, it is suspected that the time dependent behavior of convection at sufficiently large mush solutal Rayleigh number is rule rather than exception, particularly for low Prandtl number melt cases[4-6].

The materials presented in the paragraph above suggests that we may expect a range of Prandtl number and degree of unsteadiness of convection where Worster[1] model is appropriate, while a different range of Prandtl number and a different degree of unsteadiness of convection may lead to an unsteady type model. The results of the present study, indeed, lead to such conclusion. Our aim here, however, is similar to that of Worster[1]. That is, to understand the nature of the flow and the structure of the mushy layer that occurs once chimneys are fully developed.

II. FORMULATION

We consider a thin layer of a binary alloy melt of some constant composition C_0 and temperature T_∞ solidified at a constant rate V_0, with the eutectic temperature T_e at the position $z=0$ held fixed in a frame moving with speed V_0 in z-direction, where the z-axis is anti-parallel to the gravity vector. The physical model is based on the assumptions of the type considered by Worster[1], and we refer the reader to this reference for details of such assumptions. Within the layer of melt, there is a very thin mushy layer adjacent to the solidifying surface and of thickness h.

The governing system of equations for momentum, continuity, heat and solute for both liquid and mushy layers in the moving frame $oxyz$, whose origin o is centered on the solid-mush interface, is non-dimensionalized using $V_0, K/V_0, K/V_0^2, \beta\Delta C\rho_0 g K/V_0, \Delta C$ and ΔT as scales for velocity, length, time, pressure, solute and temperature, respectively. Here K is the thermal diffusivity, ρ_0 is a reference (constant) density, $\beta = \beta^* - \tau\alpha^*$, where α^* and β^* are the expansion coefficients for heat and solute respectively and τ is the slope of the liquidus curve[1], which is assumed to be constant, $\Delta C \equiv C_0 - C_e$, C_e is the eutectic concentration of the alloy, $\Delta T \equiv T_L(C_0) - T_e$, and T_L is the local liquidus temperature. Following Roberts and Loper[7], Fowler[8], and Worster[1], we treat the mushy layer as a porous medium where Darcy's law is adopted. In general, a constitutive equation for the permeability $\Pi = \Pi(1-\phi)$ as a function of the local solid fraction ϕ of the mushy layer is needed to relate Π to ϕ. However, we do not need such relation in the present study, since we will be dealing with asymptotic states in which the term involving ϕ does not require specific form to leading terms and will not enter our analysis.

Since we will be concerned mainly with the convective flow in cylindrical chimneys[1] within the mushy layer, we consider cylindrical coordinate system. In order to make further simplification, which can lead to theoretical progress in the present study, we shall consider axisymmetric asymptotic state for strong buoyancy force, due to solute concentration, and in the limit of $\beta = \beta^*$, insignificant solute diffusivity $D(D << K)$ and negligible thermal buoyancy[1].

The non-dimensional form of the equations for the momentum, continuity, temperature and solute concentration in the liquid layer are

$$\frac{1}{\Pr}\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial z} + \underset{\sim}{u}.\nabla\right)\underset{\sim}{u} = -R_l\left(\nabla \mathrm{P} + S\hat{K}\right) + \nabla^2 \underset{\sim}{u}, \alpha^*, \text{ where } \alpha^* \text{ and } \beta^* \quad (1\text{ a})$$

$$\nabla.\underset{\sim}{u} = 0, \quad (1\text{ b})$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial z} + \underset{\sim}{u}.\nabla\right)\theta = \nabla^2\theta, \quad (1\text{ c})$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial z} + \underset{\sim}{u}.\nabla\right)S = 0, \quad (1\text{ d})$$

where $\underset{\sim}{u}$ is the velocity vector, P is the pressure, S is the solute concentration, θ is the temperature, $\hat{K}$ is a unit vector along axial (z) axis, $\Pr = \mu/K$ is the Prandtl number, μ is the kinematic viscosity, $R_l = \beta^* \Delta C g K^2 \big/ \left(V_0^3 \mu\right)$ is the liquid solutal Rayleigh number, g is acceleration due to gravity and t is the time variable. The boundary conditions for the liquid layer are

$$\theta - S = \frac{\partial}{\partial z}(\theta - S) = \left[\hat{n}.\underset{\sim}{u}\right] = \underset{\sim}{u}.\hat{n}.\underset{\sim}{u} = 0 \text{ at } z = h, \quad (2\text{ a})$$

$$\theta \to \theta_\infty, \; S \to 0, \; \underset{\sim}{u} \to 0 \text{ as } z \to \infty, \quad (2\text{ b})$$

where θ_∞ is the non-dimensional form of T_∞, the square brackets denote the jump in the enclosed quantity across the interface, $\hat{n}$ is a unit vector normal to the interface $h(r,t)$ and r is the radial variable. The non-dimensional form of the equations for momentum, continuity, temperature and solute concentration in the mushy layer are

$$\frac{-\underset{\sim}{u}}{\Pi} = R_m\left(\nabla \mathrm{P} + S\hat{K}\right), \tag{3 a}$$

$$\nabla.\underset{\sim}{u} = 0, \tag{3 b}$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial z} + \underset{\sim}{u}.\nabla\right)\theta = \nabla^2\theta + S_t\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial z}\right)\phi, \tag{3 c}$$

$$\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial t}\right)\left[(1-\phi)\left(C_r - S\right)\right] + \underset{\sim}{u}.\nabla S = 0, \tag{3 d}$$

where $S_t = L/(C\Delta T)$ is the Stefan number, C is the specific heat per unit volume, L is the latent heat of solidification per unit volume, $C_r = \left(C_S - C_O\right)/\Delta S$ is a concentration ratio, C_S is the composition of the solid phase forming the dendrites, Π is the permeability of the mushy layer, $R_m = \beta\Delta C g \Pi_0/\left(V_0\mu\right)$ is the mush solutal Rayleigh number and Π_0 is a reference value of the permeability. The boundary conditions for the mushy layer are

$$\theta = -1,\ \ \underset{\sim}{\mathrm{u}}.\hat{K} = 0 \text{ at } z = 0, \tag{4 a}$$

$$[\theta] = [\hat{n}.\nabla\theta] = [\mathrm{P}] = 0 \ \text{ at } z = h. \tag{4 b}$$

Analysis in Worster[1] indicates that $\theta = S$ in the mushy layer. This result is also valid here. For details regarding (1)-(4) and the assumptions involved the reader is referred to Worster[1].

In the following four sections, we shall apply scaling analysis for (1)-(4) to determine the axisymmetric behavior and structure of the convective mushy layer and mainly the flow features in chimneys once they are fully developed within the mushy layer in the asymptotic limit of sufficiently large R_m. Following Worster[9], we shall also assume that $H \equiv R_l/R_m$ is large.

III. ANALYSIS FOR HIGH Pr AND WEAK TIME DEPENDENCE

Let us designate $a(r,t)$ to be the radius of a chimney under consideration whose axis coincides with z-axis. It is assumed that a is small $(a \ll 1)$. Define a stream function $\psi(r,z,t)$ for the flow in the chimney

$$\underset{\sim}{u} = (u,w) = \left(-\frac{1}{r}\frac{\partial \psi}{\partial z}, \frac{1}{r}\frac{\partial \psi}{\partial r}\right), \tag{5}$$

where u and w are the radial and axial components of the velocity vector, respectively. We assume that the orders of magnitude of r and z are a and 1, respectively. Let us consider the asymptotic regime of large $R_m \left(R_m \gg 1\right)$. The analysis presented in this section is for the case where the inertia terms in (1 a) can be, at most, as significant as the viscous terms in (1 a). Assuming $|\underset{\sim}{u}| \sim 1$ in the mushy layer, then (3 a) implies that to the leading terms pressure field is hydrostatic and $\theta = S$ is independent of r. Using these results in (3 c) - (3 d) implies that $W \equiv W_0(z,t)$ and $\phi = \phi_0(z,t)$ at most. Assuming that $C_r \gg \theta$ in the mushy layer[1] (3 c) - (3 d) then yield

$$\left[\frac{\partial}{\partial t} + \left(w_0 - 1\right)\frac{\partial}{\partial z} - \frac{\partial^2}{\partial z^2}\right]\theta_0 + S_t\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial t}\right)\phi_0 = 0, \tag{6 a}$$

$$\left[\left(1 - \phi_0\right)\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial t}\right) - w_0\frac{\partial}{\partial z}\right]\theta_0 + C_r\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial t}\right)\phi_0 = 0, \tag{6 b}$$

where $\theta_0(z,t)$ and $\phi_0(z,t)$ are the leading order terms for θ and ϕ, respectively.

Next, consider the flow in the chimney which is described by (1 a) - (1 d) since $\phi = 0$ there. Using (5), we have $u \sim \psi/a$, $w \sim \psi/a^2$ and thus $u \sim aw$. It is also reasonable to assume that $S \sim 1$ and $w \gg 1$[1]. It also follows from (1 a) that $w \sim R_l a^2$, $u \sim R_l a^3$, $\psi \sim R_l a^4$. Now, θ_0 is the leading order temperature solution in the mushy zone outside the chimney and its surrounding boundary layer[1]. Designate $\theta_1(r,z,t)$ to be the deviation of θ from θ_0. From (1) and the condition $1/a^2 \ll R_l \ll 1/a^4$, we find $\theta_1 \ll 1$. Using these results, (1 a) - (1 d) in the chimney are reduced to the form

$$\frac{1}{\Pr}\left(\frac{\partial}{\partial t} + \underset{\sim}{u}.\nabla\right)w = -R_l\left(S + \frac{\partial P}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right), \tag{7 a}$$

$$\nabla.\underset{\sim}{u} = 0, \tag{7 b}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta_1}{\partial r}\right)=\frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial\theta_0}{\partial z}+\frac{\partial\theta_0}{\partial t}, \tag{7 c}$$

$$\left(\frac{\partial}{\partial t}+\underset{\sim}{u}.\nabla\right)S=0. \tag{7 d}$$

Using (3 a) in the mushy zone outside the chimney, we find that

$$\frac{\partial \mathrm{P}}{\partial z}=-\theta_0 \tag{8}$$

to the leading terms. Using (8) in (7 a), we find

$$\frac{1}{\mathrm{Pr}}\left(\frac{\partial}{\partial t}+\underset{\sim}{u}.\nabla\right)w=-R_l\left(S-\theta_0\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right). \tag{9}$$

Integrate (7 c) in r from $r=0$ to $r=a$ and follow Worster[1], we find the following result

$$\theta_1\sim\psi_a\frac{\partial\theta_0}{\partial z}\ln r+\frac{\partial\theta_0}{\partial t}r^2/4, \tag{10 a}$$

where

$$2\pi\psi_a=\int_0^a 2\pi rw\,dr \tag{10 b}$$

is the vertical volume flux in the chimney. Using (5), we have

$$u\sim(1/r)\frac{\partial\psi_a}{\partial z}\quad \text{as } r\to a. \tag{11}$$

From (3 a), we have

$$u/\Pi\sim -R_m\frac{\partial \mathrm{P}}{\partial r}. \tag{12}$$

Use (11) in (12), we find

$$\Delta\mathrm{P}\sim\left(\frac{\partial\psi_a}{\partial z}\right)\ln a/R_m, \tag{13}$$

where it is assumed that $\Pi=0(1)$ and $\Delta\mathrm{P}$ represents the pressure near the chimney. Using (10 a) and (3 a), we find that

$$w\sim R_m\psi_a\ln a \tag{14}$$

holds near the wall of the chimney. Using (3 d) and the condition $C_r>>1$, we find

$$C_r\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial t}\right)\phi \sim \underset{\sim}{u}.\nabla S. \tag{15}$$

Within the mushy zone outside the chimney, we have

$$C_r\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial t}\right)\phi \sim u\frac{\partial\theta}{\partial r}+w\frac{\partial\theta_0}{\partial z}. \tag{16}$$

For $u << 1$, (16) reduces to

$$C_r\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial t}\right)\phi \sim w\frac{\partial\theta_0}{\partial z}. \tag{17}$$

Near the wall of the chimney, $\frac{\partial\theta_0}{\partial z} \sim 1$, $C_r >> 1$ and $\phi C_r \sim 1^1$, which implies that

$$W \sim 1. \tag{18}$$

From (14), (18) and earlier result on the order of magnitude of u, we find

$$R_m R_l a^4 \ln a \sim 1. \tag{19}$$

Using these results, (13), (14) and (18), we find that

$$\Delta P \sim 1/R_m^2. \tag{20}$$

Using (10 a) and the above results, we find that

$$\theta-\theta_0 \sim 1/R_m. \tag{21}$$

From (10 a), (19) and the earlier result on the order of magnitude of ψ_a, we find

$$u\frac{\partial\theta_1}{\partial r} \sim \left[R_l/\left(R_m \ln a\right)^3\right]^{\frac{1}{2}}. \tag{22}$$

Thus $u\frac{\partial\theta_1}{\partial r}$ is negligible if $R_l << \left(R_m \ln a\right)^3$ with a given by (19). For

$$R_l >> \left(R_m \ln a\right)^3, \tag{23}$$

the term in the left hand side of (16) must balance with both terms in the right hand side of (16), and using (10 a), (11), (16) and (19) yield

$$W \sim R_l^2 a^6. \tag{24}$$

From (14), (24) and earlier result on the order of magnitude of ψ_a, we find

$$R_m / R_l \sim a^2 / \ln a. \tag{25}$$

From (13), (25) and earlier result on the order of magnitude of ψ_a, we find

$$\Delta P \sim (\ln a)^3 R_m / R_l. \tag{26}$$

From (25) and the results based on the derivation of (21) and the order of magnitude of ψ_a, we find

$$\theta - \theta_0 \sim R_m^2 (\ln a)^3 / R_l. \tag{27}$$

Vertical and horizontal advection of solute balance here in this regime. Using (3 c) and (24) - (26), we find

$$\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial t}\right)\phi \sim \left(R_m \ln a\right)^3 / R_l \tag{28}$$

which is small based on the condition (23). Now, the wall of the chimney can be defined by

$$\phi[a(z,t), z, t] = 0 \quad \text{at } r = a(z,t). \tag{29}$$

Taking derivative with respect to z of (29) implies

$$\left[\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial t}\right)a\right] \Big/ a \sim \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial t}\right)\phi << 1, \tag{30}$$

which implies that, based on the scalings of the type (25) - (27), the walls of the chimney are vertical to leading order terms, so that the radius of the cylindrical chimney is independent of the axial variable to leading order terms.

Applying the scalings (25) - (27), using a Polhausen type method suggested by Lighthill[10] and following Worster[1], (appendix), we find that the total volume flux in the chimney, due to upward flow[1], is given by

$$2\pi\psi_a = 2\pi\lambda a^4 R_l[1+\theta_0(z,t)], \tag{31}$$

where λ is a constant which depends generally on a, h and Pr. To satisfy the mass conservation, downward flow through the mushy zone $W_0(z,t)$ must be equal to the total upflow through all the chimneys per unit horizontal area. Thus

$$W_0 = -2\pi\psi_a N, \tag{32}$$

where N is the number density of chimneys.

IV. ANALYSIS FOR LOW P_r AND WEAK TIME DEPENDENCE

With the same basic assumptions as in the case presented in the previous section, the results (5) - (6) are found to be valid again for the present case where the order of magnitude of the nonlinear inertia terms in (1 a) is large than that of the viscous terms in (1 a), and the order of magnitude of the time derivative inertia term in (1 a) is at most as large as the nonlinear inertia terms in (1 a). However, order of magnitudes of the velocity components and the stream function in the chimney are now

$$W \sim (\mathrm{Pr}\, R_l)^{\frac{1}{2}}, u \sim a(P_r R_l)^{\frac{1}{2}}, \psi \sim a^2 (P_r R_l)^{\frac{1}{2}}. \tag{33}$$

From (1) and the condition

$$1 << (P_r R_l)^{\frac{1}{2}} << 1/a^2, \tag{34}$$

we find $\theta_1 << 1$. Using these results, (1 a) in the chimney is reduced to the following equation

$$\frac{1}{\mathrm{Pr}}\left(\frac{\partial}{\partial t} + \underset{\sim}{u}.\nabla\right)W = -R_l\left(S + \frac{\partial P}{\partial z}\right). \tag{35}$$

The results (7 b) - (7 d) and (8) are found to be valid again here. Using (8) in (35), we find

$$\frac{1}{\Pr}\left(\frac{\partial}{\partial t}+\underset{\sim}{u}.\nabla\right)W+R_l\left(S-\theta_0\right)=0. \tag{36}$$

The results (10) - (18) are valid again here. From (14), (18) and (33), we find

$$R_m a^2 \ln a\sqrt{R_l P_r} \sim 1. \tag{37}$$

The results (20) - (21) are valid again here. From (10 a), (19) and (33), we find

$$u\frac{\partial\theta_1}{\partial r} \sim \sqrt{\Pr R_l}/\left(R_m \ln a\right). \tag{38}$$

Thus $u\frac{\partial\theta_1}{\partial r}$ is negligible if $\sqrt{P_r R_l} << R_m \ln a \sim$ with a given by (37). For

$$\sqrt{P_r R_l} \geq R_m \ln a, \tag{39}$$

the term in the left hand side of (16) must balance with both terms in the right hand side of (16), and using (10 a), (11), (16) and (37) yield

$$W \sim a^2 P_r R_l. \tag{40}$$

From (14), (33) and (40), we find

$$R_m \sim \sqrt{P_r R_l}/\ln a. \tag{41}$$

From (13), (33) and (41), we find

$$\Delta \mathrm{P} \sim (a \ln a)^2. \tag{42}$$

From (41) and the results based on the derivation of (21) and the order of magnitude of ψ_a, we find

$$\theta-\theta_0 \sim a^2 \ln a\sqrt{P_r R_l}. \tag{43}$$

Using (3 c) and (40) - (42), we find

$$\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial t}\right)\phi \sim \left(a \ln a R_m\right)^2. \tag{44}$$

Using (29), we have

$$\left[\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial t}\right)a\right]\Big/ a \sim \left(\frac{\partial}{\partial z}-\frac{\partial}{\partial t}\right)\phi. \tag{45}$$

Using (44) - (45), we find that for

$$a \ll \left(R_m \ln R_m\right)^{-1}, \tag{46}$$

the walls of the chimney are vertical and a is independent of z to leading order terms, while for

$$\left(R_m \ln R_m\right)^{-1} \leq a \ll 1, \tag{47}$$

the walls of the chimney cannot be vertical and a depends on z.

For vertical chimney, where (46) holds, we can determine the total volume flux in the chimney, due to upward flow, by using a procedure similar to that described in the previous section and in the appendix, provided we set $A(X)=0$ in (A4) and ignore (A5) in the appendix. The results (31) - (32) then follow where now λ is independent of P_r but still depends on a and h.

V. ANALYSIS FOR HIGH Pr AND STRONG TIME DEPENDENCE

With the same basic assumptions as in the cases presented in the previous two sections, the basis system (1)-(4) is analyzed here for the present case where the order of magnitude of the time derivative inertia term in (1 a) is larger than that of the nonlinear inertia terms in (1 a), but the order of magnitude of the time derivative inertia term in (1 a) is at most as large as the one of the viscous terms in (1 a). The order of magnitudes of the velocity components and the stream function in the chimney are now

$$W \sim R_l a^2, \ u \sim R_l a^3, \ \psi \sim \mathrm{R}_1 a^4. \tag{48}$$

From (1) and the condition

$$\left(1/a^2\right) << R_l << \left(1/a^4\right), \tag{49}$$

we find $\theta_1 << 1$. Using these results, (1 a) and (1 c) - (1 d) in the chimney are reduced to the following equations

$$\frac{1}{\mathrm{Pr}}\frac{\partial W}{\partial t} = -R_l\left(S + \frac{\partial P}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial W}{\partial r}\right), \tag{50 a}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta_1}{\partial r}\right) = \frac{\partial \theta_0}{\partial r}, \tag{50 b}$$

$$\frac{\partial S}{\partial t} = 0. \tag{50 c}$$

The results (7 b) and (8) are found to be valid again here. Using (8) in (50 a), we find

$$\frac{1}{\mathrm{Pr}}\frac{\partial W}{\partial t} + R_l\left(S - \theta_0\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial W}{\partial r}\right). \tag{51}$$

Integrate (50 b) in r from $r = o$ to r = a and follow Worster[1], we find

$$\theta_1 \sim a^2\frac{\partial \theta_0}{\partial t}. \tag{52}$$

The results (11) - (13) are valid again here. Using (3 a) and (52), we find that

$$W \sim R_m a^2\frac{\partial \theta_0}{\partial t} \tag{53}$$

holds near the walls of the chimney. Using (3 d) and the condition $C_r >> 1$, we find

$$\frac{\partial \phi}{\partial t} = 0, \tag{54 a}$$

$$C_r\frac{\partial \phi}{\partial z} \sim \underset{\sim}{u}.\nabla S. \tag{54 b}$$

Within the mushy zone outside the chimney, we have (54 a) and

$$C_r\frac{\partial \phi}{\partial z} \sim u\frac{\partial \theta_1}{\partial r} + W\frac{\partial \theta_0}{\partial z}. \tag{55}$$

For $u << 1$, (55) reduces to

$$C_r \frac{\partial \phi}{\partial z} \sim W \frac{\partial \theta_0}{\partial z}. \tag{56}$$

The result (18) is valid again here. From (18), (53) and earlier result on the order of magnitude of u, we find

$$R_m a^2 \frac{\partial \theta_0}{\partial t} \sim 1. \tag{57}$$

Using these results, (13), (18) and (53), we find

$$\Delta \mathrm{P} \sim R_l \ln a \Big/ \left[R_m^3 \left(\frac{\partial \theta_0}{\partial t} \right)^2 \right]. \tag{58}$$

The result (21) is valid again here. From (52), (57) and earlier result on the order of magnitude of ψ_a, we find

$$u \frac{\partial \theta_1}{\partial r} \sim R_l a^4 \frac{\partial \theta_0}{\partial t}. \tag{59}$$

Thus $u \frac{\partial \theta_1}{\partial r}$ is negligible if $R_l a^4 \frac{\partial \theta_0}{\partial t} << 1$ with a given by (57). For

$$R_l a^4 \frac{\partial \theta_0}{\partial t} \geq 1, \tag{60}$$

we use (11), (52), (55) and (57), and we find

$$W \sim R_l a^4 \frac{\partial \theta_0}{\partial t}. \tag{61}$$

From (53), (61) and earlier result on the order of magnitude of ψ_a, we find

$$R_m \sim R_l a^2. \tag{62}$$

From (13), (62) and earlier result on the order of magnitude of ψ_a, we find

$$\Delta \mathrm{P} \sim a^2 \ln a. \tag{63}$$

Using (3 c) and (61) - (63), we find

$$\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial t} \right) \phi \sim R_m^2 \frac{\partial \theta_0}{\partial t} \Big/ R_l, \tag{64}$$

which is not small based on the condition (60). Using (29) - (30) and (64), we find that the walls of the chimney cannot be concluded to be vertical to leading order terms, and the radius of the chimney may depend on z to leading order terms.

VI. ANALYSIS FOR LOW Pr AND STRONG TIME DEPENDENCE

Following the same basic assumptions and the system (1) - (4) used in the previous three sections, we consider the case where the order of magnitude of the time derivative inertia term in (1 a) is larger than those of the nonlinear and viscous terms in (1 a). The order of magnitudes of the velocity components and the stream function in the chimney are now satisfied by

$$\frac{\partial W}{\partial t} \sim R_l P_r, \quad u \sim Wa, \quad \psi \sim Wa^2. \tag{65}$$

From (1) and the condition

$$1 \ll R_l P_r W \Big/ \frac{\partial W}{\partial t} \ll 1/a^2, \tag{66}$$

we find $\theta_1 \ll 1$. Using these results, (1 a) in the chimney is reduced to

$$\frac{1}{\Pr}\frac{\partial W}{\partial t} = -R_l\left(S + \frac{\partial P}{\partial z}\right). \tag{67}$$

The results (7 b), (50 b) - (50 c) and (8) are found to be valid again here. Using (8) in (67), we find

$$\frac{1}{\Pr}\frac{\partial W}{\partial t} + R_l\left(S - \theta_0\right) = 0. \tag{68}$$

The results (11) - (13), (18) and (52) - (57) are valid again here. Using these results, (13), (18) and (53), we find

$$\Delta P \sim R_l P_r a^4 \ln a. \tag{69}$$

The result (21) is valid again here. From (52), (57) and earlier result on the order of magnitude of ψ_a, we find

$$u\frac{\partial\theta_1}{\partial r} \sim R_l P_r a^2. \tag{70}$$

Thus $u\frac{\partial\theta_1}{\partial r}$ is negligible if $R_l P_r a^2 << 1$ with a given by (57). For

$$R_l P_r a^2 \geq 1, \tag{71}$$

we use (11), (52), (55) and (57), and we find

$$W \sim R_l P_r a^2. \tag{72}$$

From (53), (72) and earlier result on the order of magnitude of ψ_a, we find

$$R_m \sim R_l P_r \Big/ \left(\frac{\partial\theta_0}{\partial t}\right). \tag{73}$$

From (13), (73) and earlier result on the order of magnitude of ψ_a, we find that (63) follows. Using (3 c), (63) and (72) - (73), we find

$$\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial t}\right)\phi \sim R_l P_r a^2, \tag{74}$$

which is not small based on the condition (71). Using (29) - (30) and (74), we find that the walls of the chimney cannot be concluded to be vertical to leading order terms, and a may depend on z to leading order terms.

VIII. DISCUSSION

In this paper, we applied scaling analysis in the asymptotic state of large $R_l >> R_m >> 1$ and identified four distinct regions corresponding to strongly or weakly time dependent flow of high or low Prandtl number melt. Using the description of these regions, given in sections **III** - **VI**, and the results (33), (48) and (65), we find that these four regions can correspond to the following cases:

1) High P_r and weak time dependence regime corresponds to

$$P_r \geq R_l a^4 \text{ and } \left(\frac{1}{W}\frac{\partial W}{\partial t}\right) \leq \sqrt{R_l P_r}.$$

2) Low P_r and weak time dependence regime corresponds to

$$P_r << R_l a^4 \text{ and } \left(\frac{1}{W}\frac{\partial W}{\partial t}\right) \leq \sqrt{R_l P_r}.$$

3) High P_r and strong time dependence regime corresponds to

$$P_r \geq R_l a^4 \text{ and } \left(\frac{1}{W}\frac{\partial W}{\partial t}\right) >> \sqrt{R_l P_r}.$$

4) Low P_r and strong time dependence regime corresponds to

$$P_r << R_l a^4 \text{ and } \left(\frac{1}{W}\frac{\partial W}{\partial t}\right) >> \sqrt{R_l P_r}.$$

It is of interest to note that some of the main results of the present study regarding the structure of the chimney and the convective flow within the chimney depend critically on the particular regime in the above list which subsequently depends on relative importance of viscous, nonlinear inertia and temporal acceleration terms of the momentum equation in the liquid layer. This result reinforces the conclusion reached by other authors[5] that the flow in the fully melted and mushy zones are coupled together and can strongly influence important properties of melt and solid, which subsequently determine the mechanical properties of the solidified system.

An essential consideration in the present work has been the time dependence aspect of the flow within the solidified system. The numerical and experimental results by a number of authors[4-6,11], indicate that such consideration is quite realistic. As a result of strong time dependence features of the convective flow, we found that a cylindrical chimney may have an axially dependent radius which may also vary with time. This result agrees with the numerical simulation of a $Pb-19$ °/o S_n alloy undergoing solidification phase change[4] where it is found that channel within the mushy zone grows both in z and r directions as t increases in some interval. The experimental results[6] indicate that channel segregates are aligned primarily in the vertical direction and incline radially inward, but they also tend to be oriented with a slight azimuthal component. The present model is, however, based on axisymmetric conditions, and the non-axisymmetric studies, which can predict the three-dimensional nature and structure of the time dependent flow and chimneys, is hoped to be accomplished by the present author in the near future.

Another interesting result obtained numerically[11], which may be related to the present result of possible time and axial dependence of the chimney's radius for strongly time dependent flow case, is that some channel within the mushy zone grows for some internal in time beyond which it closes. This result is a natural consequence of the solidification process and has been described in details by Neilson and Incropera[12].

APPENDIX

In this appendix we describe a procedure to determine ψ_a through a chimney, given by (31), using (7 d) and (9) and subject to the following boundary conditions[1]:

$$S+1=\frac{\partial S}{\partial r}=\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial S}{\partial r}\right)=\psi=\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right)=0 \ \text{ at } r=0, \tag{A1}$$

$$S-\theta_0=\frac{\partial S}{\partial r}=\frac{\partial \psi}{\partial r}=0 \ \text{ at } r=a, \tag{A2}$$

$$S+1=\psi=0 \ \text{ at } z=0. \tag{A3}$$

Following Worster[1], we introduce a trial function for S

$$1+S=\left(1+\theta_0\right)A(X)+\frac{\partial\theta_0}{\partial z}\left(1+\theta_0\right)B(X)+\frac{\partial\theta_0}{\partial t}C(X), \ X\equiv r/a, \tag{A4}$$

where

$$A(X)=\left(2X^2-X^4\right)+\left(4+\tilde{\mu}/5\right)X^2\left(1-X^2\right)^2-\tilde{\mu}X^4\left(1-X^2\right)^3, \tag{A5}$$

$$B(X)=\left[(G')^2/X^2-GG''/X^2+GG'/X^3\right]\Big/\left(P_r a^4 R_l\right), \tag{A6}$$

$$C(X)=-G'\Big/\left(P_r a^2 R_l X\right), \tag{A7}$$

$$\psi/\left(1+\theta_0\right) = G(X) = a^4 R_l\left[\left(-\frac{1}{48}X^8 + \frac{1}{12}X^6 - \frac{1}{8}X^4\right) + (4+\tilde{\mu}/5)\left(\frac{1}{8}X^{10}\right.\right.$$

$$\left.\left. -\frac{1}{24}X^8 + \frac{1}{24}X^6\right) + \tilde{\mu}\left(\frac{1}{168}X^{14} - \frac{1}{40}X^{12} + \frac{3}{80}X^{10} - \frac{1}{48}X^8\right)\right], \qquad \text{(A8)}$$

$\tilde{\mu}$ is a constant parameter and a prime denotes $\frac{d}{dX}$. The expressions for S and ψ, given by (A4) and (A8), satisfy (9) and (A1) - (A3). Evaluating (A8) at $X=1$ leads to (31) which depends on $\tilde{\mu}$. An approximation for $\tilde{\mu}$ can be obtained[1] by using (A4) and (A8) in (7 d), and integrate the resulting equation first in z, from $z=h$ to $z=0$, and then in X, from $X=1$ to $X=0$, and in time, from $t=\infty$ to $t=0$. Hence, $\tilde{\mu}$ will be, in general, a function of Pr and some integrated forms of a and h.

References

1. M.G. Worster, Natural Convection in a Mushy Layer, *J. Fluid Mech.* **224**, 335 (1991).

2. S.M. Copley, A.F. Giamei, S.M. Johnson and M.F. Hornbecker, The Origin of Freckles in Unidirectionally Solidified Castings, *Metall. Trans.* **1**, 2193 (1970).

3. T.L. Sayre and D.N. Riahi, Oscillatory Instabilities of the Liquid and Mushy Layers During Solidification of Alloys Under Rotational Constraint, submitted for publication (1995).

4. P.J. Prescott and F.P. Incropera, Binary Solid-Liquid Phase Change with Fluid Flow, Advances in Transport Processes IX, edited by A.S. Mujumdar and R.A. Mashelkar (Elsevier Science Publishers B.V., 1993) pp. 57-101.

5. P.J. Prescott and F.P. Incropera, Convective Transport Phenomena and Macrosegretation During Solidification of a Binary Metal Alloy: I-Numerical Predictions, *J. Heat Transfer* **116**, 735 (1994).

6. P.J. Prescott, F.P. Incropera and D.R. Gaskell, Convective Transport Phenomena and Macrosegregation During Solidification of a Binary Metal Alloy: II-Experiments and Comparisons with Numerical Predictions, *Transac. ASME* **116**, 742 (1994).

7. P.H. Roberts and D.E. Loper, Towards a Theory of the Structure and Evolution of a Dendrite Layer, In Stellar and Planetary Magnetism, edited A.M. Soward (Gordon and Breach, 1983) pp. 329-349.

8. A.C. Fowler, The Formation of Freckles in Binary Alloys, *IMA J.Appl. Maths.* **35**, 159 (1985).

9. M.G. Worster, Instabilities of the Liquid and Mushy Regions During Solidification of Alloys, *J. Fluid Mech.* **237**, 649 (1992).

10. M.J. Lighthill, Theoretical Considerations on Free Convection in Tubes, *Q. J. Mech. Appl. Maths.* **6**, 398 (1953).

11. D.G. Neilson and F.P. Incropera, Numerical Study of the Effects of Transport Phenomena on Macrosegregation During Unidirectional Solidification of a Binary Substance, *Wärme-und Stoffübertragung* **27**, 1 (1992)

12. D.G. Neilson and F.P. Incropera, Unidirectional Solidification of a Binary Alloy and the Effects of Induced Fluid Motion, *Int. J. Heat and Mass Transfer* **34**, 1717 (1991).

List of Recent TAM Reports

No.	*Authors*	*Title*	*Date*
714	Birnbaum, H. K., and P. Sofronis	Hydrogen-enhanced localized plasticity—A mechanism for hydrogen-related fracture	July 1993
715	Balachandar, S., and M. R. Malik	Inviscid instability of streamwise corner flow	July 1993
716	Sofronis, P.	Linearized hydrogen elasticity	July 1993
717	Nitzsche, V. R., and K. J. Hsia	Modeling of dislocation mobility controlled brittle-to-ductile transition	July 1993
718	Hsia, K. J., and A. S. Argon	Experimental study of the mechanisms of brittle-to-ductile transition of cleavage fracture in silicon single crystals	July 1993
719	Cherukuri, H. P., and T. G. Shawki	An energy-based localization theory: Part II—Effects of the diffusion, inertia and dissipation numbers	Aug. 1993
720	Aref, H., and S. W. Jones	Chaotic motion of a solid through ideal fluid	Aug. 1993
721	Stewart, D. S.	Lectures on detonation physics: Introduction to the theory of detonation shock dynamics	Aug. 1993
722	Lawrence, C. J., and R. Mei	Long-time behavior of the drag on a body in impulsive motion	Sept. 1993
723	Mei, R., J. F. Klausner, and C. J. Lawrence	A note on the history force on a spherical bubble at finite Reynolds number	Sept. 1993
724	Qi, Q., R. E. Johnson, and J. G. Harris	A re-examination of the boundary layer attenuation and acoustic streaming accompanying plane wave propagation in a circular tube	Sept. 1993
725	Turner, J. A., and R. L. Weaver	Radiative transfer of ultrasound	Sept. 1993
726	Yogeswaren, E. K., and J. G. Harris	A model of a confocal ultrasonic inspection system for interfaces	Sept. 1993
727	Yao, J., and D. S. Stewart	On the normal detonation shock velocity–curvature relationship for materials with large activation energy	Sept. 1993
728	Qi, Q.	Attenuated leaky Rayleigh waves	Oct. 1993
729	Sofronis, P., and H. K. Birnbaum	Mechanics of hydrogen–dislocation–impurity interactions: Part I—Increasing shear modulus	Oct. 1993
730	Hsia, K. J., Z. Suo, and W. Yang	Cleavage due to dislocation confinement in layered materials	Oct. 1993
731	Acharya, A., and T. G. Shawki	A second-deformation-gradient theory of plasticity	Oct. 1993
732	Michaleris, P., D. A. Tortorelli, and C. A. Vidal	Tangent operators and design sensitivity formulations for transient nonlinear coupled problems with applications to elasto-plasticity	Nov. 1993
733	Michaleris, P., D. A. Tortorelli, and C. A. Vidal	Analysis and optimization of weakly coupled thermo-elasto-plastic systems with applications to weldment design	Nov. 1993
734	Ford, D. K., and D. S. Stewart	Probabilistic modeling of propellant beds exposed to strong stimulus	Nov. 1993
735	Mei, R., R. J. Adrian, and T. J. Hanratty	Particle dispersion in isotropic turbulence under the influence of non-Stokesian drag and gravitational settling	Nov. 1993
736	Dey, N., D. F. Socie, and K. J. Hsia	Static and cyclic fatigue failure at high temperature in ceramics containing grain boundary viscous phase: Part I—Experiments	Nov. 1993
737	Dey, N., D. F. Socie, and K. J. Hsia	Static and cyclic fatigue failure at high temperature in ceramics containing grain boundary viscous phase: Part II—Modeling	Nov. 1993
738	Turner, J. A., and R. L. Weaver	Radiative transfer and multiple scattering of diffuse ultrasound in polycrystalline media	Nov. 1993
739	Qi, Q., and R. E. Johnson	Resin flows through a porous fiber collection in pultrusion processing	Dec. 1993

List of Recent TAM Reports (cont'd)

No.	*Authors*	*Title*	*Date*
740	Weaver, R. L., W. Sachse, and K. Y. Kim	Transient elastic waves in a transversely isotropic plate	Dec. 1993
741	Zhang, Y., and R. L. Weaver	Scattering from a thin random fluid layer	Dec. 1993
742	Weaver, R. L., and W. Sachse	Diffusion of ultrasound in a glass bead slurry	Dec. 1993
743	Sundermeyer, J. N., and R. L. Weaver	On crack identification and characterization in a beam by nonlinear vibration analysis	Dec. 1993
744	Li, L., and N. R. Sottos	Predictions of static displacements in 1–3 piezocomposites	Dec. 1993
745	Jones, S. W.	Chaotic advection and dispersion	Jan. 1994
746	Stewart, D. S., and J. Yao	Critical detonation shock curvature and failure dynamics: Developments in the theory of detonation shock dynamics	Feb. 1994
747	Mei, R., and R. J. Adrian	Effect of Reynolds-number-dependent turbulence structure on the dispersion of fluid and particles	Feb. 1994
748	Liu, Z.-C., R. J. Adrian, and T. J. Hanratty	Reynolds-number similarity of orthogonal decomposition of the outer layer of turbulent wall flow	Feb. 1994
749	Barnhart, D. H., R. J. Adrian, and G. C. Papen	Phase-conjugate holographic system for high-resolution particle image velocimetry	Feb. 1994
750	Qi, Q., W. D. O'Brien Jr., and J. G. Harris	The propagation of ultrasonic waves through a bubbly liquid into tissue: A linear analysis	March 1994
751	Mittal, R., and S. Balachandar	Direct numerical simulation of flow past elliptic cylinders	May 1994
752	Anderson, D. N., J. R. Dahlen, M. J. Danyluk, A. M. Dreyer, K. M. Durkin, J. J. Kriegsmann, J. T. McGonigle, and V. Tyagi	Thirty-first student symposium on engineering mechanics, J. W. Phillips, coord.	May 1994
753	Thoroddsen, S. T.	The failure of the Kolmogorov refined similarity hypothesis in fluid turbulence	May 1994
754	Turner, J. A., and R. L. Weaver	Time dependence of multiply scattered diffuse ultrasound in polycrystalline media	June 1994
755	Riahi, D. N.	Finite-amplitude thermal convection with spatially modulated boundary temperatures	June 1994
756	Riahi, D. N.	Renormalization group analysis for stratified turbulence	June 1994
757	Riahi, D. N.	Wave-packet convection in a porous layer with boundary imperfections	June 1994
758	Jog, C. S., and R. B. Haber	Stability of finite element models for distributed-parameter optimization and topology design	July 1994
759	Qi, Q., and G. J. Brereton	Mechanisms of removal of micron-sized particles by high-frequency ultrasonic waves	July 1994
760	Shawki, T. G.	On shear flow localization with traction-controlled boundaries	July 1994
761	Balachandar, S., D. A. Yuen, and D. M. Reuteler	High Rayleigh number convection at infinite Prandtl number with temperature-dependent viscosity	July 1994
762	Phillips, J. W.	Arthur Newell Talbot—Proceedings of a conference to honor TAM's first department head and his family	Aug. 1994
763	Man., C. S., and D. E. Carlson	On the traction problem of dead loading in linear elasticity with initial stress	Aug. 1994
764	Zhang, Y., and R. L. Weaver	Leaky Rayleigh wave scattering from elastic media with random microstructures	Aug. 1994
765	Cortese, T. A., and S. Balachandar	High-performance spectral simulation of turbulent flows in massively parallel machines with distributed memory	Aug. 1994

List of Recent TAM Reports (cont'd)

No.	Authors	Title	Date
766	Balachandar, S.	Signature of the transition zone in the tomographic results extracted through the eigenfunctions of the two-point correlation	Sept. 1994
767	Piomelli, U.	Large-eddy simulation of turbulent flows	Sept. 1994
768	Harris, J. G., D. A. Rebinsky, and G. R. Wickham	An integrated model of scattering from an imperfect interface	Sept. 1994
769	Hsia, K. J., and Z. Xu	The mathematical framework and an approximate solution of surface crack propagation under hydraulic pressure loading	Sept. 1994
770	Balachandar, S.	Two-point correlation and its eigen-decomposition for optimal characterization of mantle convection	Oct. 1994
771	Lufrano, J. M., and P. Sofronis	Numerical analysis of the interaction of solute hydrogen atoms with the stress field of a crack	Oct. 1994
772	Aref, H., and S. W. Jones	Motion of a solid body through ideal fluid	Oct. 1994
773	Stewart, D. S., T. Aslam, J. Yao, and J. B. Bdzil	Level-set techniques applied to unsteady detonation propagation	Oct. 1994
774	Mittal, R., and S. Balachandar	Effect of three-dimensionality on the lift and drag of circular and elliptic cylinders	Oct. 1994
775	Stewart, D. S., T. D. Aslam, and J. Yao	The evolution of detonation cells	Nov. 1994
776	Aref, H.	On the equilibrium and stability of a row of point vortices	Nov. 1994
777	Cherukuri, H. P., T. G. Shawki, and M. El-Raheb	An accurate finite-difference scheme for elastic wave propagation in a circular disk	Nov. 1994
778	Li, L., and N. R. Sottos	Improving hydrostatic performance of 1–3 piezocomposites	Dec. 1994
779	Phillips, J. W., D. L. de Camara, M. D. Lockwood, and W. C. C. Grebner	Strength of silicone breast implants	Jan. 1995
780	Xin, Y.-B., K. J. Hsia, and D. A. Lange	Quantitative characterization of the fracture surface of silicon single crystals by confocal microscopy	Jan. 1995
781	Yao, J., and D. S. Stewart	On the dynamics of multi-dimensional detonation	Jan. 1995
782	Riahi, D. N., and T. L. Sayre	Effect of rotation on the structure of a convecting mushy layer	Feb. 1995
783	Batchelor, G. K., and TAM faculty and students	A conversation with Professor George K. Batchelor	Feb. 1995
784	Sayre, T. L., and D. N. Riahi	Effect of rotation on flow instabilities during solidification of a binary alloy	Feb. 1995
785	Xin, Y.-B., and K. J. Hsia	A technique to generate straight surface cracks for studying the dislocation nucleation condition in brittle materials	March 1995
786	Riahi, D. N.	Finite bandwidth, long wavelength convection with boundary imperfections: Near-resonant wavelength excitation	March 1995
787	Turner, J. A., and R. L. Weaver	Average response of an infinite plate on a random foundation	March 1995
788	Weaver, R. L., and D. Sornette	The range of spectral correlations in pseudointegrable systems: GOE statistics in a rectangular membrane with a point scatterer	April 1995

List of Recent TAM Reports (cont'd)

No.	*Authors*	*Title*	*Date*
789	Anderson, K. F., M. B. Bishop, B. C. Case, S. R. McFarlin, J. M. Nowakowski, D. W. Peterson, C. V. Robertson, and C. E. Tsoukatos	Thirty-second student symposium on engineering mechanics, J. W. Phillips, coord.	April 1995
790	Figa, J., and C. J. Lawrence	Linear stability analysis of a gravity-driven Newtonian coating flow on a planar incline	May 1995
791	Figa, J., and C. J. Lawrence	Linear stability analysis of a gravity-driven viscosity-stratified Newtonian coating flow on a planar incline	May 1995
792	Cherukuri, H. P., and T. G. Shawki	On shear band nucleation and the finite propagation speed of thermal disturbances	May 1995
793	Harris, J. G.	Modeling scanned acoustic imaging of defects at solid interfaces	May 1995
794	Sottos, N. R., J. M. Ockers, and M. J. Swindeman	Thermoelastic properties of plain weave composites for multilayer circuit board applications	May 1995
795	Aref, H., and M. A. Stremler	On the motion of three point vortices in a periodic strip	June 1995
796	Barenblatt, G. I., and N. Goldenfeld	Does fully-developed turbulence exist? Reynolds number independence versus asymptotic covariance	June 1995
797	Aslam, T. D., J. B. Bdzil, and D. S. Stewart	Level set methods applied to modeling detonation shock dynamics	June 1995
798	Prasad N.B.R. and P. Sofronis	The effect of interface slip and diffusion on the creep strength of fiber and particulate composite materials	July 1995
799	Hsia, K. J., T.-L. Zhang, and D. F. Socie	Effect of crack surface morphology on the fracture behavior under mixed mode loading	July 1995
800	Adrian, R. J.	Stochastic estimation of the structure of turbulent fields	Aug. 1995
801	Riahi, D. N.	Perturbation analysis and modeling for stratified turbulence	Aug. 1995
802	Thoroddsen, S. T.	Conditional sampling of dissipation in high Reynolds number turbulence	Aug. 1995
803	Riahi, D. N.	On the structure of an unsteady convecting mushy layer	Aug. 1995